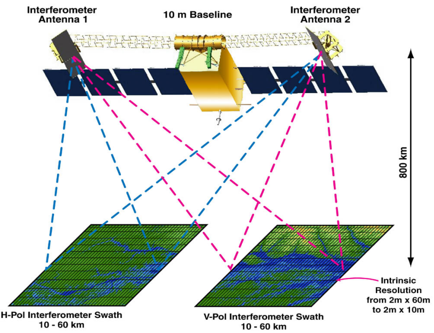
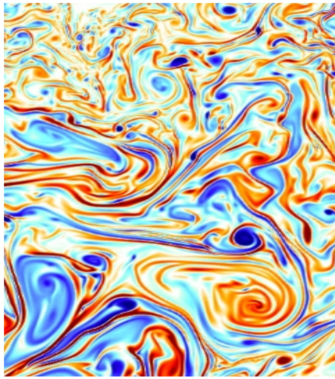
”Ocean Turbulence from SPACE”

Patrice Klein (Caltech/JPL/Ifremer)

(VI) - 2-D Turbulence (b)

[Inverse KE cascade (importance of small scales) and direct enstrophy cascade]





webpage: http://oceanturbulence.github.io/
class email: oceanturbulencefromspace@gmail.com
dropbox: oceanturbulence (pswd: patriceklein)

Reading class: two papers:

- Munk et al. (2000) : Spirals on the Sea. Proc. R. Soc. London A. 456, 1217-1280
- Capet et al. (2008): Surface KE transfer in Surface Quasi-Geostrophic flows; J.F.M., 604, 165-174.

Next Thursday?

Before moving to the inverse KE cascade …

Another application of the Okubo-Weiss criterion:

Partition of the relative vorticity field into vortices and filaments

(may be a homework topic using Dimitris' simulation at 1/48th degree?)

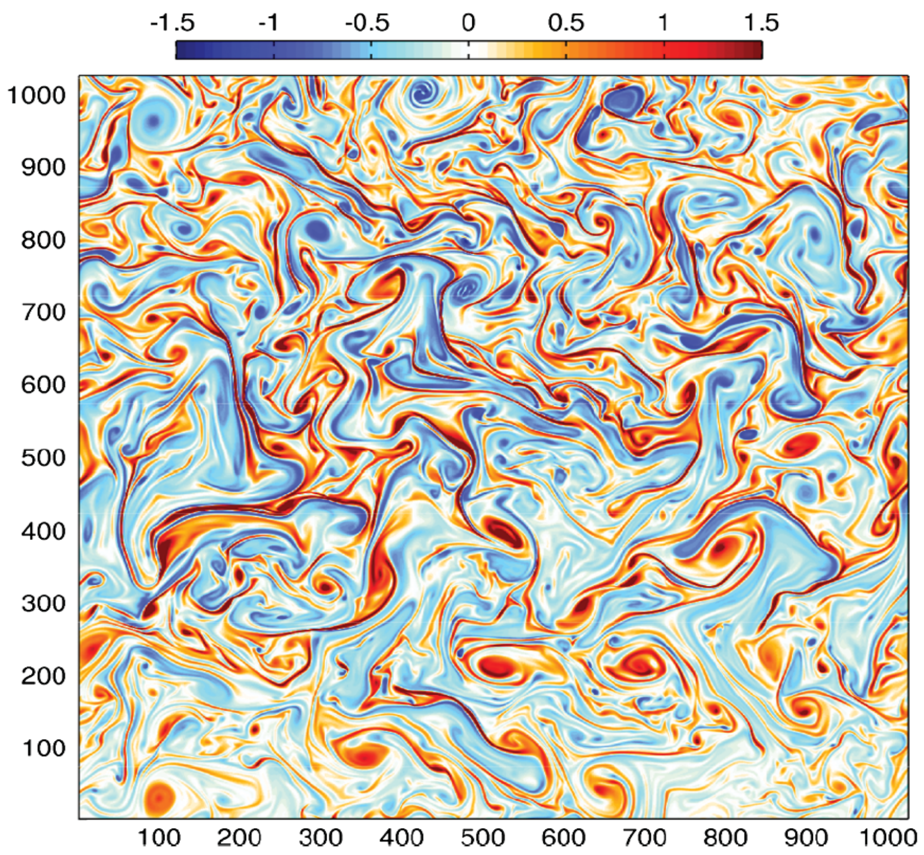


FIG. 3 (color). Snapshots of ζ/f at the upper boundary. Color scale range is chosen to better emphasize the vorticity patterns.

$W = [S_1^2+S_2^2 - \zeta^2]/f^2$ quantifies the magnitude of ζ relative to S_1 and S_2

Using W allows to partition the relative vorticity field into vortices and filaments.

- Magnitude of ζ is usually much larger in vortices which allows them to remain coherent and persistent in time
- Magnitude of ζ within the filaments is usually close to or smaller than S_1 and S_2 , which causes them to deform rapidly and explains their elongation patterns,.

- Here, vortices are defined as the structures where $W < - 0.05$ and filaments as the structures with $W > - 0.05$.

- Vortices occupy 20 % of the total area. All the other structures are stretched filaments. Sensitivity of the results to the choice of the threshold is weak.

