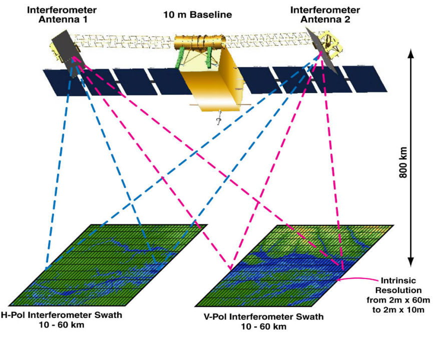
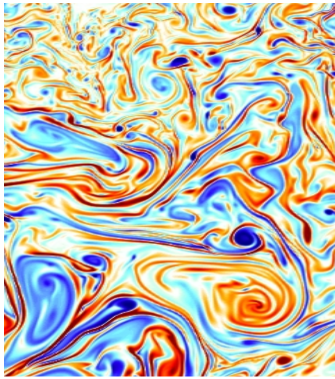
”Ocean Turbulence from SPACE”

Patrice Klein (Caltech/JPL/Ifremer)

(VIII) – <u>Surface Quasi-Geostrophic (SQG) turbulence (a)</u>





2-D turbulence is a very simple dynamical framework that captures the basic characteristics of eddy interactions and in particular the inverse KE cascade and direct enstrophy and tracer cascades.
Flows are assumed to be non-divergent (the vertical velocity, w, is zero).

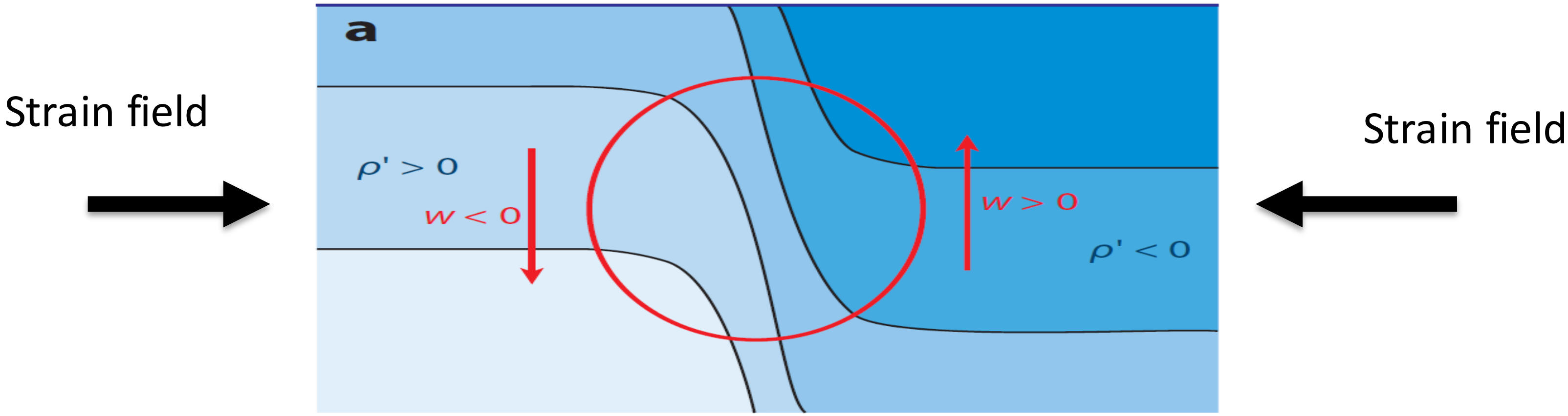
Some characteristics of eddy interactions may differ when flows are divergent ($w \neq 0$).

The quasi-geostrophic (3-D) framework is suitable for non-divergent turbulent flows characterized by a small Rossby number (Charney 1971). Usually it does not take into account the boundary dynamics (dynamics associated with surface density gradients, or surface fronts).

However experimental data strongly emphasize the need to take into account the boundary dynamics and its coupling with the interior …

Why to take into account density fronts at boundaries ?

Density fronts at boundaries are affected by FRONTOGENESIS and therefore associated with energetic 3-D motions



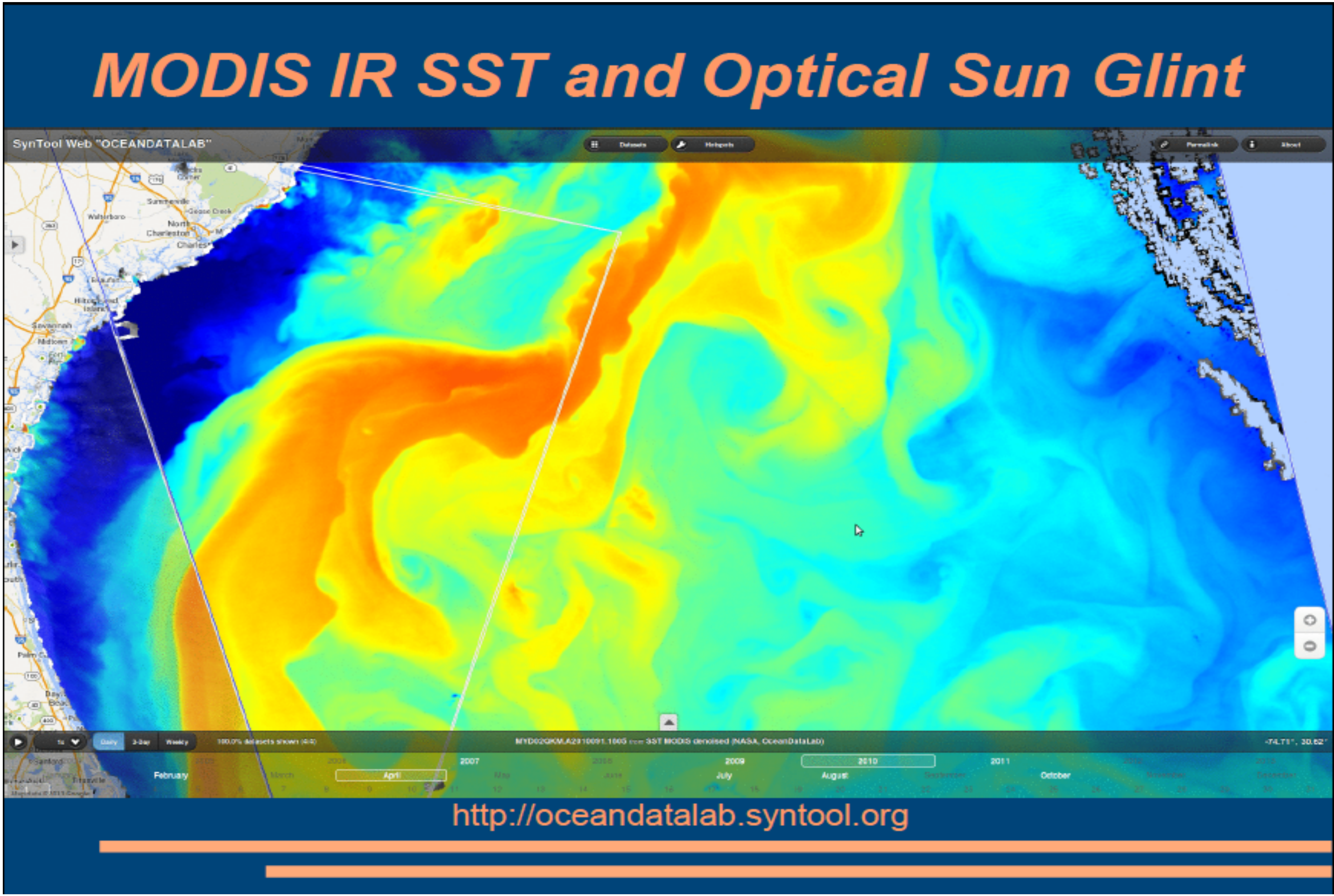
Stirring mechanisms intensify a density front but decrease the current vertical shear, which creates **a thermal wind unbalance**.

