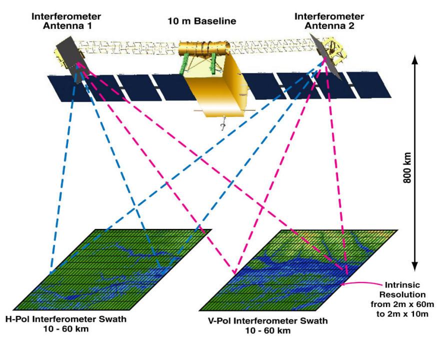
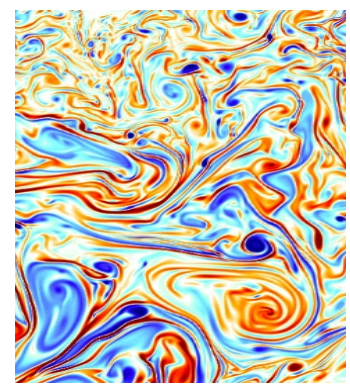
"Ocean Turbulence from SPACE"

Patrice Klein (Caltech/JPL/Ifremer)

(II) – <u>Basic properties and non-dimensional numbers</u>

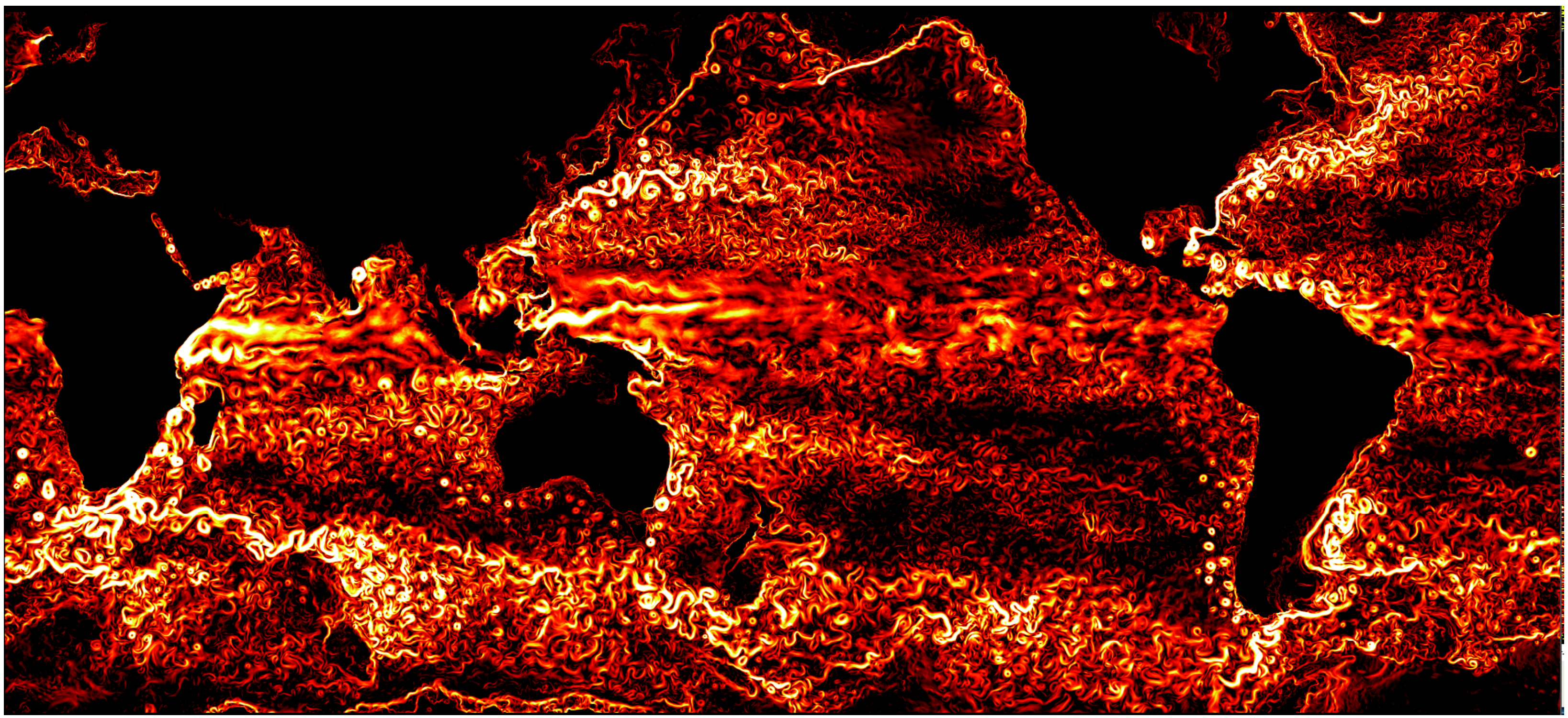




Our vision of the ocean dynamics has significantly changed in the last 20 years ago:

Numerical studies and satellite observations have revealed that all the oceans are **crowded with a large number of mesoscale eddies (with 200-300km scales) => 80% of the total kinetic energy ! Recent**

A fully turbulent ocean !



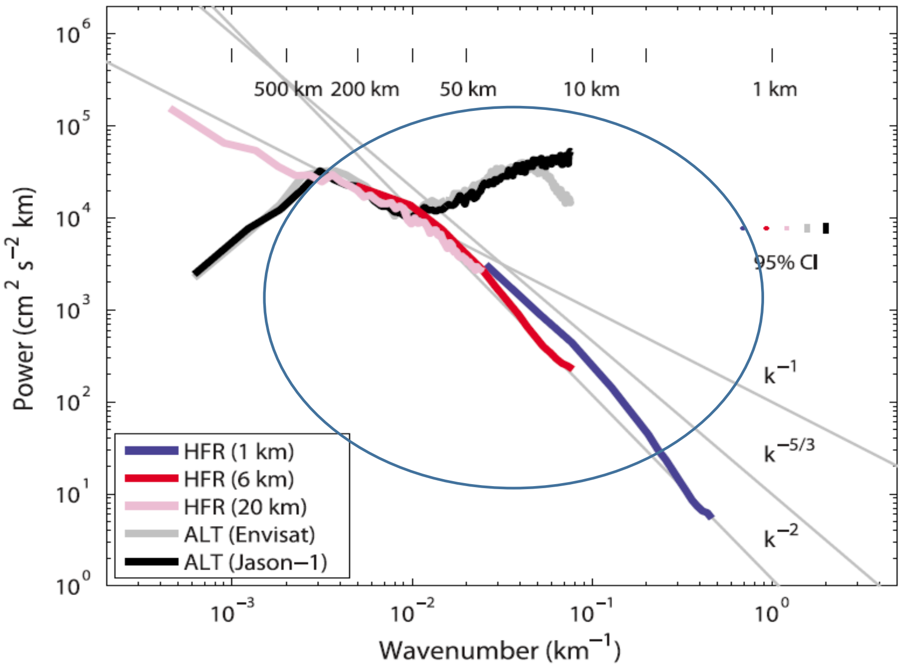
KE at the ocean surface from an OGCM [ECCO2 Courtesy Raf Ferrari MIT]

Satellite oceanography further reveals a large range of scales in all regions of the World Ocean including not only mesoscale eddies (100–300 km) but also submesoscales (5–40 km) and even smaller scales