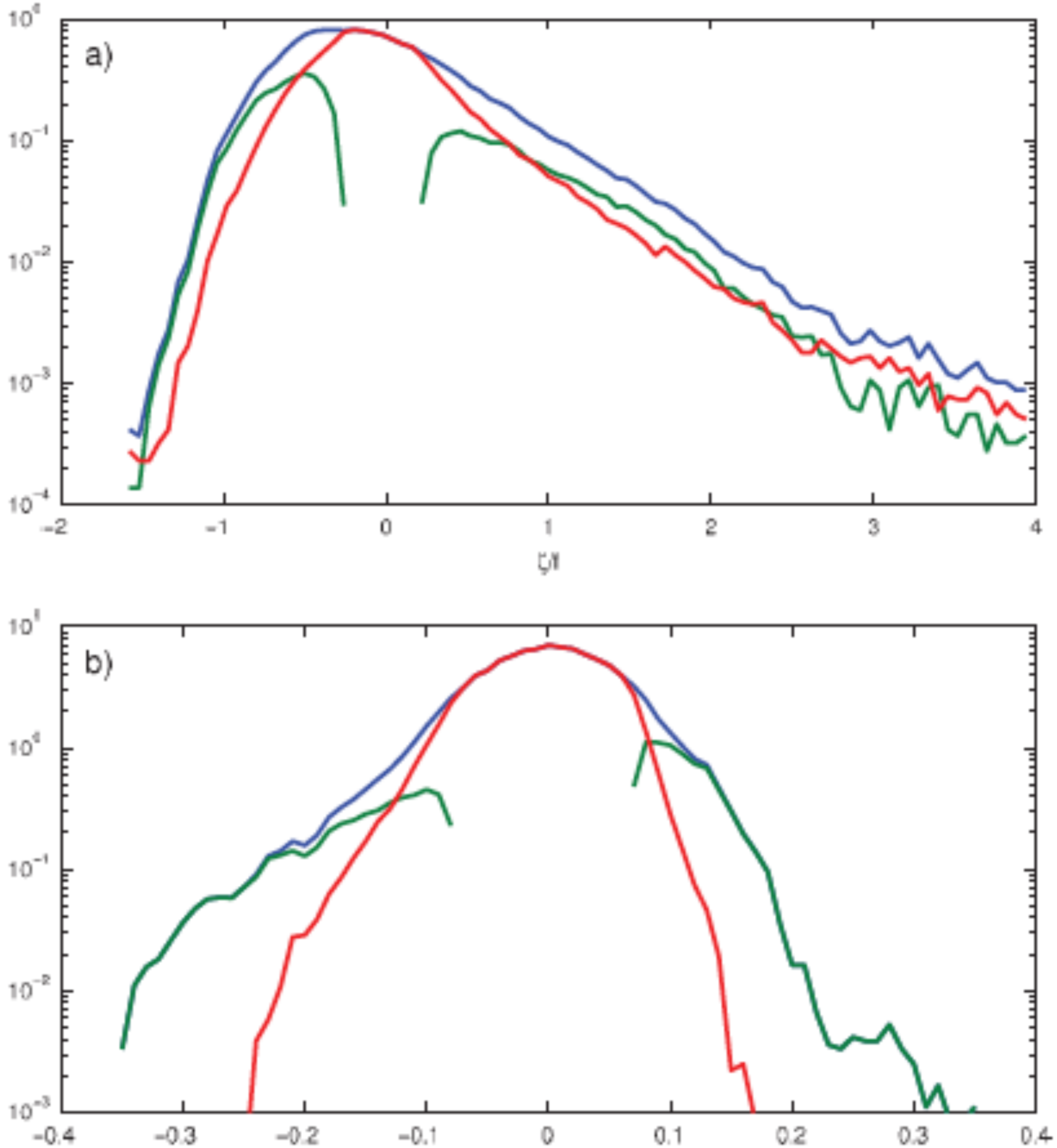
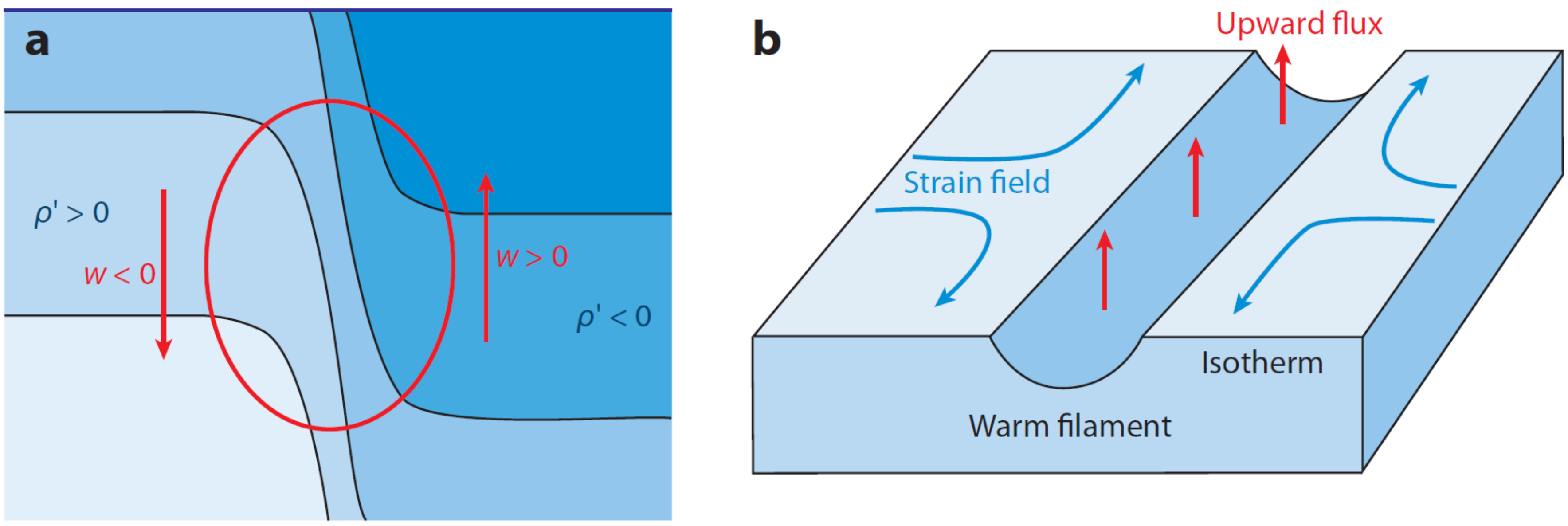


FIG. 5 (color). PDF of ζ/f at the upper boundary (top) and 800 m (bottom). The blue curve is the total PDF. The green (red) curve is the PDF of the relative vorticity related to the vortices (filaments); see text for the partitioning.

Frontogenesis

SST (density) anomalies are stirred by mesoscale eddies => SST fronts
Then, because of frontogenesis, an ageost. circulation develops for the SST front to be in thermal wind balance

$\boxed{\zeta < 0}$



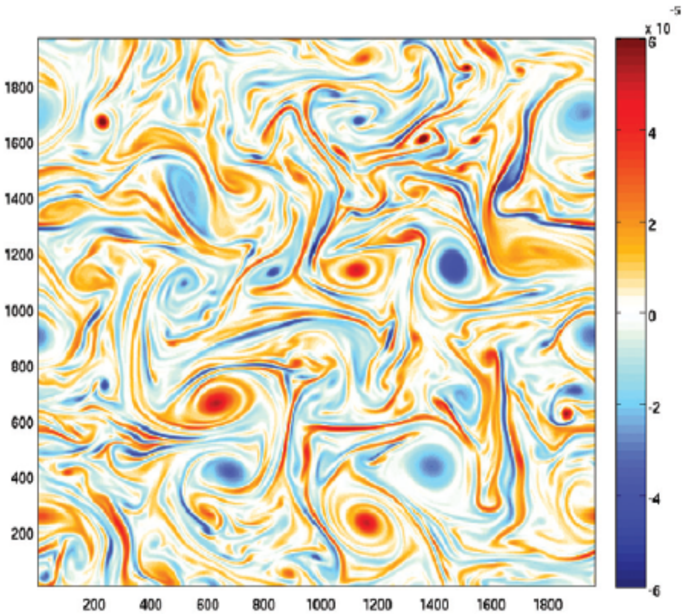
=> A warm filament in a strain field corresponds to a negative RV filament

=> The W-field in this filament is upward (>0).

