Nuclear Systems Thermal Hydraulics

Single Phase flow and convection Heat Transfer

Objectives In This Part

Fluid mechanics analysis

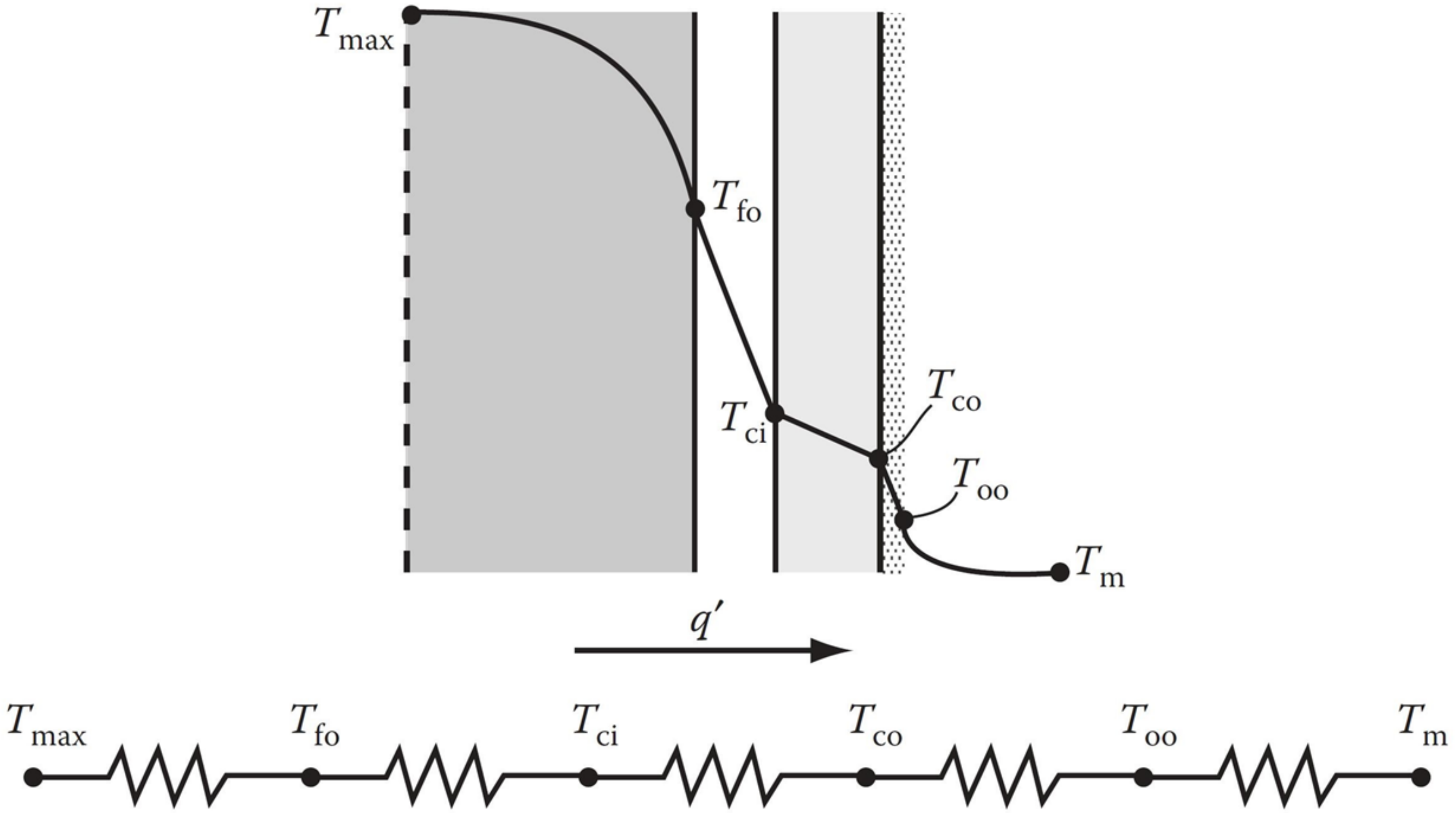
The objective of the fluid mechanics analysis is to provide the velocity (flow rate) and pressure distributions (pressure drop) in a given geometry for specified boundary and initial conditions.

By applying the accumulated engineering experience (correlations), the needs of obtaining the detailed distribution of the fluid velocity or density in the tube are eliminated.

Heat transfer analysis for single-phase flows

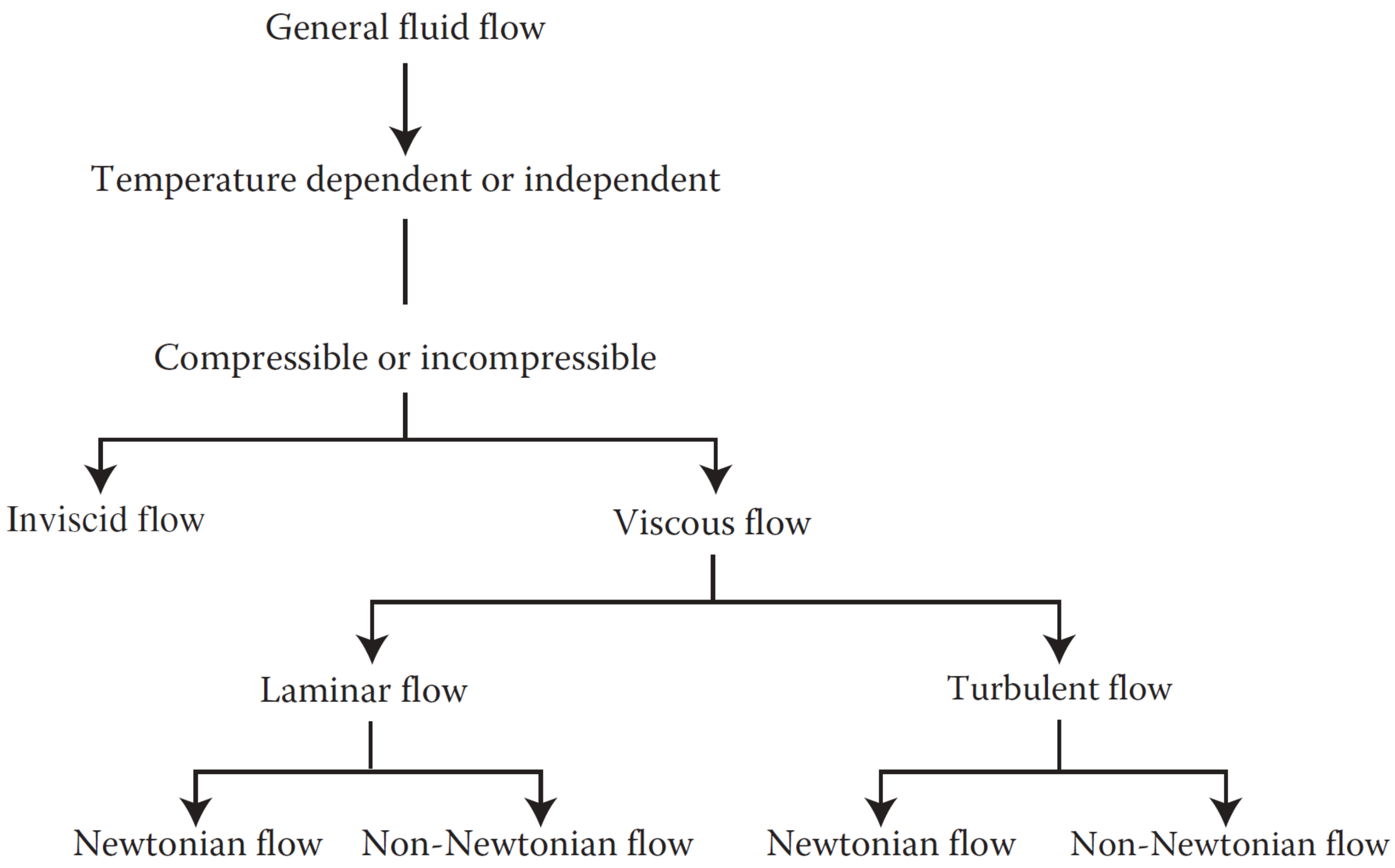
(1)determination of the temperature field in the wall (wall T) of the coolant channel so as to ensure that the operating temperatures are within the specified limits, and (2) determination of the parameters governing the heat transport rate (h) at the channel walls including importantly the coolant temperature field.

Fluid Mechanics Analysis

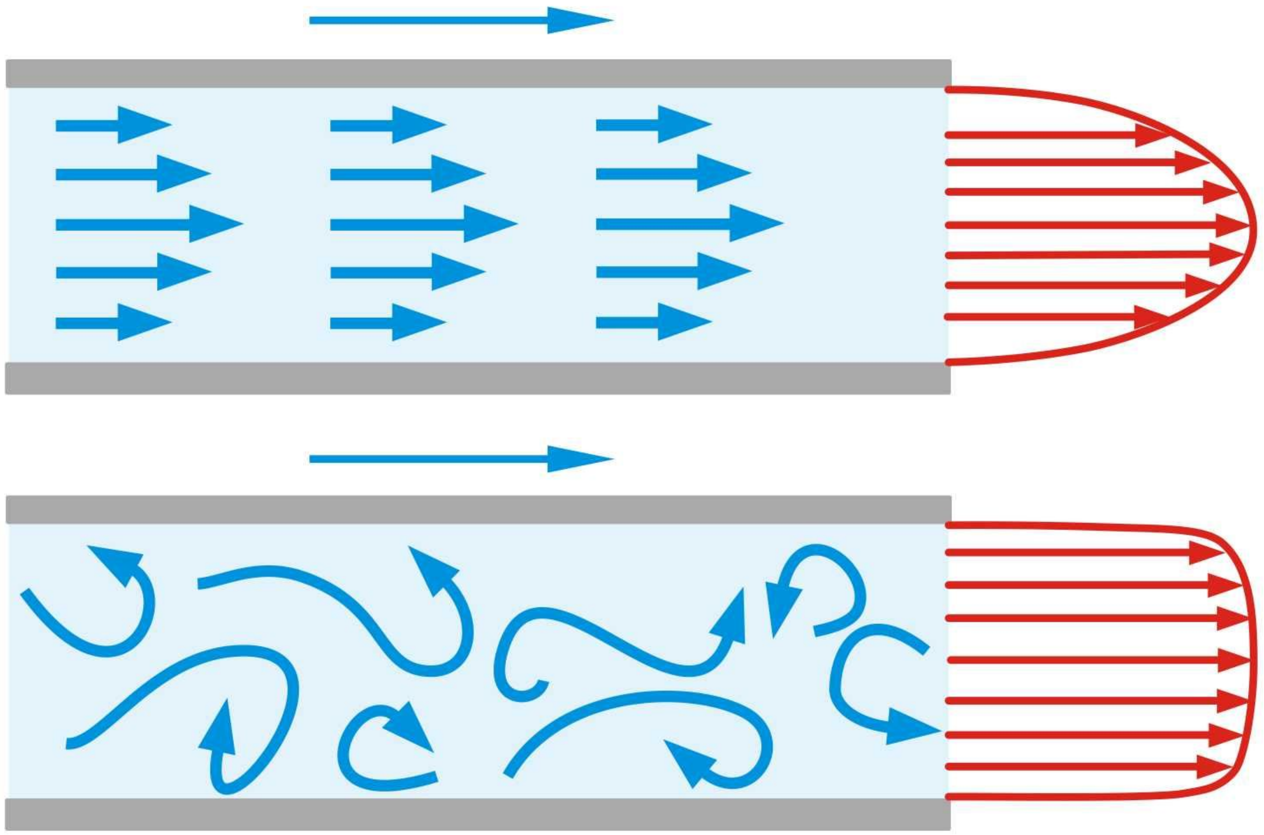


$$\frac{1}{4\pi\bar{k}_f} \qquad \frac{1}{2\pi R_g h_g} \qquad \frac{\ln\left(\frac{R_{co}}{R_{ci}}\right)}{2\pi k_c} \qquad \frac{1}{2\pi R_{oo}\frac{k_o}{\delta_o}} \qquad \frac{1}{2\pi R_{oo} h}$$

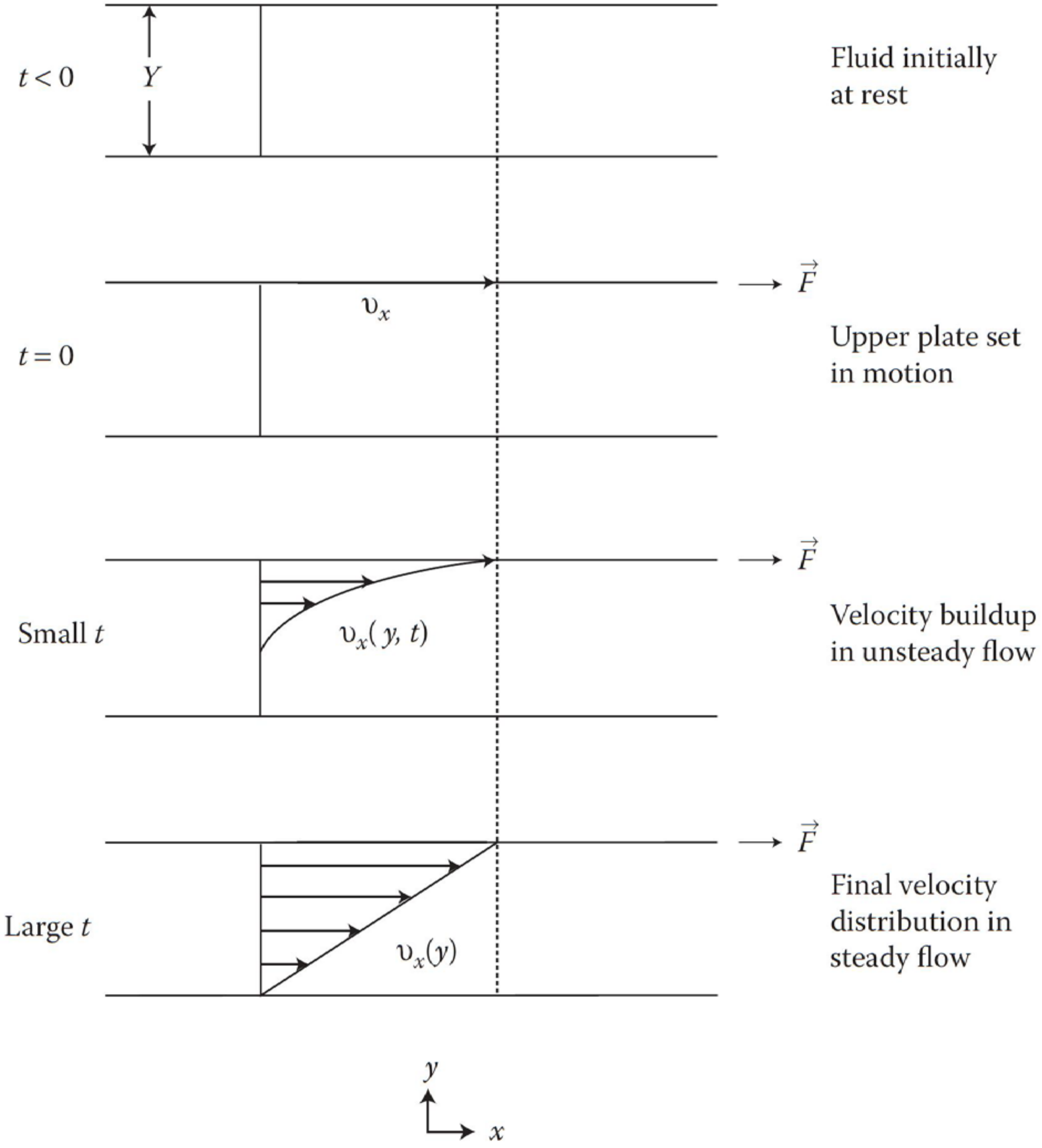
Categorization of Fluid Flow Situations

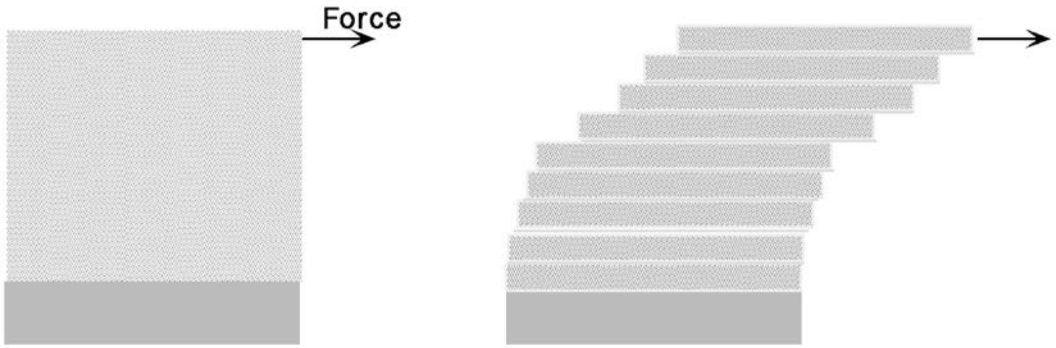


Laminar and Turbulent Flow



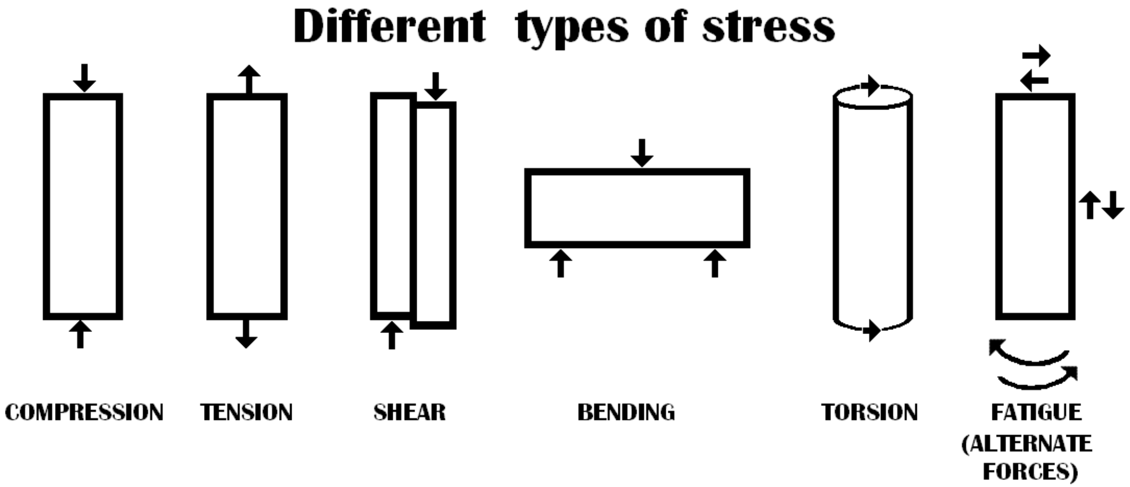
Viscous flow - Couette Flow Experiment





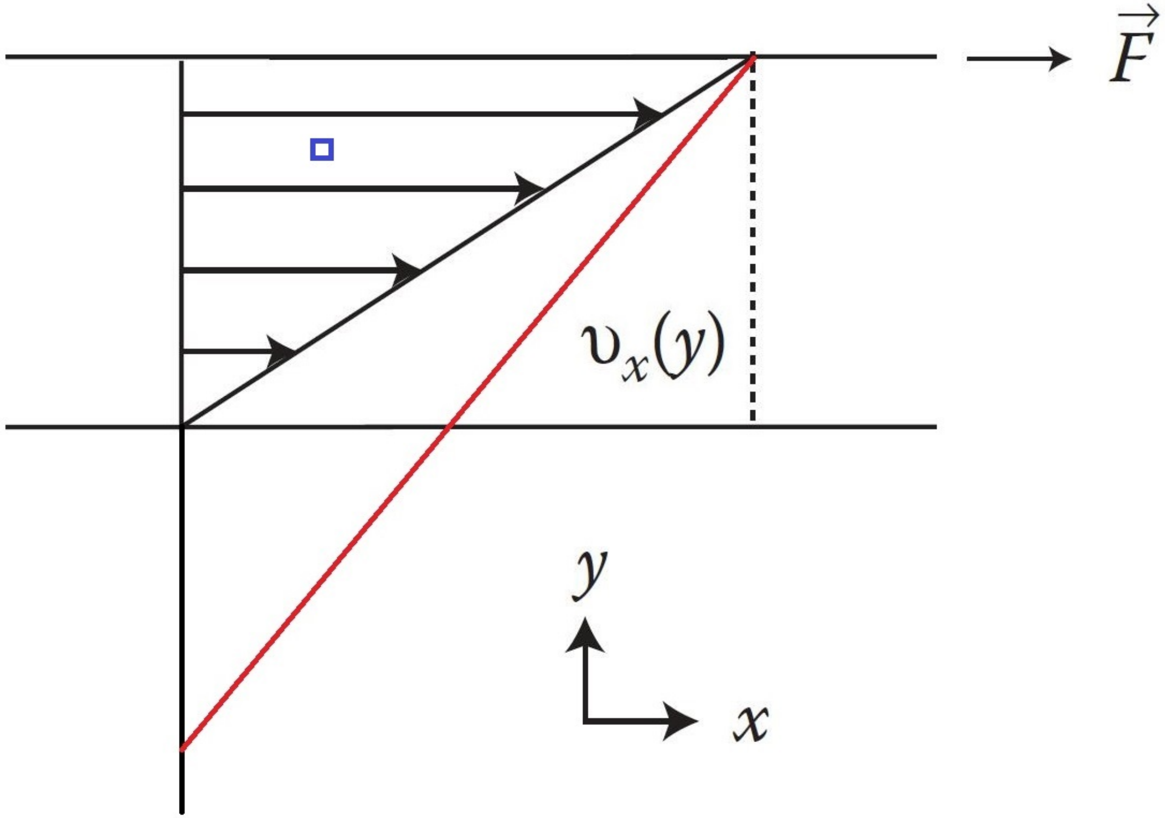
Stress

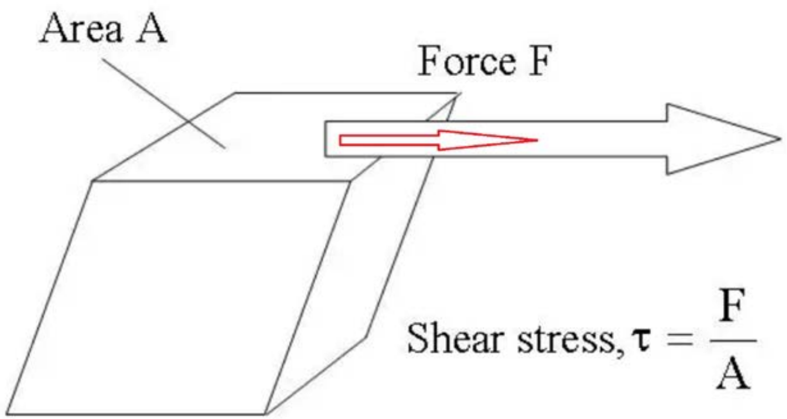
- Stress is an internal (always internal) resistance offered by the body against any kind of deformation.



- Force is not the cause of stress (though its unit is force per area), instead strain (measure of deformation) is the cause of stress.

