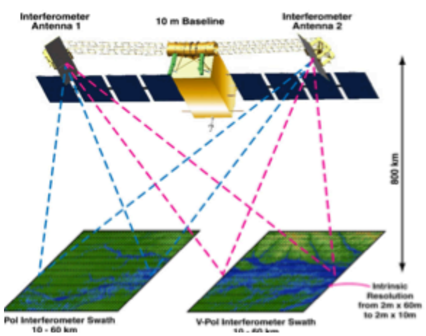
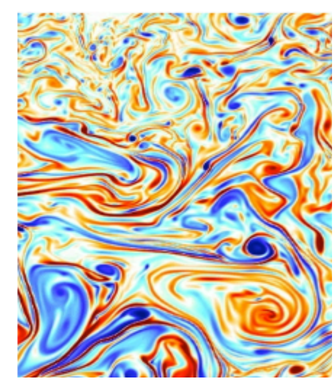
“Ocean Turbulence from SPACE”

Zhan Su (Caltech) and Patrice Klein (Caltech/JPL/Ifremer)

(XVII) Submesoscale instability (a):

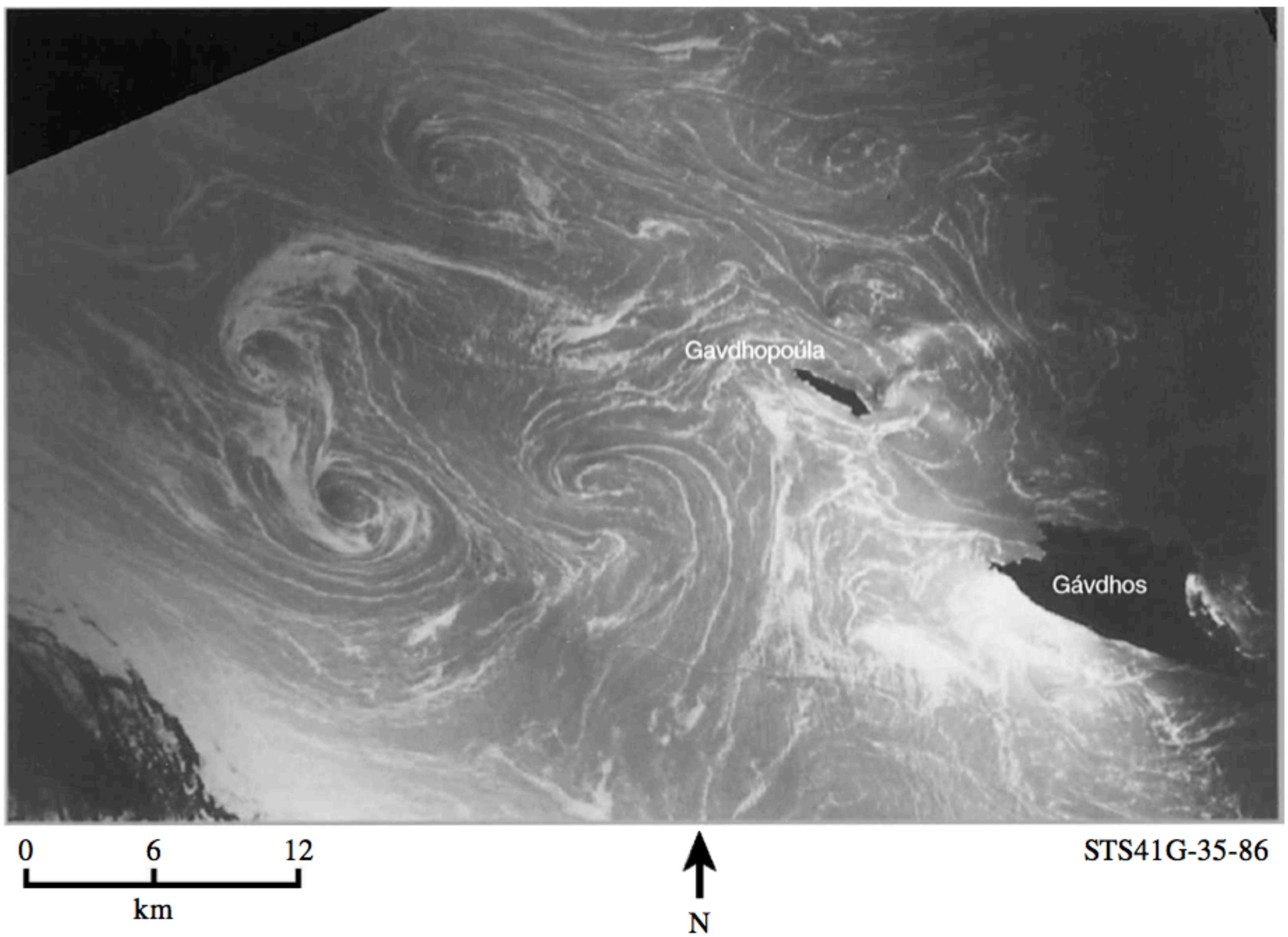
Spirals on the sea





"Spirals on the sea" by Munk et al. (2000)

