

v



1)

if we accelerate tank water  
 in tank needs P. G. F. to  
 accelerate it. water piles up  
 on upstream side

$$\frac{\partial u}{\partial t} = a = -\frac{1}{\rho} \frac{\partial p}{\partial x}$$

$$\therefore -g \frac{\partial \eta}{\partial x} = a$$

$$\Rightarrow \tan \theta = \frac{a}{g}$$

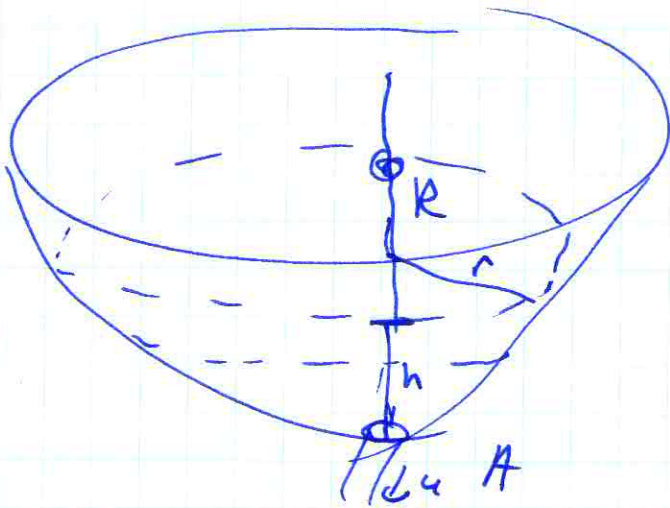
2) If tank is cylindrical?

No difference. Tank still water

still needs to have  $\frac{\partial \eta}{\partial x} = -\frac{a}{g}$

## Q2 Draining Tank

Assy 2 Lang



$$uA = \frac{dh}{dt} \pi r^2$$

$$r^2 + (R-h)^2 = R^2$$

$$r^2 - 2Rh + h^2 = 0$$

$$r^2 = 2Rh - h^2$$

$$\frac{P_{atm}}{\rho} + \frac{u^2}{2} = \frac{P_{atm}}{\rho} + \left(\frac{dh}{dt}\right)^2 \frac{1}{2} + gh$$

assume small

$$u^2 = 2gh$$

$$u = \sqrt{2gh}$$

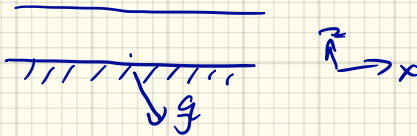
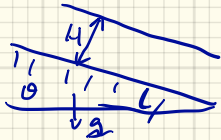
$$\text{so } \frac{dh}{dt} \left[ \frac{2Rh - h^2}{h^{3/2}} \right] = \frac{\sqrt{2g} A}{\pi}$$

$$\int_{h_1}^{h_2} 2R h^{1/2} - h^{3/2} = \frac{\sqrt{2g} A}{\pi} (t_2 - t_1)$$

$$\left. \frac{4R}{3} h^{3/2} - \frac{2h^{5/2}}{5} \right|_{h_1}^{h_2} = \frac{\sqrt{2g} A}{\pi} \Delta t$$

$$\therefore \Delta t = \frac{2\pi}{A\sqrt{2g}} \left[ \frac{2R}{3} (h_1^{3/2} - h_2^{3/2}) - \frac{1}{5} (h_1^{5/2} - h_2^{5/2}) \right]$$

A63 #2 Q3



$g' = g \sin \theta$  acts to accelerate flow.  
 $= g \frac{\partial h}{\partial x}$

x-momentum:

$$\frac{Du}{Dt} = g \sin \theta - \frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \frac{\partial^2 u}{\partial z^2}$$

B.C.:  $u = 0$  at  $z' = 0$

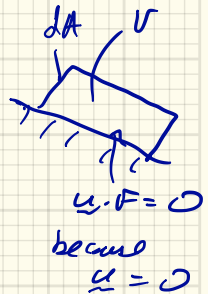
$\frac{\partial u}{\partial z} = 0$  at  $z' = H$

$$\Rightarrow u = \frac{g'}{2} z' \left( H - \frac{z'}{2} \right)$$

$$Q = \int_0^H u dz' = \frac{g'}{3} H^3$$

Energy Drop?

