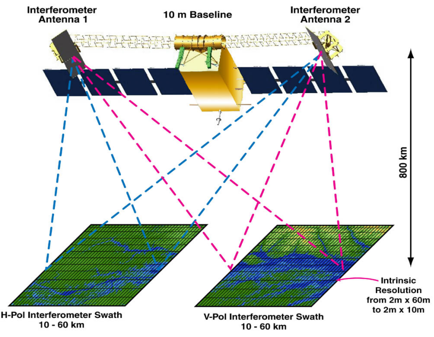
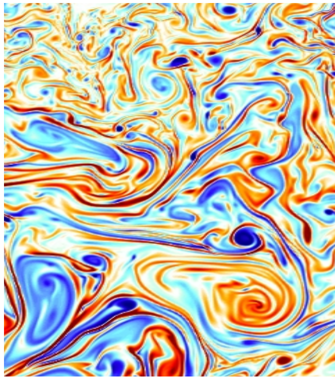
"Ocean Turbulence from SPACE"

Patrice Klein (Caltech/JPL/Ifremer)

(X) – Frontogenesis and Omega equation





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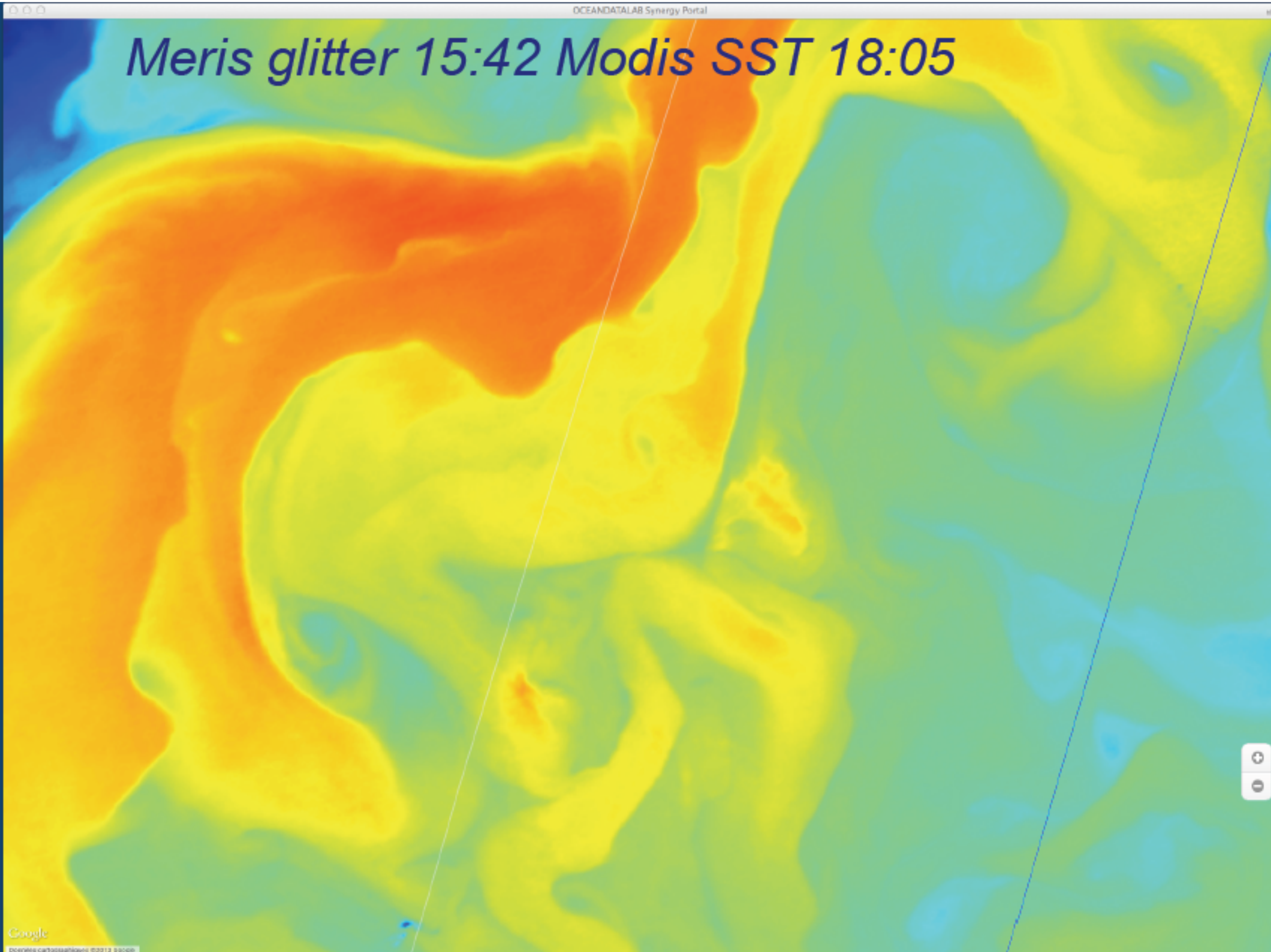