Shear Stress Inside Fluid

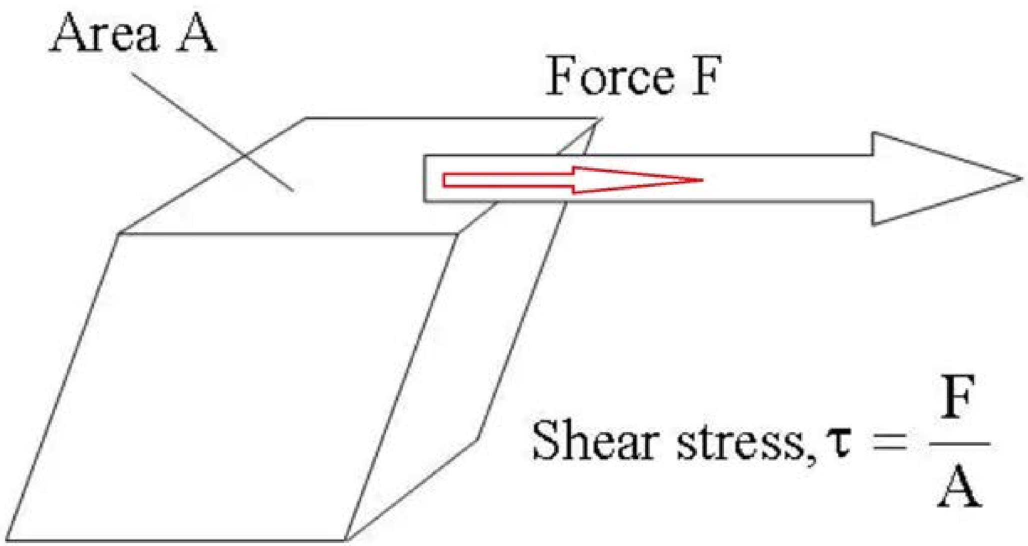




Force $F =$
$F\ (v_x, Y, A, \text{fluid propertiy})$

where A = plate area, Y is the distance between two plates and v_x is the velocity of upper plate moving in x direction

Definition of Viscosity



$$F = \tau A = \mu\left(\frac{v_x}{Y}\right)A$$

τ = shear force per unit area (i.e., shear stress).

This shear stress describes the resistance to flow

μ = dynamic viscosity of the fluid.

Shear rate: $\dot{\gamma} = \frac{v_x}{Y}$

Definition of Viscosity

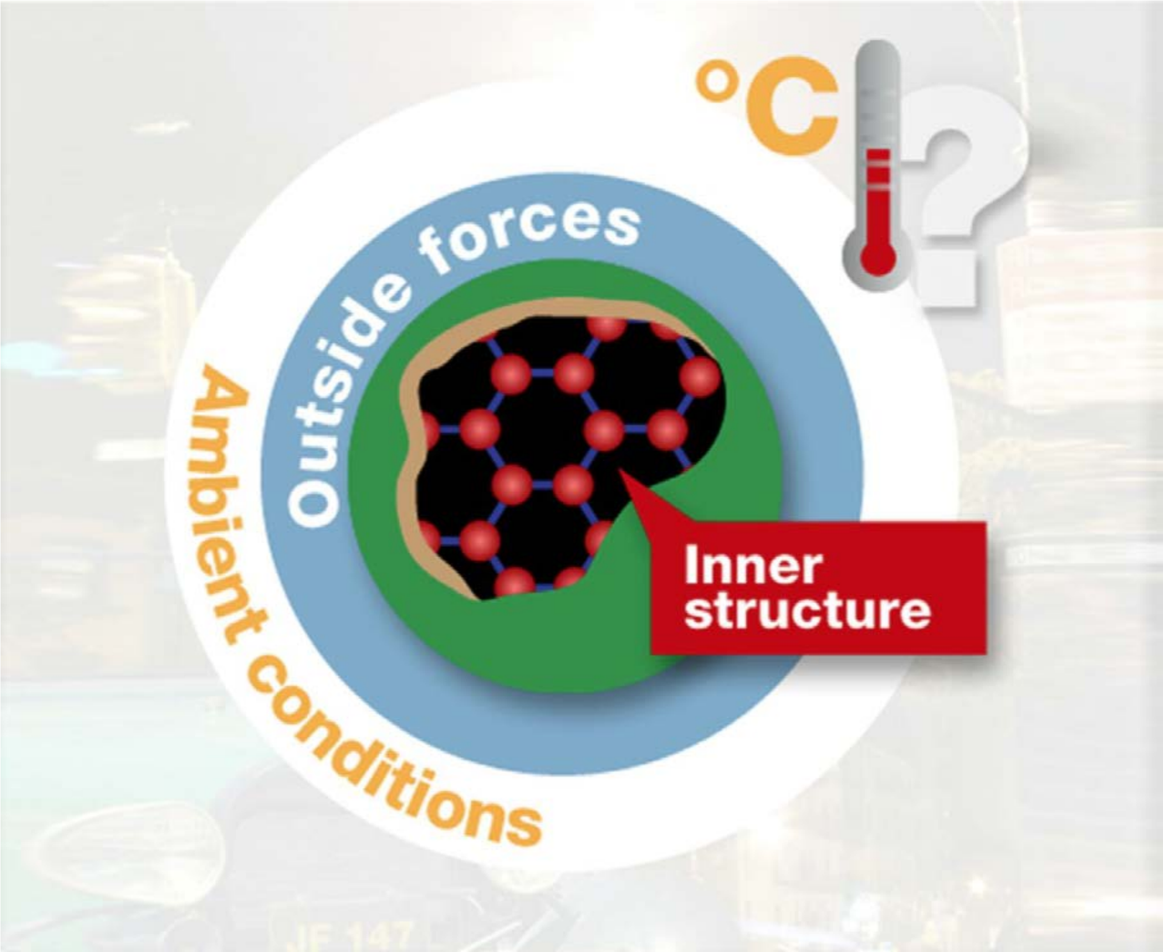
Shear stress: $\tau = \mu \frac{dv}{dy}$

The Dynamic viscosity: μ

The kinematic viscosity: $\nu = \frac{\mu}{\rho}$

The viscosity describe the degree of resistance of fluid flow in inner molecular structure level; it’s a property of the fluid.

Factors Affecting Viscosity (i.e. a substance's flow behavior)



A substance's flow behaviour depends on three factors:

- The substance's inner molecular structure.
- The external forces acting upon the substance
- The temperature and the pressure conditions

http://www.viscopedia.com/basics/factors-affecting-viscometry/

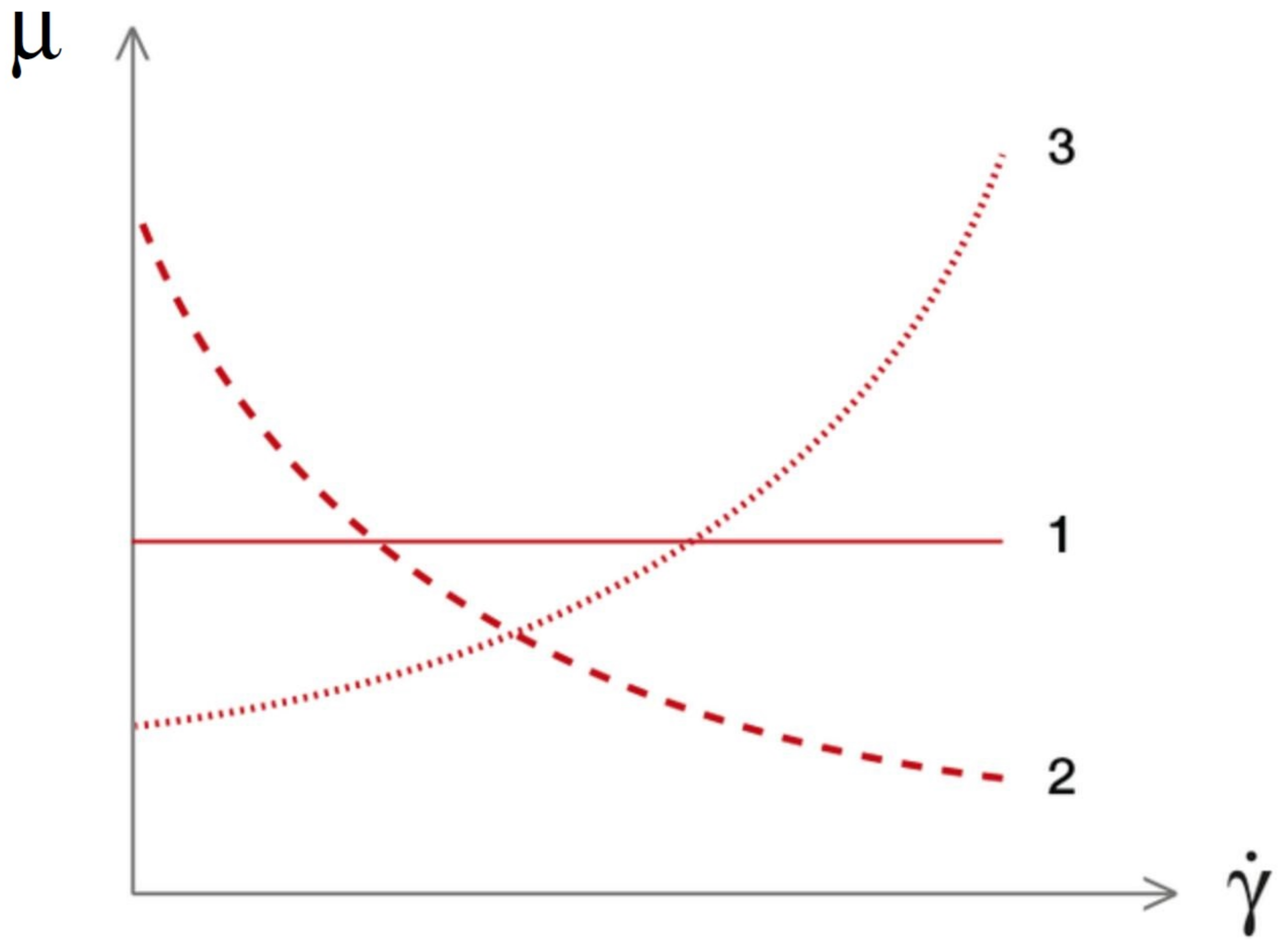
Factors Affecting Viscosity - The substance's inner molecular structure.

The tighter the molecules are linked, the more the substance will resist deformation, i.e. the less it will be willing to flow.

Factors Affecting Viscosity - The external forces

1. The external forces acting upon the substance that deform it or make it flow
2. The intensity of this external forces is described by shear rate.
3. Only Newtonian liquids are independent of the external force.
4. Newtonian liquids = Ideally viscous fluid.
5. Non-Newtonian liquids is not ideally viscous, its viscosity changes with the shear rate.

Factors Affecting Viscosity - The external forces



Viscosity function (dynamic viscosity over shear rate):

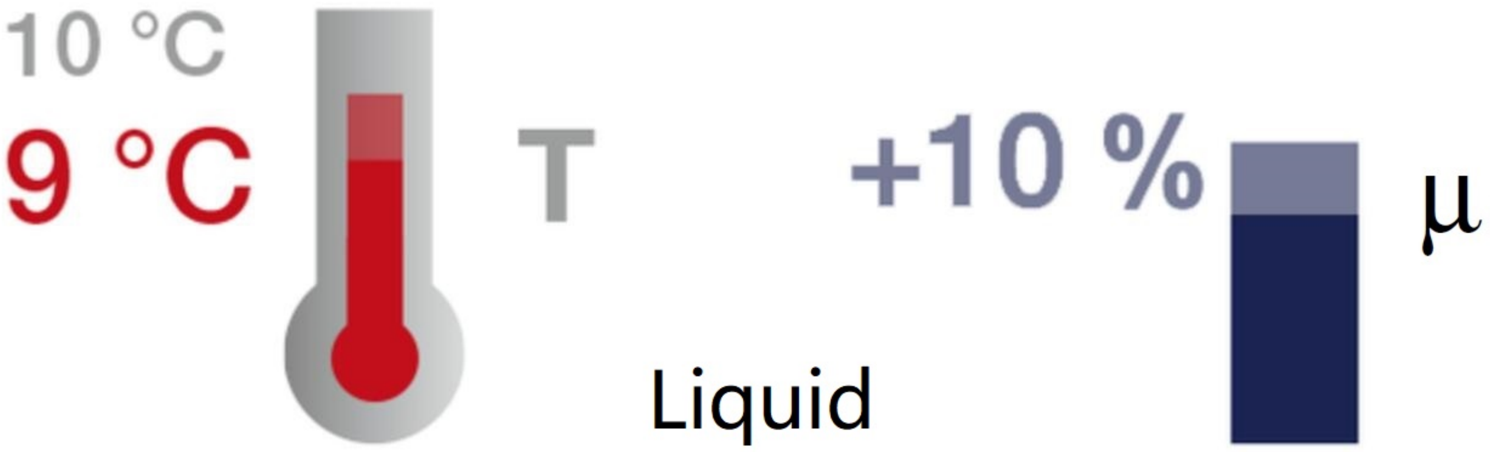
1. Newtonian fluid;
2. shear-thinning substance;
3. shear-thickening substance;

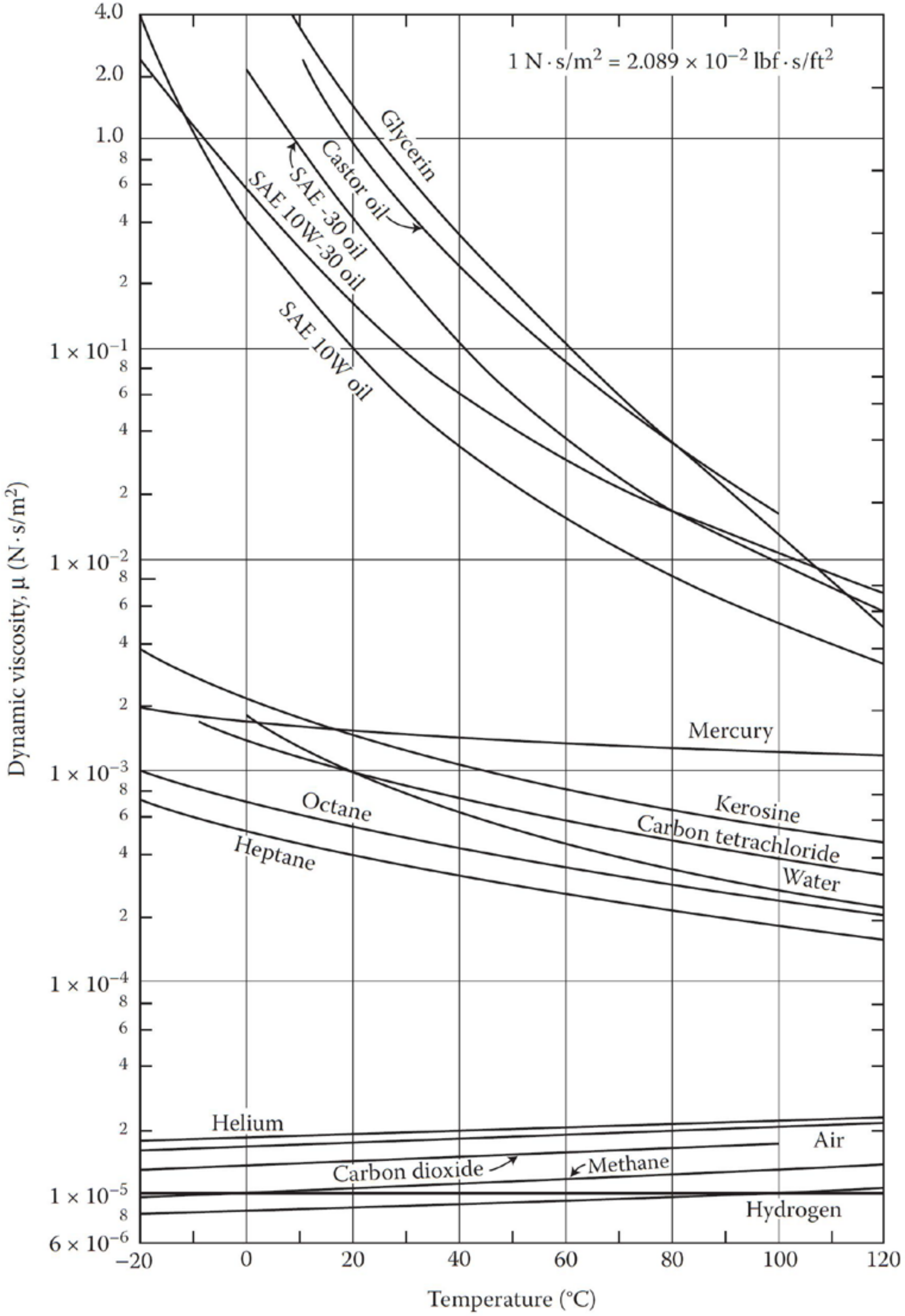
We only look at Newtonian fluid

https://www.youtube.com/watch?v=bu0ocRVmkjc

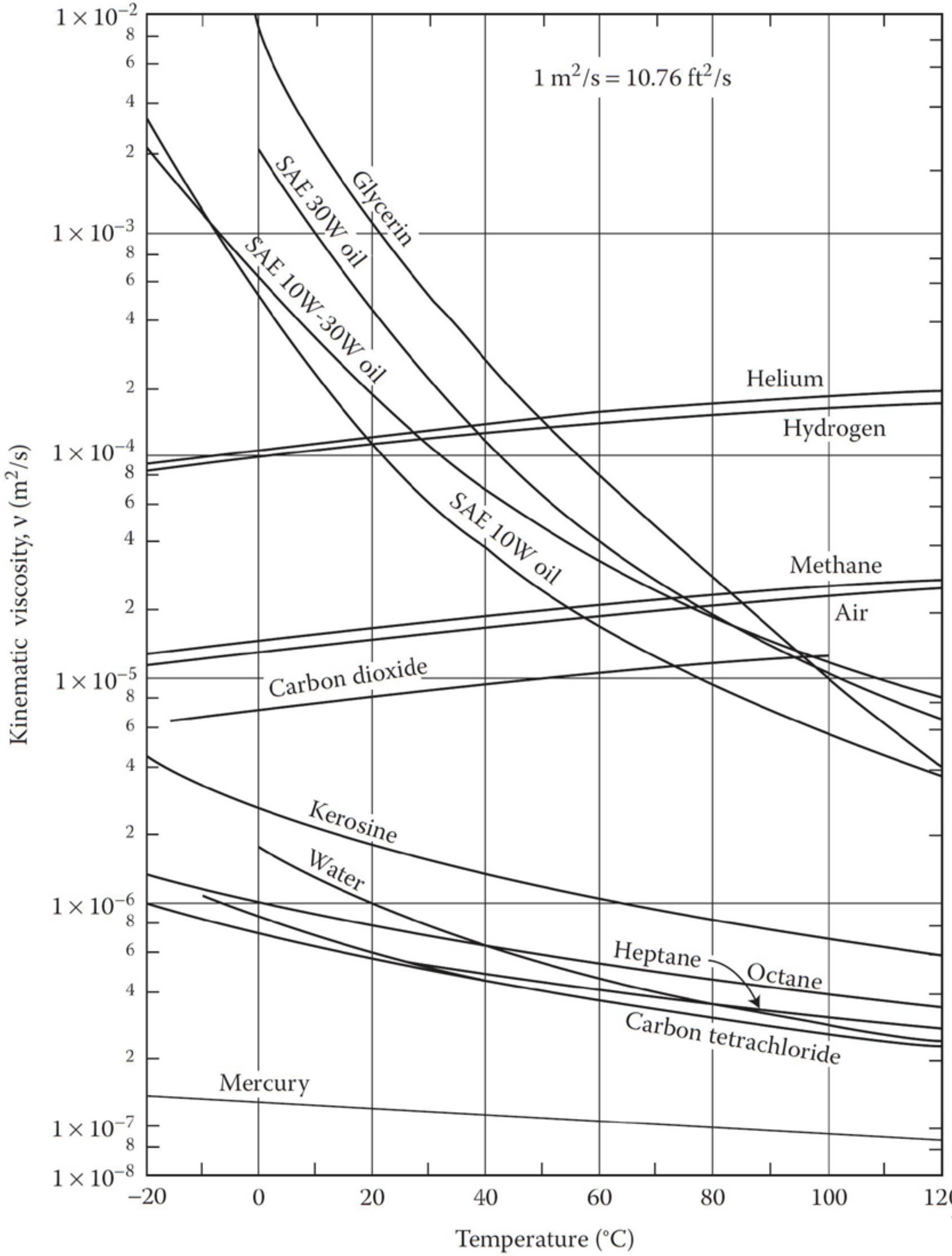
Factors Affecting Viscosity - The temperature

1. A liquid's viscosity strongly depends on its temperature.
2. The higher the temperature is, the lower a liquid's viscosity is.
3. For some liquids a increase of 1°C already causes a 10 % decrease in viscosity.
4. Along with the shear rate, temperature really is the dominating influence for liquid.
5. For most gases, increasing temperature lead to an increase in gas viscosity





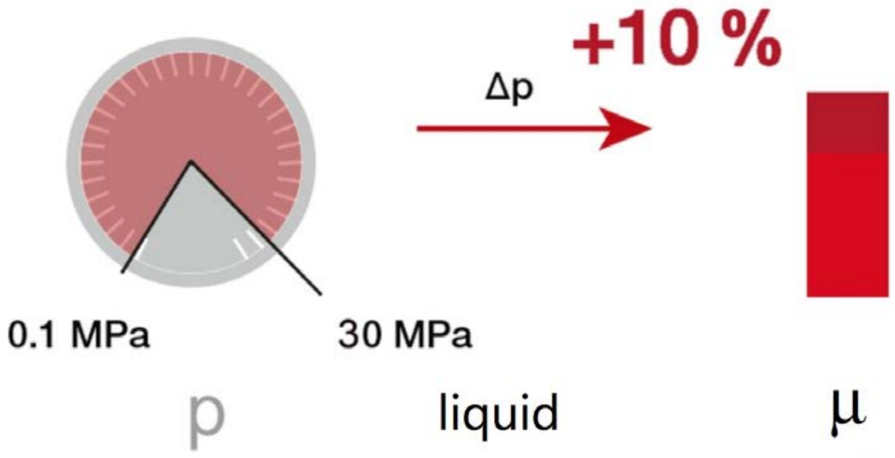
1 N · s/m² = 2.089 × 10⁻² lbf · s/ft²
Dynamic viscosity, μ (N · s/m²)
Temperature (°C)
Glycerin
Castor oil
SAE 30 oil
SAE 10W-30 oil
SAE 10W oil
Mercury
Kerosine
Carbon tetrachloride
Water
Octane
Heptane
Helium
Air
Carbon dioxide
Methane
Hydrogen



1 m²/s = 10.76 ft²/s
Kinematic viscosity, ν (m²/s)
Temperature (°C)
Glycerin
SAE 30W oil
SAE 10W-30W oil
SAE 10W oil
Helium
Hydrogen
Methane
Air
Carbon dioxide
Kerosine
Water
Heptane
Octane
Carbon tetrachloride
Mercury

Factors Affecting Viscosity - The pressure

1. In most cases, a fluid's viscosity (gas and liquid) increases with increasing pressure.
2. Compared to the temperature influence, liquids are influenced very little by the applied pressure.
3. For most liquids, a considerable change in pressure from 0.1 to 30 MPa causes about the same change in viscosity as a temperature change of about 1 K.



Inviscid Flow

Flow Rate

The steady-flow Bernoulli equation

$$z^{\circ} = \frac{p}{\rho g} + \frac{\upsilon^2}{2g} + z \qquad \text{Eq. 4.1a}$$

Total head ($z°$) is usually defined as the actual height (z) plus the static head and the velocity head.

