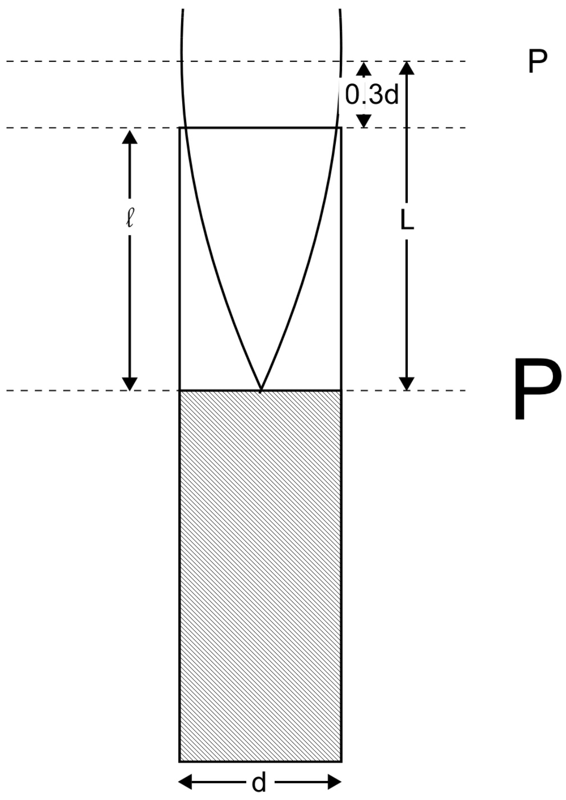
Numerical Analysis Assessed Problems 2018/19

School of Physics and Astronomy
MSc Physics and Technology of Nuclear Reactors

The assessed problems for this year's Numerical Analysis module are modified forms of exercises C.4, F.1, G.2 and J.4. The exercises are given below. You should attempt all the problems and prepare a short written report together with a copy of your EXCEL spreadsheets, where applicable. Take care to show your working clearly. You are required to submit an electronic copy of your report and spreadsheets via the assignment link on the LM PH605 PRACTICAL SKILLS FOR REACTOR PHYSICS - NUMERICAL ANALYSIS page on Canvas. Please note that as this is an open-book assessment, you are not being asked to reproduce any derivations. Marks will be assigned for the accuracy of your solutions and the clarity of your report. Like the FORTRAN assessment, it is worth half the weight of a normal laboratory report. A printed copy of your report should be handed in to the Teaching Support Office by 4 pm on **Tuesday 5th March 2019**.

Please note that this is a formal item of assessment and you must work **individually**. You are not permitted to discuss your solutions with any other student. Collaborative work is not allowed.

Good luck!



1. Exercise C.4

In a simple experiment, a tube of diameter *d* is filled with water, leaving a gap of depth *l* at the top. An audio signal generator above the tube is adjusted until the fundamental resonant frequency ν of the air column is found. It was found that the amplitude antinode (pressure node) is not at the top of the tube but a distance approximately $0.3d$ above it. The velocity of sound than can then be found from the relations

$$
\begin{aligned}
L &= l + 0.3d &= \lambda/4 \quad , \\
v_s &= \nu\lambda & \quad ,
\end{aligned}
$$

where v_s is the velocity of sound. The resonant frequency is very well measured, while the depth l has a measurement error δl.

Rewrite equation (1) to find the equation relating the inverse frequency $1/\nu$ and the depth l. Table 1 has a set of measurements of l, δl and ν taken in an experiment, using a tube of diameter 2.85 cm. Plot the points in terms of l and $1/\nu$, and find the best straight line fit through them using a minumum χ^2 method. Hence determine the speed of sound with its associated error.

Table 1:

Resonant frequency ν (Hz)	depth of air gap l (cm)	$\sigma(cm)$
490	16.4	0.09
521	15.5	0.09
553	14.3	0.09
629	13.1	0.09
729	10.6	0.09
775	10.0	0.09
824	9.5	0.09
876	8.7	0.09
931	8.2	0.09
990	7.7	0.09

