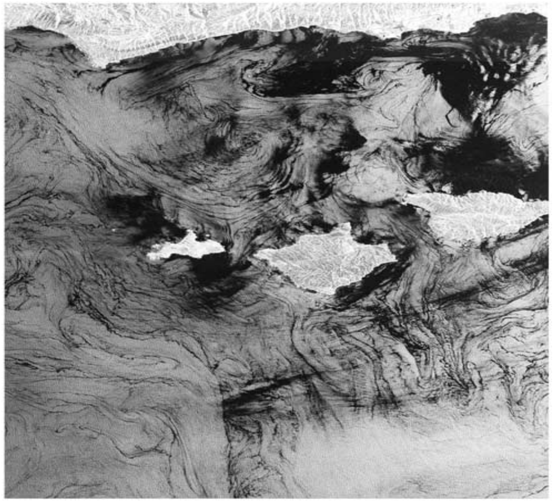
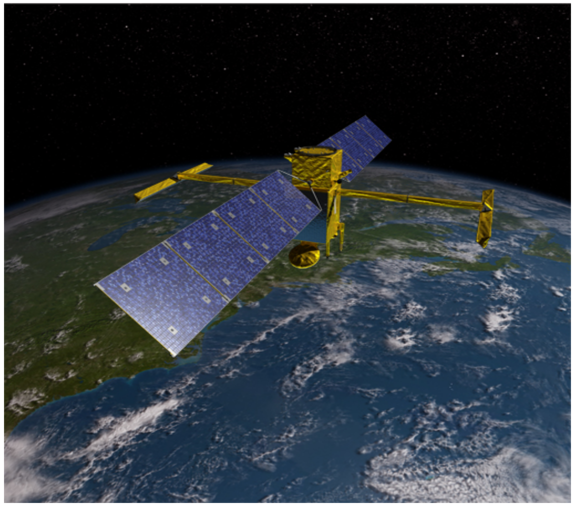
"Wave-Turbulence Interactions in the Oceans"

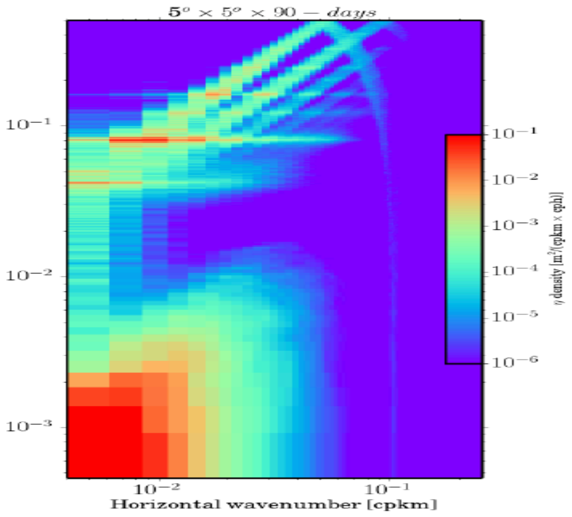
Patrice Klein (Caltech/JPL/Ifremer)

(XI) Interactions between wave and balanced motions (b)





WHY SUPER-INERTIAL MOTIONS NEAR THE SURFACE ARE SO ENERGETIC ?



IMPACT OF BALANCED MOTIONS ON THE WAVE DYNAMICS

Eric Kunze: Near-inertial wave propagation in a geostrophic shear. JPO, 1985

Motions in the oceans

Waves (near-inertial, tidal, internal gravity waves):

- Fast motions
- assumed to explain **most of the mixing** (at small-scale) **in the ocean interior**
- strong signature in in-situ (moorings, gliders, ADCP, surface drifters) and satellite observations [SAR] at high-resolution.

Balanced motions: geostrophic (or gradient wind) turbulence [10-500 km]:

- Slow motions
- explains **most of the kinetic energy in the oceans,** well captured by satellite observations on a global scale [SSH (> 100 km), SST, Ocean Color, …]

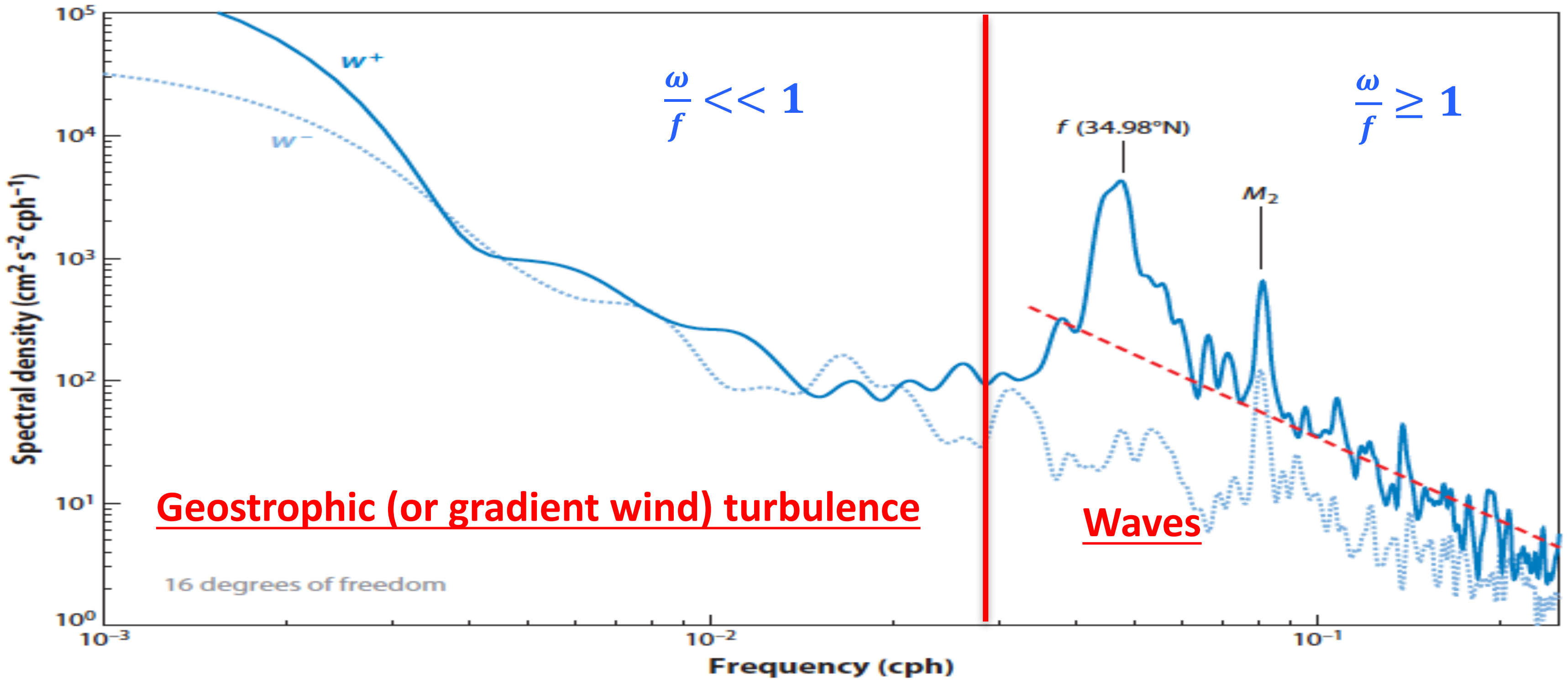


Figure 1

Rotary velocity spectrum at 261-m depth from current-meter data from the WHOI699 mooring gathered during the WESTPAC1 experiment (mooring at 6,149-m depth.) The solid blue line (w^+) is clockwise motion, and the dashed blue line (w^-) is counterclockwise motion; the differences between these emphasize the downward energy propagation that often dominates the near-inertial band. The dashed red line is the line $E_0 N \omega^{-p}$ with $N = 2.0$ cycles per hour (cph), $E_0 = 0.096\ cm^2\ s^{-2}\ cph^{-2}$, and $p = 2.25$, which is quantitatively similar to levels in the Cartesian spectra presented by Fu (1981) for station 5 of the Polygon Mid-Ocean Experiment (POLYMODE) II array.

A frequency spectrum displays different properties between fast and slow motions

Momentum equations

Rossby number: U/fL $\leq$ 1

<u>Slow motions</u>: $\frac{\omega}{f} << 1$

$$\cancel{\frac{\partial U}{\partial t}} + \mathbf{U}.\nabla U - fk\times U = -\frac{\nabla p}{\rho_o}$$

$\Rightarrow$ **Geostrophic balance or gradient wind balance at zero order** $[c = \frac{\omega}{k} \ll U]$

<u>Fast motions</u>: $\frac{\omega}{f} \geq 1$

$$\frac{\partial U}{\partial t} + \cancel{\mathbf{U}.\nabla U} - fk\times U = -\frac{\nabla p}{\rho_o}$$

=> Wave equations $[c = \frac{\omega}{k} >> U]$

Interactions waves - balanced motions: full non-linear momentum equations

$$\frac{\partial U}{\partial t} + \boldsymbol{UU_x} + \boldsymbol{VU_y} - \mathrm{f}V = -\frac{\partial P}{\partial x}$$

