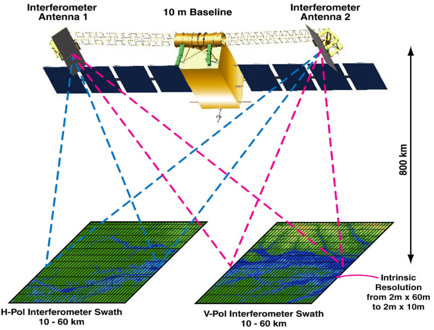
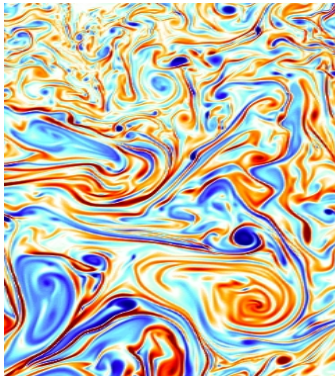
“Ocean Turbulence from SPACE”

Patrice Klein (Caltech/JPL/Ifremer)

(XI) – SQG turbulence (b)

<u>Stirring versus mixing</u>





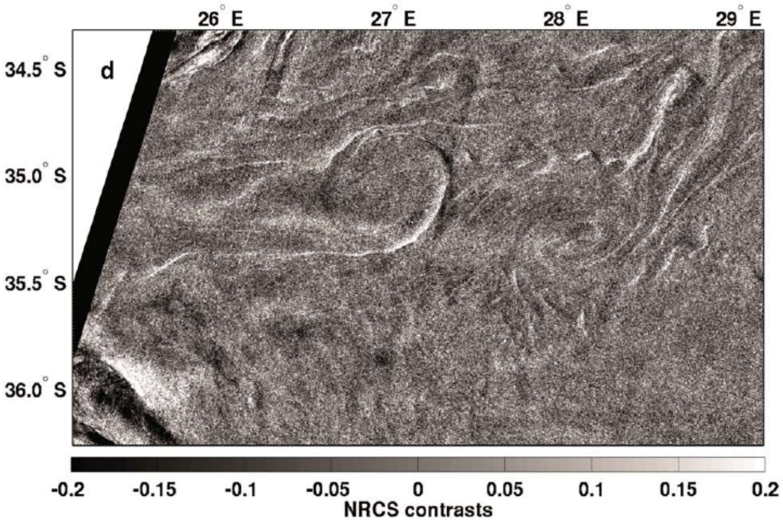
Before talking « Stirring versus Mixing »:

How to exploit the synergy between SAR and SST images using a dynamical framework?

SST at H.R. (Modis): 1 km but affected by clouds

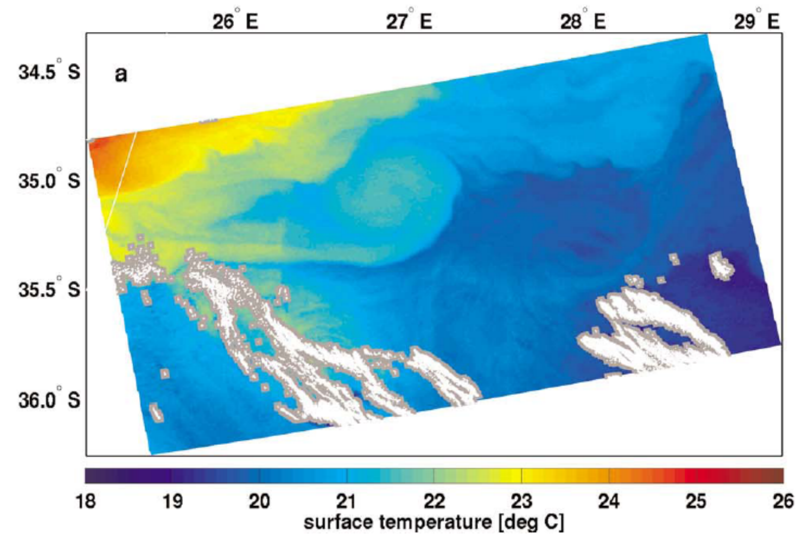
SAR images: 10-100 m but do not give useful information with too low or too strong wind speed.

