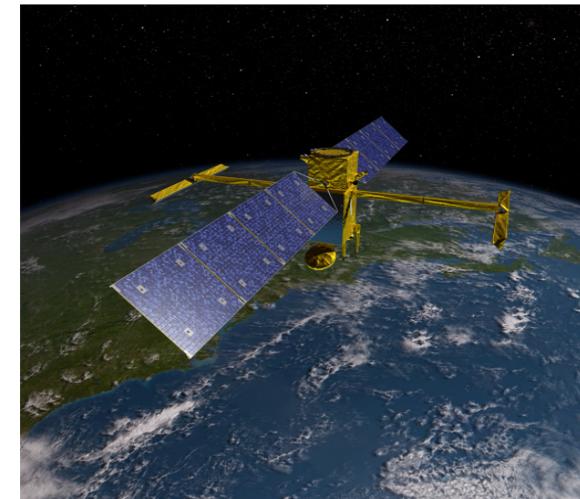


“Wave-Turbulence Interactions in the Oceans”

<https://oceanturbulence.github.io>

Patrice Klein (Caltech/JPL/Ifremer)

(XVII) A synthesis: Near-Inertial Waves



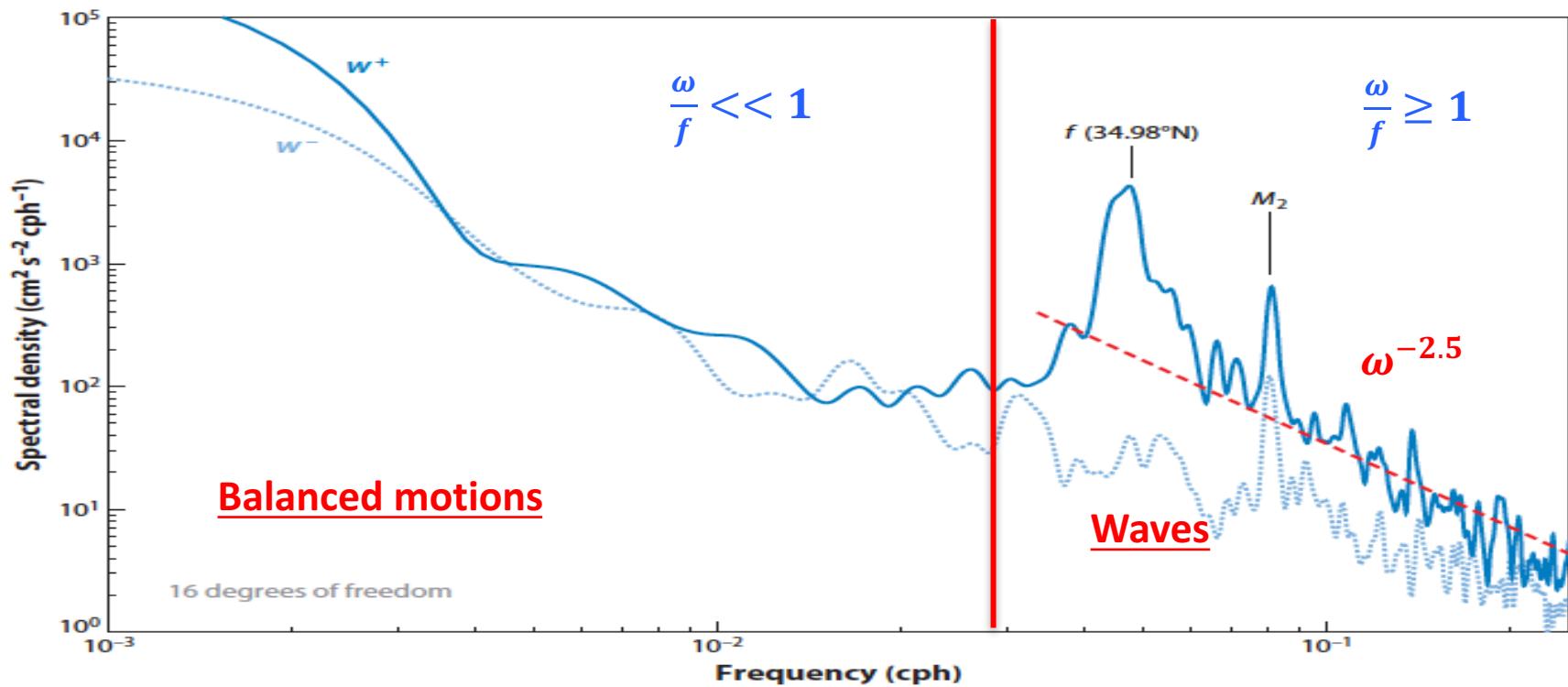


Figure 1

Rotary velocity spectrum at 261-m depth from current-meter data from the WHOI699 mooring gathered during the WESTPAC1 experiment (mooring at 6,149-m depth.) The solid blue line (w^+) is clockwise motion, and the dashed blue line (w^-) is counterclockwise motion; the differences between these emphasize the downward energy propagation that often dominates the near-inertial band. The dashed red line is the line $E_0 N \omega^{-p}$ with $N = 2.0$ cycles per hour (cph), $E_0 = 0.096 \text{ cm}^2 \text{s}^{-2} \text{cph}^{-2}$, and $p = 2.25$, which is quantitatively similar to levels in the Cartesian spectra presented by Fu (1981) for station 5 of the Polygon Mid-Ocean Experiment (POLYMODE) II array.

SPECTRAL GAP AT FREQUENCIES (ω) LARGER (BUT CLOSE TO) f

Waves (near-inertial, tidal, internal gravity waves):

- Fast motions $f \geq \omega \geq N$
- **assumed to explain most of the mixing (at small-scale) in the ocean interior**
- strong signature in in-situ observations (moorings, gliders, ADCP, surface drifters) and satellite observations [SAR, altimetry] at high-resolution.

Geostrophic turbulence [10-500 km]:

- Slow motions
- explains **most of the kinetic energy in the oceans**, well captured by satellite observations on a global scale [SSH (> 100 km), SST, Ocean Color, ...]