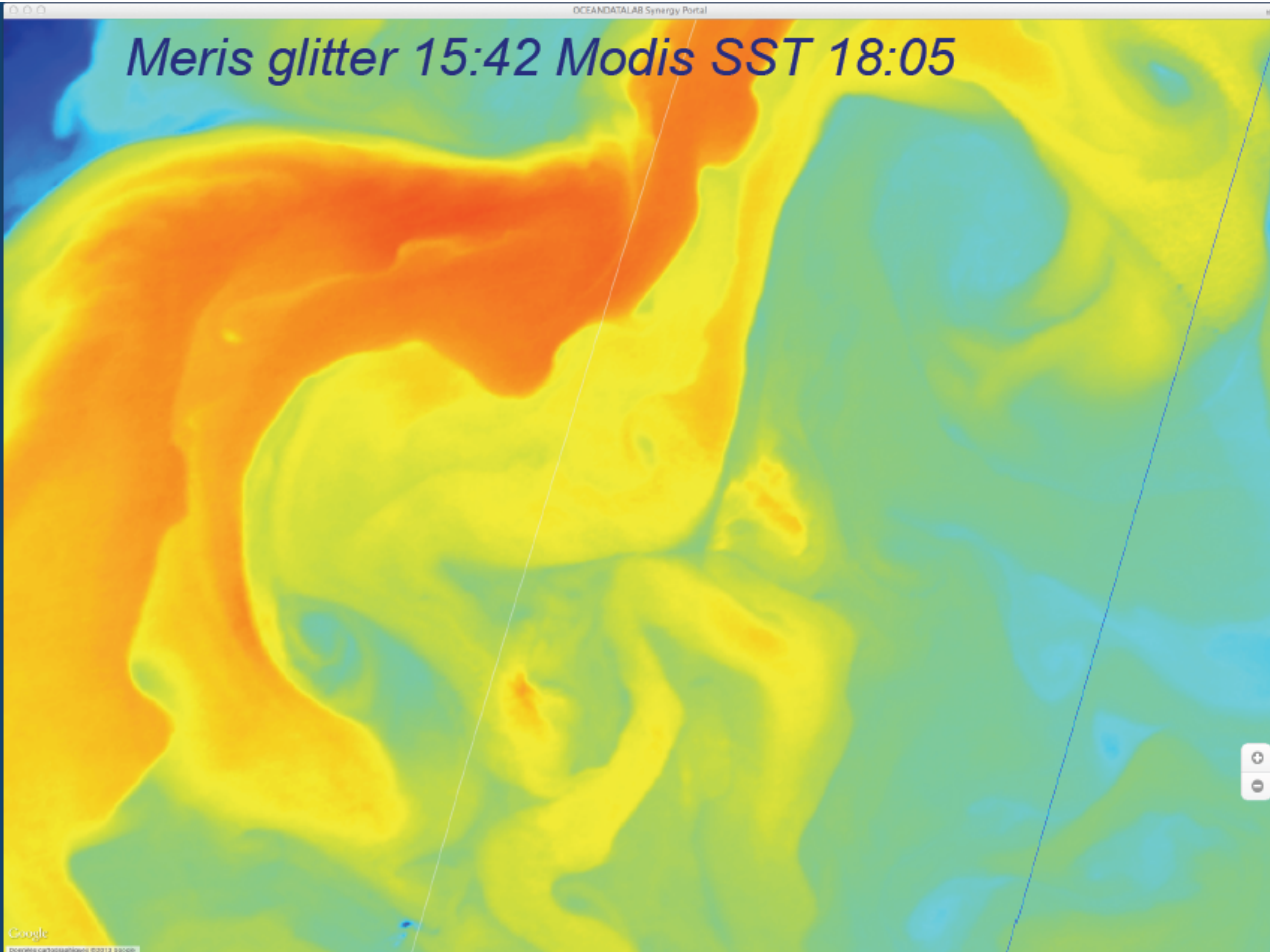
Then an ageostrophic circulation including a w-field, develops for the density fronts to be again in thermal wind balance. This w usually is such that $|\mathbf{w}| \sim |\boldsymbol{\Delta\rho}|$ and can extend at large depths.

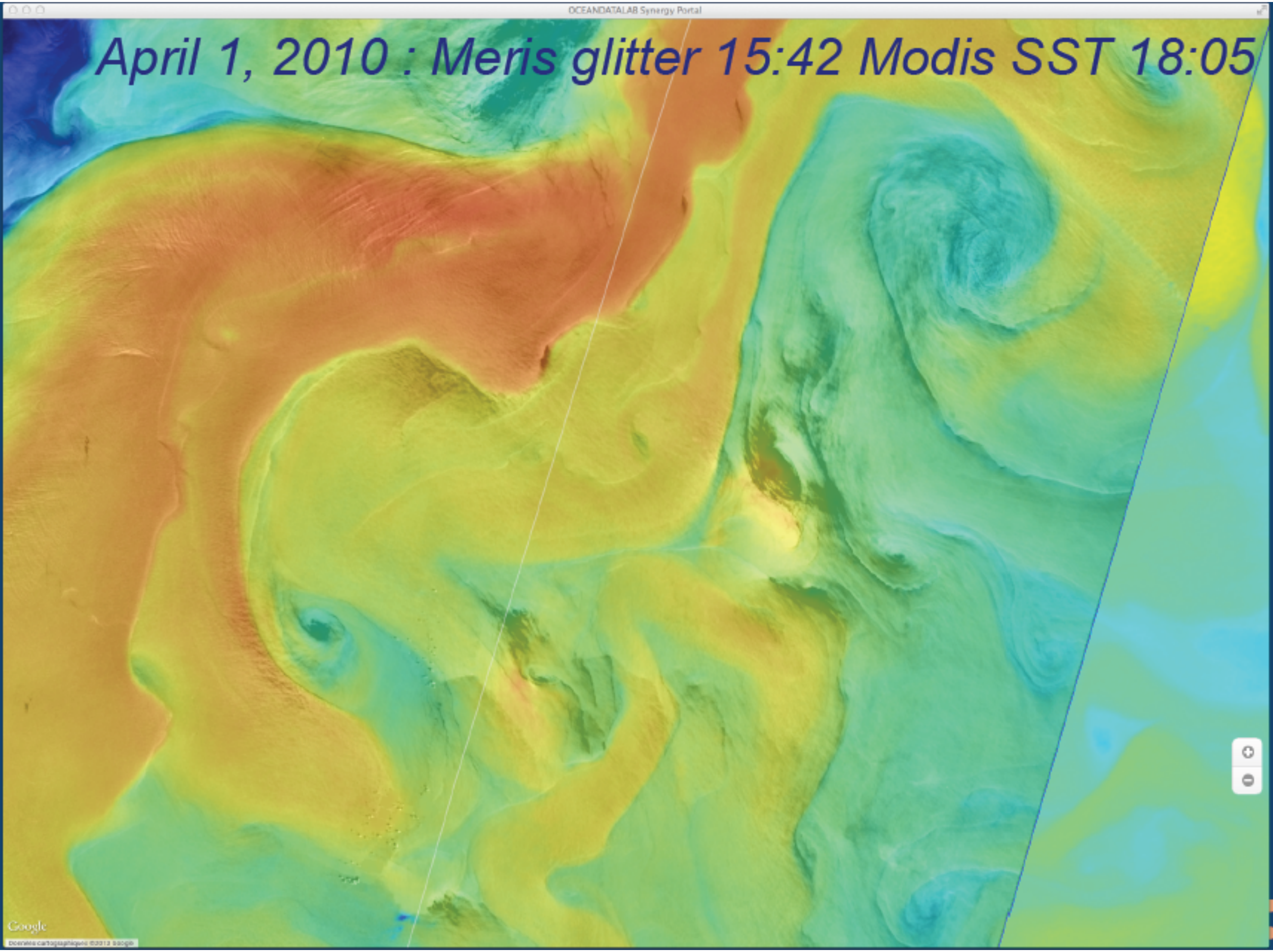
Depending on the spectrum slope of surface density, this w-field can be quite significant or not!