The total head of an incompressible and inviscid flow in a channel is constant

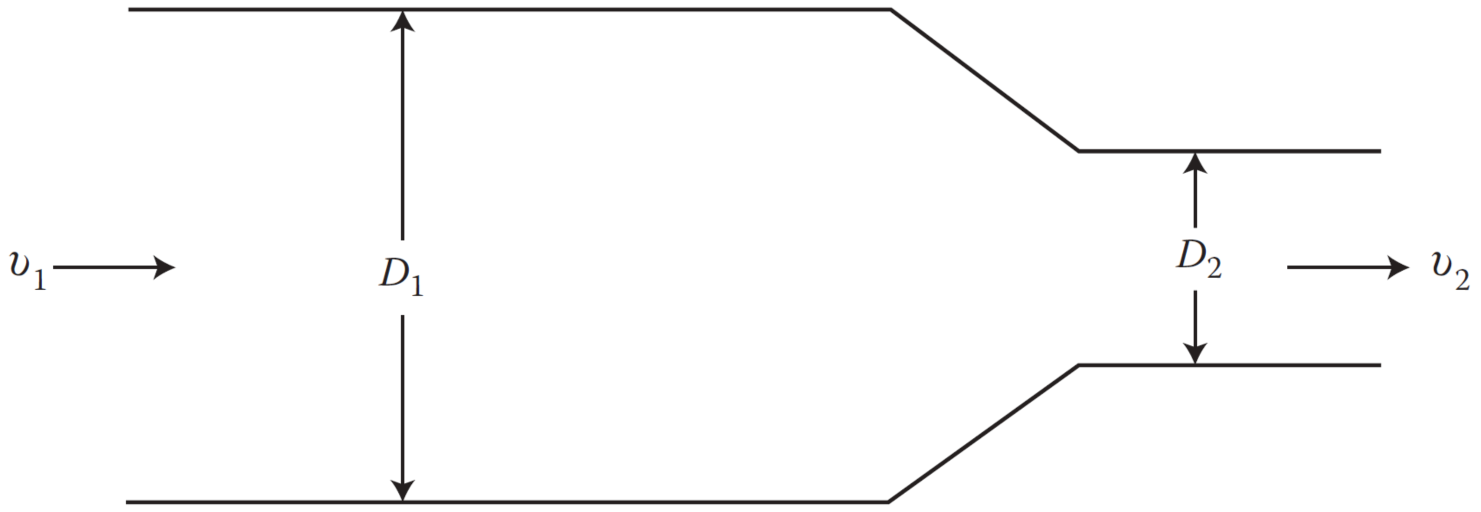
Hence, between channel locations we can write:

$$\Delta\left(\frac{p}{\rho g} + \frac{\upsilon^2}{2g} + z\right) = 0 \qquad \text{Eq. 4.1b}$$

Flow through a Nozzle

The flow of an incompressible, inviscid fluid through a nozzle can be predicted by application of the steady-flow Bernoulli equation.

Consider a nozzle of diameter D_2 attached to a pipe of diameter D_1



Nozzle.

Flow through a Nozzle

when gravity is the only body force, we get

$$\left(\frac{p}{\rho g}+\frac{\upsilon^2}{2g}+z\right)_1=\left(\frac{p}{\rho g}+\frac{\upsilon^2}{2g}+z\right)_2 \qquad \text{Eq. 4.1c}$$

For a horizontal nozzle, however

$$z_1 = z_2$$

From the mass balance consideration, we get

$$\rho\upsilon_1\frac{\pi D_1^2}{4}=\rho\upsilon_2\frac{\pi D_2^2}{4} \qquad \text{Eq. 4.2}$$

Flow through a Nozzle

which leads to

$$\upsilon_1 = \upsilon_2 \left(\frac{D_2}{D_1} \right)^2 \qquad \text{Eq. 4.3}$$

Substituting, we get

$$\frac{\upsilon_2^2}{2} \left[\left(\frac{D_2}{D_1} \right)^4 - 1 \right] = \frac{p_2 - p_1}{\rho} \qquad \text{Eq. 4.4}$$

Flow through a Nozzle

Or

$$\upsilon_2 = \sqrt{\frac{2(p_1 - p_2)}{\rho[1-(D_2/D_1)^4]}} \qquad \text{Eq. 4.5}$$

It should be noted that under real conditions the velocity in the nozzle is less than that predicted by the above equation. The departure from ideal conditions due to fluid viscous forces is accounted for by introducing a nozzle coefficient (C_D) such that the velocity v_2 is given by:

$$\upsilon_2 = C_D\sqrt{\frac{2(p_1 - p_2)}{\rho[1-(D_2/D_1)^4]}} \qquad \text{Eq. 4.6}$$

Flow through a Nozzle

The values C_D are typically 0.7 to 0.95 depending on the nozzle geometry

The flow rate is given by:

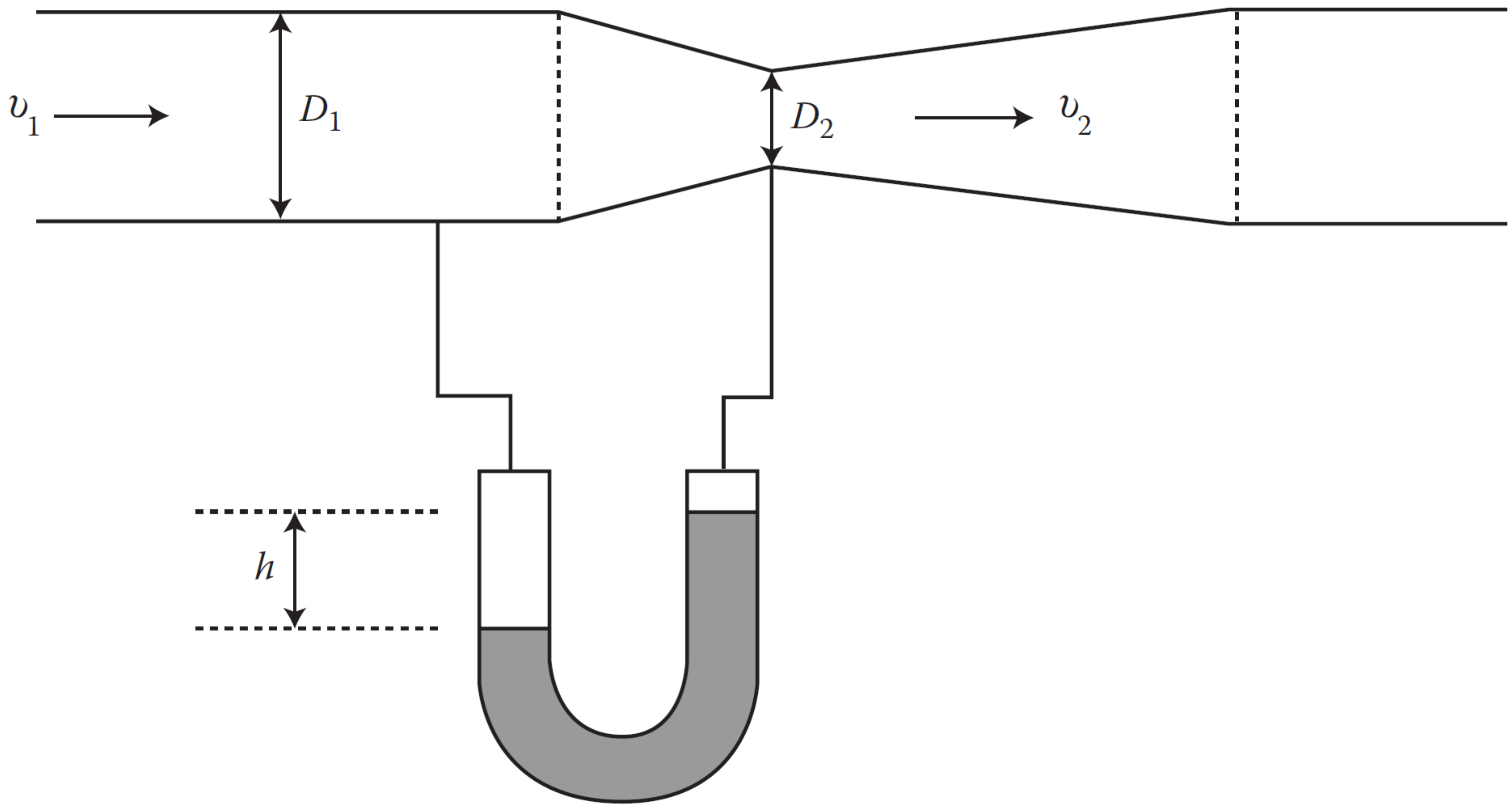
$$\dot{m} = \rho \upsilon_2 \left(\frac{\pi D_2^2}{4} \right) = C_D \left(\frac{\pi D_2^2}{4} \right) \sqrt{\frac{2\rho(p_1 - p_2)}{[1-(D_2/D_1)^4]}}$$

Eq. 4.7

Example 4.1: Venturi Meter for Flow Measurement

A venturi meter is inserted into one flow loop of a PWR as shown in the Figure in next page. The dimensions are D_1 = 0.711 m and D_2 = 0.686 m. The venture meter is mostly filled with stagnant water that is separated from the primary flow by an air bubble. This setup enables visual determination of the difference in water elevation levels (*h*) and hence the pressure difference between the contraction and the loop. Given that the height (*h*) is 0.924 m and the water density is approximately $\rho_w \approx 1000\ kg/m^3$ what are the velocity and the mass flow rate in the loop? (The Venturi Meter provide a quick way to estimate the loop flow rate. This example explains the principle behind it.)

Example 4.1: Venturi Meter for Flow Measurement

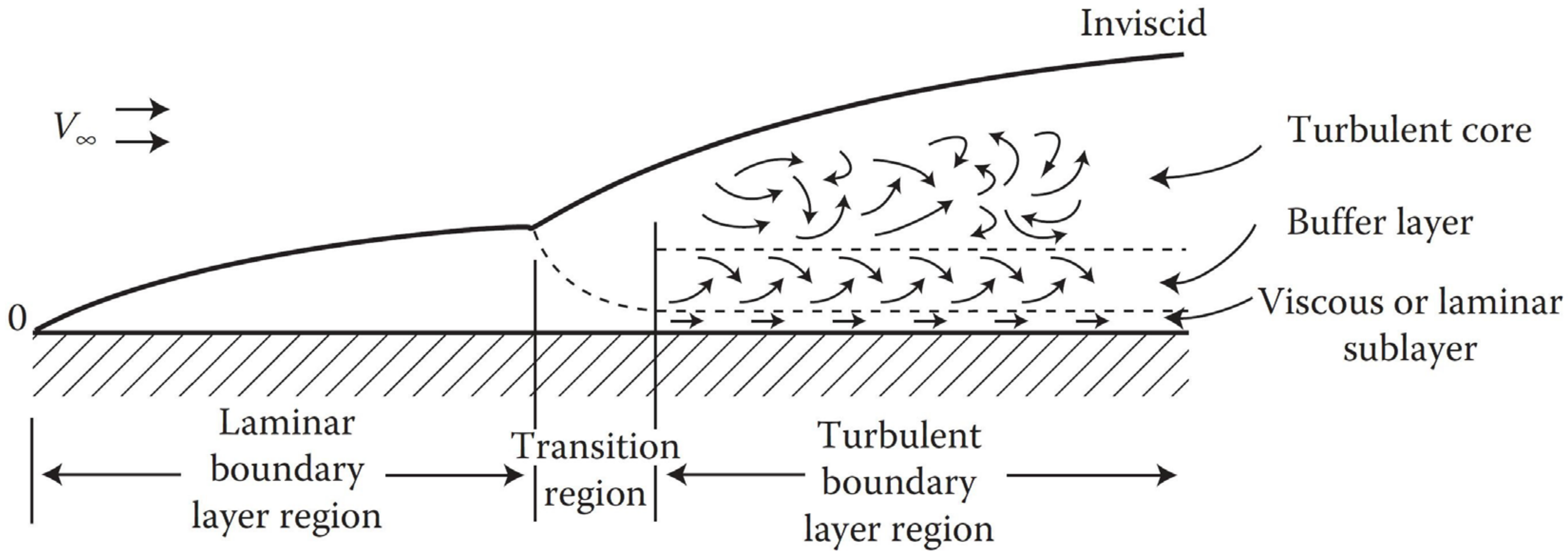


Venturi meter.

Viscous Flow

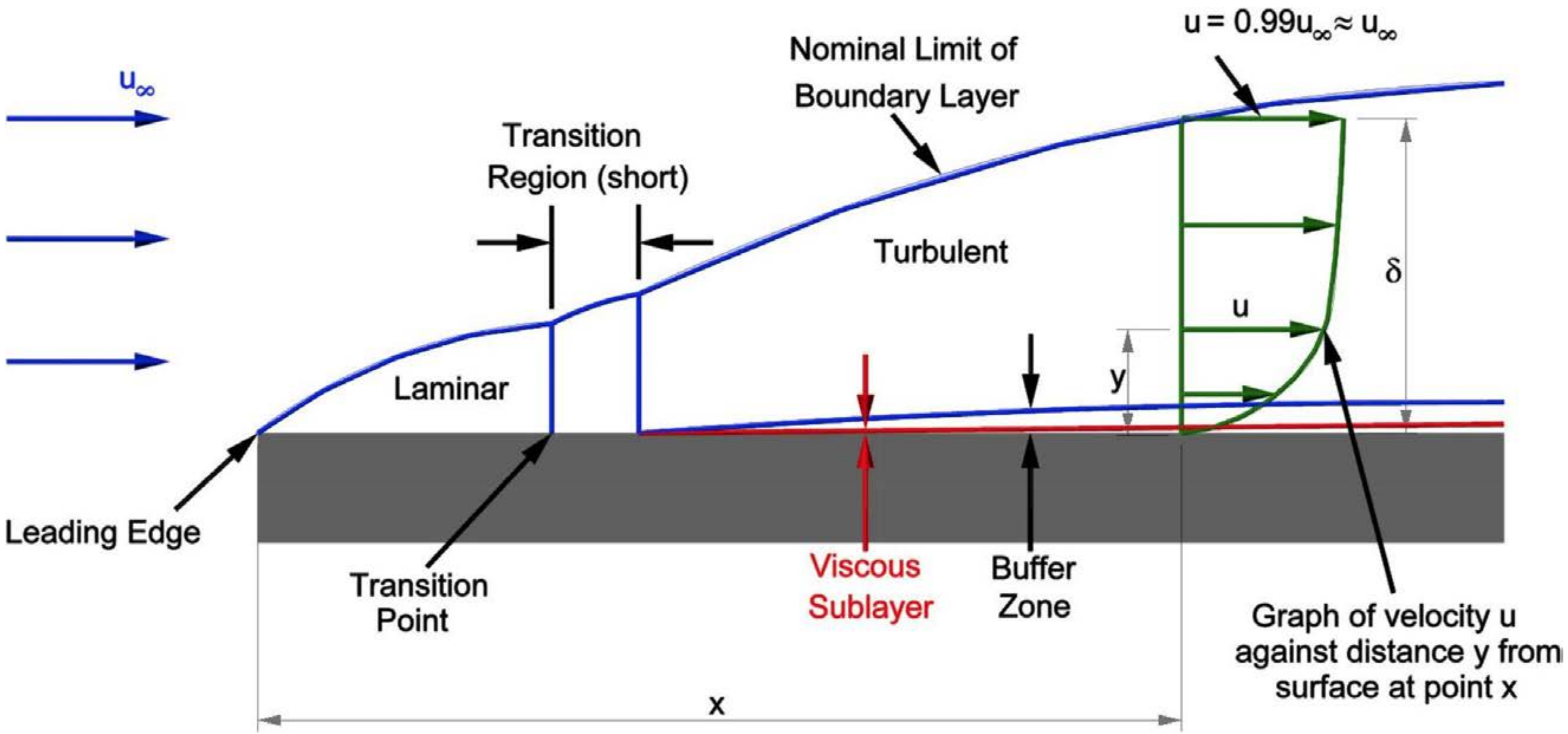
Boundary layer

Viscus/velocity Boundary layer velocity distribution for flow on an **external** surface



Boundary layer velocity distribution for flow on an external surface.

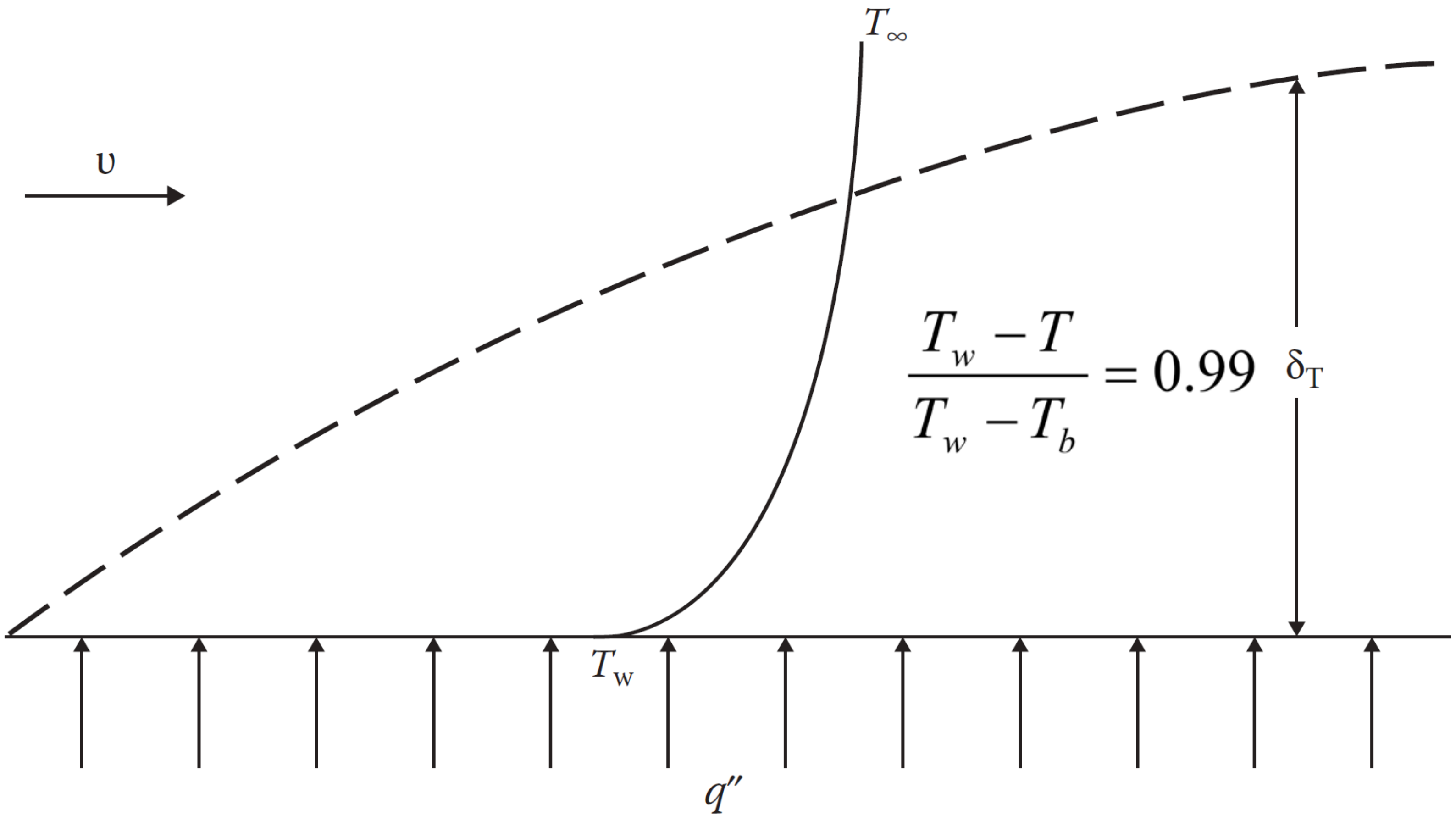
Viscus/velocity Boundary layer velocity distribution for flow on an **external** surface



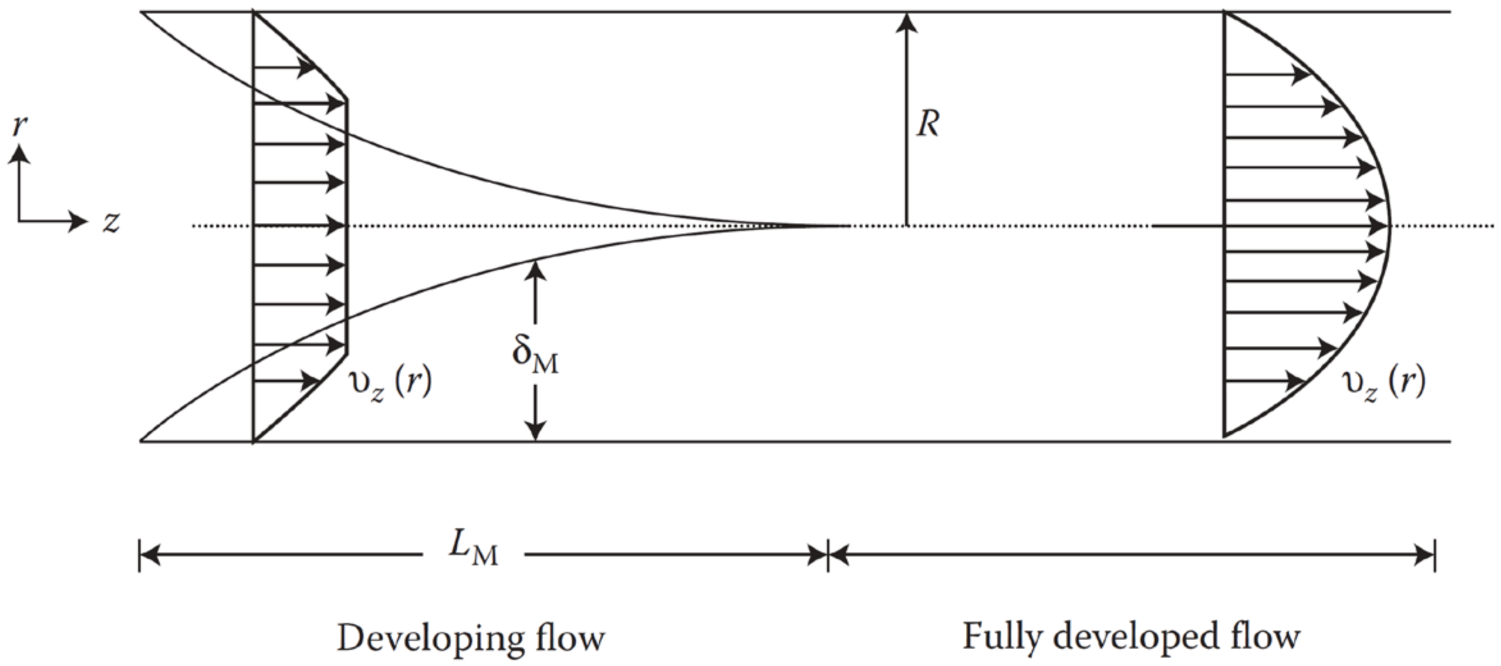
The boundary layer thickness is formally defined as the value of y at which $U = 0.99U_\infty$

Thermal boundary layer for **external** flows.

The thickness of the thermal boundary layer at any position x is determined by δ_T defined as:

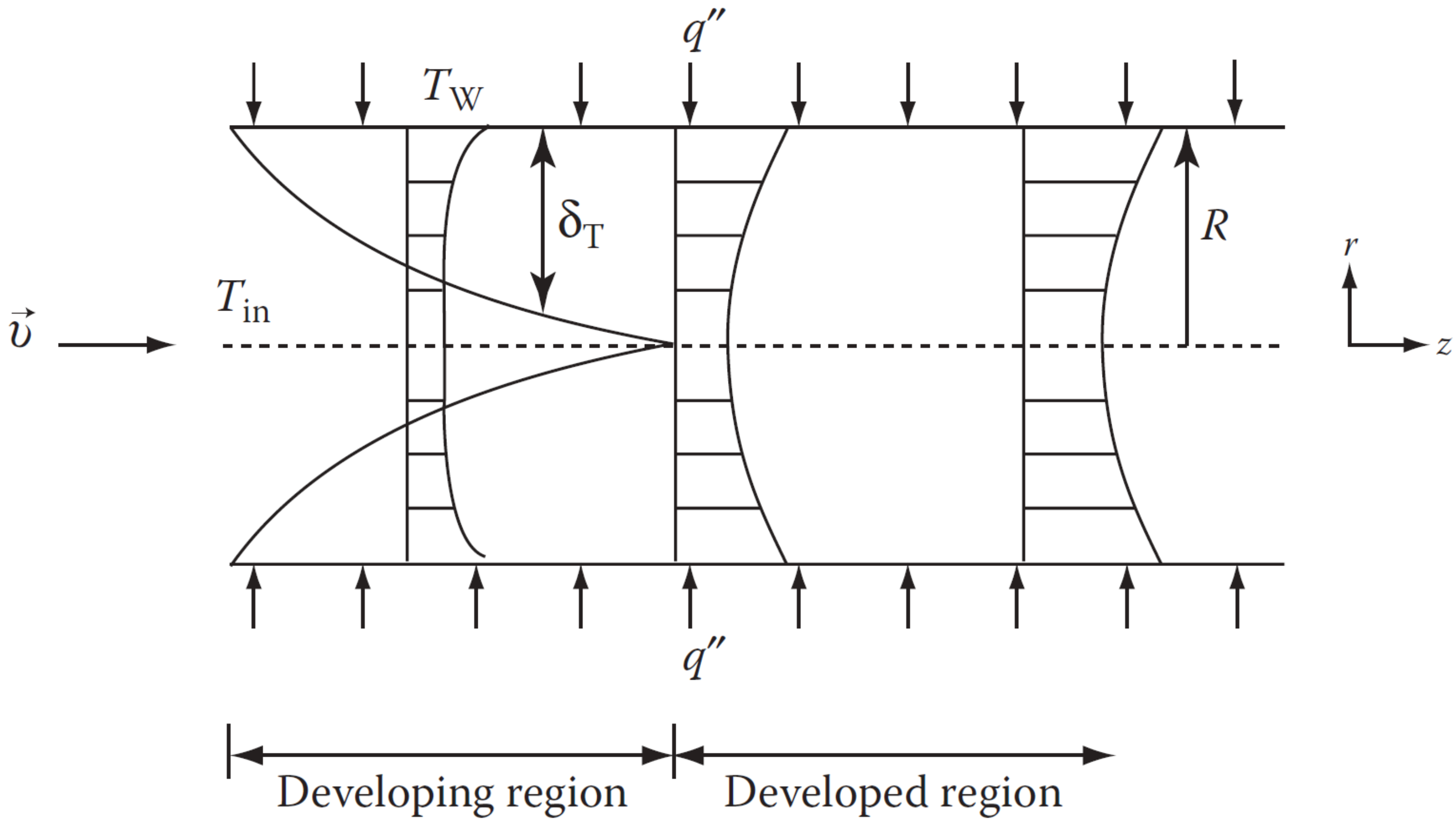


Viscus/velocity Boundary layer development **in a tube**.