spirals in the Mediterranean Sea

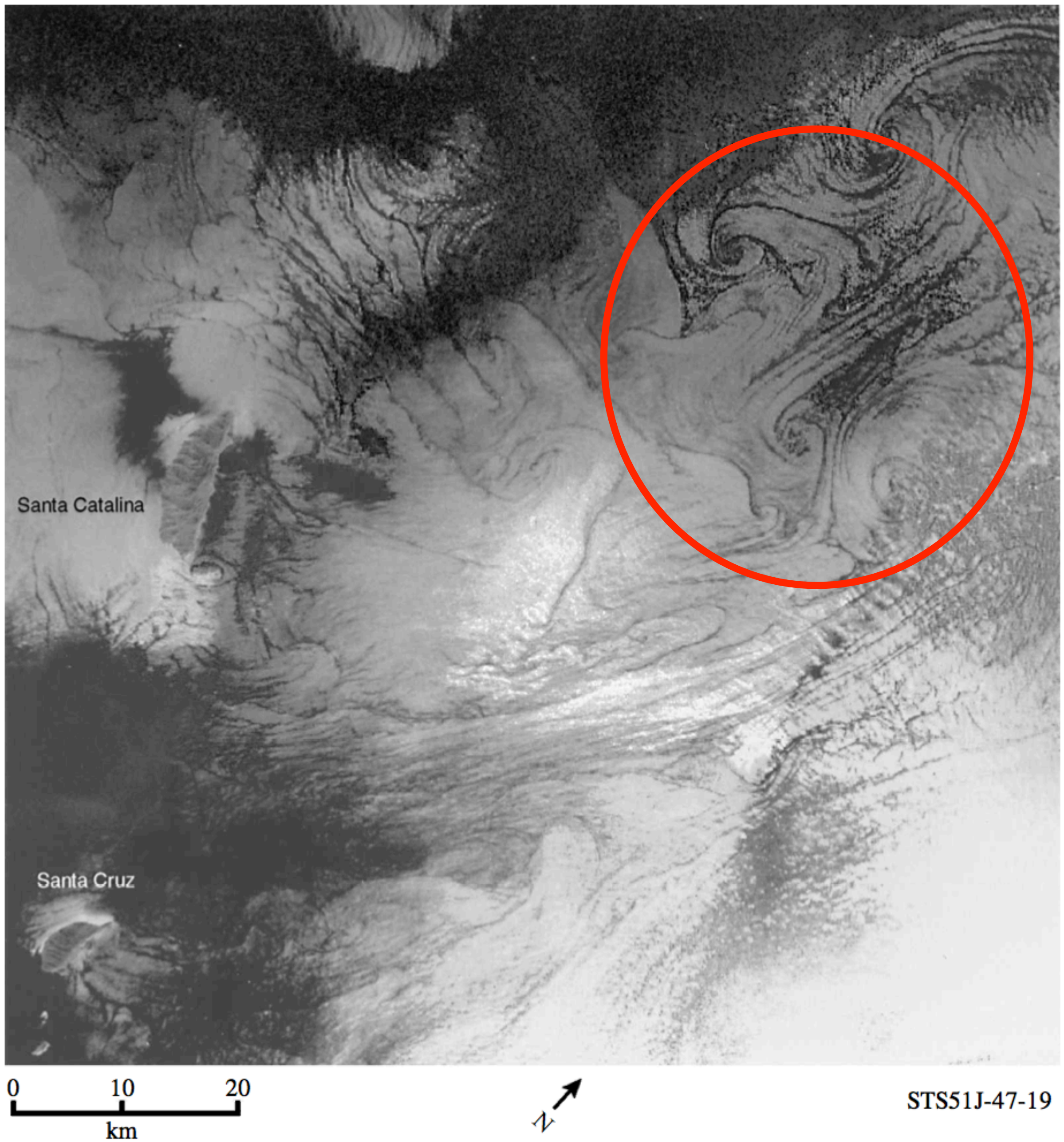


by sunglitter:
spiral (~10km)
& filament

ocean surfactants
as tracer

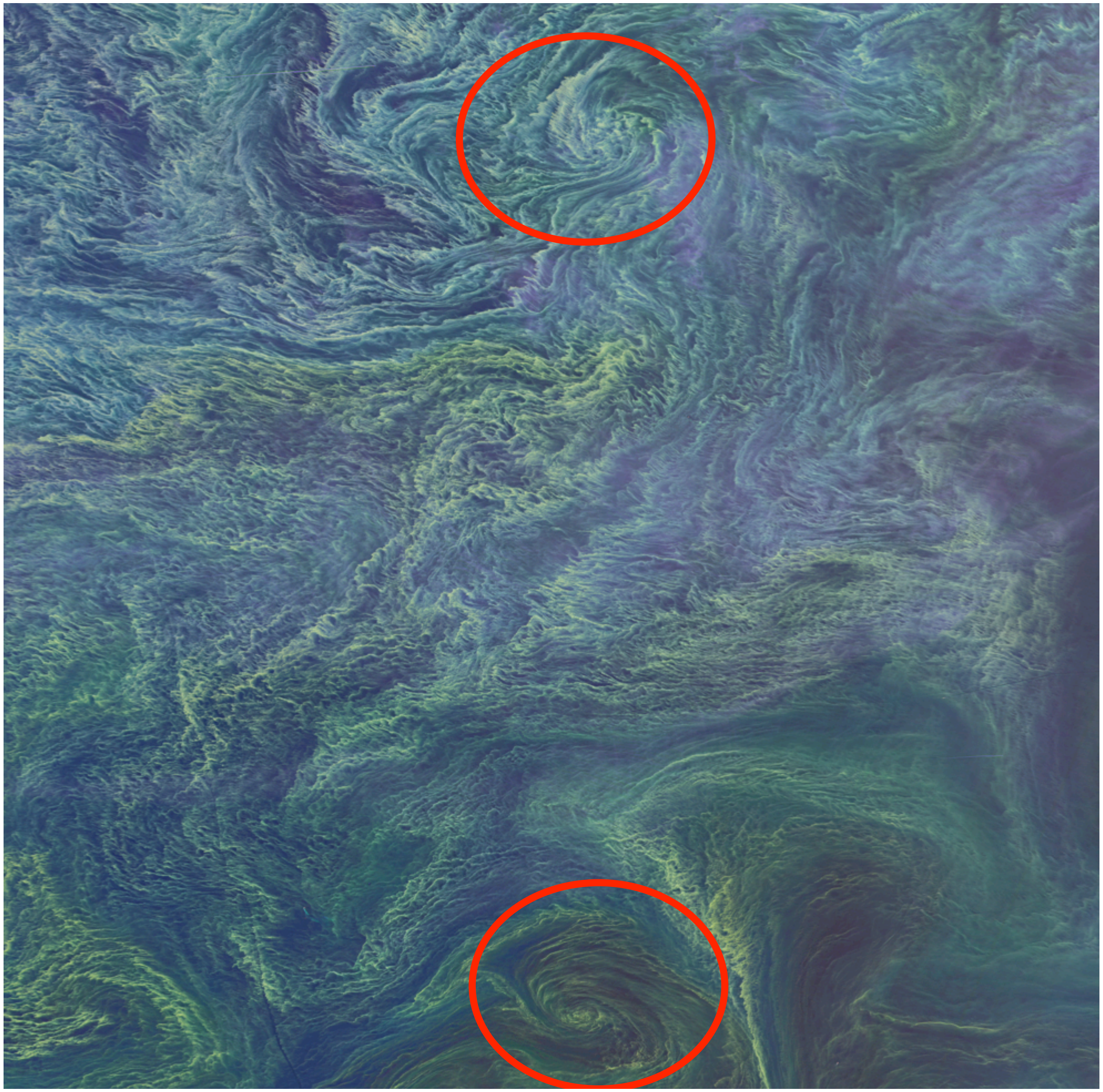
three questions of
this paper

Also around California



shows eddies

Recall Patrice's previous slide: Algal bloom in the Baltic Sea (tracer distribution)



why study it?

submesoscale is highly divergent and transport neutrient vertically….

vs mesoscale (horizontal)

0 if Ro<<1

$$\frac{D\boldsymbol{u}}{Dt} + \boldsymbol{f} \times \boldsymbol{u} = -\frac{1}{\rho}\nabla_h P$$

Ro 1 1

if ϱ is constant and *f* is constant, by taking curl, we get dw/dz=0 if Ro<<1

Recall the famous QG-omega equation, at $O(R_o)$ order not $O(1)$ order derive from basic equations (… a little like internal wave equation):

$$\nabla_H^2 w + \frac{f_0^2}{N^2}\partial_z^2 w = -\frac{2g}{\rho_0 N^2}\boldsymbol{\nabla}_H \cdot \mathbf{Q}$$

$$\mathbf{Q} = (\boldsymbol{\nabla}_H \mathbf{u})^{T} \nabla_H \rho$$

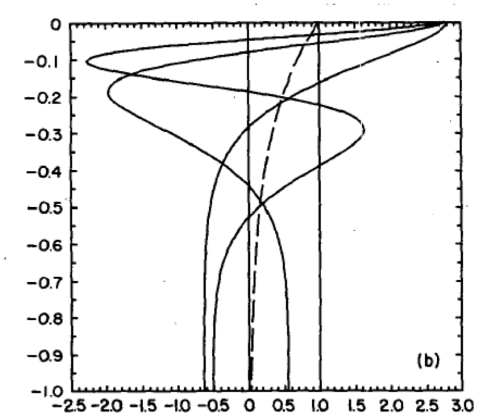
For the strain field (filament area), due to the inverse cascade of KE and direct cascade of density, the scale for ∇U is much larger than the scale for the density gradient. Therefore ∇Q is dominated by $\nabla^2 \varrho$