(Kudratyev et al. '12)



SAR

SST



- SAR images show the divergence and convergence fields. Their resolution is between 10 and 100 m.

- SST images at high resolution (1 km) display SST fronts

- Is there a relation between both ?

From Q G approximation: thermal wind balance

$$\nabla \rho = \frac{f\rho_0}{g} \cdot \vec{k} \times U_{0z} \qquad (1)$$

Both components evolve with time !

Eqs. for each component of (1) are:

$$\frac{d}{dt}\nabla\rho = -\left[\nabla U_0\right]^T \cdot \nabla\rho + \frac{N^2\rho_0}{g}\nabla w_1 \qquad (2) \quad \text{with } \left[\nabla U_0\right]^T = \begin{bmatrix} u_{0x} & v_{0x} \\ u_{0y} & v_{0y} \end{bmatrix}$$

$$\frac{d}{dt}\left[\frac{f\rho_0}{g}\cdot\vec{k}\times U_{0z}\right] = \left[\nabla U_0\right]^T\left[\frac{f\rho_0}{g}\cdot\vec{k}\times U_{0z}\right] + \frac{\rho_0}{g}U_{1z} - \frac{f\rho_0}{g}\vec{k}\times\nabla p_{1z} \qquad (3)$$

If the thermal wind balance is verified:

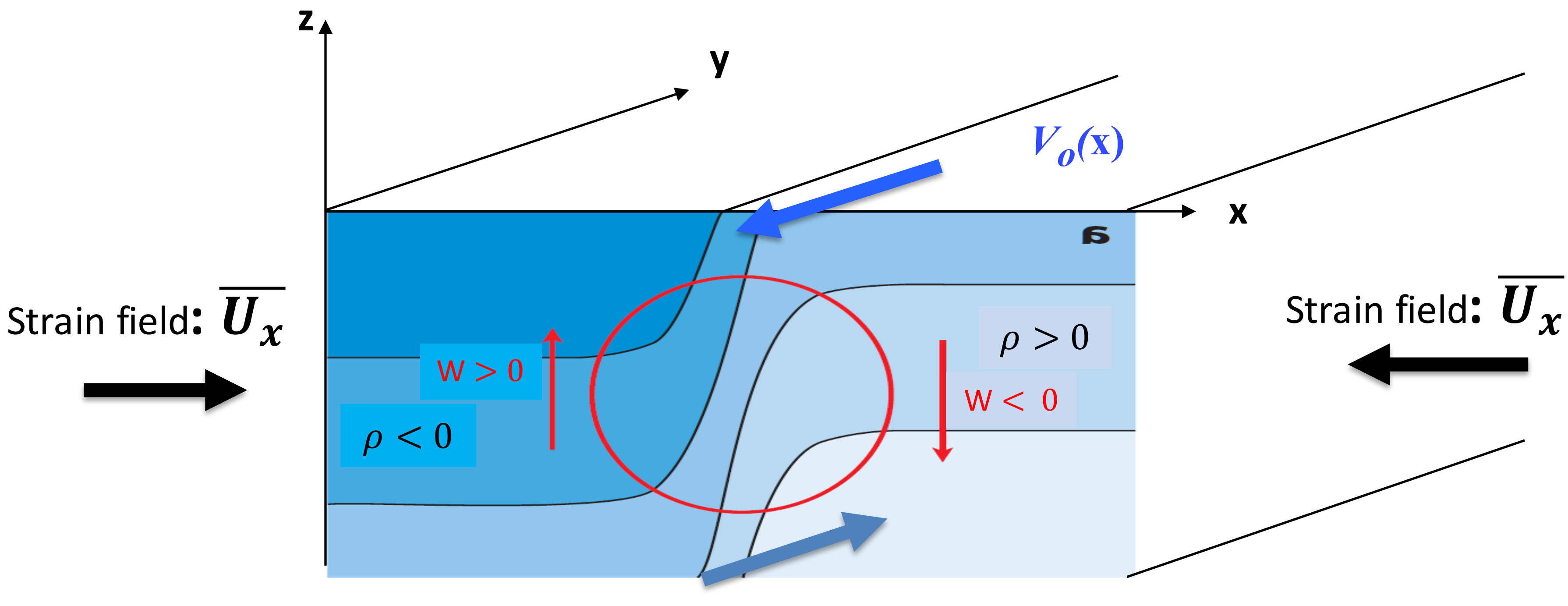
$$\frac{d}{dt}\nabla\rho = \frac{d}{dt}\left[\frac{f\rho_0}{g}\cdot k\times U_{0z}\right] \neq 0. \qquad (4)$$

