GM Spectra

Zachary Erickson

25 May 2017

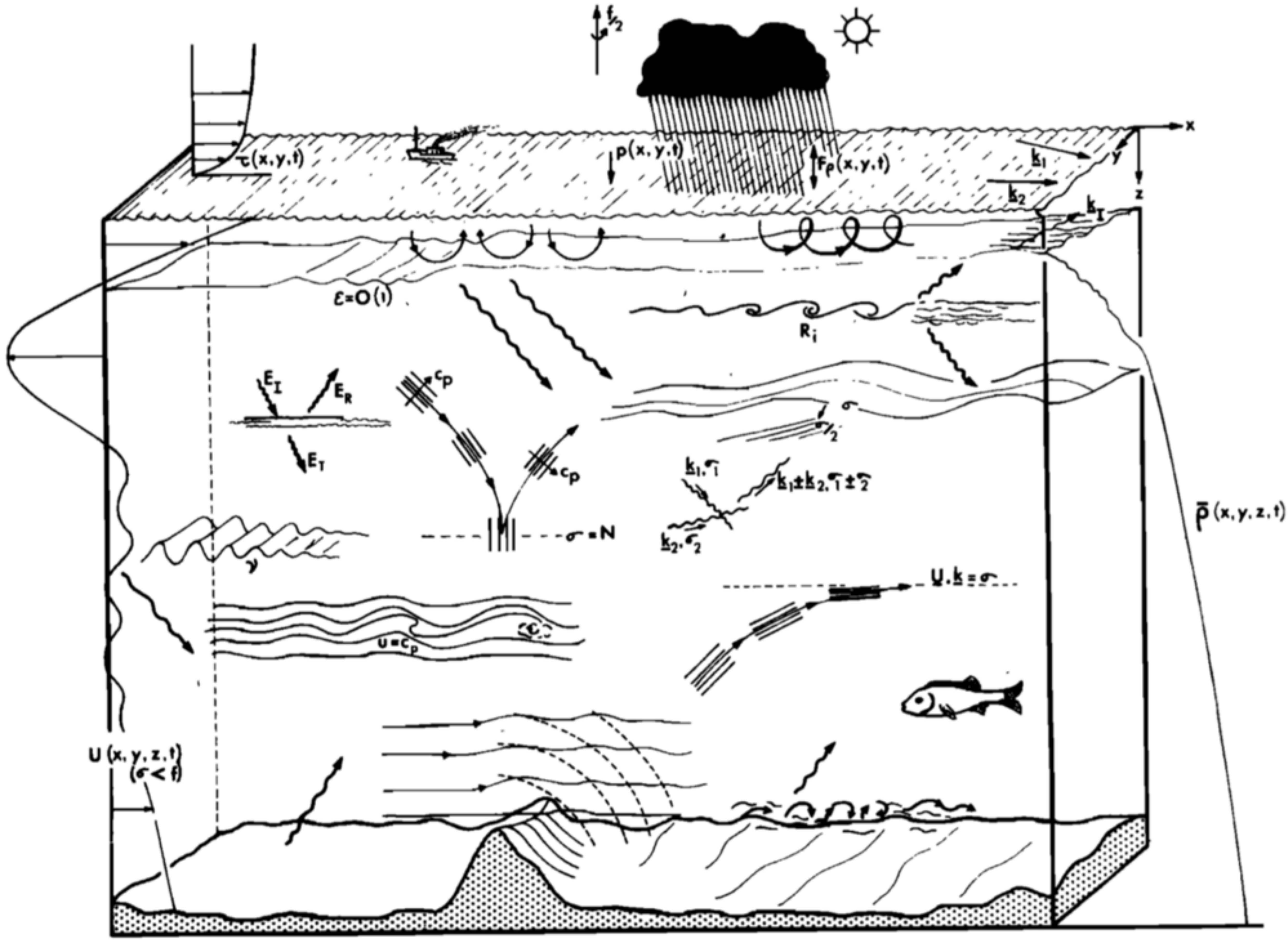
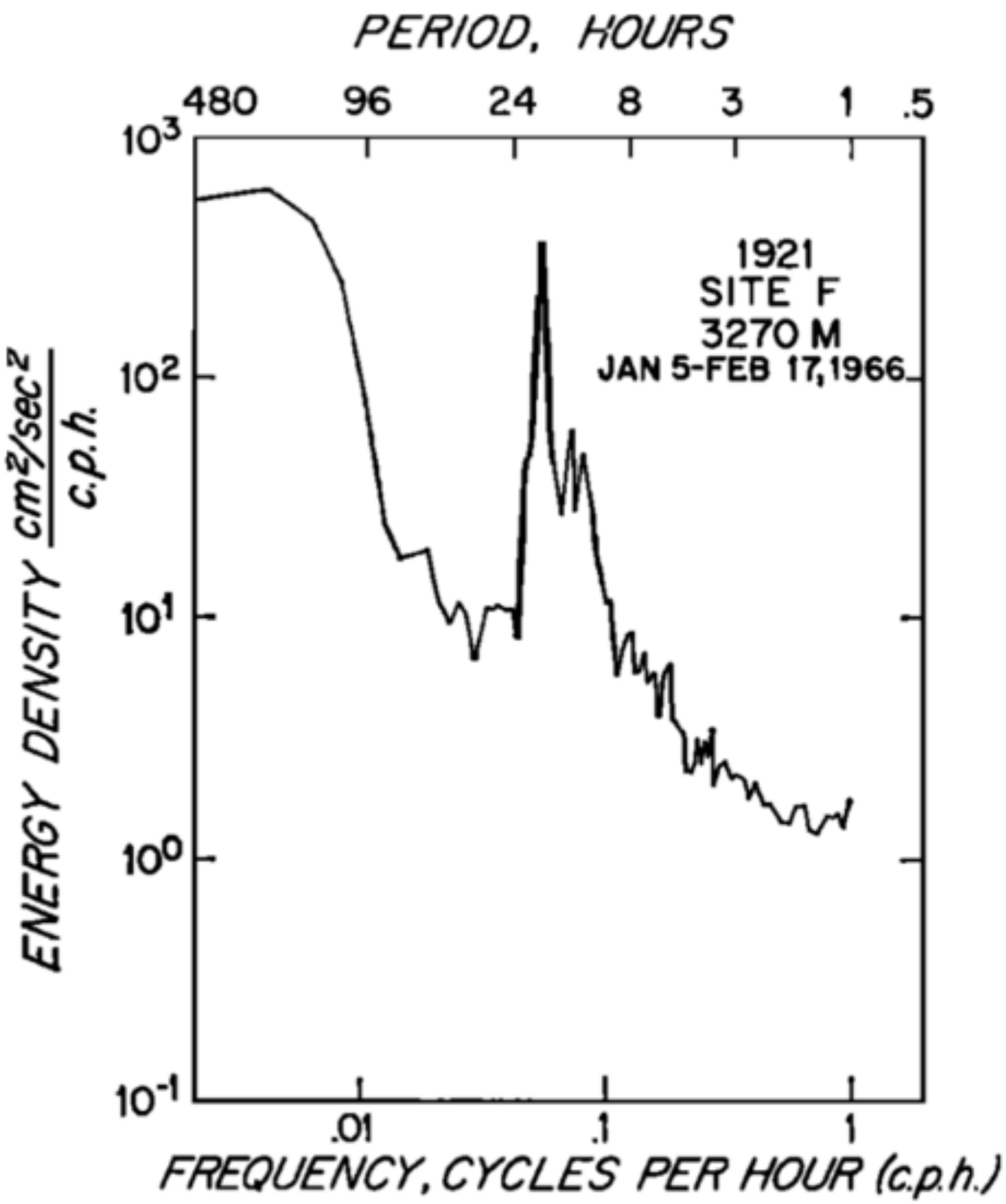


Fig. 5. Physical processes affecting internal waves.

(Thorpe, JGR, 1975)



(Webster, Rev. Geophys., 1968)

"A generation of oceanographers has suffered the disappointments of ending up with uncorrelated measurements in what was intended as a coherent experiment" (Garrett and Munk, JGR, 1975)

GM spectra are

- descriptions of internal wave properties in the interior ocean,
- agnostic to generation and dissipation mechanisms,
- empirical, and
- improved over time largely due to better measurements.

(Müller, Olbers, and Willebrand, JGR, 1978)

GM spectra are

- descriptions of internal wave properties in the interior ocean,
- agnostic to generation and dissipation mechanisms,
- empirical, and
- improved over time largely due to better measurements.

GM spectra assume internal wave field is

1. statistically stationary,
2. horizontally homogenous,
3. a linear superposition of waves,
4. well-approximated by WKB theory (spectral gap between internal waves and jets/eddies),
5. horizontally isotropic, and
6. vertically symmetric.

(Müller, Olbers, and Willebrand, JGR, 1978)

“*The spectrum was developed on the basis of rank empiricism, with no trace of underlying theory*” (Munk, 1981)

A timeline view of the GM spectra

GM72 (Garrett and Munk, Geophys. Fl. Dyn., 1972)
- first formulated spectra

GM75 (Garrett and Munk, JGR, 1975)
- extended to larger wavenumbers (smaller scales)

GM76 (Cairn and Williams, JGR, 1976)
- better fidelity around the buoyancy frequency

IWEX78 (Müller, Albers, and Willebrand, JGR, 1978)
- rigorous determination of best-fit parameters
- tests assumptions 1-6

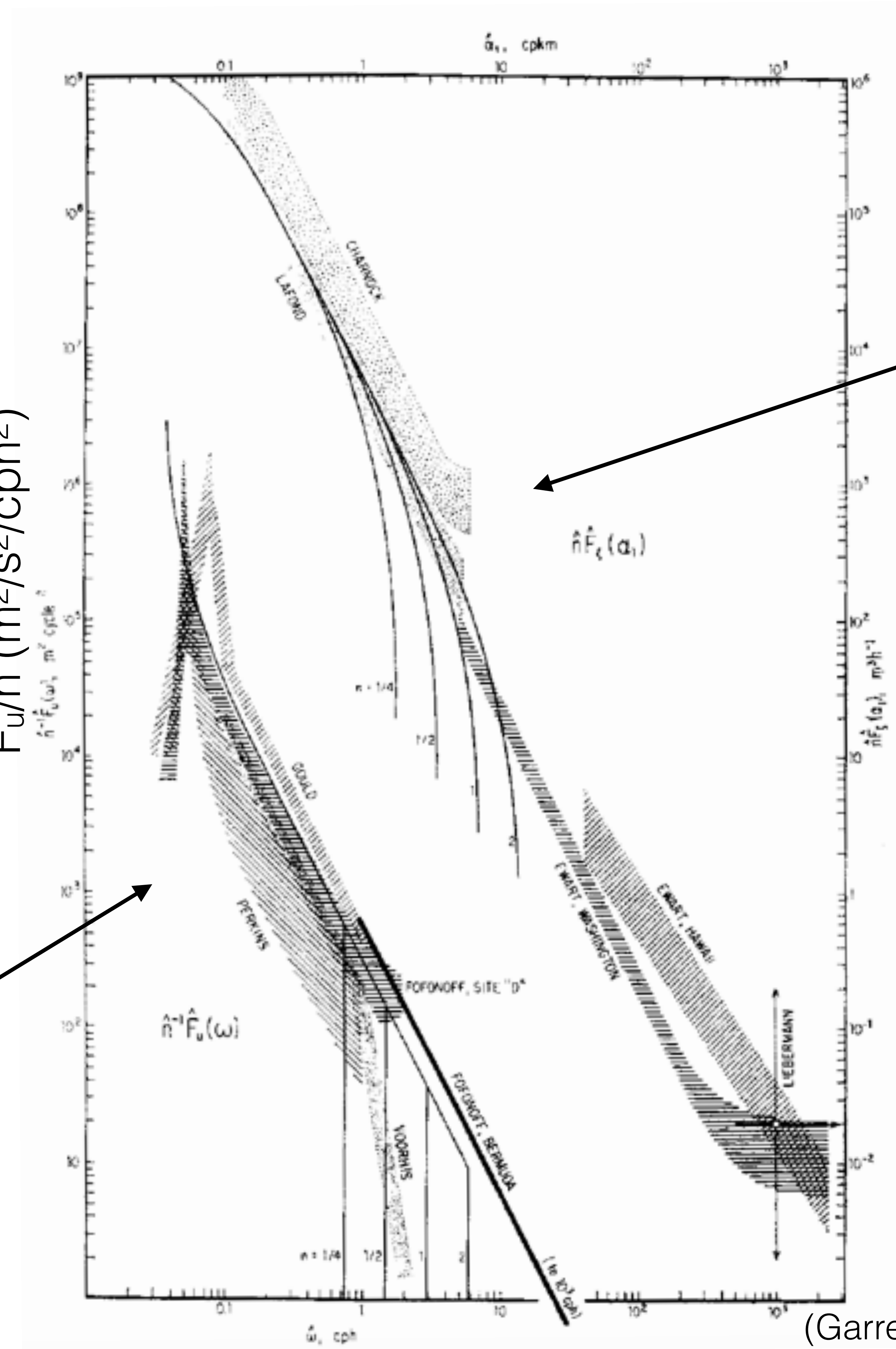
GK91 (Gregg and Kunze, JGR, 1991; also Munk, 1981)
- high wavenumber roll-off

A few notes first…

The literature uses different variables in each paper, making it somewhat difficult to compare. For this lecture,

k as horizontal wavenumber (cycles per length)
m as vertical wavenumber (cycles per length)
ω as frequency (cycles per time)
(.) as dimensionless variable
(ˆ) as having dimensions (dimensionful?)