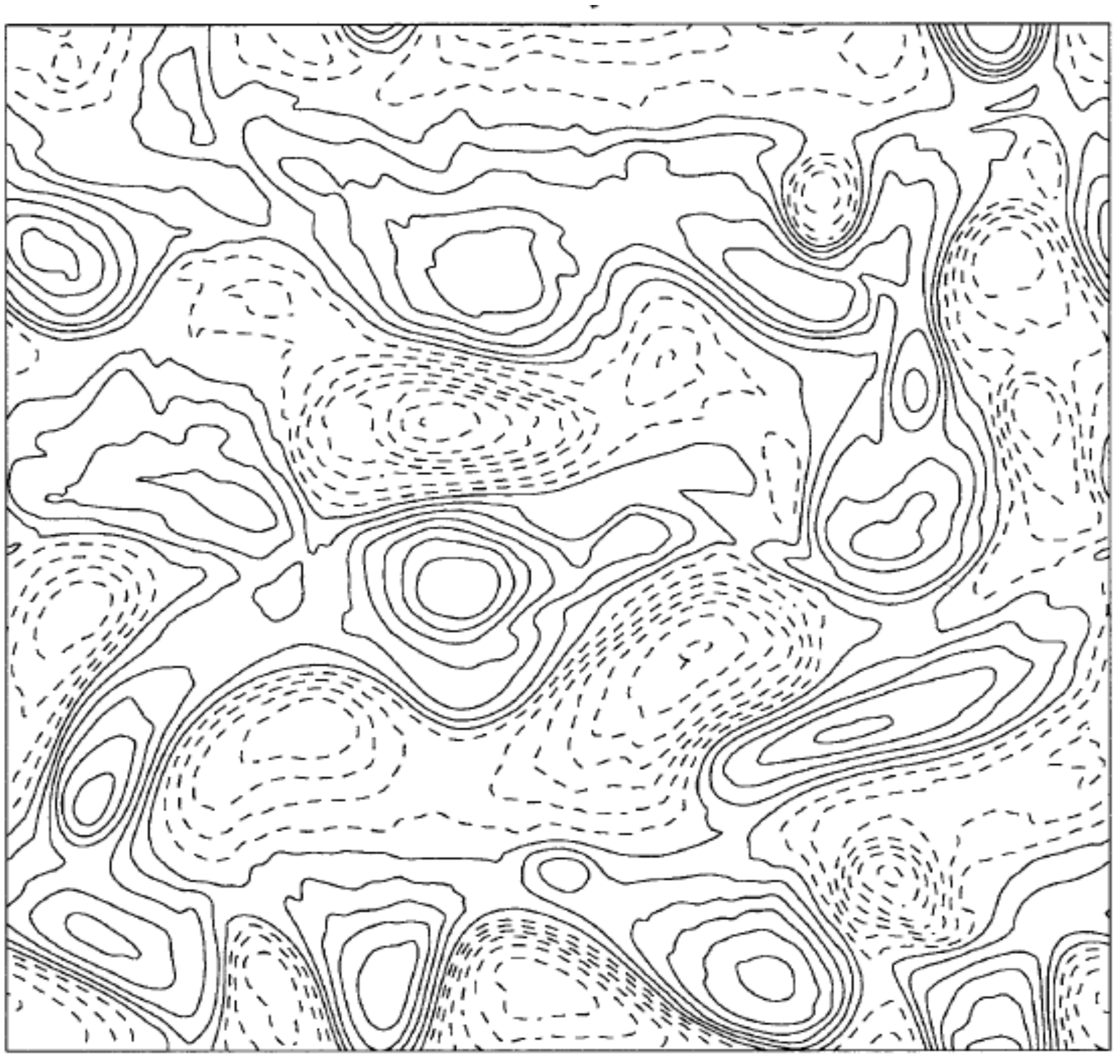
$$\frac{\partial V}{\partial t} + \boldsymbol{UV_x} + \boldsymbol{VV_y} + \mathrm{f}U = -\frac{\partial P}{\partial y}$$

We use: $U=u+U_g$, $V=v+V_g$ $P=p+P_g$ [**u =wave, U_g =balanced motions**] and assume the geostrophic (or gradient wind) assumption for balanced motions and: $u \ll U_g$, $v \ll V_g$. This leads to *(replacing U_g by U to simplify the notations for the sake of simplicity)*:

$$\frac{\partial u}{\partial t} + \boldsymbol{Uu_x} + \boldsymbol{Vu_y} + \boldsymbol{uU_x} + \boldsymbol{vU_y} - fv = -\frac{\partial p}{\partial x}$$

$$\frac{\partial v}{\partial t} + \underbrace{\boldsymbol{Uv_x} + \boldsymbol{Vv_y}}_{\text{Doppler shift}} + \underbrace{\boldsymbol{uV_x} + \boldsymbol{vV_y}}_{\text{Refraction}} + fu = -\frac{\partial p}{\partial y}$$

Let us consider a 2-D (non-divergent) mesoscale eddy field



STREAM FUNCTION $\psi(x, y)$

$$U = -\psi_y$$
$$V = \psi_x$$

$$V_x - U_y = \Delta\psi$$

$$p(x,y,z,t) = \sum_m p_m(x,y).F_m(z)$$

For baroclinic mode m:

$$\frac{\partial u_m}{\partial t} + \boldsymbol{u_m U_x} + \boldsymbol{v_m\, U_y} - \mathrm{f}v_m = -\frac{\partial p_m}{\partial x}$$

$$\frac{\partial v_m}{\partial t} + \boldsymbol{u_m V_x} + \boldsymbol{v_m\, V_y} + \mathrm{f}u_m = -\frac{\partial p_m}{\partial y}$$

$$\frac{\partial p_m}{\partial t} + f^2.r_m^2\left(u_{mx} + v_{my}\right) = 0$$

Doppler terms are not considered for the sake of simplicity. Searching for plane wave solutions as, $p_m(x,y,t), u_m(x,y,t), v_m(x,y,t) \approx e^{i.(k.x+l.y-\omega t)}$, leads to: $\omega = \omega_r + i.\omega_i$ with:

$$\omega_r^2 \approx f^2 + \boldsymbol{f.\left[V_x - U_y\right] - V_x U_y - U_x^2} + f^2 r_m^2.(k^2 + l^2)$$

$$\omega_i \approx \boldsymbol{i.\left[k.l.\left(V_x + U_y\right) + \left(k^2 - l^2\right).U_x\right]}$$

Physics involved in the dispersion of waves by balanced motions

$$\frac{\partial}{\partial t}\begin{bmatrix} u \\ v \end{bmatrix} + \begin{bmatrix} \boldsymbol{U_x} & \boldsymbol{U_y} - f \\ \boldsymbol{V_x} + f & \boldsymbol{V_y} \end{bmatrix}\begin{bmatrix} u \\ v \end{bmatrix} = -\begin{bmatrix} p_x \\ p_y \end{bmatrix}$$

$$\frac{\partial}{\partial t}\begin{bmatrix} u \\ v \end{bmatrix} + \begin{bmatrix} \boldsymbol{S_1/2} & \boldsymbol{S_2/2} - (f + \boldsymbol{\zeta/2}) \\ \boldsymbol{S_2/2} + (f + \boldsymbol{\zeta/2}) & \boldsymbol{-S_1/2} \end{bmatrix}\begin{bmatrix} u \\ v \end{bmatrix} = -\begin{bmatrix} p_x \\ p_y \end{bmatrix}$$

$$\frac{\partial p}{\partial t} + f^2 . r^2 \left(u_x + v_y\right) = 0$$

with: $\boldsymbol{S_1 = U_x - V_y, S_2 = V_x + U_y, \zeta = V_x - U_y}$

$$\omega_r^2 \approx f^2 + \boldsymbol{f.\zeta} - \underbrace{\boldsymbol{\left(S_1^2 + S_2^2 - \zeta^2\right)/4}}_{\text{Okubo-Weiss quantity}} + f^2 r_m^2 . \left(k^2 + l^2\right)$$

Physics involved in the dispersion of waves by balanced motions

$$\frac{\partial}{\partial t}\begin{bmatrix} u \\ v \end{bmatrix} + \underbrace{\begin{bmatrix} S_1/2 & S_2/2 \\ S_2/2 & -S_1/2 \end{bmatrix}}_{\substack{\text{Strain} \\ \text{(growth or decay)}}} + \underbrace{\begin{bmatrix} 0 & -(f+\zeta/2) \\ (f+\zeta/2) & 0 \end{bmatrix}}_{\text{Rotation}} \begin{bmatrix} u \\ v \end{bmatrix} = -\begin{bmatrix} p_x \\ p_y \end{bmatrix}$$

with: $S_1 = U_x - V_y,\ S_2 = V_x + U_y,\ \zeta = V_x - U_y$

$$\frac{\partial p}{\partial t} + f^2.r^2(u_x + v_y) = 0$$

$$\omega_r^2 \approx f^2 + f.\zeta - (S_1^2 + S_2^2 - \zeta^2)/4 + f^2 r_m^2.(k^2 + l^2)$$

$$\omega_i \approx i.[k.l.(V_x + U_y) + (k^2 - l^2).U_x]$$

Physics involved in the dispersion of waves by a balanced motions

Let us consider: $S_1=S_2=0$, $\boldsymbol{\zeta = V_x - U_y \neq 0}$

$$\frac{\partial}{\partial t}\begin{bmatrix} u \\ v \end{bmatrix} + \begin{bmatrix} \mathbf{0} & -(f+\boldsymbol{\zeta/2}) \\ (f+\boldsymbol{\zeta/2}) & \mathbf{0} \end{bmatrix}\begin{bmatrix} u \\ v \end{bmatrix} = -\begin{bmatrix} p_x \\ p_y \end{bmatrix}$$

$$\frac{\partial p}{\partial t} + f^2.r^2(u_x + v_y) = 0$$

$\Rightarrow \boldsymbol{u}(\boldsymbol{x},\boldsymbol{t}) = u_o.\cos[(k_o.x - (f+\zeta)t] \approx u_o.\cos[(k.x - (f+\zeta_o)t]$ with $\boldsymbol{k = k_o - \frac{\partial \zeta}{\partial x} t}$

$$=> \frac{\partial p}{\partial t} = -f^2.r^2(\frac{\partial \zeta}{\partial x}t - k_o).\sin[k_o.x - (f+\zeta_o)\mathrm{t}]$$

Interface anomalies and w increase in time, whatever the sign of $\frac{\partial \zeta}{\partial x}$ is, but there is a feedback of the interface gradients (p_x, p_y) on u **and** v **in terms of wave propagation …**