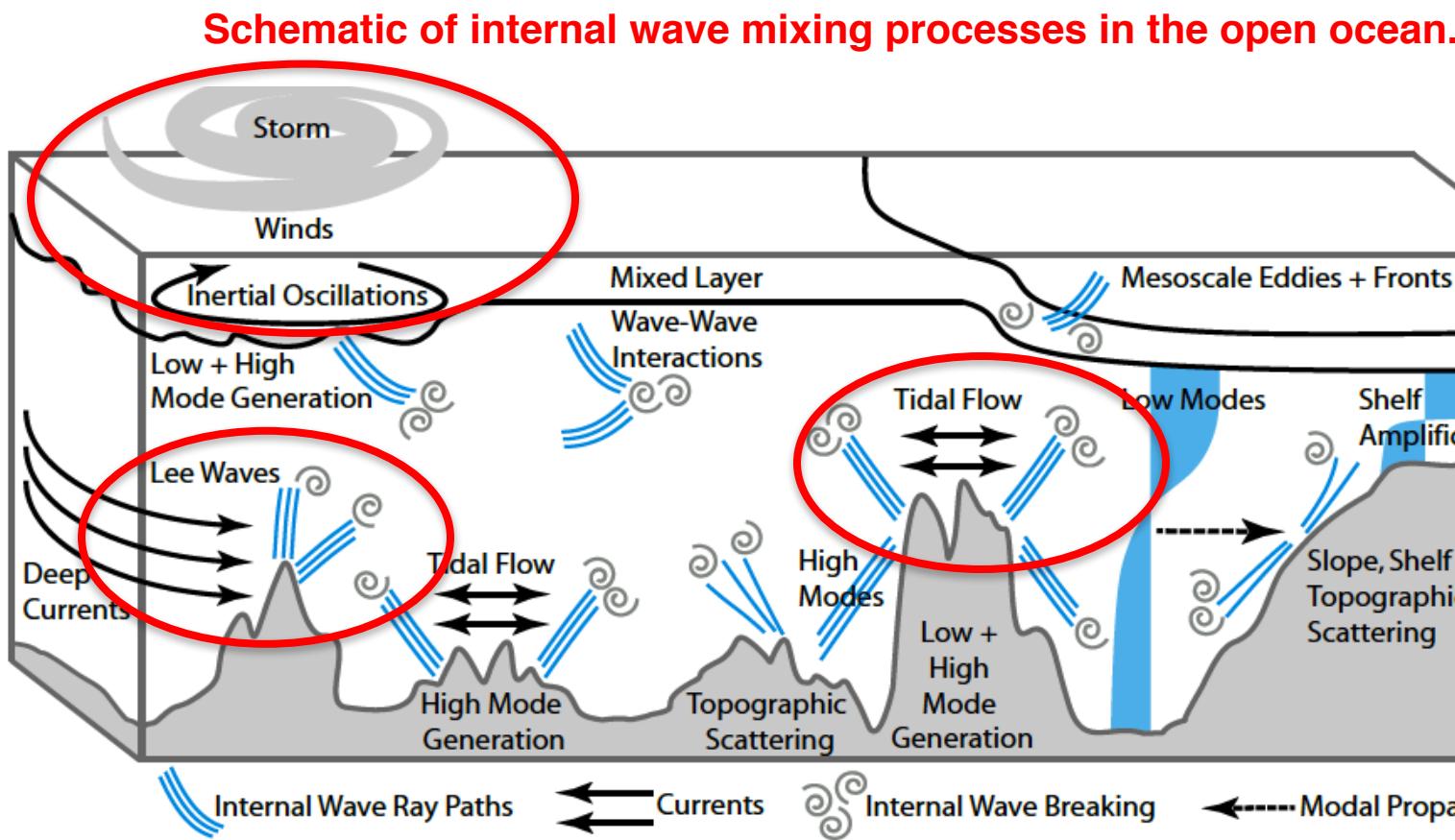
Open questions. see: Alford et al. ARMS 2016, Garrett and Kunze, ARFM 2007, Polzin and Lvov, Review of Geophysics 2011 ...

Another review paper: MacKinnon et al. BAMS 2017 (in press):

« Climate Process Team on Internal-Wave Driven Ocean Mixing »

« We have sufficient evidence from theory, process models, laboratory experiments, and field measurements to conclude that **away from ocean boundaries** (atmosphere, ice, or the solid ocean bottom), **diapycnal mixing is largely related to the breaking of internal gravity waves.** »

« Ocean internal gravity waves propagate through the stratified interior of the ocean. They are generated by a variety of mechanisms, with the most important being, **wind variations at the sea-surface, tidal flow over topography, and flow of ocean currents and eddies over topography leading to lee-waves** »



Tides interact with topographic features to generate high-mode internal waves (e.g. at mid-ocean ridges) and low-mode internal waves (e.g. at tall steep ridges such as the Hawaiian Ridge). Deep currents flowing over topography can generate lee waves (e.g. in the Southern Ocean). Storms cause inertial oscillations in the mixed layer, which can generate both low and high mode internal waves (e.g. beneath storm tracks). In the open ocean these internal waves can scatter off of rough topography and potentially interact with mesoscale fronts and eddies, until they ultimately dissipate through wave-wave interactions. Internal waves that reach the shelf and slope can scatter, or amplify as propagate towards shallower water.

From MacKinnon et al. BAMS 2017 (in press):

« IGWs are generated by a variety of mechanisms, with the most important being, **wind variations at the sea-surface**, tidal flow and flow of ocean currents and eddies over topography leading to lee-waves »

Wind-driven near-inertial motions

The largest uncertainties are associated with:

- **the poorly known high frequency and wavenumber part of the wind spectrum,** and
- the partitioning between locally dissipated energy and **the amount radiated away (interactions with mesoscale eddies).**

- OGCM 0.4° resolution 40 levels

- Two different wind stress forcings:

NCEP reanalysis:

- 6-hourly, T63 grid = 1.875° resolution

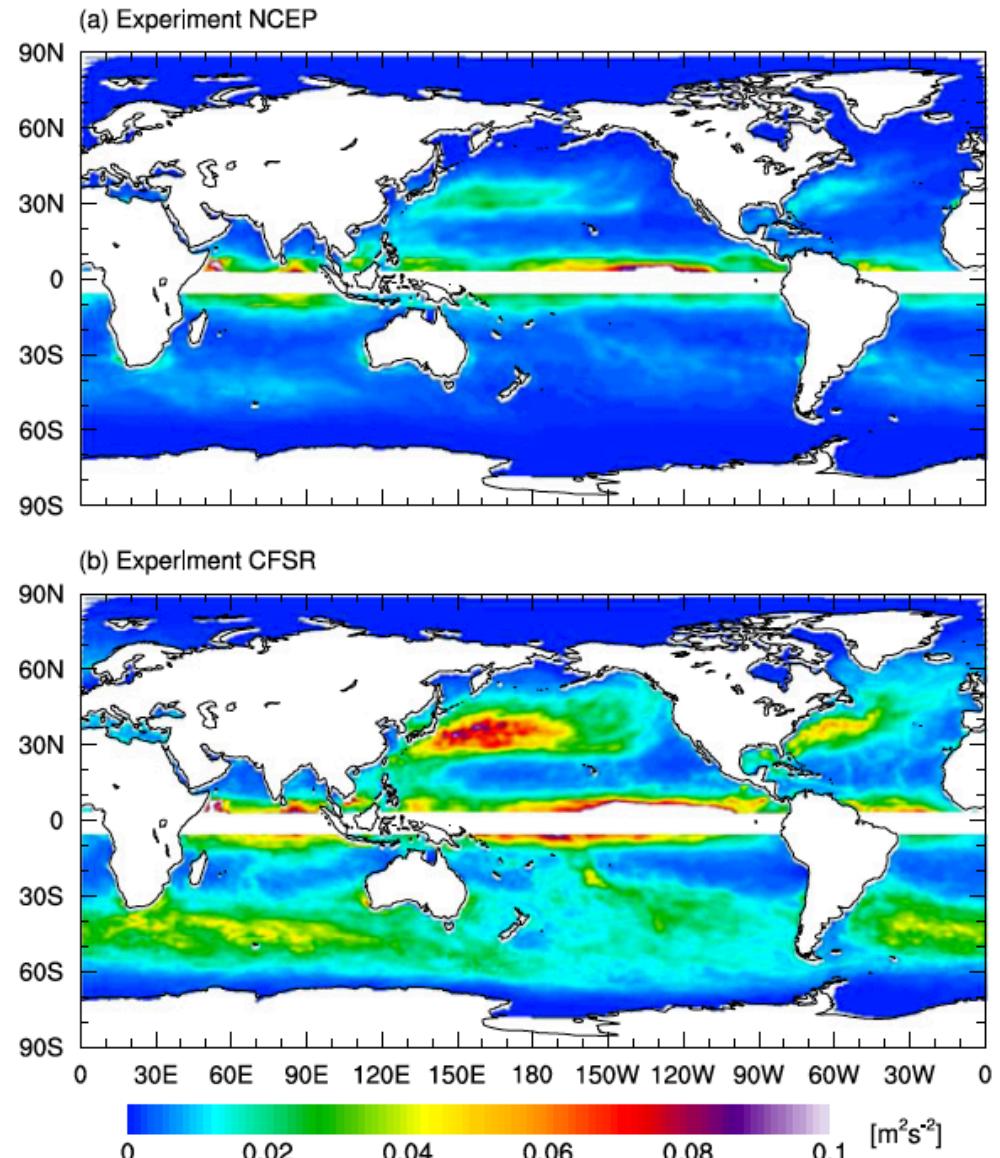
CFSR reanalysis:

- 1-hourly, T383 grid = 0.35° resolution

Globally averaged value of NI KE:

NCEP: 0.006 m²/s²

CFSR: 0.02 m²/s²



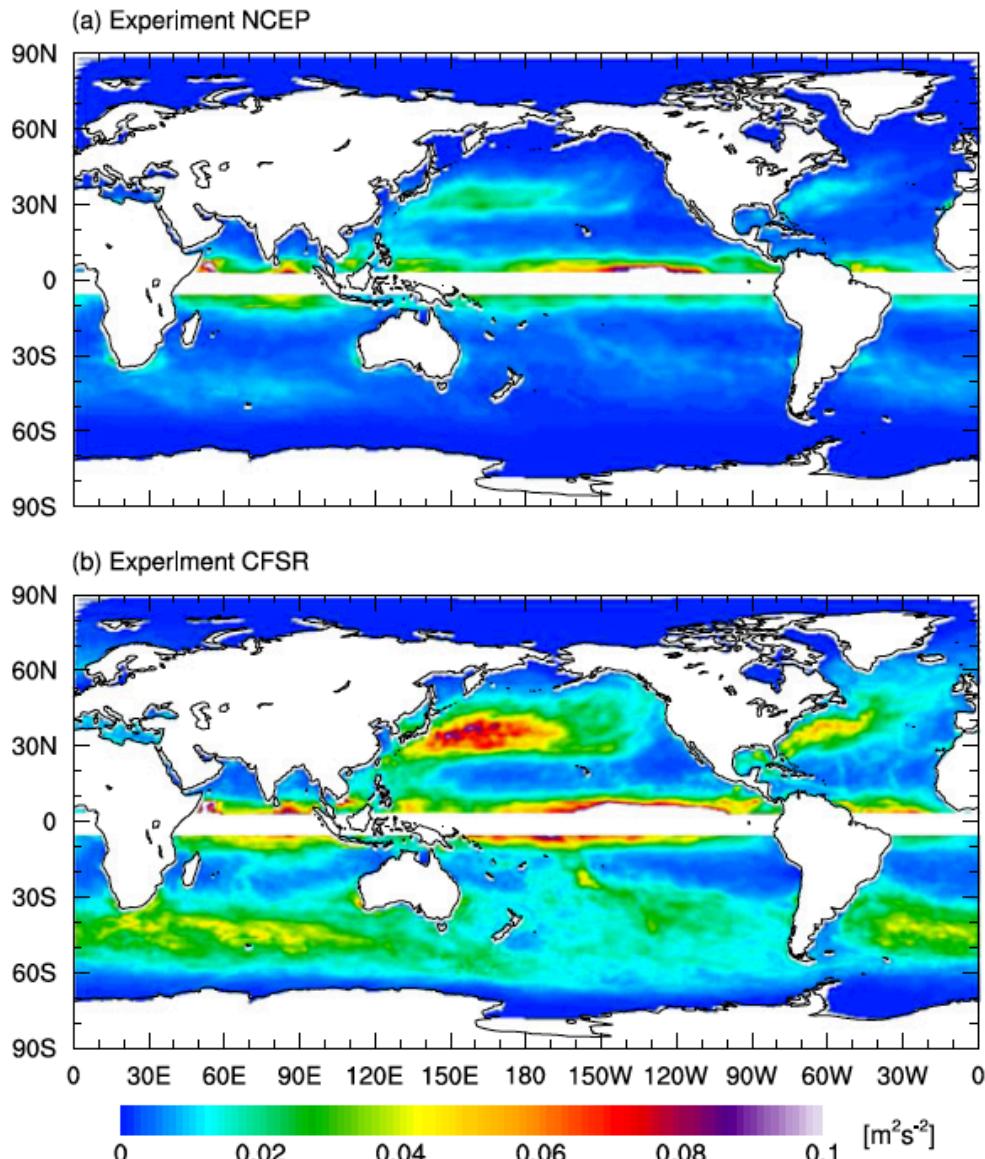
Spatial distribution of the near-inertial KE calculated from the bandpass-filtered model velocities

OGCM 0.4° resolution 40 levels

Table 1. Experiment Name, Temporal and Spatial Resolution of the Wind Stress Data Used to Force the Ocean Model MPIOM

	Experiment Name	Temporal Resolution	Spatial Resolution
TFX:	NCEP	6-hourly	T63 grid
	CFSR	1-hourly	T383 grid
	TF04	4-hourly	T383 grid
	TF06	6-hourly	T383 grid
	TF12	12-hourly	T383 grid
	TF24	daily	T383 grid
	SF0.7	1-hourly	0.7°grid
SFX:	SF1.1	1-hourly	1.125°grid
	SF1.8	1-hourly	1.875°grid
	SF2.8	1-hourly	2.8°grid

T63 grid = 1.875° resolution
T383 grid = 0.35° resolution



Spatial distribution of the near-inertial KE calculated from the bandpass-filtered model velocities

Causes of the differences in NI energy levels between CFSR ($0.02\text{m}^2/\text{s}^2$) and NCEP ($0.006\text{m}^2/\text{s}^2$)