GM72 GM75 GM76 IWEX78 GK91



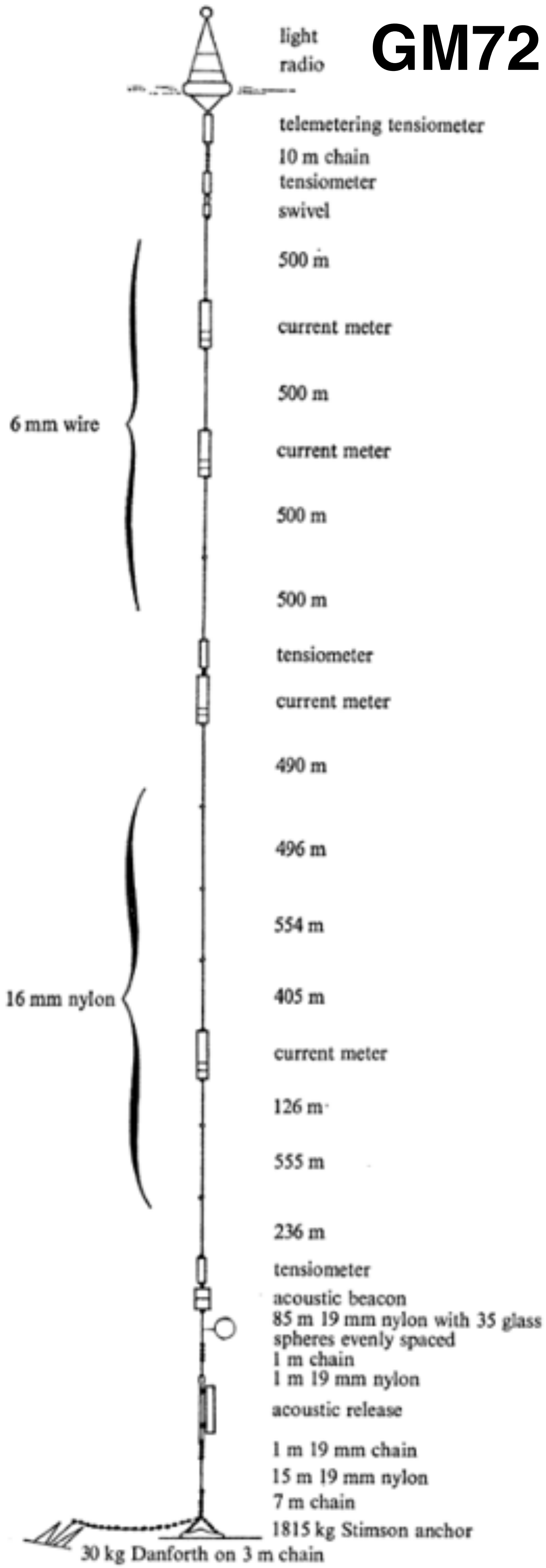
towed spectra

moored spectra

F_u/n ($m^2/s^2/cph^2$)

nF_ζ (m^3/hr)

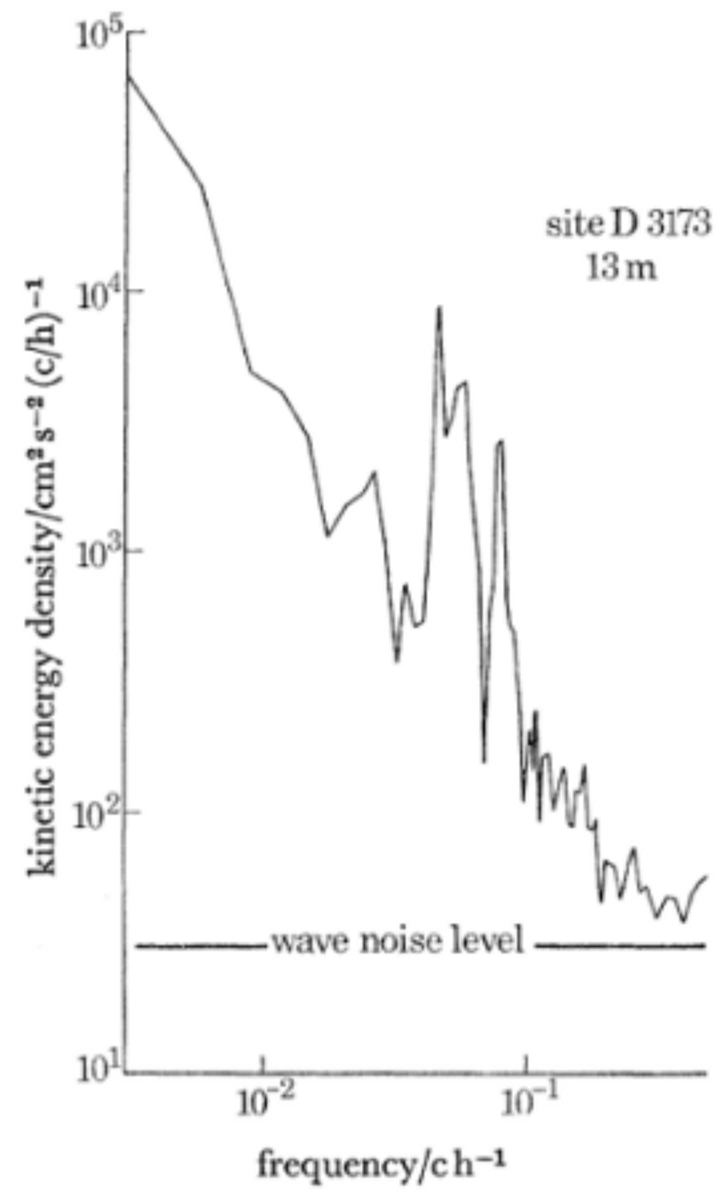
(Garrett and Munk, Geophys. Fl. Dyn., 1972)



GM72 GM75 GM76 IWEX78 GK91

Types of data:

- Moored spectra $F(\omega)$



(Fofonoff and Webster, Ph. Trans. Roy. Soc. London Ser. A, 1971)

Types of data:

- Towed spectra $F(k)$

Typically, thermistor chains towed behind vessels, with the depth of a given isotherm found through interpolation. 8 meter vertical resolution was considered amazing.

Coherence

Coherence

co-variance of time series

time series

$$\rho_{mn}(\tau) = \langle f_m(t) f_n(t+\tau) \rangle$$

Coherence

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time series

$$\rho_{mn}(\tau) = \langle f_m(t) f_n(t+\tau) \rangle$$

co-spectrum

quadrature-spectrum

$$C(\omega)_{mn} + iQ(\omega)_{mn} = \frac{2}{\pi}\int_0^{\infty} \rho_{mn}(\tau) e^{-i\omega\tau}\mathrm{d}\tau$$

Coherence

co-variance of time series   time series

$$\rho_{mn}(\tau) = \langle f_m(t) f_n(t+\tau) \rangle$$

co-spectrum   quadrature-spectrum

$$C(\omega)_{mn} + iQ(\omega)_{mn} = \frac{2}{\pi} \int_0^\infty \rho_{mn}(\tau) e^{-i\omega\tau} \mathrm{d}\tau$$

coherence   phase lag

$$R_{mn} e^{i\gamma_{mn}} = (C_{mn} + iQ_{mn})(C_{mm} C_{nn})^{-1/2}$$

Coherence

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Simple example: $m = n$

Coherence

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Simple example: $m = n$

$$Q_{mn} = 0$$

$$C_{mn} = F_n(\omega)$$

$$R_{mn} e^{i\gamma_{mn}} = (C_{mn} + iQ_{mn})(C_{mm} C_{nn})^{-1/2} = 1e^{i0}$$

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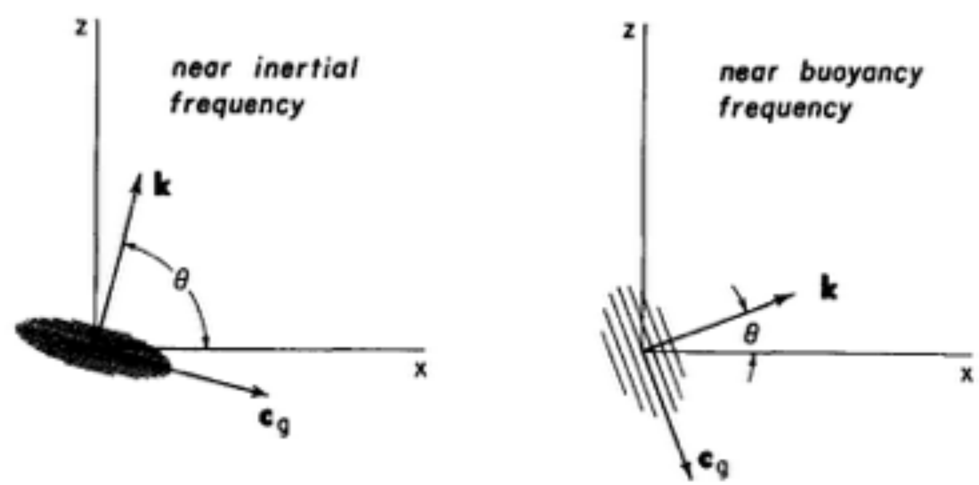
What scales are we interested in?

1. Frequency

   A non-hydrostatic ocean follows the dispersion relation:

$$\omega^2 = \frac{k^2 N^2 + m^2 f^2}{m^2 + k^2} = N^2 \cos^2\theta + f^2 \sin^2\theta$$



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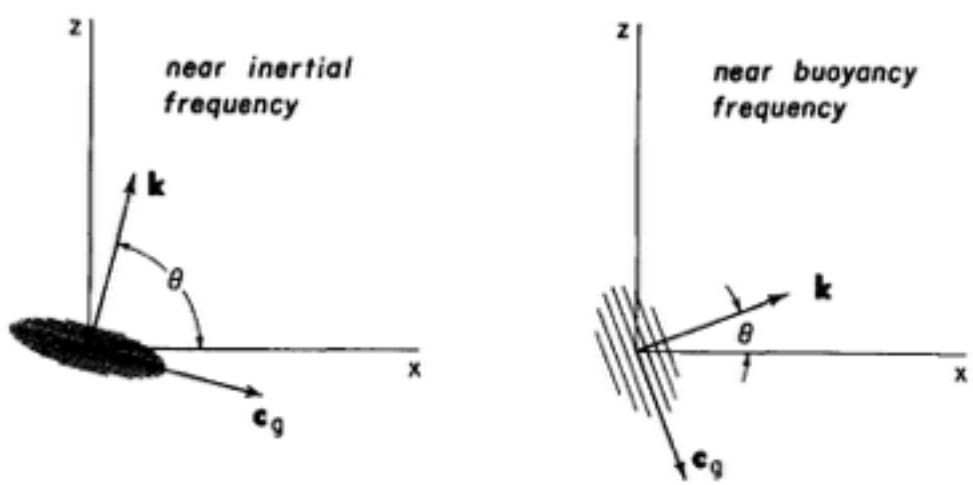

$$|f| \leq \omega \leq N$$

GM72 GM75 GM76 IWEX78 GK91

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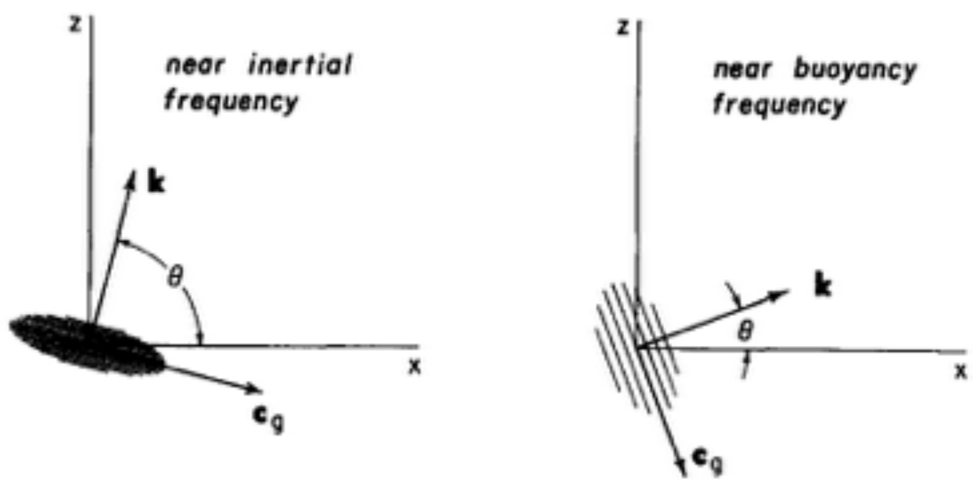


$$|f| \leq \omega \leq N$$

2. Horizontal/vertical wavenumber
   ??? (depends on how many modes are important)

Ideally, we will find some equation

$$E(k, \omega) = E_0 A(k) B(\omega)$$

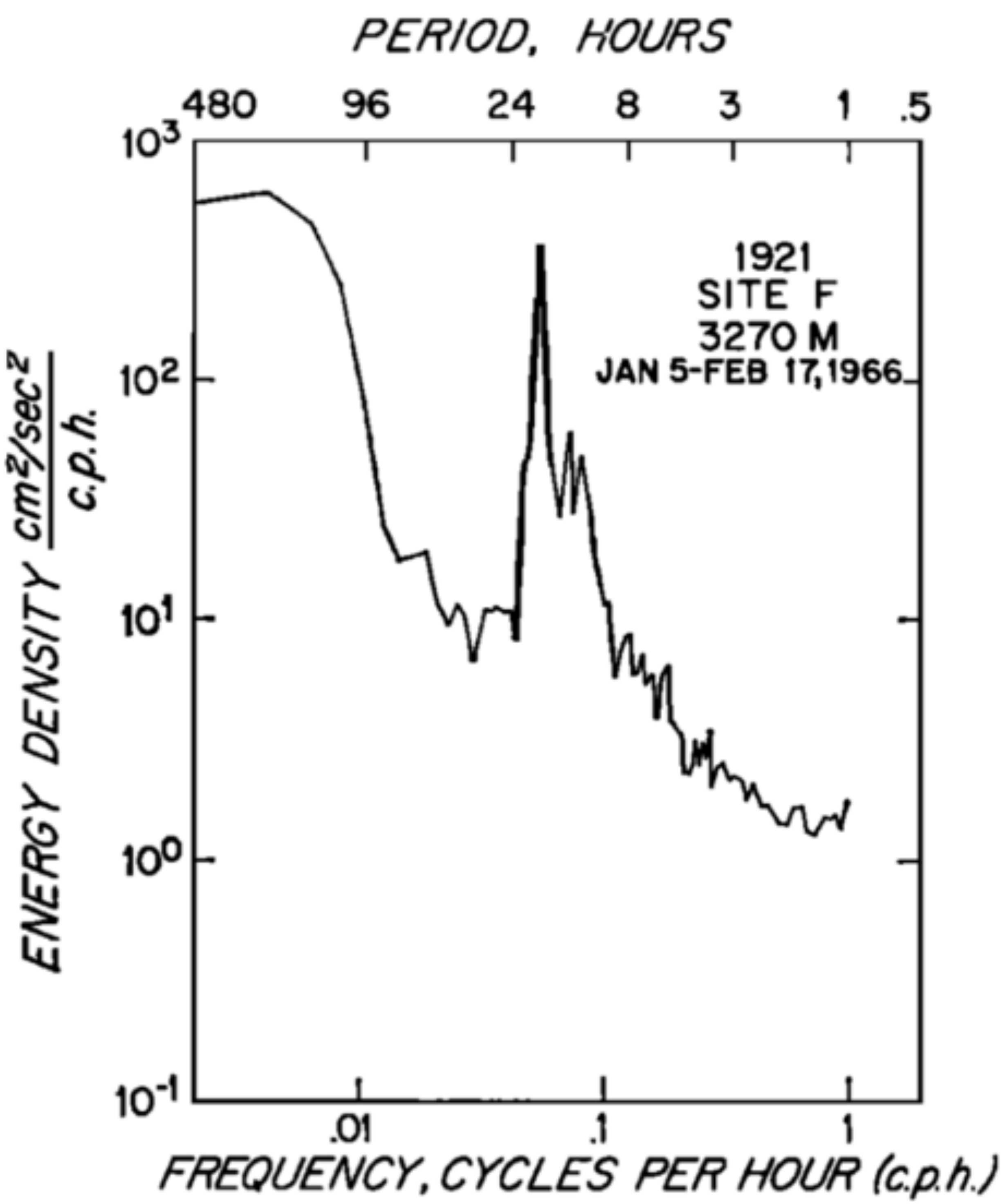
Unfortunately, observations are inconsistent with this, and we must use the more general

$$E(k, \omega) = E_0 A(\lambda) B(\omega) k{*}^{-1}$$

$$\lambda = k/k^*$$

$$k^*(\omega)$$

GM72 GM75 GM76 IWEX78 GK91



(Webster, Rev. Geophys., 1968)