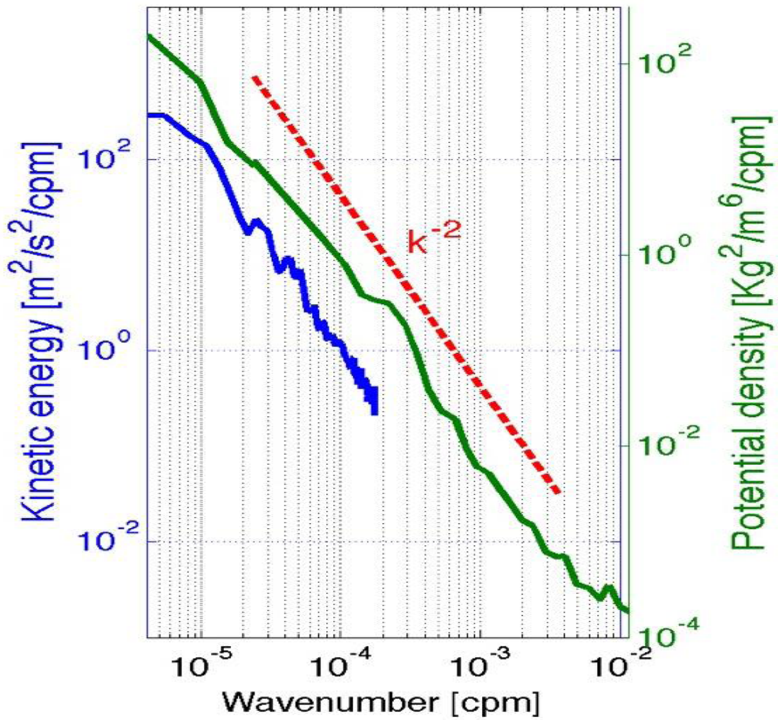
These scales are strongly interacting as indicated by the spatial patterns displayed by the SST images. But small scales are not just passively advected by horizontal motions. SAR and Ocean Color images suggest that they are associated with an energetic vertical velocity field.

These **ocean scale interactions** appear to be confirmed by some in-situ experiments ...

Kim et al.,'11, using **HF radar observations,**
indicate that the velocity spectrum (NE Pacific) is
in $\mathbf{k^{-2}}$, in particular in the range of submesoscales



Observations: from Ferrari & Rudnick, 2000:
both velocity and density spectra in $\mathbf{k^{-2}}$

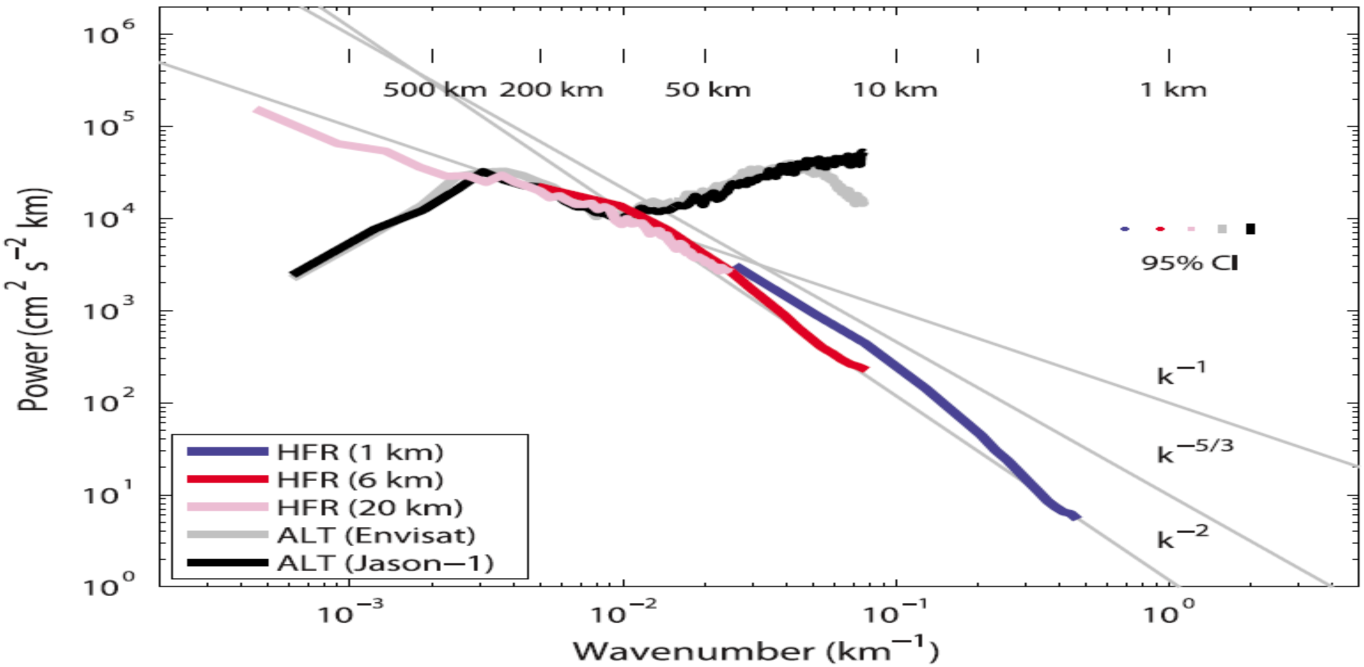


These high-resolution observations reveal that submesoscales are more energetic than previously thought: velocity and density spectra have a shallower slope (**k^{-2} instead of k^{-3}** near the surface)

These continuous spectra indicate that the different scales are strongly interacting with the consequences that: **significant energy transfers (due to the nonlinear interactions)** **can occur between the different scales!**

But we need a dynamical framework to analyse these satellite data (in connection with in-situ data) that goes beyond the geostrophic approximation in order to take into account these nonlinearities.

………



Focus on scales from 10 km to 300-500 km

This concerns turbulence in rotating and stratified flows

What are the basic properties and nondimensional numbers that characterize the scale range (between 10 km and 300-500 km) ?

1 - Boussinesq approximation:

$$\rho = \rho_0 + \rho'(x, y, z, t). \quad \text{with} \quad |\rho'| \ll \rho_0.$$

$$\rho_0 = 1028\ kg/m^3 \qquad \rho' \sim 3\ kg/m^3.$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0.$$

$$\frac{\partial \rho'}{\partial t} + u\frac{\partial \rho'}{\partial x} + v\frac{\partial \rho'}{\partial y} + w\frac{\partial \rho'}{\partial z} = 0$$

2 - Aspect ratio:

$$\frac{H}{L} \ll 1. \qquad H = 5000\,m \quad L = 10^5\,m \quad \frac{H}{L} \sim 5.10^{-3}$$

hydrostatic balance:

$$\frac{1}{\rho_0}\frac{\partial p}{\partial z} = -g\frac{\rho}{\rho_0}$$

… This balance may have to be revisited for submesoscales within the mixed-layer …