In the case of the flow inside a tube, the boundary layer is assumed to start developing at the entrance of the channel and to grow **from the surface until it reaches the tube centreline**

Thermal boundary layer for internal flow in pipe



Importance of The Boundary Layer Concept

The boundary layer concept is of great value because it allows for simplification of the flow equations.

The greatest simplification arises from the ability to assume that the flow in the boundary layer is predominantly parallel to the surface.

Thus, the pressure gradient in the perpendicular directions to the flow can be taken as zero.

$$\frac{\partial p}{\partial r} \simeq 0 \quad \text{and} \quad \frac{\partial p}{r \partial \theta} = 0 \qquad \text{Eq. 4.8}$$

Importance of The Boundary Layer Concept

The velocity conditions are:

$$\upsilon_z \gg \upsilon_r$$ For laminar flow, $$v_r = 0$$ Eq. 4.9

and

$$\frac{\partial \upsilon_z}{\partial r} \gg \frac{\partial \upsilon_z}{\partial z}$$ For laminar flow, $$\frac{\partial v_z}{\partial z} = 0$$ Eq. 4.10

The second condition arises because the velocity changes from zero at the wall to the free stream value across the boundary.

Viscous Flow

Pressure Drop Terms In Channels

Pressure Drop Terms In Channels

For incompressible, inviscid flow at steady state, we have:

$$\Delta\left(\frac{p}{\rho g}+\frac{\upsilon^2}{2g}+z\right)=0$$

More general Bernoulli equation (compressible, viscous and time dependent) takes the form:

$$\left(\frac{\ell}{A}\right)_{\mathrm{T}}\frac{\mathrm{d}\dot{m}}{\mathrm{d}t}+p_{\mathrm{out}}-p_{\mathrm{in}}+\rho g(z_{\mathrm{N}}-z_1)+\frac{\dot{m}^2}{2\rho}\left(\frac{1}{A_{\mathrm{N}}^2}-\frac{1}{A_1^2}\right)+\Delta p_{\mathrm{loss}}=0$$

Eq. 4.11

Fully Developed Laminar Flow in a Circular Tube

Equation 4.11 can be rearranged into the form:

$$p_{\text{in}} - p_{\text{out}} = \Delta p_{\text{inertia}} + \Delta p_{\text{acceleration}} + \Delta p_{\text{gravity}} + \Delta p_{\text{form}} + \Delta p_{\text{friction}}$$

Where,

$$\Delta p_{\text{inertia}} = \left(\frac{\ell}{A}\right)_{\text{T}} \frac{\text{d}\dot{m}}{\text{d}t}$$

$$\Delta p_{\text{form}} \equiv K\left(\frac{\rho \upsilon_{\text{ref}}^2}{2}\right)$$

$$\Delta p_{\text{acceleration}} = \frac{\dot{m}^2}{2\rho}\left(\frac{1}{A_2^2} - \frac{1}{A_1^2}\right)$$

$$\Delta p_{\text{friction}} = \bar{f}\,\frac{L}{D_{\text{e}}}\left(\frac{\rho \upsilon_{\text{ref}}^2}{2}\right)$$

$$\Delta p_{\text{gravity}} = \rho g(z_2 - z_1)$$

Summary of Pressure Changes for Multiple Flow Conditions

Flow Geometry and Condition	Incompressible	Applicable to Both Incompressible and Compressible	Compressible
Constant Area Channels			
Frictionless	$\Delta p_{acc} = 0$	$\Delta p_{form} = 0$ $\Delta p_{fric} = 0$	$\Delta p_{acc} = \rho_2 \upsilon_2^2 - \rho_1 \upsilon_1^2$
Viscous	$\Delta p_{acc} = 0$ $\Delta p_{fric} = \bar{f} \frac{L}{D_e} \rho \frac{\upsilon_{ref}^2}{2}$	$\Delta p_{form} = 0$	$\Delta p_{acc} = \rho_2 \upsilon_2^2 - \rho_1 \upsilon_1^2$ $\Delta p_{fric} \approx \int_0^L \frac{f(\ell)}{2D_e} \rho(\ell) \upsilon^2(\ell)\, d\ell$
Abrupt Area Change[a]			
Frictionless	$\Delta p_{acc} = \rho\left(\frac{\upsilon_2^2}{2} - \frac{\upsilon_1^2}{2}\right)$	$\Delta p_{form} = 0$ $\Delta p_{fric} = 0$	$\Delta p_{acc} \approx \rho\left(\frac{\upsilon_2^2}{2} - \frac{\upsilon_1^2}{2}\right)$
Viscous	$\Delta p_{acc} = \rho\left(\frac{\upsilon_2^2}{2} - \frac{\upsilon_1^2}{2}\right)$ $\Delta p_{form,c} = K_c \rho \frac{\upsilon_2^2}{2}$ $\Delta p_{form,e} = K_e \rho \frac{\upsilon_1^2}{2}$ $\Delta p_{fric} = 0$		Not covered

[a] $\Delta p_{fric} = 0$ in this part for viscous flow because the effect of friction is included in Δp_{form}.

For pressure drop through the core, only the friction loss is important

Viscous Flow

Friction Pressure Drop for Laminar Flow

Fully Developed Laminar Flow in a Circular Tube

Differential transport equation of momentum for incompressible fluids:

$$\rho \frac{D\vec{\upsilon}}{Dt} = -\nabla p + \mu \nabla^2 \vec{\upsilon} + \rho \vec{f} \qquad \text{Eq. 4.12}$$

It is easily seen that it is a restatement of Newton's law of motion, whereby the left-hand side is the mass times the acceleration, and the right-hand side is the sum of the forces acting on that mass. We only look at the z direction for a horizontal circular tube:

$$\rho \upsilon_z \frac{\partial \upsilon_z}{\partial z} + \rho \upsilon_r \frac{\partial \upsilon_z}{\partial r} = -\frac{\partial p}{\partial z} + \mu \frac{\partial^2 \upsilon_z}{\partial z^2} + \frac{\mu}{r} \frac{\partial}{\partial r}\left(r \frac{\partial \upsilon_z}{\partial r} \right) \qquad \text{Eq. 4.13}$$

Material Derivative: https://www.youtube.com/watch?v=l4F2bZgwcpU

Fully Developed Laminar Flow in a Circular Tube

Applying the conditions of Equations 4.9 and 4.10 yields

$$\frac{\mu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \upsilon_z}{\partial r}\right) = \frac{\partial p}{\partial z} \qquad \text{Eq. 4.17}$$

Because from the conditions of Equation 4.8 $\partial p/\partial r = 0$ and $\partial p/\partial \theta = 0$

$$\frac{\partial p}{\partial z} = \frac{\mathrm{d}p}{\mathrm{d}z} \qquad \text{Eq. 4.18}$$

Fully Developed Laminar Flow in a Circular Tube

Hence the equation to be solved for v_z as a function of *r* is:

$$\frac{\mu}{r}\frac{d}{\mathrm{d}r}\left(r\frac{\mathrm{d}\upsilon_z}{\mathrm{d}r}\right)=\frac{\mathrm{d}p}{\mathrm{d}z} \qquad \text{Eq. 4.19}$$

Note that the total derivatives are used because v_z and *p* are only functions of *r* and *z*, respectively.

Equation 4.19 can be directly integrated twice over *r*, as the pressure is not a function of *r* and d*p*/d*z* is not a function of *r*. Applying the boundary conditions at

$$r=R \quad \upsilon_z=0 \qquad \text{Eq. 4.20}$$

Fully Developed Laminar Flow in a Circular Tube

And

$$r = 0 \quad \upsilon_z \text{ remains finite} \qquad \text{Eq. 4.21}$$

Condition 4.21 yields

$$\left.\frac{\partial \upsilon_z}{\partial r}\right|_r = 0 \text{ (i.e., } \upsilon_z \text{ is maximum at } r = 0)$$

Hence Equation 4.19 yields

$$\upsilon_z = \frac{R^2}{4\mu}\left(-\frac{dp}{dz}\right)\left(1 - \frac{r^2}{R^2}\right) \qquad \text{Eq. 4.22}$$

Fully Developed Laminar Flow in a Circular Tube

Hence, when d*p*/d*z* is negative, the velocity is positive in the axial direction.

It is useful to obtain the mean (mass weighted) velocity (V_m):

$$V_{\mathrm{m}} = \frac{\int_0^R \rho \upsilon_z (2\pi r)\, \mathrm{d}r}{\int_0^R \rho (2\pi r)\, \mathrm{d}r} \qquad \text{Eq. 4.23}$$

For a constant density:

$$V_{\mathrm{m}} = \frac{\int_0^R \upsilon_z (2\pi r)\, \mathrm{d}r}{\pi R^2} \qquad \text{Eq. 4.24}$$

Fully Developed Laminar Flow in a Circular Tube

Substituting v_z from Equation 4.22 and performing the integration, we obtain

$$V_m = \frac{R^2}{8\mu}\left(-\frac{dp}{dz}\right) \qquad \text{Eq. 4.25}$$

From Equations 4.22 and 4.25, the local velocity can be written as

$$\frac{\upsilon_z}{V_m} = 2\left(1 - \frac{r^2}{R^2}\right) \qquad \text{Eq. 4.26}$$

Fully Developed Laminar Flow in a Circular Tube

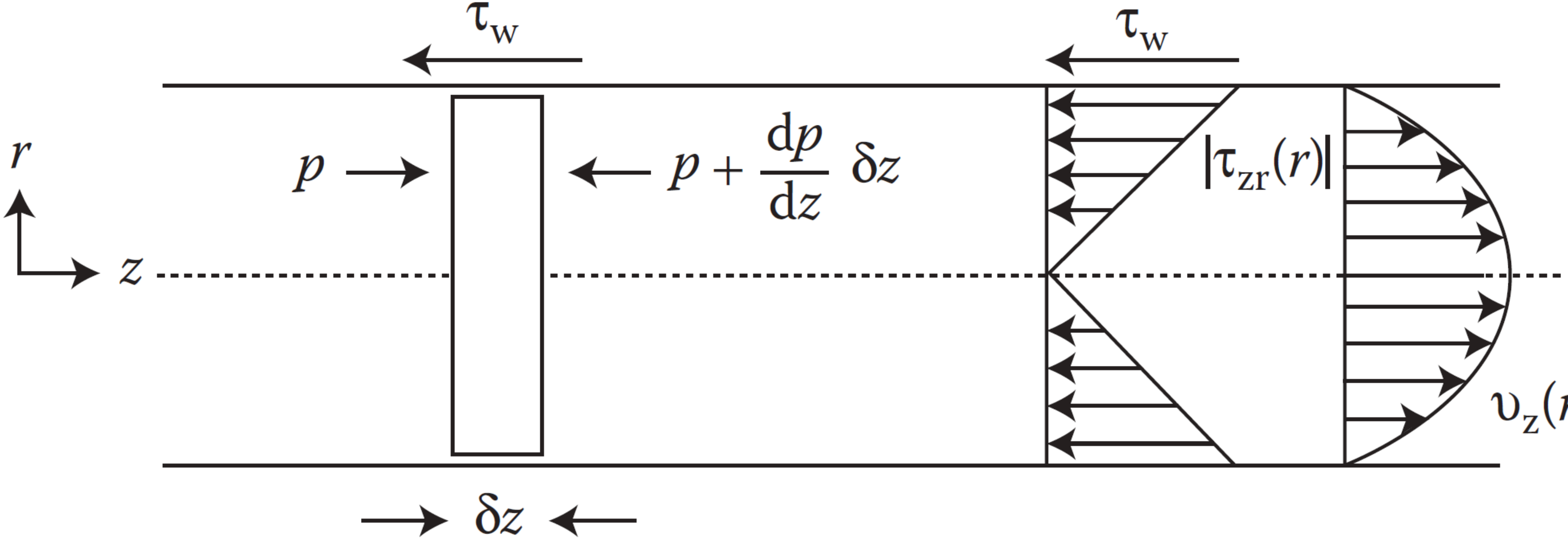
The parabolic nature of the velocity profile leads to a linear profile of the shear stress τ_{rz}, as

$$\tau_{rz} = +\mu \frac{d\upsilon_z}{dr} = -4\mu \frac{V_m}{R}\left(\frac{r}{R}\right) \qquad \text{Eq. 4.27}$$

Note that τ_{rz} is opposite in sign to V_m, which means that it acts opposite to the flow direction, as expected. The maximum shear stress is given by specifying the value of $r = R$ in Equation 4.27 to get:

$$\tau_{rz}\Big|_{max} = -4\mu \frac{V_m}{R} = \frac{R}{2}\left(+\frac{dp}{dz}\right) \qquad \text{Eq. 4.28}$$

Shear stress distribution and velocity profile in fully developed pipe flow



Fully Developed Laminar Flow in a Circular Tube

Since the wall shear stress, τ_w, is always opposite to the direction of flow, it is convenient to define it as positive in the opposite direction of the flow. Thus, as

$$\tau_w = \left|\tau_{rz}\right|_{max} \qquad \text{Eq. 4.29}$$

We have:

$$\tau_w = -\frac{R}{2}\left(\frac{dp}{dz}\right) \qquad \text{Eq. 4.30}$$

Note d*p*/d*z* has negative values for flow in the positive *z* direction.

Fully Developed Laminar Flow in a Circular Tube

Of great practical importance is the definition of a friction factor (*f*) such that the pressure gradient is related to the kinetic pressure based on the average velocity and the diameter of the pipe. From Equation 4.25, we can see that

$$-\frac{\mathrm{d}p}{\mathrm{d}z} = \frac{8\mu}{R^2}V_\mathrm{m} = \frac{64\mu}{\rho D^2 V_\mathrm{m}}\left(\frac{\rho V_\mathrm{m}^2}{2}\right) \qquad \text{Eq. 4.31}$$

Which can be recast in the form originally proposed by Darcy and defined as

$$-\frac{\mathrm{d}p}{\mathrm{d}z} = \frac{f}{D}\frac{\rho V_\mathrm{m}^2}{2} \qquad \text{Eq. 4.32}$$

Fully Developed Laminar Flow in a Circular Tube

Where *f*, the friction factor, in this case is given by:

$$f = \frac{64}{\rho D V_m / \mu} = \frac{64}{\mathrm{Re}} \qquad \text{Eq. 4.33}$$

The result of this analysis leads to a condition usually observed for laminar flow, that is, that the product *f*Re is a constant dependent only on the geometry of the flow.

Fully Developed Laminar Flow in a Circular Tube

Experience shows that laminar flow in a tube exists up to a Reynolds number of about 2100. By combining Equations 4.30 and 4.32, we get a relation for the wall shear stress and the friction factor f:

$$\tau_w = \frac{R}{2}\frac{f}{2R}\frac{\rho V_m^2}{2} = \frac{f}{4}\frac{\rho V_m^2}{2} \qquad \text{Eq. 4.34}$$

Unfortunately, there is another friction factor that appears in the literature, the Fanning friction factor, which is defined in terms of the shear stress (τ_w) relation to the kinetic pressure, such that

$$\tau_w = f'\frac{\rho V_m^2}{2} \qquad \text{Eq. 4.35}$$

Fully Developed Laminar Flow in a Circular Tube

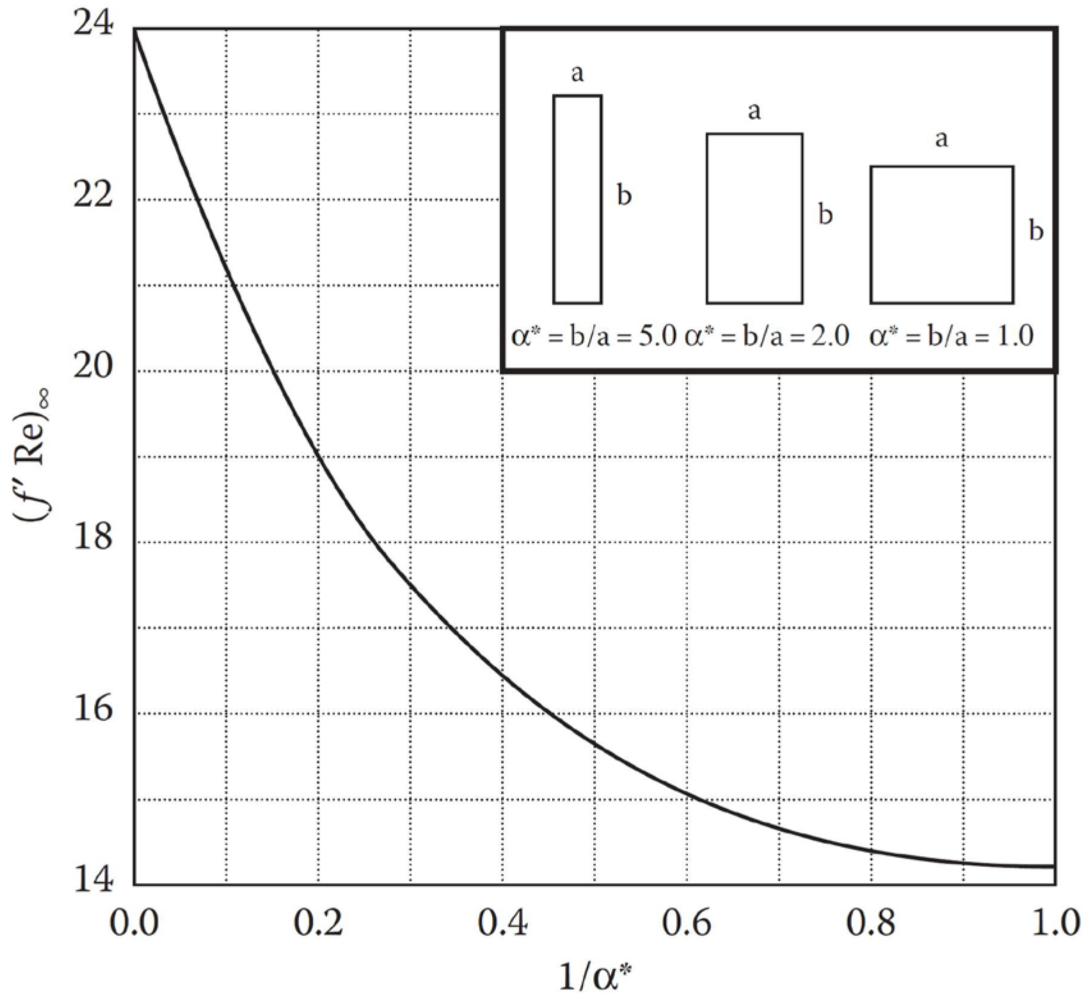
Thus, the Darcy factor *f* is related to the Fanning factor *f ′* as

$$f = 4f' \qquad \text{Eq. 4.36}$$

The friction pressure drop across a pipe of length *L*, when the developing flow region can be ignored, is given by

$$\Delta p_{\text{friction}} = \int_{z_{\text{in}}}^{z_{\text{out}}} \left(-\frac{\mathrm{d}p}{\mathrm{d}z} \right) \mathrm{d}z = p_{\text{in}} - p_{\text{out}} = \frac{\bar{f} L}{D_{\text{e}}} \frac{\rho V_{\text{m}}^2}{2} \qquad \text{Eq. 4.37}$$

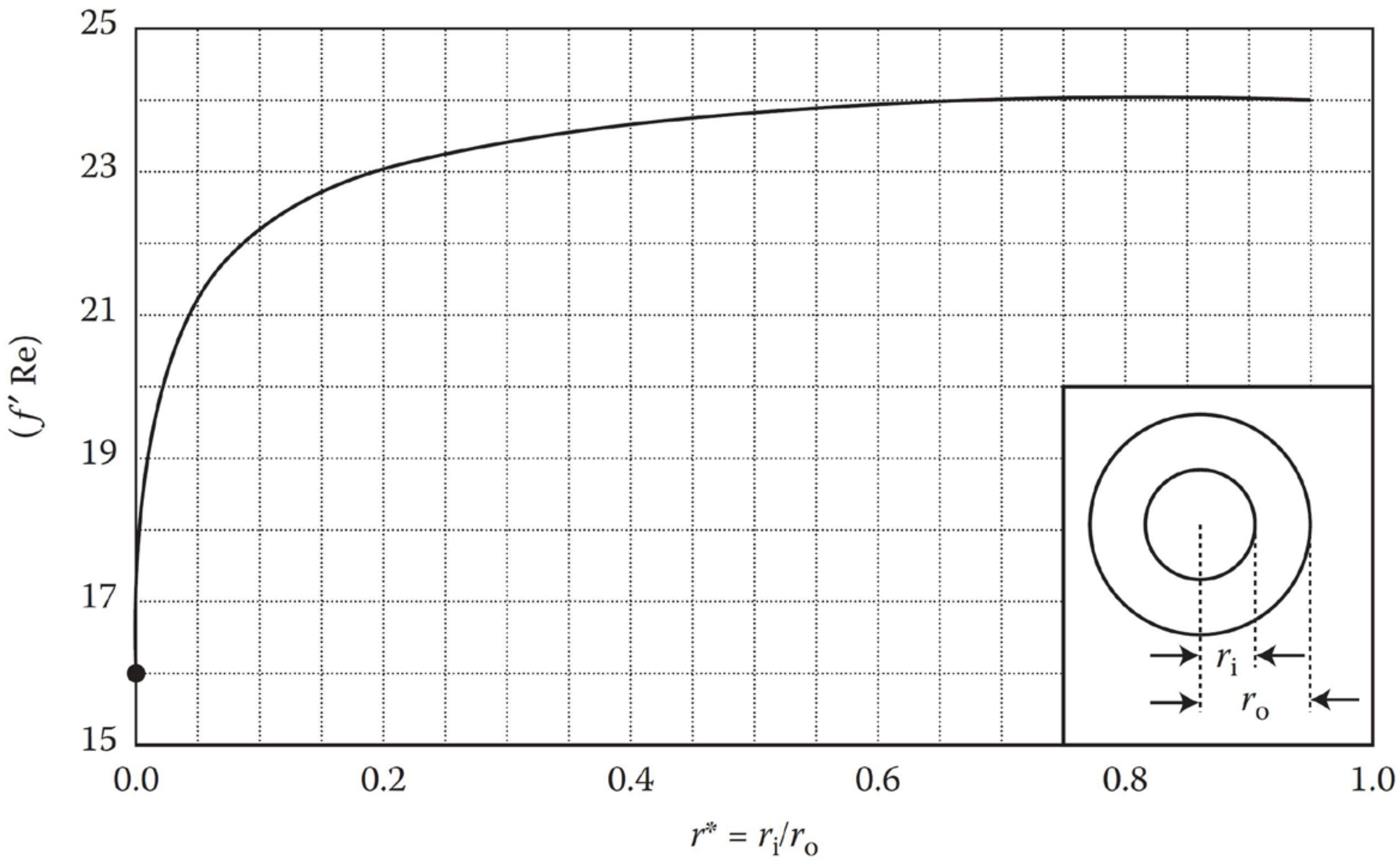
Product of laminar friction factor (Fanning) and Reynolds number for fully developed flow with rectangular geometry.