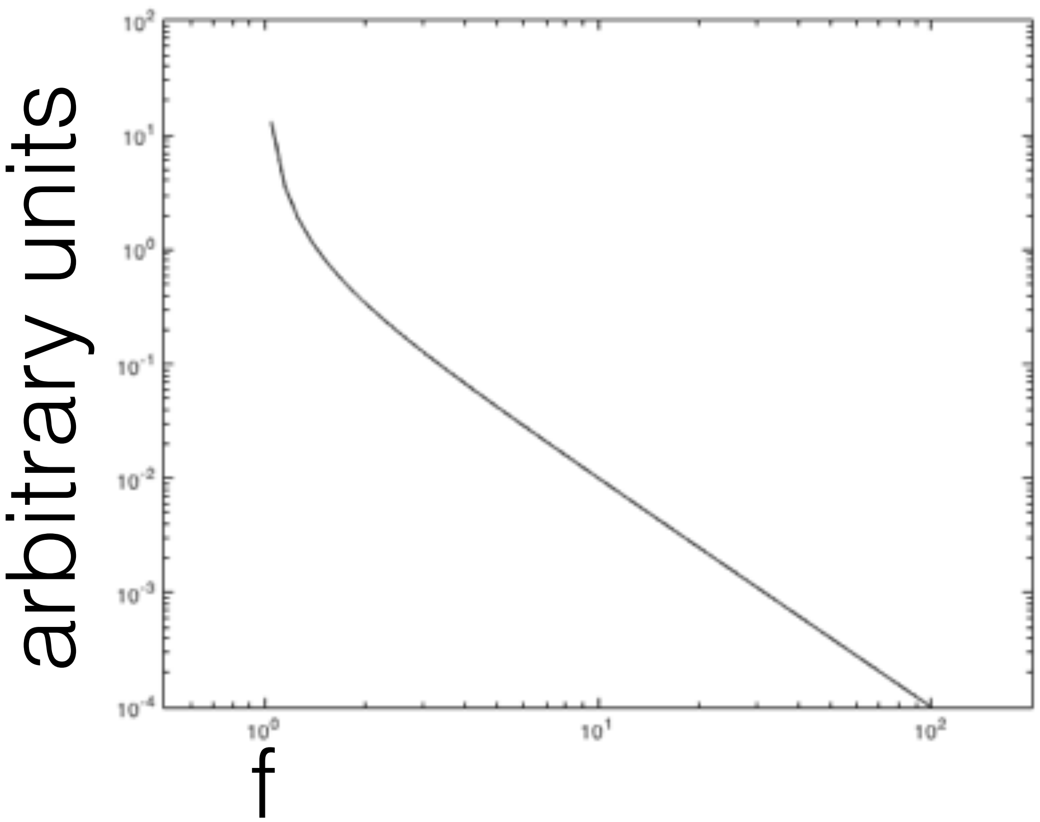
What is $B(\omega)$?

Easiest would be a simple power law, but that ignores the cusp seen near *f*.

Instead choose:

$$B(\omega) = \omega^{-p+2s}(\omega^2 - f^2)^{-s}$$

What is $B(\omega)$?



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Instead choose:

$$B(\omega) = \omega^{-p+2s}(\omega^2 - f^2)^{-s}$$

s must be between 0 and 1;
"arbitrarily choose the mid-point, s=1/2".

Empirical fit leads to $p = 2$

GM72 GM75 GM76 IWEX78 GK91

What is $A(\lambda) = A(k/k^*)$?

What is k^*?

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What is k^*?

Simplest form for A: a plateau

$$A(\lambda) = 1 \text{ for } 0 < \lambda \leq 1$$

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What is $A(\lambda) = A(k/k^*)$?
What is k^*?

Simplest form for A: a plateau

$$A(\lambda) = 1 \text{ for } 0 < \lambda \leq 1$$
$$A(\lambda) = 0 \text{ for } \lambda \leq 0; \lambda > 1$$

$$k^* = j_i \pi \left(\frac{\omega}{f}\right)^{r-1} (\omega^2 - f^2)^{1/2}$$

where j_i is the number of modes

Empirical fit leads to $r = 1$

GM72:

$$E(k,\omega) = \frac{2}{\pi} E_0 f k^{*-1} (\omega^2 - f^2)^{-1/2},$$

$$k^* = j_i \pi (\omega^2 - f^2)^{1/2}$$

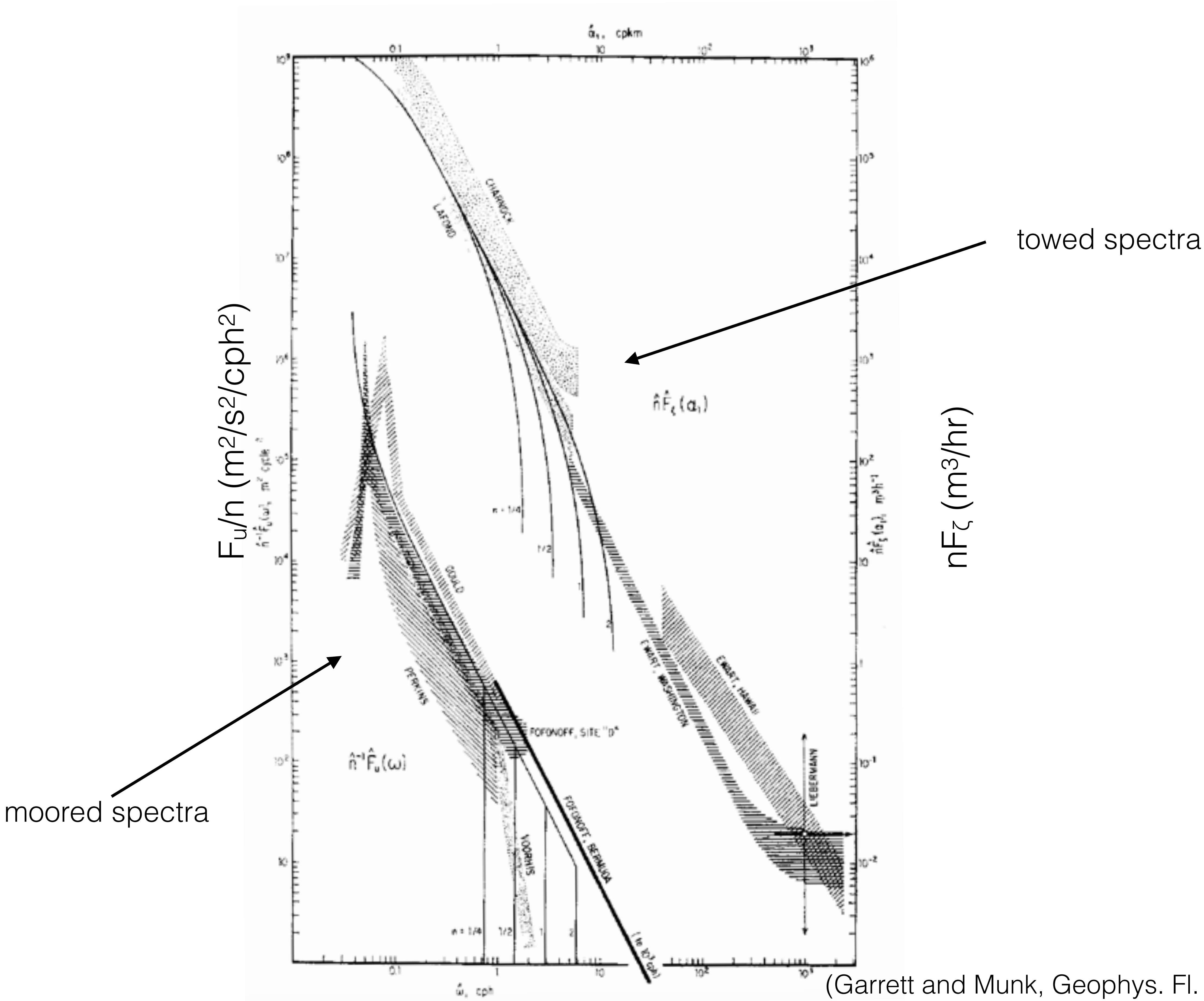
$$E_0 = 2\pi \times 10^{-5}$$

$$j_i = 20$$

$$\text{for } k < k^*$$

$$|f| < \omega < n$$

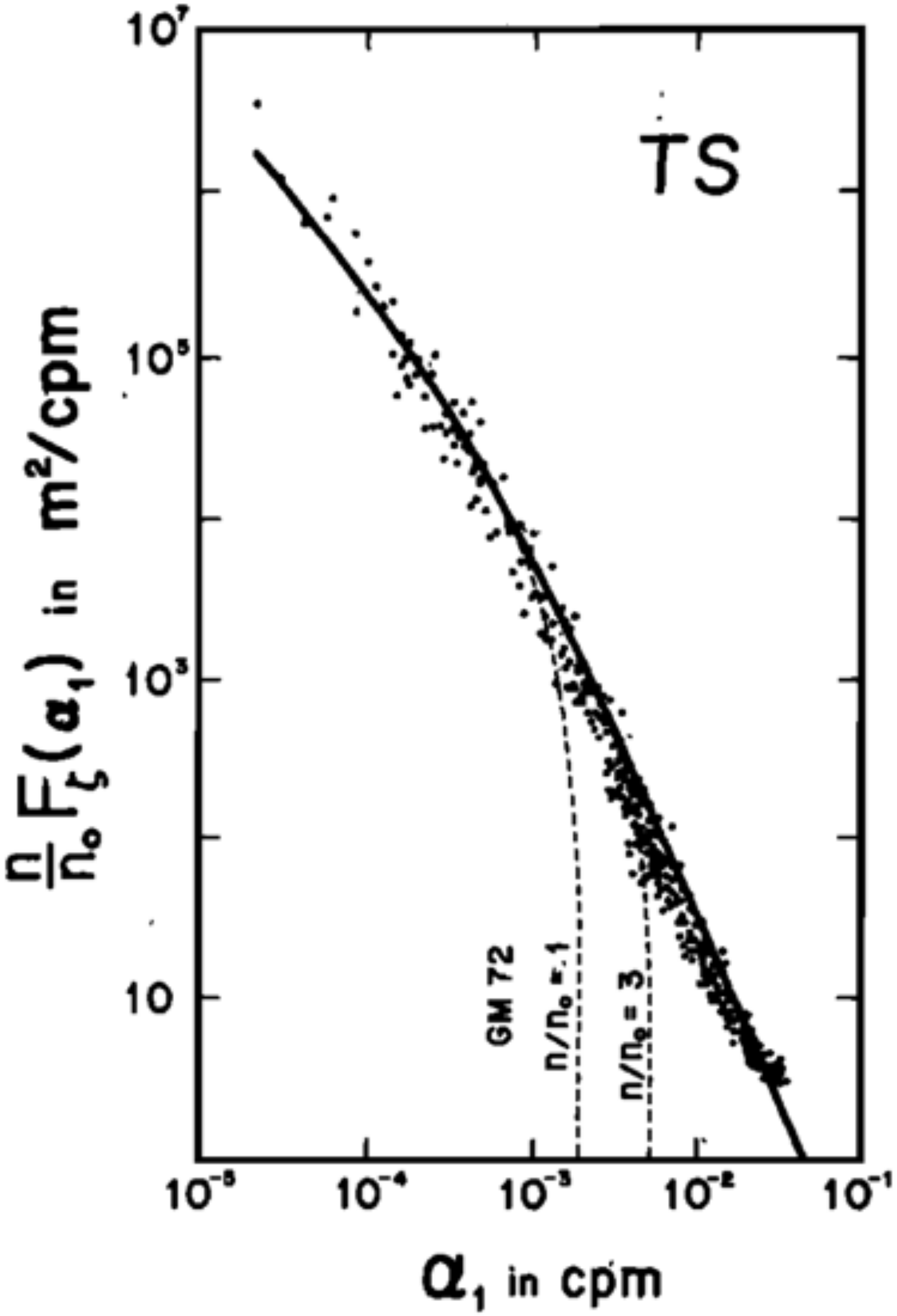
GM72 GM75 GM76 IWEX78 GK91

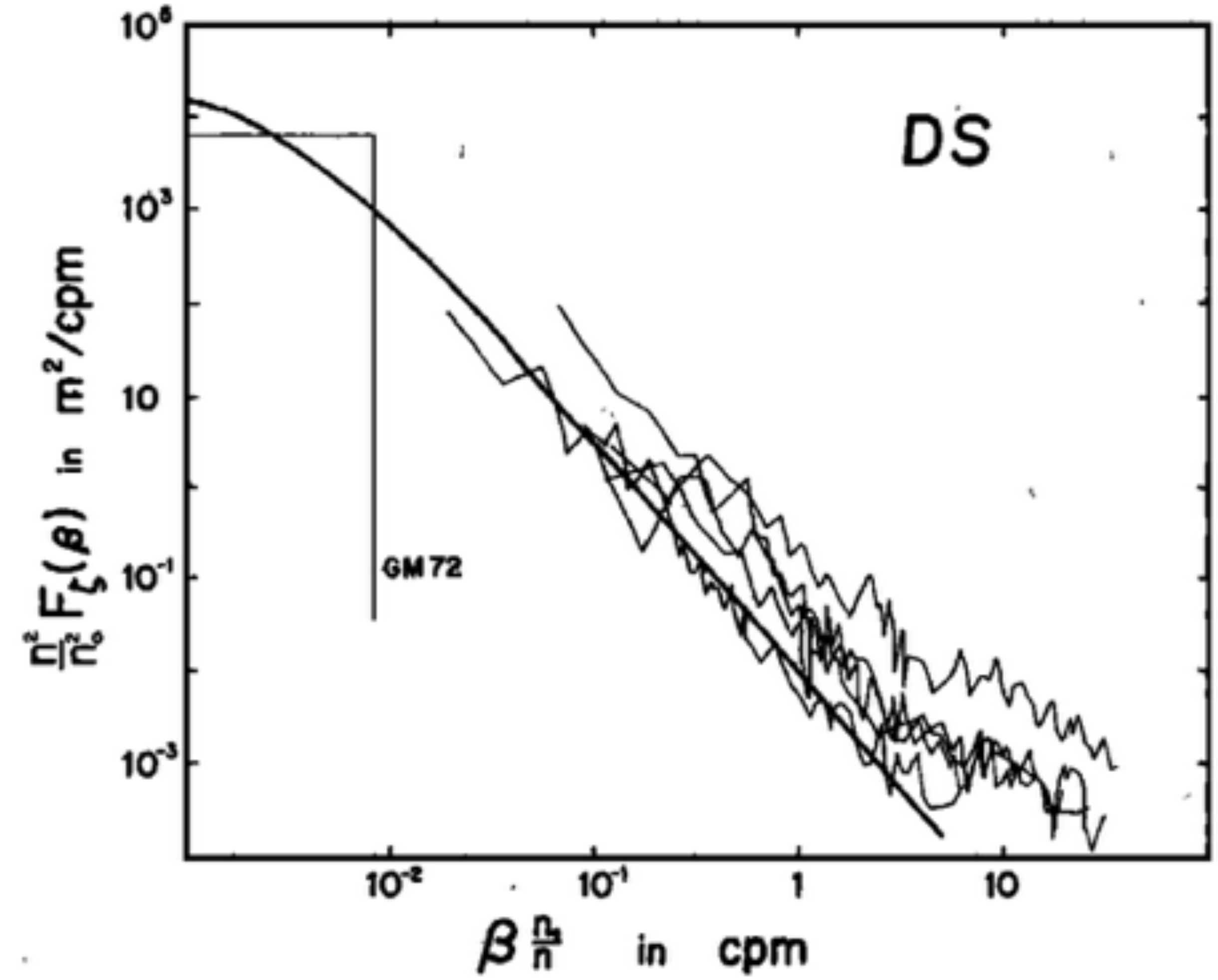


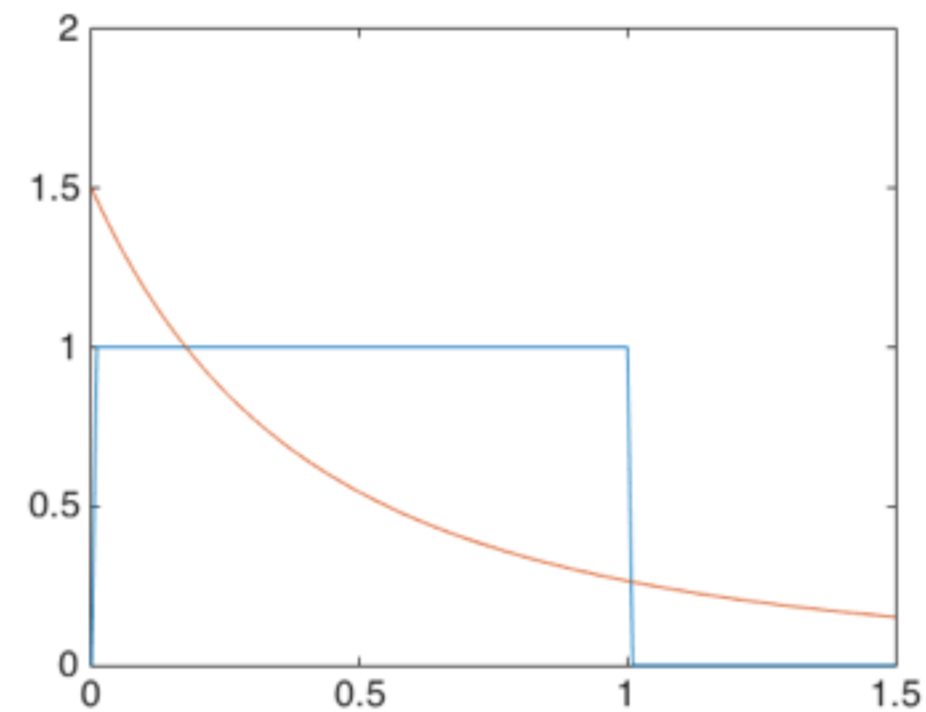
(Garrett and Munk, Geophys. Fl. Dyn., 1972)

GM72 **GM75** GM76 IWEX78 GK91

New measurements: towed and dropped spectra







GM72: $A(\lambda) = 1$ for $0 < \lambda \leq 1$

$A(\lambda) = 0$ for $\lambda \leq 0; \lambda > 1$

GM75: $A(\lambda) = (t-1)(1+\lambda)^{-t}, \quad t = 2.5$

GM72 **GM75** GM76 IWEX78 GK91

$$E(k,\omega) = E_0 A(k/k^*) B(\omega) k^{*-1}$$

$$E(m,\omega) = E_0 A(m/m^*) B(\omega) m^{*-1}$$

$$E(k,m) = \frac{2\pi^{-1} f E_0 n (m/m^*) A(m/m^*)}{n^2k^2 + f^2m^2}$$

for

$$k^* = j_i \pi (\omega^2 - f^2)^{1/2}$$

$$m^* = j_i \pi n$$

$$j^* = 6$$

$$A(\lambda) = (t-1)(1+\lambda)^{-t}$$

$$B(\omega) = 2\pi^{-1} f \omega^{-1} (\omega^2 - f^2)^{-1/2}$$

GM72 **GM75** GM76 IWEX78 GK91

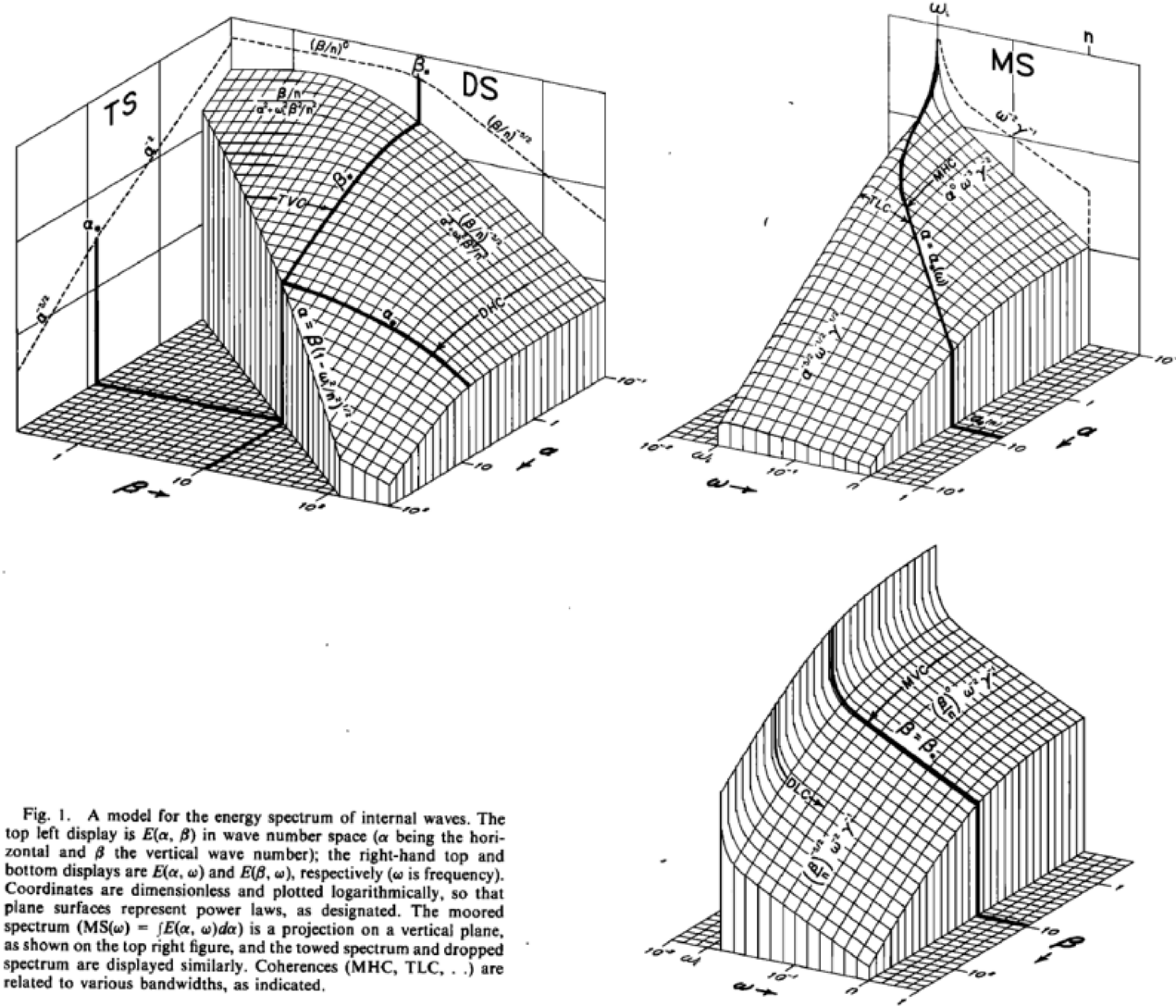
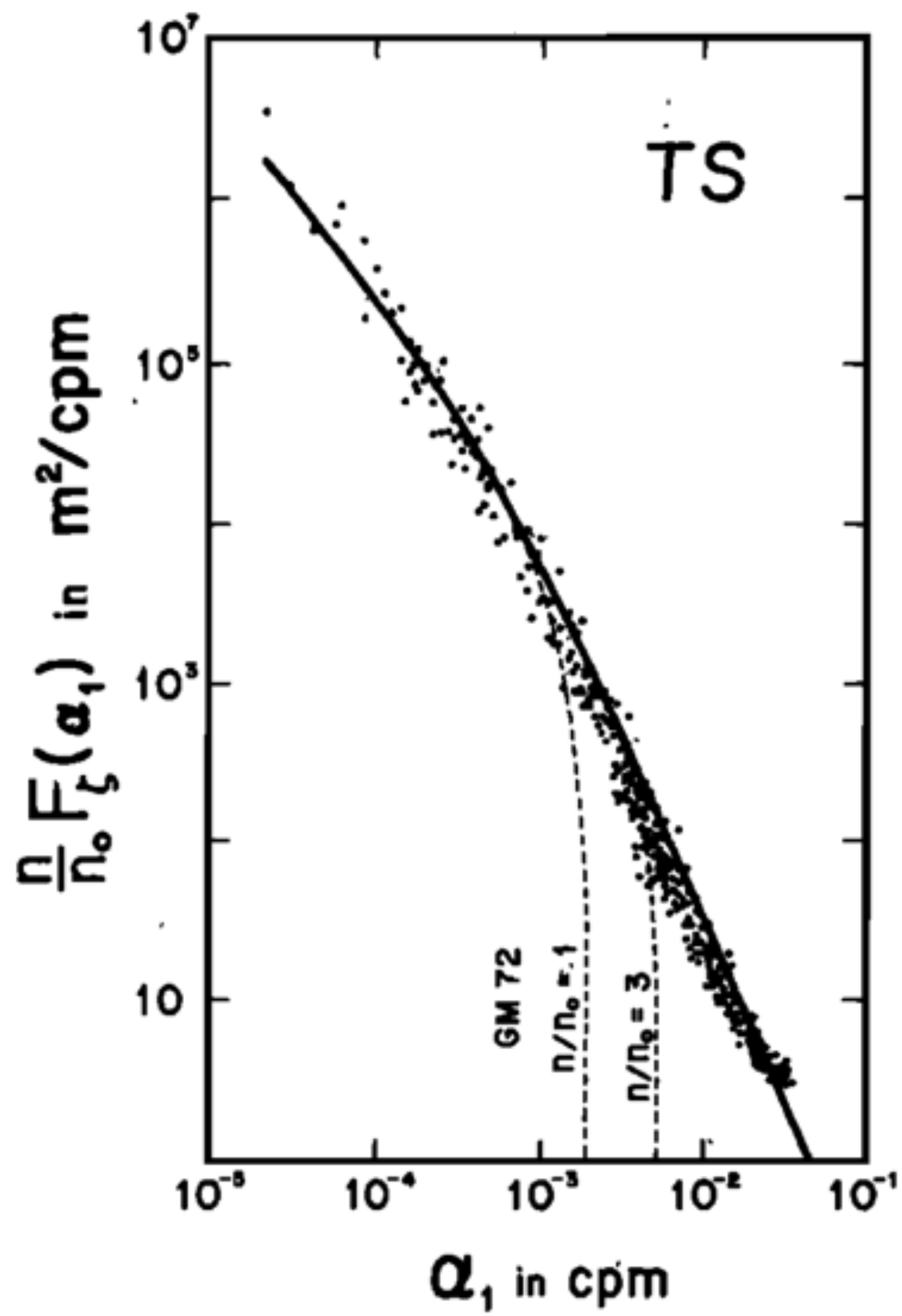
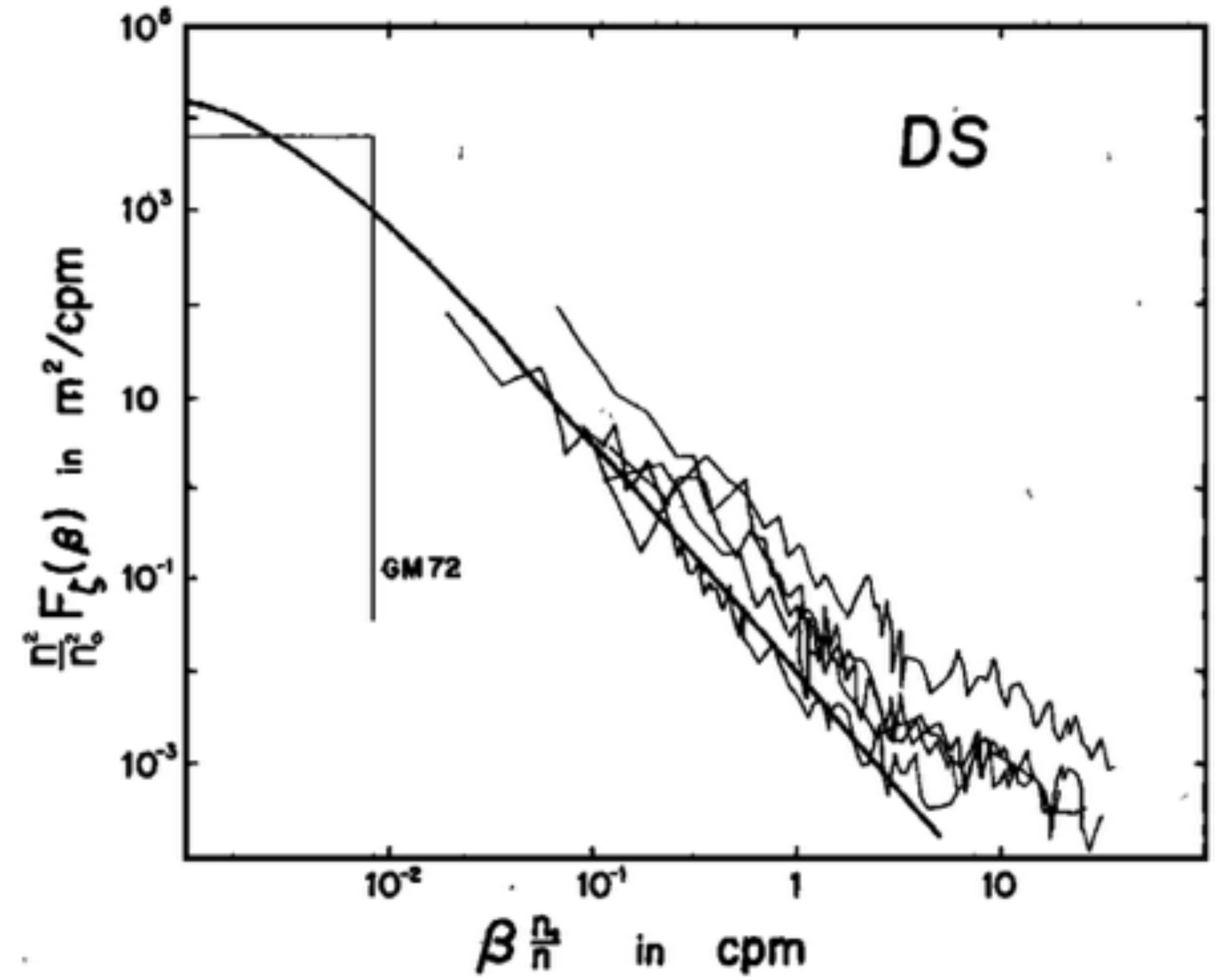