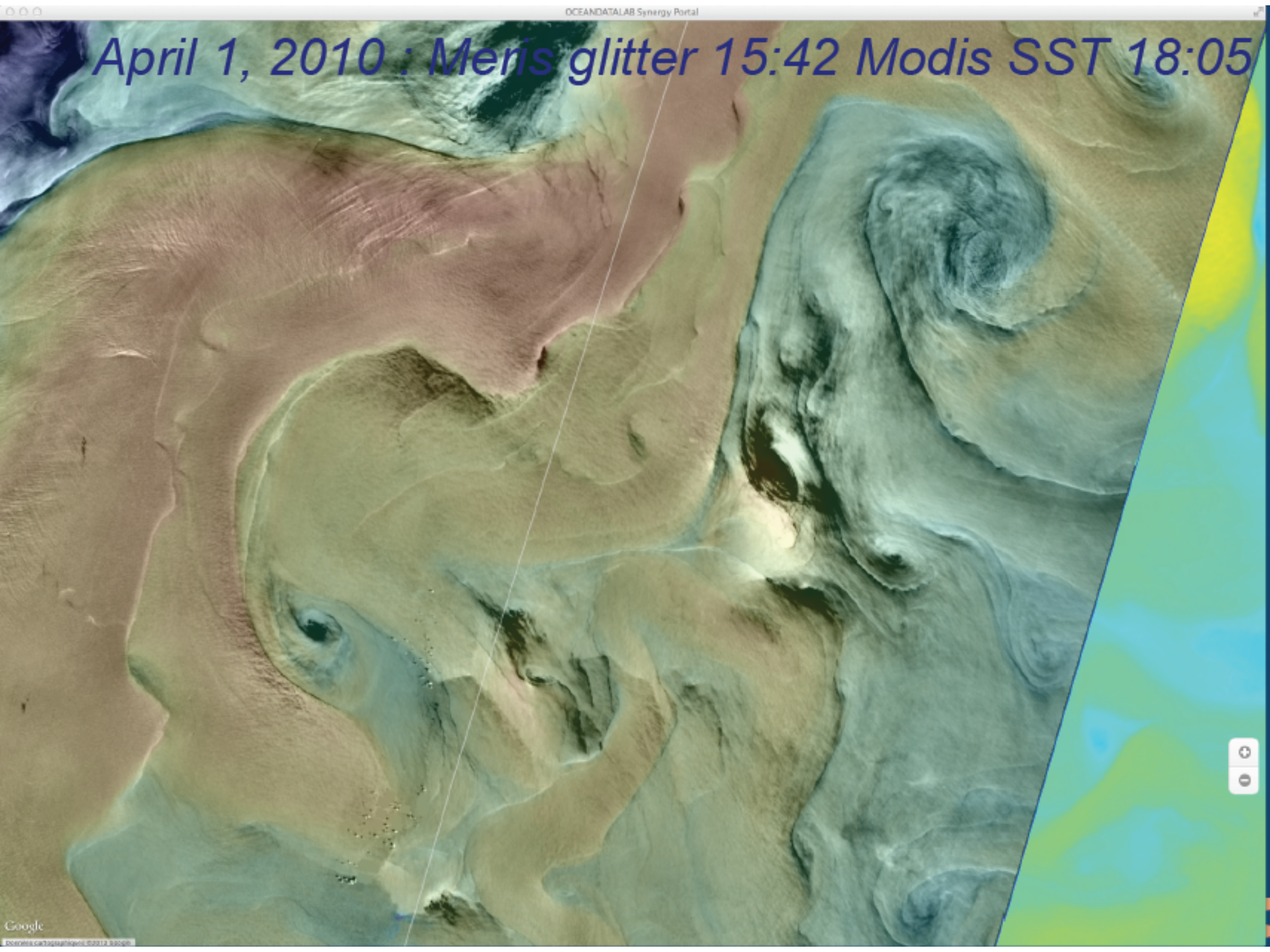
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