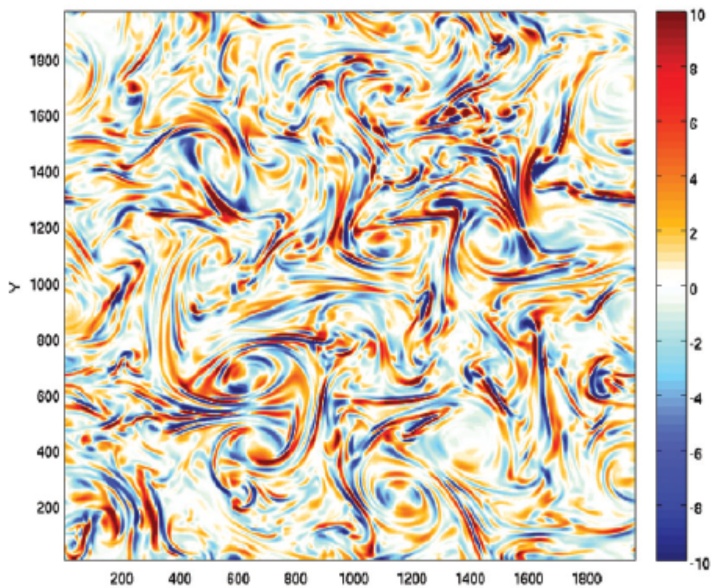
Submesoscale impacts on phytoplankton competition

Model of meso/submesoscale turbulence coupled with a NP1P2ZD model

(Perruche, Rivière,Pondaven,Carton, Lapeyre, JMR'11)



Relative vorticity field



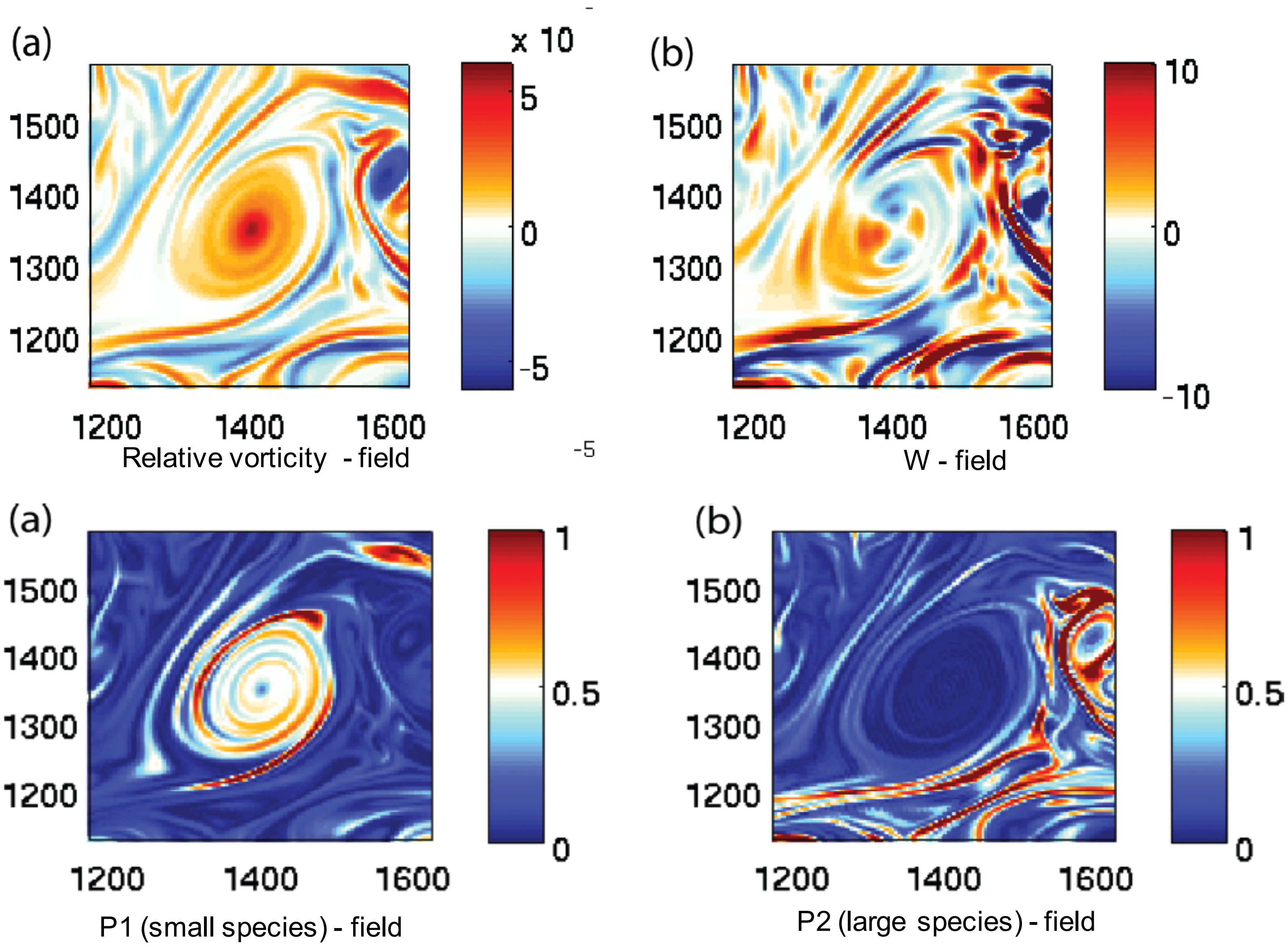
W - field

W – field: only 40 % within mesoscale eddies; 60% within filaments (submesoscales).