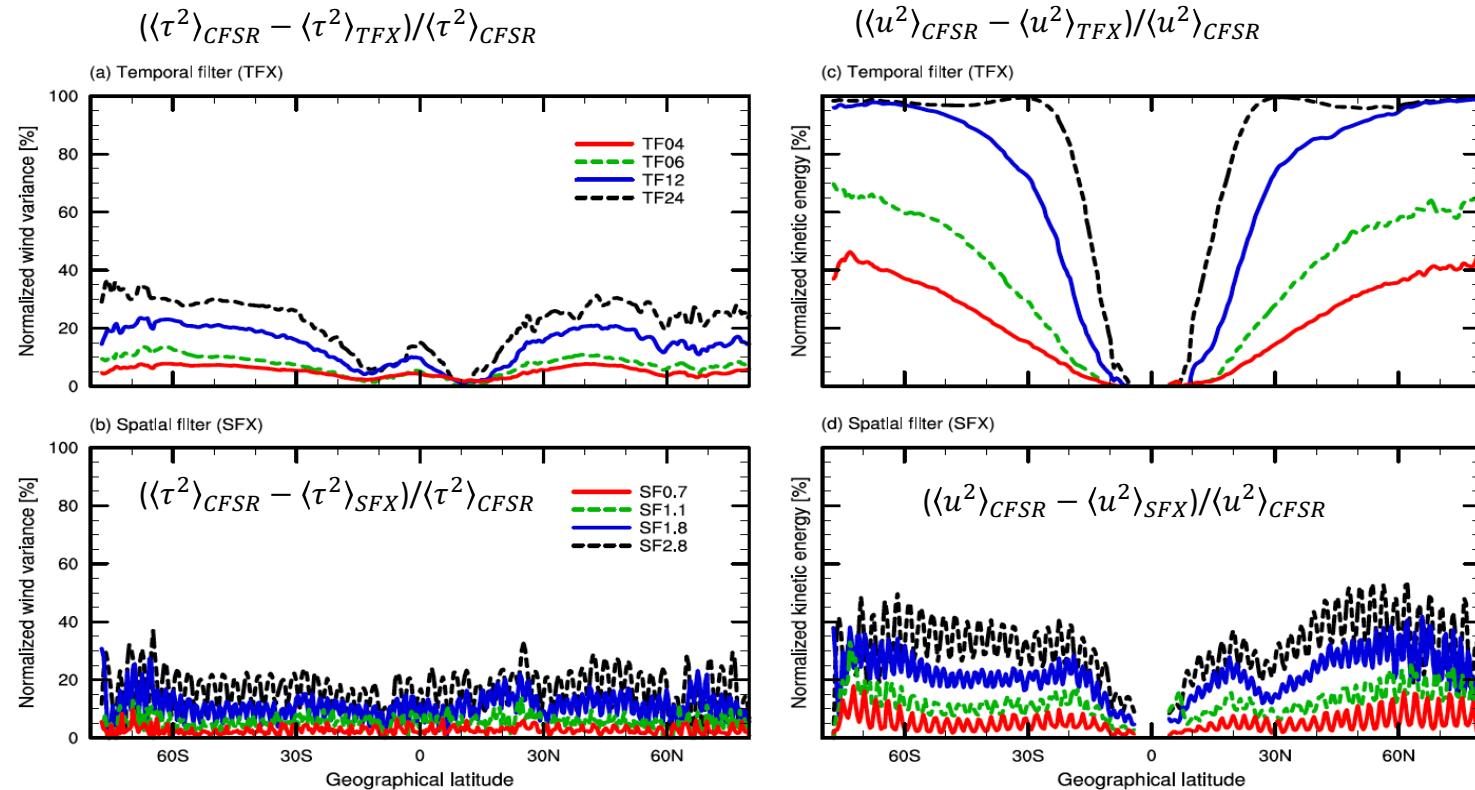


Figure 2. (left) Zonal mean difference in wind stress variance between experiment CFSR and experiments (a) TFX and (b) SFX normalized by the respective wind stress variance in experiment CFSR. (right) Zonal mean difference in the near-inertial kinetic energy between experiment CFSR and experiments (c) TFX and (d) SFX normalized by the near-inertial kinetic energy from experiment CFSR.

Experiment Name	Temporal Resolution	Spatial Resolution
NCEP	6-hourly	T63 grid
TFX:	1-hourly	T383 grid
	4-hourly	T383 grid
	6-hourly	T383 grid
	12-hourly	T383 grid
	daily	T383 grid
SFX:	1-hourly	0.7°grid
	1-hourly	1.125°grid
	1-hourly	1.875°grid
	1-hourly	2.8°grid

TOP
BOTTOM

T63 grid = 1.875° resolution
T383 grid = 0.35° resolution

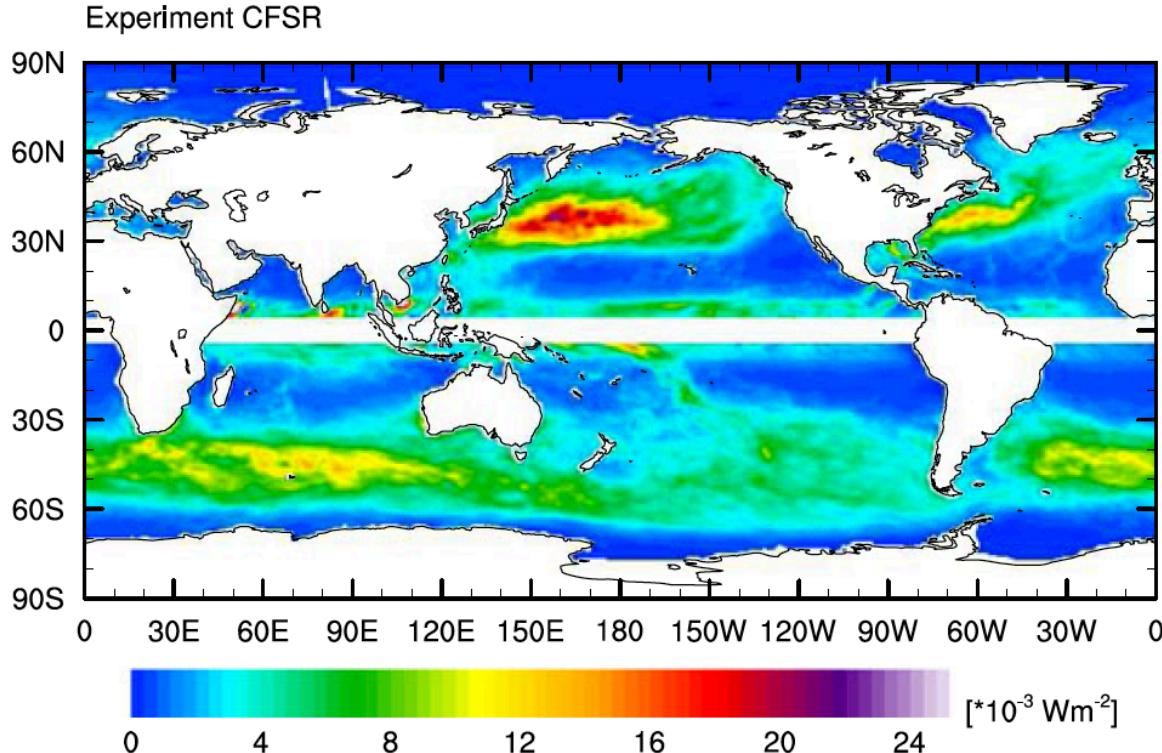


Figure 3. Spatial distribution of the wind power input to near-inertial (NI) motions calculated as the cross-spectrum between current velocities and wind stress and integrated over the NI frequency range in Wm^{-2} . A Daniell estimator is used to estimate the cross-spectrum [von Storch and Zwiers, 1999]. Note that the method based on the integrated cross-spectrum (defined by a raw cross-periodogram) is identical to the method based on data filtered in respect to the same frequency range.

Eliot and Lumpkin GRL 2008:
Estimation of the NI kinetic energy from surface drifters