Fig. 1. A model for the energy spectrum of internal waves. The top left display is $E(\alpha, \beta)$ in wave number space (α being the horizontal and β the vertical wave number); the right-hand top and bottom displays are $E(\alpha, \omega)$ and $E(\beta, \omega)$, respectively (ω is frequency). Coordinates are dimensionless and plotted logarithmically, so that plane surfaces represent power laws, as designated. The moored spectrum ($\mathrm{MS}(\omega) = \int E(\alpha, \omega)d\alpha$) is a projection on a vertical plane, as shown on the top right figure, and the towed spectrum and dropped spectrum are displayed similarly. Coherences (MHC, TLC, . .) are related to various bandwidths, as indicated.

GM72 **GM75** GM76 IWEX78 GK91

New measurements: towed and dropped spectra





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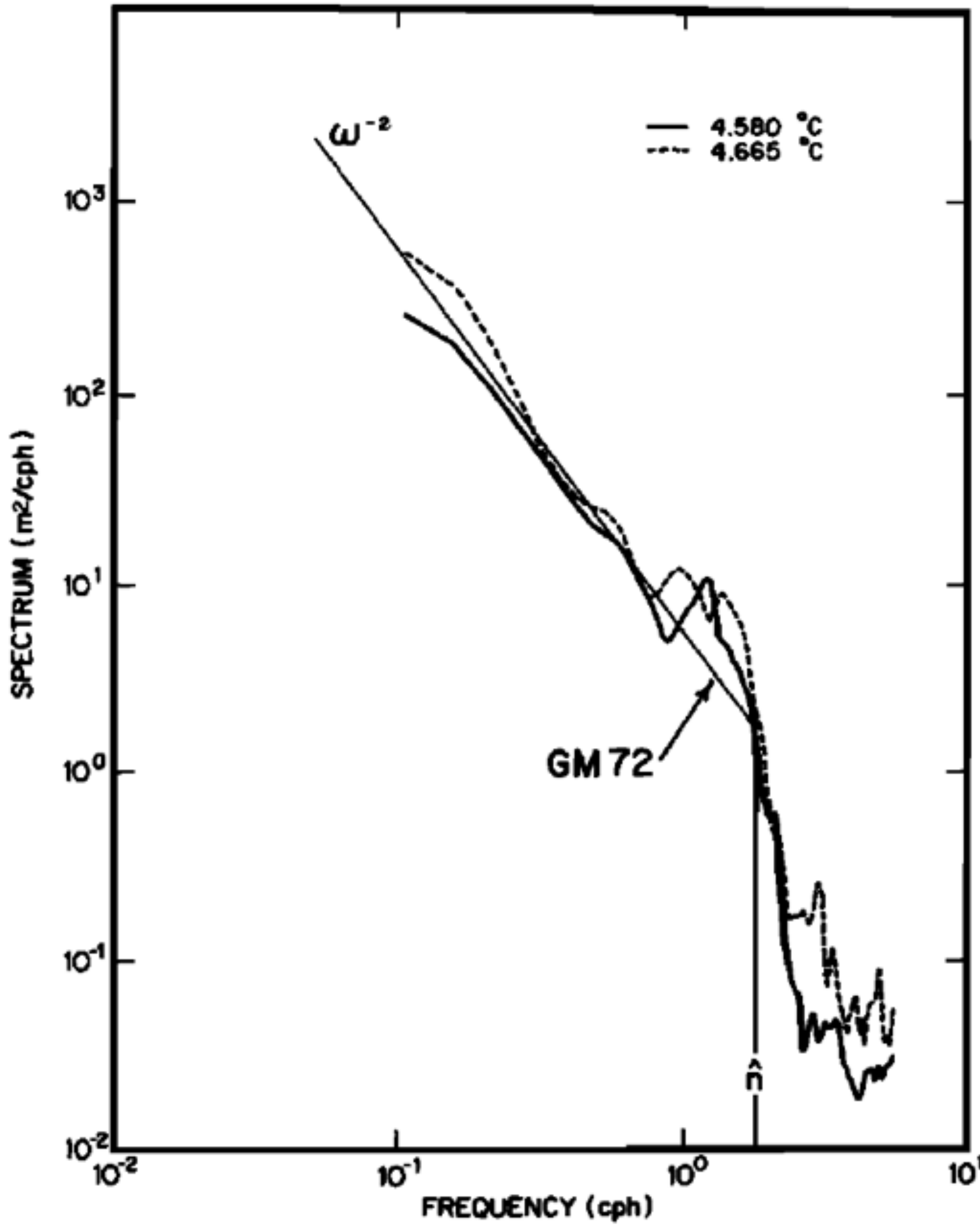

Cairns, JGR, 1975 (also reproduced in Garrett and Munk, JGR, 1975)

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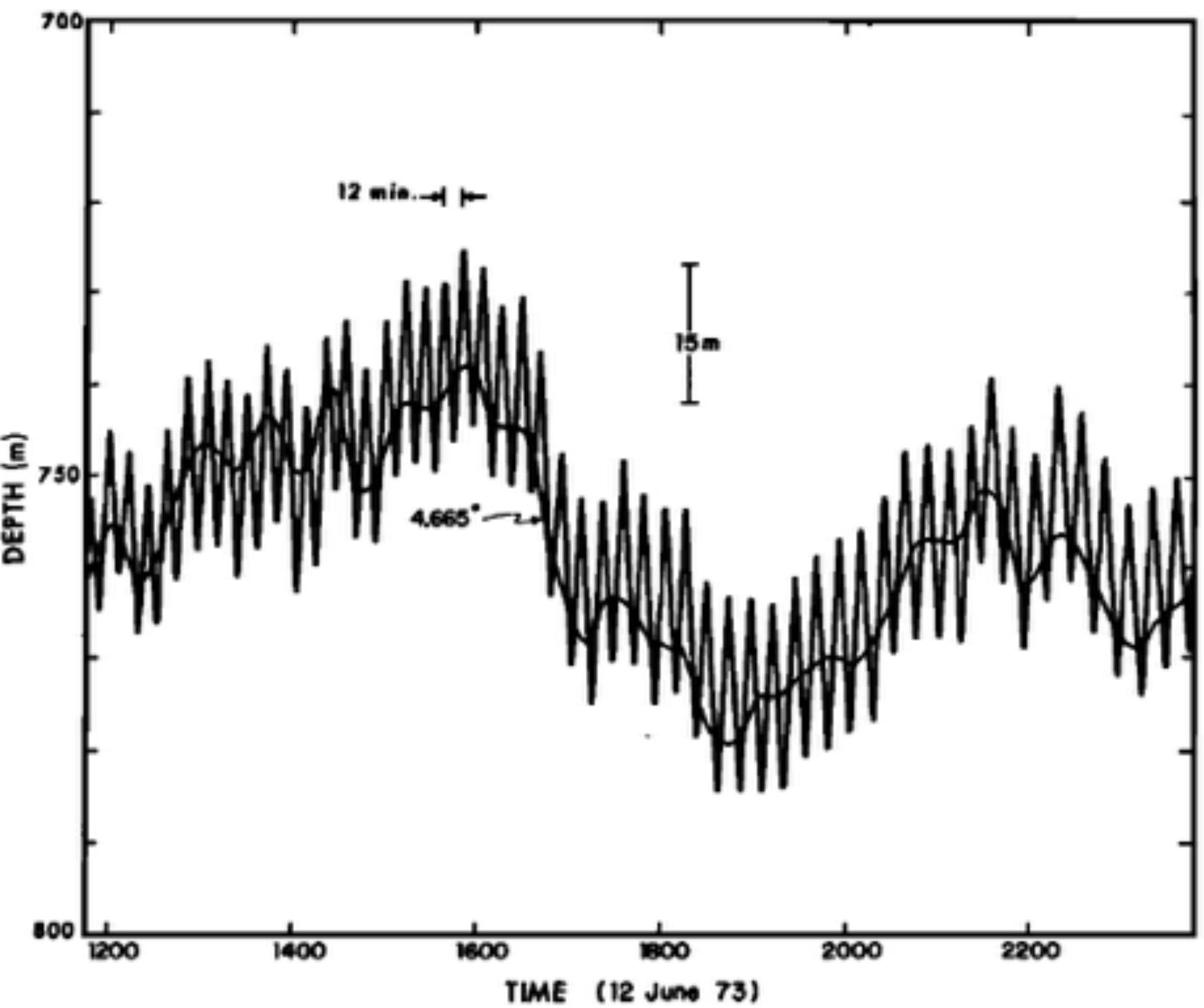
New measurements:



(Cairns, JGR, 1975)

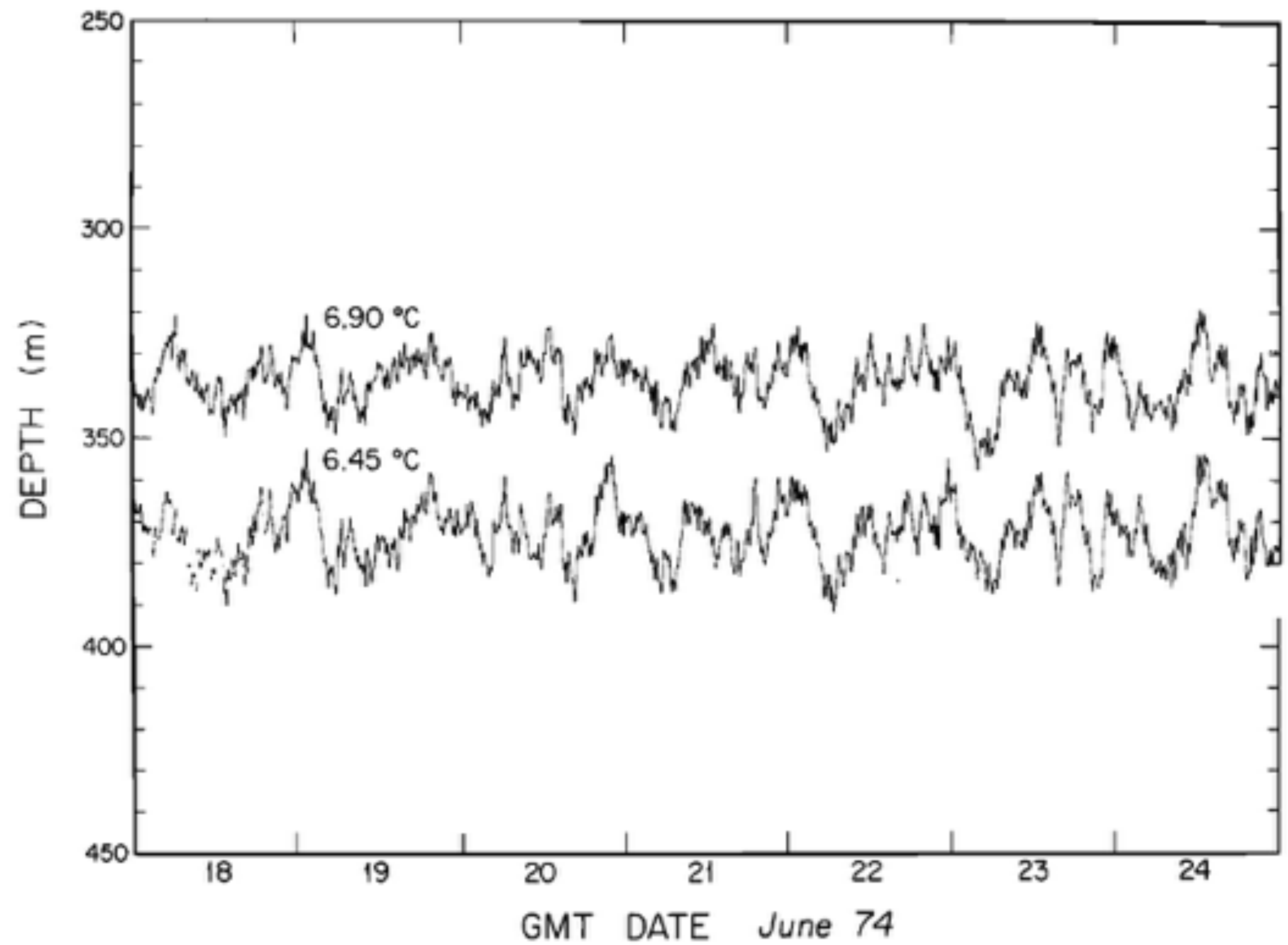


Fig. 1. Segments of the 6.90°C and 6.45°C isotherm records from Misery 1.