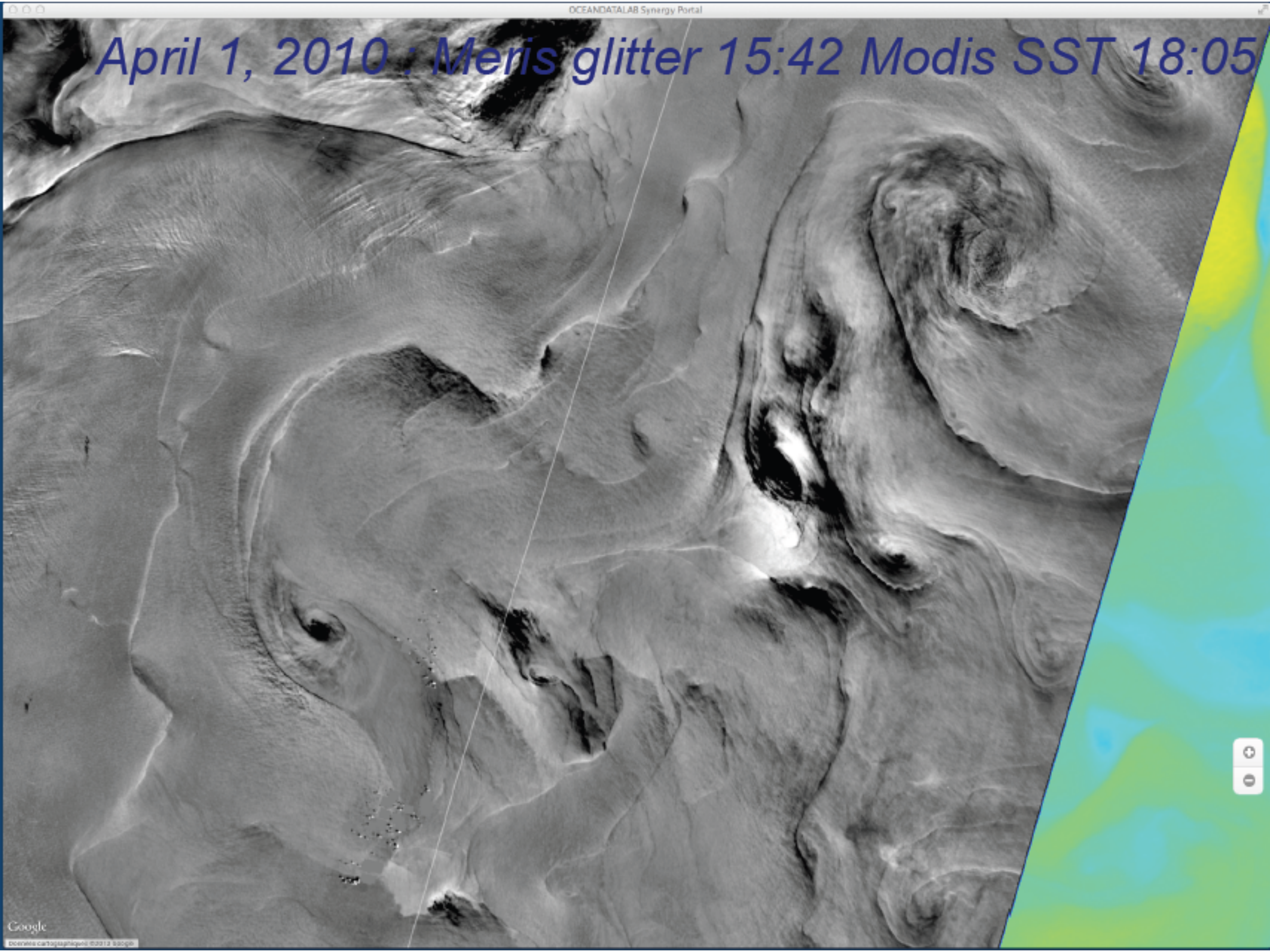