3 - Reynolds number:

$$\frac{\partial u_H}{\partial t} + u_H \cdot \nabla u_H + w \frac{\partial u_H}{\partial z} + f \cdot \vec{k} \times u_k = -\frac{\nabla p}{\rho_0} + \nu \Delta u_H .$$

$$Re = \frac{\text{NL terms}}{\text{Viscous terms}} \sim \frac{U^2}{L} \cdot \frac{L^2}{\nu U} = \frac{UL}{\nu} \gg 1$$

$$U \sim 1\,m\,s^{-1} \quad L = 10^5\,m \quad \nu = 10^{-6}\,m^2 s^{-1} \Rightarrow Re \sim 10^{10}.$$

4 - Rossby number:

$$\frac{\partial u_H}{\partial t} + u_H \cdot \nabla u_H + w \frac{\partial u_H}{\partial z} + f \cdot \vec{k} \times u_k = -\frac{\nabla p}{\rho_0} + \nu \Delta u_H .$$

$$R_o = \frac{\text{NL terms}}{\text{Coriolis term}} = \frac{U}{fL} \leqslant 1.$$

$$U = 1\,m.s^{-1} \quad L = 10^5\,m \quad f = 10^{-4}\,s^{-1} \Rightarrow R_o \sim 0.1.$$

… Rossby number may be O(1) at submesoscales …

5 – Nonlinear parameter (Rhines scale):

$$\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}. \qquad f = f(y).$$

$$\frac{\partial \zeta}{\partial t} + \underbrace{u \cdot \nabla \zeta}_{\text{NL term}} + \underbrace{\beta v}_{\beta\text{-effect}} = 0 \qquad \text{(2D version)},$$

$$\text{NL param.} = \frac{\text{NL term}}{\beta\ \text{effect}} = \frac{U}{\beta L^2} \gg 1$$

$$L_\beta = \left(\frac{U}{\beta}\right)^{1/2} \quad \text{Rhines scale.}$$

$$\text{NL param.} = \left(\frac{L_\beta}{L}\right)^2.$$

… Rossby waves propagation is not considered…

6 – Burger number (deduced from hydrostaticity and geostrophy):

$$Bu = \frac{\text{Stratching}}{\text{Rel. Verticity}} = \frac{N^2 H^2}{f^2 L^2} = 1$$

$\Rightarrow L \sim L_d$. (1st Rossby radius of deformation)

… May be questionable …

7 – Richardson number:

$$Ri = \frac{\text{Stabilizing forces}}{\text{Destabilizing force}} = \frac{N^2 H^2}{U^2} \geq 1$$

… Ri < 1 is observed in some numerical simulations at submesoscales…

Nondimensional numbers that characterize the scale range 10 km to 300 km

1- Boussinesq approximation: $|\rho'| \ll \rho_0$.

2- Aspect ratio: $H/L \ll 1$.

3- Reynolds number: $Re = \frac{UL}{\nu} \gg 1$.

4- Rossby number: $Ro = \frac{U}{fL} \leq 1$

5- Nonlinear parameter: $\frac{U}{\beta L^2} \gg 1$.

6- Burger number: $Bu = \frac{N^2H^2}{f^2L^2} \sim 1$.

7- Richardson number: $Ri = \frac{N^2H^2}{U^2} \geq 1$.

Departure from the geostrophic balance ???

Momentum equations:

$\vec{v} = (u, v)$

$$\boxed{\frac{\partial \vec{v}}{\partial t} + \vec{v}\cdot\nabla\vec{v} + f\cdot\vec{k}\times\vec{v} = -\frac{1}{\rho_0}\nabla p}$$

$$R_0 f U \qquad R_0 f U \qquad f U \qquad f U$$

(… see the momentum balance in the next slide …)

Divergence equation:

$$-\frac{\partial}{\partial t} w_z + \nabla\cdot(\vec{v}\cdot\nabla\vec{v}) - f\cdot\zeta = -\Delta p. \qquad \zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}.$$

$$R_0^2 \frac{fU}{L} \qquad R_0 \frac{fU}{L} \qquad \frac{fU}{L} \qquad \frac{fU}{L}$$

$$\Rightarrow \boxed{\nabla\cdot(\vec{v}\cdot\nabla\vec{v}) - f\zeta = -\Delta p.}$$

$$O(R_0) \qquad O(1) \qquad O(1)$$

$\nabla\cdot(\vec{v}\cdot\nabla\vec{v})$ is the Okubo–Weiss quantity.

Some calculations lead to:

$$\nabla\cdot(\vec{v}\cdot\nabla\vec{v}) = \frac{1}{2}\left[S_1^2 + S_2^2 - \zeta^2\right].$$

$$S_1 = \left[\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}\right] \quad S_2 = \left[\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right] \quad \zeta = \left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right].$$

$$OW = \nabla\cdot(\vec{v}\cdot\nabla\vec{v}).$$

$OW < 0$ in vorticity regions

$OW > 0$ in strain regions.

An old problem: including inertia in the derivation of velocities from pressure

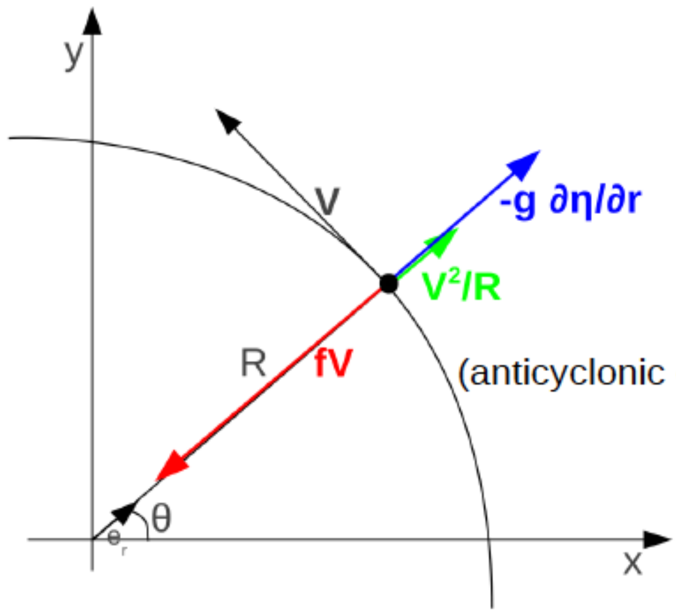
(here in a stationary case)

$$\boldsymbol{u}.\nabla\boldsymbol{u}+f\boldsymbol{k}\times\boldsymbol{u}=-g\nabla\eta$$

Which can be re-written as a function of geostrophic velocities:

$$\boldsymbol{u}-\frac{\boldsymbol{k}}{f}\times(\boldsymbol{u}.\nabla\boldsymbol{u})=\boldsymbol{u_g}$$

Gradient wind



(anticyclonic curve in the southern hemisphere)

Cross-stream balance of forces

$$V + \frac{V^2}{fR} = V_g$$

Solutions: $$V = \frac{2V_g}{1 \pm \sqrt{1 + 4V_g/(fR)}}$$

("normal" solution corresponds to the positive sign)

From Penven et al. Ocean Sciences 2016

Effect of centrifugal force on eddies:

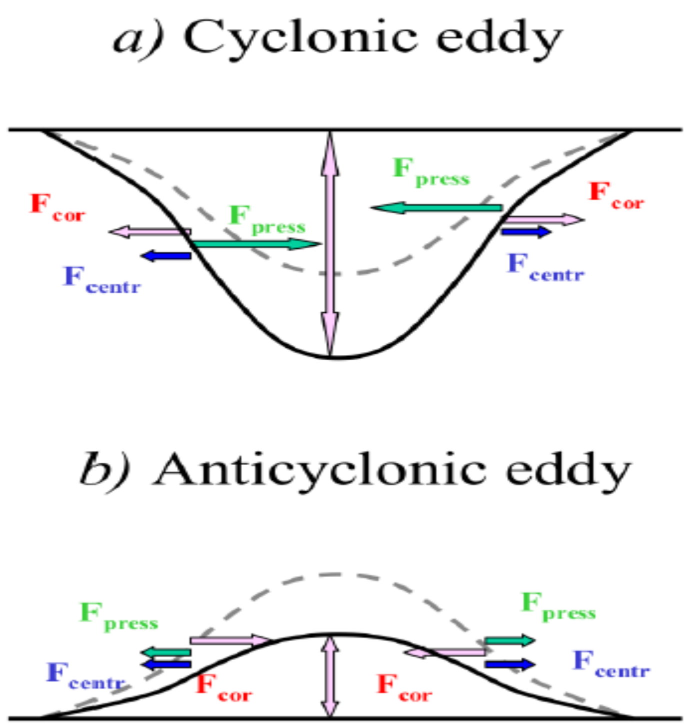


Figure adapted from Maximenko and Niiler (2006)

Cyclones : larger pressure gradient to compensate for centrifugal force

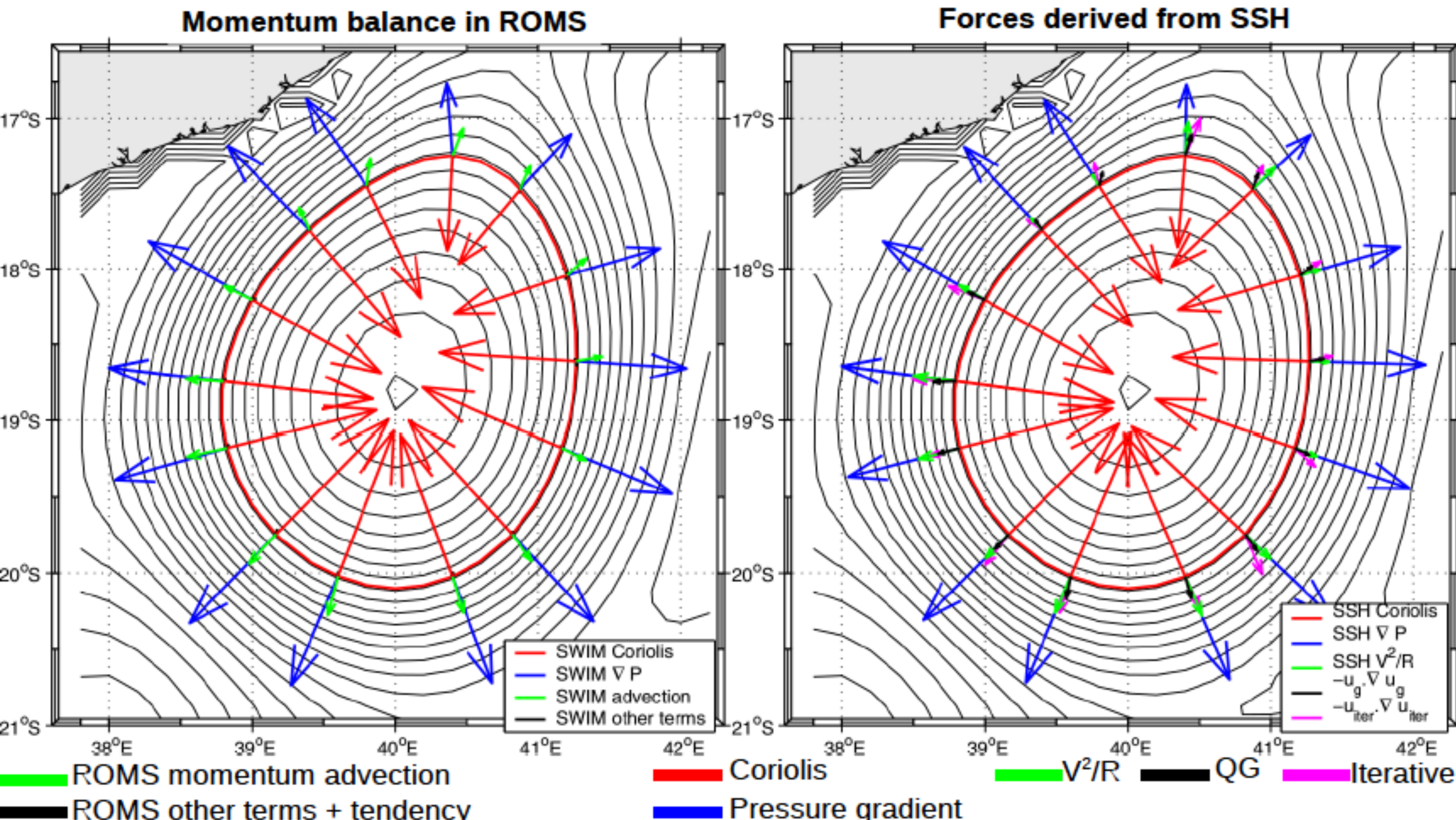
(here in a stationary case)

$$\boldsymbol{u}.\nabla\boldsymbol{u}+f\boldsymbol{k}\times\boldsymbol{u}=-g\nabla\eta$$

From Penven et al. Ocean Sciences 2016

Test 1: momentum balance around a modeled Mozambique Channel Ring

ROMS model simulation of the Mozambique Channel (Halo et al., 2014)



From Penven et al. Ocean Sciences 2016

Momentum equations:

$\vec{v} = (u, v)$

$$\boxed{\begin{array}{ccccccc} \dfrac{\partial \vec{v}}{\partial t} & + & \vec{v}\cdot\nabla\vec{v} & + & f\cdot\vec{k}\times\vec{v} & = & -\dfrac{1}{\rho_0}\nabla p \\ R_0 f U & & R_0 f U & & f U & & f U \end{array}}$$

(… see the momentum balance in the next slide …)

Divergence equation:

$$-\frac{\partial}{\partial t} w_z + \nabla\cdot(\vec{v}\cdot\nabla\vec{v}) - f\cdot\zeta = -\Delta p \qquad \zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$$

$$R_0^2 \frac{fU}{L} \qquad R_0 \frac{fU}{L} \qquad \frac{fU}{L} \qquad \frac{fU}{L}$$

$$\longrightarrow \boxed{\nabla\cdot(\vec{v}\cdot\nabla\vec{v}) - f\zeta = -\Delta p}$$

$$O(R_0) \qquad O(1) \qquad O(1)$$

$\nabla\cdot(\vec{v}\cdot\nabla\vec{v})$ is the Okubo–Weiss quantity.