By following a procedure similar to the one in circular tube, solutions for the velocity distribution and wall friction factor coefficient have been obtained for a variety of flow geometries.

The values of f' Re for fully developed flow in rectangular and annular channels are no longer constant

Product of laminar friction factor (Fanning) and Reynolds number for fully developed flow in an annular channel.



EQUIVALENT DIAMETER

The equivalent diameter is defined as:

$$D_e = \frac{4A_f}{P_w}$$

Where A_f is the flow area, and P_w is the wetted/heated perimeter

f Re is no longer constant in Noncircular Geometries laminar flow

1. Although the values of f' Re for these cases are expressed in terms of the equivalent hydraulic diameter, these results are not equivalent to simply transforming the circular tube results utilizing the equivalent diameter concept.
2. The molecular shear effects are significant throughout the flow cross section so that the governing equations have to be solved for each specific geometry.
3. The need to solve for each specific geometry is also the case for heat transfer in laminar flow.

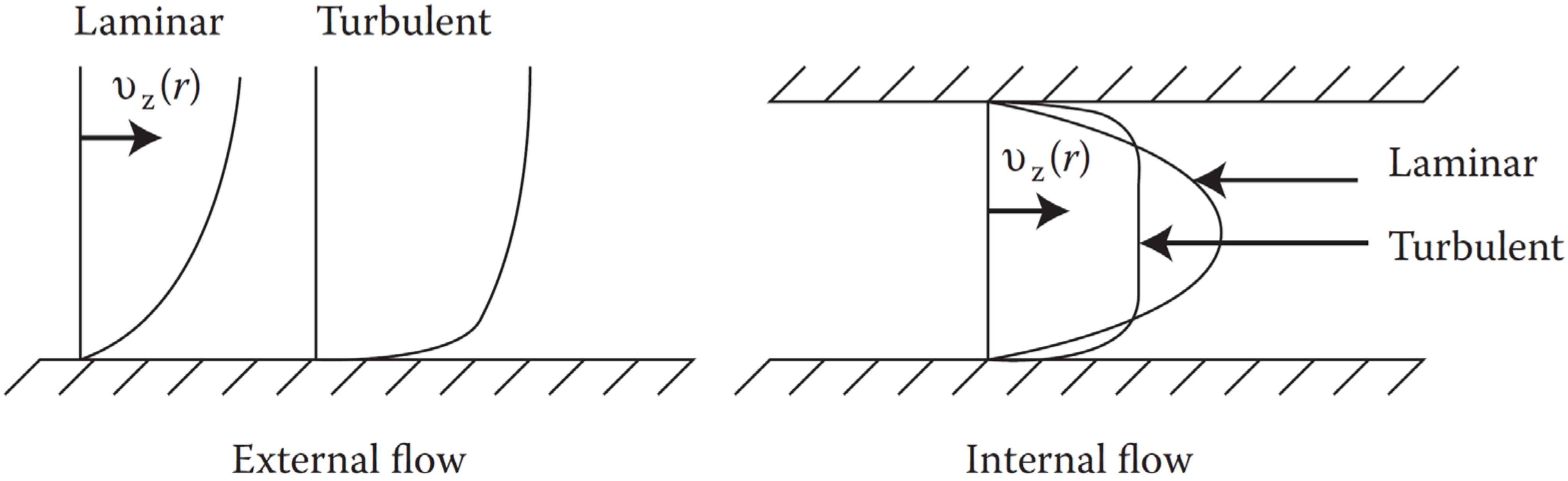
Viscous Flow

Friction Pressure Drop for Turbulent Flow

Turbulent Flow

1. At sufficiently long distances or high velocities (i.e., high Re values), the smooth flow of the fluid is disturbed by the irregular appearance of eddies.
2. The turbulence enhancement of the momentum and energy lateral transport above the rates possible by molecular effects alone flattens the velocity and temperature profiles
3. The enhancement, by the eddies, leads to the practical preference for the use of turbulent flow in most heat transfer equipment.

Velocity profiles of fully developed laminar and turbulent flows



Note that the velocity profile is flatter than in laminar flow.

FULLY DEVELOPED TURBULENT FLOW WITH NONCIRCULAR GEOMETRIES

In the turbulent flow case, the velocity gradient is principally near the wall. Hence, the flow channel geometry does not have an important influence on the friction factor. The hydraulic diameter of noncircular channels:

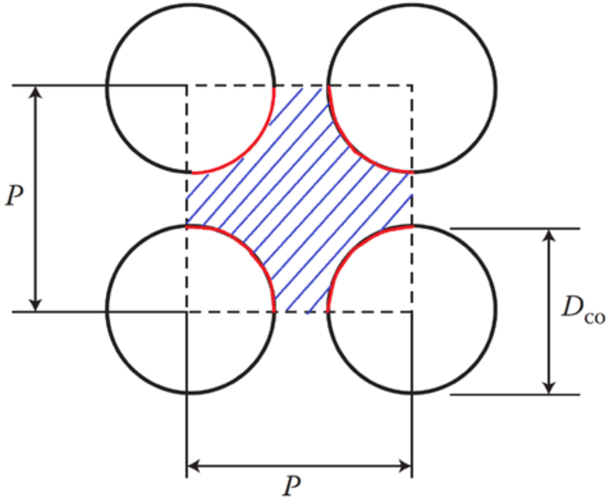
$$D_e = \frac{4A_f}{P_w}$$

Can be used in place of the circular channel diameter *D* in **circular channel friction factor correlations**. this is not the case for laminar flow in noncircular channels.

HYDRAULIC DIAMETER OF NONCIRCULAR CHANNELS

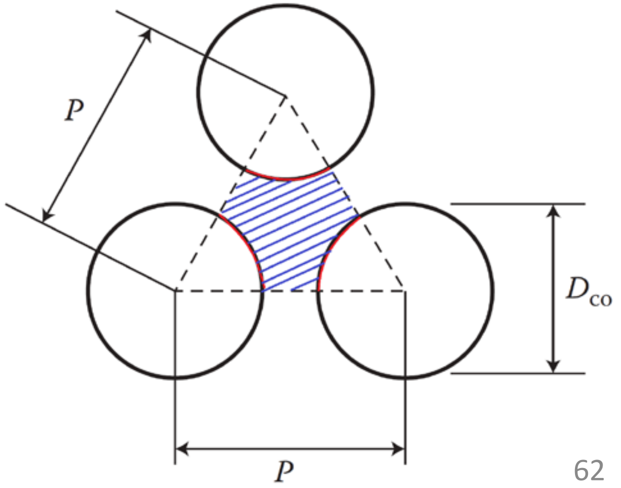
- **For uniformly spaced rectangle rod arrays**

$$D_e = \frac{4A_f}{P_w} = \left[\frac{4}{\pi}\left(\frac{P}{D}\right)^2 - 1\right]D$$



- **For uniformly spaced triangular rod arrays**

$$D_e = \frac{4A_f}{P_w} = \left[\frac{2\sqrt{3}}{\pi}\left(\frac{P}{D}\right)^2 - 1\right]D$$



Turbulent Velocity Distribution

The velocity profile can be approximated by:

$$\frac{\upsilon_z}{\upsilon_{c\ell}} = \left(\frac{y}{R}\right)^{1/7} = \left(\frac{R-r}{R}\right)^{1/7}$$

Where v_{cl} is the velocity at the pipe centreline.

Consequently the average velocity is given by:

$$V_m = 0.817\,\upsilon_{c\ell}$$

Recall for laminar flow:

$$\frac{\upsilon_z}{V_m} = 2\left(1 - \frac{r^2}{R^2}\right)$$

$$V_m = 0.5 v_{cl}$$

TURBULENT FRICTION FACTOR

McAdams relation (suitable For $3 \times 10^4 <$ Re $< 10^6$, smooth tube):

$$f = 0.184\,\mathrm{Re}^{-0.20}$$

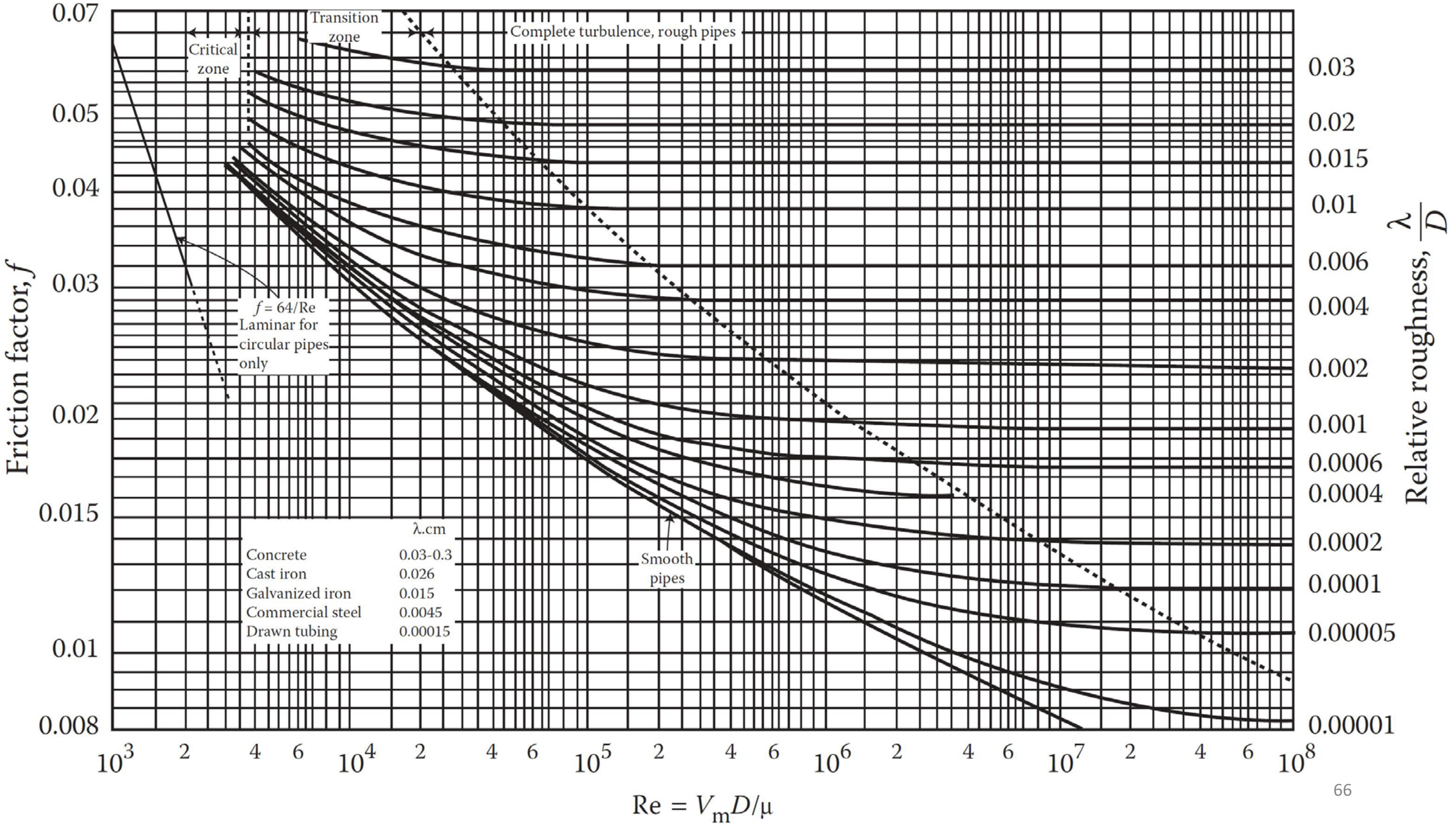
Blasius relation (suitable For $4 \times 10^3 <$ Re $< 10^5$, smooth tube):

$$f = 0.316\,\mathrm{Re}^{-0.25}$$

The Moody chart (next page)

1. The Moody chart is commonly used to obtain **Darcy friction** factors for flow inside **circular pipes**
2. In Moody's chart, the effect of the **roughness** depends on the pipe diameter (*D*), as is reasonable to expect.
3. The Moody diagram is a graphic representation of the Colebrook equation which need iteration to calculate. Explicit approximated relation given by Churchill provides friction factors within 1%:

$$f_{\text{Churchill}} = \frac{8}{6.0516}\left\{\ell n\left[\frac{\lambda/D}{3.7}+\left(\frac{7}{\text{Re}}\right)^{0.9}\right]\right\}^{-2}$$

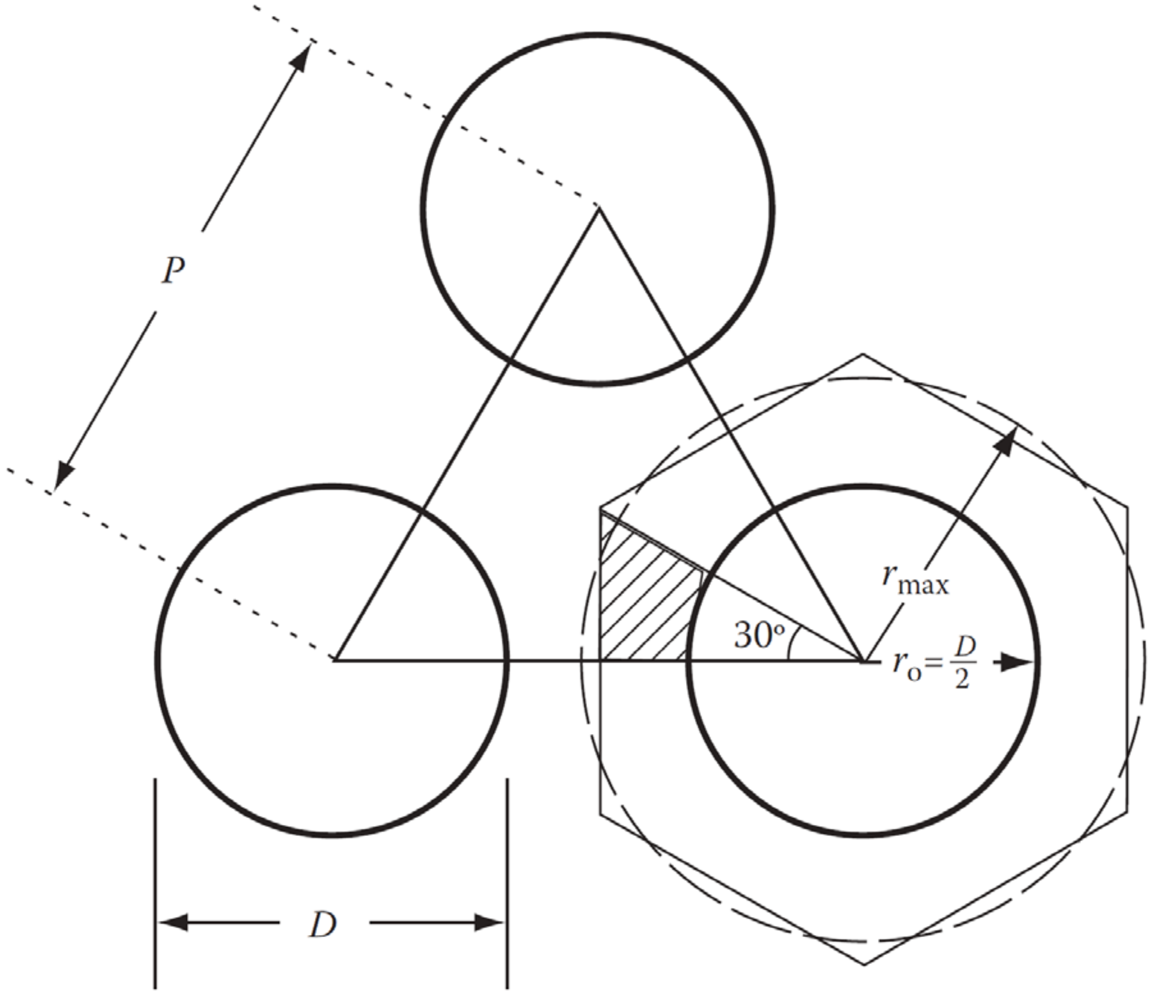


Friction factor, f
Relative roughness, $\frac{\lambda}{D}$
Re = $V_m D/\mu$
Critical zone
Transition zone
Complete turbulence, rough pipes
f = 64/Re
Laminar for circular pipes only
Smooth pipes
λ.cm
Concrete 0.03-0.3
Cast iron 0.026
Galvanized iron 0.015
Commercial steel 0.0045
Drawn tubing 0.00015
0.07
0.05
0.04
0.03
0.02
0.015
0.01
0.008
0.03
0.02
0.015
0.01
0.006
0.004
0.002
0.001
0.0006
0.0004
0.0002
0.0001
0.00005
0.00001
10^3 2 4 6 10^4 2 4 6 10^5 2 4 6 10^6 2 4 6 10^7 2 4 6 10^8

Viscous Flow

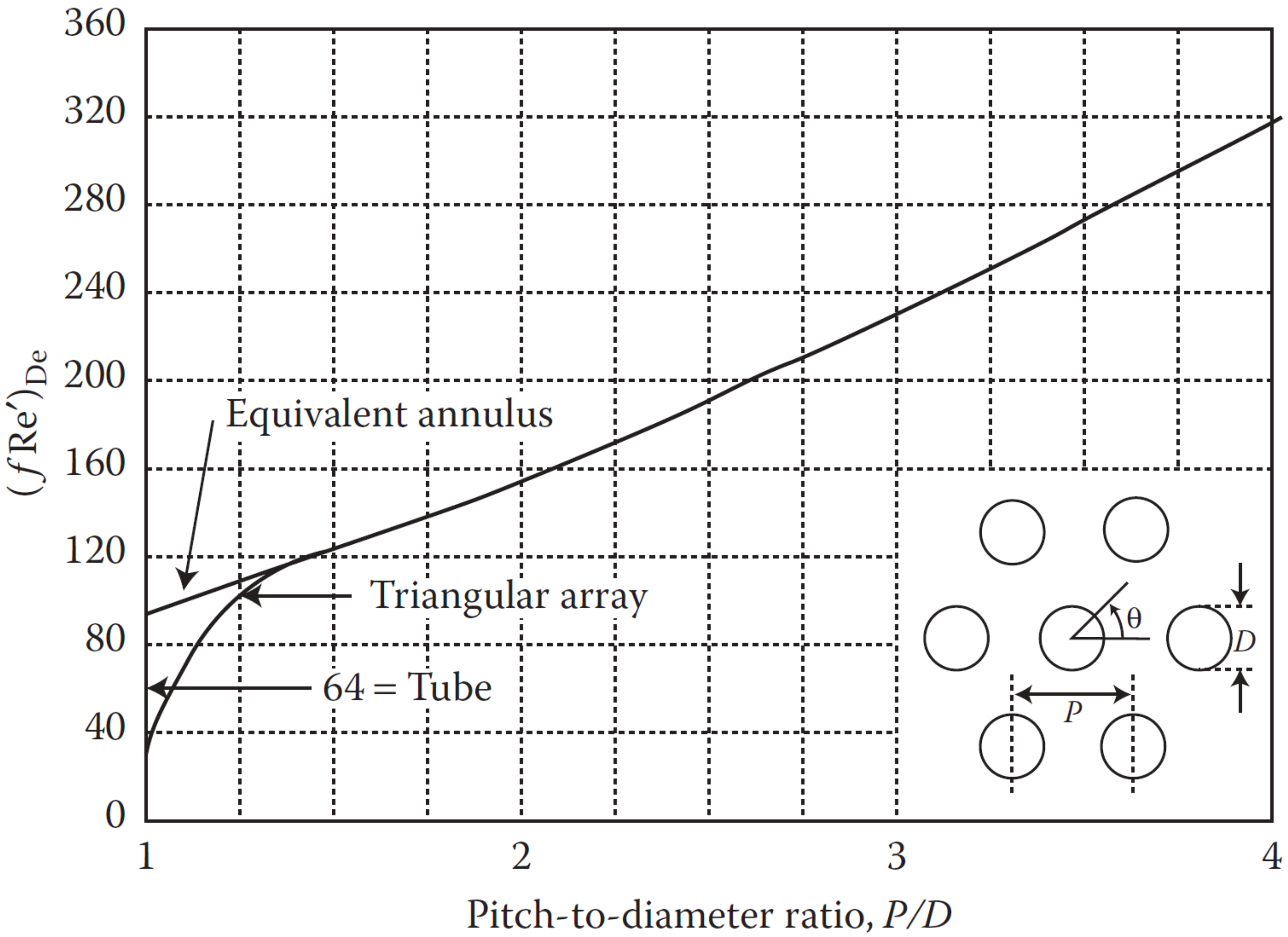
Friction Pressure Drop in Rod Bundles

Equivalent annulus in a triangular array

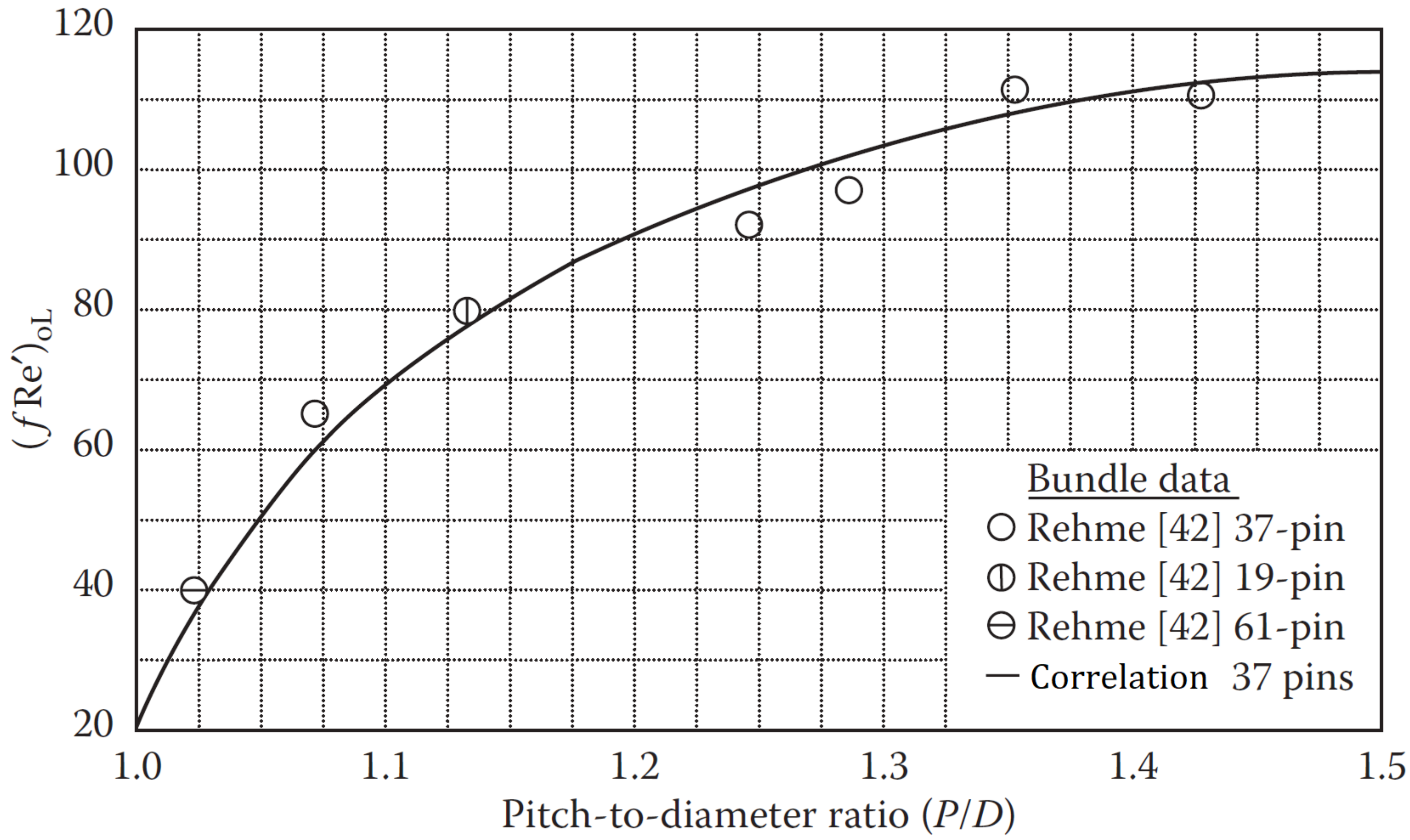


Definition of an equivalent annulus in a triangular array. *Cross-hatched area* represents an elemental coolant flow section. Circle of radius r_{max} represents the equivalent annulus with equal flow area.

