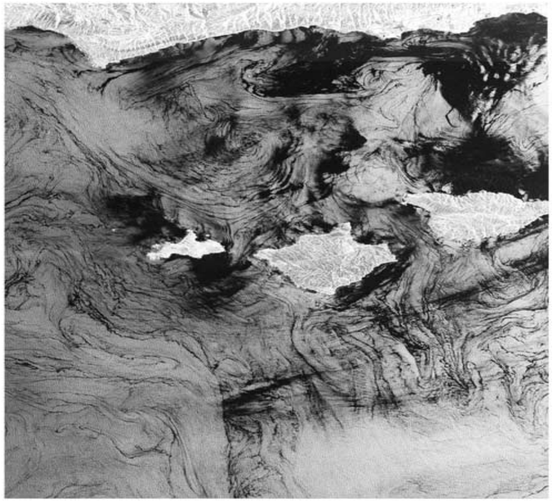
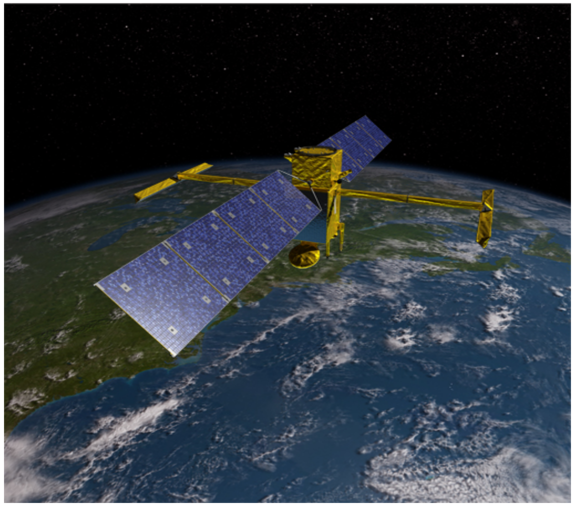
“Wave-Turbulence Interactions in the Oceans”

Patrice Klein (Caltech/JPL/Ifremer)

(XIII) Interactions between waves and balanced motions

Propagation of waves in a 3-D balanced flow field





Coming back to the Okubo-Weiss quantity ...

$$\frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \nabla \vec{v} - f \cdot \vec{k} \times \vec{v} = -\frac{\nabla p}{\rho_0}$$

$$\rightarrow \boxed{\nabla \cdot (\vec{v} \cdot \nabla \vec{v}) - f \zeta = -\frac{\Delta p}{\rho_0}} \qquad \textbf{(1)}$$

$O(R_0) \qquad O(1) \qquad O(1).$

$\nabla \cdot (\vec{v} \cdot \nabla \vec{v})$ is the Okubo-Weiss quantity

$$\rightarrow \boxed{\nabla \cdot (\vec{v} \cdot \nabla \vec{v}) = \frac{1}{2} \left[S_1^2 + S_2^2 - \zeta^2 \right]}$$

$$S_1 = \left[\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}\right], \quad S_2 = \left[\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right], \quad \zeta = \left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right]$$

O.W. < 0 in vorticity regions.
O.W. > 0 in strain regions.

→ **Dispersion of tracers**

(1) is called the gradient wind balance

Effect of centrifugal force on eddies:

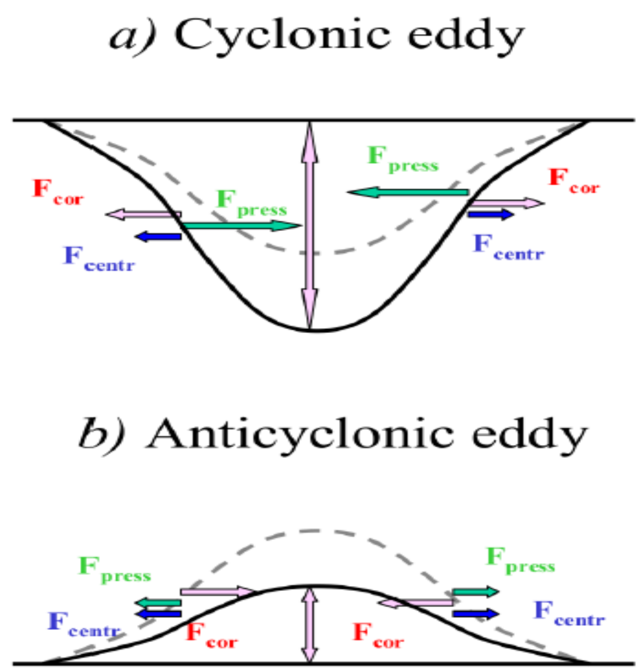


Figure adapted from Maximenko and Niiler (2006)