Cairns and Williams, JGR, 1976)

Internal Wave Observations From a Midwater Float, 2

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Naval Undersea Center, San Diego, California 92156

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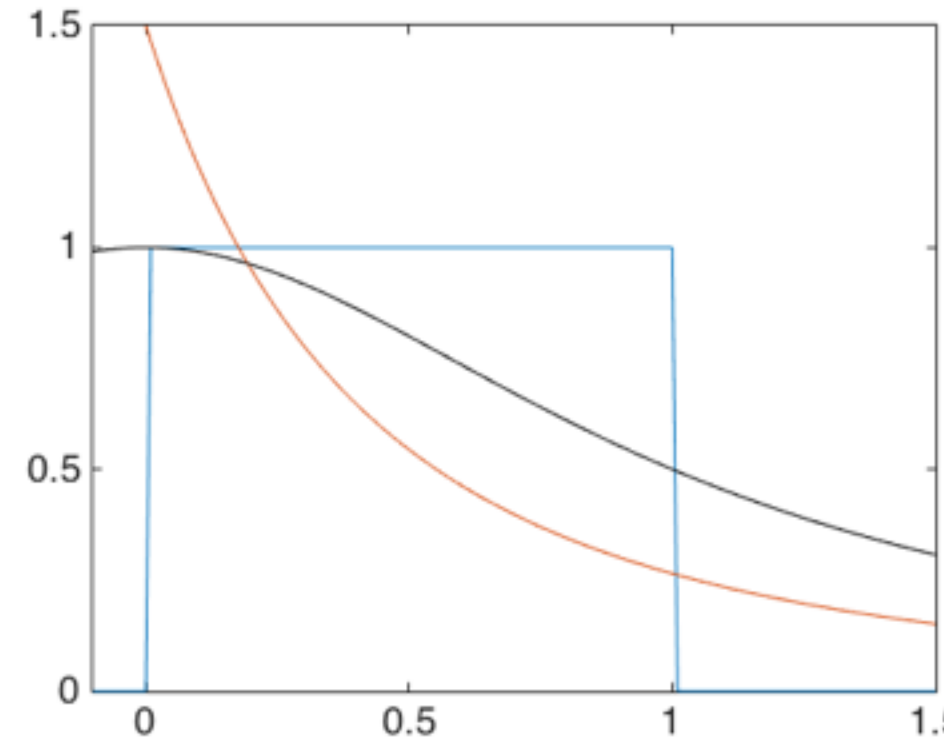
La Jolla, California 92037

MISERY AND THE GARRETT-MUNK MODEL

This section of the paper was suggested by Christopher Garrett and Walter Munk and was written in collaboration with them.



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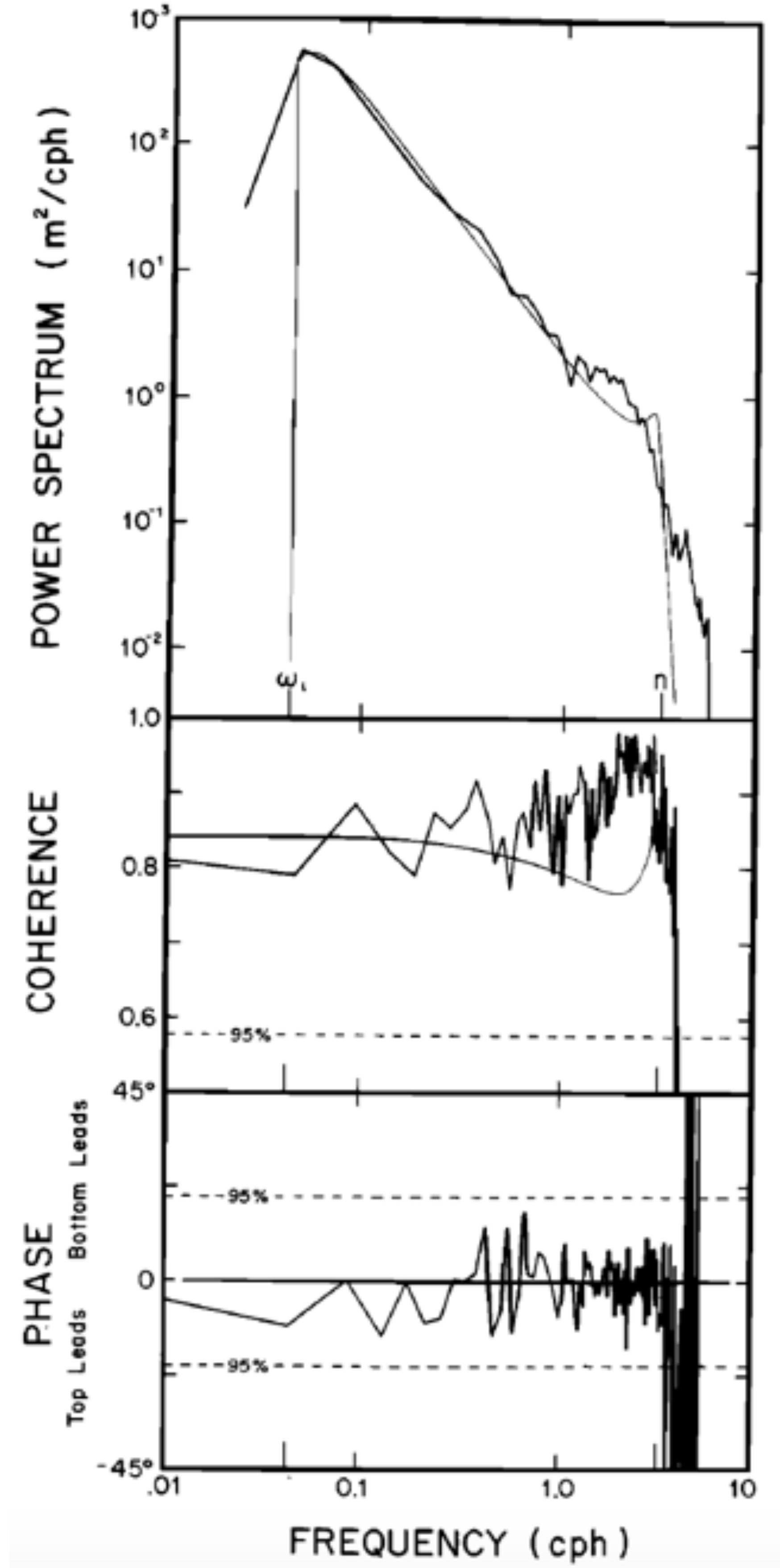
$A(\lambda) = 0 \text{ for } \lambda \leq 0; \lambda > 1$

GM75: $A(\lambda) = (t-1)(1+\lambda)^{-t}, \quad t = 2.5$

GM76: $A(\lambda) \sim (1+\lambda^2)^{-t/2}, \quad t = 2$

GM72 GM75 **GM76** IWEX78 GK91



Let's look at one of our assumptions! Stationarity.

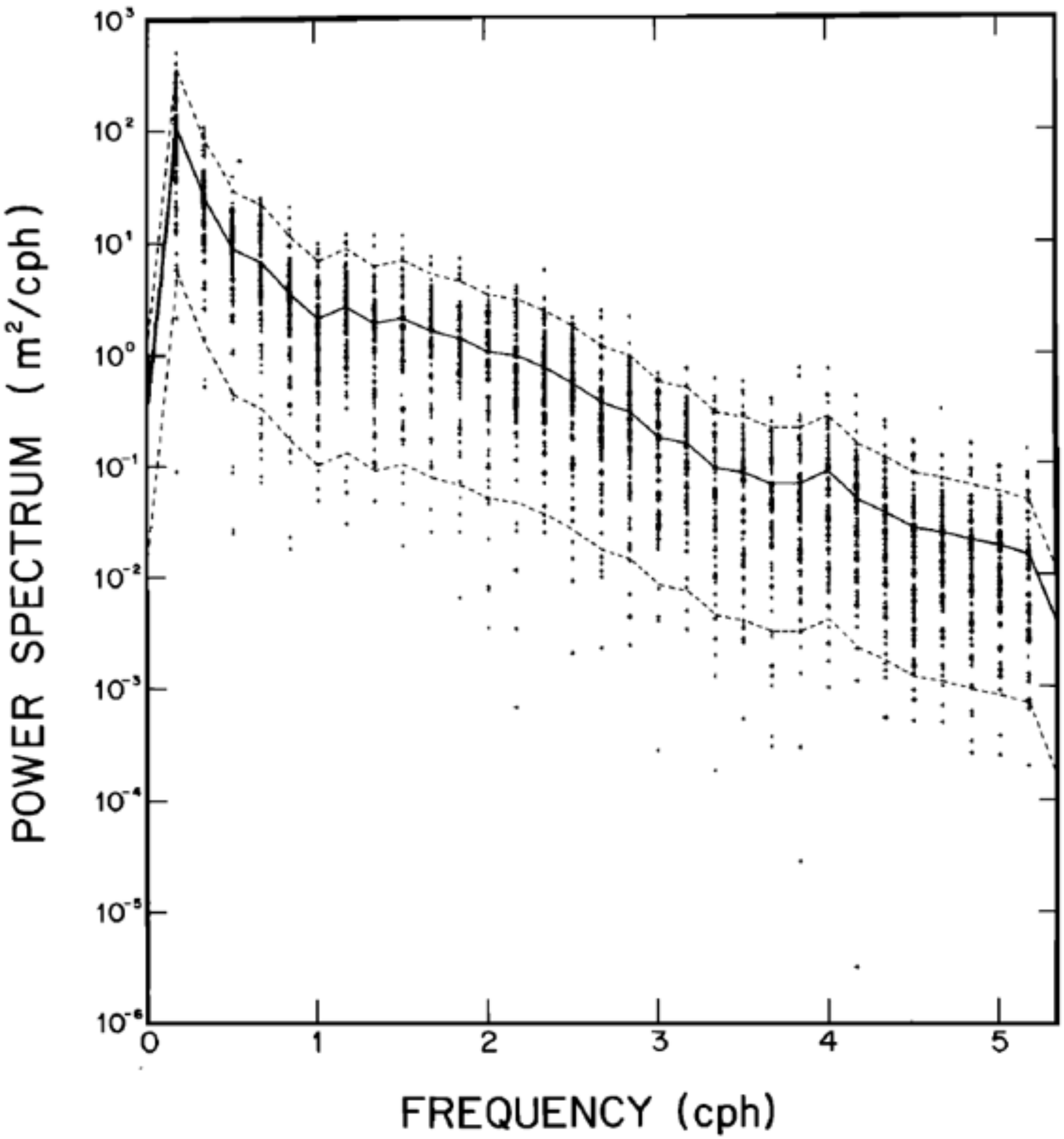
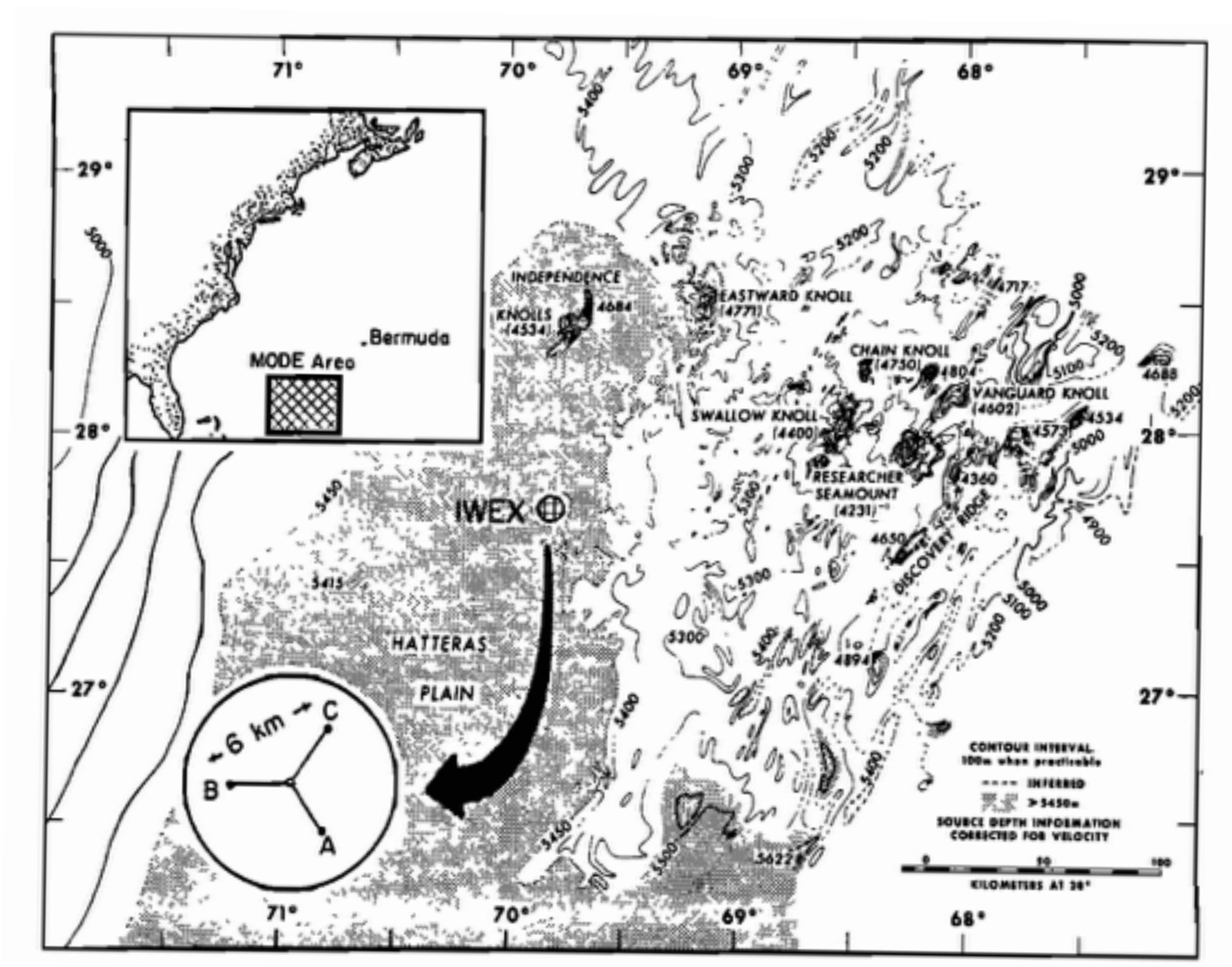
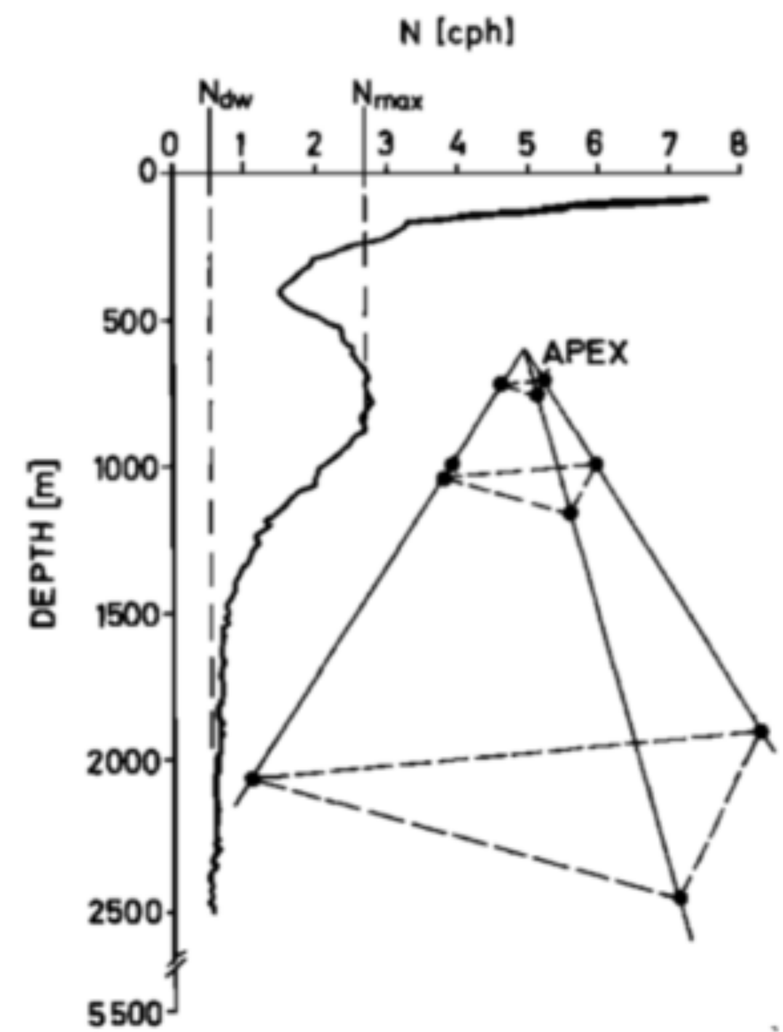


Fig. 6. Displacement spectra of 5.8-hour segments of the 6.60°C isotherm record. There are 74 estimates at each frequency, each with 2 d.f. The solid line is the average spectrum, and the dashed lines are the 90% confidence limits.

