Take-home Final
Phy 426, 2017
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DUE: Thu 11 Apr, 2017, 1000 Exam to be completed independently. Open book, open notes are fine. Show all work, define any constants you need that I don't provide, check your units, etc. Except as noted, the density of the fluid is ρ, gravity is g, the kinematic viscosity ν, and the fluid can be assumed Bousinesque and incompressible.

Please try to make it readable. I will deduct up to 10% for illegible chicken scratches, so please take the time to recopy your work.

The value of each question is indicated in square brackets, the total is out of 75.

Question 1. Viscous flow around a rotating cylinder

Consider an infinite viscous fluid with a solid cylinder of radius a spinning with rotation rate Ω:

1. [7] Show that the velocity of the fluid is given $u_{theta} = \Omega a^2/r$, where r is the distance from the axis of rotation of the cylinder.
2. [10] Show that the rate of work done by the cylinder on the flow is equal to the viscous dissipation in the flow.
3. [8] *Sketch* what happens to the velocity in time field if the cylinder suddenly stops rotating. Indicate an appropriate relationship between the spatial and temporal scales in your sketch.

Question 2. Lossy standing waves

Consider the quasi-steady response in a rectangular basin H deep, W wide, with a vertical wall at one end to forcing at a tidal frequency ω.

1. [4] A distance L from the far end, the pressure is measured with a gauge to vary as $p = p_o \cos \omega t$. Assuming no energy losses in the basin, what is the water height as a function of x, and t?
2. [4] What would the functional dependence of $u(x, t)$ be?
3. [10] Suppose we measure u at $x = L$ from the far end, and it is given my $u = u_o \sin(\omega t + \phi)$. what is the average rate of energy loss inside the embayment in terms of p_o and u_o?
4. [7] If $\phi << 1$, $p_o << \rho g H$, and $L << \frac{2\pi\sqrt{gH}}{\omega}$, approximate the above just in terms of p_o and ϕ.

Question 3. Flow around a Headland

Consider a headland protruding into a rectangular channel of free-stream depth H. The headland is approximated by a circle with radius R. Assume that R is much less than the width of the channel, and that the free-stream speed far from the headland is U along the channel. The flow separates from the headland at the tip.

1. [4] What is the fastest speed along the headland?
2. [4] Assume that the pressure in the separation bubble behind the tip is the same as at the tip. What is the total drag on the flow exerted by the headland?
3. [4] Assuming that the separation vortex is a Rankine vortex with a radius similar to the headland (i.e. approximately R) and outer speed given by the speed of the flow past the headland, what does the surface of the water look like?
4. [4] Suppose the mean flow suddenly turns off. Where and how fast will this eddy start to move?
5. [5] Suppose the eddy travels around, with its vorticity core largely intact, and that it slowly shoals into water h deep, where $h < H$. Describe the surface height as a function of distance from the centre of the eddy at this point.