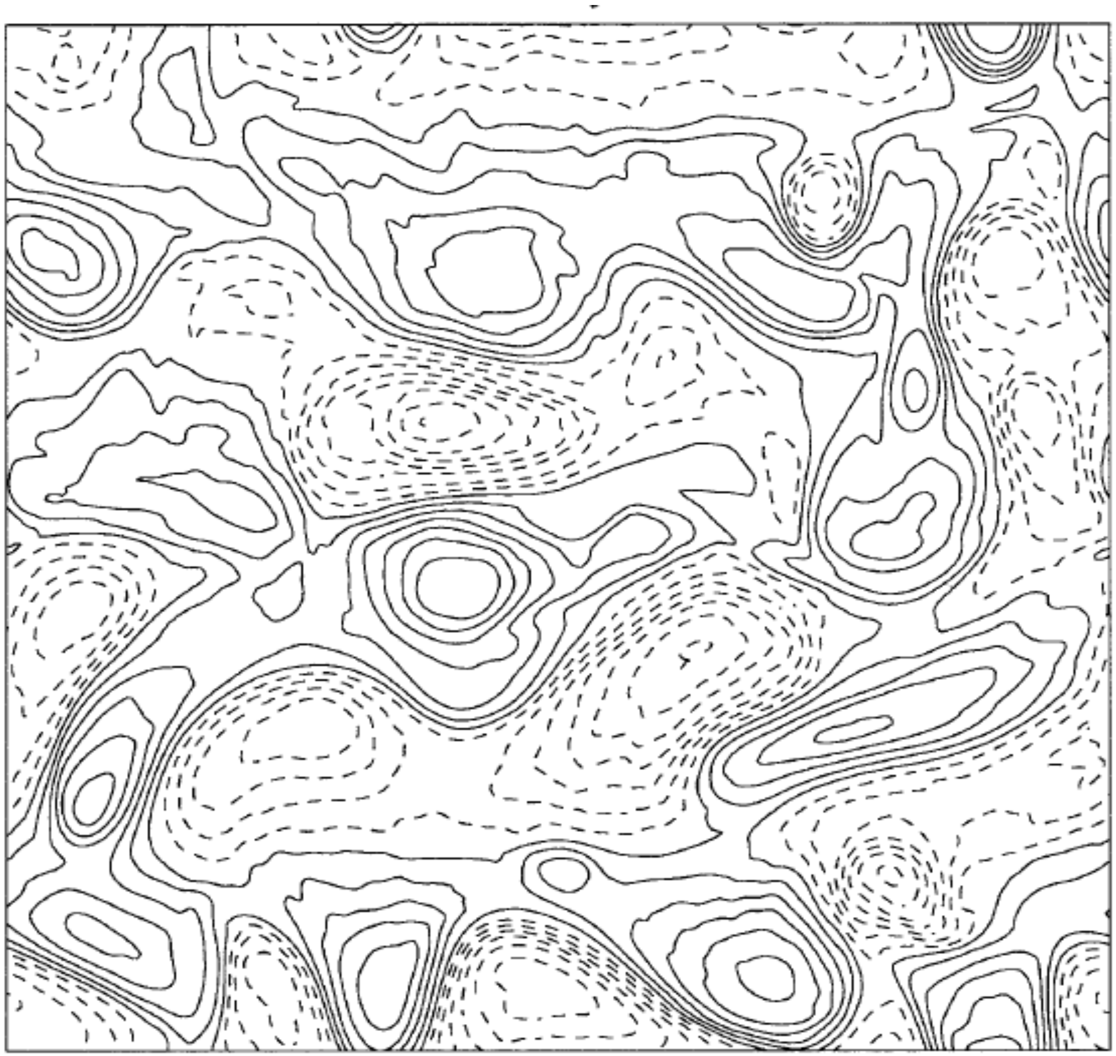
Cyclones : larger pressure gradient to compensate for centrifugal force

(here in a stationary case)