Some calculations lead to:

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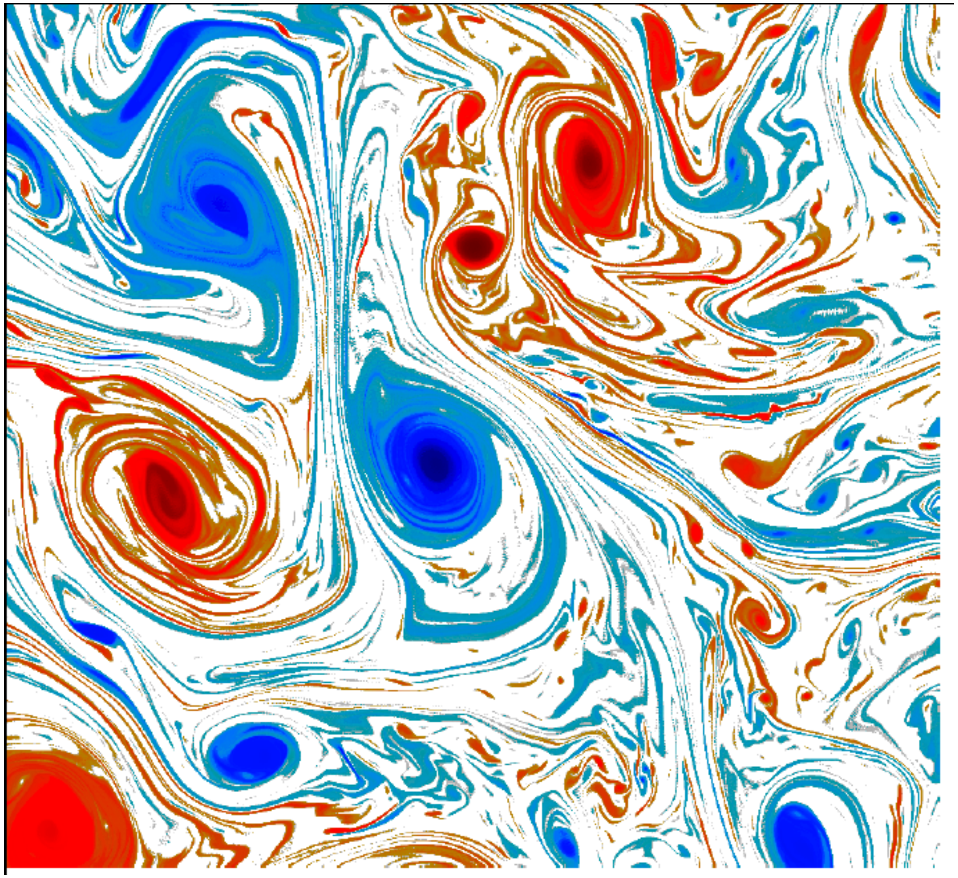


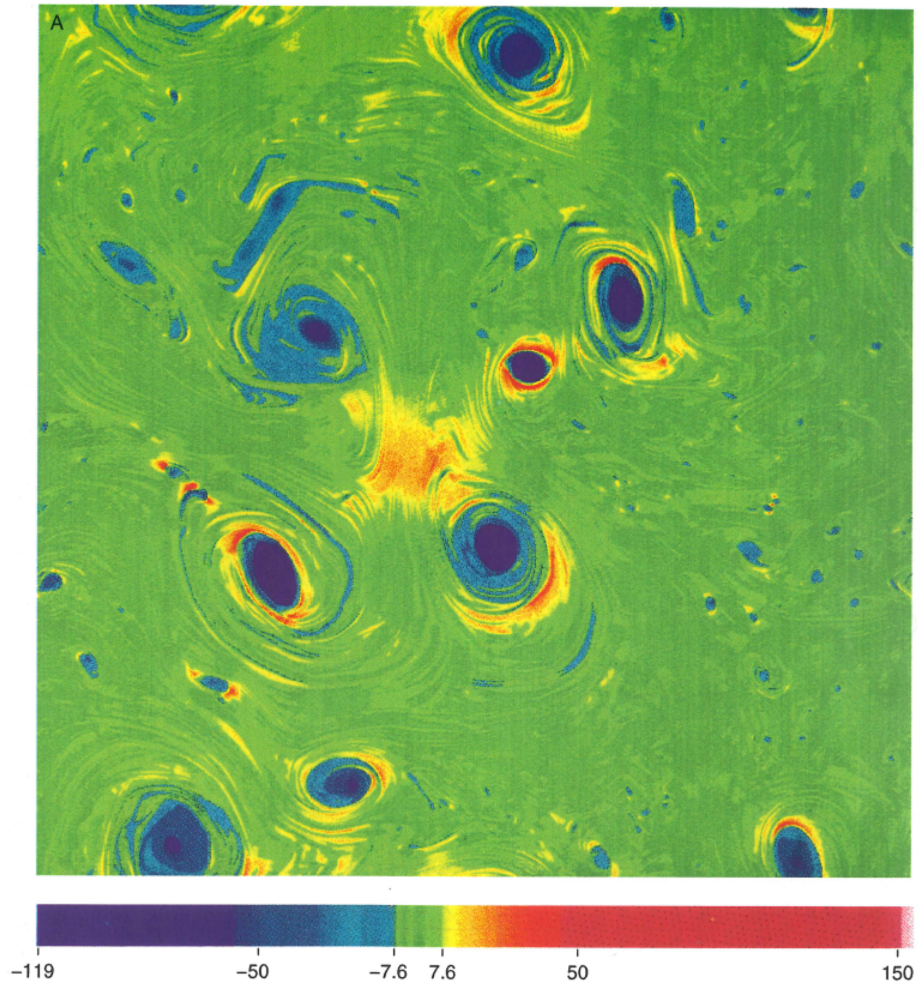
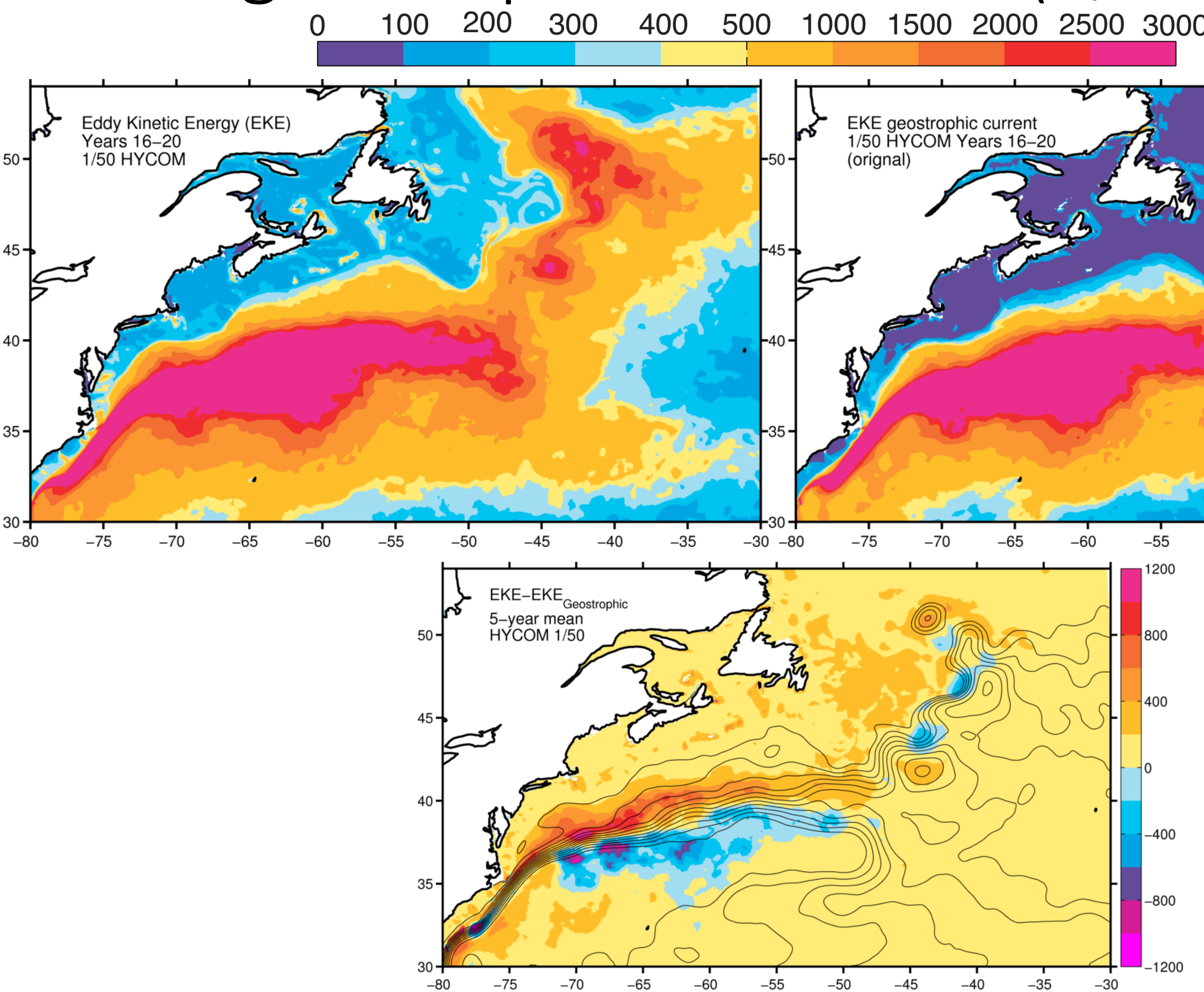