(2) - (4) lead to the Omega equation:

$$\boxed{\Delta w_1 + \frac{f^2}{N^2} w_{1zz} = \frac{2f}{N^2}\nabla\cdot Q} \qquad (5).$$

with $Q = \left[\nabla U_0\right]^T \cdot \nabla\rho$

Let us consider a density front embedded in a strain field …



Strain field $\overline{U_x} < 0$ is assumed to be large-scale. Density gradient $\boldsymbol{\rho_x > 0}$

The density front is in thermal balance:

$$\boxed{\rho_x = -\frac{f\,\rho_o}{g} v_{oz}}$$

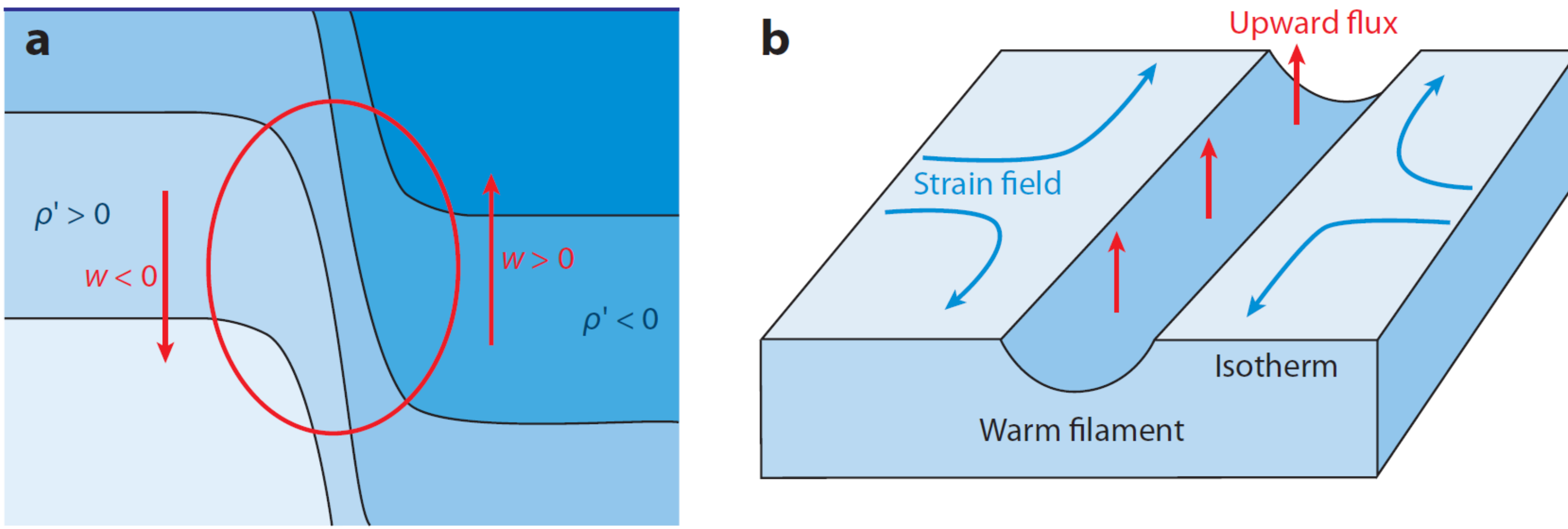
Assume $\overline{U}_x$ is large-scale.