$$\boxed{\mathbf{u}.\nabla\mathbf{u}+f\mathbf{k}\times\mathbf{u}=-g\nabla\eta}$$

From Penven et al. Ocean Sciences 2016

Let us consider a 2-D (non-divergent) mesoscale eddy field

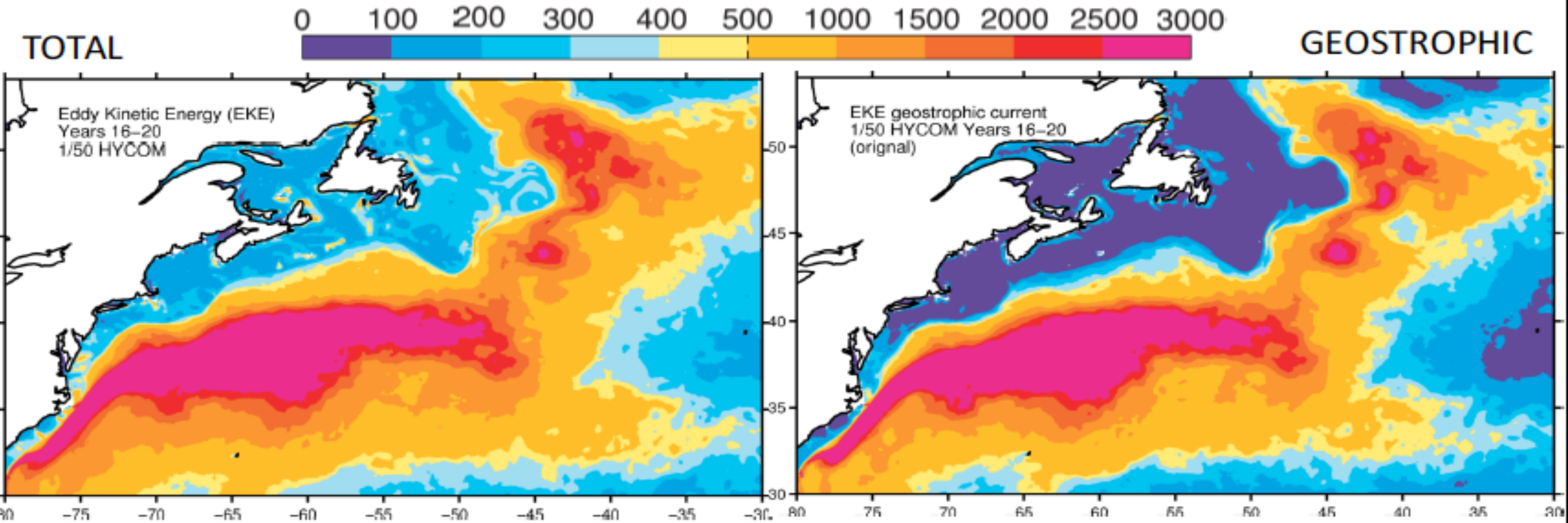


$$U = -\psi_y$$
$$V = \psi_x$$

$$P = f\psi\rho_o + \tilde{p}$$

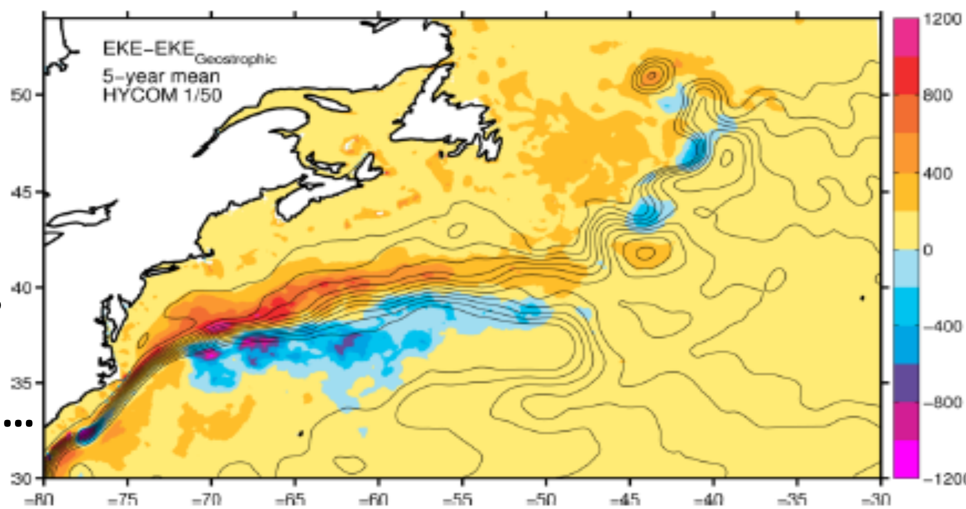
STREAM FUNCTION $\psi(x, y)$

EKE geostrophic difference (1/50°)



$$f.\zeta = \frac{\Delta SSH}{\rho_o} + OW$$

⇒ But « geostrophic » anticyclones may be less energetic than « geostrophic » cyclones…

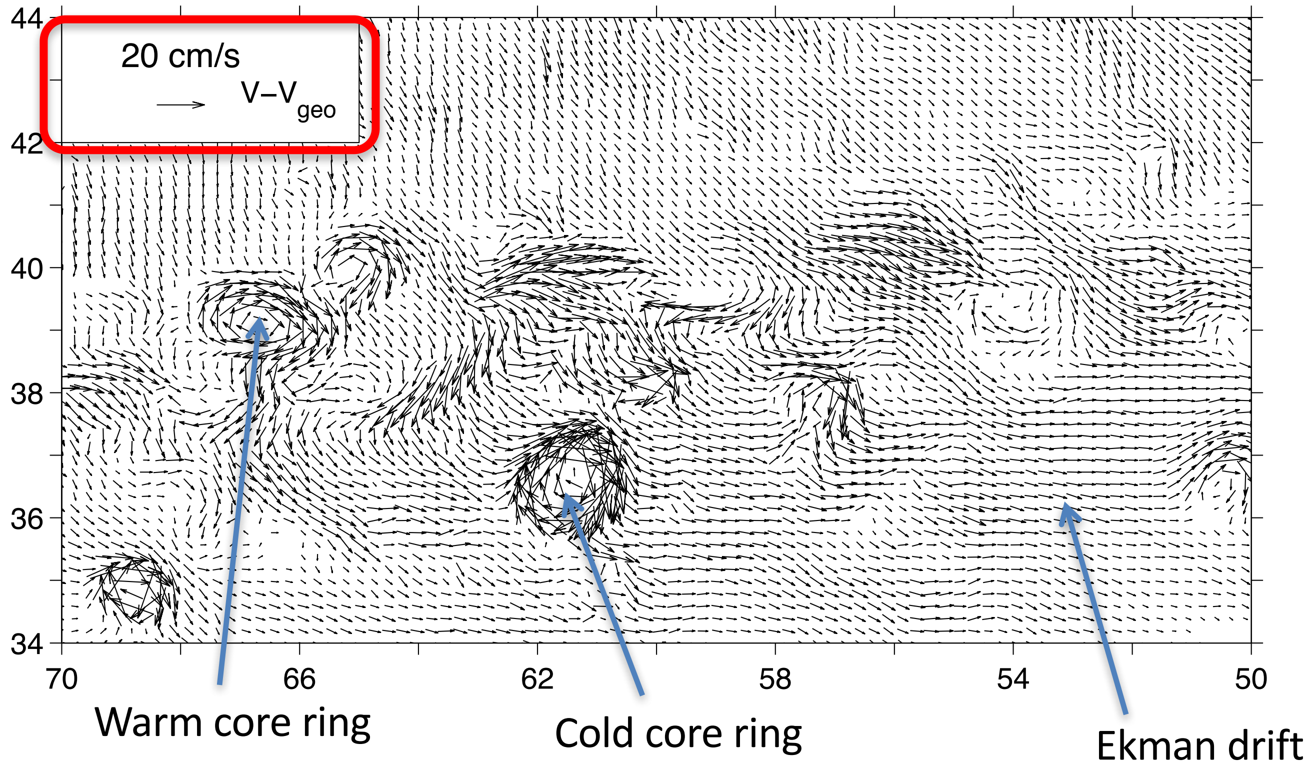


SSH is total pressure

OW is always <0 within coherent eddies

From Chassignet & Xu JPO 2017, In press

Nonlinear terms are important !



$$f\zeta = g\Delta\eta + \nabla(U.\nabla U)$$

$\nabla(U.\nabla U)$ is always <0 within coherent (axisymmetric) eddies. Rotation of anticyclones is faster and slower in cyclonic eddies. Hence more total EKE north of the Gulf Stream compared with the »geostrophic EKE ».

But eddies are not always axisymmetric. They can be sheared by other eddies leading to the emergence of non-zero strain field in their interior, which can strongly reduce the OW magnitude.

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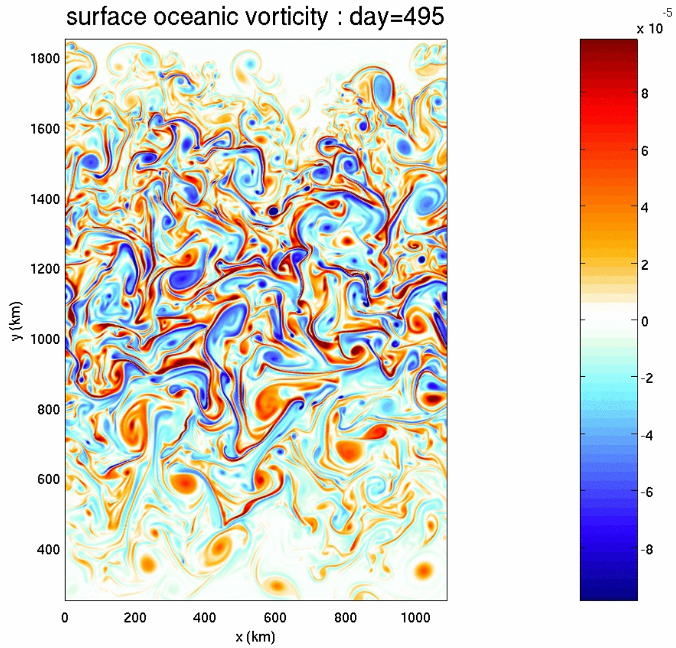
. An example ...