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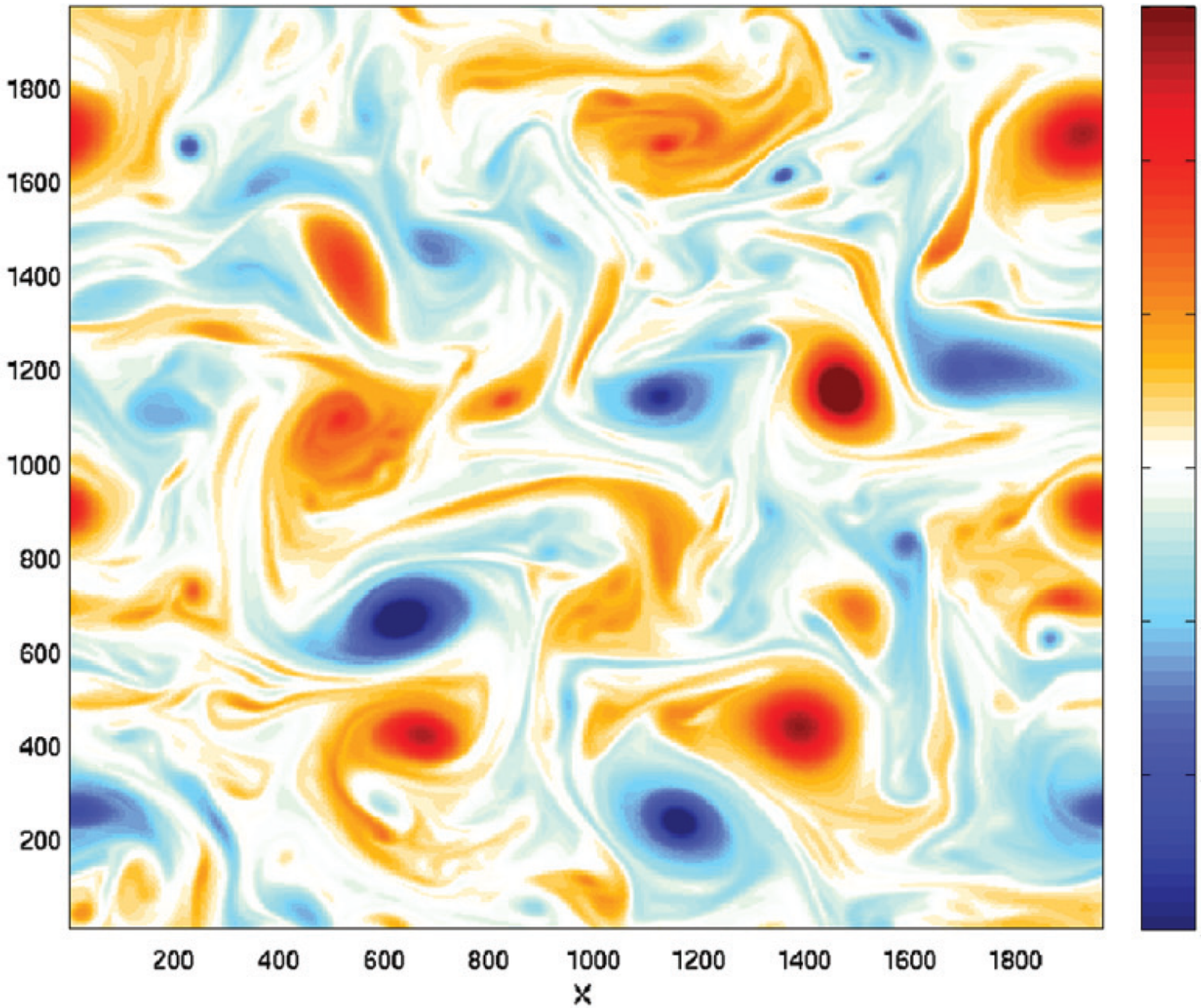