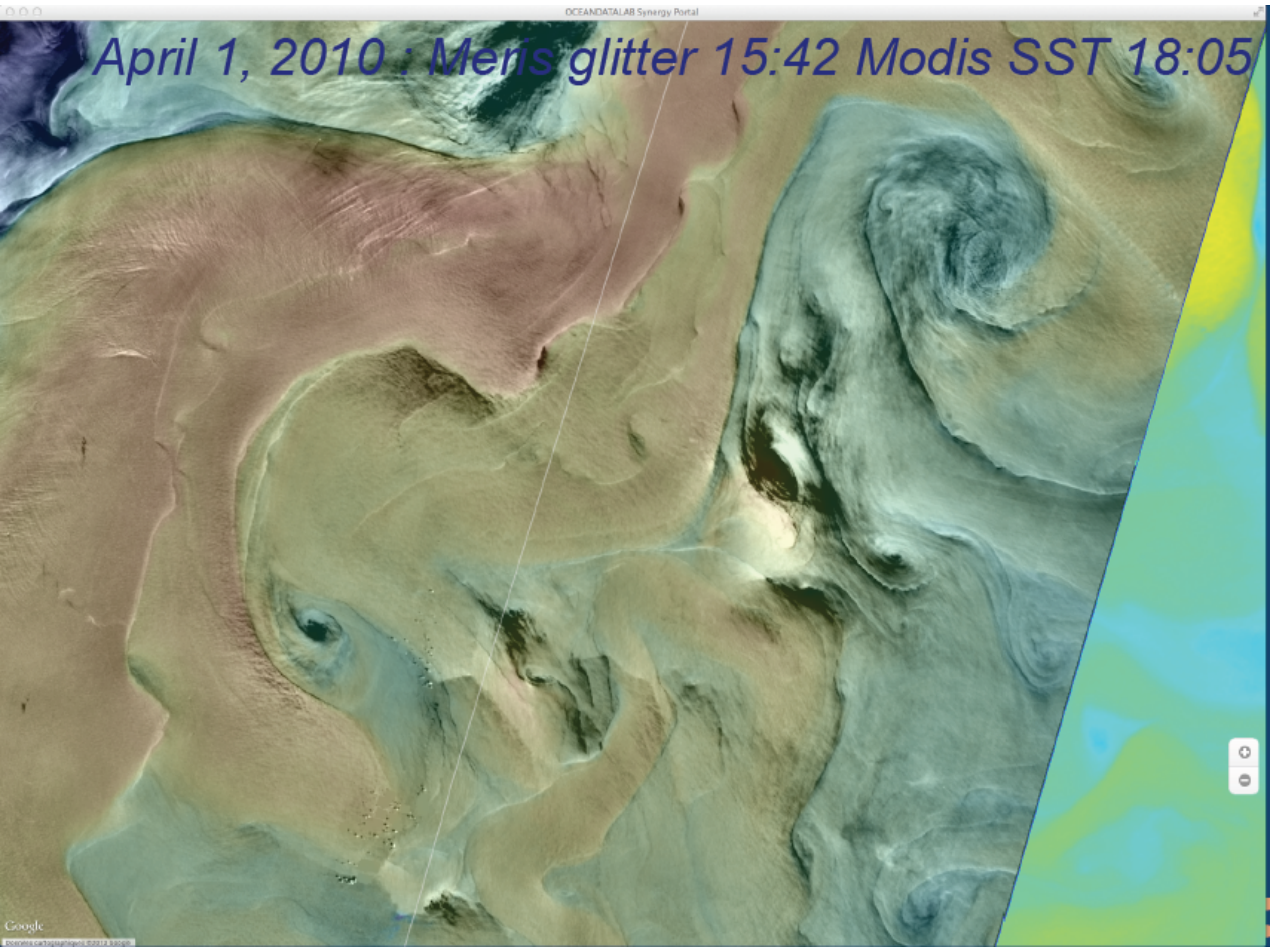
$\Rightarrow$ Density (or SST) fronts at the ocean surface ?

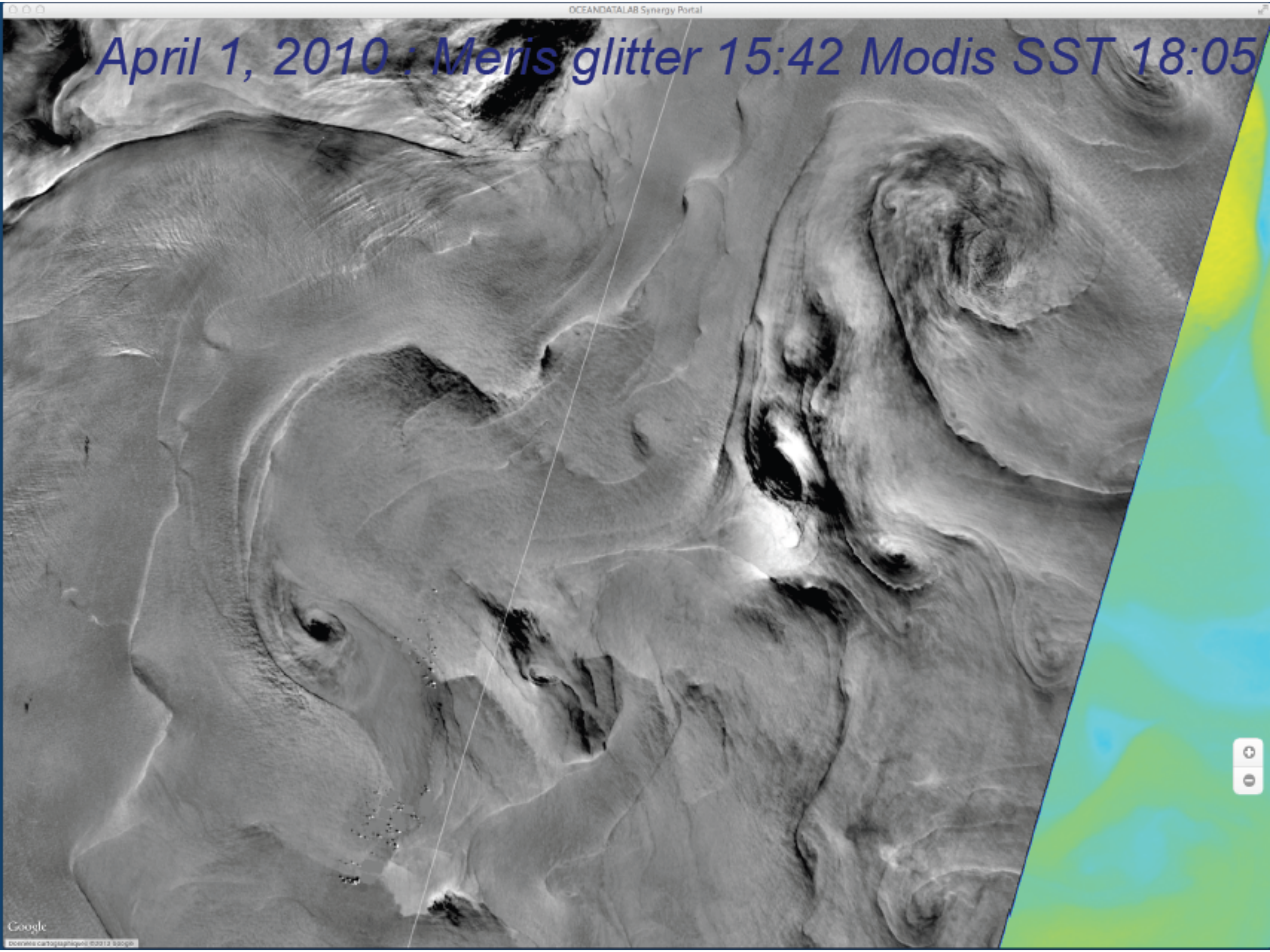
Density gradients at both small and large scales are quite energetic at the ocean surface.