Ecosystem model: 2 phytoplankton species [P1 (P2): small (large) species with large (small) growth rate]

What is the impact of mesoscale eddy and submesoscale dynamics on the competition between P1 and P2?

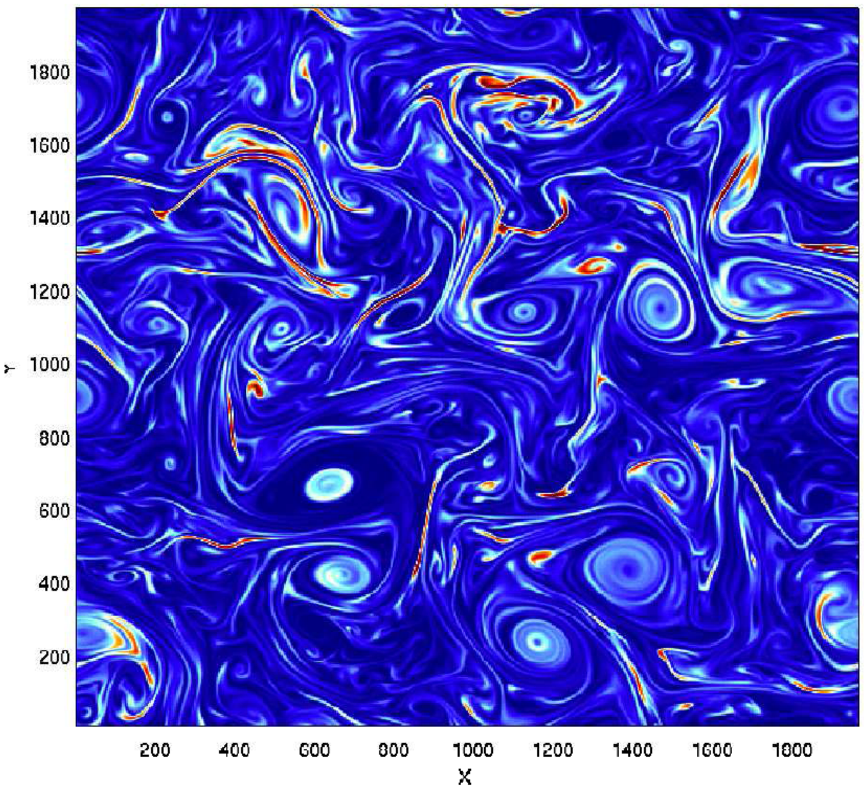
Results : mesoscale eddies (100-200 km) and submesoscales (1-10 km) are ecological niches that shelter different phytoplankton species



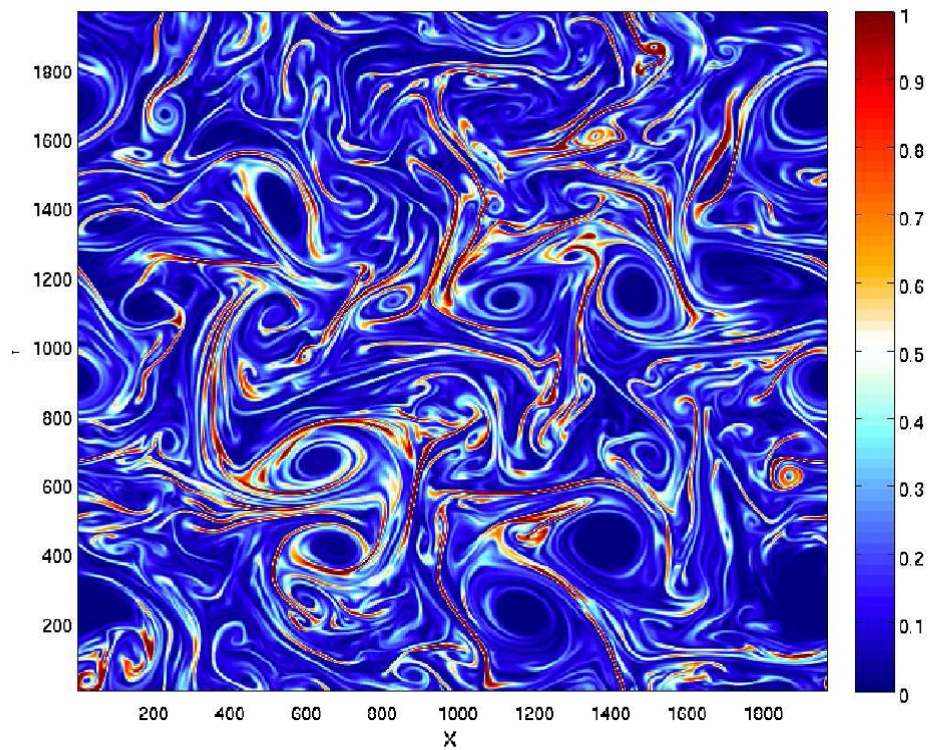
Meso/submesoscale turbulence and phytoplankton competition:

Results

Small phytoplankton (P_1)



Large phytoplankton (P_2)



. 65% of the biomass is inside filaments

. Competition : P_1 : inside eddies P_2 : inside filaments

(Perruche et al. JMR 2011)