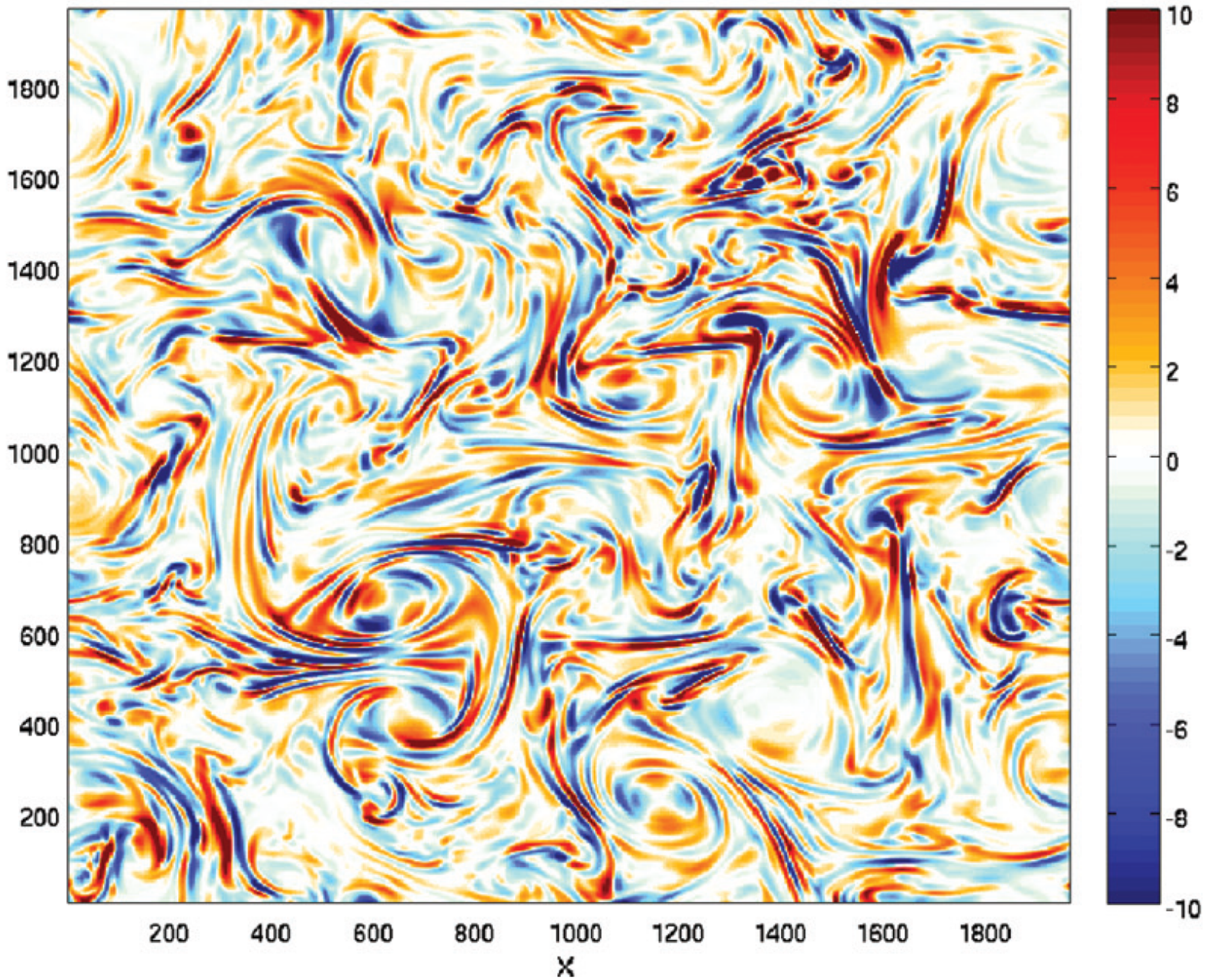
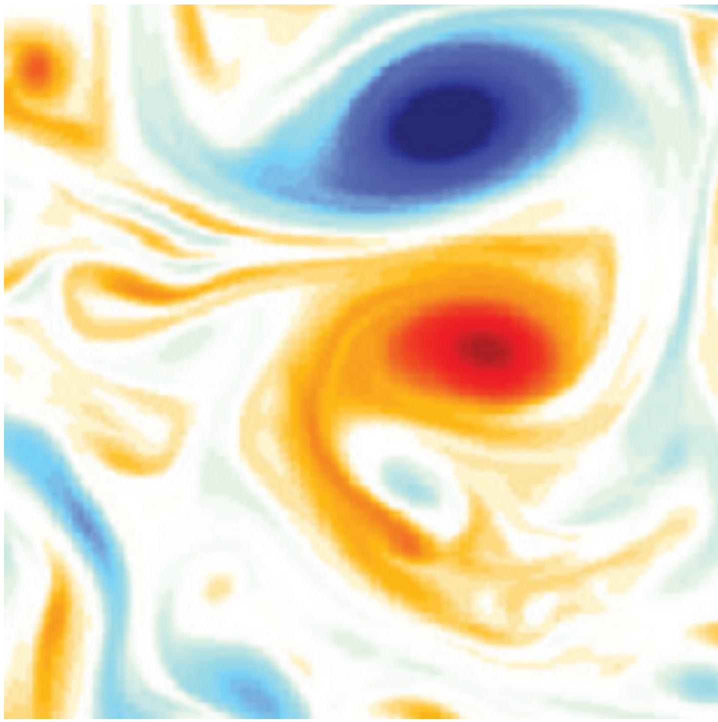