OCEANDATALAB Synergy Portal
Meris glitter 15:42 Modis SST 18:05
Google


OCEANDATALAB Synergy Portal
April 1, 2010 : Meris glitter 15:42 Modis SST 18:05
Google


OCEANDATALAB Synergy Portal
April 1, 2010 : Meris glitter 15:42 Modis SST 18:05
Google


OCEANDATALAB Synergy Portal
April 1, 2010 : Meris glitter 15:42 Modis SST 18:05
Google

… just a qualitative information

on the strength of density fronts.

Need data from in-situ
Experiments ...

Figure 1. RADARSAT SAR image of the Santa Barbara Channel at 1400 UT on January 8, 2003. The area is 100 km by 110 km. The image was provided by B. Holt, Jet Propulsion Laboratory, and processed at the Alaska Satellite Facility. (Copyright by the Canadian Space Agency (2003)).

Results from in-situ data

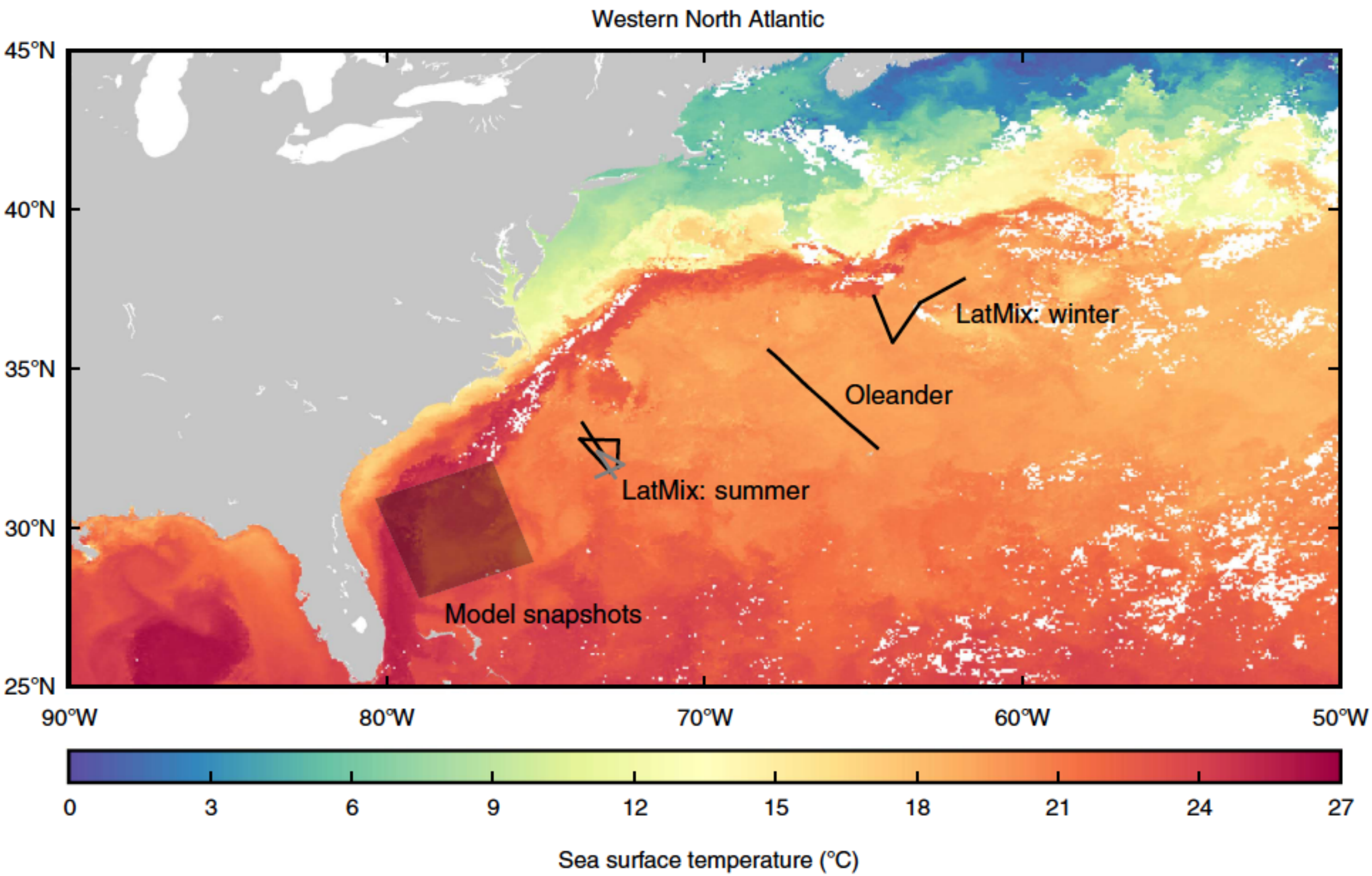


Figure 2 | Measurement locations. Locations of velocity transects (black lines), of additional buoyancy transects (dark gray lines) and of the model snapshots shown in Fig. 1 (transparent shading). The colour shading shows sea surface temperatures on 13–20 March 2012 (8-day L3 MODIS Aqua composite of 4 µm nighttime temperature). Missing data are indicated by white shading.

Northwestern Atlantic: Callies et al. NC'15

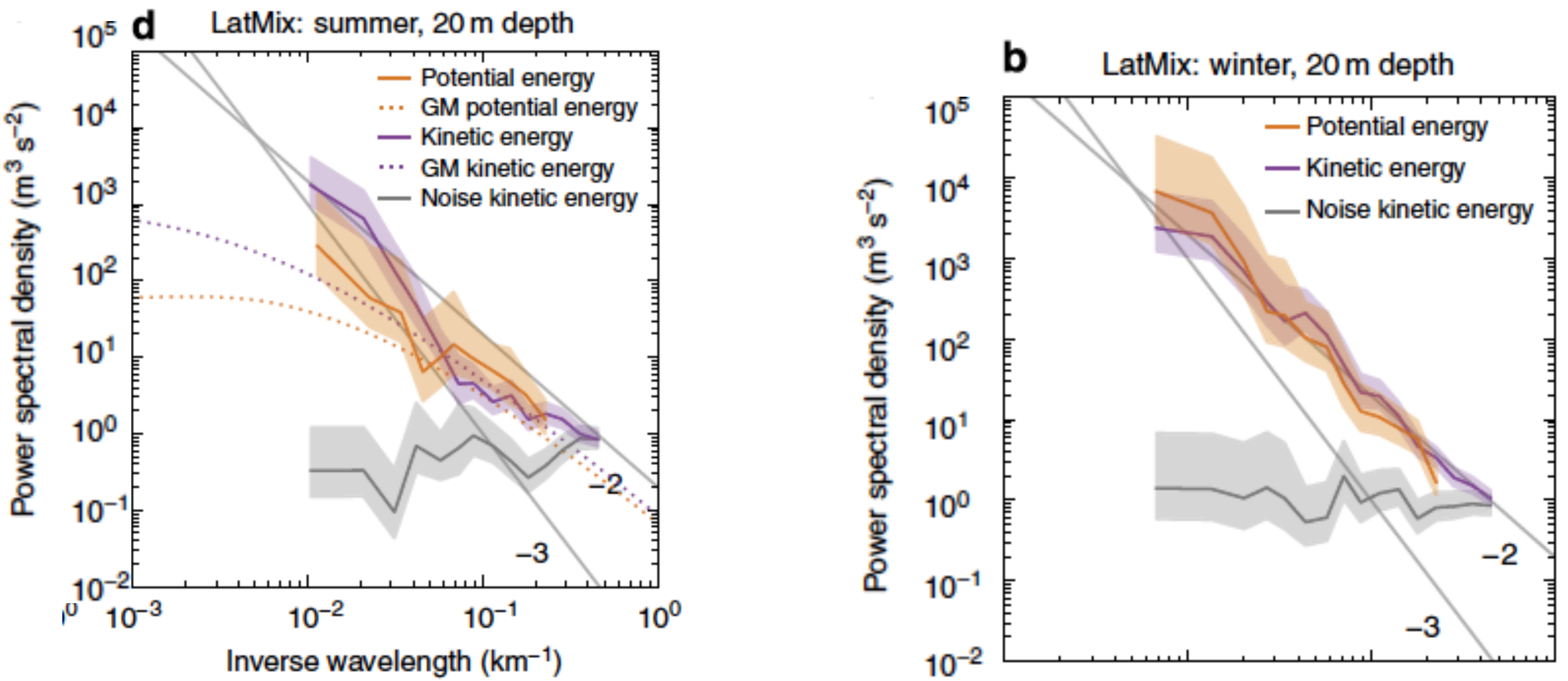


Figure 3 | Seasonality in observations. (**a**) Kinetic energy spectrum at 50 m depth for the Oleander winter data. (**b**) Potential and kinetic energy spectra at 20 m depth for the LatMix winter experiment. (**c**) Kinetic energy spectrum at 50 m depth for the Oleander summer data. (**d**) Potential and kinetic energy spectra at 20 m depth for the LatMix summer experiment. The light shadings are 95% confidence intervals. Also shown are the GM model spectra for internal waves in the seasonal thermocline (with parameters from ref. 30), estimates for the noise level of the LatMix velocity data and reference lines with slopes −2 and −3.