Inverse KE cascade in a turbulent geophysical flow

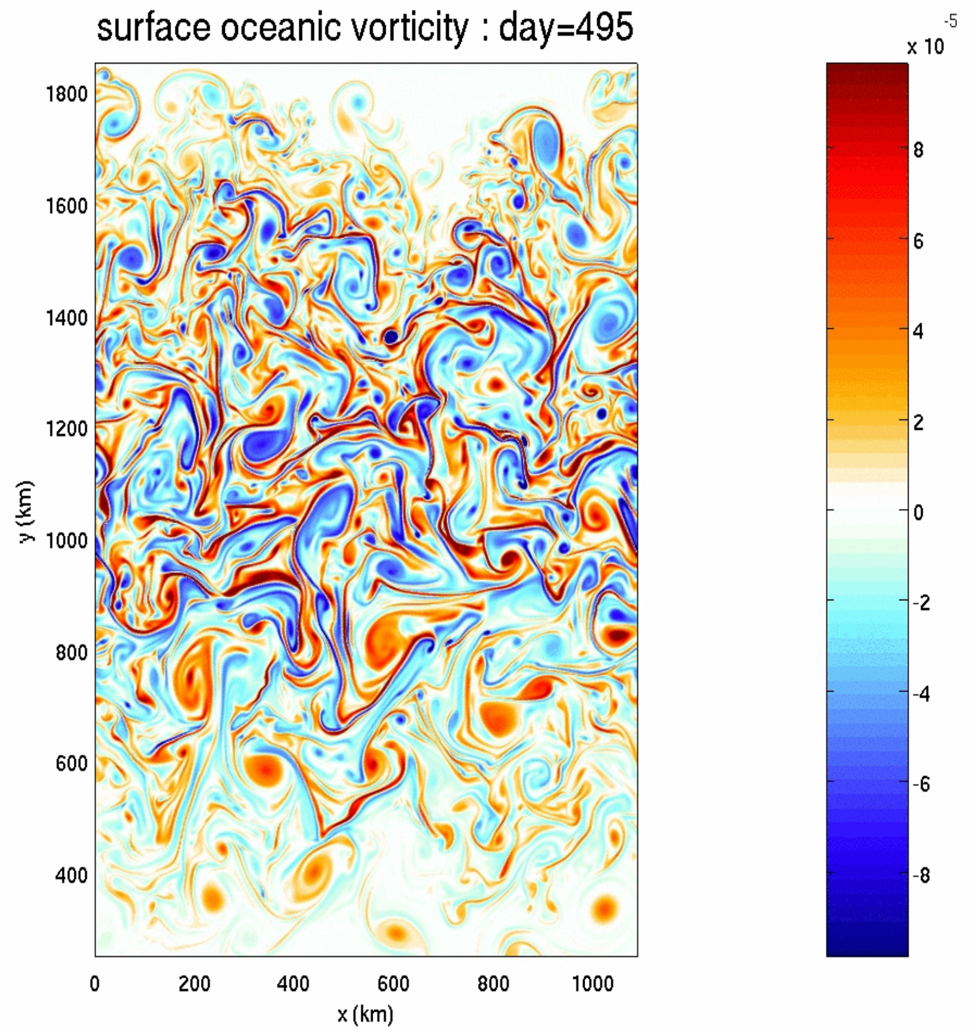
Simulation of eddy turbulence

PE model 1/100e degree, 200 levels
*[3000km*2000km*4000m]*

Eddy turbulent field near the the surface:

*** thin (<10km) filaments**

*** Vorticity field is quickly evolving**



2-D Turbulence:

It is a simple framework (involving simple equations) that captures many of the main characteristics of the balanced (QG) 3-D turbulence.

Inverse kinetic energy cascade

Two dimensional turbulence

Euler equations

$$\mathbf{U} = (u, v).$$

$$\nabla \cdot \mathbf{U} = 0$$

$$\frac{\partial u}{\partial t} + \mathbf{U} \cdot \nabla u - (f_0 + \beta y) v = -\frac{1}{\rho_0} \frac{\partial P}{\partial x} + [\text{mixing}].$$

$$\frac{\partial v}{\partial t} + \mathbf{U} \cdot \nabla v + (f_0 + \beta y) u = -\frac{1}{\rho_0} \frac{\partial P}{\partial y} + [\text{mixing}].$$

In 2-D flow we can define a stream function ψ:

$$u = -\psi_y \qquad v = \psi_x$$

We use $\tilde{p} = \dfrac{P}{\rho_0} - (f_0 + \beta y)\psi$

$\tilde{p}$

$$\Rightarrow \quad \frac{\partial u}{\partial t} + \mathbf{U} \cdot \nabla u = -\frac{\partial \tilde{p}}{\partial x} + [\text{mixing}]$$

$$\frac{\partial v}{\partial t} + \mathbf{U} \cdot \nabla v = -\frac{\partial \tilde{p}}{\partial y} - \beta \psi + [\text{mixing}].$$

$$\boxed{\underline{\text{Two invariants:}} \quad KE = \frac{u^2+v^2}{2} \qquad Z = \frac{\zeta^2}{2}}$$

Z is the enstrophy

(I) $\dfrac{\partial \vec{U}}{\partial t} + \vec{U}\cdot\nabla\vec{U} = -\nabla\tilde{p} - \beta\psi\vec{j} \qquad \vec{j} = \begin{bmatrix}0\\1\end{bmatrix}$

$\Rightarrow \dfrac{\partial KE}{\partial t} + \nabla\cdot[\vec{U}\, KE] = -\nabla\cdot[\vec{U}\tilde{p}] - \beta\left(\dfrac{\psi^2}{2}\right)_x$

$\Rightarrow \displaystyle\iint_{x,y} \frac{\partial KE}{\partial t}\, dx\,dy = 0. \qquad \boxed{KE = cst}$

KE and Z conservation

(II) $\dfrac{\partial \zeta}{\partial t} + \vec{U}\cdot\nabla\zeta + \beta v = 0$

$\Rightarrow \dfrac{\partial Z}{\partial t} + \nabla\cdot(\vec{U}\, Z) = 0$

$\Rightarrow \displaystyle\iint_{x,y} \frac{\partial Z}{\partial t}\, dx\,dy = 0 \qquad \boxed{Z = cst.}$

<u>Comment:</u>

The vorticity eq. can also be written as:

$$\frac{\partial \zeta}{\partial t} + J(\psi, \zeta) + \beta v = 0$$

or $\dfrac{\partial(\zeta + \beta y)}{\partial t} + J(\psi, \zeta + \beta y) = 0.$

with $J(a,b) = a_x b_y - b_x a_y$

<u>The Jacobian $J(a,b) = 0$ when isolines of a are // to isolines of b!</u>

KE and Z cascades

In 2-D turbulence both KE and Z are conserved. Because of these two invariants, there is:

- a KE transfer between small to larger scales,

- a Z transfer between small to larger scales,

These transfers occur through the nonlinear terms. The classical way to explain them is to move to the spectral space …

It is convenient to express variables (ψ) in terms of double Fourier integral.

$$\psi(x,y) = \frac{1}{2\pi} \iint_{k,l} \hat{\psi}(k,l)\, e^{i(kx+ly)}\, dk\, dl.$$

with k and l respectively the zonal and meridional wavenumbers.

