Quiz
Phys 411 2012
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Name: _____

Student Number: _____

This quiz is 50 minutes long, closed-book, calculators allowed.
There are 4 questions over 4 pages, each question is equal weight.

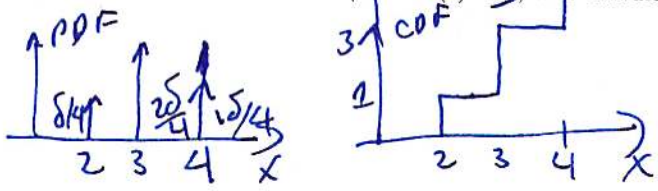
Section 1. Short Answers

1. Imagine we have two coins, each labeled "1" on one side, and "2" on the other. A *time series* $x_k(t)$ is generated by randomly flipping one of the two coins alternately every 3 s and adding the results For example, a realization of this process might be that at $t = 0$ the coins might read $(1, 1)$, and $x(t) = 2$. At time $t = 3$, the first coin is not flipped, and the second one is, and happens to flip to "2", so the state is $(1, 2)$, and $x(t) = 3$; at time $t = 6s$, the first coin is flipped, and happens to flip to "2", so $(2, 2)$ and $x(t) = 4$. (Again, note, this is just an example, and each coin flip should be random, so other realizations may differ).

a) Sketch the probability density function, and the cumulative distribution function of $x(t)$.

b) Demonstrate that the mean of $x(t) = 3$, and ~~calculate~~ the variance of $x(t)$.

a)



b) $\text{mean} = \int_{-\infty}^{\infty} p(x)\, x\, dx = \frac{2 + 2\cdot 3 + 4}{4} = 3$

$\text{var} = \int_{-\infty}^{\infty} p(x)\,(x-3)^2\, dx = \frac{1}{4}\left((-1)^2 + 2(0)^2 + (1)^2\right)$

$= 1/2$

3. If a digital time series $x(t_n) = x_n$ has N samples, demonstrate that $X_m^* = X_{N-m}$, where $m < N/2$, and X_m is the m-th Fourier co-efficient of a orthogonal Fourier decomposition.

$$X_{n-m} = \sum_{n=0}^{\infty} x_n e^{-j2\pi \frac{(N-m)}{N} n}$$

$$= \sum x_n e^{-j2\pi} e^{+j2\pi \frac{m}{N} n}$$

$$= \sum x_n e^{j2\pi mn/N}$$

$$= X_m^*$$

V^2 1000

4. a) Quantitatively graph the spectrum $G_{xx}(f_{k)})$ of ~~N~~ samples of a Gaussian white-noise process $x(t)$ with mean of zero, and variance $\sigma_x^2 = 5$ sampled at 1 Hz.

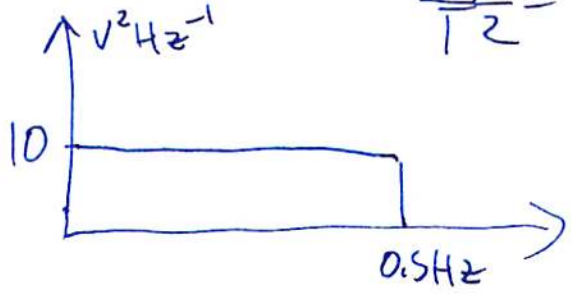
   b) Do the same for $G_{yy}(f)$, if $y_n = 0.1 \sum_{k=0}^{9} x_{n-k}$ (you can just sketch, but make sure the sketch is as accurate as you can make it).

a) note $\sum G_{xx}\, \Delta f = \text{var}(x)$

$= \frac{A N}{T_2} = \text{var}(x)$

$\Rightarrow A = T \text{var} x 2$

$= \frac{N \cdot (1_s)\, \text{var} x 2}{N}$



b) $G_{YY} = |H|^2 G_{XX}$

where H is sinc fcn representy de filter, or linear systemy it has a zero at 5 samples $\Rightarrow f = \frac{1}{5}$ Hz and then a second zero at $\frac{2}{5}$ Hz