A k^{-2} density spectrum slope means a flat spectrum for density gradients!

=> **Density fronts at small scales are as strong as those at larger scales. Frontogenesis occur at all scales!** [remember $|w| \sim |\Delta\rho|$]

In-situ data further highlight the strength of density fronts at any scales

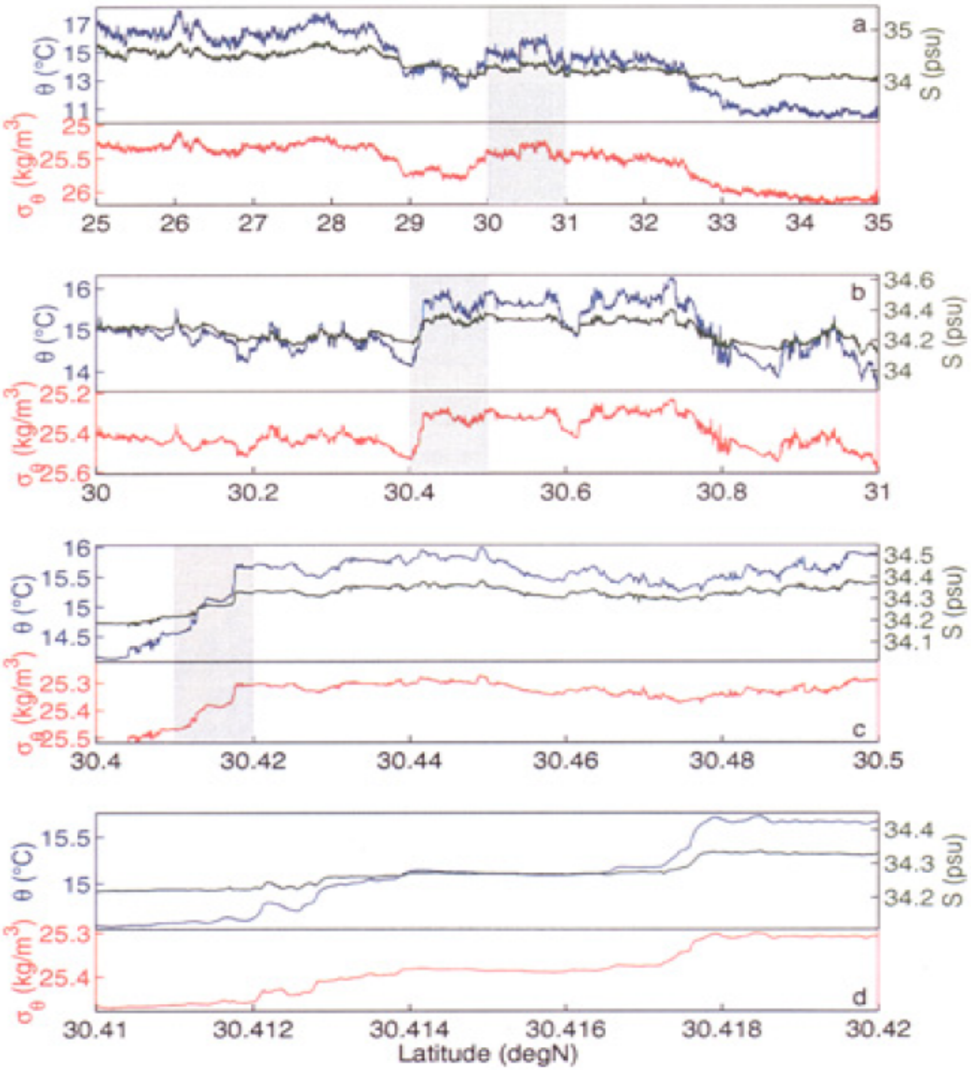


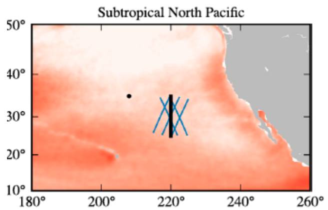
Plate 7. Potential temperature θ (blue lines), salinity S (green lines), and potential density σ_θ (red lines) as in Plate 2 but along the 200 dbar pressure surface. Salinity fluctuations are proportional and opposite to those of temperature at all scales but do not cancel completely the effect of temperature on density because horizontal tows in the thermocline include diapycnal signal due to the tilt of isopycnal surfaces.

Ferrari and Rudnick (JGR'00):

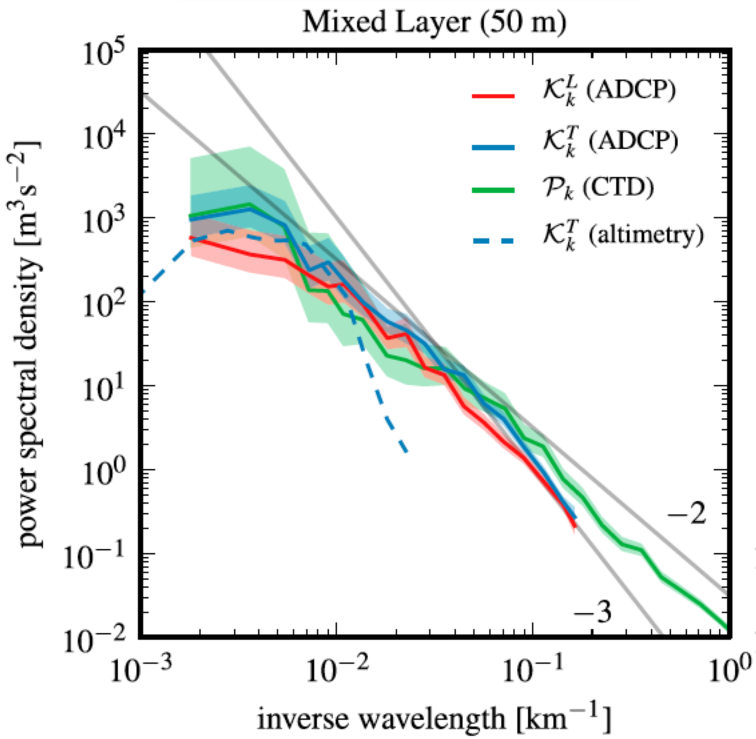
North Pacific along 140°W between 25°N and 35°N

… towing a Seasoar during the winter of 1997 …

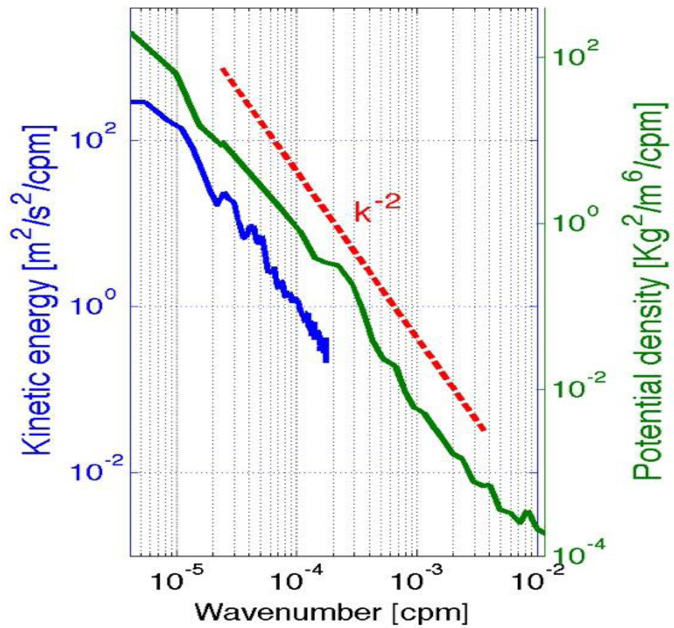
$$\nabla\rho = \frac{\delta\rho}{\delta x}$$



Callies et al. JPO’13



Ferrari & Rudnick JGR’00



A k^{-2} density spectrum slope means a flat spectrum for density gradients!