~0 because at z=0, w=0, but d^2w/dz^2 is nonzero at z=0

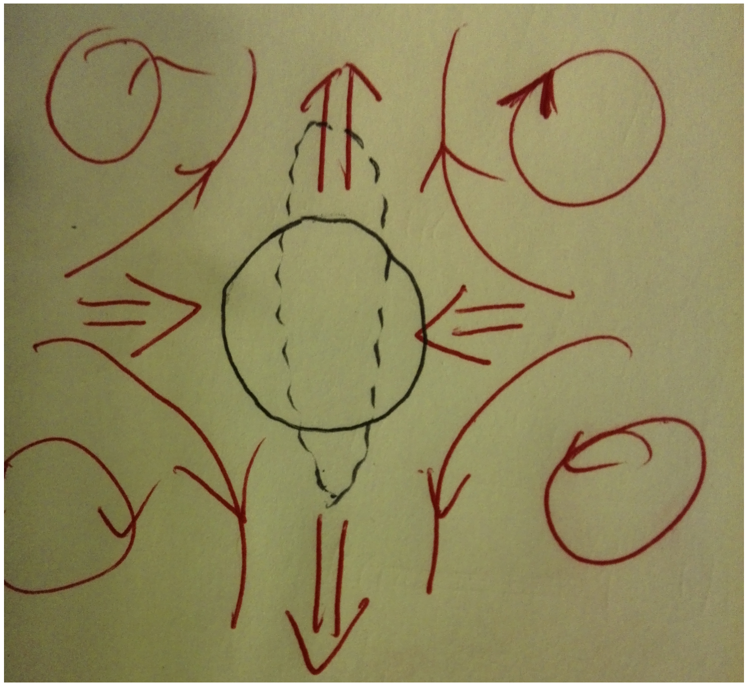
~w

$$\nabla_H^2 w + \frac{f_0^2}{N^2}\partial_z^2 w \propto \nabla^2 \varrho \implies w \propto \nabla^2 \varrho$$

true for the filaments, not necessarily true for the eddies



turbulence is trapped at surface
(left figure shows the mode of horizontal motions)
=> w also has an exponential decay
vertically => d^2w/dz^2 ~w



where does $\nabla \varrho$ (or $\nabla^2 \varrho$) come from?

strain field (red)

material line (black)

solid $\xrightarrow[\text{direct cascade of density}]{\text{frontogenesis} \atop \text{stirring by strain field}}$ dash

$\nabla \varrho$ increases at small scales,

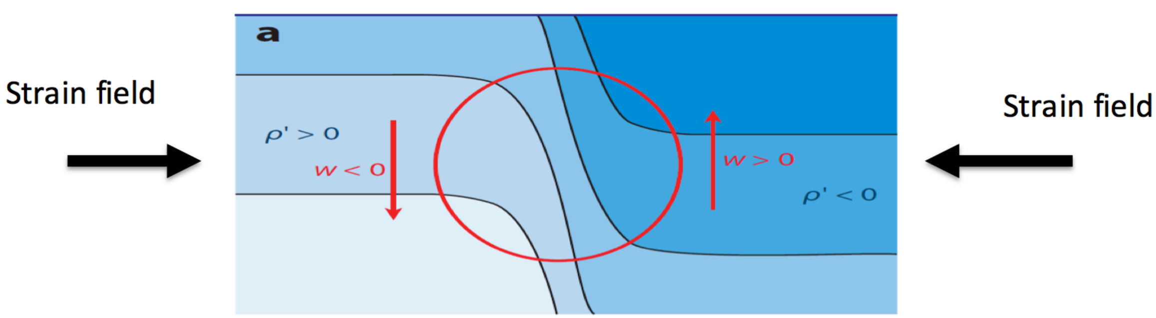
w $\propto \nabla^2 \varrho \propto k^2 \varrho$ increases with k

so submesoscale is key for w

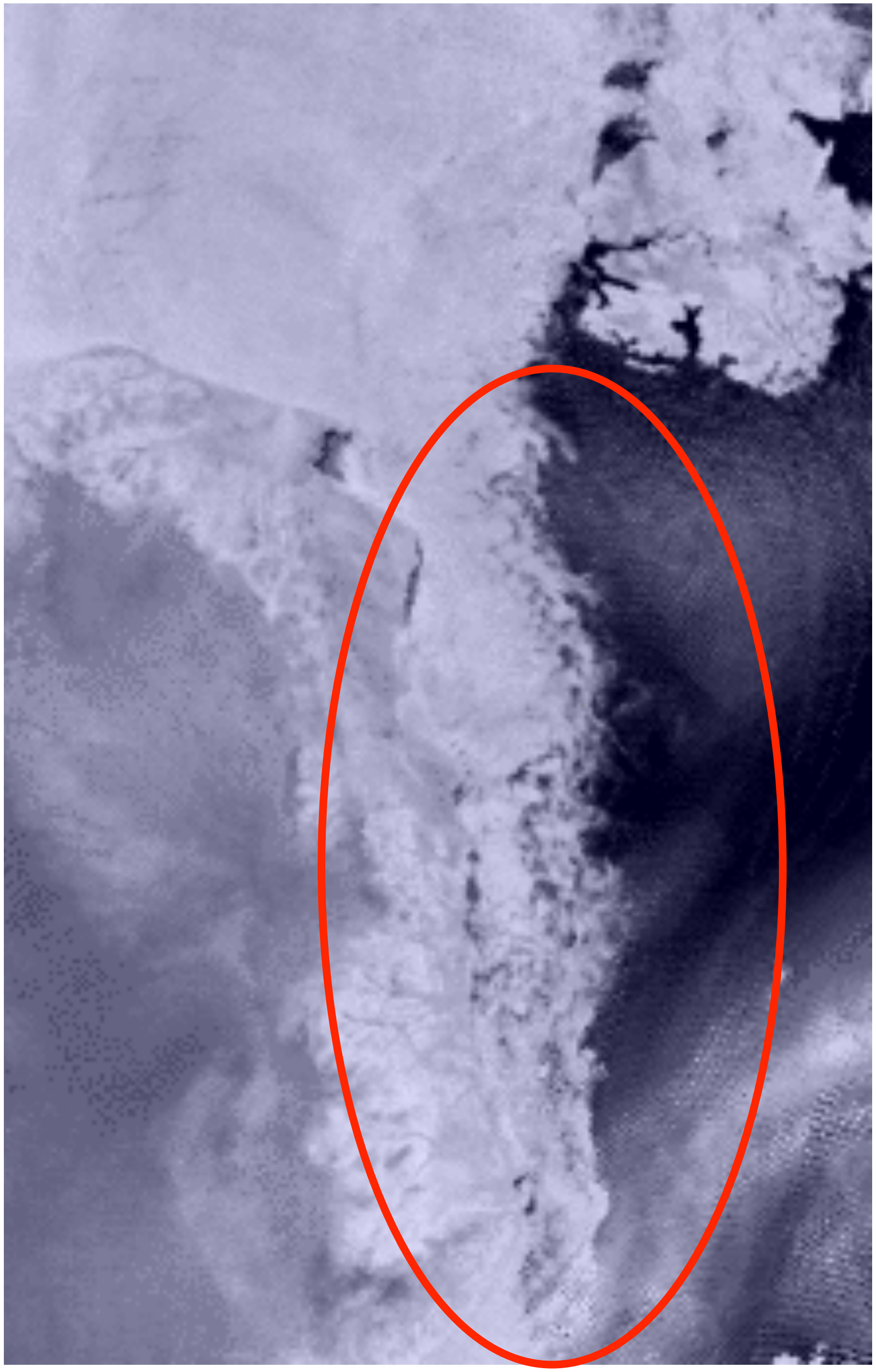
Recall Georgy's teaching: for SQG

$$|\widehat{\rho}_s|^2(k,l) = |\widehat{\boldsymbol{u}_s^0}|^2(k,l) \sim k^{-5/3}$$

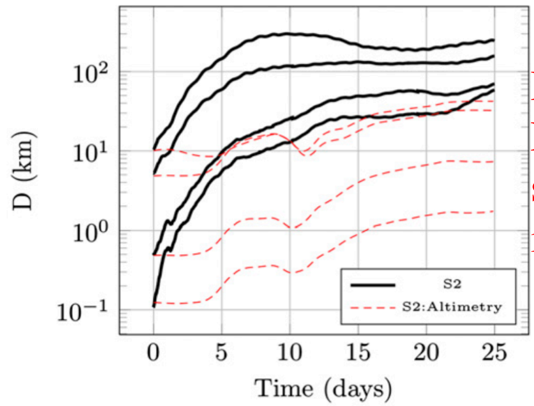
pretty flat, i.e, lots of density gradient are distributed in small scales



again, w is due to ASC, (stirring for v and b) restoring the thermal wind



around Greenland Sea,
sea-ice as tracer,
again in submesoscale (10-50km),
why study it?
submesoscale is key for dispersion
(turbulent mixing) of tracers (sea-ice)