$$\frac{DR}{Dt} = \underbrace{\phi}_{\text{dissipation}} + \underbrace{u \cdot \underline{F}}_{\text{stress}} + \underbrace{\frac{1}{\rho} \frac{\partial p}{\partial x}}_{\text{steady}}$$



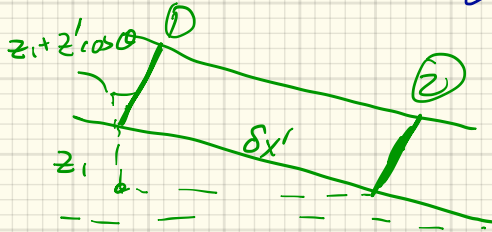
Integrate around volume

$$\int u \cdot \underline{R} dV = \int \phi dV$$

$$\int u \cdot \underline{R} \cdot d\underline{A} = \text{Dissipation.}$$

By choosing volume at top + bottom of flow, only the in-flow + out-flow contribute

$$\oint \mathbf{R} \cdot d\mathbf{r} = \int_1 \mathbf{R} \cdot d\mathbf{z}' + \int_2 \mathbf{R} \cdot d\mathbf{z}'$$



$$z_1 = \delta x' \sin \theta$$

$$= \int_0^H \left( \cancel{\frac{1}{2} u^3} + \cancel{\frac{u p}{\rho}} + \underline{u g z} \right) dz' \Big|_1 - \int_0^H \left( \cancel{\frac{1}{2} u^3} + \cancel{\frac{u p}{\rho}} + \underline{u g z} \right) dz' \Big|_2$$

Note:  $z$  is real  $z$ ,  $g$  is real  $g$ . and these are different at ① + ②

$$= \int_0^H u g (z_1 + z' \cos \theta) dz' - \int_0^H u g (z' \cos \theta) dz'$$

$$= g \delta x' \sin \theta \int_0^H \frac{1}{5} H^3 / 3 = \delta x' \frac{g}{5} H^3 / 3$$

Dissipation??

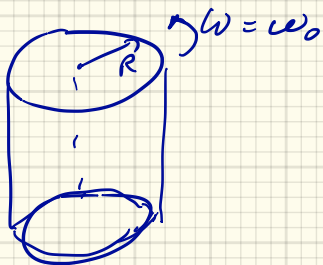
$$= \delta x' \nu \int_0^H \left( \frac{\partial u}{\partial z'} \right)^2 dz'$$

$$= \delta x' \frac{g'^2}{\nu} \int_0^H (H - z')^2 dz'$$

$$= \delta x' \frac{g'^2}{\nu} \frac{H^3}{3}$$

So loss of P.E. = dissipation  
in flow.

## Assign 2 Q:4



Remains in solid body rotation?

Debye-interacted momentum  
says that

$$\hbar \frac{\partial u}{\partial t} = \frac{\tau}{\rho} = -k u$$

$r \rightarrow$  component

$$\hbar \frac{\partial u_r}{\partial t} = -k u_r$$

But  $u_r = 0$  so  $\frac{\partial u_r}{\partial t}$  remains 0

$\theta \rightarrow$  component

$$\hbar \frac{\partial u_\theta}{\partial t} = -k u_\theta$$

$$\Rightarrow u_\theta = u_{\theta 0} e^{-\frac{k t}{\hbar}}$$

so if  $u_{\theta 0}$  is solid  
body

$u_\theta$  is solid-body

2) Timescale is simply given by  
 $\frac{H}{k}$

3) Work = loss KE?

$$\text{Work} = \int_A \underline{\underline{z}} \cdot \underline{\underline{u}} \, dA$$

at bottom

$$= \int_A -k u_0^2 \, dA$$

$$= -k \int_A \omega_0^2 r^2 \, dA$$

Rate loss KE?

$$\frac{2}{\pi} \int \frac{1}{2} u^2 \, dV = \frac{H}{2} \int z u \frac{\partial u}{\partial t} \, dV$$

$$= H \int \omega_0^2 r^2 \frac{k}{H} \, dA$$

$$= \text{work}$$