Fig. 3. (a) The Okubo–Weiss eigenvalue λ_0 and (b) the exact eigenvalue λ_+ at the same time as Fig. 1.

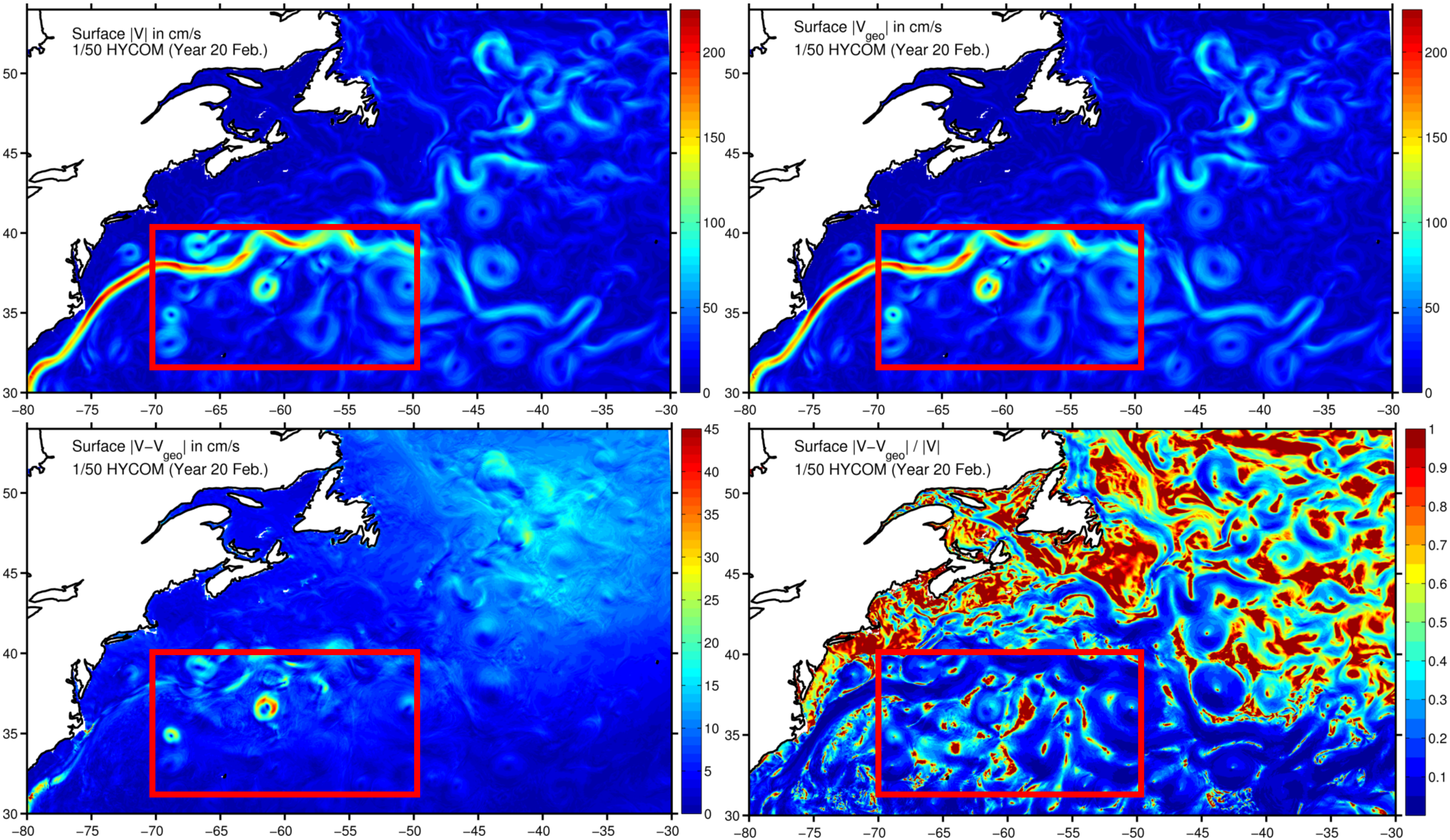
From Hua & Klein Physica D 1998

EKE geostrophic difference (1/50°)