So we get:

$$|\hat{u}(k,l)|^2 + |\hat{v}(k,l)|^2 = (k^2+l^2)\, |\hat{\psi}(k,l)|^2$$

$$|\hat{\zeta}(k,l)|^2 = -(k^2+l^2)^2 |\hat{\psi}(k,l)|^2 = -(k^2+l^2)\left[|\hat{u}(k,l)|^2 + |\hat{v}(k,l)|^2\right]$$

From the Parseval equality, we have:

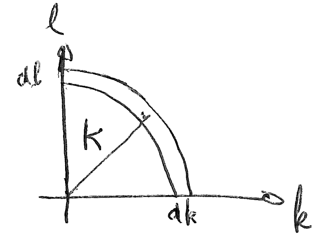
$$KE = \frac{1}{2} \iint_{x,y} (u^2+v^2)\, dx\, dy = \frac{1}{2} \iint_{k,l} \left[|\hat{u}(k,l)|^2 + |\hat{v}(k,l)|^2\right] dk\, dl.$$

$$Z = \frac{1}{2} \iint_{x,y} \zeta^2\, dx\, dy = \frac{1}{2} \iint_{k,l} (k^2+l^2) \left[|\hat{u}(k,l)|^2 + |\hat{v}(k,l)|^2\right] dk\, dl.$$

So the KE and Z transfers between small and large scales can be understood through the KE and Z transfers between wavenumbers.

let us use $K = [k^2 + l^2]^{1/2}$

and $|\hat{u}(K)|^2 = \iint_{k,l} |\hat{u}(k,l)|^2 \, dk\, dl$ for $k^2 + l^2 = K^2$



$$\Rightarrow \quad KE = \iint_K \left[|\hat{U}(K)|^2 + |\hat{V}(K)|^2 \right] dK,$$

$$Z = \iint_K K^2 \left[|\hat{U}(K)|^2 + |\hat{V}(K)|^2 \right] dK.$$

KE and Z transfers between small and large scales are due to the nonlinear interactions!

For the sake of simplicity, let us consider the simple eq.:

$$\frac{\partial u}{\partial t} = u.u.,$$

and. $u(x) = \sum_k \cos kx \, \hat{u}_k$. $\Rightarrow \hat{u}_k = \frac{1}{2\pi} \int_{-\pi}^{\pi} \cos kx \, u(x) \, dx.$

The time evolution of $\hat{u}_k$ is:

$$\frac{\partial \hat{u}_k}{\partial t} = \frac{1}{2\pi} \int_{-\pi}^{\pi} u(x).u(x) \cos kx \, dx.$$

$$= \frac{1}{2\pi} \int_{-\pi}^{\pi} \left[\sum_n \cos nx . \hat{u}_n \right] \left[\sum_m \cos mx . \hat{u}_m \right] \cos kx \, dx.$$

$$= \frac{1}{8\pi} \int_{-\pi}^{\pi} \sum_n \sum_m \cos(m \pm n \pm k)x \; \hat{u}_n \hat{u}_m \, dx.$$

The R.H.S. is non zero only if $m \pm n \pm k = 0$!

This is the classical triad interaction!

- let us consider the triad $n = k + m$ with $k < m < n$.

(for example $k = 1$ $m = 2$ $n = 3$),

with the initial condition:

$$|\hat{u}_k|^2(0) = \varepsilon \quad |\hat{U}_m|^2(0) = U_0^2 \quad |\hat{U}_n|^2(0) = \varepsilon.$$

- Assume there is KE and Z transfers between wavenumbers such that at time t:

$$|\hat{U}_m|(t) = \varepsilon.$$

So $|\hat{U}_k|^2(t)$?, $|\hat{U}_n|^2(t)$?

- KE and Z are conserved. So:

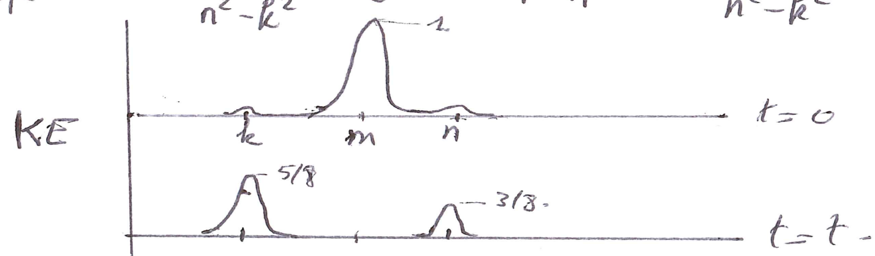
$$KE \approx |\hat{U}_k|^2(t) + |\hat{U}_n|^2(t) = U_0^2.$$

$$Z \approx k^2 |\hat{U}_k|^2(t) + n^2 |\hat{U}_n|^2(t) = m^2 U_0^2$$

This leads to:

$$|\hat{U}_k|^2(t) = \frac{n^2 - m^2}{n^2 - k^2} \cdot U_0^2 \qquad |\hat{U}_n|^2(t) = \frac{m^2 - k^2}{n^2 - k^2} U_0^2$$

$$k^2 |\hat{U}_k|^2(t) = \frac{k^2 (n^2 - m^2)}{n^2 - k^2} U_0^2 \qquad n^2 |\hat{U}_n|^2(t) = \frac{n^2 (m^2 - k^2)}{n^2 - k^2} U_0^2$$



So there is a dominant KE transfer from small to larger scales!

" " " Z transfer from large to smaller scales!

These transfers occur through the nonlinear terms.

How to characterize the KE transfer in a fully turbulent flow?

$$\frac{\partial \hat{U}_k}{\partial t} = -[U \cdot \nabla U]_k + \boxed{[\text{Sources}]_k + [\text{mixing}]_k}.$$