Dispersion is 10-100 times LARGER when submesoscales are taken into account !

if the initial distance of two parcels has a scale within the enstropy inertial range (E_k ~k^{-3}), it is stirred by nonlocal strain field and has an exponential growth, i.e, δx~exp(t). In contrast, if the initial distance of two parcels has a scale within the energy inertial range (E_k ~$k^{-5/3}$), it is stirred by local strain field and has a smaller growth like δx~$t^{2.2}$.
Aviso data (red curve) can not resolve submeoscale so its strain field is weak, hence the growth rate for the disperson is also weaker in contrast to the resolved case (black)

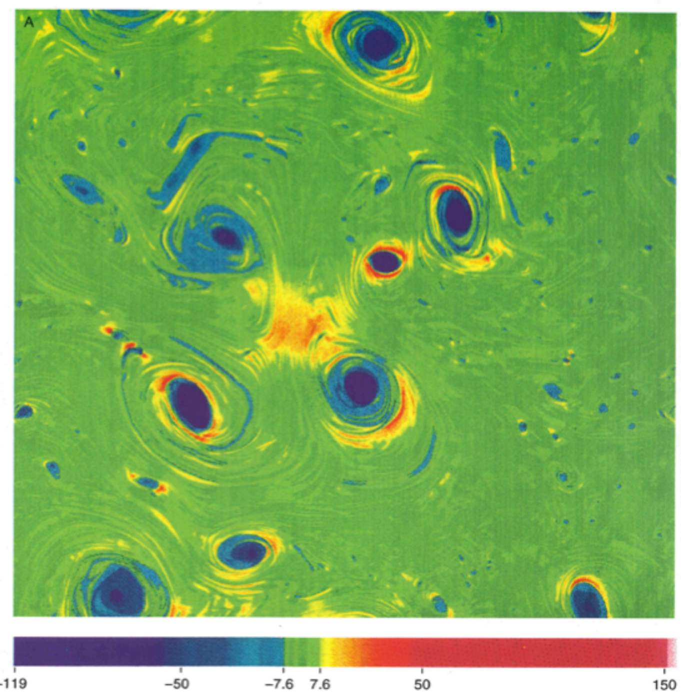
Lumpkin and Elipot 2010_JGR
Bennet 1984_JAS
Scott 2006_PF

Stirring of the strain field=> the dispersion of tracer

$$\frac{d\,\delta X}{dt} = [\nabla U]\,\delta X \qquad\qquad \frac{d\nabla C}{dt} = -[\nabla u]^T \nabla C$$

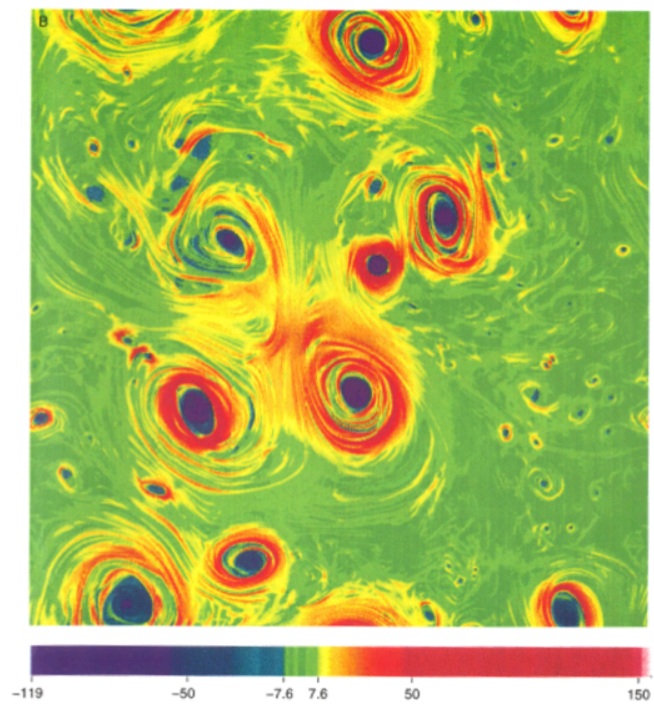
$$\nabla C(t) \approx \nabla C(t=0) \cdot \exp\left[\pm (d_\pm)^{1/2}\, t\right]$$

This is for the enstropy inertial range (exponential growth)



$$\lambda_+ = -\frac{1}{2}\Delta\tilde{p}$$

assuming a slow time evolution of the strain field

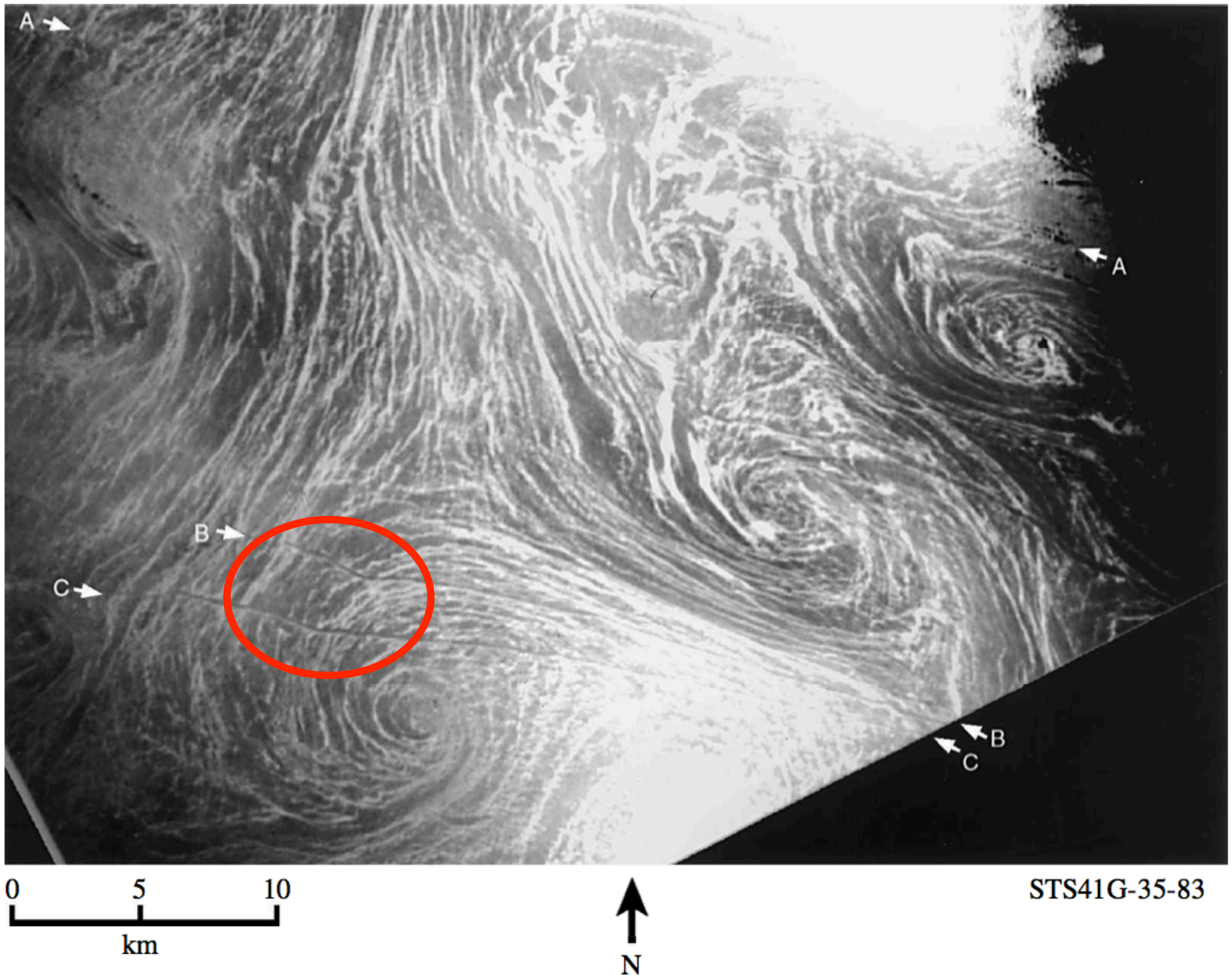


$$\lambda_+ = -\frac{1}{2}\Delta\tilde{p} + \frac{1}{2}\sqrt{\{(\tilde{p}_{xx} - \tilde{p}_{yy})^2 + 4\tilde{p}_{xy}^2\}}$$