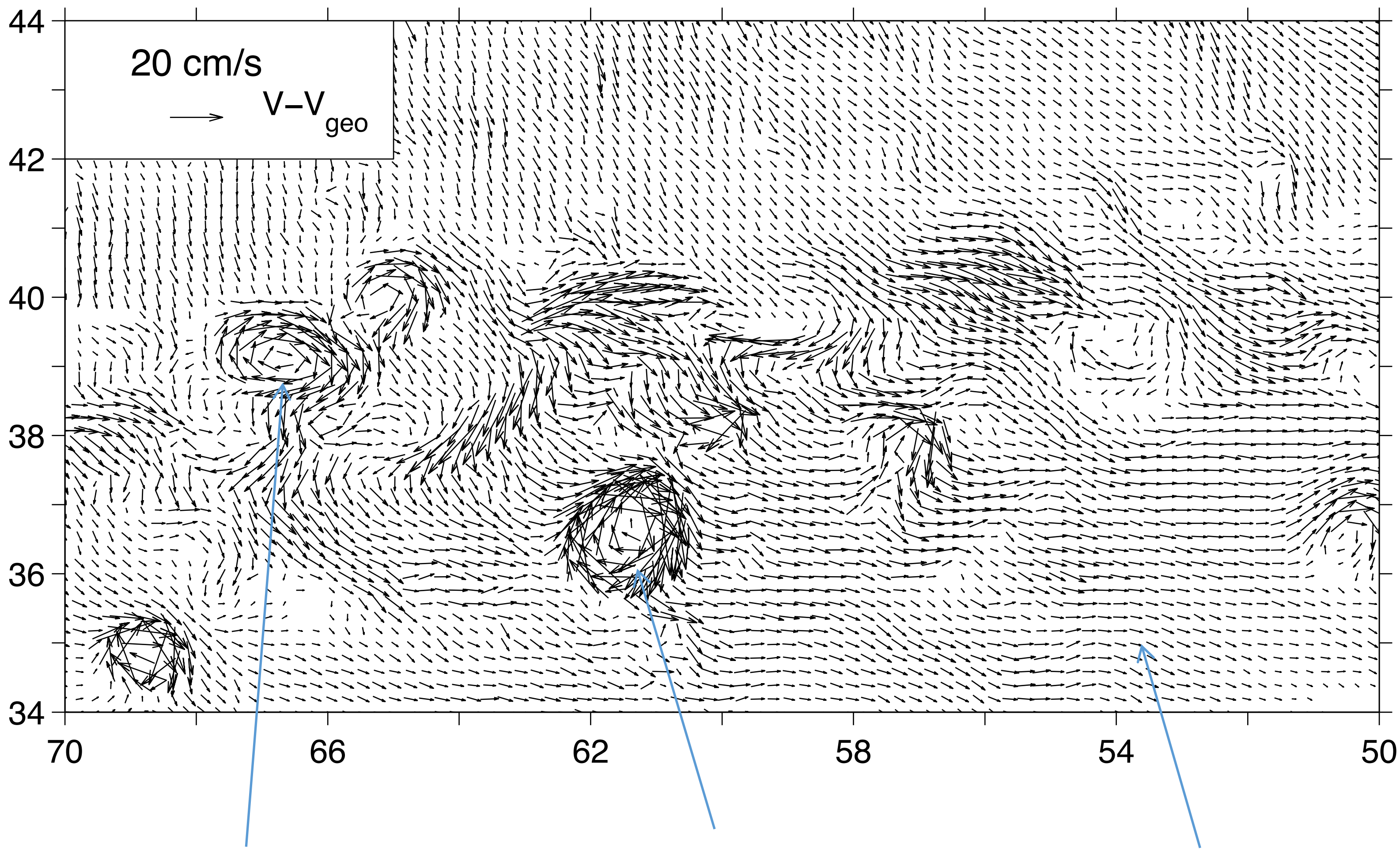
From Eric Chassignet: Ocean Sciences 2016

For a typical month



From Eric Chassignet: Ocean Sciences 2016

Nonlinear terms are important !



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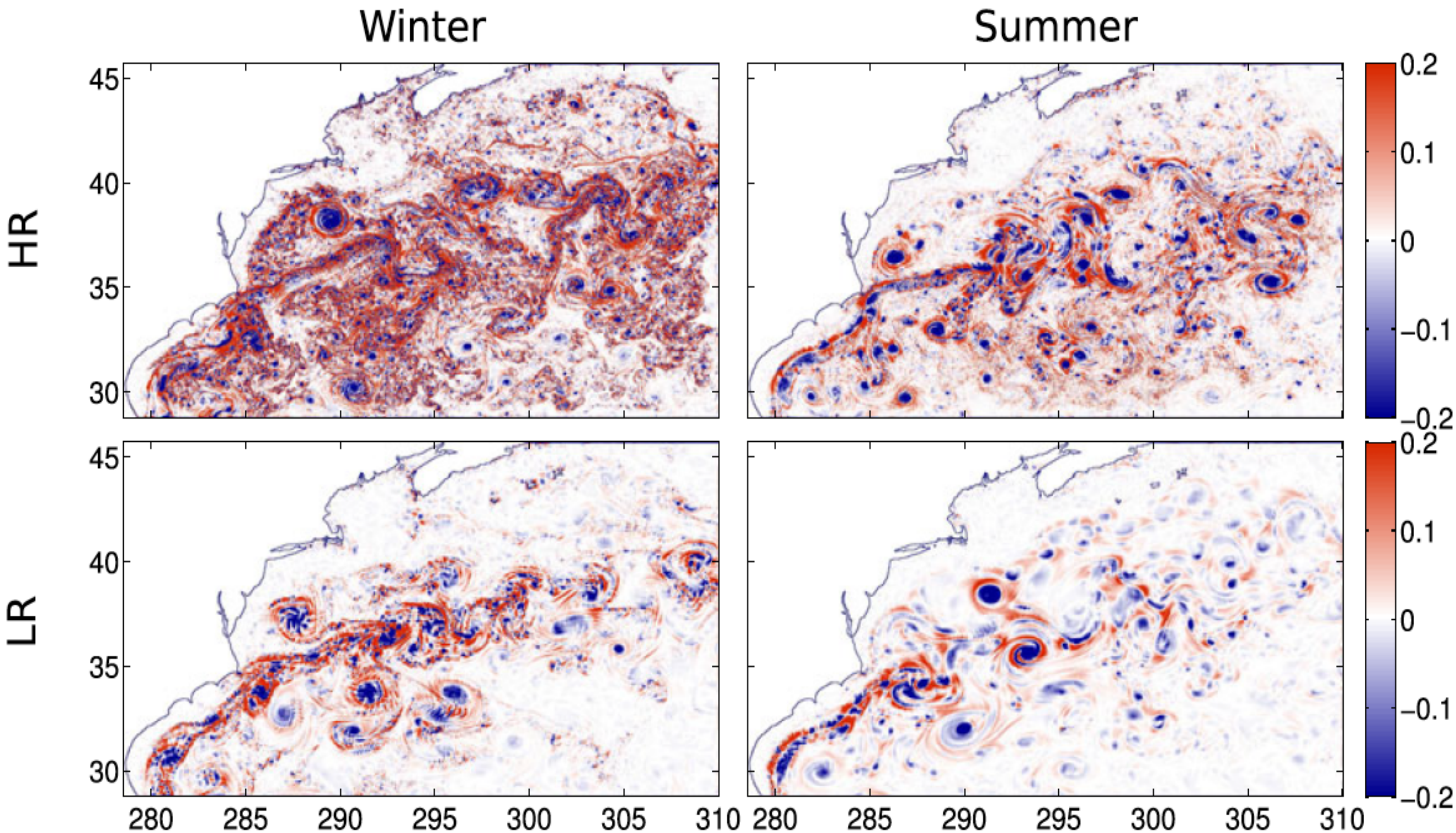


Fig. 14 Okubo–Weiss parameter normalized by f_0^2 computed at the surface (5 m) for winter (*left column*), summer (*right column*), HR (*top row*), and LR (*bottom row*)