The Omega equation at the surface (for a density front in the x-direction) is:

$$\boxed{w_{,zz} = 2 \frac{g}{f^2 \rho_0} \overline{U}_x \cdot \rho_{xx}} \qquad (5).$$

The w-field is proportional to the density Laplacian,

Frontogenesis within submesoscales

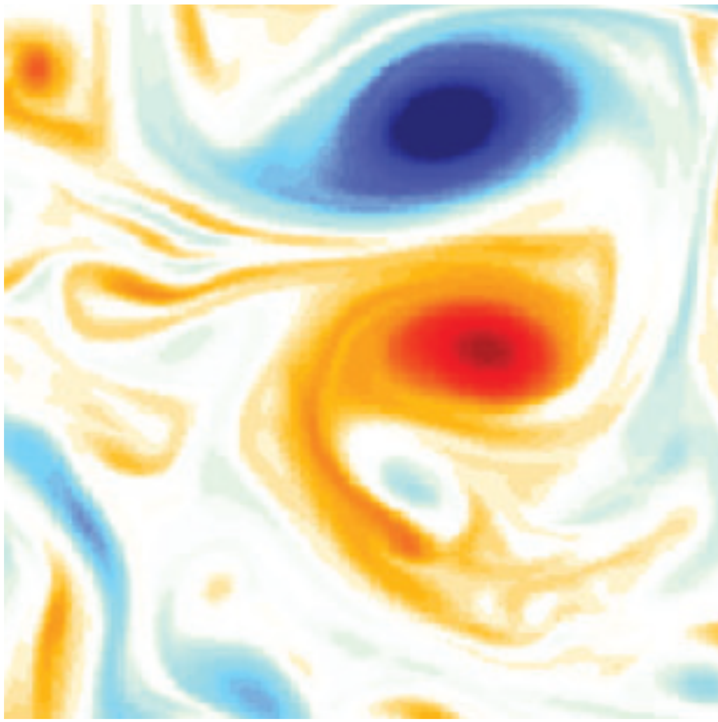


The previous relationship: $\underline{w \approx \alpha . \Delta\rho}$ with $\underline{\alpha}$ a function of the large-scale strain, works well for elongated surface density fronts as well as for filaments where w is intensified.

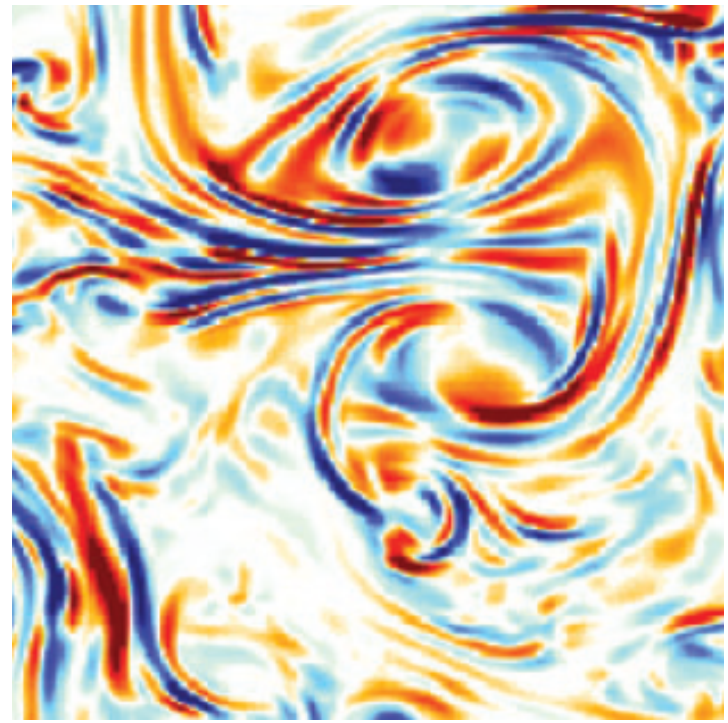
It does not work inside mesoscale eddies because of the rotation effects …

$$\frac{d}{dt}\nabla\rho = -\left[\nabla U_0\right]^T \cdot \nabla\rho + \frac{N^2 \rho_0}{g}\nabla w_1 \quad (2) \quad \text{with } \left[\nabla U_0\right]^T = \begin{bmatrix} u_{0x} & v_{0x} \\ u_{0y} & v_{0y} \end{bmatrix}$$