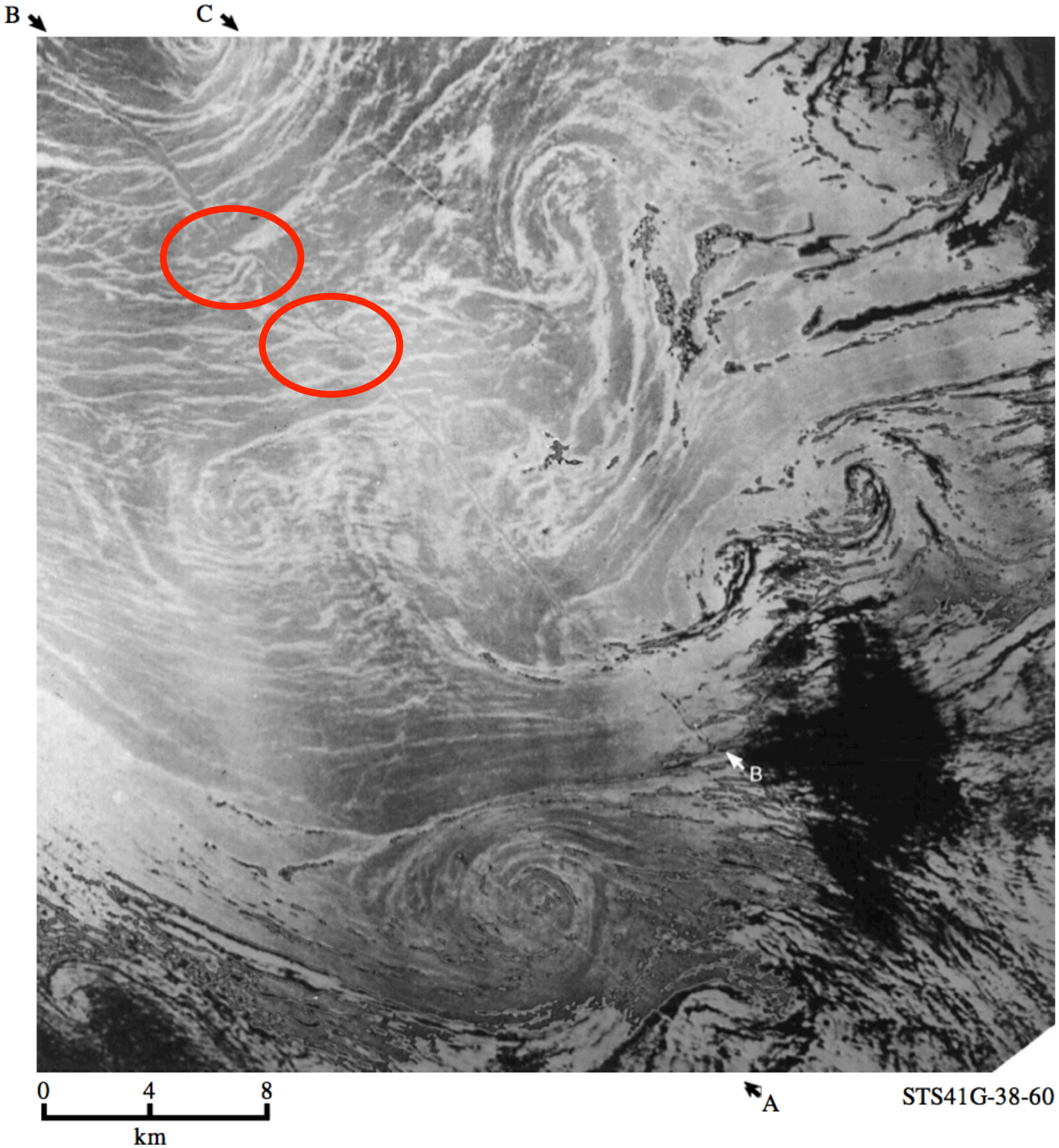
accounting for the evolution of the strain field

How the submesocale spirals/eddies are generated?
through shear (K-H) instability!