=> **Density fronts at small scales are as strong as those at larger scales. Frontogenesis occur at all scales!** [remember |w| ~ |$\Delta\rho$|]

Same observations at the atmospheric tropopause (12000 m): density fronts at both small and large scales are quite energetic

See Tulloch & Smith PNAS'06

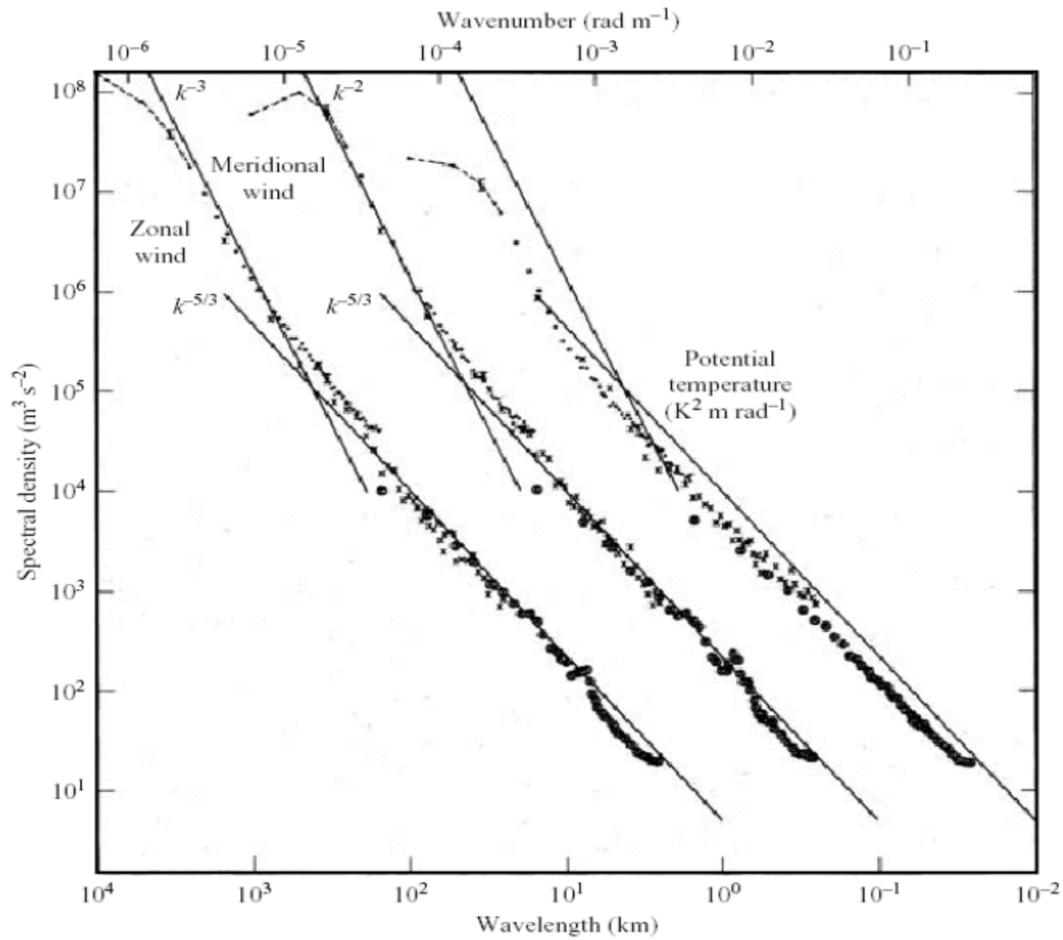


Figure 1: Variance power spectra of horizontal wind and potential temperature near the tropopause from GASP aircraft data, reproduced from ref. 3. The spectra for meridional wind and temperature are shifted one and two decades to the right; lines with slopes -3 and -5/3 are drawn for reference.

Density fronts at boundaries (either at the surface ocean or at the atmospheric tropopause) are strongly energetic. They are **characterized by the spectrum slope in k^{-2}**.

Usually density fronts at small scale are produced by the stirring of large-scale density anomalies by mesoscale eddies.

Surface frontogenesis leads to vertical velocity intensified at the smallest scales since: $w \approx \Delta\rho$. Their depth scale can attain 200 to 500 m.

How to take into account these active boundaries and their interactions with the ocean interior within the QG framework ?

=> **Surface Quasi-Geostrophic dynamics coupled to interior Quasi-Geostrophic dynamics.**

(cf. Charney, JAS'71 & Blumen, JAS'78)

Quasi-geostrophic equations

Momentum eqs. [see 2nd Class].

$R_o = \frac{U}{fL} < 1.$ $\quad \frac{U}{\beta L^2} > 1.$

$$\frac{\partial \vec{U}}{\partial t} + \vec{U} \cdot \nabla \vec{U} + f \cdot \vec{k} \times \vec{U} = -\frac{\nabla P}{\rho_0} + [\text{mixing}] \qquad \vec{U} = (u, v).$$

We assume that the non-divergent part is entirely captured by the $O(1)$ dynamics. So the $O(R_o)$ dynamics need only to contain the divergent part.

$$\Rightarrow \boxed{\vec{U} = \vec{U}_0 + R_o \vec{U}_1} \quad \text{with } \vec{U}_0 = \vec{k} \times \nabla \psi$$

and $\text{Curl}\, \vec{U}_1 = 0 \quad \boxed{\nabla \cdot \vec{U}_1 = -\frac{\partial w_1}{\partial z}} \qquad \vec{U}_1 = (u_1, v_1).$

We also define an ageostrophic pressure (as for 2-D flows)

$$p_1 = \frac{P}{\rho_0} - (f_0 + \beta y)\psi.$$

Hydrostatic approximation [using $\rho = \bar{\rho}(z) + R_o \rho'$]

$$\boxed{-g \frac{\rho'}{\rho_0} = f_0 \frac{\partial \psi}{\partial z}}$$

Note that $\vec{U}_0 = \vec{k} \times \nabla \psi$ and $-g \frac{\rho'}{\rho_0} = f_0 \frac{\partial \psi}{\partial z}$ leads to:

$$\boxed{\vec{U}_{0z} = -\frac{g}{\rho_0 f_0} \vec{k} \times \nabla \rho'.} \qquad [\text{Thermal wind balance}]$$

The resulting eqs [retaining only $O(R_o)$ terms] are:

$$\boxed{\begin{aligned} &\frac{\partial U_0}{\partial t} + U_0 \cdot \nabla U_0 = -\nabla p_1 - f_0 k \times U_1 - \beta \psi \vec{j} \\ &\frac{d\psi_z}{dt} = -\frac{N^2}{f} w_1 \quad (2). \quad \text{with } N^2 = -\frac{g}{\rho_0}\frac{d\bar{\rho}}{dz} \end{aligned}} \quad (1)$$

Curl of (1) leads to [using $\zeta = V_{0x} - U_{0y} = \Delta\psi$]

$$\boxed{\frac{d\zeta}{dt} + \beta V_0 = f \cdot w_{1z}} \quad (3).$$

From (2) and using the thermal wind balance we get:

$$\boxed{f w_{1z} = -\frac{\partial}{\partial z}\left[\frac{f^2}{N^2}\frac{d\psi_z}{dt}\right] = -\frac{d}{dt}\frac{\partial}{\partial z}\left[\frac{f^2}{N^2}\frac{\partial\psi}{\partial z}\right]} \quad (4).$$

(3) and (4) lead to:

$$\boxed{\frac{dq}{dt} + \beta\psi_x = 0 \quad (5) \quad \text{with } q = \Delta\psi + \frac{\partial}{\partial z}\left[\frac{f^2}{N^2}\frac{\partial\psi}{\partial z}\right]} \quad (6)$$

q is the QG potential vorticity.