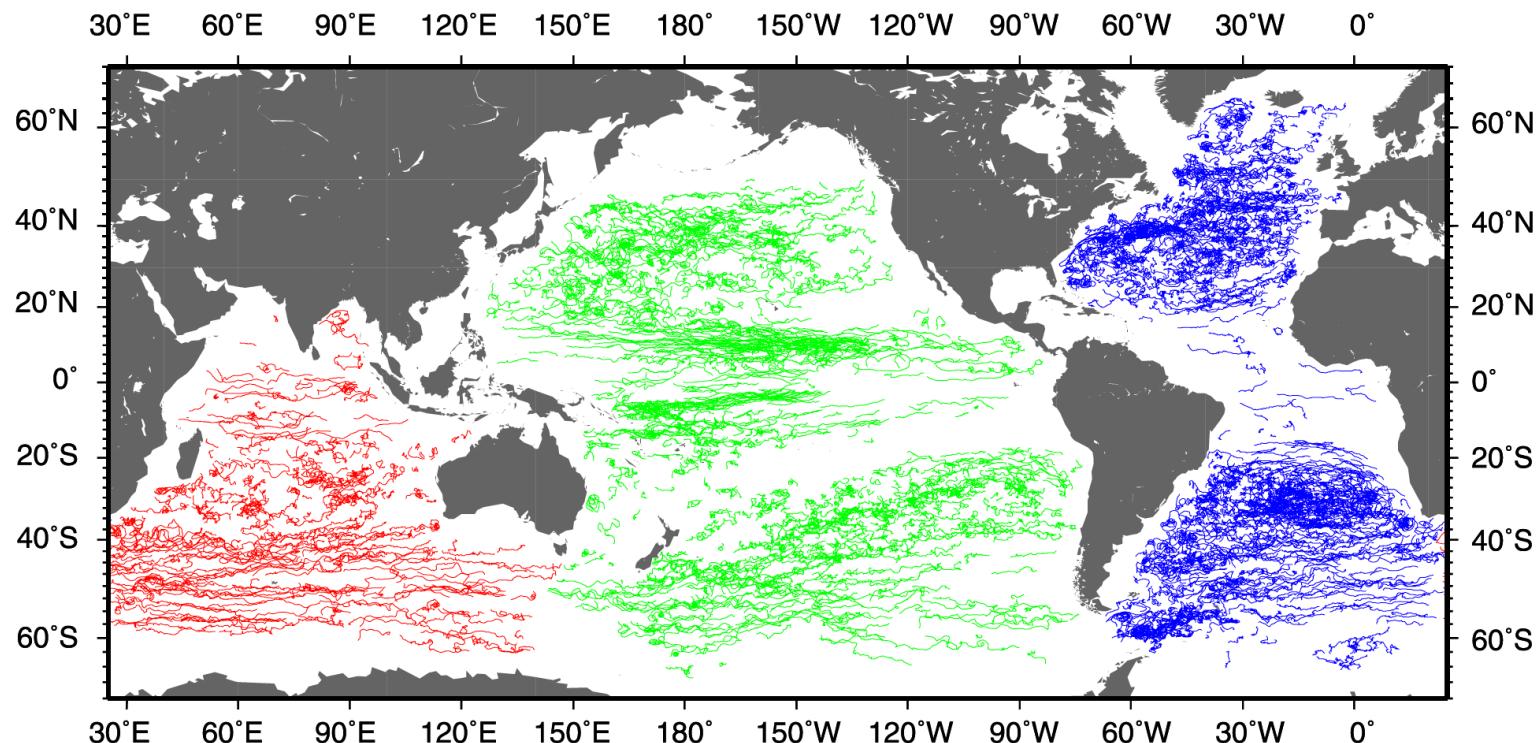


Figure 1. Drifter trajectory segments used for this study.

January 2000 to June 2007

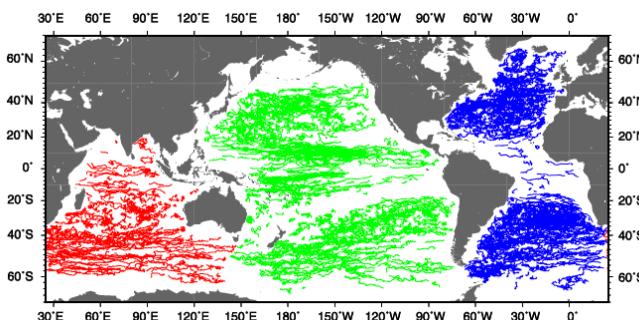
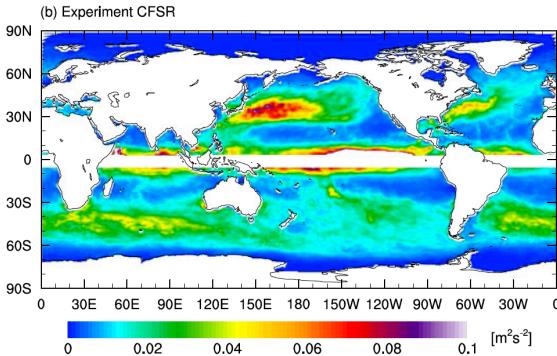


Figure 1. Drifter trajectory segments used for this study.

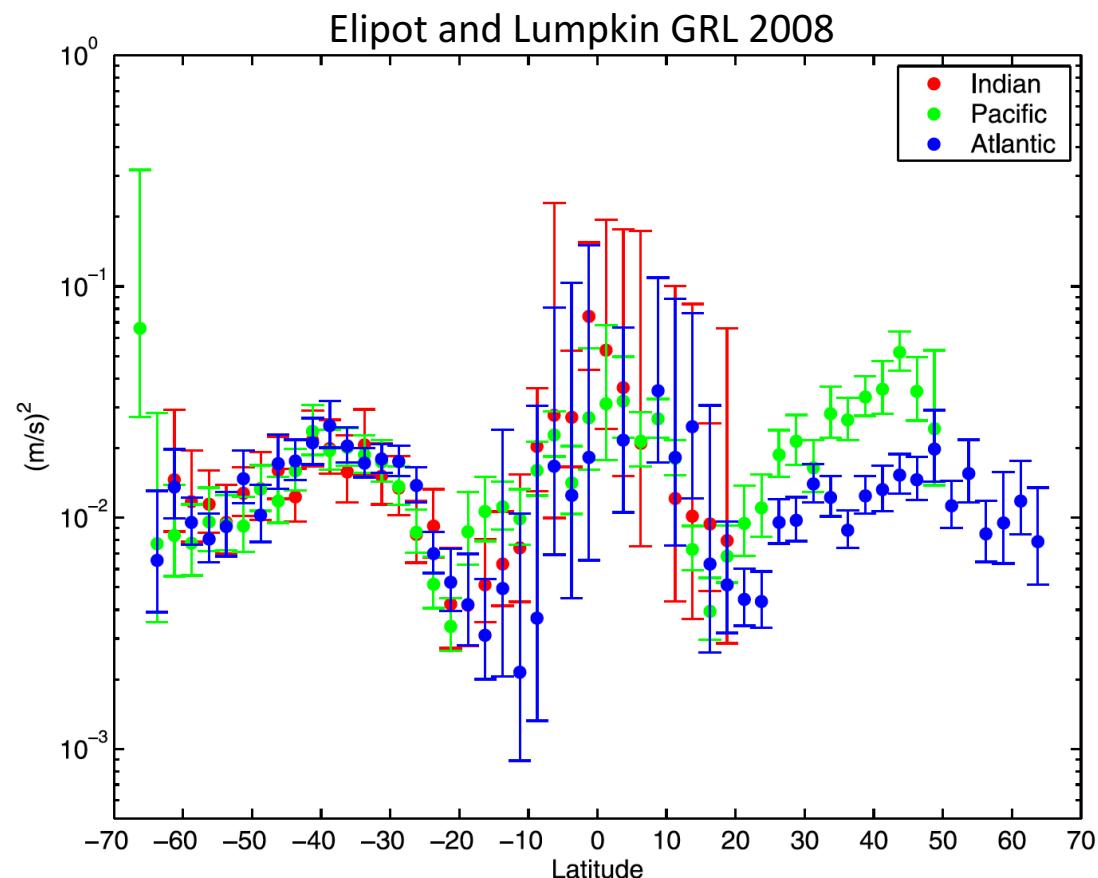


Figure 3. Drifter velocity variance in the near-inertial band for the Indian, Pacific and Atlantic basins in 2.5 degrees latitudinal bands. Error bars are derived from the 95% confidence intervals of the rotary spectrum estimates.

Propagation of near-inertial waves into the deep ocean

Experimental evidence

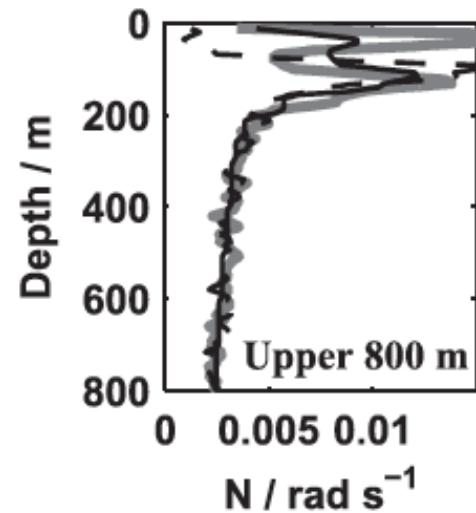
WKB APPROXIMATION

N² USUALLY DEPENDS ON Z (N²=N²(Z)). THIS AFFECTS THE PROPAGATION OF NIW KE. IN THAT CASE WE NEED TO SOLVE:

$$\frac{\partial}{\partial t} \left[\left(\frac{p'_z}{N^2} \right)_{ztt} + f^2 \left(\frac{p'_z}{N^2} \right)_z + \Delta p' \right] =$$

Using $p'(x,y,z,t) = P(z).e^{-i(k.x+l.y-\omega t)}$, leads to:

$$\left(\frac{P_z}{N^2} \right)_z - \frac{k^2 + l^2}{f^2 - \omega^2} P = 0 \quad (1)$$



- ONE SOLUTION IS TO USE THE NORMAL VERTICAL MODES FOR P(z) (SEE NEXT CLASS). THIS IS POSSIBLE WHEN THE NIW KE IS KNOWN FROM THE SURFACE DOWN TO THE BOTTOM.
- **ONE SOLUTION IS TO USE THE WKB APPROXIMATION (POSSIBLE IF N² IS SMOOTHLY VARYING ON THE VERTICAL \approx N² is constant locally).**

USING $P(Z)=P_m(z).\cos[m(z).z]$ IN (1) LEADS TO:

$$m^2(z) = \frac{N^2(z)(k^2 + l^2)}{\omega^2 - f^2}$$

WKB APPROXIMATION

$$m^2(z) = \frac{N^2(z)(k^2 + l^2)}{\omega^2 - f^2}$$

So, $m(z)$ is roughly proportional to $N(z)$ (since ω does not change following a wave packet). This means that the vertical scales become small for large N and large for small N .