High resolution simulation (1km, 200 levels) of mesoscale turbulence in a beta-plane channel forced by the Charney instability (surface) of a vertically sheared mean zonal flow

SSH statistics display the opposite asymmetry for positive and negative values.

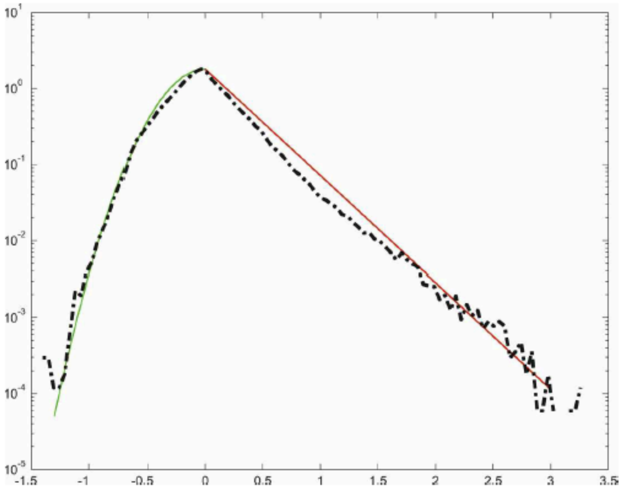


FIG. 4. PDF of surface vorticity (black curve) estimated in the area concerned by the turbulent flow. Exponential law is shown by the red curve and the Gaussian law (with std dev equal to the observed) by the green curve. Abscissa units are vorticity values normalized by f_0.

For this specific dynamics, where a a strong asymmetry cyclone-anticyclone is present, geostrophic equilibrium is still well observed. The OW quantity is usually one order of magnitude smaller than the pressure term, except in filaments and small-scale vortices (where Ro>1)!

See next slide ...

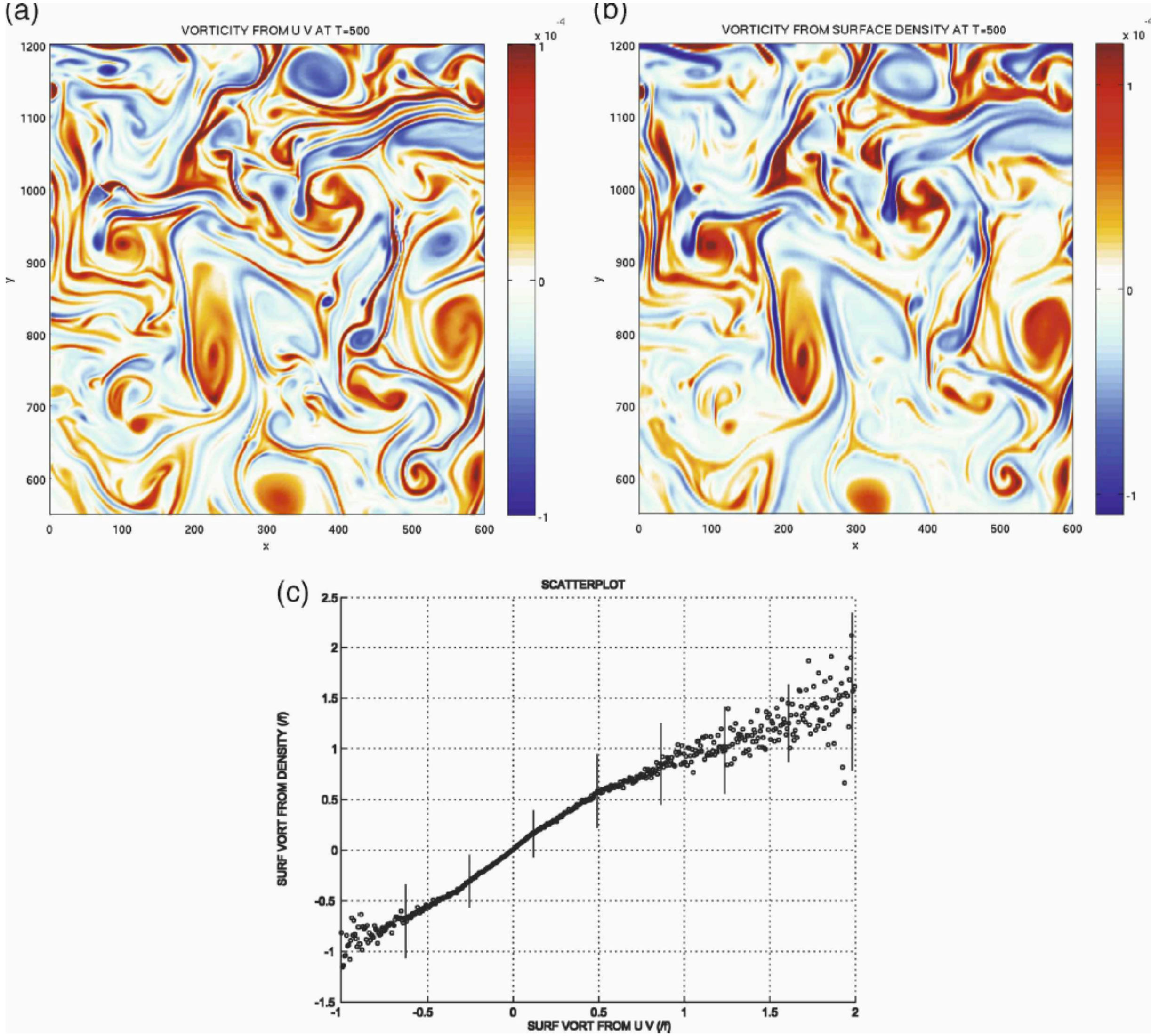


FIG. 7. Zoom of the surface relative vorticity field (a) deduced from u and v and (b) reconstructed from the surface density using (4); units in the x and y directions in are km and vorticity units are s^{-1}. (c) Scatterplot between the original surface vorticity and its reconstruction from the surface density over the whole domain. In (c), each point represents the average over each grid interval on the abscissa (that has a total of 2000 grid intervals), and thin vertical lines show std dev around the averages.