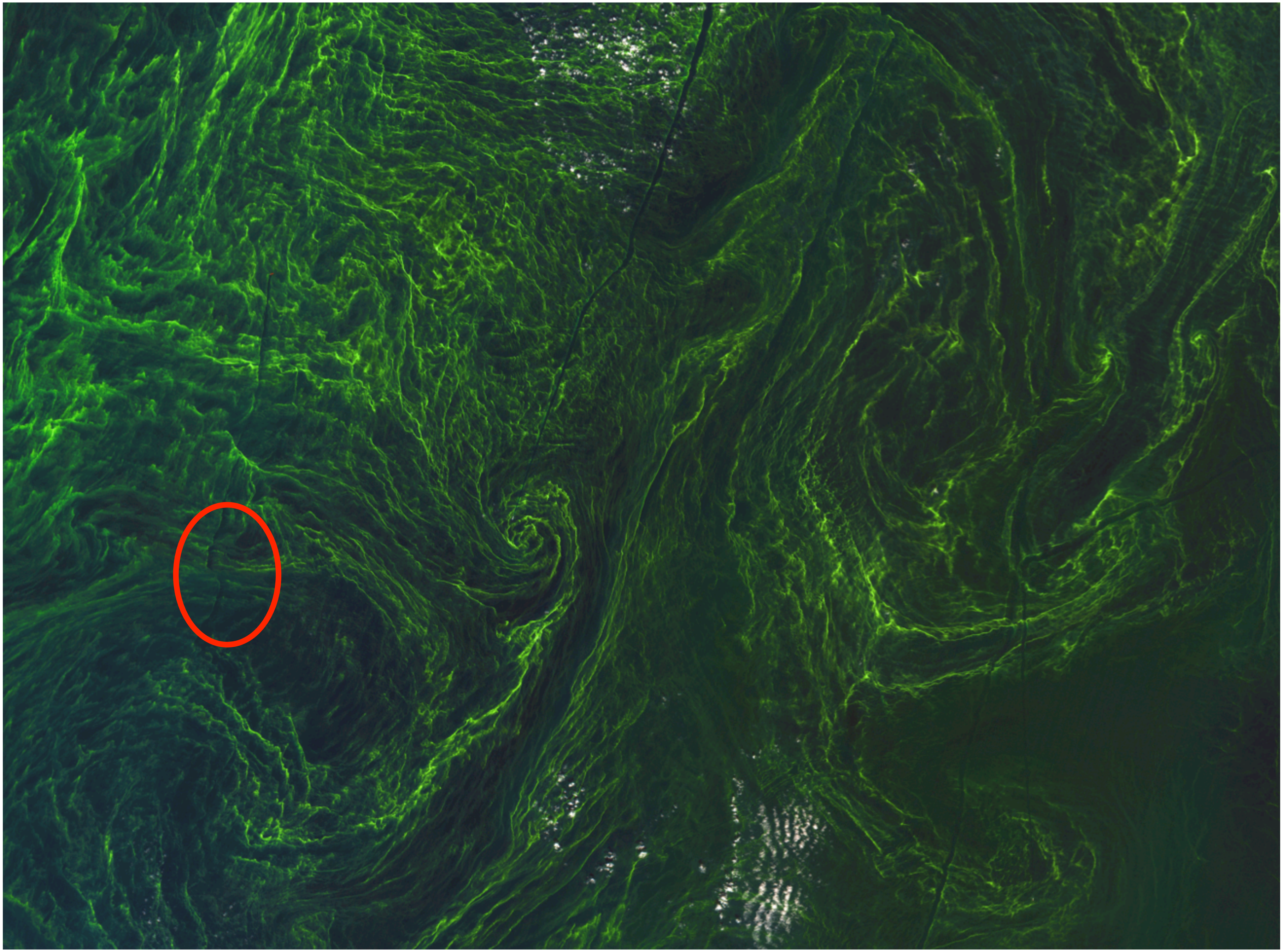
MKE (shear, U_{yy}) => EKE



ship track: showing strong velocity shear



ship track: showing strong velocity shear



Kelvin–Helmholtz instability examples

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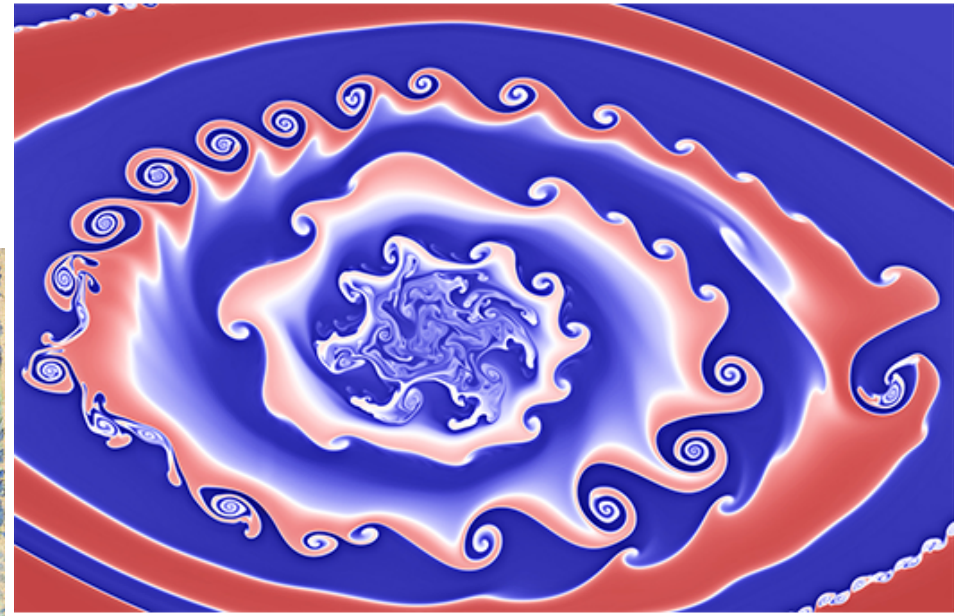
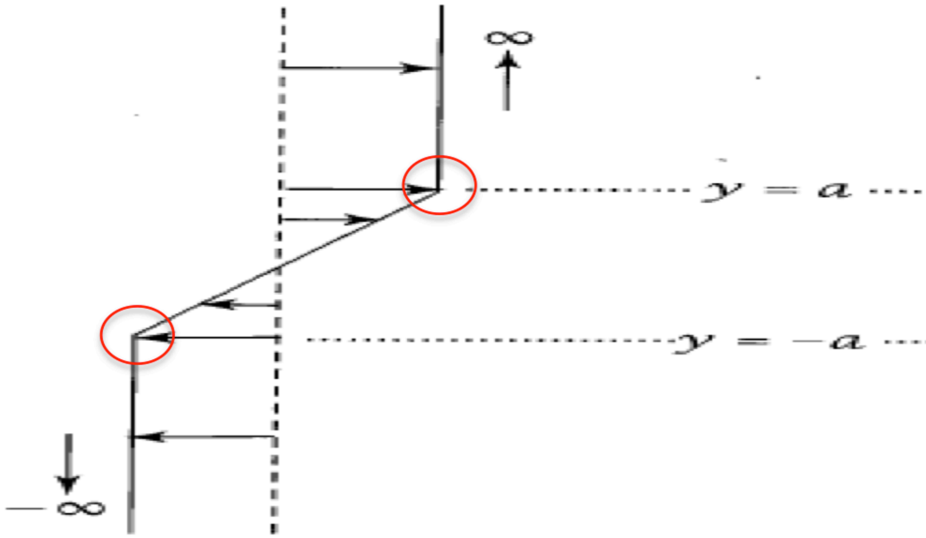
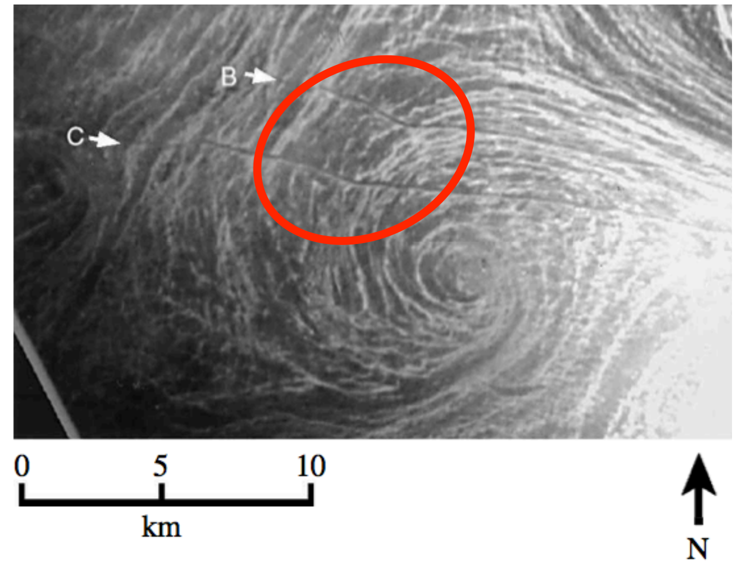
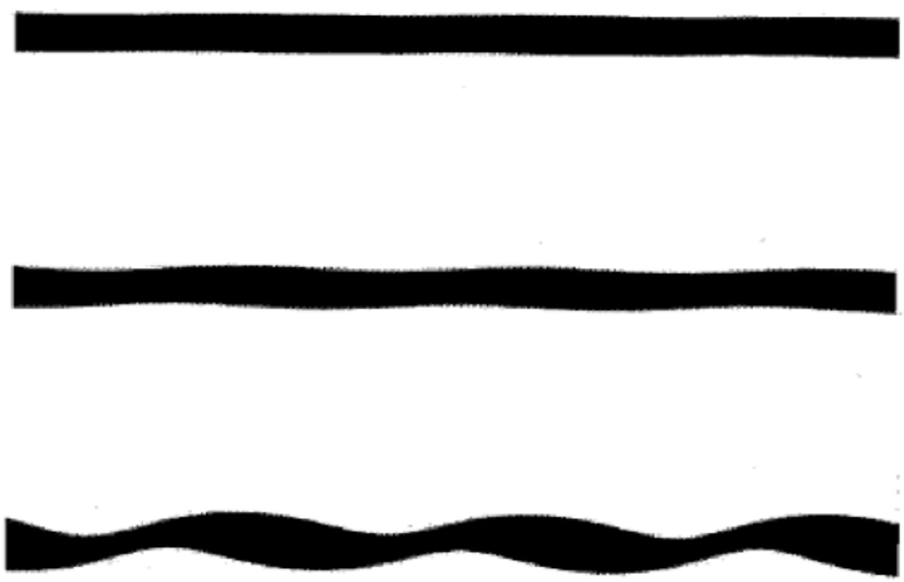