Surface temperature (density) field

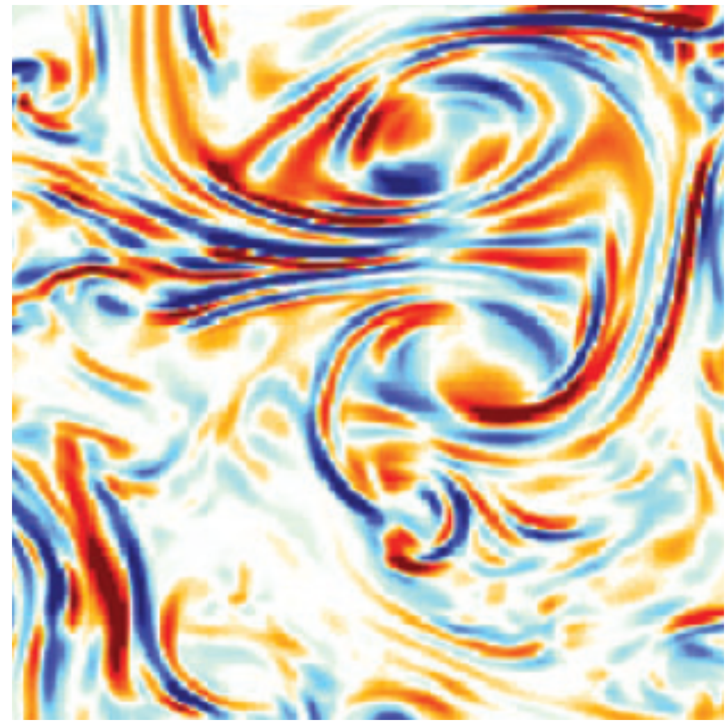


W-field associated with the previous temperature field

T



W



What are the mechanisms that explain the W-field?

Come back to the QG eqs. for momentum and density.
(assuming $R_0 = \frac{U}{fL} < 1$ and $\beta = 0$ (for this class))

$\mathbf{U} = [u, v]$ is decomposed as:

$$\mathbf{U} = \mathbf{U}_0 + R_0 \mathbf{U}_1$$

assuming again that the non-divergent part is entirely captured by the $O(1)$ dynamics

$\Rightarrow$, $\mathbf{U}_0 = k \times \nabla \psi \quad w_0 = 0.$

$\mathrm{curl}.\mathbf{U}_1 = 0 \quad \mathrm{div}.\mathbf{U}_1 = -\frac{\partial w_1}{\partial z}$

$\Rightarrow \quad p_1 = \frac{p}{\rho_0} - f\psi.$

Hydrostatic approximation leads to:

$$\rho = -\frac{f\rho_0}{g}\psi_z.$$

Geostrophic + Hydrostatic approximation lead to the thermal wind balance:

$$\nabla \rho = -\frac{f\rho_0}{g}(\nabla\psi)_z = \frac{f\rho_0}{g} k \times \mathbf{U}_{0z} \quad (1).$$

Momentum eqs:

$$\frac{d\mathbf{U}_0}{dt} = -\nabla p_1 - f.k \times \mathbf{U}_1 \quad (2)$$

Density eq.:

$$\frac{d\rho}{dt} = \frac{\rho_0}{g} N^2 w_1 \quad (3).$$

(2) + (3) lead to the QG PV equation

But properties of the flow field [stirring, dispersion, ...] are better understood by looking at the ageostrophic quantities: p_1, $\mathbf{U}_1$ and w_1!

Momentum eqs.:

$$\text{curl } (2) \Rightarrow \frac{d\zeta}{dt} = f w_{1z} \quad \text{with } \zeta = v_{0x} - u_{0y} \qquad (4)$$

$$\text{div } (2) \Rightarrow \nabla\cdot(\mathbf{U}_0\cdot\nabla \mathbf{U}_0) = -\Delta p_1 = \frac{1}{2} OW \quad [\text{ex } W] \quad (5)$$

$\Rightarrow p_1$ is diagnosed from (5) using ψ.

Thermal wind balance:

Diagnosis of $\mathbf{U}_1$ and w_1 is usually done by considering the thermal wind balance: $\nabla\rho = \frac{f\rho_0}{g} \mathbf{k}\times \mathbf{U}_{0z}$

and more precisely through the eqs. that govern each component.