From Mensa et al., OD 2013

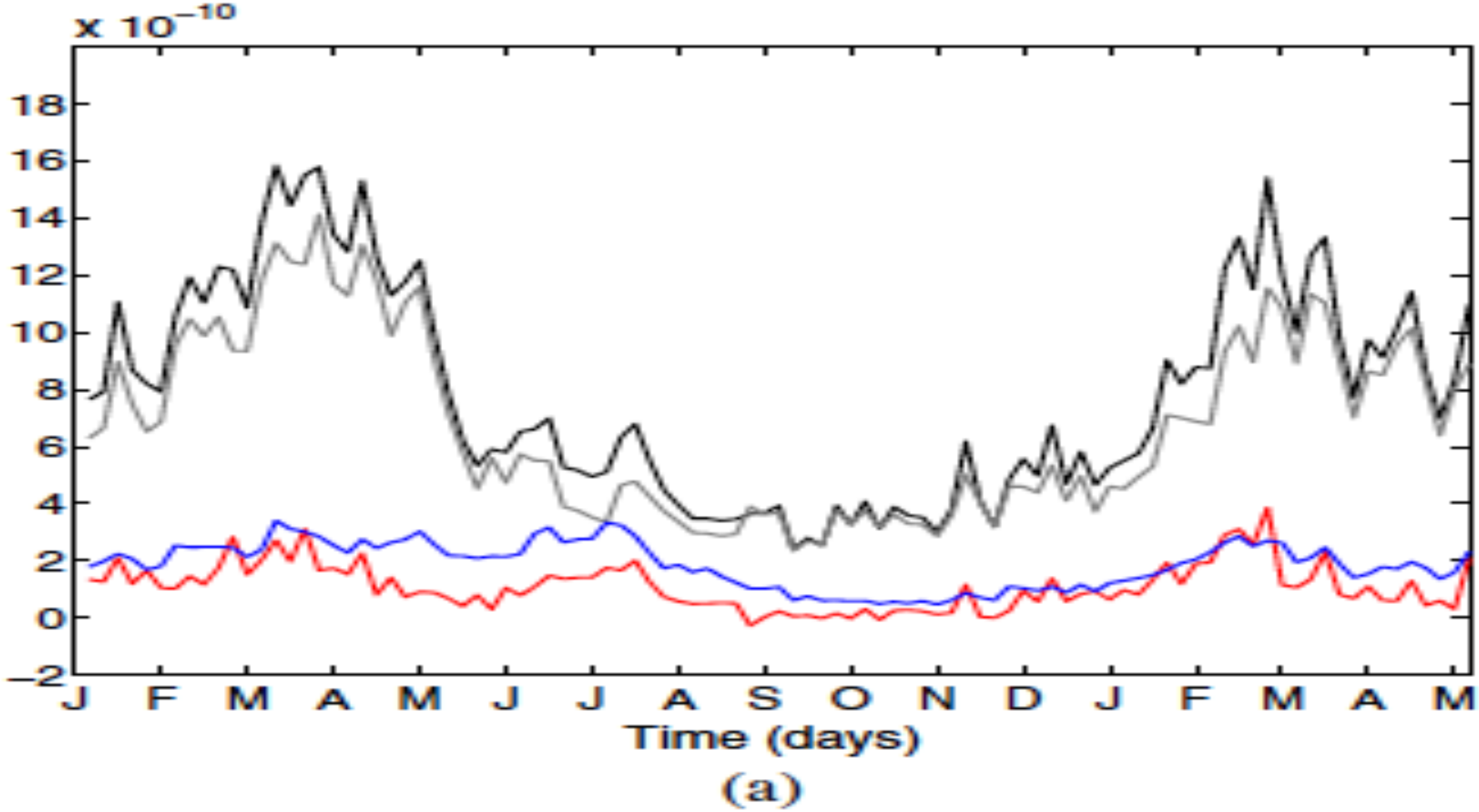
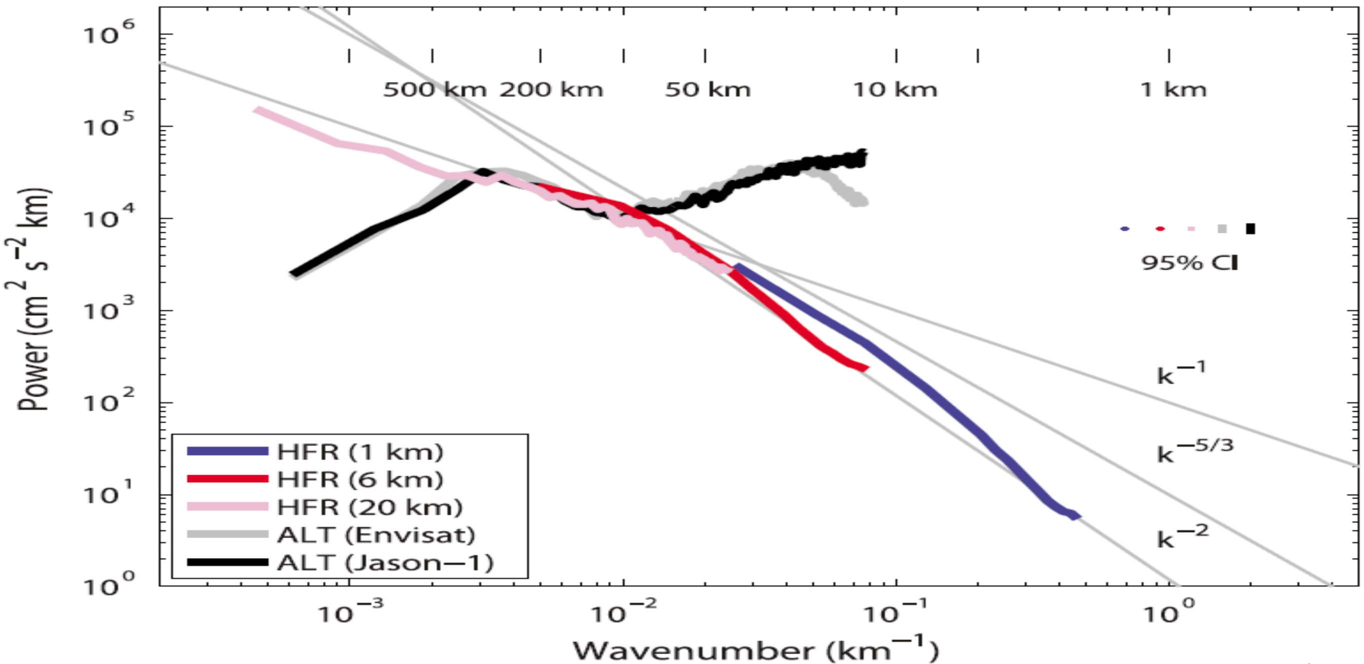


Fig. 15 Temporal evolution of the components of the Okubo–Weiss parameter (per square second, from Eq. 9) integrated over region A at a 5-m depth for **a** HR and **b** LR: OW parameter (*red line*), S^2 (strain rate, *black line*), ξ^2 (relative vorticity squared, *gray line*), and δ^2 (divergence squared, *blue line*)

From Mensa et al., OD 2013



From Kim et al. JGR'11

Focus on scales from 10 km to 300-500 km

Energy transfers between scales ?

Nonlinear terms drive the kinetic energy transfer between scales!