T



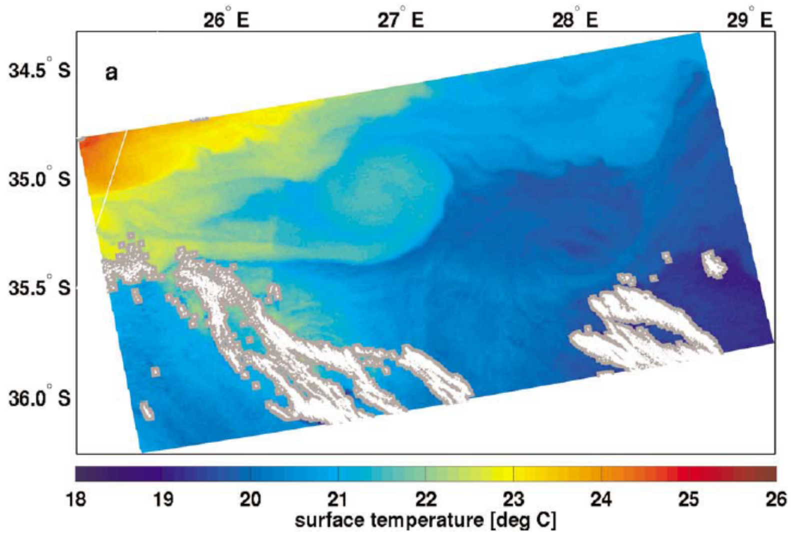
W



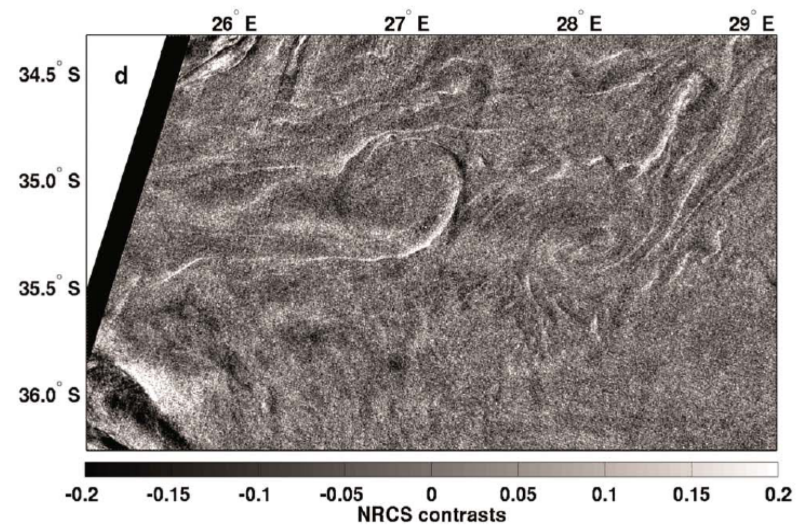
From numerical simulations, the previous relationship works well for elongated filaments.

What about SAR and SST satellite images?