Shear (K-H) instability, due to interaction of edge waves





vallis 2006

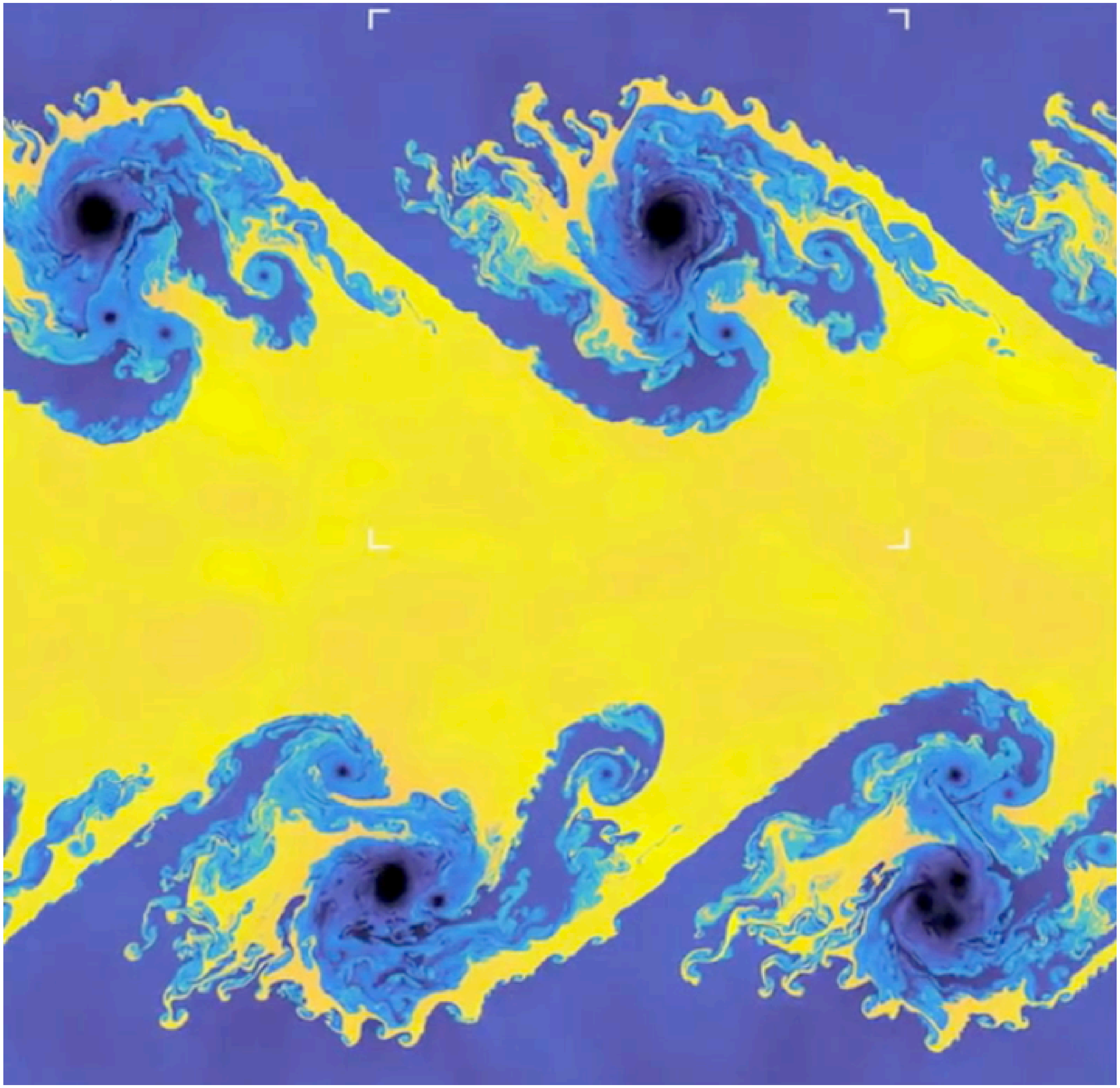


filament is unstable

K-H instability in 2D flow; showing conserved tracer (e.g., vorticity); not much submesocale features; inverse cascade of KE

for 2D, by scaling zeta(k)~(E_k*k^3)^0.5 ~k^0, so the zeta strength does not change with k

$E_k \sim k^{-3}$
for small scales



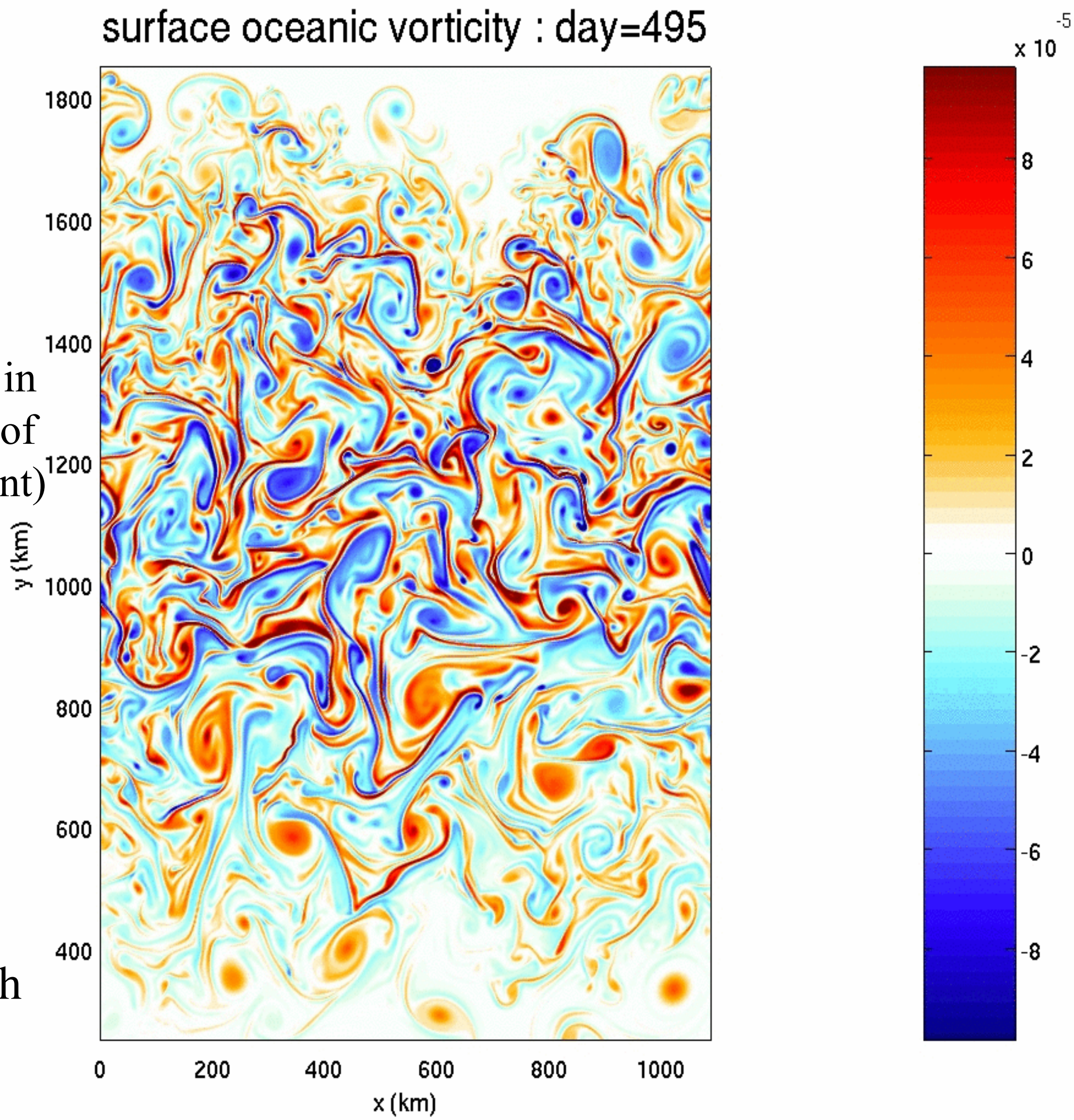
K-H instability in SQG induces an energetic submesoscale field