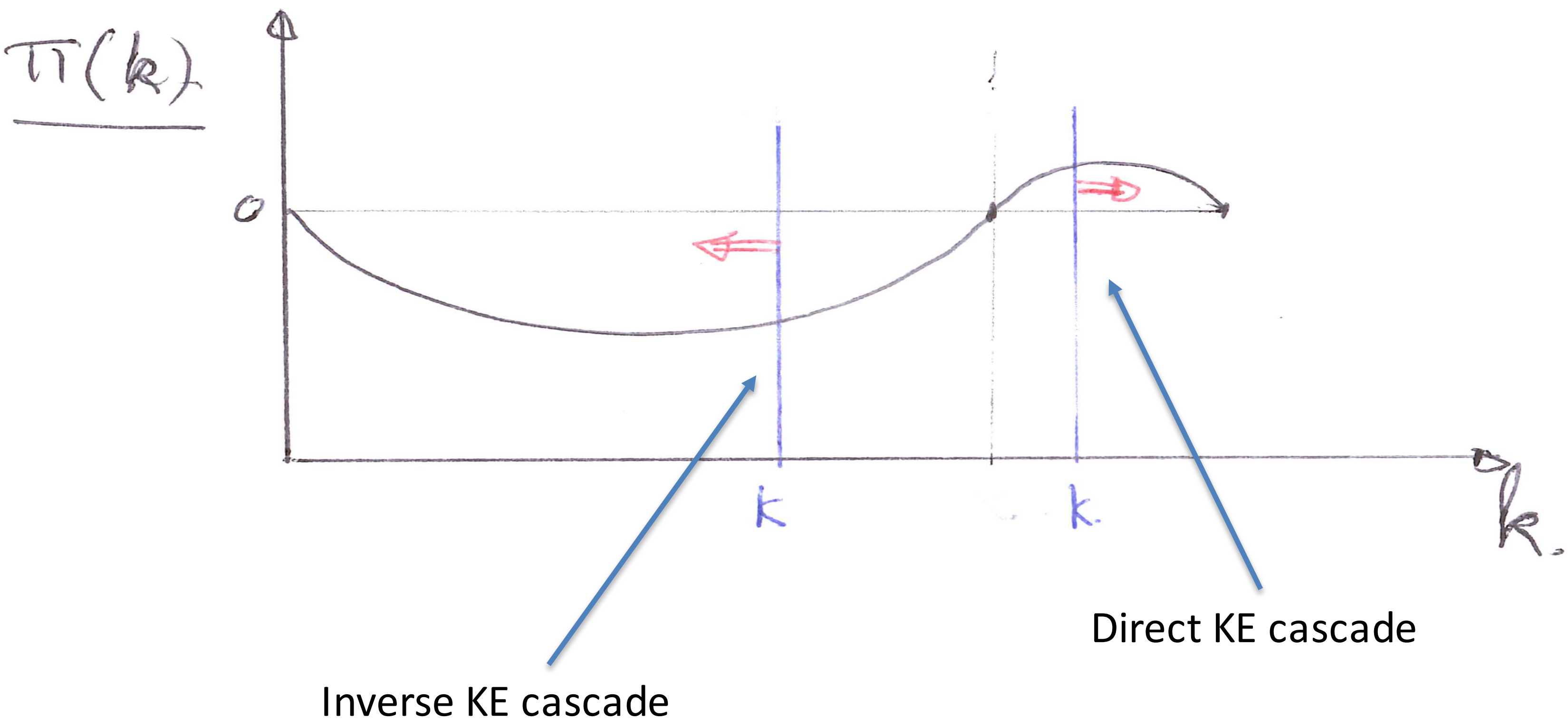
$$\frac{\partial KE_k}{\partial t} = -Re\left[\hat{U}_k^* \cdot [U \cdot \nabla U]_k\right] + [\cdots]_k + [\cdots]_k$$

$Re\left[\hat{U}_k^* \cdot [U \cdot \nabla U]_k\right]$ can be a source or a sink of KE_k

KE transfer between small and larger scales is assessed through:

$$\frac{\partial \int_K KE_k}{\partial t} = \boxed{\underbrace{-\int_K^{\infty} Re\left[\hat{U}_k^* [U \cdot \nabla U]_k\right] dk}_{\Pi(K)}} + \cdots$$

IF $\Pi(K) < 0$, wavenumbers larger than K (smaller scales) have lost KE and therefore wavenumbers smaller than K (larger scales) have gained KE (total KE is conserved).



Π(k)
0
k
k
k
Direct KE cascade
Inverse KE cascade

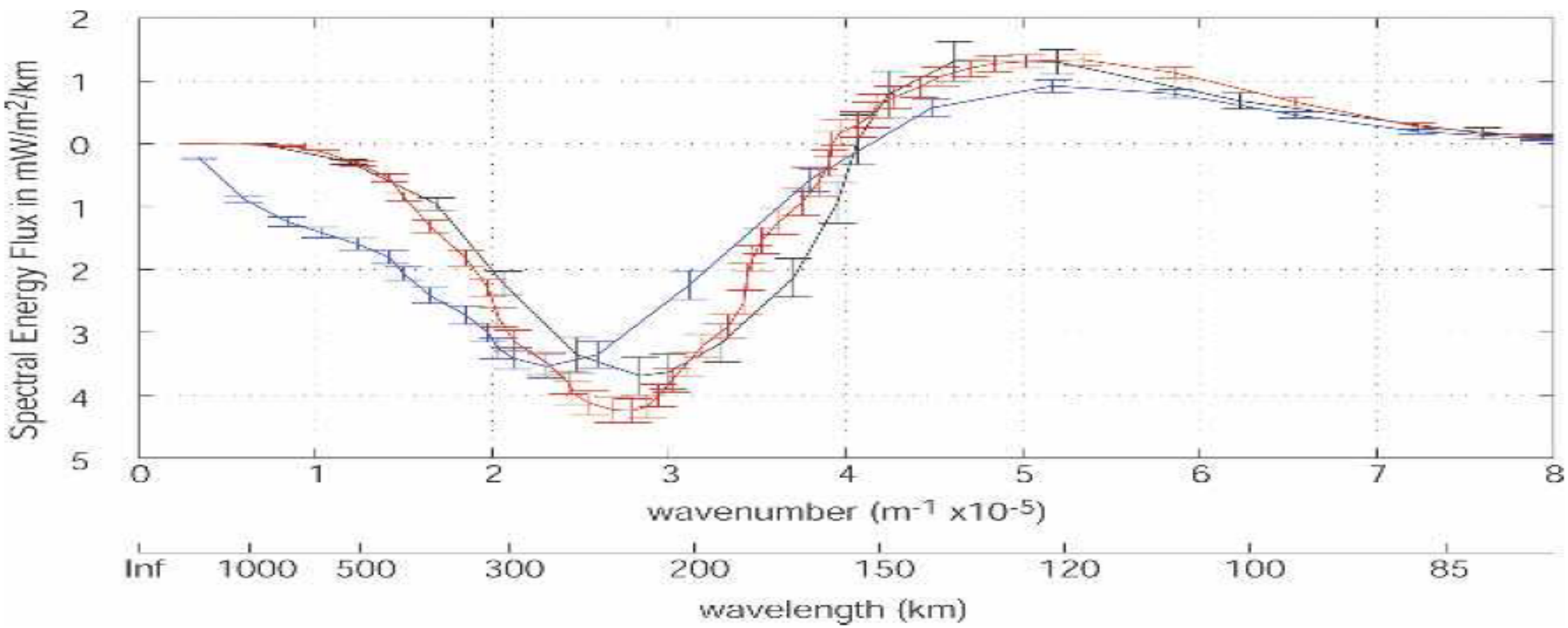


FIG. 2. Time mean, spectral kinetic energy flux $\overline{\Pi}(K)$ vs total wavenumber K in a homogeneous ACC region (rectangles centered at 57°S, 120°W): black curve using SSH on a 32 × 32 grid, red curve using SSH on a 64 × 64 grid, blue curve using velocity on a 64 × 64 grid. Positive slope reveals a source of energy. The larger negative lobe reveals a net inverse cascade to lower wavenumber. Error bars represent standard error.