Let us consider a geostrophic jet

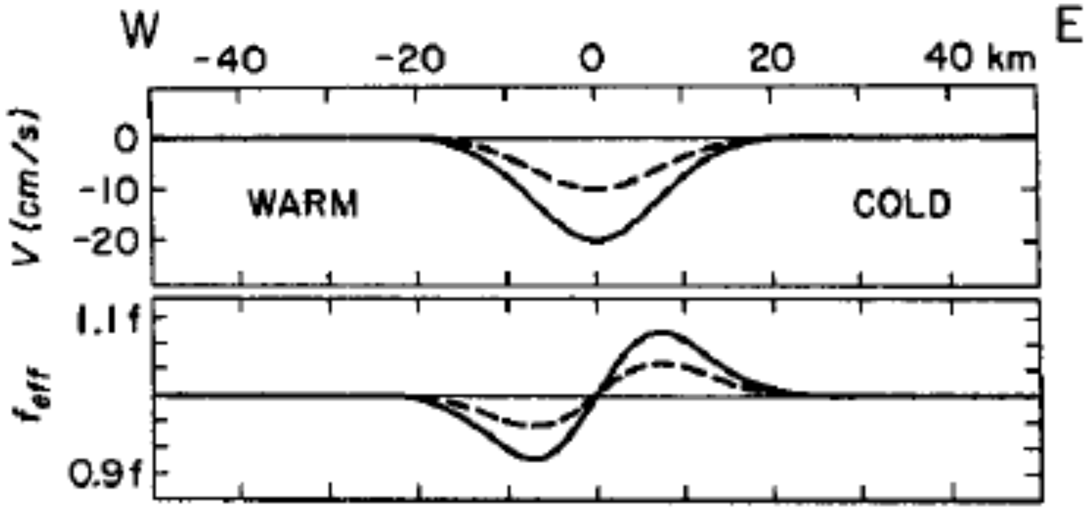


FIG. 4. A model southward baroclinic jet (upper frame) analogous to the North Pacific Subtropical Front, and the associated effective Coriolis frequency $f_{eff} = f + \zeta/2$ (lower frame). The solid curves represent values at the surface, and dashed curves values at a depth of 100 m. Internal waves propagate freely only for frequencies lying above the f_{eff}-curve.

We have: $\partial V/\partial x = 0.1f/20km$

Kunze JPO 1985

Let us consider time evolution of a wave packet in a geostrophic jet

group velocity is: $\overrightarrow{C_g} = \nabla_k \omega$

$$C_{gx} = \frac{\partial \omega}{\partial k} = \frac{N^2 k}{\omega m^2} = \frac{\omega^2 - f^2}{\omega} \frac{k}{k^2 + l^2}$$

$$C_{gy} = \frac{\partial \omega}{\partial l} = \frac{N^2 l}{\omega m^2} = \frac{\omega^2 - f^2}{\omega} \frac{l}{k^2 + l^2}$$

$$C_{gz} = \frac{\partial \omega}{\partial m} = -\frac{N^2 (k^2 + l^2)}{\omega m^3} = -\frac{\omega^2 - f^2}{\omega m}$$ (downward propagation if $\omega > 0$)

Example:

$m = \frac{\pi}{200}$ m^{-1}, $\omega = 1.02f$, leads to: $\boldsymbol{C_{gz} \sim -22\, m.d^{-1}}$

$k = \frac{2.\pi}{20km}$, $\omega = 1.02f$, leads to: $\boxed{\boldsymbol{C_{gx} \sim 0.013\, m.s^{-1}}}$

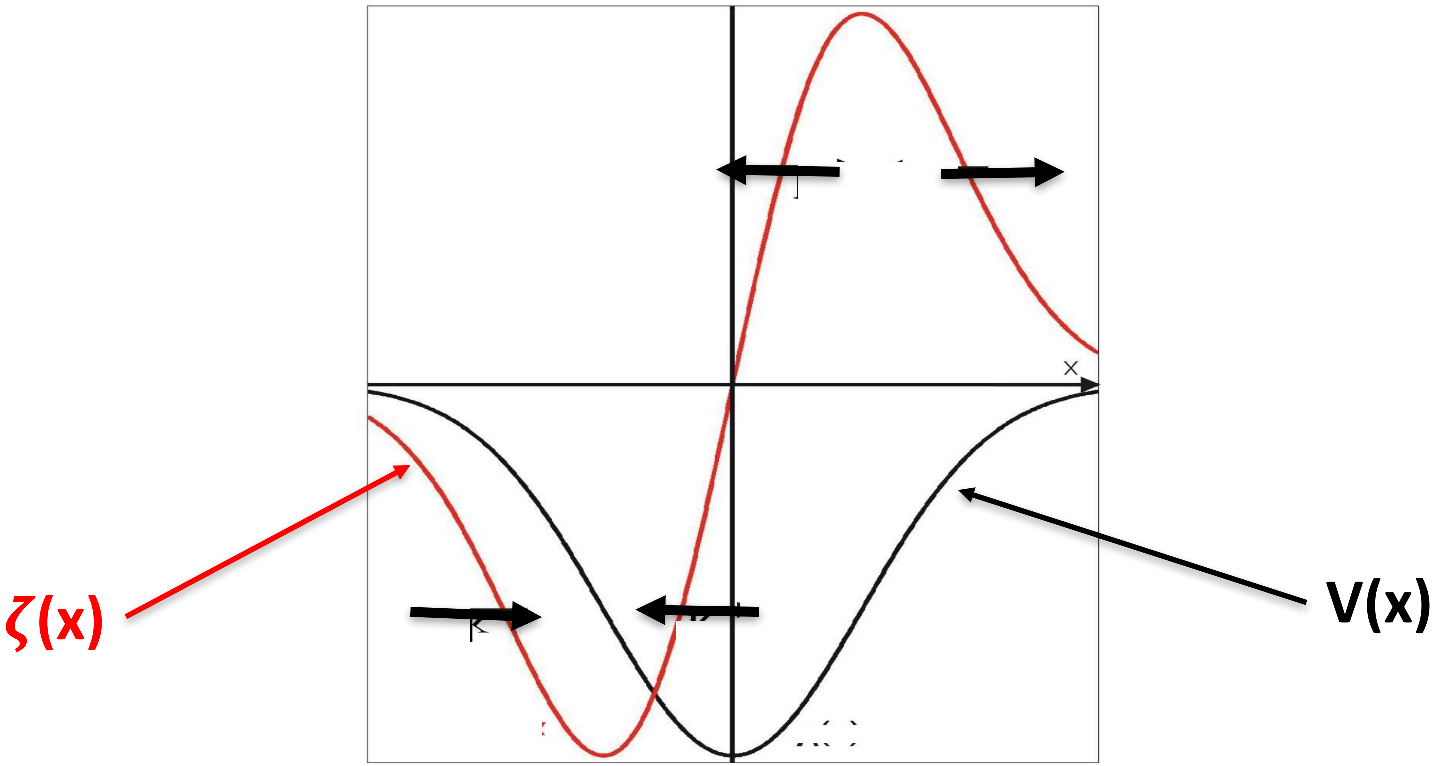
[10 days to propagate over 10 km!]

Waves propagate in the direction of the wavevector!

Propagation is due to the impact of pressure gradients on u and v!

Let us consider several wave packets in a geostrophic jet

We have: $k = k_o - \frac{\partial \zeta}{\partial x} t$. Let us assume that k_o is very small => $\boldsymbol{k \sim -\frac{\partial \zeta}{\partial x} t}$



Schematic representation of the wave propagation induced by a geostrophic jet V(x). The velocity, V(x), (in black) and the relative vorticity, ζ(x) (in red) are shown. We have:

$\partial\zeta/\partial x = \pm 0.1 f/20km$, $\boldsymbol{k} \approx -\frac{\partial \zeta}{\partial x}\boldsymbol{t} \sim \boldsymbol{2\,\pi\,/20km}$ in 6 days.!

The wavenumbers (black arrows) are proportional to $d\zeta/dx$.

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Wave packets propagating east or west …
… and downward (taken into account C_{gz}) trapped in $\zeta < 0$ structures