SST



From Kudratyev et al' 12



SAR

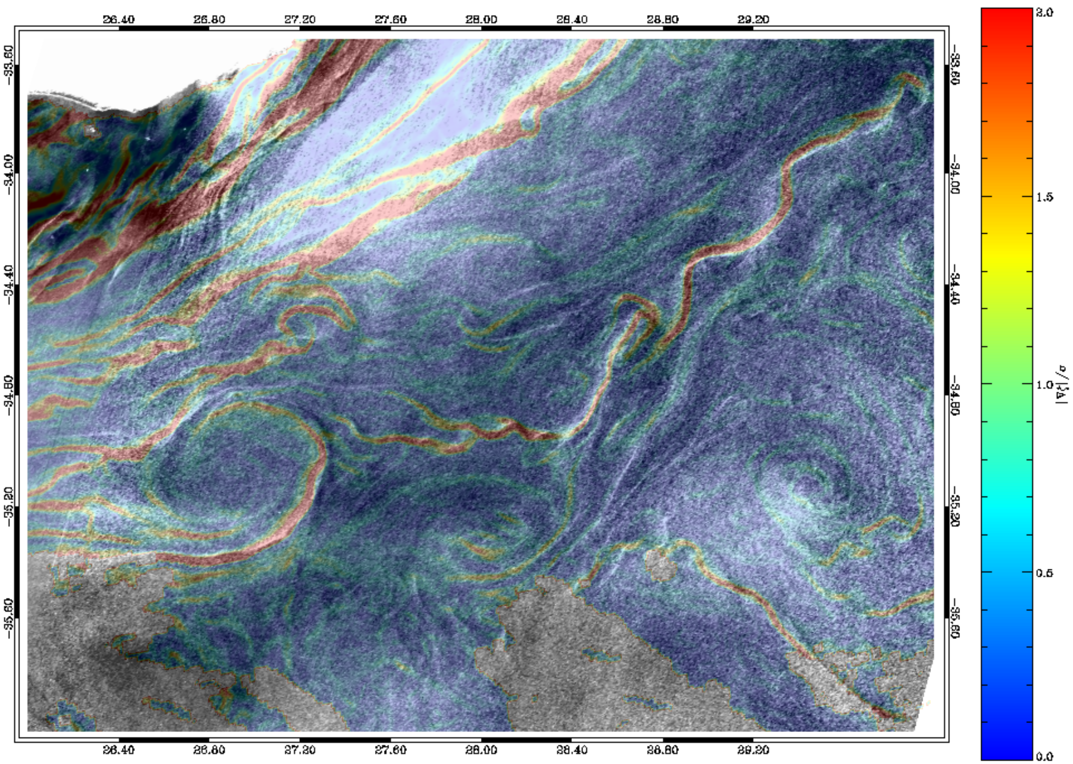
- SST images at high resolution are used to estimate the SST Laplacian

- Divergence and convergence fields displayed by SAR images should be related to the SST Laplacian

- Is the previous relationship working ?

Meso/submesoscales from Space :

SAR (grey scale) + SST Laplacian (color scale)



SAR images are not affected by clouds (SST images are)

Information from SAR images can be used to identify the surface density fronts

STIRRING VERSUS MIXING in SQG (or 2D) turbulence

In the ocean interior, increase of the density gradients is compensated by the vertical velocity.

$$\boxed{\frac{d\nabla\rho}{dt} = -\left[\nabla U_o\right]^T \cdot \nabla\rho} + \frac{N^2}{g}\rho_o \nabla w,$$

At the surface $w_1 = 0$. So other mechanisms should act to decrease (or arrest) the formation of density gradients. Mixing (or dissipation) is one candidate.

To understand the competition between stirring and mixing, let us consider again the simple tracer gradient equation …

Eq. for the tracer gradient can be written as:

$$\frac{d\nabla C}{dt} = -[\nabla U]^T \nabla C + \eta_e \Delta[\nabla C]$$

If λ is one of the eigenvalues of $[\nabla U]^T$ and η_e the diffusivity, a dimensional analysis leads to:

$$L_s = [\eta_e / \gamma]^{1/2}$$

L_s is the width of filaments resulting from the competition between stirring and mixing.

What are the mechanisms that drive the horizontal mixing in the ocean?

(ii) STIRRING VERSUS MIXING

What are the mechanisms that drive the horizontal mixing ?

One of them is the shear dispersion by internal waves (including internal tides, near-inertial waves, …).

In that case, the horizontal mixing is produced by the **combined action of the vertical shear of internal waves and vertical mixing.** (see Young et al. JPO 1982)

Both are usually significant within the upper oceanic layers.

To better understand the mechanisms involved, let us consider the following simple case (Young et al. JPO'82) …

Let us consider a tracer field initially homogeneous in the vertical and varying in the horizontal:

$$C(x, z, 0) = \cos kx \qquad (1)$$

No large-scale strain is considered!