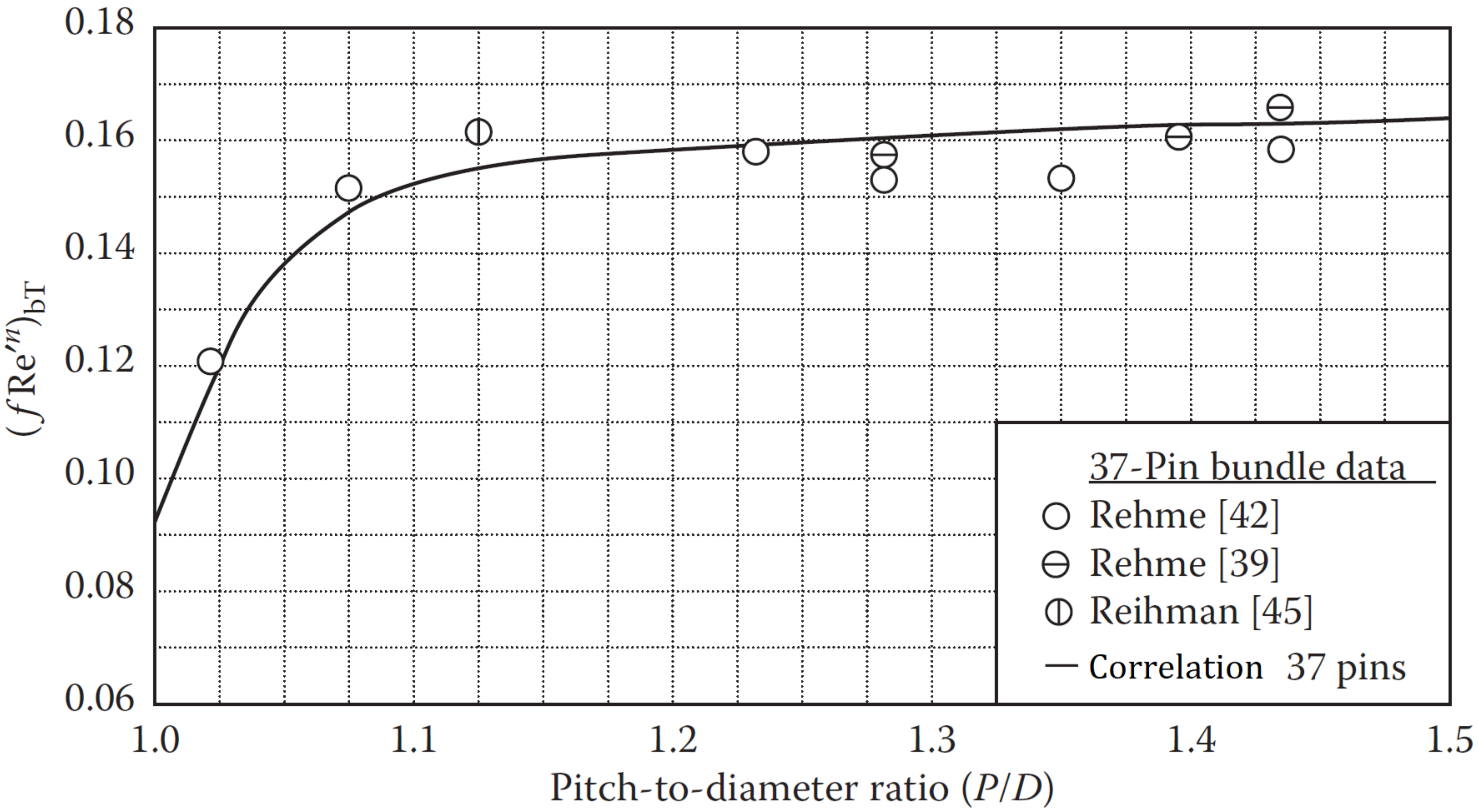
Laminar Flow Results In
An **Infinite** Triangular Array Of Rods



Laminar Flow Results In **Finite** Triangular Array Bare Rod Bundles



Turbulent Flow Results In **Finite** Triangular Array Bare Rod Bundles



Correlation For Turbulent Flow In Triangular Array Bare Rod Bundles

For a triangular array, the following **equivalent annulus** solutions is obtained:

$$\text{For } \mathrm{Re}'_{\mathrm{De}} = 10^4: \qquad \frac{f}{f_{\mathrm{c.t.}}} = 1.045 + 0.071(P/D - 1)$$

$$\text{For } \mathrm{Re}'_{\mathrm{De}} = 10^5: \qquad \frac{f}{f_{\mathrm{c.t.}}} = 1.036 + 0.054(P/D - 1)$$

where $f_{\mathrm{c.t.}}$ = circular tube friction factor

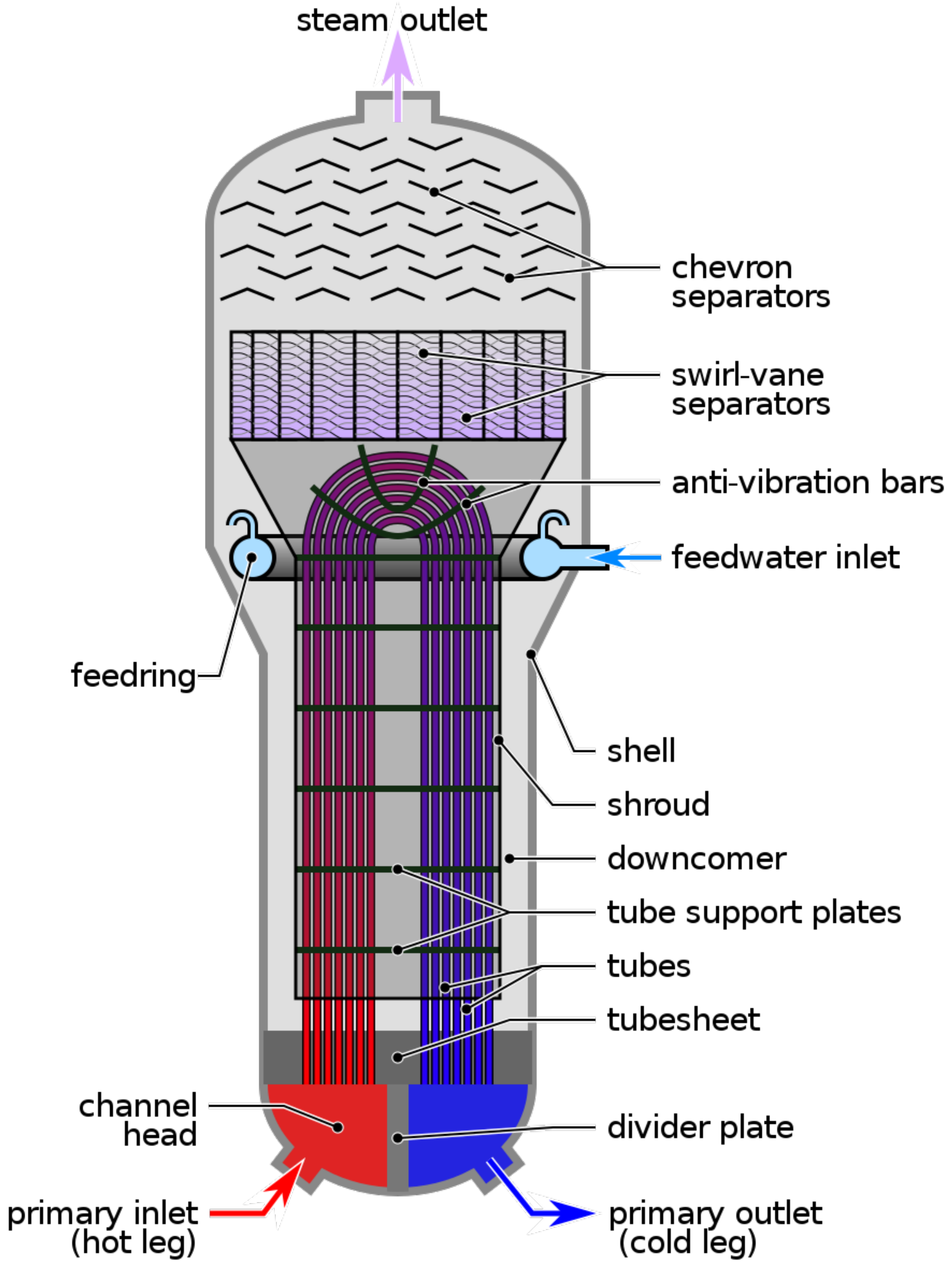
Example 4.2 Laminar Flow Characteristics in a Steam Generator Tube

During a shutdown condition in a PWR, the flow is driven through the loop by natural circulation at a rate corresponding to about 1% of the full flow rate provided by the pumps. Assuming that the total flow rate is m_T = 4686 kg/s in the full flow condition and that there are approximately 3800 tubes of 0.0222 m inside diameter in the steam generator of average length 16.5 m, determine

1. Whether the flow is turbulent or laminar.
2. The value of the friction factor.
3. The friction pressure loss between the inlet and outlet of one tube (neglecting the pressure loss due to the tube bend of 180°).

Note: 1. Use ρ = 1000 kg/m3 and μ = 0.001 Pa·s

2. Re < 2100, stable laminar flow



Inside of Steam Generator

Heat Transfer Analysis for Single-phase Flows

FOURIER'S LAW

At (or approximately near) the solid wall, Fourier's law states that

$$\vec{q}'' = -k \frac{\partial T}{\partial n} \vec{n}$$

- where k is the thermal conductivity of the coolant (W/m K)
- n is the unit vector perpendicular to the surface
- $\partial T/\partial n$ is the temperature gradient (K/m)

NEWTON LAW FOR HEAT TRANSFER

In engineering analyses, where only the heat transfer coefficient (h) is desired, the heat flux is related to the bulk or mean temperature of the flow (T_b), via the so-called Newton law for heat transfer:

$$\vec{q}'' \equiv h(T_\mathrm{w} - T_\mathrm{b})\vec{n}$$

Where T_w is the wall temperature (K), and *h* is the heat transfer coefficient (W/m2 K).

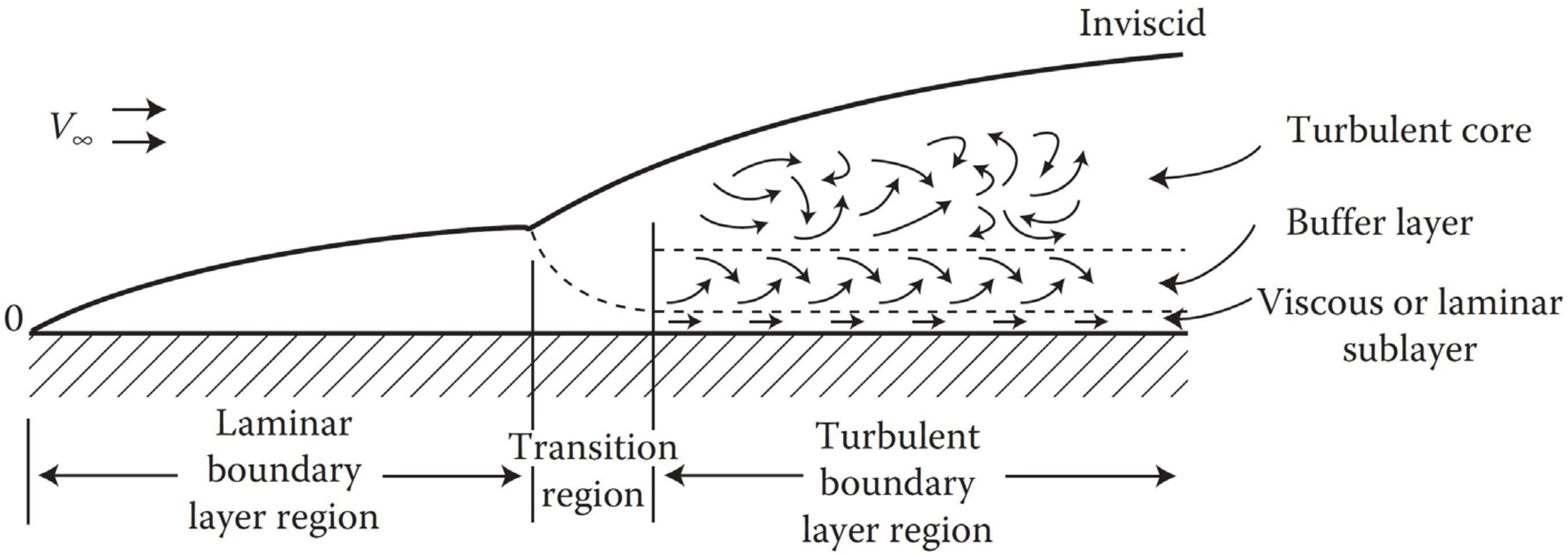
This Equation is applied in the engineering analysis when it is possible to determine *h* for the flow conditions based on prior engineering experience.

Often the heat transfer coefficient is a semi-empirical function of the coolant properties and velocity as well as of the flow channel geometry.

Laminar Flow Heat Transfer

1. When the flow is laminar, the boundary layer will be laminar
2. Fluid particles move along streamlines. That is there is no mixing and heat is **only transferred by conduction**
3. However, the calculation procedure is more complicated than the case of solids: the velocity of each layer is different from the others.
4. Variations of velocity affect the amount of heat going to different layers of the flow
5. Therefore, it is the shape of the velocity profile that is important not the main-stream velocity.

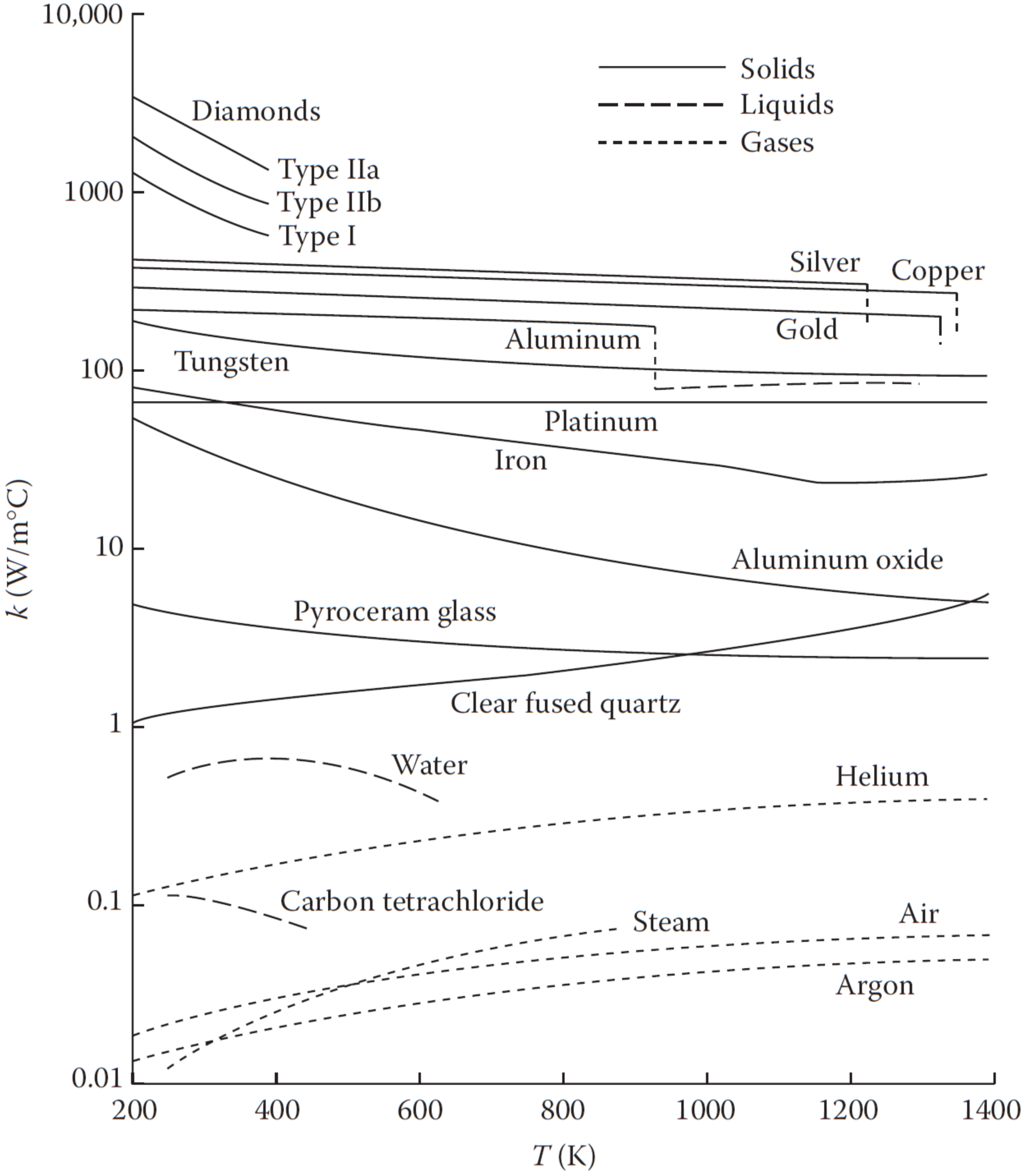
Viscous/velocity Boundary layer velocity distribution for flow on an external surface



Boundary layer velocity distribution for flow on an external surface.

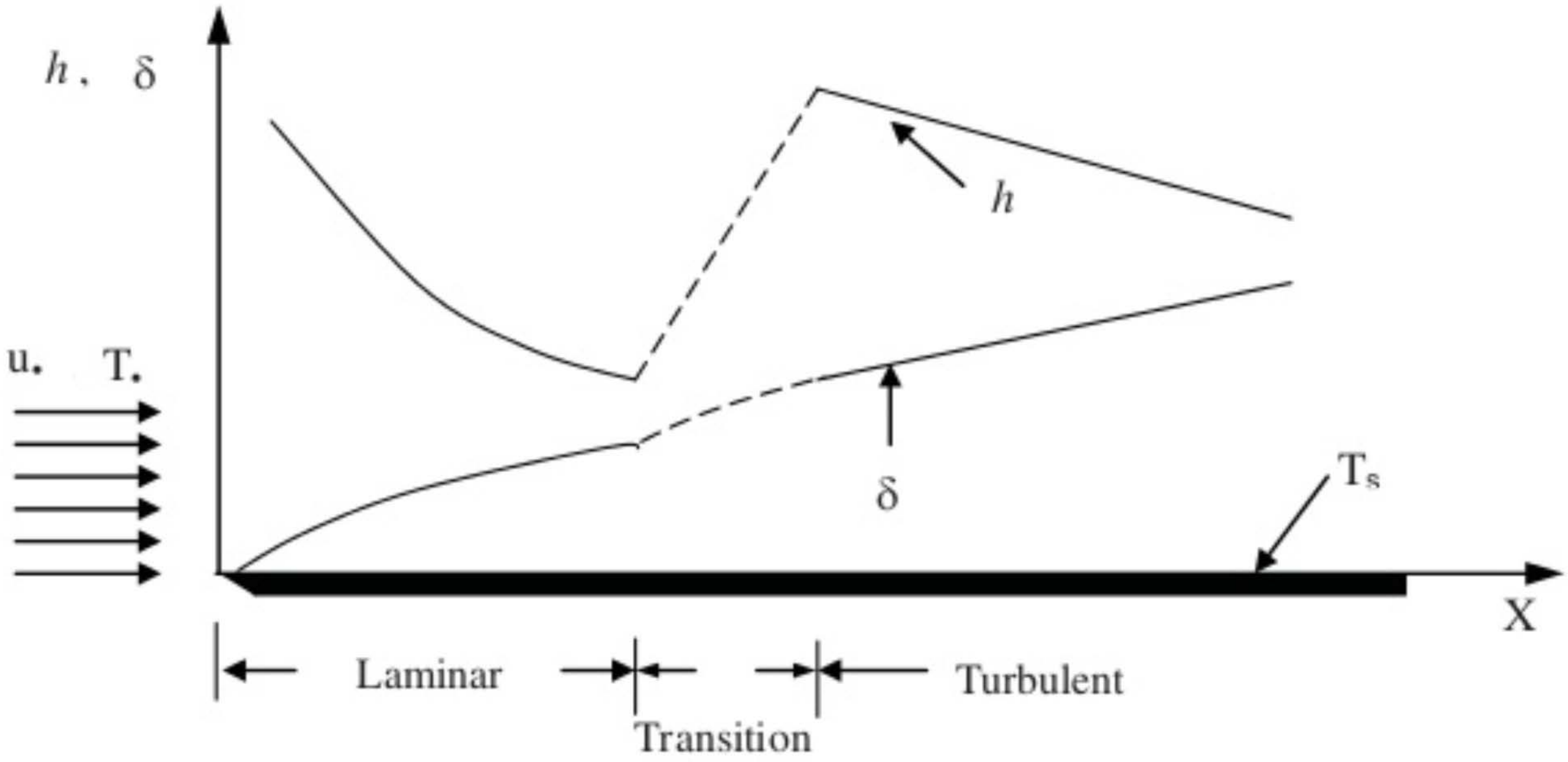
Turbulent Flow Heat Transfer

1. When the flow is turbulent, the boundary layer is also turbulent
2. Here the fluid motion is highly irregular. **Convection heat transfer is important**
3. However, the main **barrier** to heat transfer is conduction through the laminar sublayer since gases and most liquids are bad thermal conductors.
4. Thus heat transfer is mainly determined by **the thickness** of the laminar sublayer.
5. A high flow velocity means a reduced sublayer thickness and improved heat transfer.



Variation of the thermal conductivity of various solids, liquids, and gases with temperature

Variation of the velocity boundary layer thickness and the local convective heat transfer coefficient.



Single-phase Heat Transfer

Dimensionless Numbers

BUCKINGHAM $\boldsymbol{\pi}$ THEOREM

1. Dimensional analysis using Buckingham π theorem is a commonly used technique to solve engineering problems.
2. The Buckingham π theorem loosely states that if we have a physically meaningful equation involving a certain number, n, of physical variables, and these variables are expressible in terms of k independent fundamental physical quantities, then the original expression is equivalent to an equation involving a set of $p = n - k$ dimensionless parameters constructed from the original variables.
3. The dimensional analysis is used to solve engineering problems that can not be solved theoretically such as forced convection heat transfer.
4. The method is conducted in a number of steps as follows:

DIMENSIONAL ANALYSIS OF CONVECTION HEAT TRANSFER

Step 1: List the variables that affect the convection heat transfer h as follows:

1. Fluid velocity U (m/s)
2. Characteristic length L (m)
3. The fluid conductivity k (W/mK)
4. The fluid viscosity μ(kg/ms)
5. The specific heat capacity of the fluid C_p (kJ/kgK)
6. Fluid density ρ (kg/m3)

$$h = f(U, L, k, \mu, C_p, \rho)$$

DIMENSIONAL ANALYSIS OF CONVECTION HEAT TRANSFER

Step 2: Write the variables in terms of the basic units : M for mass, L for length, T for time and θ for temperature.