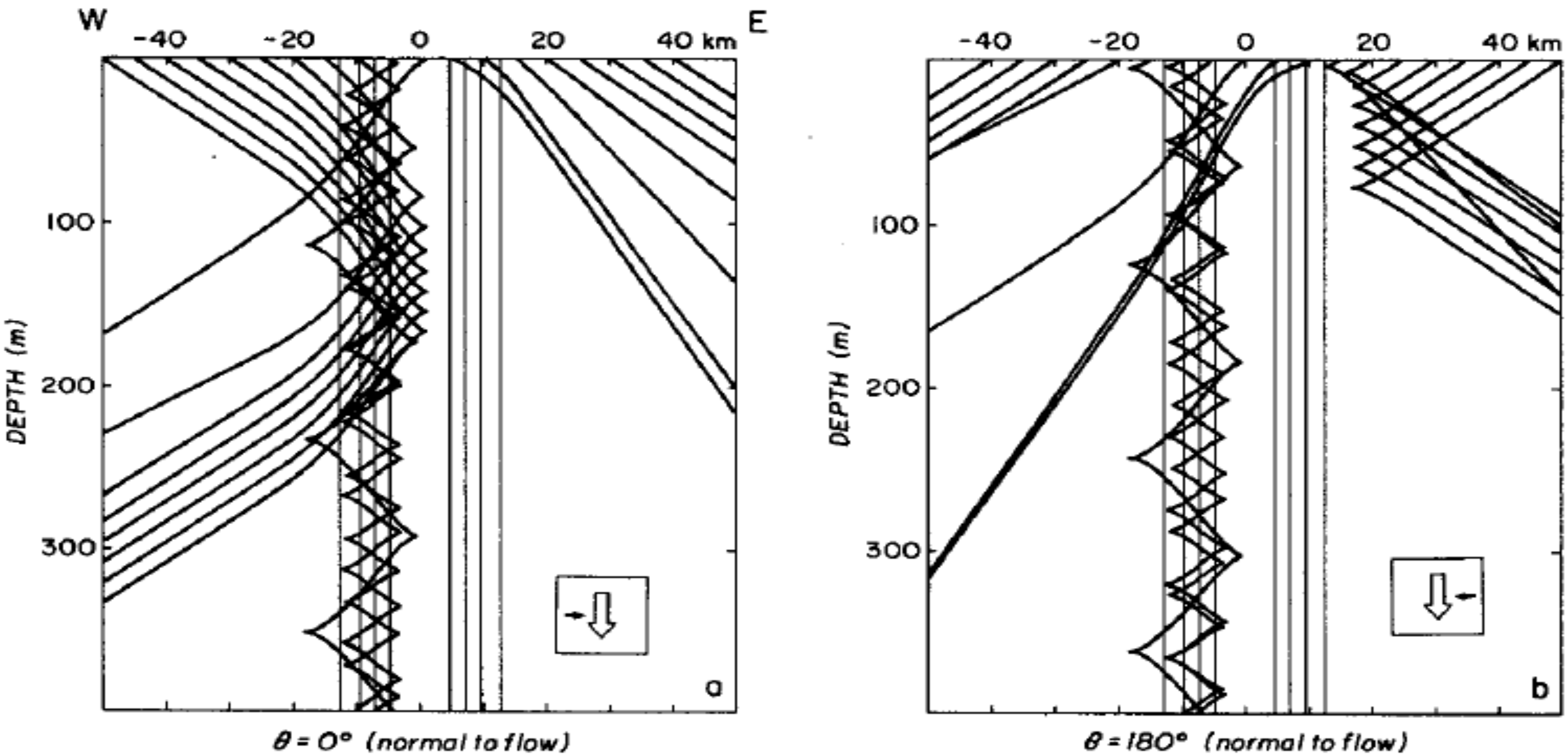


FIG. 5. Ray paths (thick solid lines) for near-inertial waves propagating downward and into a barotropic jet. The thin vertical lines are isotachs. Flow is out of the page. In this and Figs. 7–9, the horizontal wavevector $(k_x, k_y) = k_H(\cos\theta, \sin\theta)$ where k_x is the acrossfront and k_y the alongfront wavenumber. The legend insert in the lower right indicates the orientation of the horizontal wavevector with respect to the mean flow in each case. The two cases above are for normal incidence. Waves originating outside the jet are deflected away from the positive vorticity ridge on the eastern side of the jet (Fig. 4). Waves originating in the negative vorticity trough on the western side of the jet are trapped.

Kunze JPO 1985

An example:

$$\frac{\partial u}{\partial t} - fv + u\frac{\partial u}{\partial x} = -g'\frac{\partial h}{\partial x} + \nu\frac{\partial^2 u}{\partial x^2} \qquad (1)$$

$$\frac{\partial v}{\partial t} + (f + \zeta)u + u\frac{\partial v}{\partial x} = \nu\frac{\partial^2 v}{\partial x^2} \qquad (2)$$

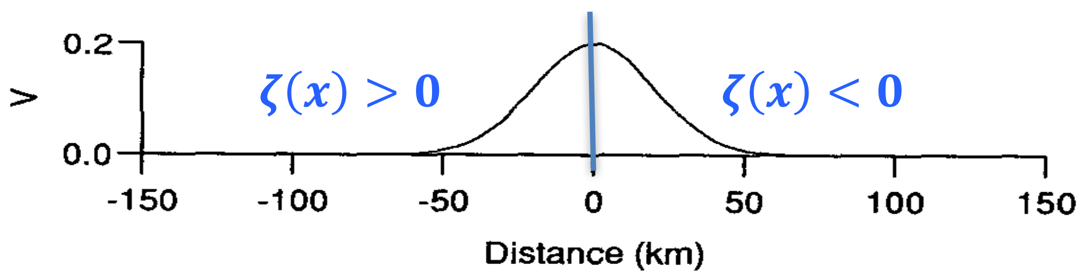
$$\frac{\partial h}{\partial t} + \frac{\partial hu}{\partial x} = 0, \qquad (3)$$

with $f = 10^{-4}\ \mathrm{s}^{-1}$ the Coriolis parameter, g the gravitational acceleration, $g' = g\Delta\rho/\rho$ the reduced gravity, $\zeta = \partial V/\partial x$ the jet vorticity, and ν the friction coefficient.

In the numerical experiments $V(x)$ is a Gaussian:

$$V(x) = V_0 e^{-(x^2/2\lambda^2)} \qquad (4)$$

with $V_0 = 0.2\ \mathrm{m\ s^{-1}}$ and $\lambda = 20$ km. The Gaussian is



An initial value problem: *h* and *v* initially homogeneous with:
interface $h_i = 50$ m, $v_i = 0.2\ m/s$

$h^* = h - h_{i,}$ with $h_i = 50m$

$v_i = 0.2\, m/s$

$h^* = 8\ m$ after 6 inertial periods!

Intensified in $\zeta(x) < 0$ regions

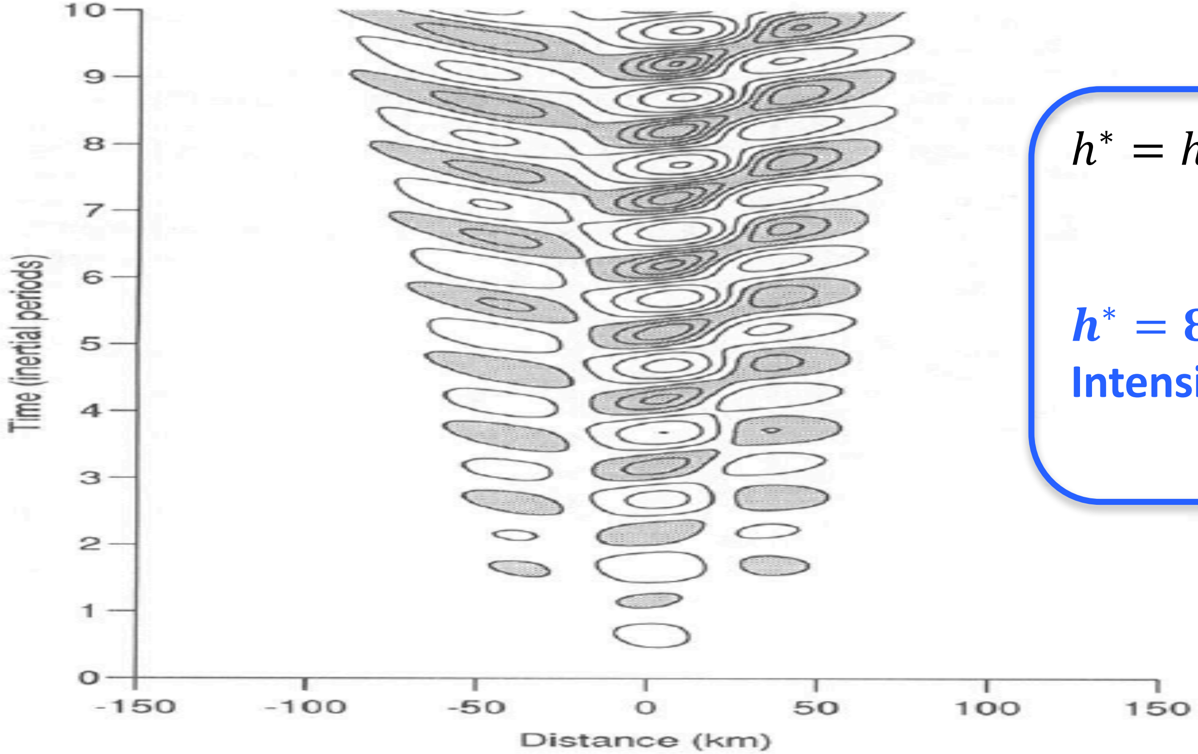


FIG. 4. Time–longitude plot of the mixed-layer depth perturbation for the nonlinear equations, with $g' = 2\ 10^{-3}$ m s^{-2} and $\nu = 10$ m^2 s^{-1}. Contour interval is 2 m, from −7 to 9 m; positive values (downwelling) are shaded.

