Assignment 4 Phy 426, 2017 J. Klymak

DUE: 4 April, 2017

Question 1. Zhukhovski Airfoil

Using the Zhukhovski Transform, numerically demonstrate the lift law for an airfoil.

- 1. For suitable choice of an offset in the ζ -plane, transform a circle into an airfoil in the z-plane. Note that for full marks it should look like an airfoil!
- 2. Show the streamlines for a flow with mean speed U and circulation $-\Gamma$, where you chose the appropriate values for those constants around the sphere and airfoil. Note that you should chose Γ to satisfy the Kutta condition. (HINT: you want to define your cylinder in the ζ -plane, but you want to plot your streamlines in the z-plane where the cylinder is an airfoil. So on an even grid in x and y, for each value of z = x + iy determine the value of z = z + iy determine the z = z + iy determine
- 3. Repeat the above for two other angles of attack α , where you chose α .
- 4. Calculate and plot the pressure field (up to an arbitrary constant) around the circle and airfoil. ($|\mathbf{u}(z)|=|dw/dz|\approx |dw/dx|$ which you can calculate from first difference from your grid (or analytically for the cylinder) Then you need to evaluate on the edge of the body.).
- 5. Calculate the lift from the pressure field for all three values of the angle of attack. Compare to the Zhukovski lift law.

Question 2. Blasius profile

Numerically solve Blasius' equation:

$$\frac{1}{2}f\frac{d^2f}{d\eta^2} + \frac{d^3f}{d\eta^3} = 0\tag{1}$$

subject to the boundary conditions f(0) = f'(0) = 0 and $f(\infty) = 1$. (HINT: make sure you let η go "high enough" so that "infinity" is suitably far from $\eta = 0$. Here "numerically" means to use a shooting method with Newton-Raphson method to ensure rapid convergence. For full marks, do not use a built in optimizer (though you are welcome to check your work with one)).

- 1. Plot the resulting *velocity* profile as a function of y at three different times.
- 2. Contour streamlines ψ in x-y space. (HINT: to get good looking streamlines for small x I needed to extend my solution $f(\eta)$ to very large η . Rather than doing this numerically, I note that $df/f\eta$ is approximately constant, so the extrapolation is just linear.)
- 3. Contour v and show that it approaches a constant as $y/\delta >> 1$.