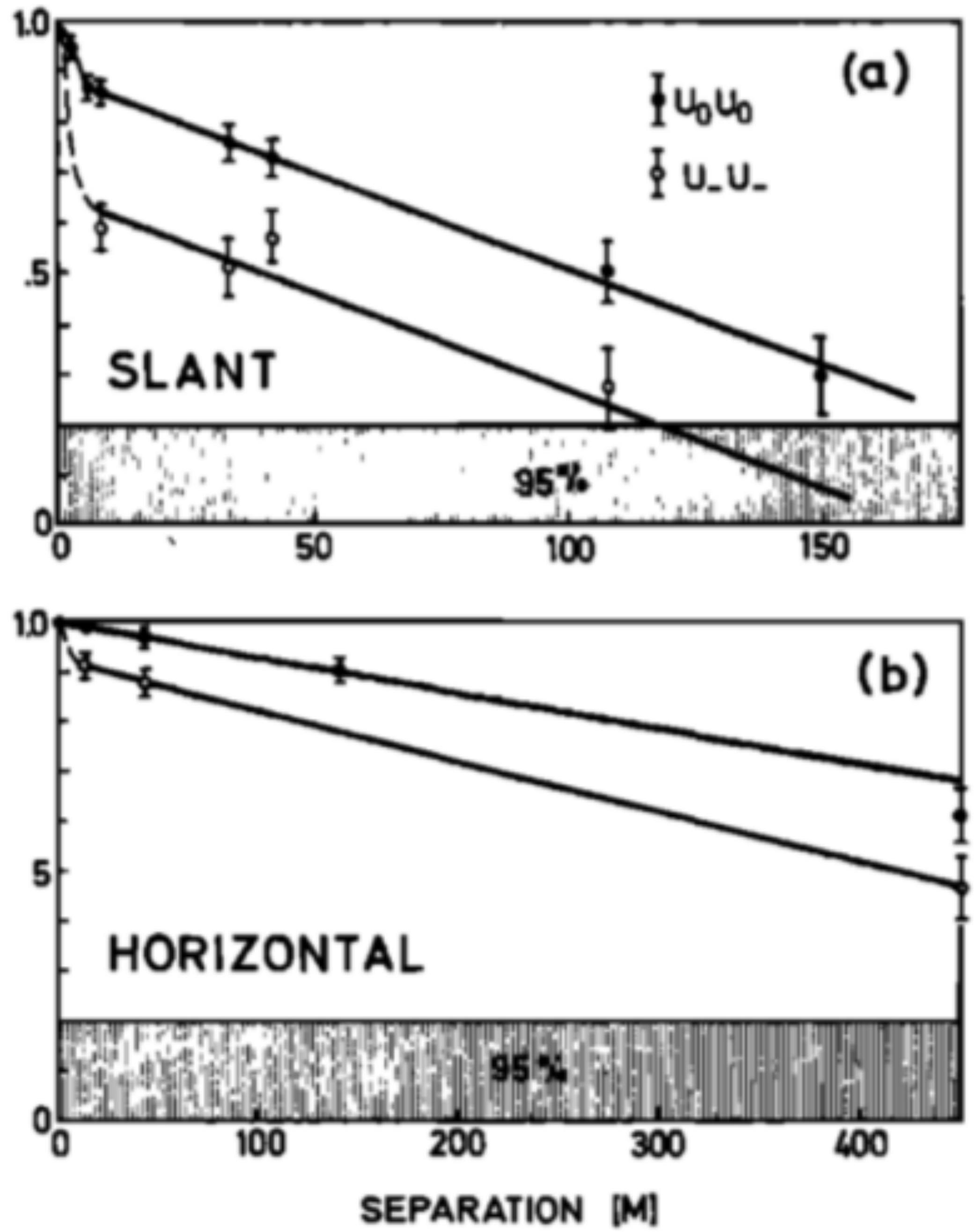


(Briscoe, JGR, 1975)



(Müller, Olbers, and Willebrand, JGR, 1978)

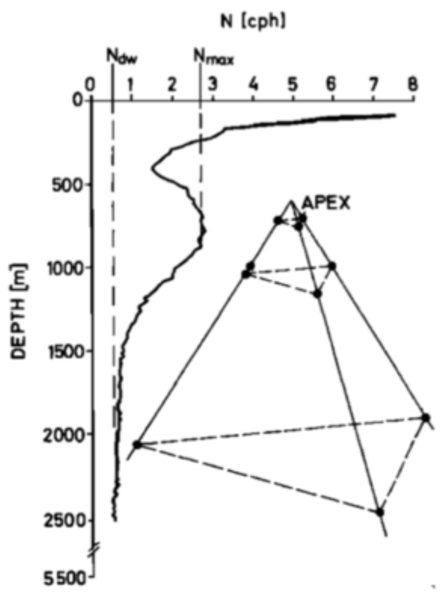
Deployed for 40 days in late 1973 in the main thermocline.
Sensor spacings 1.4-1600 m (horizontal), 2.1-1447 m (vertical)



(Müller, Olbers, and Willebrand, JGR, 1978)

Expect linear loss of coherence with increased separation — effect of fine structure at small scales!

Test GM assumptions



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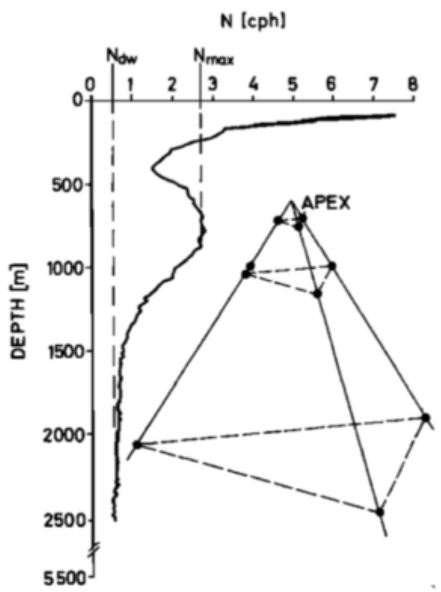

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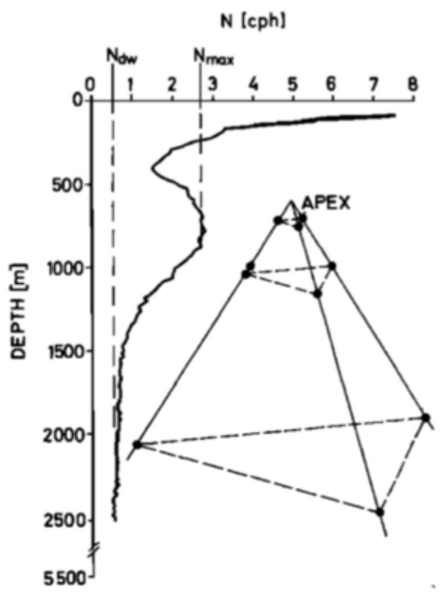

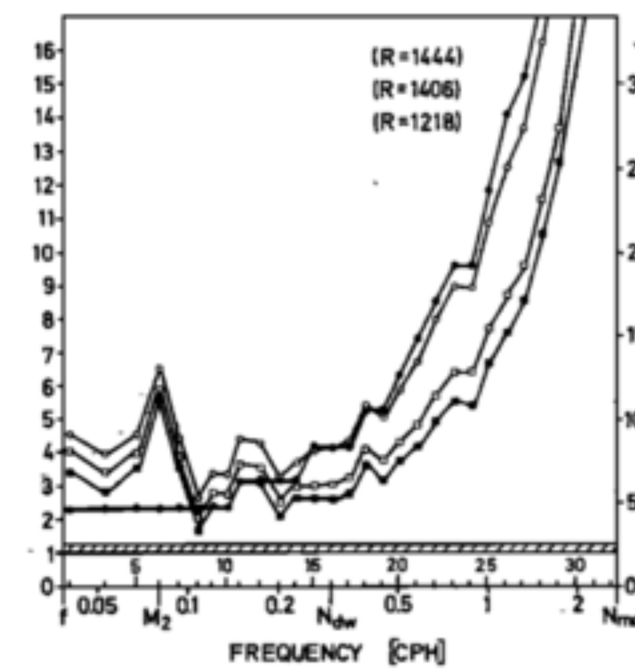

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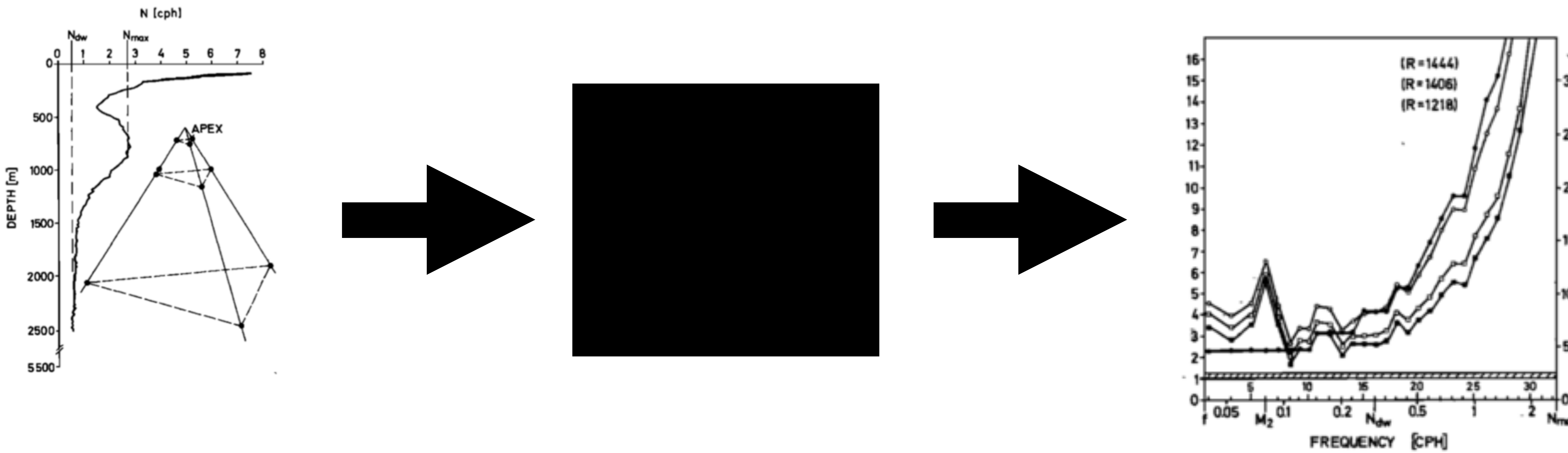
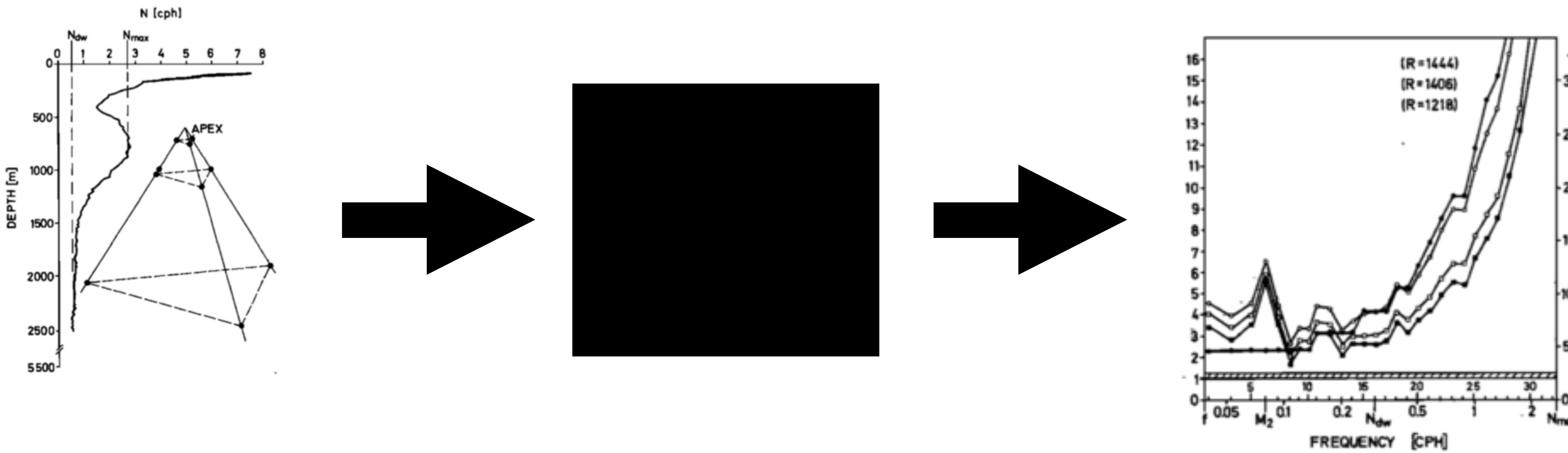