$h: MT^{-3}\theta^{-1}$ U: LT^{-1}

L: L k: $MLT^{-3}\theta^{-1}$

$\mu: ML^{-1}T^{-1}$ $C_p: L^2T^{-2}\theta^{-1}$

$\rho: ML^{-3}$

We have 7 variables and 4 basic units. Thus we can formulate 7-4 = 3 dimensionless groups (3 π groups) of the 7 variables.

DIMENSIONAL ANALYSIS OF CONVECTION HEAT TRANSFER

Step 3: For each basic unit, choose a representing variable. These are known as repeating variables:

- For mass: density (ρ)
- For length: characteristic length (L)
- For Time: Velocity (U)
- For temperature: Conductivity (k)

DIMENSIONAL ANALYSIS OF CONVECTION HEAT TRANSFER

Step 4: Write the dimensionless groups by multiplying the repeating variables raised to a specific power with one of the remaining ones:

$$\pi_1 = L^{a_1} \rho^{b_1}\ U^{c_1}\ k^{d_1} \mathrm{h}$$

$$\pi_2 = L^{a_2} \rho^{b_2}\ U^{c_2}\ k^{d_2} \mu$$

$$\pi_3 = L^{a_3} \rho^{b_3}\ U^{c_3}\ k^{d_3} C_p$$

π_1, π_2 and π_3 are dimensionless groups that is they have no units $M^0 L^0 T^0 \theta^0$.

NUSSELT NUMBER

Step 5: Express the three dimensionless groups in terms of the basic units and solve for the value of the powers.

$$\pi_1 = L^{a_1} \rho^{b_1} \, U^{c_1} \, k^{d_1} \mathrm{h} = M^0 L^0 T^0 \theta^0$$

$$\pi_1 = L^{a_1} (ML^{-3})^{b_1} \, (LT^{-1})^{c_1} \left(MLT^{-3} \theta^{-1}\right)^{d_1} \left(MT^{-3} \theta^{-1}\right) = M^0 L^0 T^0 \theta^0$$

Equating the powers:

For length: $a_1 - 3b_1 + c_1 + d_1 = 0$

For mass: $b_1 + d_1 + 1 = 0$

For time: $-c_1 - 3d_1 - 3 = 0$

For temperature: $-d_1 - 1 = 0$

NUSSELT NUMBER

$$a_1 = 1$$
$$b_1 = 0$$
$$c_1 = 0$$
$$d_1 = -1$$

$$\pi_1 = L^{a_1}\rho^{b_1}\, U^{c_1}\, k^{d_1}\mathrm{h} = L^1\, k^{-1}\mathrm{h} = \frac{hL}{k} = \frac{hD}{k}$$

This dimensionless group is known as the **Nusselt Number**

REYNOLDS NUMBER

Similarly:

$\pi_2 = L^{a_2} \rho^{b_2}\ U^{c_2}\ k^{d_2} \mu = M^0 L^0 T^0 \theta^0$

$\pi_2 = L^{a_2} (ML^{-3})^{b_2}\ (LT^{-1})^{c_2}\ \left(MLT^{-3} \theta^{-1}\right)^{d_2} (ML^{-1} T^{-1}) = M^0 L^0 T^0 \theta^0$

Equating the powers:

For length: $a_2 - 3b_2 + c_2 + d_2 - 1 = 0$

For mass: $b_2 + d_2 + 1 = 0$

For time: $-c_2 - 3d_2 - 1 = 0$

For temperature: $-d_2 = 0$

REYNOLDS NUMBER

$$a_2 = -1$$
$$b_2 = -1$$
$$c_2 = -1$$
$$d_2 = 0$$

$$\pi_2 = L^{a_2}\rho^{b_2}\, U^{c_2}\, k^{d_2}\mu\ = L^{-1}\rho^{-1}U^{-1}\mu = \frac{\mu}{\rho UL} = \frac{\mu}{\rho UD}$$

Raise π_2 to a power of -1 results:

$$\pi_2 = \frac{\rho UD}{\mu}$$

This is known as **Reynolds Number**

PRANDTLE NUMBER

Similarly:

$\pi_3 = L^{a_3}\rho^{b_3}\, U^{c_3}\, k^{d_3}C_p \; = M^0L^0T^0\theta^0$

$\pi_3 = L^{a_3}(ML^{-3})^{b_3}\, (LT^{-1})^{c_3}\left(MLT^{-3}\theta^{-1}\right)^{d_3}\left(L^2T^{-2}\theta^{-1}\right) = M^0L^0T^0\theta^0$

Equating the powers:

For length: $a_3 - 3b_3 + c_3 + d_3 + 2 = 0$

For mass: $b_3 + d_3 = 0$

For time: $-c_3 - 3d_3 - 2 = 0$

For temperature: $-d_3 - 1 = 0$

PRANDTLE NUMBER

$$a_3 = 1$$
$$b_3 = 1$$
$$c_3 = 1$$
$$d_3 = -1$$

$$\pi_3 = L^{a_3}\rho^{b_3}\ U^{c_3}\ k^{d_3}C_p\ = L^1\rho^1U^1k^{-1}C_p = \frac{C_p\rho UL}{k} = \frac{C_p\rho UD}{k}$$

Divide π_3 by Reynolds Number:

$$\pi_3\ = \frac{C_p\rho Ud}{k} \times \frac{\mu}{\rho Ud} = \frac{C_p\mu}{k}$$

This is known as **Prandtle number**

DIMENSIONAL ANALYSIS OF CONVECTION HEAT TRANSFER

$$h = f(U, L, k, \mu, C_p, \rho)$$

$$f(\pi_1, \pi_2, \pi_3) = 0$$

$$\pi_1 = F(\pi_2, \pi_3)$$

$$\frac{hd}{k} = F\left(\frac{\rho U d}{\mu}, \frac{C_p \mu}{k}\right)$$

$$Nu = F(\mathrm{Re}, \mathrm{Pr})$$

REYNOLDS NUMBER

Reynold's number controls the nature of the flow in the channel i.e. ***the degree of turbulence and the flow pattern***

It is the ratio of ***inertia forces*** *tending to push the fluid forward* (ρU^2) *per unit area to the* ***viscous forces*** *tending to slow the fluid down* ($\frac{\mu U}{d}$) *per unit area.*

$$\mathrm{Re} = \frac{\rho U^2 / Area}{\mu U / d(Area)} = \frac{\rho U^2 d}{\mu U} = \frac{\rho U d}{\mu}$$

REYNOLDS NUMBER

For flow in pipes:

- Re < 2100, stable laminar flow
- Re between 2100 and 10,000, transition
- Re > 10,000, stable fully turbulent flow

PRANDTLE NUMBER

Prandtle number determines the effect of fluid properties on the heat transfer process. It can be rewritten as:

$$\Pr = \frac{\mu C_p / \rho}{k / \rho} = \frac{\mu / \rho}{k / \rho C_p} = \frac{\nu}{\alpha}$$

ν =μ/ρ is the kinematic viscosity which controls the diffusion of momentum and $\alpha = k/\rho C_p$ controls the diffusion of thermal.

Thus Pr is the ratio of molecular diffusivities of momentum and heat.

Prandtle Number and BCs

1. For the nonmetallic fluids (Pr ≥1), convection heat transfer predominates and h is not very sensitive to BCs. In the liquid metals case (Pr < 0.4), where heat transfer by conduction is important, h is sensitive to BC and channel shape.
2. Similar to laminar flow case, direct use of the equivalent diameter is not satisfactory for liquid metal cases.

$$D_e = \frac{4A_f}{P_w}$$

3. As long as the heat conduction near the wall is important, the equivalent diameter concept become less useful.

Prandtl number, viscous and thermal boundary layers

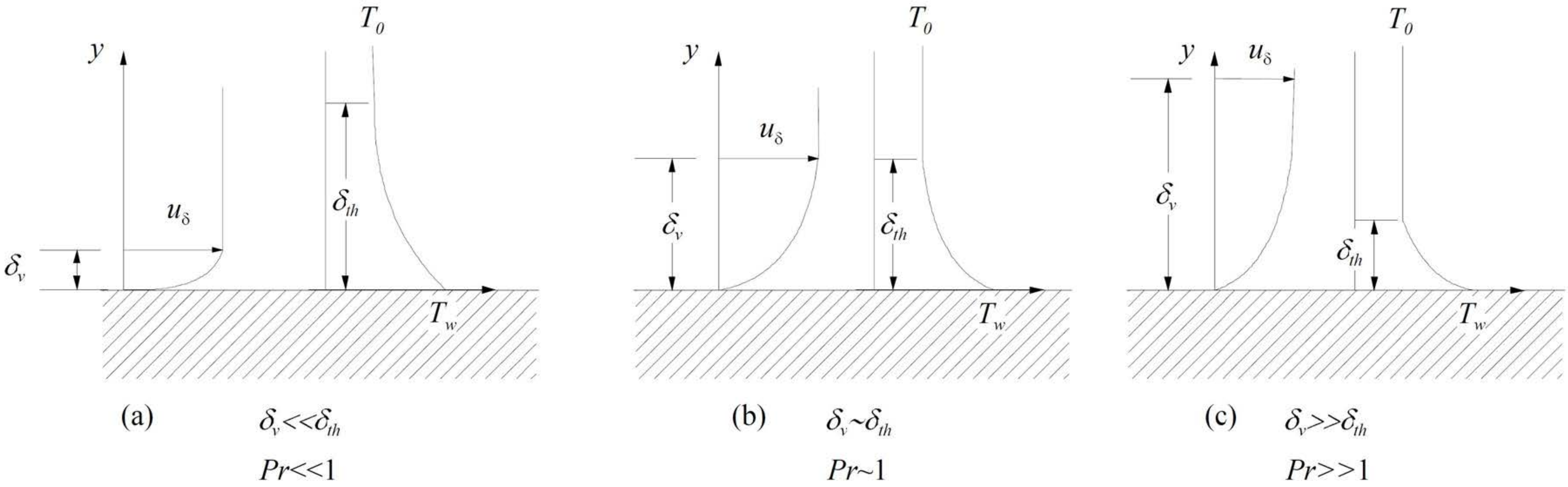
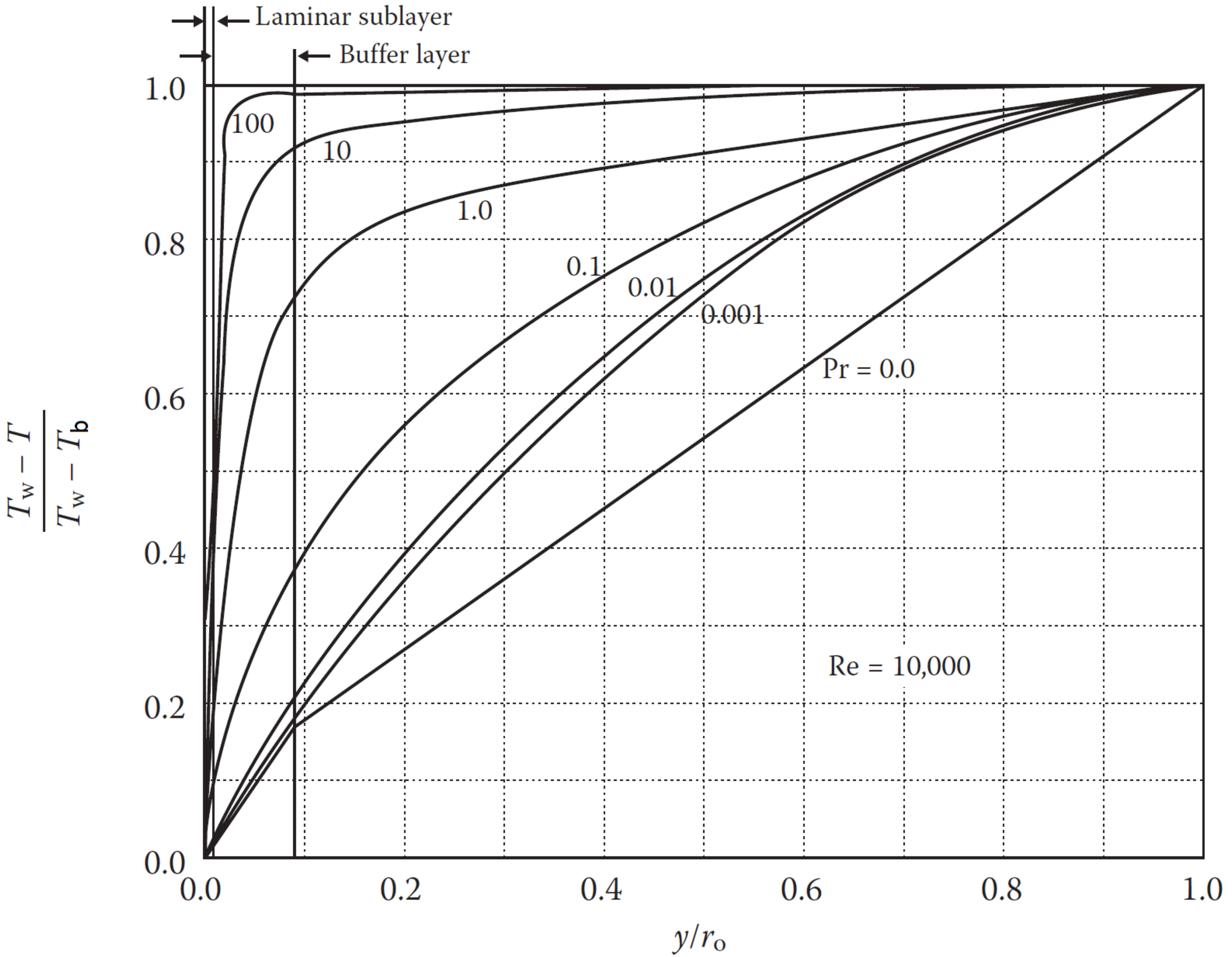


Illustration of the influence of the Prandtl number on the magnitude of the viscous and thermal boundary layers in a two-dimensional flow over plate with the constant wall temperature Tw



the temperature distribution in a uniformly heated round tube.

PECLET NUMBER

Peclet Number (**Pe**) is the product of **Re** and **Pr**

$$Pe = Re \times Pr = \frac{U\rho C_p L}{k}$$

$$h = f(U, L, k, \mu, C_p, \rho)$$

It describes the ratio of heat transfer by convection to heat transfer by conduction

$$Pe = \frac{\text{heat transfer by convection}}{\text{heat transfer by conduction}}$$

Example 4.3: Importance of Terms in the Energy Equation under Various Flow Conditions

Consider the following two flow conditions in a pressurized water reactor steam generator on the primary side.

	Forced Flow	**Natural Circulation**
Flow per tube	1.184 kg/s	0.01184 kg/s
Characteristic temperature difference	15°C	25°C

For a tube of inner diameter 0.0222 m (7/8 in.) and an average temperature of 305°C, evaluate the various dimensionless parameters in the **dimensionless form of energy equation** and determine which terms are important under both flow conditions.

Example 4.3: Importance of Terms in the Energy Equation under Various Flow Conditions

Under the follow approximations:

1. Incompressible fluids
2. No heat generation in coolant
3. The material properties are temperature independent and pressure independent
4. The heat flux from wall is due to conduction alone

The energy transport equation:

$$\rho c_p \frac{DT}{Dt} = k\nabla^2 T + \mu\phi'$$

can be transform to dimensionless form. Where $\mu\phi'$ is the dissipation function. It’s describes the energy dissipation due to, e.g. internal friction.

Example 4.3: Importance of Terms in the Energy Equation under Various Flow Conditions

The energy equation in dimensionless form is:

$$\frac{\partial T^*}{\partial t^*} + \vec{\upsilon}^* \cdot \nabla * T^* = \frac{1}{\mathrm{Re}\,\mathrm{Pr}} \nabla^{*2}\, T^* + \frac{\mathrm{Br}}{\mathrm{Re}\,\mathrm{Pr}} \phi^*$$

$$\mathrm{Re} = \frac{\rho V D}{\mu}$$

$$\mathrm{Pr} = \frac{\mu c_p}{k}$$

$$\mathrm{Br} = \frac{\mu V^2}{k(T_\ell - T_o)}$$

Br is called the Brinkmann number, is the ratio of heat production by viscous dissipation to heat transfer by conduction.

Solution

Evaluating ρ, μ, c_p, and k for saturated water at 305°C, we find

$\rho = 701.9\ \text{kg/m}^3$
$\mu = 8.9 \times 10^{-5}\ \text{kg/m s}$
$k = 0.532\ \text{W/m°C}$
$c_p = 5969\ \text{J/kg°C}$

For the forced flow condition,

$$V = \frac{\dot{m}}{\rho A} = \frac{1.184\ \text{kg/s}}{(701.9\ \text{kg/m}^3)\dfrac{\pi}{4}\left[(0.875\ \text{in.})\dfrac{0.0254\ \text{m}}{\text{in.}}\right]^2} = 4.348\ \text{m/s}$$

Solution

so that

$$\text{Re} = \frac{(701.9\ \text{kg/m}^3)(4.348\ \text{m/s})(0.0222\ \text{m})}{8.90 \times 10^{-5}\text{kg/m s}} = 7.613 \times 10^5$$

$$\text{Pr} = \frac{(8.90 \times 10^{-5}\text{kg/m s})(5969\ \text{J/kg °C})}{0.532\text{W/m °C}} = 1.00$$

$$\text{Br} = \frac{(8.90 \times 10^{-5}\text{kg/m s})(4.348\ \text{m/s})^2}{(0.532\ \text{W/m °C})(15\text{°C})} = 2.11 \times 10^{-4}$$

Solution

$$\frac{\partial T^*}{\partial t^*} + \vec{\upsilon}^* \cdot \nabla * T^* = \frac{1}{\mathrm{Re}\,\mathrm{Pr}} \nabla^{*2}\, T^* + \frac{\mathrm{Br}}{\mathrm{Re}\,\mathrm{Pr}} \phi^*$$

Then

$$\frac{1}{\mathrm{Re}\,\mathrm{Pr}} = \frac{1}{\mathrm{Pe}} = \frac{1}{(7.613 \times 10^5)(1.00)} = 1.314 \times 10^{-6}$$

$$\frac{\mathrm{Br}}{\mathrm{Re}\,\mathrm{Pr}} = \frac{\mathrm{Br}}{\mathrm{Pe}} = (2.11 \times 10^{-4})(1.314 \times 10^{-6}) = 2.773 \times 10^{-9}$$

Thus in the forced-flow condition, these parameter values are sufficiently low that the conduction term and the dissipative term in the energy equation **can often be ignored** in the energy balance.

Solution

In the **natural circulation condition**, the velocity is two orders of magnitude lower than in the forced flow case, so that

$$\mathrm{Re} = 7.613 \times 10^{3}$$

$$\mathrm{Pr} = 1.00$$

$$\mathrm{Br} = (2.11 \times 10^{-4})(1 \times 10^{-4})\left(\frac{15}{25}\right) = 1.266 \times 10^{-8}$$

Then

$$\frac{1}{\mathrm{Pe}} = \frac{1}{\mathrm{Re\,Pr}} = 1.314 \times 10^{-4}$$

$$\frac{\mathrm{Br}}{\mathrm{Pe}} = \frac{\mathrm{Br}}{\mathrm{Re\,Pr}} = (1.266 \times 10^{-8})(1.314 \times 10^{-4}) = 1.664 \times 10^{-12}$$

Solution

$$\frac{\partial T^*}{\partial t^*} + \vec{\upsilon}^* \cdot \nabla^* T^* = \frac{1}{\mathrm{Re\,Pr}} \nabla^{*2}\, T^* + \frac{\mathrm{Br}}{\mathrm{Re\,Pr}} \phi^*$$

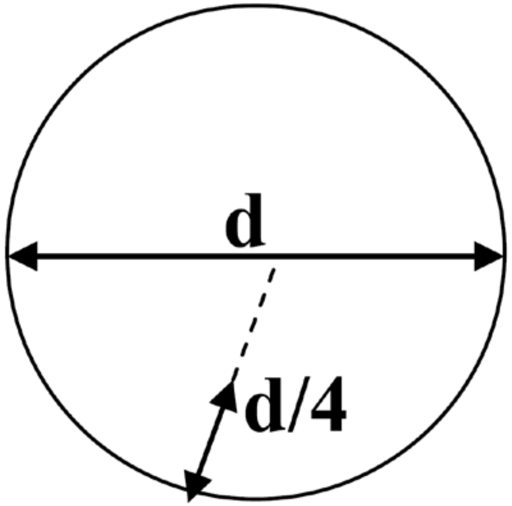
To sum up:

1. The dissipation energy term can always be ignored, and the conduction term can often be ignored.
2. In the case of sodium-cooled breeder reactors, for example, the high thermal conductivity of sodium results in a Pr number of the order 0.004. In natural circulation conditions, $1/\mathrm{Pe} \approx 1$, and conduction may become important

NUSSELT NUMBER FOR LAMINAR FLOW

In laminar flow, conduction occurs across much of the flow cross section, roughly a distance of d/4 in circular pipes. The Fourier law gives the heat flux, which in turn equal to hΔT.

$$q = \frac{k(T_w - T_b)}{d/4} = h(T_w - T_b) \qquad \Rightarrow \qquad \frac{k}{h} = \frac{d}{4}$$



$$Nu = \frac{hd}{k} = \frac{d}{d/4} = 4$$

More accurate calculations showed that Nu = 4.36 if q’ was maintained constant and 3.66 if T_w was maintained constant.

NUSSELT NUMBER FOR LAMINAR FLOW

Nusselt Number for Laminar Fully Developed Velocity and Temperature Profiles in Tubes of Various Cross Sections

Cross-Sectional Shape	b/a	Nu^a $q'' =$ Constant	Nu^a $T_w =$ Constant
Circle	—	4.364	3.66
Square	1.0	3.61	2.98
Rectangle	1.43	3.73	3.08
Rectangle	2.0	4.12	3.39
Rectangle	3.0	4.79	3.96
Rectangle	4.0	5.33	4.44
Rectangle	8.0	6.49	5.60
Infinite parallel plates	∞	8.235	7.54
Isosceles triangle	—	3.00	2.35

Source: From Kays, W. M. and Crawford, M. E., *Convective Heat and Mass Transfer*, 2nd Ed. New York, NY: McGraw-Hill, 1980.