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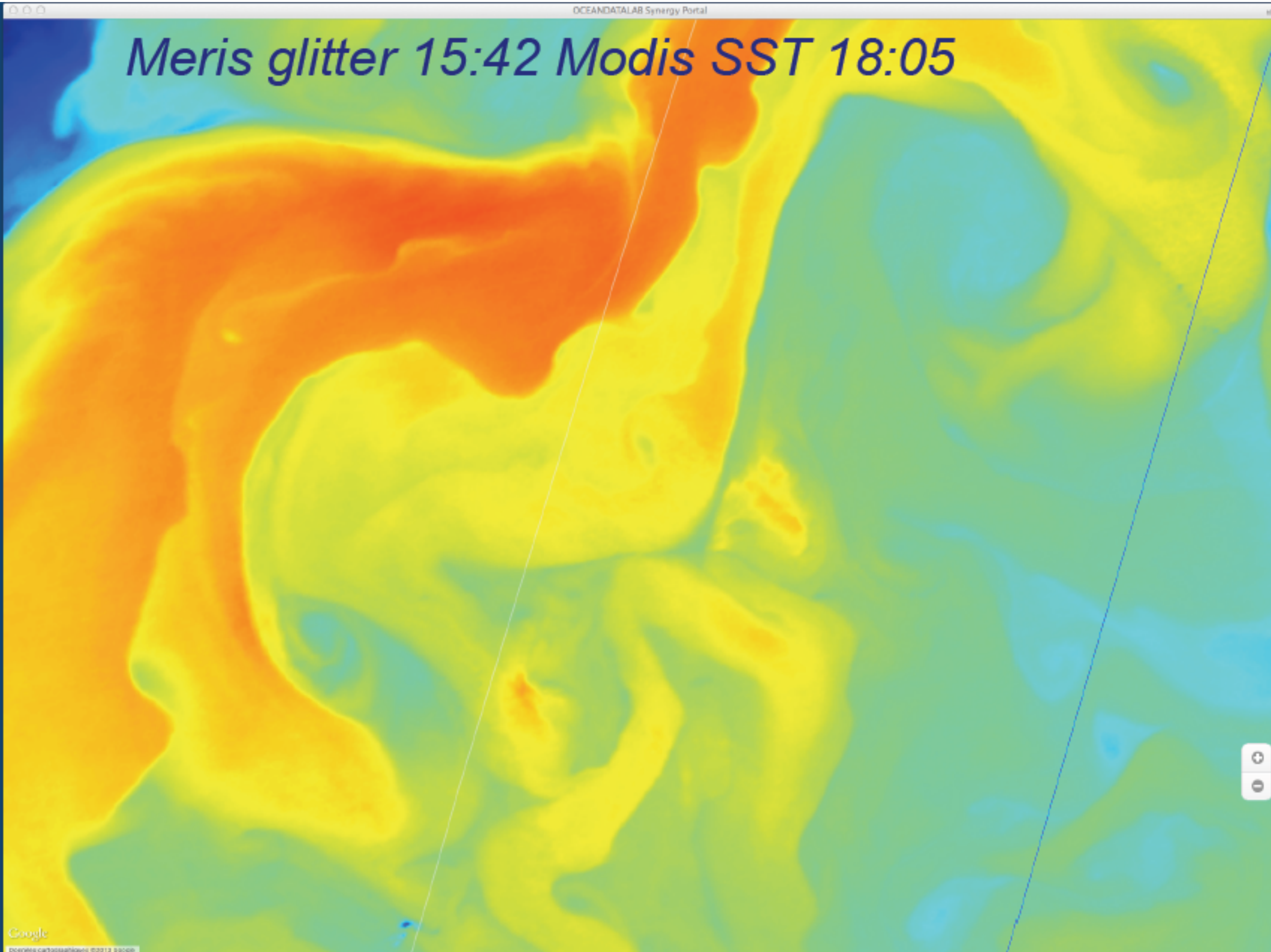
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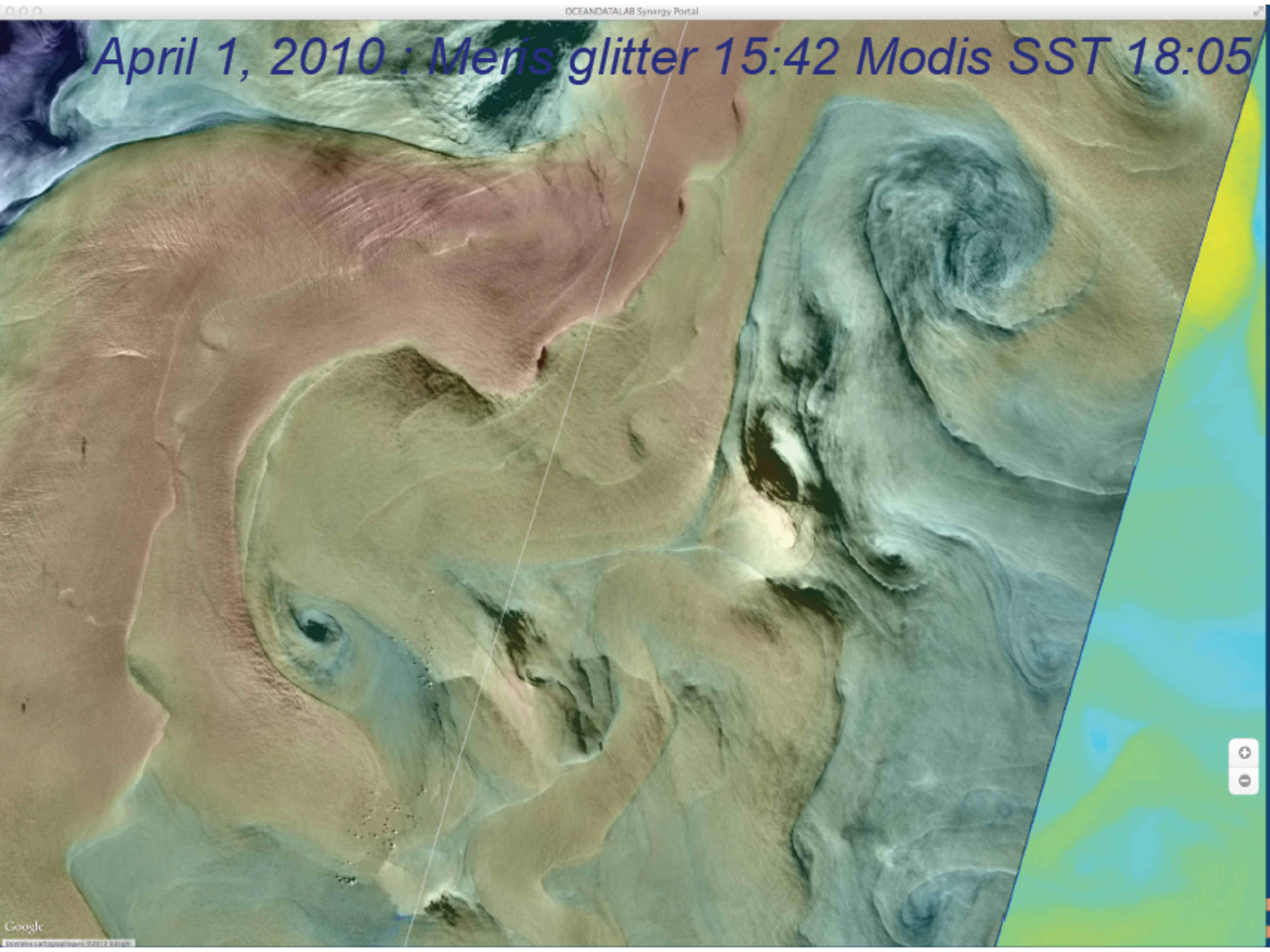
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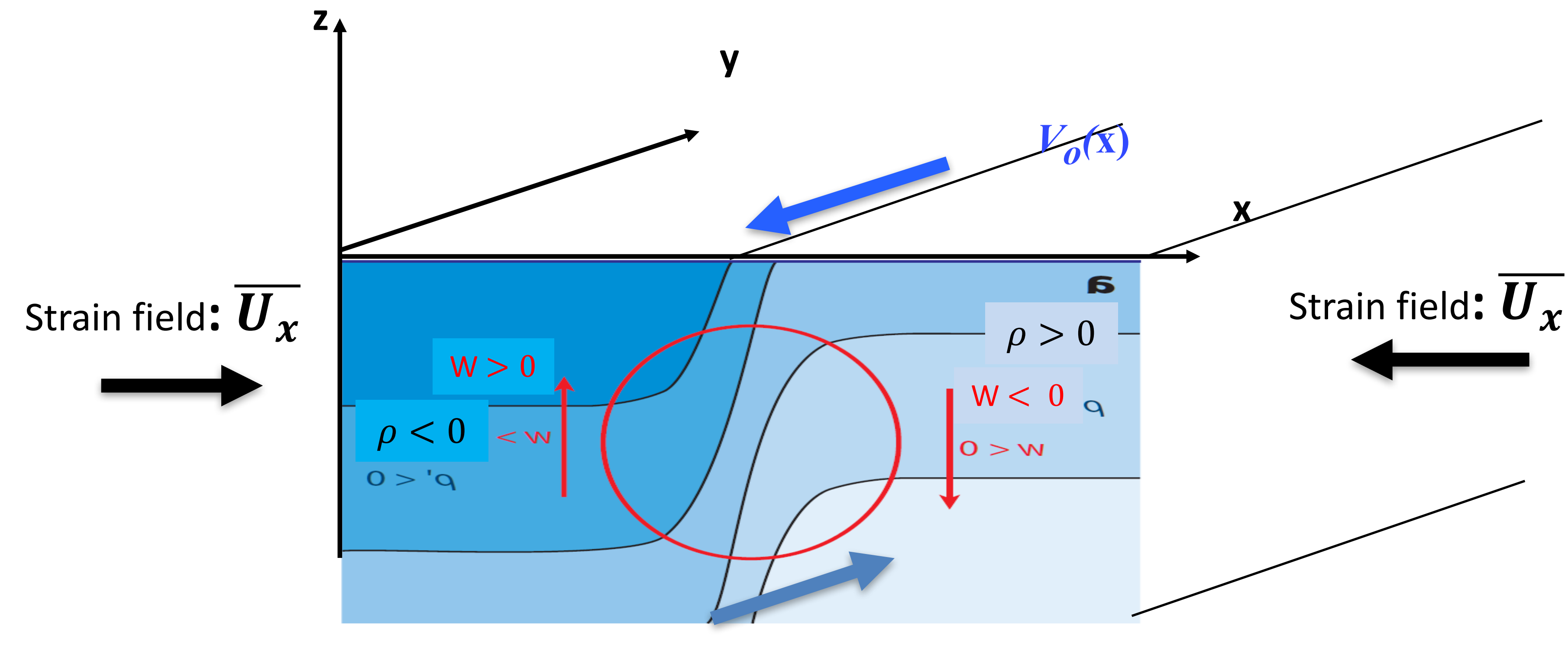
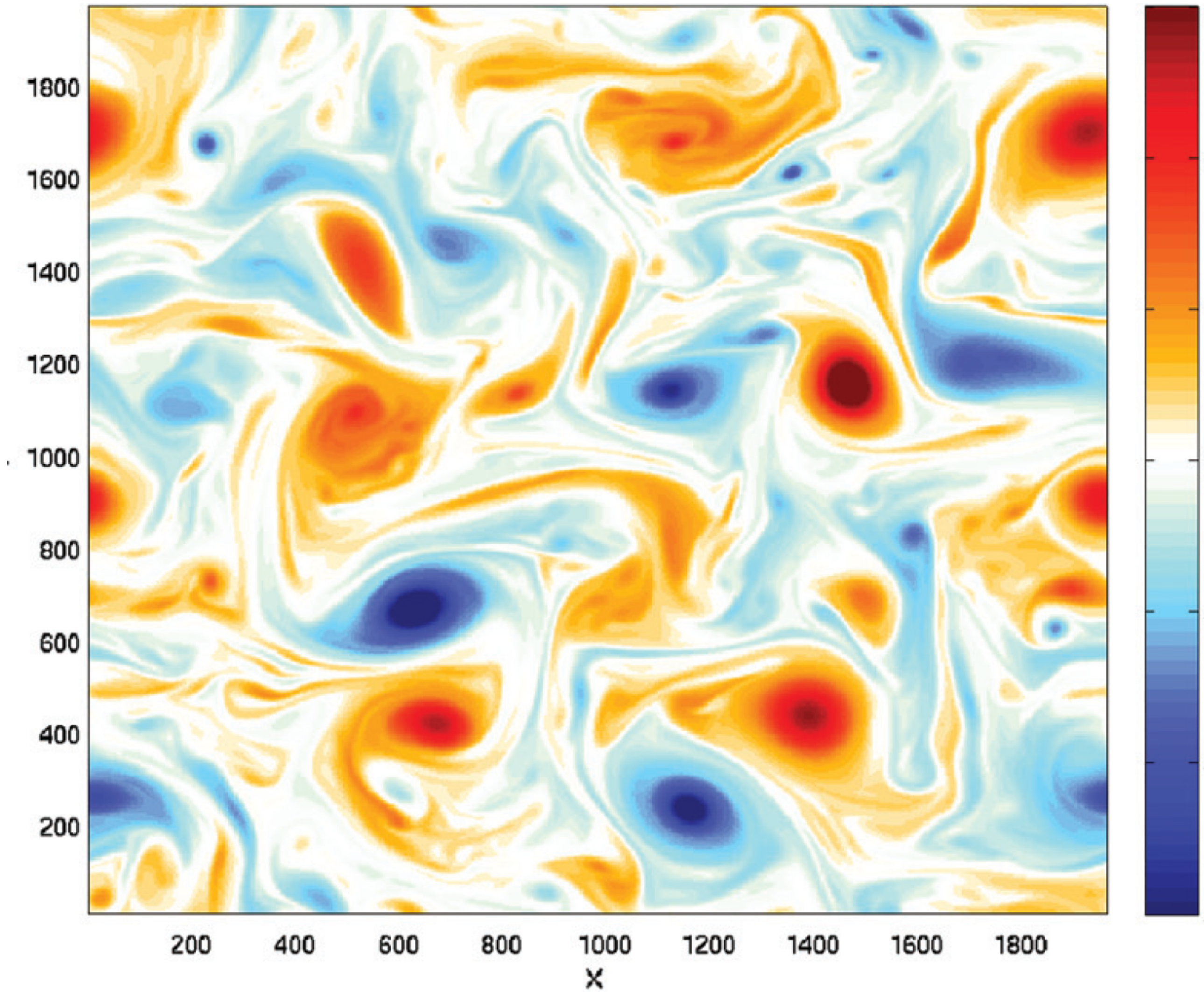
GEOSTROPHIC FRONT IN THERMAL WIND BALANCE: $W_G > 0$ WHERE $\zeta < 0$



