V

1) if we accellerate tank water in tank needs P. G. F. to accellerate it. water piles up on upstream side

$$\frac{\partial u}{\partial t} = a = -\frac{1}{\rho}\frac{\partial P}{\partial x}$$

$$\therefore \quad -g\frac{\partial \eta}{\partial x} = a$$

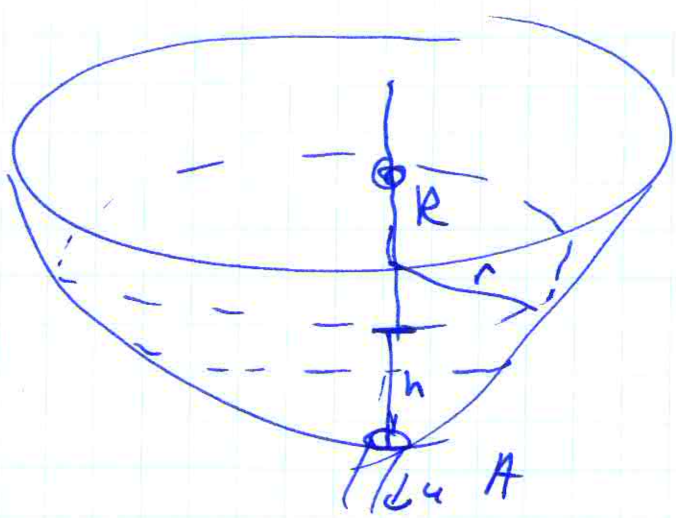
$$\Rightarrow \tan\theta = \frac{a}{g}$$

2) If tank is cylindrical?

No difference. ~~Tank still~~ Water still needs to have $\frac{\partial \eta}{\partial x} = -\frac{a}{g}$

Q2 Draining Tank

Assg 2 Key



$$uA = \frac{dh}{dt}\pi r^2$$

$$r^2 + (R-h)^2 = R^2$$

$$r^2 - 2Rh + h^2 = 0$$

$$r^2 = 2Rh - h^2$$

$$\cancel{\frac{P_{atm}}{\rho}} + \frac{u^2}{2} = \cancel{\frac{P_{atm}}{\rho}} + \left(\frac{dh}{dt}\right)^2\frac{1}{2} + gh$$

assume small

$$u^2 = 2gh$$

$$u = \sqrt{g2h}$$

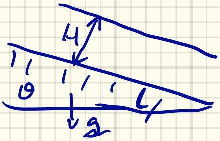
so $$\frac{dh}{dt}\left[\frac{2Rh - h^2}{h^{1/2}}\right] = \frac{\sqrt{2g}\,A}{\pi}$$

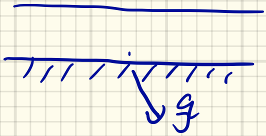
$$\int_{h_1}^{h_2} 2Rh^{1/2} - h^{3/2} = \frac{\sqrt{2g}\,A}{\pi}(t_2 - t_1)$$

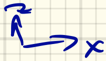
$$\frac{4R}{3}h^{3/2} - \frac{2h}{5}^{5/2}\Big|_{h_1}^{h_2} = \frac{\sqrt{2g}\,A}{\pi}\Delta t$$

$$\therefore \Delta t = \frac{2\pi}{A\sqrt{2g}}\left[\frac{2}{3}R\left(h_1^{3/2} - h_2^{3/2}\right) - \frac{1}{5}\left(h_1^{5/2} - h_2^{5/2}\right)\right]$$

Ass #2 Q 3







$g' = g\sin\theta$ acts to accelerate flow.

$= g\frac{\partial h}{\partial x}$

x-momentum:

$$\cancel{\frac{Du}{Dt}} = g\sin\theta - \cancel{\frac{1}{\rho}\frac{\partial p}{\partial x}} + \nu\frac{\partial^2 u}{\partial x^2}$$

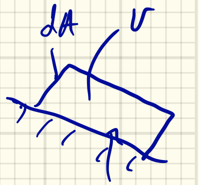
B.C.: $u = \phi$ at $z' = 0$

$\frac{\partial u}{\partial z} = \phi$ at $z' = H$

$$\Rightarrow \boxed{u = \frac{g'}{\nu_0} z'\left(H - \frac{z'}{2}\right)}$$

$$q = \int_0^H u\,dz' = g'_0\,H^3/3$$

Energy Drop?