NUSSELT NUMBER FOR TURBULENT FLOW

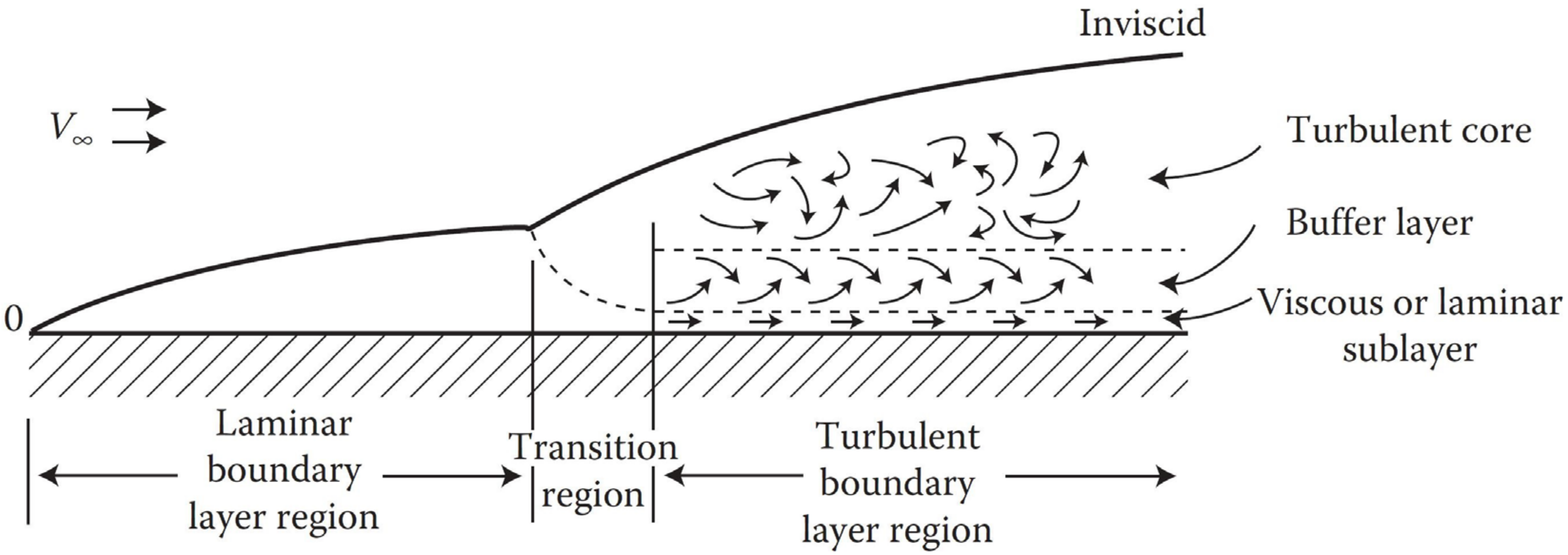
In turbulent flow, conduction occurs through the laminar sublayer that has an average thickness of δ.

$$q = \frac{k(T_w - T_b)}{\delta} = h(T_w - T_b) \quad \Rightarrow \frac{k}{h} = \delta \text{ and } Nu = \frac{hd}{k} = \frac{d}{\delta}$$

$$Nu = \frac{\text{characteristic dimension of the flow}}{\text{thickness of the laminar sublayer across which conduction occurs}}$$

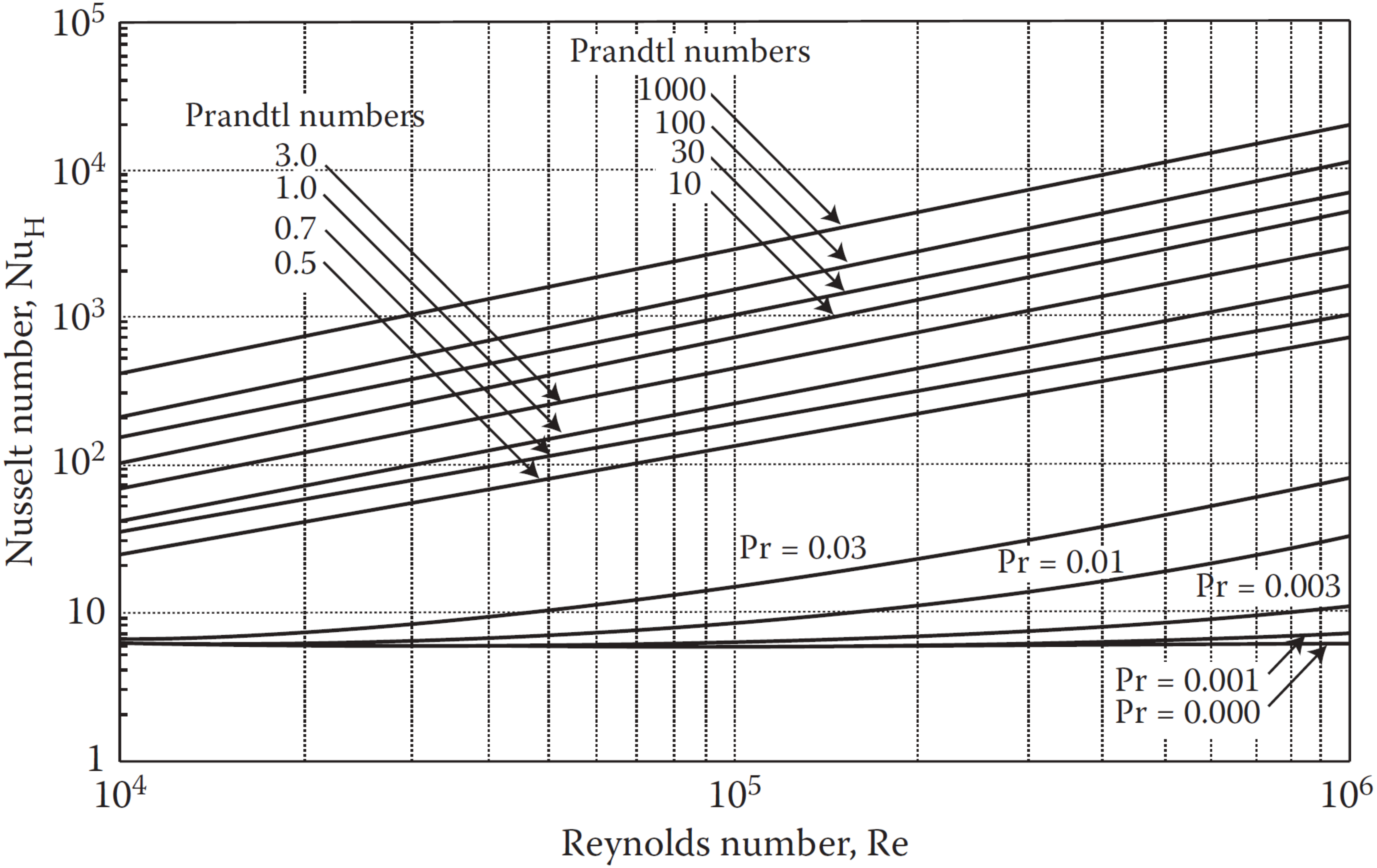
$$Nu = \frac{\text{Temperature gradient at wall}}{\text{Overall temperature difference}}$$

Viscous/velocity Boundary layer velocity distribution for flow on an external surface

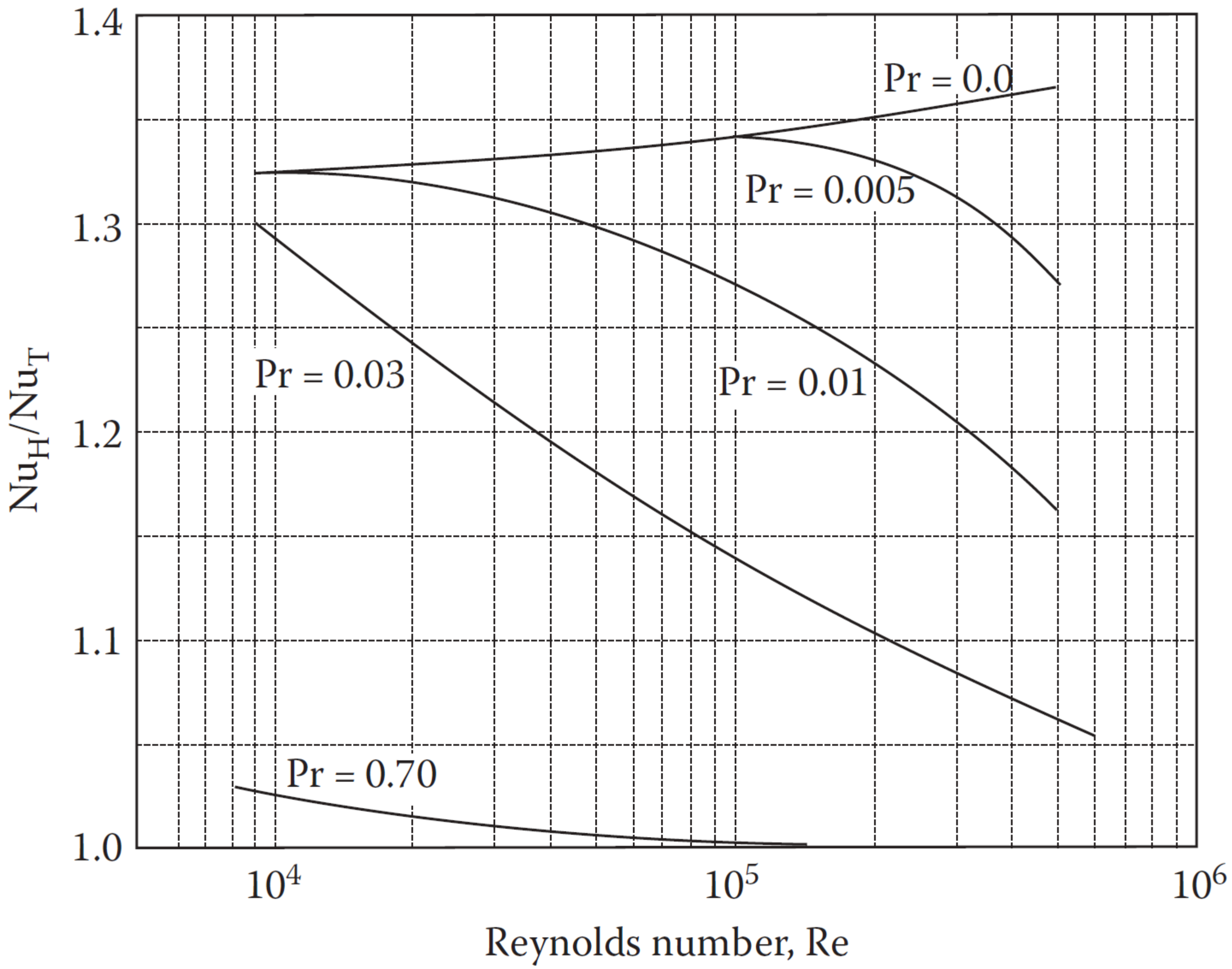


Boundary layer velocity distribution for flow on an external surface.

NUSSELT NUMBER FOR TURBULENT FLOW



Nu Under Different BCs for fluid with different **Pr** in fully developed conditions in a circular tube



- It is seen that the Nusselt number for constant q'_w is higher than that for constant *T*w but that the difference is significant only for *Pr* < 0.7.
- Thus, for metallic fluids, the wall boundary condition significantly affects the turbulent Nu number;
- But for nonmetallic fluids the Nu number is practically independent of the boundary condition.

Single-phase Heat Transfer

Heat Transfer Correlations In Turbulent Flow

Non-metallic Fluids Fully Developed Turbulent Flow on Smooth Heat Transfer Surfaces

Both experiment and theory show that for almost all nonmetallic fluids

$$\mathrm{Nu}_{\infty} = C\mathrm{Re}^{\alpha}\mathrm{Pr}^{\beta}\left(\frac{\mu_w}{\mu}\right)^{\kappa}$$

Nu_{∞} is the Nusselt number for fully developed flow. μ_w is the fluid viscosity at $T = T_w$; μ is the fluid viscosity at $T = T_b$; C, α, β, and κ are constants that depend on the fluid and the geometry of the channel.

When $T_w - T_b$ is not very large, $\mu_w \approx \mu$

$$\mathrm{Nu}_{\infty} = C\mathrm{Re}^{\alpha}\mathrm{Pr}^{\beta}$$

DITTUS–BOELTER CORRELATION

For cases when $\mu \approx \mu_w$ the Dittus–Boelter/McAdams correlation is the most universally used correlation. It is recommended by McAdams for both heating and cooling at moderate $T_w - T_b$ differences.

$$\mathrm{Nu}_\infty = 0.023\ \mathrm{Re}^{0.8}\ \mathrm{Pr}^{0.4}$$

for $0.7 < \mathrm{Pr} < 100$, $\mathrm{Re} > 10{,}000$, and $L/D > 60$. All fluid properties are evaluated at the mean bulk temperature when the correlation is used on a local basis. When used on an average basis, the arithmetic mean bulk temperature (i.e., the average of the bulk inlet and outlet temperature) is good for constant heat flux while the log mean bulk temperature applies for the constant wall temperature boundary condition.

Annuli and Noncircular Ducts

The same relations for the circular tube are used for annuli and non-circular ducts by employing the concept of the hydraulic diameter:

$$D_e = \frac{4A_f}{P_w}$$

for the Reynolds number.

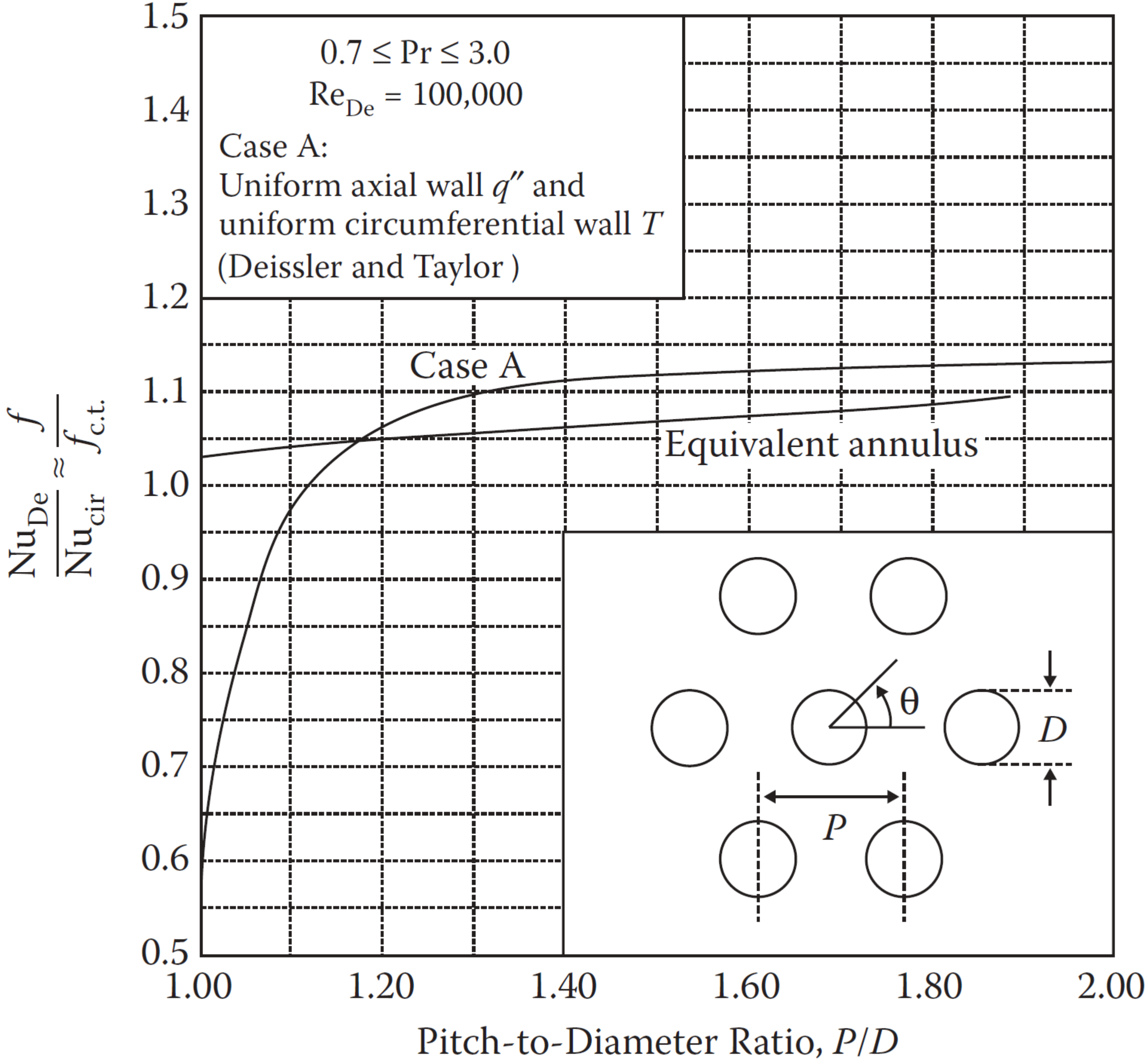
However, for geometries far from circular, this method may not be satisfactory, particularly in the regions of sharp corners.

Fully Developed Turbulent Flow For *Rod Bundles*

1. For fully turbulent flow along rod bundles, Nu values may significantly deviate from the circular geometry because of the strong geometric nonuniformity of the subchannels.

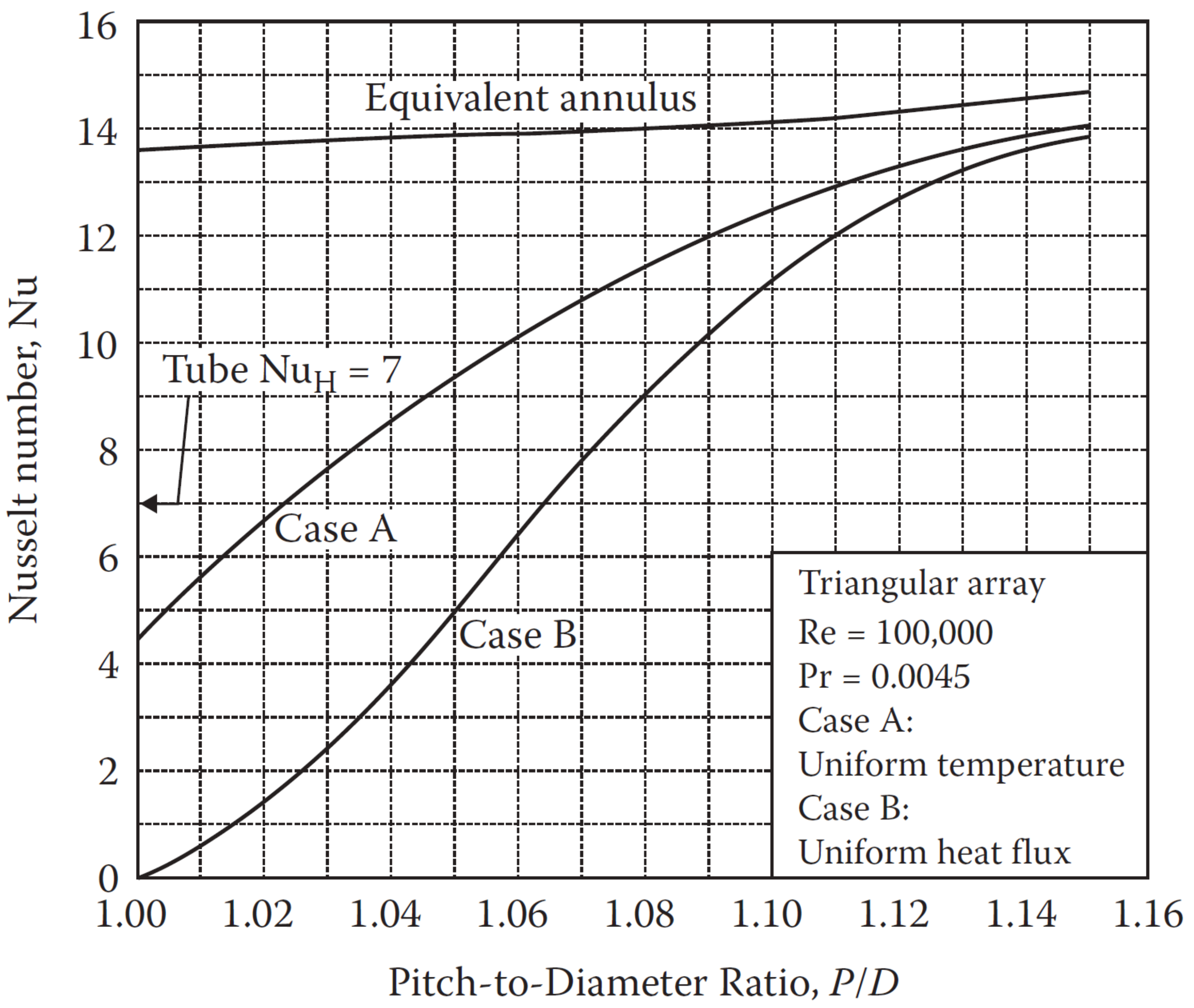
2. However, the value of Nu is insensitive to the BCs for $Pr > 0.7$

3. And for $P/D \geq 1.12$, the Nu predictions are accurate to within ±10% for both large and small Pr numbers.



1. Fully developed turbulent flow parallel to a bank of circular tubes or rods.

2. Reynolds number influence is small, and Nusselt number behavior is virtually the same as friction behavior.



Variation of Nusselt number for fully developed turbulent flow with rod spacing for Prandtl number < 0.01.

Fully Developed Turbulent Flow For Infinite Array

The usual way to represent the relevant correlation is to express the Nusselt number for fully developed conditions (Nu_{∞}) as a product of $(Nu_{\infty})_{c.t}$ for a circular tube multiplied by a correction factor:

$$\mathrm{Nu}_{\infty} = \psi(\mathrm{Nu}_{\infty})_{\mathrm{c.t.}}$$

where $(Nu_{\infty})_{c.t}$ is usually given by the Dittus–Boelter equation unless otherwise stated.

Fully Developed Turbulent Flow For Infinite Array (Presser correlation)

The problem is then formulated as the evaluation of Ψ.

For the triangular array and $1.05 \leq P/D \leq 2.2$:

$$\psi = 0.9090 + 0.0783\, P/D - 0.1283\ e^{-2.4(P/D-1)}$$

For the square array and $1.05 \leq P/D \leq 1.9$:

$$\psi = 0.9217 + 0.1478 P/D - 0.1130\ e^{-7(P/D-1)}$$

Metallic Fluids Fully Developed Flow in Smooth Heat Transfer Surfaces

The behavior of the **Nu** for liquid metals follows the relation

$$\mathrm{Nu}_\infty = A + B\ (\mathrm{Pe})^{\mathrm{C}}$$

Where Pe is the *Peclet number. A, B*, C are constants that depend on the geometry and the boundary conditions. The constant C is a number close to 0.8. The constant *A* reflects the fact that significant heat transfer by conduction in liquid metals occurs even as Re goes to zero.

Metallic Fluids Fully Developed Flow in Smooth Heat Transfer Surfaces

For Circular Tube, the following relations hold for the boundary conditions cited and fully developed flow conditions.

1. Constant heat flux along and around the tube (Lyon):

$$Nu_{\infty} = 7 + 0.025\ Pe^{0.8}$$

2. Uniform axial wall temperature and uniform radial heat flux (Seban and Shimazaki):

$$Nu_{\infty} = 5.0 + 0.025\ Pe^{0.8}$$

Metallic Fluids Fully Developed Flow in Smooth Heat Transfer Surfaces

For Rod Bundles, the nonuniformity of the subchannel shape creates substantial azimuthal variation of Nu. Therefore, the value of Nu is a function of position within the bundle.

However, for $P/D > 1.1$, the equivalent annulus concept provides an acceptable answer. Graber and Rieger correlation:

$$\mathrm{Nu} = 0.25 + 6.2\left(P/D\right) + \left[-0.007 + 0.032\left(P/D\right)\right]\mathrm{Pe}^{0.8-0.024\left(P/D\right)}$$

for $1.25 \le P/D \le 1.95$ and $150 \le \mathrm{Pe} \le 3000$.

Other correlation for Metallic Fluids Fully Developed Flow in Smooth Heat Transfer Surfaces

Kazimi and Carelli:

$$\mathrm{Nu} = 4.0 + 0.33(P/D)^{3.8}(\mathrm{Pe}/100)^{0.86} + 0.16(P/D)^{5.0}$$

for $1.1 \le P/D \le 1.4$ and $10 \le \mathrm{Pe} \le 5000$.

Example 4.4 Typical Heat Transfer Coefficients of Water and Sodium

Liquid flow velocity along rod bundles is limited principally to avoid rod vibration and possible cavitation at obstructions. Selecting a maximum velocity of 10 m/s, compare the heat transfer coefficient of water (at an average coolant temperature for a PWR) to the heat transfer coefficient of sodium (at an average temperatures for an SFBR). How do the coefficients compare at a velocity of 1 m/s?

Note: the pitch and diameter equals to 12.6 mm and 9.5 mm for PWR (square array), and 9.8 mm and 8.5 mm for SFBR (triangular array), respectively.

Example 4.4 Typical Heat Transfer Coefficients of Water and Sodium

Property Data

Parameter	Water (315°C)	Sodium (538°C)
k (W/m°C)	0.5	62.6
ρ (kg/m^3)	704	817.7
μ (kg/m s)	8.69×10^{-5}	2.28×10^{-4}
c_p (J/kg°C)	6270	1254

Example 4.4 Solution

	h(kW/m^2 °C)	
	Water	Sodium
Velocity (m/s)	Presser	Kazimi and Carelli
10	68.2	113.1
1	10.8	74.6