(Klein et al, JPO, 2008)

this slide shows vorticity, last slide shows conserved tracer (vorticity is a conserved tracer in 2D, just the initial distribution of conserved tracer can be different)

$E_k \sim k^{-5/3}$
for small scales

for SQG, by scaling zeta(k)~(E_k*k^3)^0.5 ~k^2/3, so the zeta strength increases with k

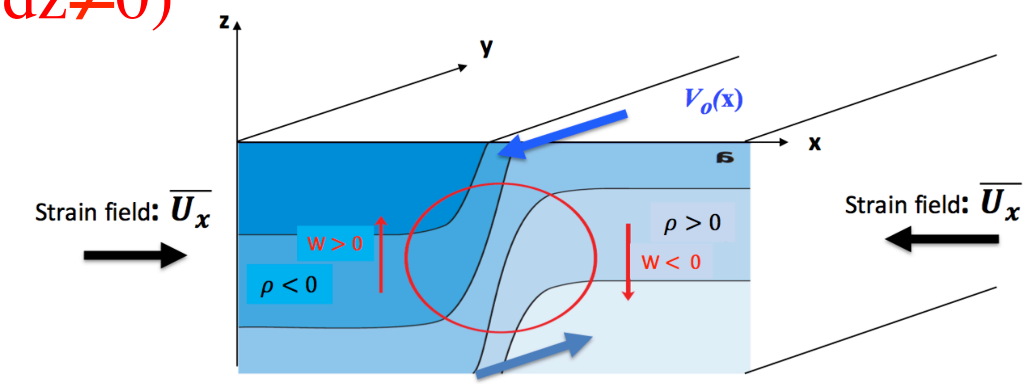


In 2D turbulence, the filaments are unstable but later is stabilized by the stirring mechanism, due to the conservation of vorticity.
i.e., there is no 3D vortex stretching to extract EKE from MKE.
i.e, 2D turbulence need extra forcing to generate eddies.

In SQG, the vorticity is not conserved and has vortex stretching.
i.e., vorticity (shear) can be generated at submesoscale by vortex stretching.

direct cascade of buoyancy (frontogenesis) by the stirring of SQG

=> ASC (vortex stretching, dw/dz≠0)
=> local jet acceleration
=> increasing of velocity shear
=> shear instability
=> eddies



i.e, SQG turbulence, in the absense of extra forcing, can still generate eddies by frontogenesis & shear instability.

why submesoscale spirals/eddies are usually cyclonic?

$$\frac{\partial v}{\partial t} + v \cdot \nabla v + f \cdot \vec{k} \times v = -\frac{1}{\rho_0} \nabla p$$

$$\nabla \cdot (v \cdot \nabla v) - f\xi = -\Delta p$$

R_o 1 1

< 0 for cyclonic
>0 for anticyclonic

< 0 for cyclonic
>0 for anticyclonic

for submesoscale, $R_o \sim 1$

$=OW=(S_1^{\,2} + S_2^{\,2} - \zeta^2) < 0$ for eddy region

$$\underbrace{\nabla\cdot(u\cdot\nabla u)}_{F_{centr}} - \underbrace{f\xi}_{F_{cor}} = -\underbrace{\Delta p}_{F_{press}}$$

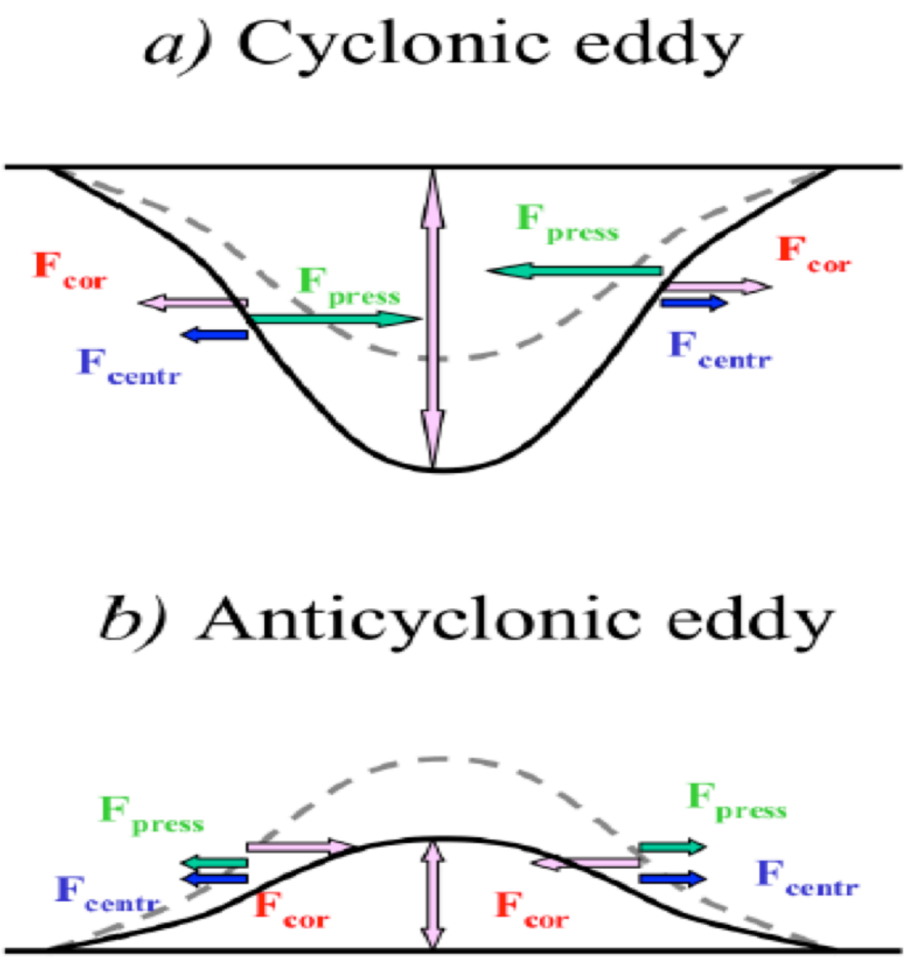


Figure adapted from Maximenko and Niiler (2006)

Another mechanism of why submesoscale spirals/eddies are usually cyclonic?

Taking curl of the momentum eq => vorticity equation in 3D

vortex stretching vortex tilting

$$\frac{\mathrm{D}}{\mathrm{D}t}(\zeta+f) = -(\zeta+f)\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)+\left(\frac{\partial u}{\partial z}\frac{\partial w}{\partial y}-\frac{\partial v}{\partial z}\frac{\partial w}{\partial x}\right)$$
$$+\frac{1}{\rho^2}\left(\frac{\partial \rho}{\partial x}\frac{\partial p}{\partial y}-\frac{\partial \rho}{\partial y}\frac{\partial p}{\partial x}\right)+\left(\frac{\partial F^y}{\partial x}-\frac{\partial F^x}{\partial y}\right).$$

baroclinic term extra forcing (wind…)

0 if beta=0

$$\frac{\mathrm{D}}{\mathrm{D}t}(\zeta+f) = -(\zeta+f)\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)$$

if just considering vortex stretching

For submesoscale, R_o~1 or |ζ| ~ f. If ζ and f have the same sign (cyclonic), then RHS≠0 and there is vorticity generation by vortex stretching. Otherwise there is no vorticity generation.