Tracer C is embedded in a field of internal waves characterized by

$$\boxed{u = \alpha z \cos \omega t} \qquad (2).$$

The field is sheared on the vertical with a period ω. Note that $u = 0$ at the surface but adding a non-zero constant does not change anything.

Eqn for $C(x, z, t)$ is:

$$\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} = \eta \frac{\partial^2 C}{\partial x^2} + \nu \frac{\partial^2 C}{\partial z^2} \qquad (3).$$

The analytical solution of (3) can be easily obtained using (1) and (2) …

I - If $\eta = \nu = 0$: C is conserved on a Lagrangian trajectory.

From (2): $\quad \dfrac{dx}{dt} = \alpha z \cos \omega t.$

$$\Rightarrow \quad x = x_0 + \frac{\alpha z}{\omega} \sin \omega t.$$

or $\boxed{x_0 = x - \dfrac{\alpha z}{\omega} \sin \omega t} \qquad (4).$

Since C is conserved on a Lagrangian trajectory, using (4):

$$\boxed{\begin{aligned} C(x, z, t) &= C(x_0, z, 0) \\ &= \cos(k x_0) \end{aligned}} \qquad (5)$$

II - If $\eta \neq 0$ and $\nu \neq 0$: C is not more conserved on a Lagrangian trajectory. The exact solution can be found by looking for a solution of the form:

$$\boxed{C(x, z, t) = A(t) \cos k x_0} \quad \text{with } A(0) = 1 \quad (6)$$

Diffusion is only going to change the amplitude of C following a particle.

Using (6): $C(x,z,t) = A(t)\cos kx_0$ (with $x_0 = x - \frac{\alpha z}{\omega}\sin\omega t$)

in (3):
$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} = \eta\frac{\partial^2 C}{\partial x^2} + \nu\frac{\partial^2 C}{\partial z^2}$$

leads to:

$$\boxed{\frac{dA}{dt} = -\left[\eta k^2 + \nu k^2\left(\frac{\alpha}{\omega}\right)^2 \sin^2\omega t\right]\cdot A} \qquad (7).$$

Solution of (7) is (using (6)),

$$\boxed{C(x,z,t) = \exp\left[-\eta k^2 t - \frac{1}{2}\nu k^2\left(\frac{\alpha}{\omega}\right)^2\left[t - \frac{\sin 2\omega t}{2\omega}\right]\right]\cdot\cos kx_0} \qquad (8)$$

Solution (8) indicates that the interaction between the vertical shear of the internal waves, (α) and the vertical diffusivity (ν) produces an "effective" horizontal diffusivity:

$$\boxed{\eta_e = \eta + \frac{1}{2}\left(\frac{\alpha}{\omega}\right)^2\nu} \qquad (9)$$