Spectral kinetic energy fluxes estimated from AVISO data:

- direct KE cascade for scales smaller than 150 km !
- inverse KE cascade for scales larger than 150 km !

(Scott and Wang, JPO 2005)

What are the results in HR numerical models ?

The small-scale ageostrophic motions (including the u-v-w-field) have a dramatic impact on the time evolution of the eddy field

KINETIC ENERGY BUDGET

$$\underbrace{\frac{1}{2}\frac{\partial|\widehat{\mathbf{u}_h}|^2}{\partial t}}_{d/dt} = \underbrace{-\mathrm{Re}[\widehat{\mathbf{u}_h}^* \cdot \widehat{(\mathbf{u}_h \cdot \nabla_H \mathbf{u}_h)}]}_{\text{advection}}$$

$$\underbrace{-\frac{1}{\rho_0}\mathrm{Re}(\widehat{\mathbf{u}_h}^* \cdot \widehat{\nabla_H p})}_{\text{pressure}} + \underbrace{MT}_{\text{mixing}}$$

$$\widehat{\mathbf{u}_h}^* \cdot \widehat{\nabla}_H p = \widehat{w_z}^* \hat{p}$$

Impact of filtering in space or time on spectral kinetic energy fluxes (Arbic et al. 2013)

Two-layer QG turbulence model

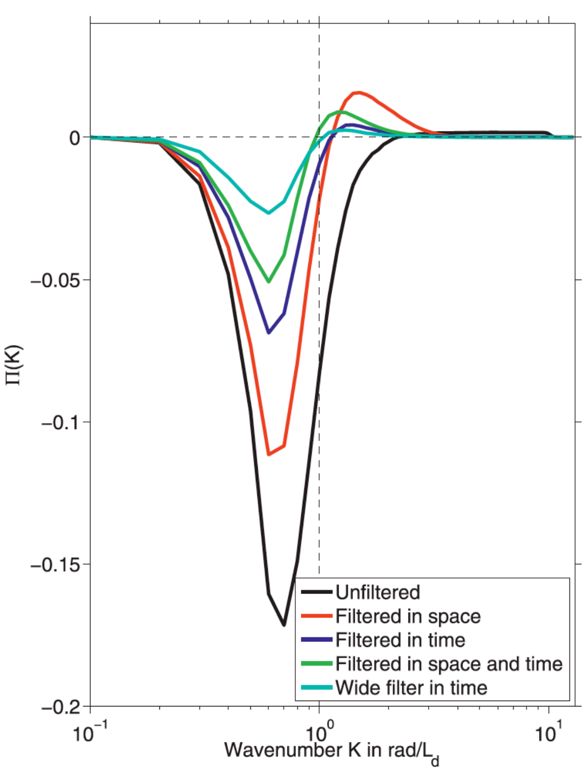
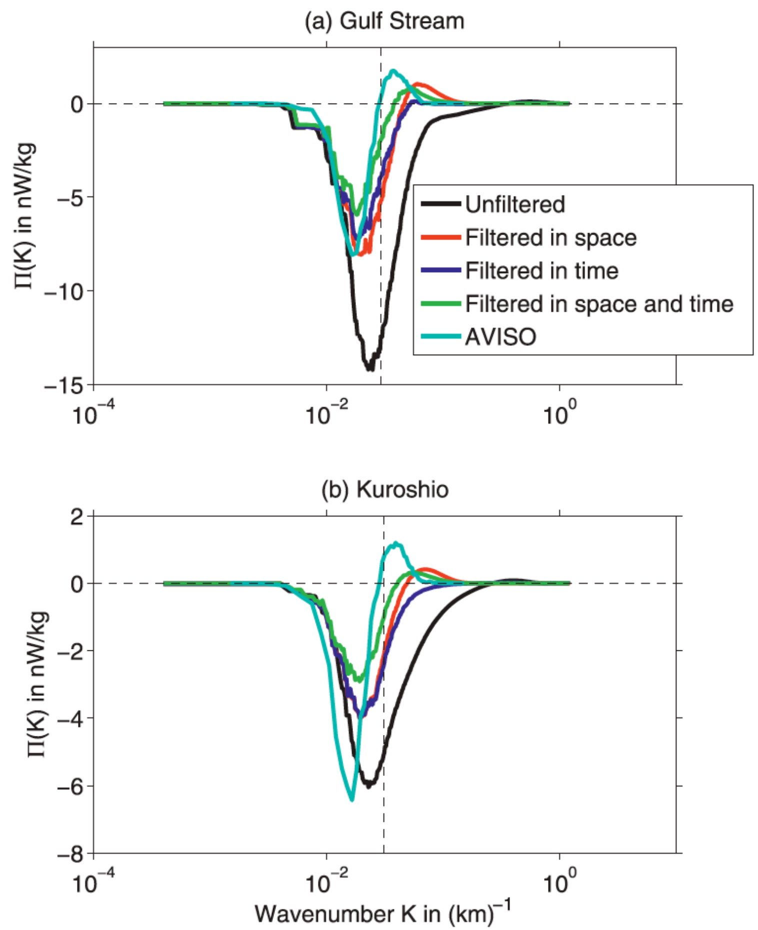


FIG. 6. Spectral flux $\Pi_{\mathrm{KE},1}(K)$ of upper-layer kinetic energy computed from the $\nu = 0$, $F_L = 0.4$ two-layer QG simulation ("unfiltered") and from filtered versions of this simulation. See text for descriptions of filters used. All fluxes normalized by $(\overline{u}_1 - \overline{u}_2)^3/L_d$.

NLOM



What is missing in the AVISO data ?

Contribution of the small scales!

SWOT should sample theses small scales

SWOT