What about the density fronts at the surface ?

ψ is obtained from q by inverting a 3-D elliptic operator (6), using appropriate boundary conditions.

The problem to solve is:

$$\left.\boxed{\begin{aligned} &\Delta\psi + \frac{\partial}{\partial z}\left[\frac{f^2}{N^2}\frac{\partial \psi}{\partial z}\right] = q. \\ &\text{with } \left.\frac{\partial \psi}{\partial z}\right|_{z=0} = -\frac{g}{\rho_0 f_0}\,\rho'|_{z=0} \text{ and } \left.\frac{\partial \psi}{\partial z}\right|_{z=-H} = 0. \end{aligned}}\right\} \quad (7).$$

Only the time evolution of $q(t)$ [eq. 5] and $\rho'|_{z=0}(t)$ [eq. 2 with $W=0$] are needed!

Mathematically, this problem can be splitted into two problems

....

The first one [noted "sur" for surface] involves zero PV in the interior and is forced by non-zero surface density:

$$\left.\begin{aligned} &\Delta \psi_{surf} + \frac{\partial}{\partial z}\left[\frac{f^2}{N^2}\frac{\partial \psi_{sur}}{\partial z}\right] = 0. \\ &\left.\frac{\partial \psi_{surf}}{\partial z}\right|_{z=0} = -\frac{g}{\rho_0 f_0}\rho'|_{z=0}(t) \qquad \left(\text{and } \left.\frac{\partial \psi_{surf}}{\partial z}\right|_{z=-H} = 0\right) \end{aligned}\right\} \quad (8).$$

The second one [noted "int" for interior] is forced by non-zero PV in the interior and involves zero surface density:

$$\left.\begin{aligned} &\Delta \psi_{int} + \frac{\partial}{\partial z}\left[\frac{f^2}{N^2}\frac{\partial \psi_{int}}{\partial z}\right] = q(t). \\ &\left.\frac{\partial \psi_{int}}{\partial z}\right|_{z=0} = \left.\frac{\partial \psi_{int}}{\partial z}\right|_{z=-H} = 0 \end{aligned}\right\} \quad (9).$$

The total streamfunction is:

$$\boxed{\psi = \psi_{surf} + \psi_{int}.}$$

The coupling occurs through the advection terms in eqs. for q(t) and $\rho'|_{z=0}(t)$.

These equations:

$$\Delta\psi_{surf} + \frac{\partial}{\partial z}\left[\frac{f^2}{N^2}\frac{\partial\psi_{surf}}{\partial z}\right] = 0.$$

$$\left.\frac{\partial\psi_{surf}}{\partial z}\right|_{z=0} = -\frac{g}{\rho_0 f_0}\,\rho'\big|_{z=0}(t) \qquad \left(\text{and } \left.\frac{\partial\psi_{surf}}{\partial z}\right|_{z=-H} = 0\right)$$

with:

$$\frac{d}{dt}\rho'\big|_{z=0} = 0$$

… describe the **Surface Quasi-Geostrophic (SQG) dynamics** or the **dynamics associated with the surface density fronts and their impact at depth!**

There is a formal resemblance with 2-D dynamics with $\boldsymbol{\rho'}|_{\boldsymbol{z}\,=\,\boldsymbol{0}}$ playing the role of ζ. But there is a different relationship between the flow ψ and the advected scalar.

The physics that drives SQG dynamics involves inverse KE cascade and direct tracer cascade as in 2-D flows but displays some important differences (see Georgy this thusday) **The key process is the frontogenesis and the associated 3-D motions.**

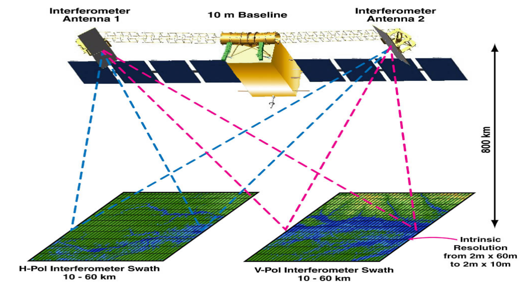
These equations:

$$\Delta \psi_{surf} + \frac{\partial}{\partial z}\left[\frac{f^2}{N^2}\frac{\partial \psi_{surf}}{\partial z}\right] = 0.$$

$$\left.\frac{\partial \psi_{surf}}{\partial z}\right|_{z=0} = -\frac{g}{\rho_0 f_0}\,\rho'|_{z=0}(t) \qquad \left(\text{and } \left.\frac{\partial \psi_{surf}}{\partial z}\right|_{z=-H} = 0\right)$$

with:

$$\frac{d}{dt}\rho'|_{z=0} = 0$$



SWOT

SQG dynamics points to the strong potential of the new generation of satellite altimeters (wide swath altimeters such as SWOT):

Using SSH at high resolution and low resolution interior density (N^2), SQG dynamics allows to diagnose not only surface motions at HR but also the 3-D dynamics, including the w-field down to 500 m!

=> See Jinbo's talk

This new potential has been successfully tested, in particular for …
diagnosis of the 3D dynamics in the first 500m including within the ML

Simulated W by an OGCM ***Diagnosed W from SSH, SST and Kv***

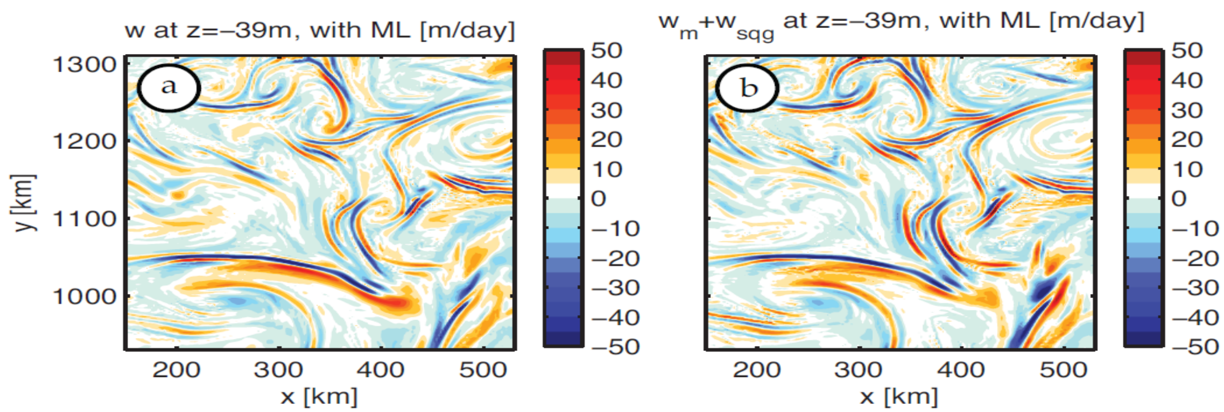


FIG. 6. Snapshots of the vertical velocity field at 40 m with ML (a) and reconstructed field $w_{sqg} + w_m$ (b). Units are m/day.

=> W diagnosis requires the knowledge of, **both**, **HR SSH, HR SST and order of magnitude of the vertical mixing (from Argo floats)**

These diagnosis results seem promising and point to the strong potential of the wide-swath

Depth scales of the SQG dynamics

Some simple analytical solutions (using $N^2 = cst$).