2. Exercise F.1

Fill an EXCEL table column with values of the function $y = \sin^{-1}(x)$ in the interval 0 to 0.99 rad in steps of 0.01 rad. Use the first-order and second-order expansions of the differential operator D expressed in forward differences according to equation (F.8) and apply it to your $\sin^{-1}(x)$ values. Compare your answers to those from the analytic derivative of $y = \sin^{-1}(x)$. Make a table of the relative difference Δ, where

$$\Delta = (\mathrm{Derivative}_{\mathrm{Numerical}} - \mathrm{Derivative}_{\mathrm{Analytic}})/\mathrm{Derivative}_{\mathrm{Analytic}}.$$

3. Exercise G.2

The normalized Breit-Wigner function is given by

$$BW(m, m_0, \Gamma) = \frac{\Gamma}{2\pi}\left(\frac{1}{(m - m_0)^2 + \Gamma^2/4}\right)$$

Evaluate this function in the range 10-14 MeV$/c^2$for an unstable state with $m_0 = 12$ MeV$/c^2$, $\Gamma = 2$ MeV$/c^2$, in steps of 0.1 MeV$/c^2$. With this mesh calculate

$$I_1 = \frac{\Gamma}{2\pi}\int_{m_0-\Gamma/2}^{m_0+\Gamma/2}\left(\frac{\mathrm{d}m}{(m - m_0)^2 + \Gamma^2/4}\right),$$

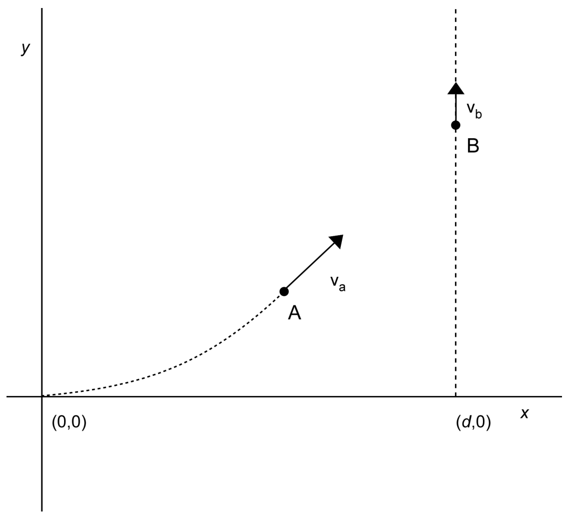
(*i*) using Simpson's rule. Next (*ii*) evaluate the integral using 4th order Gaussian quadrature by considering the interval $(m_0, m_0 + \Gamma/2)$ and doubling it.

Tables for the Gauss-Legendre method for n = 2 to n = 4:

n = 2	x_i	ω_i
	$\pm$0.577350269189626	1.000000000000000

n = 3	x_i	ω_i
	0.000000000000000	0.888888888888888
	$\pm$0.774596669241483	0.555555555555555

n = 4	x_i	ω_i
	$\pm$0.339981043584856	0.652145154862546
	$\pm$0.861136311594053	0.347854845137454



4. Exercise J.4

Two objects A and B are set in motion at time $t = 0$. Object A starts at position $(0, 0)$ and moves with speed v_a with a direction such that it always points towards object B. Object B starts at position $(d, 0)$ and moves parallel to the y axis with velocity v_b, $(v_b < v_a)$. The path taken by object A is known as a "curve of pursuit". It can be shown that the equation of motion for Object A is

$$(x - d)\frac{\mathrm{d}q}{\mathrm{d}x} = -\frac{v_b}{v_a}\sqrt{1 + q^2}, \tag{1}$$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = q. \tag{2}$$

Equation (1) can also be solved analytically, subject to the initial conditions, to give

$$q = \frac{1}{2}\left[\left(1 - \frac{x}{d}\right)^{-\frac{v_b}{v_a}} - \left(1 - \frac{x}{d}\right)^{\frac{v_b}{v_a}}\right] \tag{3}$$

In a specific example, the objects A and B have speeds 30 ms^{-1} and 10 ms^{-1} respectively, and start a distance $d = 50$ m apart. Use the Runge-Kutta method with equations (1) and (2) to determine the position and direction of object A at $x = 49$ m, using steps of 0.5 m.

Why is it not possible to use this method to determine the point of contact at $d = 50$ m?

Continue to follow the trajectory of object A in the interval $49 < x < 49.9$ m, using finer steps of 0.1 m. Integrate equation (3), subject to the initial conditions, to obtain an analytic solution for $y(x)$, and compare this to your Runge-Kutta solution. Comment on the precision of the method for both $q(x)$ and $y(x)$.