Solution (8) is very sensitive to α/ω

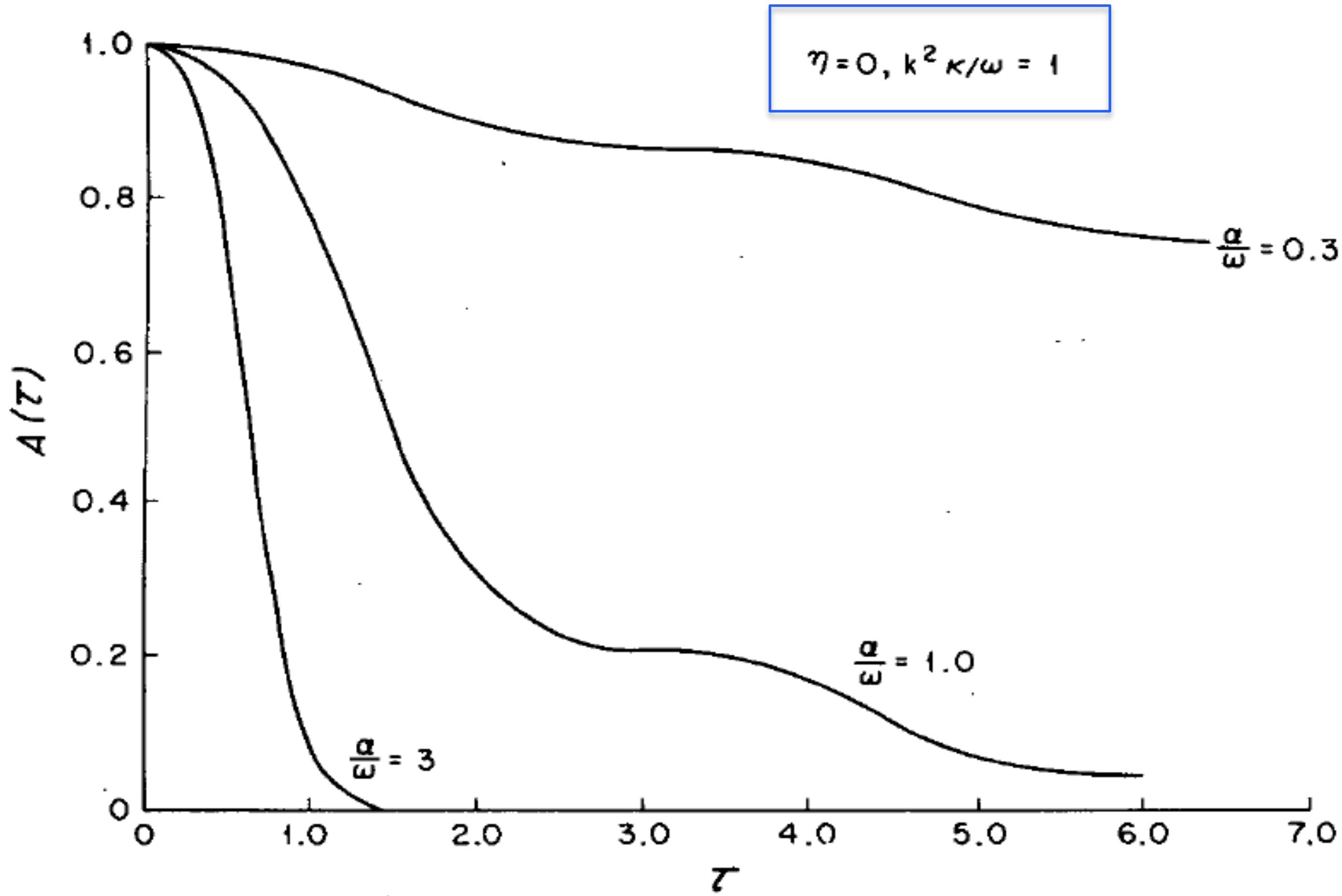


FIG. 1. The amplitude A in (5) as a function of the nondimensional time $\tau = \alpha t$. For simplicity $\eta = 0$ so that the decay is due solely to the interaction of the shear with the vertical diffusivity.

(from Young et al. JPO'82)

A simple explanation ...

If H is the depth scale over which tracer is diffused during a wave period ω^{-1}, a scaling analysis leads to -

$$H^2\omega = 2\nu .$$

ζ isopleths separated by H will be distorted but the shear flow over a horizontal scale -

$$L = H\,\alpha/\omega .$$

So vertical diffusion will mix ζ vertically over the horizontal distance L (and depth scale H) during a wave period (ω^{-1}) such ~~that~~

$$L^2\omega = 2\nu\left(\frac{\alpha}{\omega}\right)^2 .$$

The effective horizontal diffusion will be. $\boxed{\nu_e = \nu\left(\frac{\alpha}{\omega}\right)^2}$

Estimation of the effective diffusivity and of the filament width

Using $\nu = 10^{-2}$ m^2 s^{-1}, $\omega = 10^{-4}$ s^{-1}, $\alpha = 10^{-3}$ s^{-1} leads to

$$\eta_e = 1 \text{ m}^2 \text{ s}^{-1}$$

Then the order of magnitude of the filament width resulting from the competition between stirring and diffusion mechanisms ($L_s = \sqrt{\eta_e/\gamma}$), using $\gamma = 10^{-5}$ s^{-1} is:

$$\boxed{L_s \approx 300 \text{ m !}}$$

=> Horizontal diffusivity is quite small!