Note that the vertical velocity group is still equal to:

$$C_{gz} = \frac{\partial \omega}{\partial m} = -\frac{\omega^2 - f^2}{\omega \cdot m(z)}$$

with frequency, ω unchanged with z (see next week).

On the other hand, the vertical energy flux has to be constant (there is no energy accumulation at a given level). So

$$F(z) = P_m^2(z) \cdot C_{gz}(z) = -P_m^2(z) \cdot \frac{\omega^2 - f^2}{\omega \cdot m(z)} = cst.$$

$P_m^2(z)$ and $m(z)$ are roughly proportional to $N(z)$. Thus, for each mode m , where $N(z)$ is large, $P_m^2(z)$ and $m(z)$ should be large and the opposite when where $N(z)$ is small (see next figures...)

$$\frac{P_m^2(z)}{N(z)} = \text{cst}$$

WKB SCALED MEANS SCALED BY $N(z)$

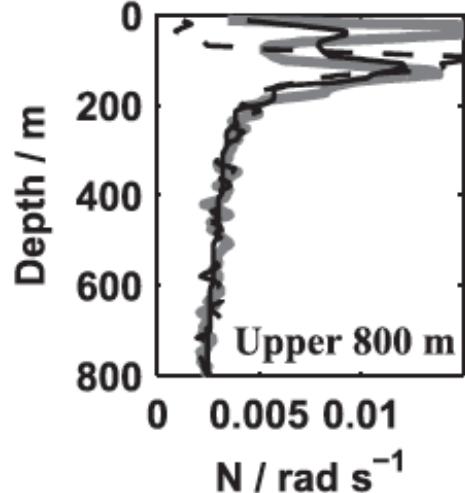
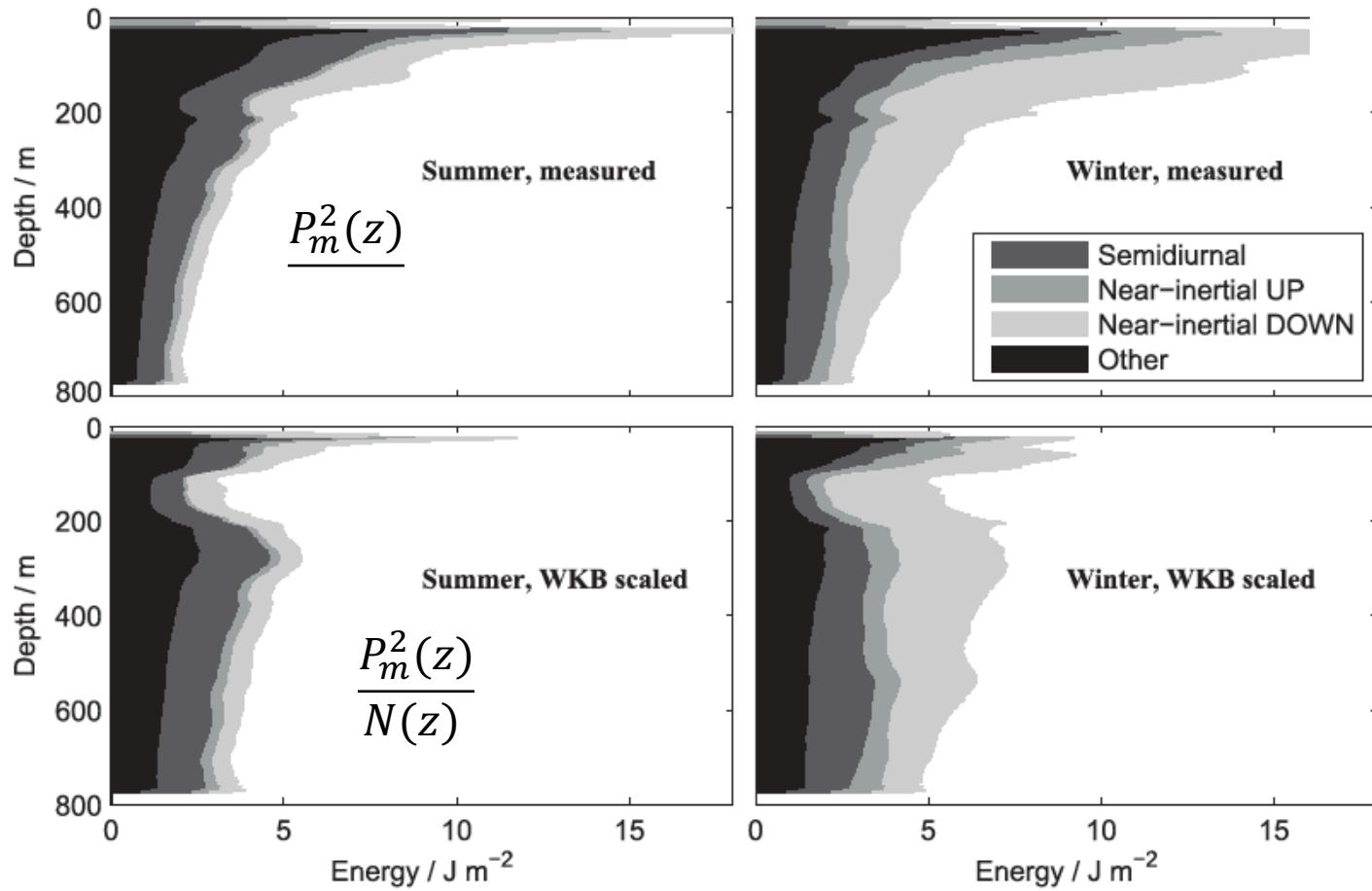
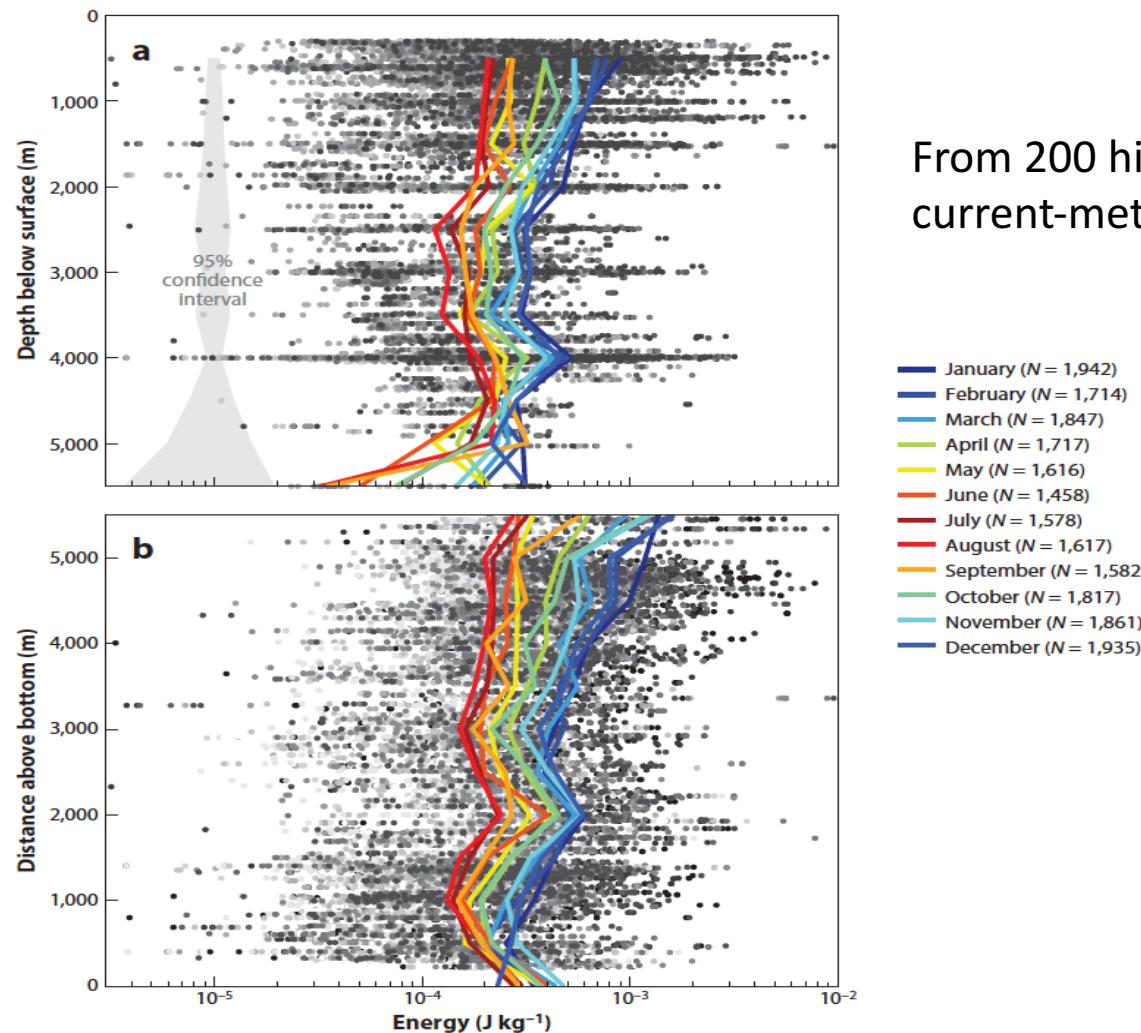