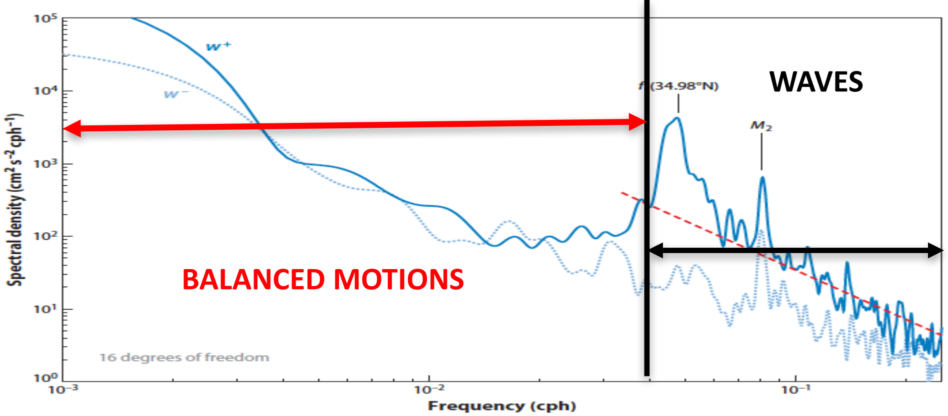
OW quantity in a balanced flow field

Three classes of structures:

- **Axisymmetric eddies** (including large-scale eddies such as Gulf Stream rings) where OW can be large;

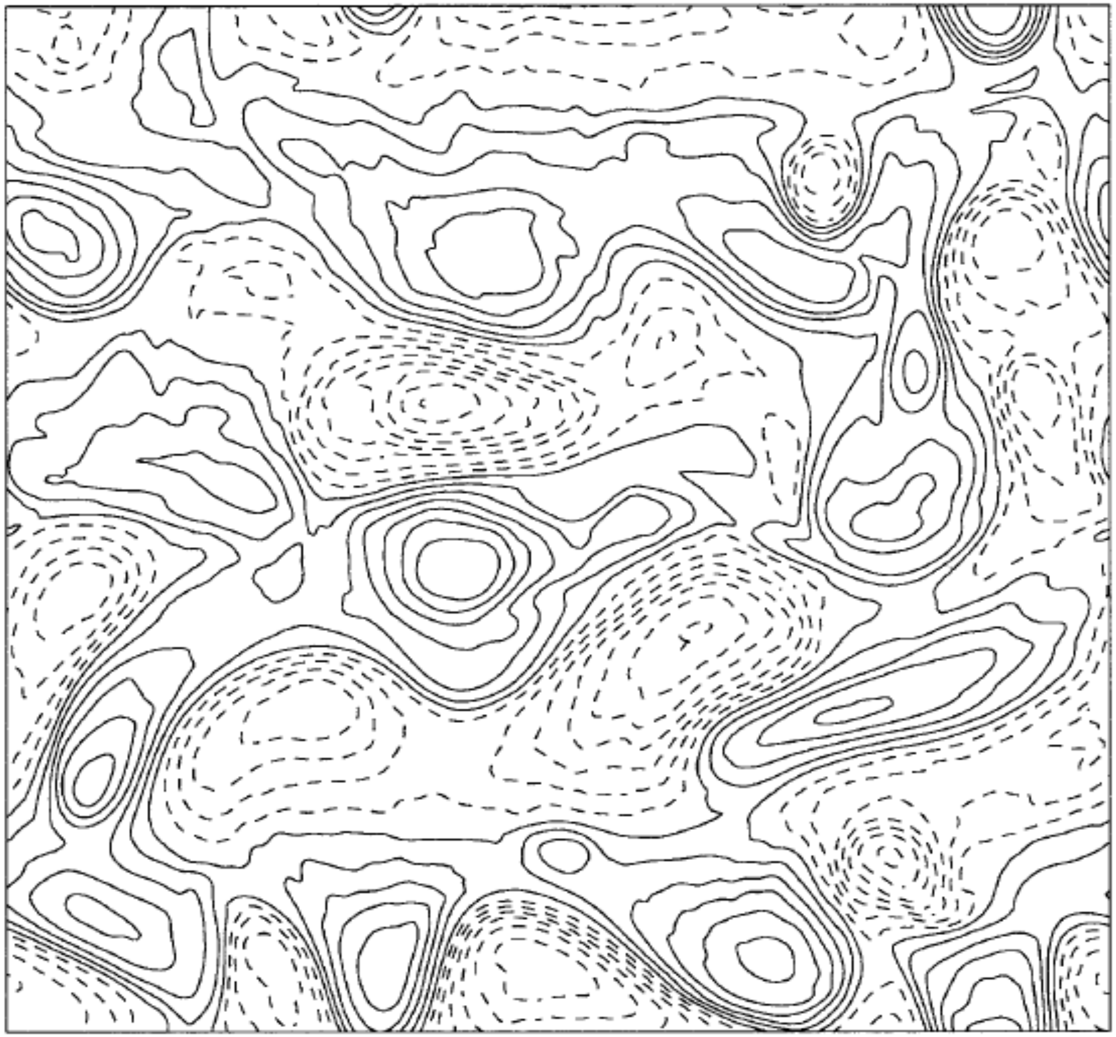
- **Strongly interacting eddies** (where both strain and vorticity are large [see merging of eddies we discussed last year]) where OW can be small and for which geostrophic equilibrium can be observed;

- **Small-scale filaments and vortices** for which OW can be large but that are usually characterized by a large Rossby number.