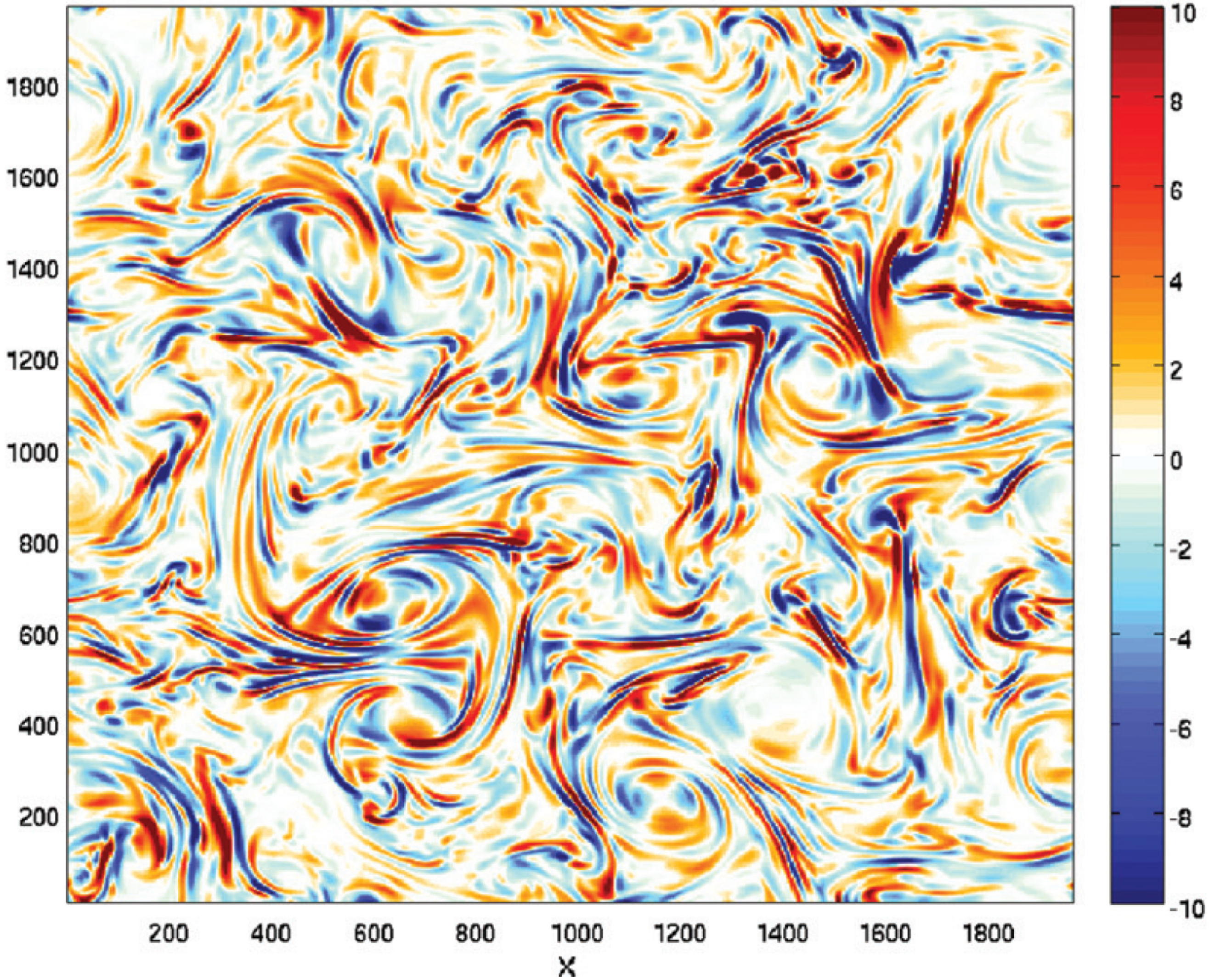
Strain field $\overline{\boldsymbol{U_x}}$ **< 0** , density gradient $\boldsymbol{\rho_x}$**>0.** The density front is in thermal balance:

$$\rho_x = -\frac{f\,\rho_o}{g} v_{oz}$$

=> **Trapping of waves in regions where $W_G > 0$. Impact on mixing ?**

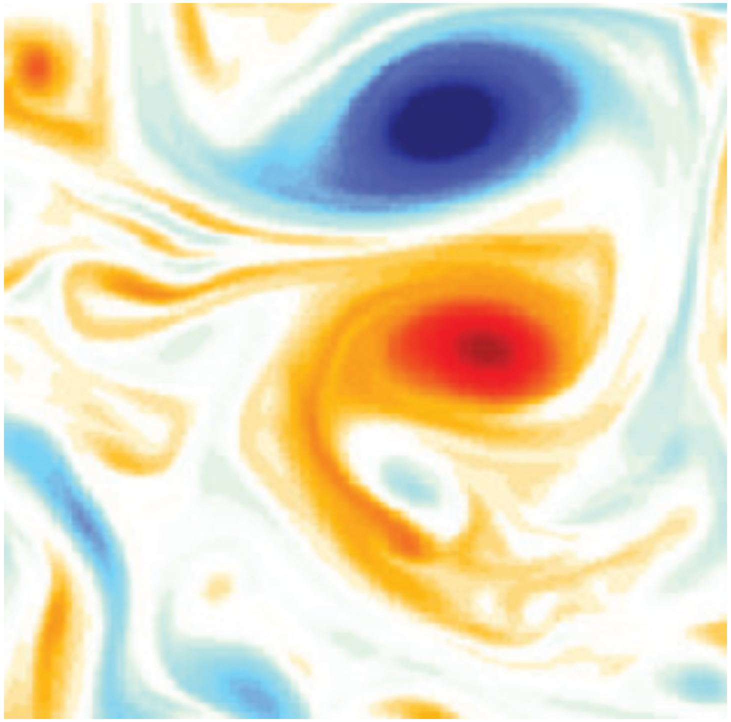


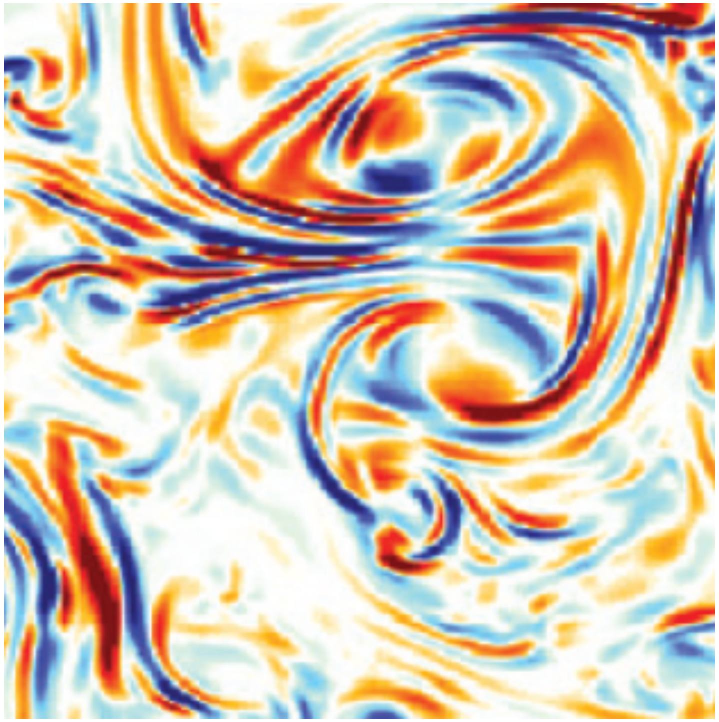
Surface temperature (density) field



W-field associated with the previous temperature field

T **W**





W-field (right) associated with the temperature field (left)