FIG. 12. Energy profiles for (left) summer and (right) winter, (top) measured and (bottom) WKB scaled.

Strong seasonality of NIWs explained by the seasonality of the atmospheric stormtracks



From 200 historical
current-meter records

Figure 6

Northern Hemisphere WKB-scaled near-inertial kinetic energy plotted versus (a) depth below the surface and (b) distance above the bottom. Individual 30-day estimates are plotted with dots, with lighter shades for summer months. Colored lines indicate the 500-m boxcar average for each month; the legend additionally indicates the number of observations in each month. The gray area on the left side of panel a shows the 95% confidence intervals on the mean computed using the number of observations in each depth range. Abbreviation: WKB, Wentzel, Kramers, Brillouin. Modified from Alford & Whitmont (2007).

What makes the near-inertial motions to propagate in the deep ocean?

Let us consider the time evolution of a wave packet

The group velocity is: $\vec{C}_g = \nabla_k \omega$

$$C_{gx} = \frac{\partial \omega}{\partial k} = \frac{N^2 k}{\omega m^2} = \frac{\omega^2 - f^2}{\omega} \frac{k}{k^2 + l^2}$$

$$C_{gy} = \frac{\partial \omega}{\partial l} = \frac{N^2 l}{\omega m^2} = \frac{\omega^2 - f^2}{\omega} \frac{l}{k^2 + l^2}$$

$$C_{gz} = \frac{\partial \omega}{\partial m} = -\frac{N^2 (k^2 + l^2)}{\omega m^3} = -\frac{\omega^2 - f^2}{\omega m} \quad (\text{downward propagation if } \omega > 0)$$

These calculations are valid for near-inertial motions, tidal motions and, higher frequency motions!

Example for near-inertial motions:

$$m = \frac{\pi}{200} \text{ m}^{-1}, \omega = 1.02f, \text{ leads to: } C_{gz} \sim -22 \text{ m. d}^{-1}$$

$$k = \frac{2\pi}{20 \text{ km}}, \omega = 1.02f, \text{ leads to: } C_{gx} \sim 0.013 \text{ m. s}^{-1}$$

[10 days to propagate over 10 km!]

Near-inertial motions propagate slowly if their length scales are small!

What makes the near-inertial motions to propagate in the deep ocean?

Interaction with balanced motions that strongly decreases their length scales

Their slow group velocity makes near-inertial waves likely to interact strongly with mesoscale/submesocale structures. These interaction can take several forms, including mesoscale/submesoscale influence on the wind generation process, **refraction and trapping of propagating waves, and two-way nonlinear energy transfers with mesoscale and submesoscale features such as fronts.**

Physics involved in the dispersion of waves by balanced motions

$$\frac{\partial}{\partial t} \begin{bmatrix} u \\ v \end{bmatrix} + \begin{bmatrix} S_1/2 & S_2/2 \\ S_2/2 & -S_1/2 \end{bmatrix} + \begin{bmatrix} 0 & -(f + \zeta/2) \\ (f + \zeta/2) & 0 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = - \begin{bmatrix} p_x \\ p_y \end{bmatrix}$$

[]

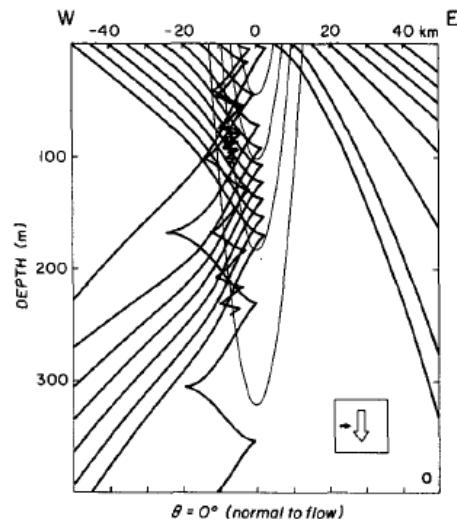
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*Strain
(growth or decay)*

Rotation

with: $S_1 = U_x - V_y$, $S_2 = V_x + U_y$, $\zeta = V_x - U_y$

$$\frac{\partial p}{\partial t} + f^2 \cdot r^2 (u_x + v_y) = 0$$



$$\omega_r^2 \approx f^2 + f \cdot \zeta - (S_1^2 + S_2^2 - \zeta^2)/4 + f^2 r_m^2 \cdot (k^2 + l^2)$$

$$\omega_i \approx i \cdot [k \cdot l \cdot S_2 + (k^2 - l^2) \cdot S_1]$$

Interaction of near-inertial waves with balanced motions

Their slow group velocity makes near-inertial waves likely to interact strongly with mesoscale/submesocale structures. These interaction can take several forms, including mesoscale/submesoscale influence on the wind generation process, refraction and trapping of propagating waves, and two-way nonlinear energy transfers with mesoscale and submesoscale features such as fronts.

These interactions make NIWs to have smaller horizontal scales and therefore to quickly propagate downward .

Analysis of satellite altimetry and surface drifters has observationally confirmed the general tendency of near-surface inertial motions to be organized by mesoscale vorticity (Elipot et al. 2010).