Interactions between waves and balanced motions

Let us consider a 2-D (non-divergent) mesoscale eddy field

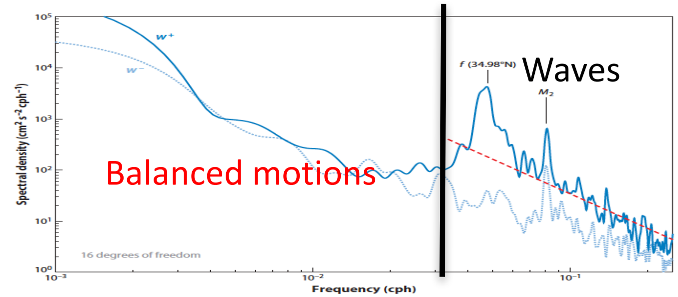


STREAM FUNCTION $\psi(x, y)$

$$U = -\psi_y$$
$$V = \psi_x$$

$$P = f\psi\rho_o + \tilde{p}$$

Interactions between waves and 2-D balanced motions



For baroclinic mode m:

$p(x,y,z,t) = \sum_m p_m(x,y).F_m(z)$

$$\frac{\partial u_m}{\partial t} + \boldsymbol{u_m U_x + v_m U_y} - \mathrm{f} v_m = -\frac{\partial p_m}{\partial x}$$

$$\frac{\partial v_m}{\partial t} + \boldsymbol{u_m V_x + v_m V_y} + \mathrm{f} u_m = -\frac{\partial p_m}{\partial y}$$

$$\frac{\partial p_m}{\partial t} + f^2.r_m^2\left(u_{mx} + v_{my}\right) = 0$$

Doppler terms are not considered for the sake of simplicity. Searching for plane wave solutions as, $p_m(x,y,t), u_m(x,y,t), v_m(x,y,t) \approx e^{i.(k.x+l.y-\omega t)}$, leads to:

$\omega = \omega_r + i.\omega_i$ with:

$$\omega_r^2 \approx f^2 + \boldsymbol{f.\left[V_x - U_y\right] - V_x U_y - U_x^2} + f^2 r_m^2.(k^2 + l^2)$$

$$\omega_i \approx \boldsymbol{i.\left[k.l.\left(V_x + U_y\right) + \left(k^2 - l^2\right).U_x\right]}$$

Physics involved in the wave dispersion by 2-D balanced motions

$$\frac{\partial}{\partial t}\begin{bmatrix} u \\ v \end{bmatrix} + \begin{bmatrix} \boldsymbol{U_x} & \boldsymbol{U_y} - f \\ \boldsymbol{V_x} + f & \boldsymbol{V_y} \end{bmatrix}\begin{bmatrix} u \\ v \end{bmatrix} = -\begin{bmatrix} p_x \\ p_y \end{bmatrix}$$

$$\frac{\partial}{\partial t}\begin{bmatrix} u \\ v \end{bmatrix} + \begin{bmatrix} \boldsymbol{S_1/2} & \boldsymbol{S_2/2} - (f + \boldsymbol{\zeta/2}) \\ \boldsymbol{S_2/2} + (f + \boldsymbol{\zeta/2}) & \boldsymbol{-S_1/2} \end{bmatrix}\begin{bmatrix} u \\ v \end{bmatrix} = -\begin{bmatrix} p_x \\ p_y \end{bmatrix}$$

$$\frac{\partial p}{\partial t} + f^2.r^2\left(u_x + v_y\right) = 0$$

with: $\boldsymbol{S_1 = U_x - V_y, S_2 = V_x + U_y, \zeta = V_x - U_y}$

$$\omega_r^2 \approx f^2 + \boldsymbol{f.\zeta} - \boldsymbol{\left(S_1^2 + S_2^2 - \zeta^2\right)/4} + f^2 r^2.(k^2 + l^2)$$

$$\omega_i \approx \boldsymbol{i.[k.l.\left(V_x + U_y\right) + \left(k^2 - l^2\right).U_x]}$$

Dispersion relation in a 3-D balanced flow field

$$\omega_0 = \omega - (\mathbf{k}\cdot\mathbf{V}) \approx \Bigg[\underset{\text{(i)}}{f_{\text{eff}}} + \underset{\text{(ii)}}{\frac{N^2 k_H^2}{2 f k_z^2}} + \underset{\text{(iii)}}{\underline{\frac{1}{k_z}\left(\frac{\partial U}{\partial z} k_y - \frac{\partial V}{\partial z} k_x\right)}} \Bigg]$$

$$+\, i\Bigg\{ \underset{\text{(iv)}}{\underline{\frac{\zeta}{2 f k_z}\left(\frac{\partial U}{\partial z} k_x + \frac{\partial V}{\partial z} k_y\right)}} + \underset{\text{(v)}}{\frac{N^2}{2 f^2 k_z^2}\left[\frac{\partial U}{\partial x} k_y^2 - \left(\frac{\partial U}{\partial y} + \frac{\partial V}{\partial x}\right) k_x k_y + \frac{\partial V}{\partial y} k_x^2\right]} \Bigg\},$$

with: $$f_{\text{eff}} \approx f + \frac{\zeta}{2} = f + \frac{1}{2}\left(\frac{\partial V}{\partial x} - \frac{\partial U}{\partial y}\right)$$

(from E. Kunze JPO 1985)

In a 3-D balanced flow field new terms involve the vertical shear of the horizontal motions, i.e. the horizontal density gradients.
So both the vertical and horizontal density gradients are involved!

Dispersion relation can be cast in a form similar to that for internal gravity waves propagating in an ocean bereft of balanced motions by replacing the Coriolis and buoyancy frequencies with effective Coriolis and buoyancy frequencies:

See Eric Kunze JPO 1985

$$\boxed{\omega_0 \approx f_{\text{eff}} + \frac{N_{\text{eff}}^2}{2f} \frac{{k_H}^2}{{k_z}^2}}$$

with: $f_{\text{eff}} \approx f + \frac{\zeta}{2} = f + \frac{1}{2}\left(\frac{\partial V}{\partial x} - \frac{\partial U}{\partial y}\right)$

$$N_{\text{eff}}^2 = N^2 + {M_x}^2 \frac{k_x k_z}{{k_H}^2} + {M_y}^2 \frac{k_y k_z}{{k_H}^2}$$

where ${M_x}^2 = g(\partial\rho/\partial x)/\rho_0$, ${M_y}^2 = g(\partial\rho/\partial y)/\rho_0$

Interpretation:

Slope of the isopycnals is: $s = \frac{M^2}{N^2}$ **=>** $N_{eff}^2 = N^2(1 + s.\frac{m}{k})$

