Assignment 4
Phy 426, 2017
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Question 1. Zhukhovski Airfoil

Using the Zhukhovski Transform, numerically demonstrate the lift law for an airfoil.

1. For suitable choice of an offset in the ζ-plane, transform a circle into an airfoil in the z-plane. Note that for full marks it should look like an airfoil!
2. Show the streamlines for a flow with mean speed U and circulation $-\Gamma$, where you chose the appropriate values for those constants around the sphere and airfoil. Note that you should chose Γ to satisfy the Kutta condition. (HINT: you want to define your cylinder in the ζ-plane, but you want to plot your streamlines in the z-plane where the cylinder is an airfoil. So on an even grid in x and y, for each value of $z = x + iy$ determine the value of ζ, and then determine the value of $\psi(z)$ from its value in ζ space.)
3. Repeat the above for two other angles of attack α, where you chose α.
4. Calculate and plot the pressure field (up to an arbitrary constant) around the circle and airfoil. ($|\mathbf{u}(z)|=|dw/dz| \approx |dw/dx|$ which you can calculate from first difference from your grid (or analytically for the cylinder) Then you need to evaluate on the edge of the body.).
5. Calculate the lift from the pressure field for all three values of the angle of attack. Compare to the Zhukovski lift law.

Question 2. Blasius profile

Numerically solve Blasius' equation:

$$\frac{1}{2} f \frac{d^2 f}{d\eta^2} + \frac{d^3 f}{d\eta^3} = 0 \tag{1}$$

subject to the boundary conditions $f(0) = f'(0) = 0$ and $f(\infty) = 1$. (HINT: make sure you let η go "high enough" so that "infinity" is suitably far from $\eta = 0$. Here "numerically" means to use a shooting method with Newton-Raphson method to ensure rapid convergence. For full marks, do *not* use a built in optimizer (though you are welcome to check your work with one)).

1. Plot the resulting *velocity* profile as a function of y at three different times.
2. Contour streamlines ψ in $x - y$ space. (HINT: to get good looking streamlines for small x I needed to extend my solution $f(\eta)$ to very large η. Rather than doing this numerically, I note that $df/f\eta$ is approximately constant, so the extrapolation is just linear.)
3. Contour v and show that it approaches a constant as $y/\delta >> 1$.