- Eq. for $\nabla\rho$ from (3):

$$\frac{d\nabla\rho}{dt} = \boxed{-[\nabla \mathbf{U}_0]^T \nabla\rho} + \frac{N^2\rho_0}{g}\nabla w_1 \quad \text{with } [\nabla\mathbf{U}_0]^T = \begin{bmatrix} u_{0x} & v_{0x} \\ u_{0y} & v_{0y}\end{bmatrix} \quad (6)$$

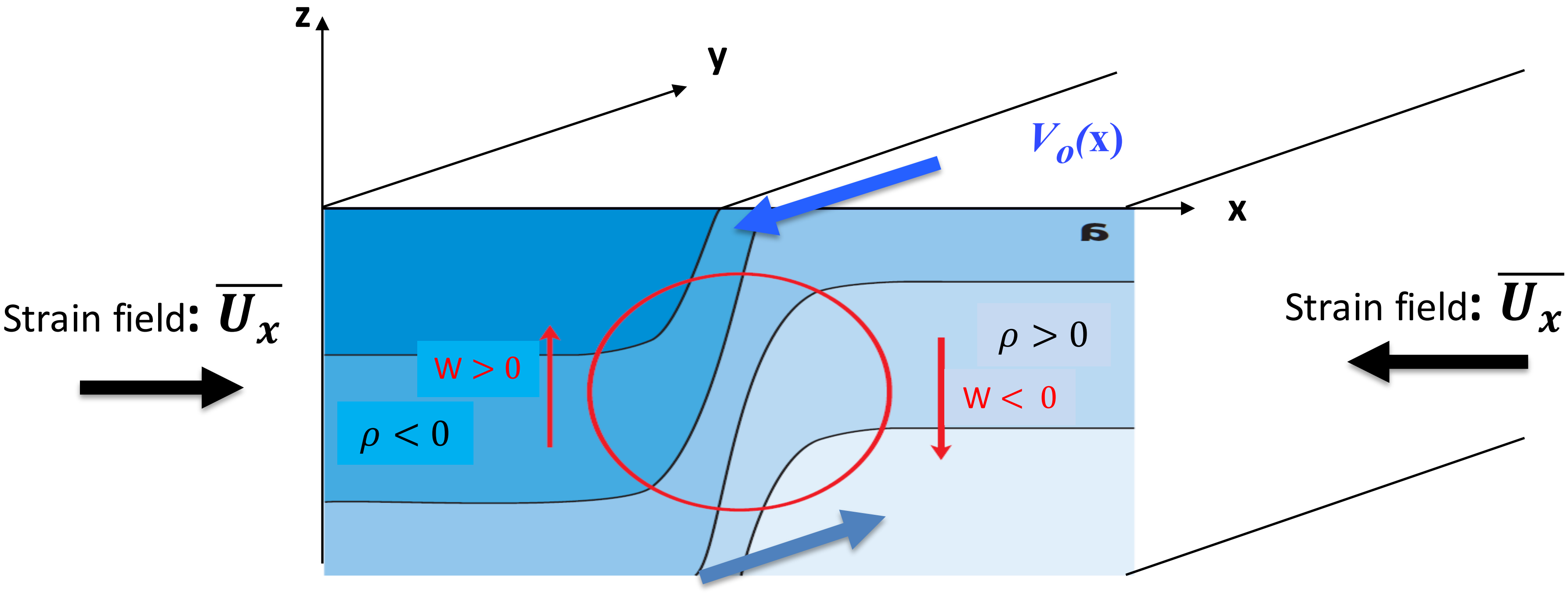
- Eq. for $\frac{f\rho_0}{g} \mathbf{k}\times \mathbf{U}_{0z}$ from (2):

$$\frac{d}{dt}\left[\frac{f\rho_0}{g}\mathbf{k}\times \mathbf{U}_{0z}\right] = \boxed{[\nabla \mathbf{U}_0]^T\left[\frac{f\rho_0}{g}\mathbf{k}\times \mathbf{U}_{0z}\right]} + \frac{\rho_0}{g}\mathbf{U}_{1z} - \frac{f\rho_0}{g}\mathbf{k}\times\nabla p_{1z} \quad (7)$$

Note that $[\nabla \mathbf{U}_0]^T$ appears in both eqs. ((6) and (7)) with the opposite sign, so the nonlinear terms will have opposite effects on the time evolution of the thermal wind components! which should quickly destroy the thermal wind balance.

This causes motions to depart from geostrophy and then to induce an ageostrophic circulation ---

--- let us consider a simple example.