Let us consider several wave packets in a geostrophic jet

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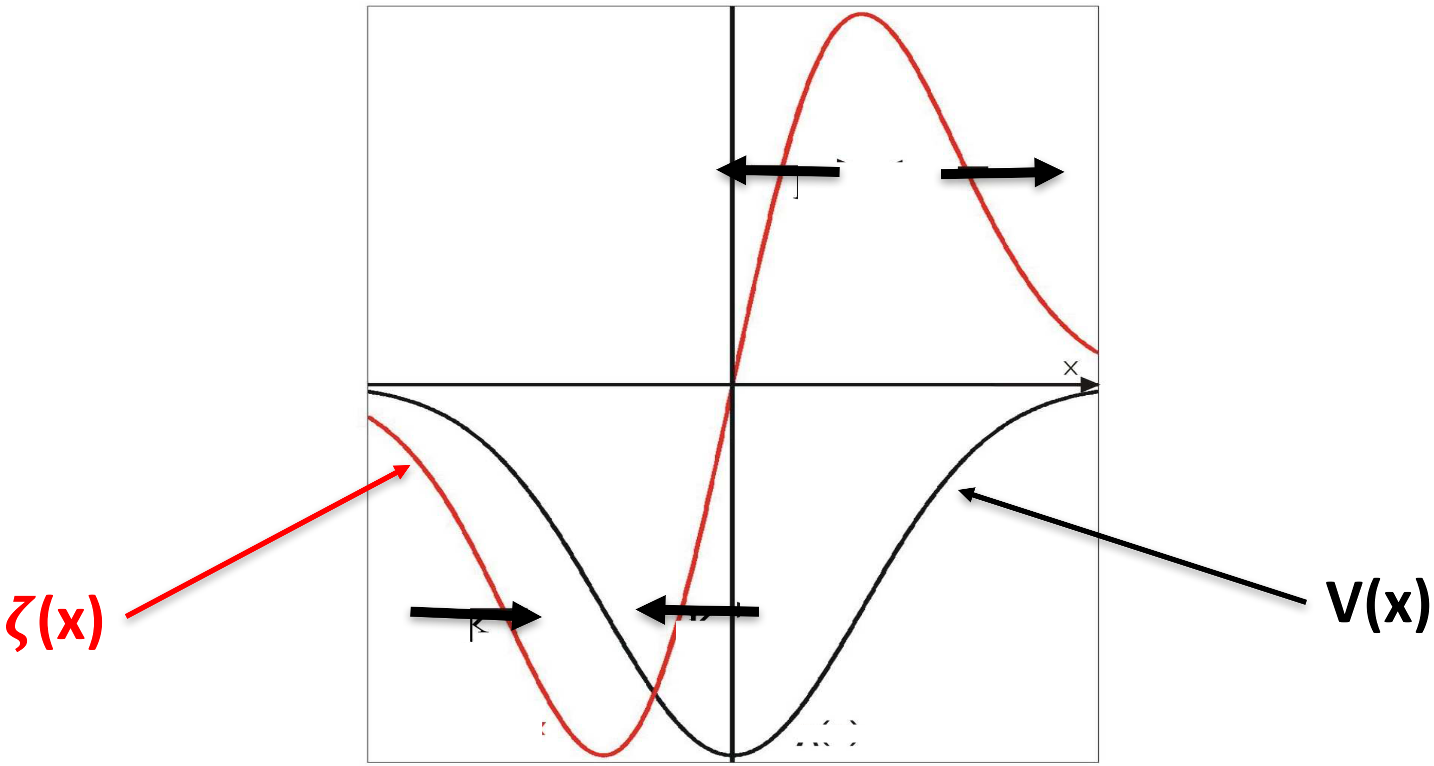

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Example:

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Propagation is due to the impact of pressure gradients on u and v!

NEAR-INERTIAL (f) AND TIDAL WAVES (M2) EXPLAIN A LARGE PART OF THE WAVE SPECTRUM

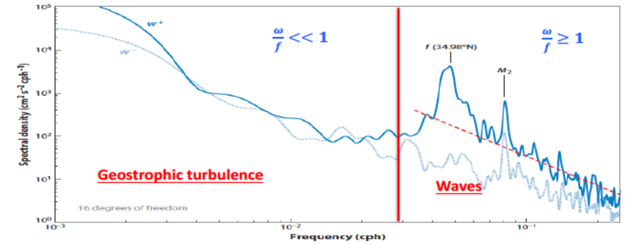
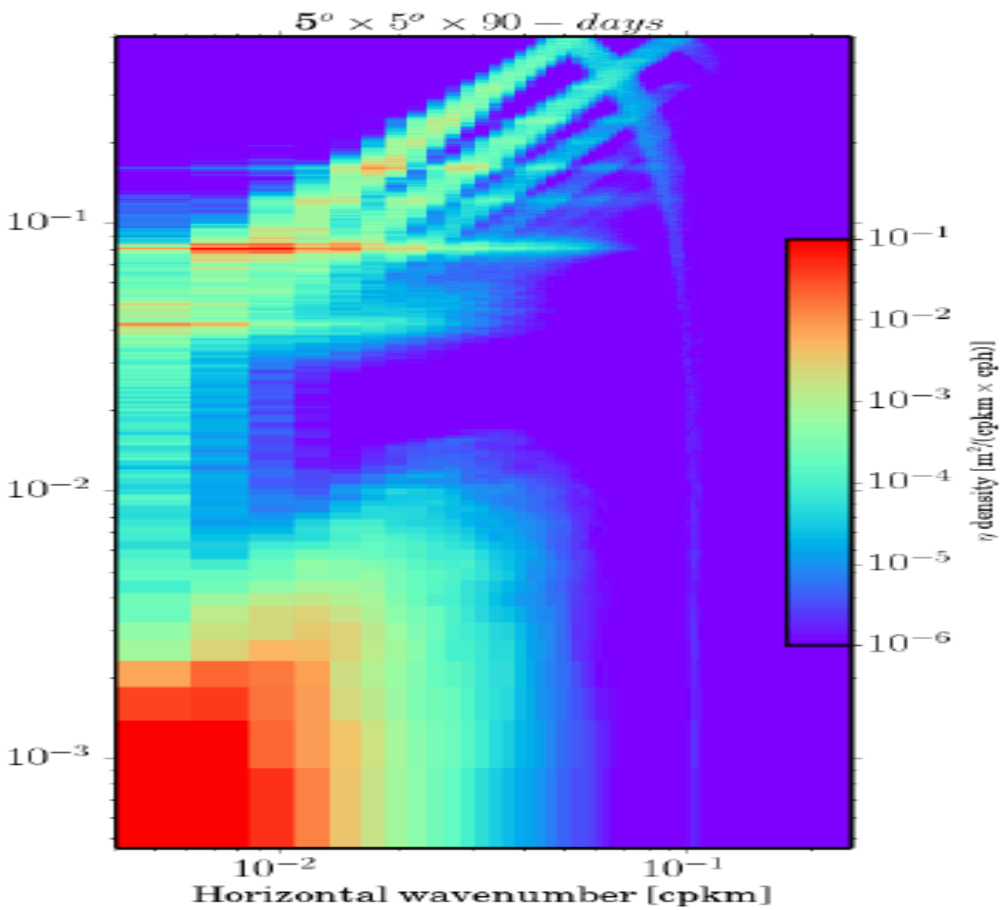


Figure 1

Rotary velocity spectrum at 261-m depth from current-meter data from the WHOI699 mooring gathered during the WESTPAC1 experiment (mooring at 6,149-m depth.) The solid blue line (w^+) is clockwise motion, and the dashed blue line (w^-) is counterclockwise motion; the differences between these emphasize the downward energy propagation that often dominates the near-inertial band. The dashed red line is the line $E_0 N \omega^{-p}$ with $N = 2.0$ cycles per hour (cph), $E_0 = 0.096$ cm^2 s^{-2} cph^{-2}, and $p = 2.25$, which is quantitatively similar to levels in the Cartesian spectra presented by Fu (1981) for station 5 of the Polygon Mid-Ocean Experiment (POLYMODE) II array.

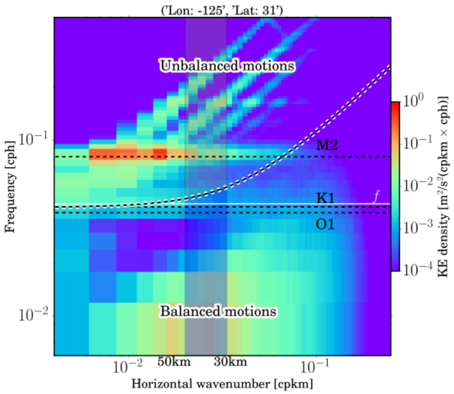


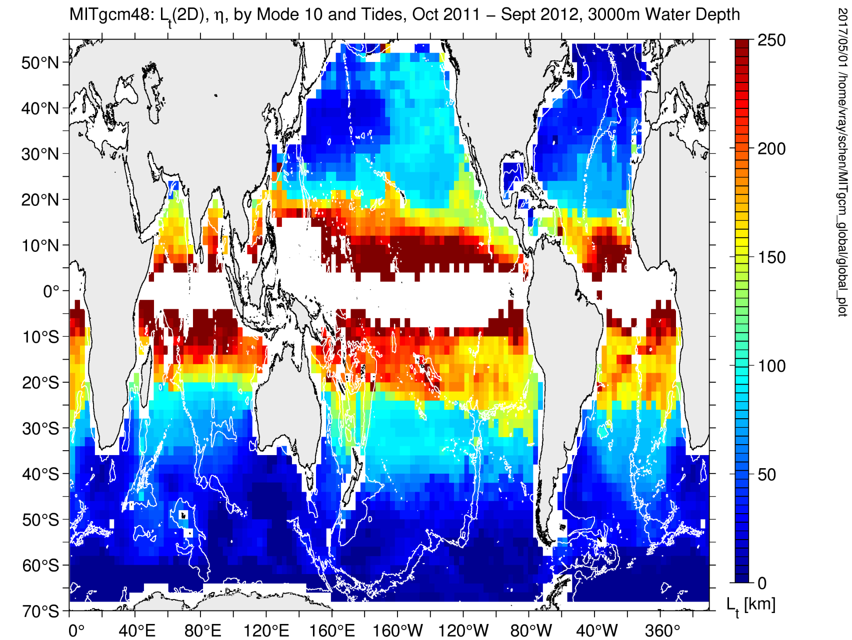
The <u>high frequency part of the wave spectrum</u> is characterized by <u>discrete bands at high wavenumbers!</u>

Next week:

<u>Ray Tracing Approach:</u>
How wavenumbers and frequencies of waves are affected by properties of meso/submesoscale turbulence

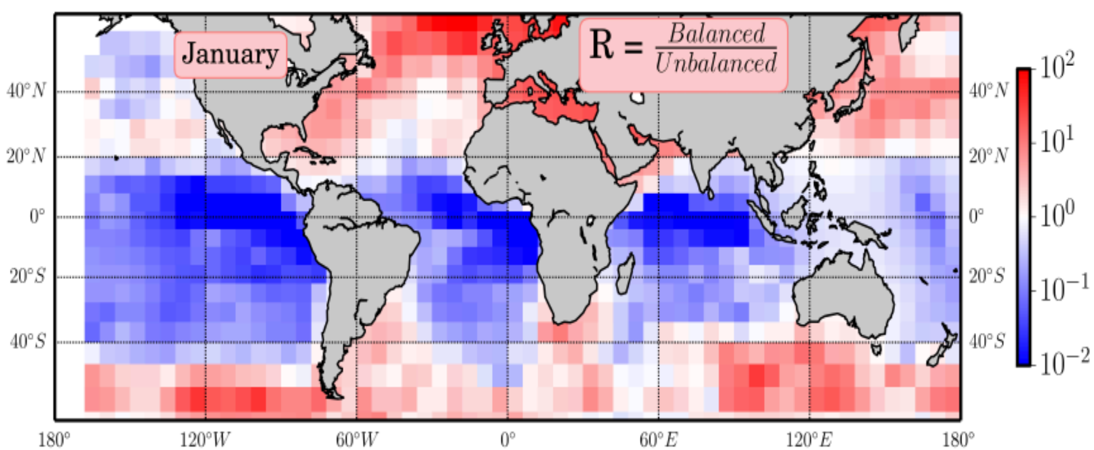
Frequency-Wavenumber spectra of KE





Bo Qiu et al., 2017

Hector T. , Dimitris M.,
Scripps, May 2-4, 2017



Strong seasonality in western boundary currents due to submesoscales:

Results have been confirmed in the Gulf Stream by experimental data [Oleander, LatMix] (Callies et al., NC'15)

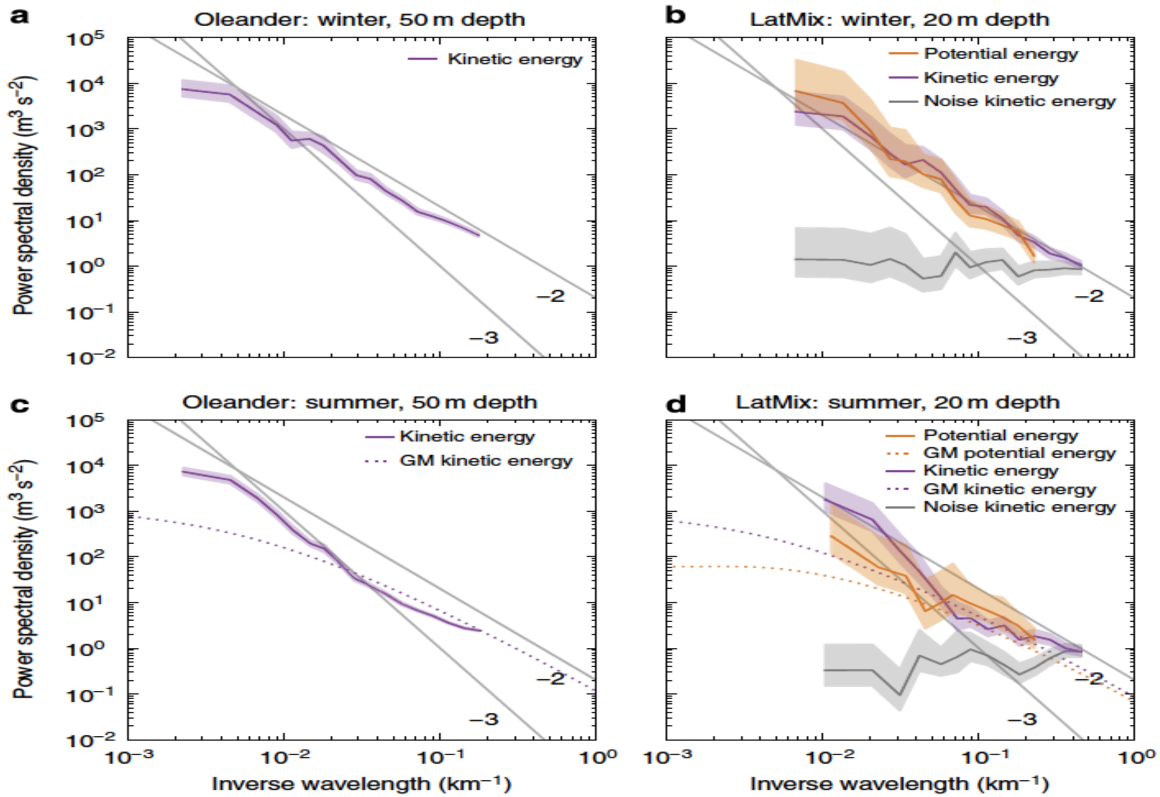


Figure 3 | Seasonality in observations. (**a**) Kinetic energy spectrum at 50 m depth for the Oleander winter data. (**b**) Potential and kinetic energy spectra at 20 m depth for the LatMix winter experiment. (**c**) Kinetic energy spectrum at 50 m depth for the Oleander summer data. (**d**) Potential and kinetic energy spectra at 20 m depth for the LatMix summer experiment. The light shadings are 95% confidence intervals. Also shown are the GM model spectra for internal waves in the seasonal thermocline (with parameters from ref. 30), estimates for the noise level of the LatMix velocity data and reference lines with slopes −2 and −3.

Internal waves affect the velocity spectrum at scales < 50-100 km in summer

See also Callies et al. JPO 2013, Qiu et al. 2017

- Without interactions with balanced motions, waves have NO potential vorticity; or have constant PV …

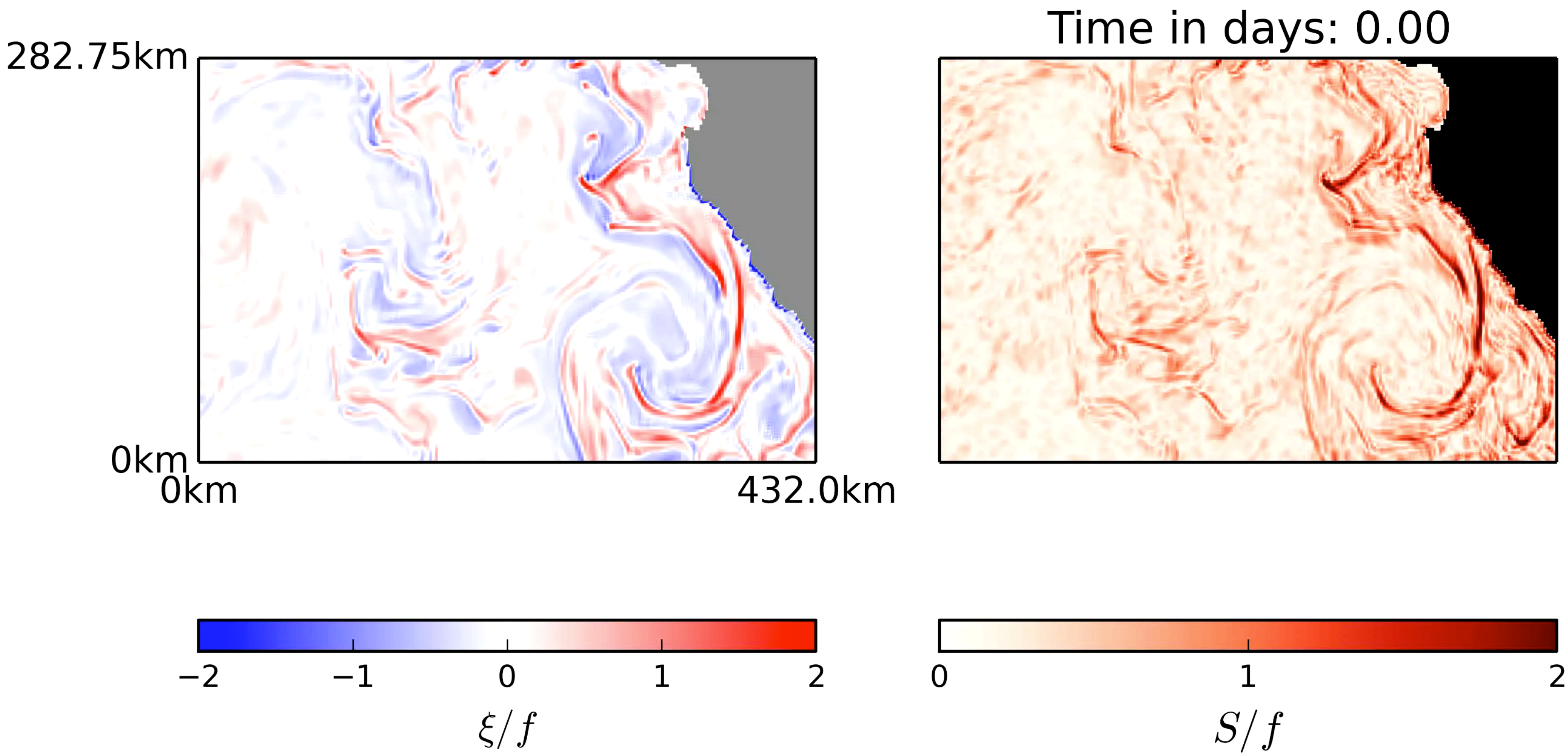
- From primitive equations, Ertel PV is conserved.

- When interacting with balanced motions, waves get PV from the balanced flow.

- If EPV’ is the wave EPV and EPV_b, balance PV, then equations are:

$$\frac{\partial \overline{EPV'^2}}{\partial t} = -\overline{u'.EPV'}.\nabla PV_b$$

Quantification: work in progress …



Time in days: 0.00
282.75km
0km
0km
432.0km
−2
−1
0
1
2
ξ/f
0
1
2
S/f