If $\psi(x,y,z,t) = \sum_k \sum_l \hat{\psi}_{k,l}(z,t)\, e^{i(kx+ly)}$

Solution of $\Delta\psi + \frac{\partial}{\partial z}\left(\frac{f^2}{N^2}\frac{\partial \psi}{\partial z}\right) = 0$ is:

$$\hat{\psi}_{k,l}(z,t) = \hat{\psi}_{k,l}(0,t)\, e^{+NK/f\, z} \qquad K = (k^2+l^2)^{1/2} \text{ and } z<0$$

or $$\hat{\psi}_{k,l}(z,t) = -\frac{g}{\rho_0}\frac{1}{NK}\hat{\rho}_s''(0,t).\, e^{\frac{NK}{f}z}$$

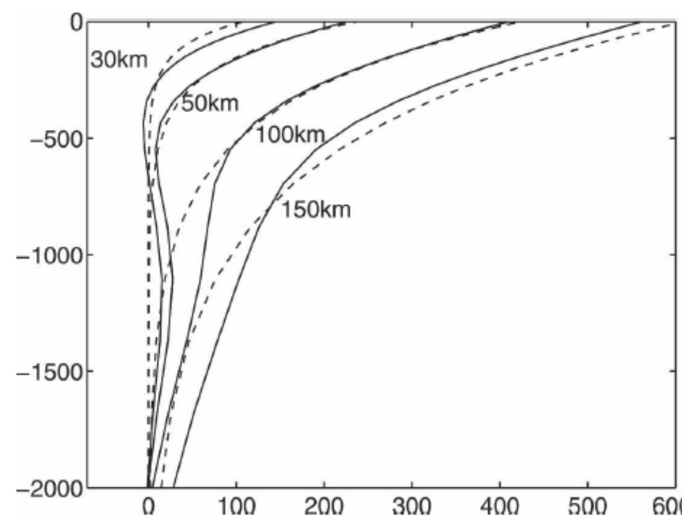
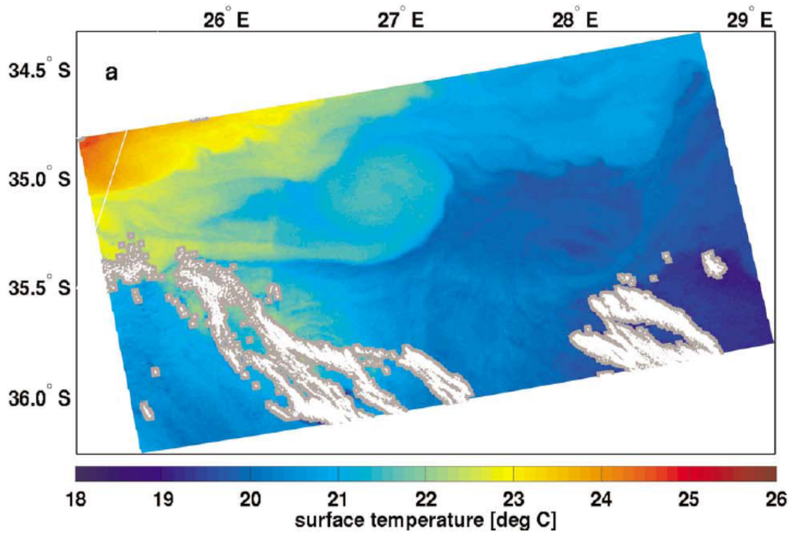
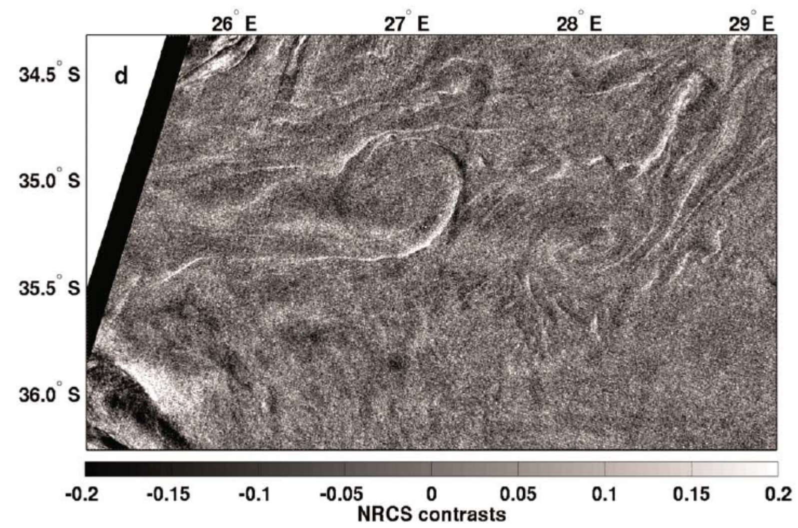


FIG. 5. Vertical profiles of φ (continuous curves) and of its exponential fit using $N/|f_0| = 45$ (dashed curves) for different wavelengths (30, 50, 100, and 150 km).

SST



(Kudratyev et al. '12)



. - SST images can be used to estimate the vorticity field using the **SQG approximation**

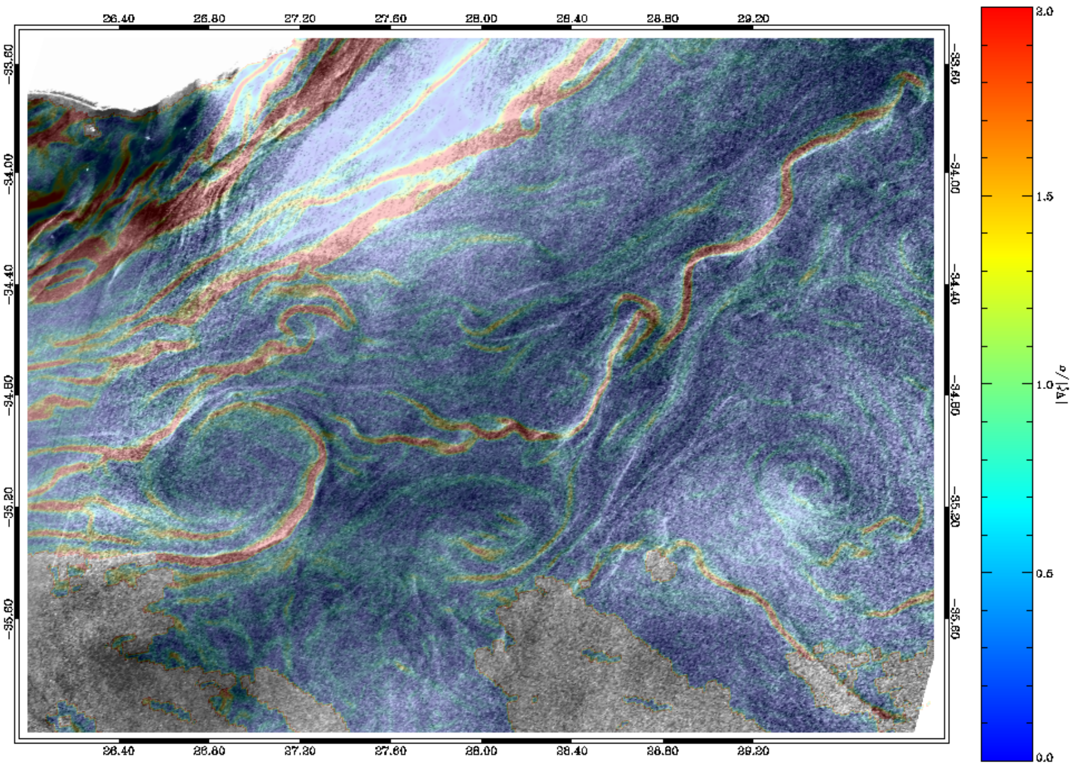
- SAR images show the divergence and convergence fields. They are related to the vorticity field.

- Is there a relation between both ?

SAR

Meso/submesoscales from Space :

SAR + vorticity (from SST using SQG) (Kudratyev et al., JGR'12)



Information provided by HR satellite data SSH, SST, ocean color and SAR images (in addition to the ARGO floats) should allow to retrieve the ocean dynamics in the first 500m