Strain field $\overline{\boldsymbol{U_x}}$ **< 0** , density gradient $\boldsymbol{\rho_x}$**>0**

The density front is in thermal balance:

$$\rho_x = -\frac{f\,\rho_o}{g}\,v_{oz}$$

Let us consider a density front embedded in a strain field.

$\rho_x > 0$ is the density gradient in thermal wind balance:

$$\rho_x = -\frac{f\rho_0}{g} v_{0z} \quad (8)$$

$\overline{U}_x$ is a large-scale and steady strain field.

From (3):

$$\boxed{\frac{d\rho_x}{dt} = -\overline{U}_x \rho_x + \frac{\rho_0}{g} N^2 w_{1x}} \quad (9)$$

From (2):

$$\frac{d v_{0z}}{dt} = \overline{U}_x v_{0z} - f u_{1z}$$

$$\Rightarrow \boxed{\frac{d}{dt}\left[-\frac{f\rho_0}{g} v_{0z}\right] = \overline{U}_x\left[-\frac{f\rho_0}{g} v_{0z}\right] + \frac{f^2\rho_0}{g} u_{1z}} \quad (10).$$

Since $\overline{U}_x < 0$, the non linear term in (9) will lead to an exponential increase of ρ_x and the one in (10) to an exponential decrease of $\left[-\frac{f\rho_0}{g} v_{0z}\right]$ leading to a rapid destruction of the thermal wind balance.

- The exponential increase of ρ_x is driven by the stirring mechanisms.
- The exponential decrease of v_{0z} can be understood as follows.

Consider a vortex tube with section A. The volume $V = A\,\delta X$ as well as $A.\zeta_h$ (with $\zeta_h = v_{0z}$, the horizontal component of the relative vorticity).

We have $\frac{d\delta X}{dt} = -\overline{U}_x \delta x \Rightarrow \frac{dA}{dt} = \overline{U}_x A \Rightarrow \frac{d\zeta_h}{dt} = -\overline{U}_x \zeta_h$.

$\Rightarrow$ A will increase and therefore $\zeta_h = v_{0z}$ will decrease!

This will cause motions to depart from geostrophy (thermal wind balance is rapidly destroyed) and then to induce an ageostrophic circulation.
In the QG approximation the role of this ageostrophic circulation is to restore geostrophy!

If we assume a steady state, eqs. 9 and 10 lead to:

$$2\overline{U}_x \rho_x = \frac{\rho_0}{g} N^2 w_{1x} - \frac{f^2 \rho_0}{g} u_{1z}$$

and using $u_{1x} + w_{1z} = 0$ ($+\overline{U}_x$ large-scale),

$$\Rightarrow \boxed{w_{1xx} + \frac{f^2}{N^2} w_{1zz} = 2\overline{U}_x \frac{g}{N^2 \rho_0} \rho_{xx}} \quad (11).$$

(11) is a diagnostic of w_1 from ψ.

- Eq. for $\nabla\rho$ from (3):

$$\frac{d\nabla\rho}{dt} = -\left[\nabla U_0\right]^T \nabla\rho + \frac{N^2\rho_0}{g}\nabla w_1 \quad \text{with } \left[\nabla U_0\right]^T = \begin{bmatrix} u_{0x} & v_{0x} \\ u_{0y} & v_{0y} \end{bmatrix} \quad (6)$$

- Eq. for $\frac{f\rho_0}{g} k \times U_{0z}$ from (2):

$$\frac{d}{dt}\left[\frac{f\rho_0}{g} k \times U_{0z}\right] = \left[\nabla U_0\right]^T \left[\frac{f\rho_0}{g} k \times U_{0z}\right] + \frac{\rho_0}{g} U_{1z} - \frac{f\rho_0}{g} k \times \nabla p_{1z} \quad (7)$$

Generalisation

$[\nabla U_0]^T \nabla \rho$ is the frontogenetic vector.

$[\nabla U_0]^T \left[\frac{f\rho_0}{g} U_{0z}\right]$ is the vortex stretching vector.

The sum of both is one-half of the conventional Q-vector

Taking the divergence of (6) and (7) leads to:

$$\boxed{\Delta w_1 + \frac{f^2}{N^2} w_{1zz} = \frac{2f}{N^2} \nabla \cdot Q.} \qquad (12).$$

(12) is the so-called QG Omega equation

The ageostrophic motions (p_1, U_1, w_1) prevent the formation of strong density fronts in the ocean interior.

What about the density fronts at the surface where W=0 ?

=> see Georgy presentation ...