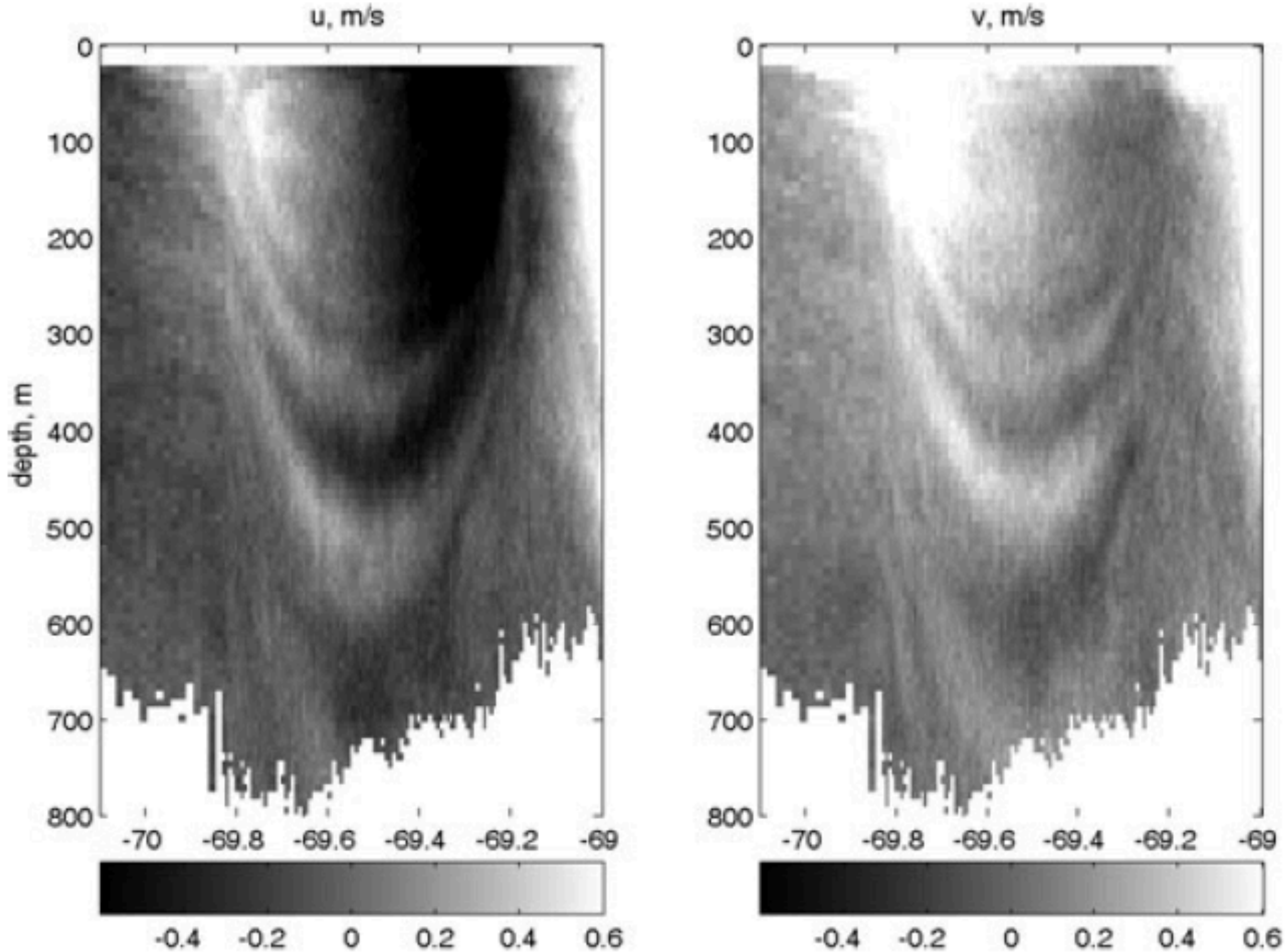


Figure 2. Grey-scale images of SADCP velocity data (zonal component on the left panel and meridional component on the right panel) plotted against longitude showing vertical banding of 100–200 m wavelength within the center of the WCR (Figure 1) with strong WSW flow on the seaward side (69.3°W) and NNW flow on the shoreward side (69.8°W). The phase of the vertical banding slopes up on both flanks of the WCR.

