# **Session C′**

1. We have seen that we can calculate the error on a function  of two variables  defined in terms of the primary variables  whose error matrix we know, following the three steps
   1. Differentiate  with respect to the variables  in terms of which it is defined.
   2. Transform the  dependence to  dependence using an appropriate transformation matrix
   3. Square the result and take expectation values for the errors, which should now be expressed in terms of  and the errors on the primary variables  .

Use this method to determine the error on the function  , and show that if the correlation between  and  is properly taken into account, the “common sense” result that  is recovered.

1. The least-squares method can be used to determine estimates for a set of variables by finding the best fit to a set of data. The error on the fit variables can be estimated at the point where the least squares function is minimised. For the two-variable case we have



Show that for a fit to a straight line,  this leads to



Where  is the covariance matrix  .

Consider the compound pendulum problem presented in last week’s problem sheed (problem C4). Data are provided in table 1 (*different from those in problem C4*) for  (x-axis) and  (y-axis). Let the best values for the intercept and gradient of the line be  and  respectively. From this we are able to obtain an estimate for the acceleration due to gravity as



and the radius of gyration,  as

 .

Determine estimates based on the data in table 1 for  and  , and calculate the corresponding errors. Note that for the radius of gyration  the correlation between the fit variables must be taken into account.

**Table 1.** Measurements from a compound pendulum experiment. σ is the error on ht2.

|  |  |  |
| --- | --- | --- |
| **h2** | **ht2** | **σ** |
| 0.170982 | 1.04030 | 0.014 |
| 0.177662 | 1.04090 | 0.01 |
| 0.126025 | 0.83300 | 0.008 |
| 0.127092 | 0.85090 | 0.004 |
| 0.093025 | 0.69930 | 0.01 |
| 0.091506 | 0.69630 | 0.011 |
| 0.060270 | 0.58850 | 0.009 |
| 0.057600 | 0.55700 | 0.015 |
| 0.042230 | 0.51650 | 0.009 |
| 0.023562 | 0.41960 | 0.025 |
| 0.013110 | 0.38850 | 0.01 |
| 0.041820 | 0.51410 | 0.007 |
| 0.022500 | 0.41340 | 0.02 |
| 0.013572 | 0.38900 | 0.007 |