Assignment 2
Phy 426, 2017
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Question 1. Accelerating tank

A rectangular tank is placed on wheels and is given a constant acceleration a.

1. Show that, at steady state, the angle made by the free surface with the horizontal is given by $tan\theta = a/g$.
2. What angle does the surface make if the tank is cylindrical?

Question 2. Draining tank

A hemispherical vessel of radius R has a small rounded orifice of area A at the bottom.

1. Show that the time required to lower the level from h_1 to h_2 is given by

$$t = \frac{2\pi}{A\sqrt{2g}}\left[\frac{2}{3}R\left(h_1^{3/2} - h_2^{3/2}\right) - \frac{1}{5}\left(h_1^{5/2} - h_2^{5/2}\right)\right] \tag{1}$$

Question 3. Viscous flow down an incline

Consider steady viscous flow down a shallow incline given by dh/dx. Assuming the incline is infinite. Assume a constant viscosity and that gravity g points downwards.

1. Derive an expression for the flow velocity as a function of z', where z' is the distance perpendicular to the incline.
2. Show that the viscous dissipation in the fluid balances the rate of loss of potential energy of the flow. (Hint: integrate the equation for the conservation of energy over a volume, and note that $\int_V \mathbf{u}\cdot\nabla R\; dV = \oint_A R\mathbf{u}\cdot d\mathbf{A}$. Most of the terms from the upstream side of this integral cancel the downstream, except for a potential energy term).

Question 4. Spin-down of an eddy

Consider an eddy in solid body rotation, or radius R and rotation rate $\frac{1}{2}\omega_0$, in water depth H. If the bottom stress is parameterized as *linear* drag (i.e. $\tau/\rho(z=-H) = -k\mathbf{u}$, where $k > 0$ is the drag co-efficient), then:

1. Show that the eddy remains in solid body rotation.

2. How long does it take for the eddy to slow down so that $\omega = e^{-1}\omega_0$?

3. Show that the work done by the bottom drag is equivalent to the rate of the loss of kinetic energy of the eddy. (Note that this is a approximation. In real life, the eddy changes potential energy as well, because it needs a different parabolic shape at the surface, and we are ignoring that term here).