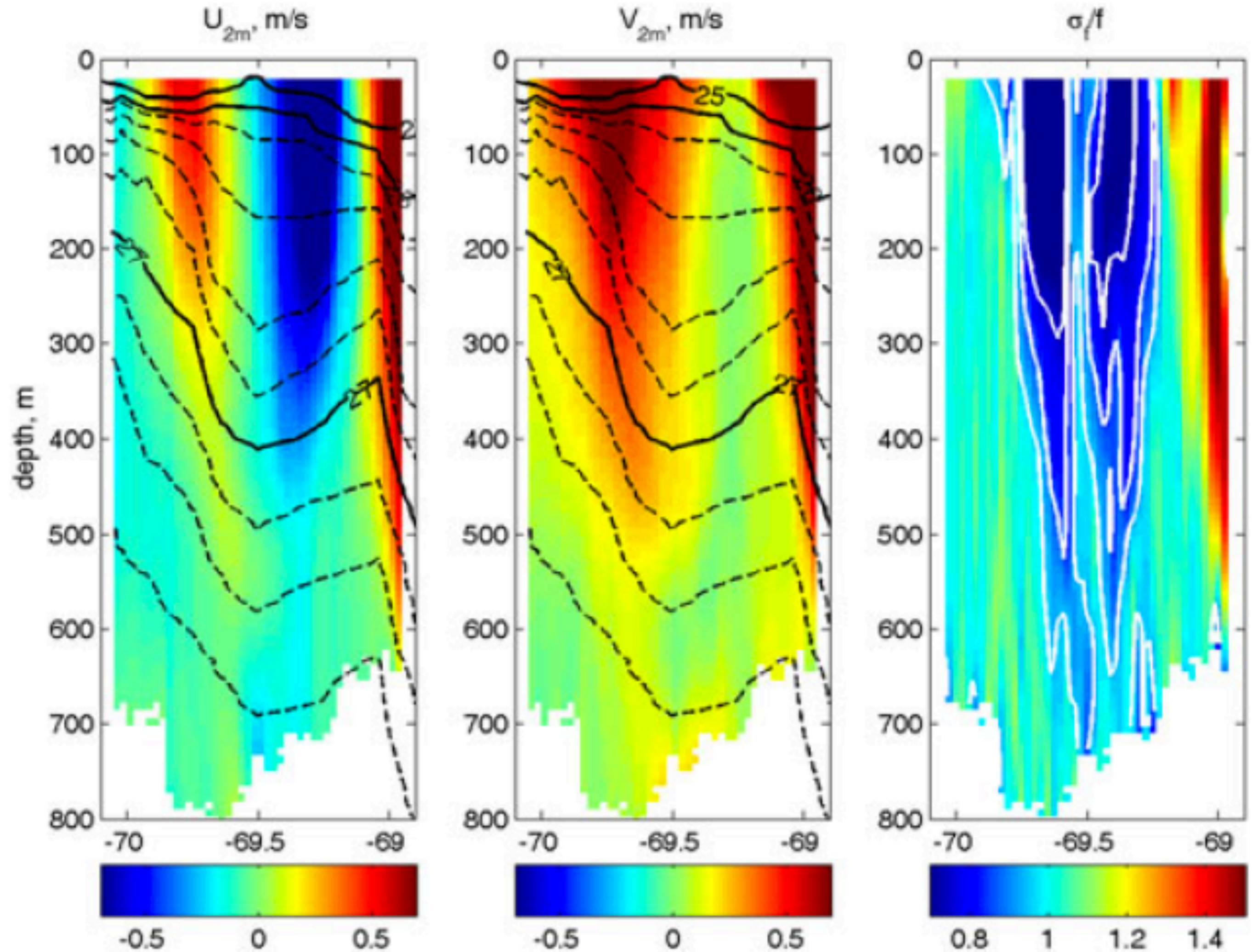


Figure 4. Smoothed velocity components and vortex-modified inertial frequency associated with the circulation of the WCR. In the latter, the white contours illustrate the sub-inertial structure of σ_f/f (equation (B10), contours of 0.75, 0.85, and 0.95), and this can be seen to encompass the region of anomalous velocities and spice (see Figure 5) within the WCR.

PHASE LINES FOLLOW THE SLOPE $S=-M^2/N^2$

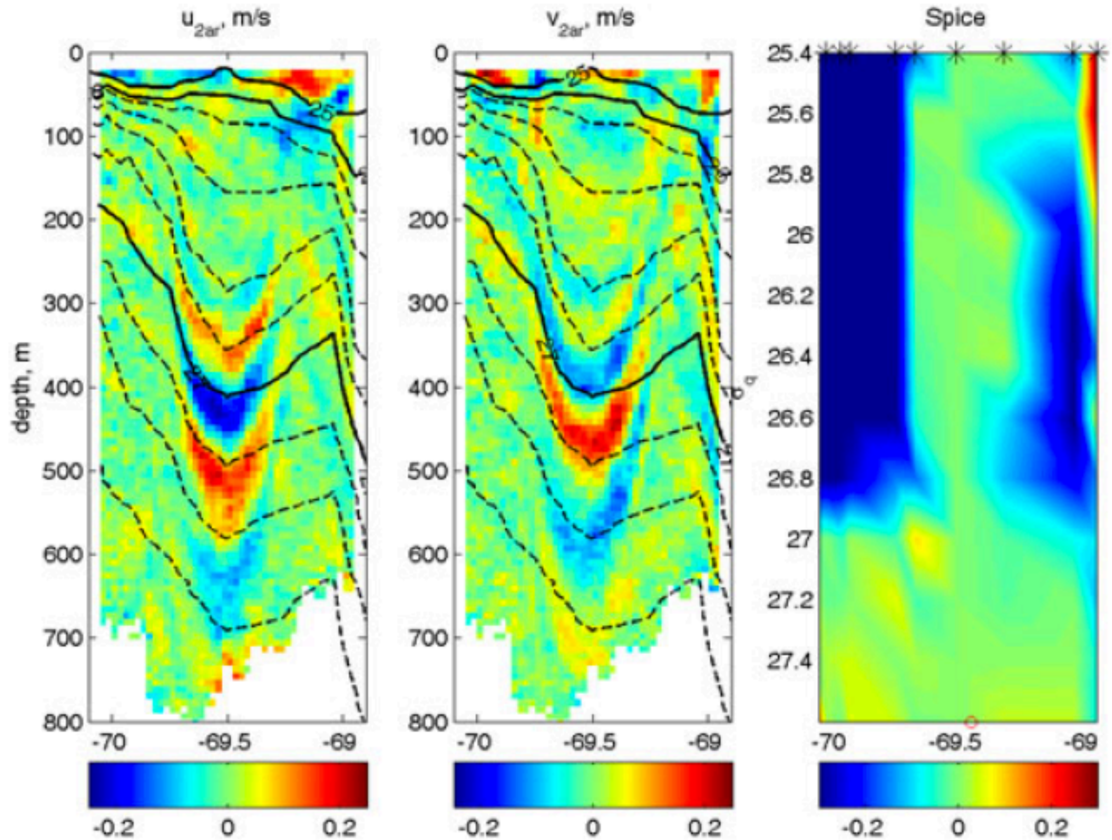


Figure 5. Phase-adjusted anomalies of velocity and spice (relative to ring center; right panel). During the velocity anomaly phase adjustment, the upward phase propagation altered the rotated velocity anomalies, and the constant phase lines (Figure 2) moved slightly upward on the right and downward on the left of ring center. Also plotted are constant σ_θ surfaces at a 0.2 kg m^{-3} contour interval. Solid lines denote the 25, 26, and 27 σ_θ surfaces. These are from the CTD stations taken on the seaward leg and have been shifted westward in longitude an amount of 0.13° to account for translation of the WCR between the two legs of the cruise. The Gulf Stream north wall can be seen intruding at the far right of all panels, consistent with the contact between the Gulf Stream and WCR indicated in Figure 1. The vertical axis is depth for the left and middle panels and σ_θ for the right panel. Longitude of the mooring W2 is indicated by the open circle at the bottom of the right panel.

Next class: Ray tracing approach.