$$\frac{DR}{Dt} = \Phi + \nabla\cdot(\underset{\sim}{\tau}\cdot\underset{\sim}{u}) + \cancel{\frac{1}{\rho}\frac{\partial p}{\partial t}}$$

dissipation

stress divergence

steady

$\underset{\sim}{u}\cdot F = 0$ because $\underset{\sim}{u} = 0$

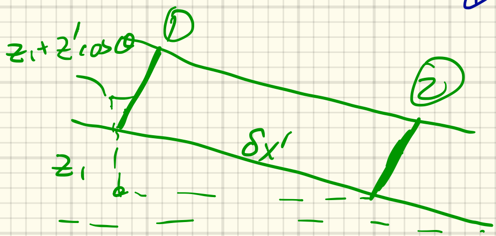
Integrate around volume

$$\int \underset{\sim}{u}\cdot\nabla R\,dV = \int \Phi\,dV$$

$$\oint \underset{\sim}{u}R\cdot d\underset{\sim}{A} = \text{Dissipation.}$$

By choosing volume at top + bottom of flow, only the inflow + outflow contribute

$$\oint R\,\underline{u}\cdot d\underline{A} = \int_{①} R u\, dz' + \int_{②} R u\, dz'$$



$$z_1 = \delta x' \sin\theta$$

$$= \int_0^H \left(\cancel{\tfrac{1}{2}u^3} + \cancel{\frac{uP}{\rho}} + u g z \right) dz' \Big|_{①}$$

$$- \int_0^H \left(\cancel{\tfrac{1}{2}u^3} + \cancel{\frac{uP}{\rho}} + u g z \right) dz' \Big|_{②}$$

Note: z is real z, g is real g. and these are different at ① + ②

$$= \int_0^H u g \left(z_1 + z'\cos\theta\right) dz' - \int_0^H u g \left(z'\cos\theta\right) dz'$$

$$= g\,\delta x' \sin\theta \; \frac{g'}{\nu} H^3/3 = \delta x' \frac{g'^2}{\nu} H^3/3$$

Dissipation??

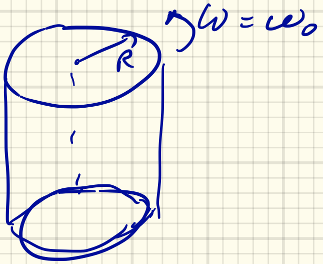
$$= \delta x' \, \nu \int_0^H \left(\frac{\partial u}{\partial z'}\right)^2 dz'$$

$$= \delta x' \, \frac{g'^2}{\nu} \int_0^H (H - z')^2 \, dz'$$

$$= \delta x' \, \frac{g'^2}{\nu} \, \frac{H^3}{3}$$

So loss of P.E. = dissipation in flow.

Assign 2 Q: 4



Remains in solid body rotation?

Depth-integrated momentum says that

$$H \frac{\partial \underline{u}}{\partial t} = \frac{\underline{\tau}}{\rho} = -k \frac{\underline{u}}{}$$

r → component

$$H \frac{\partial u_r}{\partial t} = -k u_r$$

But $u_r = \phi$ so $\frac{\partial u_r}{\partial t}$ remains ϕ

θ → component

$$H \frac{\partial u_\theta}{\partial t} = -k u_\theta$$

$$\Rightarrow u_\theta = u_{\theta 0} e^{-\frac{kt}{H}}$$

so if $u_{\theta 0}$ is solid body

u_θ is solid-body

2) Timescale is simply given by

$$\frac{H}{k}$$

3) Work = loss KE?

$$\text{Work} = \int_A \underset{\sim}{\tau} \cdot \underset{\sim}{u} \, dA$$

at bottom

$$= \int_A -k u_0^2 \, dA$$

$$= -k \int_A \omega_0^2 r^2 \, dA$$

Rate loss KE?

$$\frac{\partial}{\partial t}\int \frac{1}{2} u^2 \, dV = \frac{H}{2} \int 2u \frac{\partial u}{\partial t} \, dV$$

$$= H \int \omega_0^2 r^2 \frac{k}{H} \, dA$$

$$= \text{work}$$