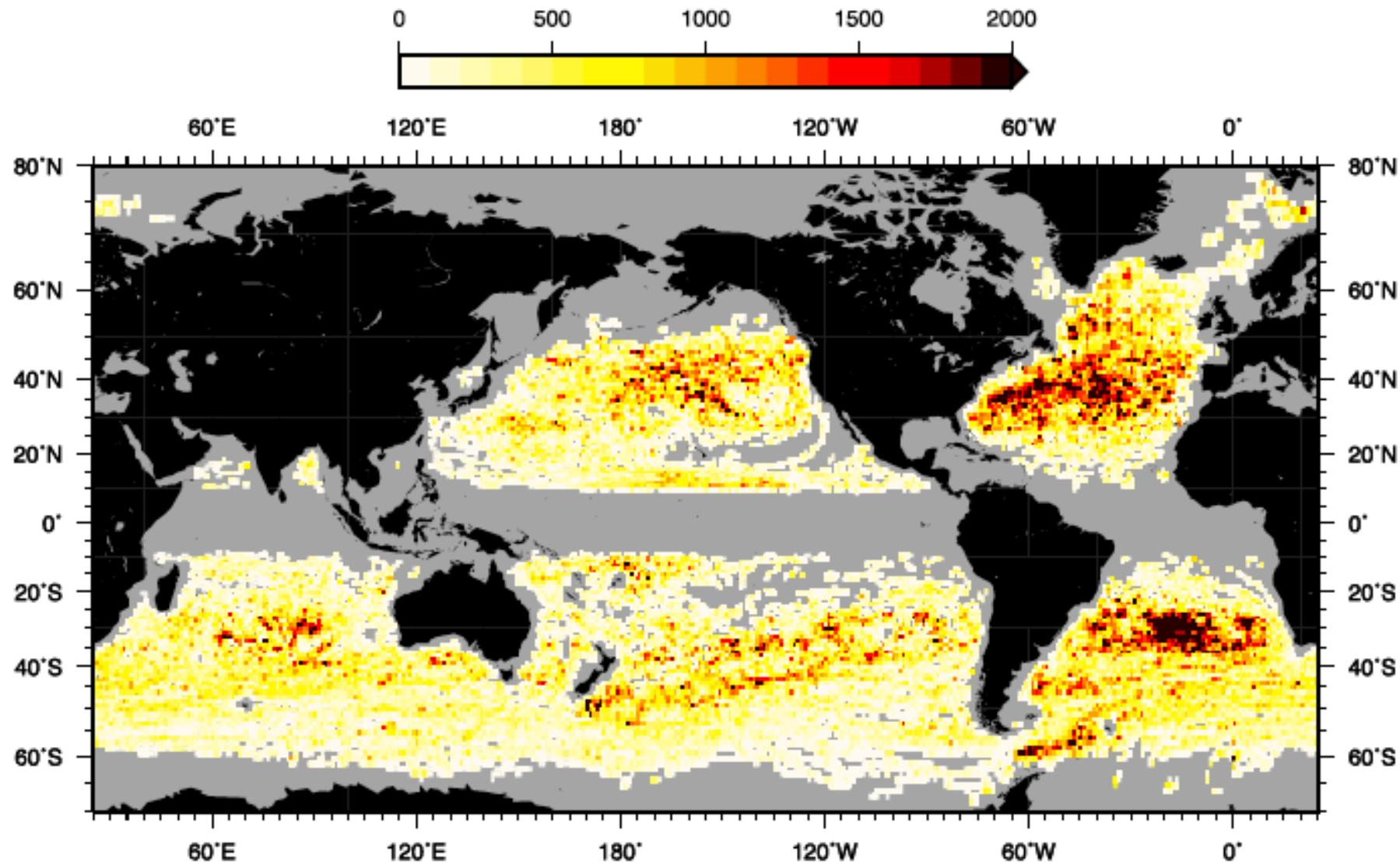
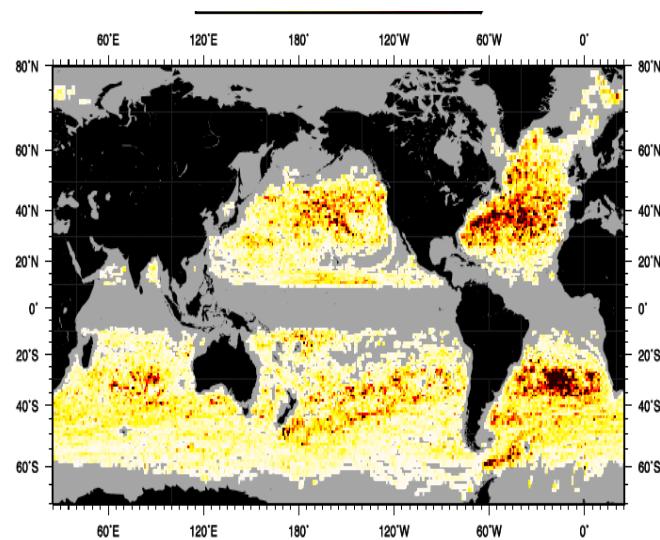
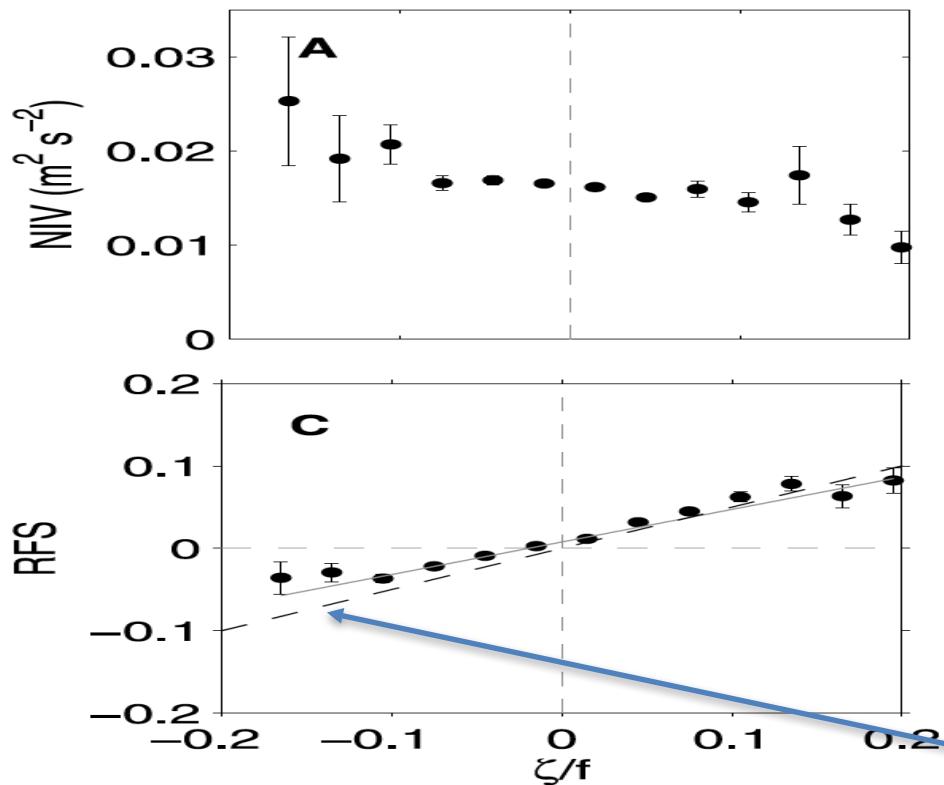


Figure 1. Number of hourly velocity observations in 1° bins. Gray indicates absence of data or areas where no data were selected for the analysis.



$$RFS = (\omega - f)/f$$

From Kunze (1985):

$$(\omega - f)/f = 0.5\zeta/f$$

Figure 15. (a) Mean near-inertial variance, (b) mean inverse excess bandwidth, and (c) mean relative frequency shift when the data are binned by values of ζ/f . The solid gray line in Figure 15c corresponds to the weighted least-square regression on the data $\Delta f/f = 0.39\zeta/f + 0.007$. The dashed black line corresponds to the theoretical $\Delta f/f = 0.5\zeta/f$. Positive ζ/f correspond to cyclonic vorticity anomalies. Error bars correspond to the standard error of the mean in each bin.

NEXT:

- **INTERNAL TIDES**
- **GM SPECTRUM REVISITED ...**