$$R_{mn}e^{i\gamma_{mn}} = (C_{mn} + iQ_{mn})(C_{mm}C_{nn})^{-1/2}$$

Test GM assumptions



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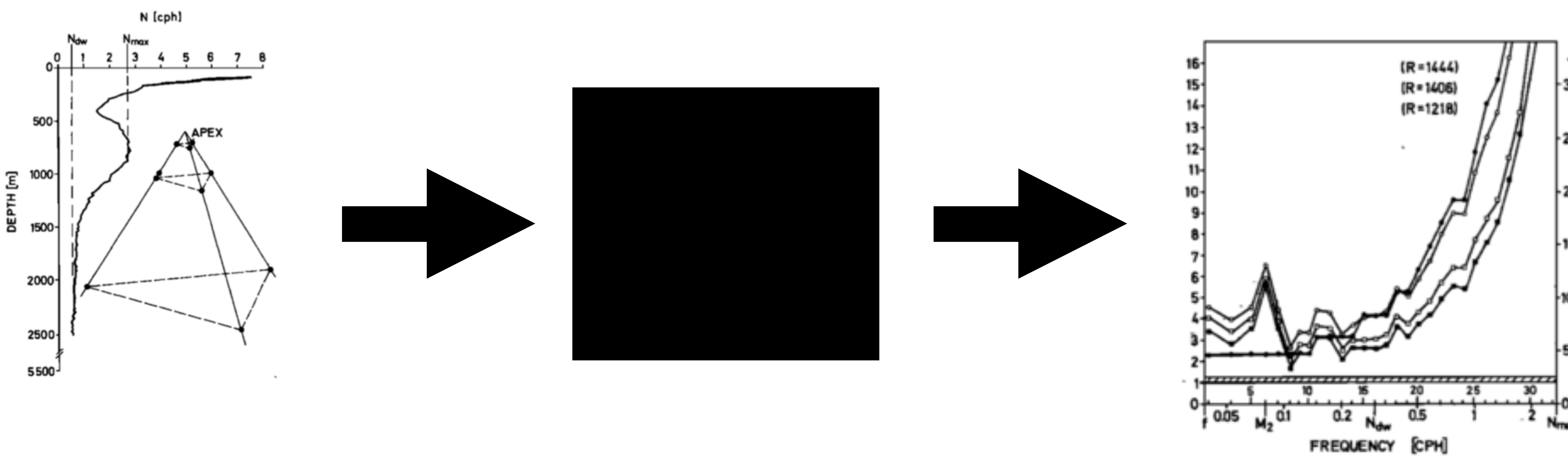
$$R_{mn} = 0$$ (no correlations; “zero model”)

Test GM assumptions



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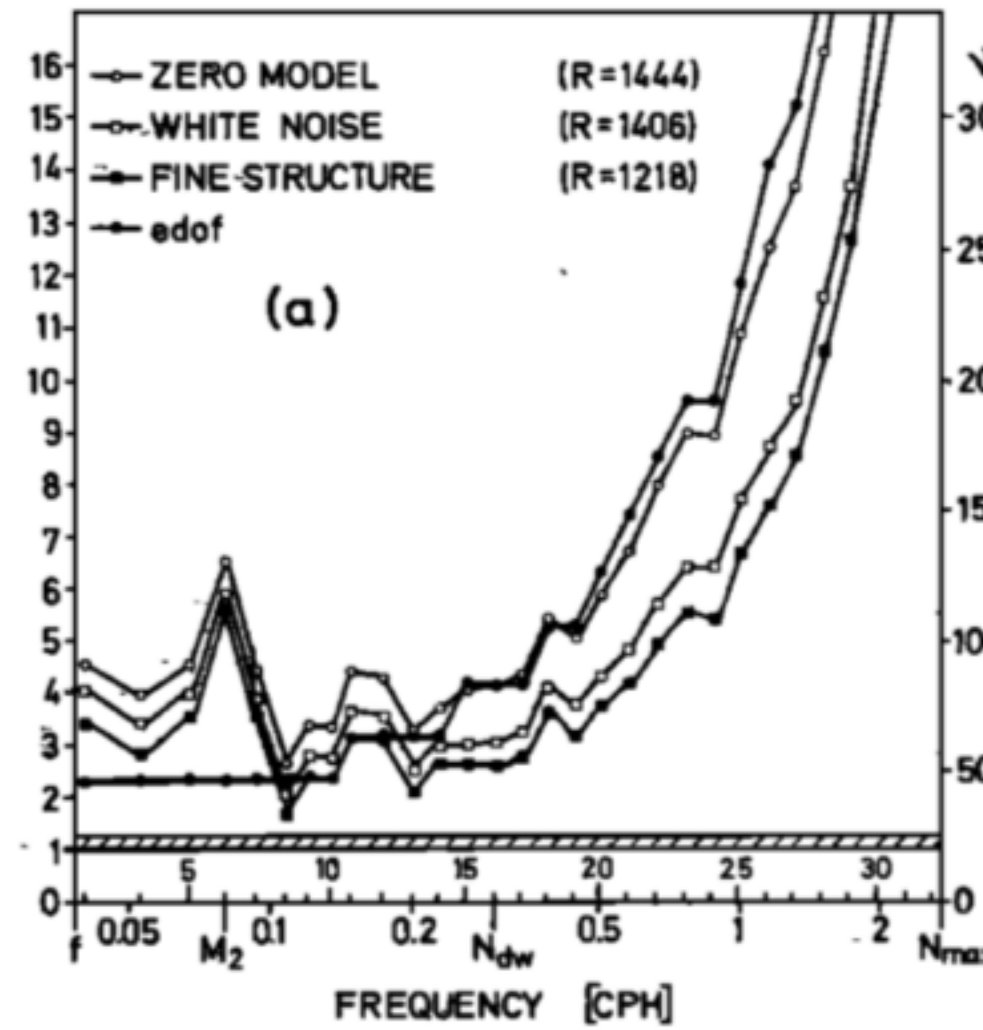
$R_{mn} = 0$ (no correlations; “zero model”)

$\Delta^2 = \frac{1}{L}\sum (R_{mn})^2/\mathrm{Var}(R_{mn})$ (expect =1)

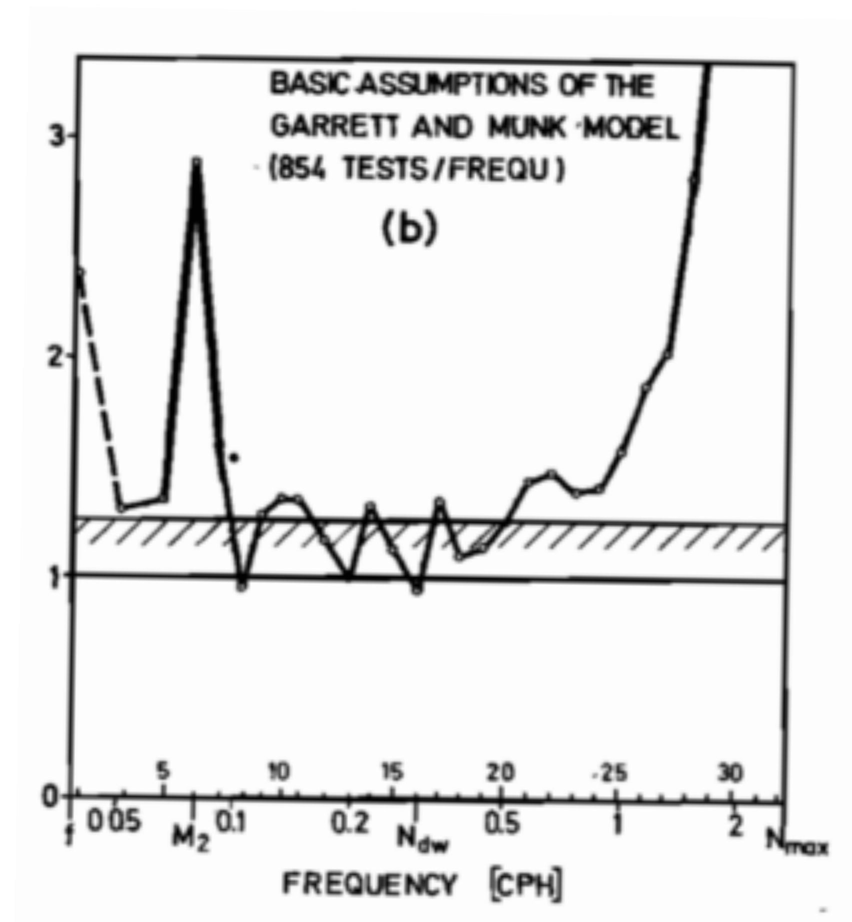
Test GM assumptions

Zero model: all coherences vanish
White noise: all coherences vanish except for auto spectra
Fine structure: all coherences from vertically separated instruments vanish



GM72 GM75 GM76 **IWEX78** GK91

Test GM assumptions



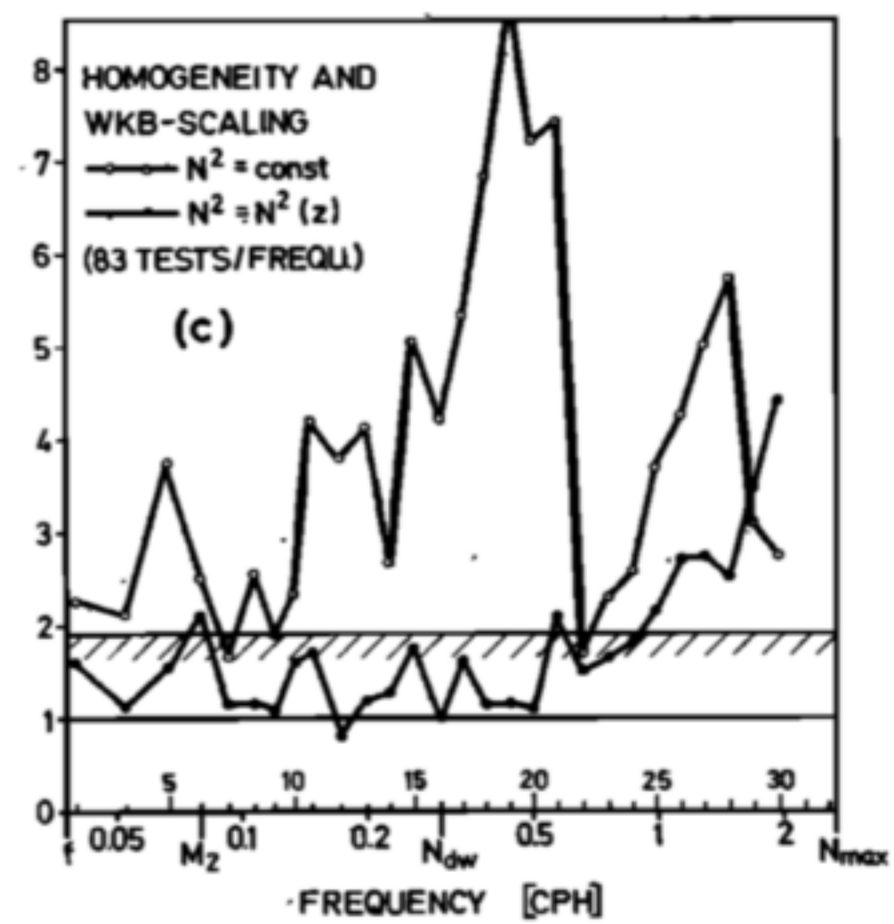
Test GM assumptions

1. statistically stationary
2. horizontally homogenous
3. linear (superposition of waves)
4. well-approximated by WKB theory (spectral gap between internal waves and jets/eddies)
5. horizontally isotropic
6. vertically symmetric

Test GM assumptions

2. horizontally homogenous

4. well-approximated by WKB theory (spectral gap between internal waves and jets/eddies)

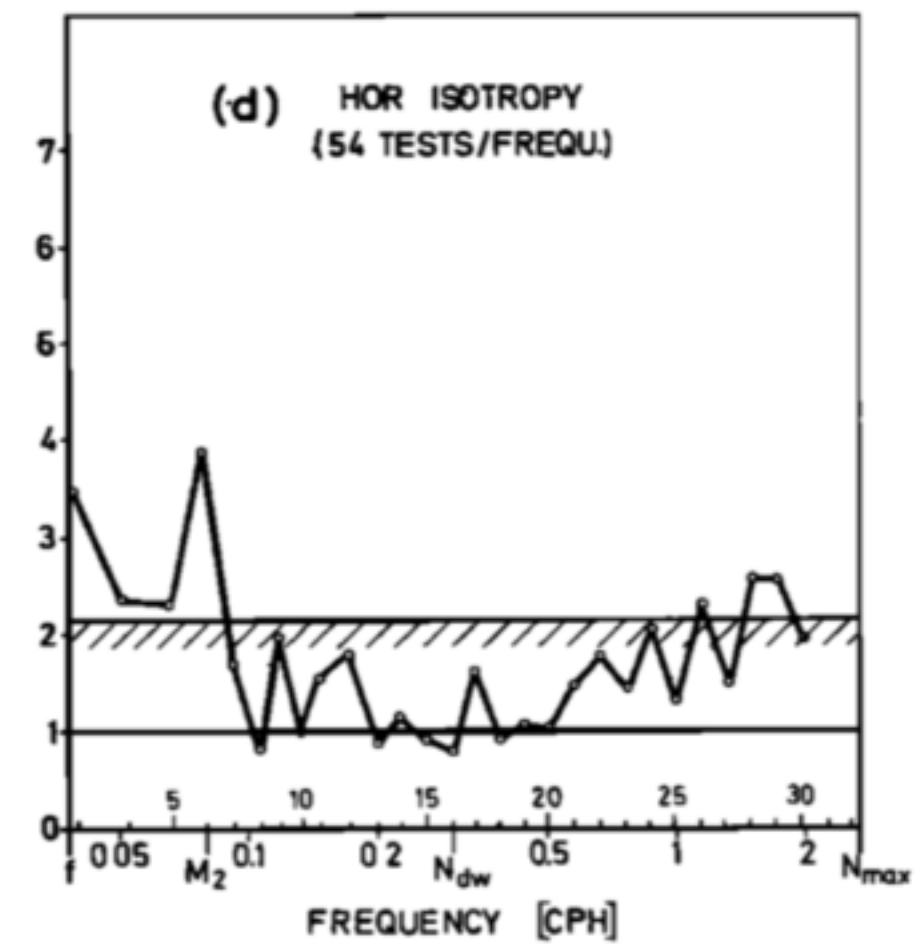


Test: coherences from different instruments are similar

GM72 GM75 GM76 **IWEX78** GK91

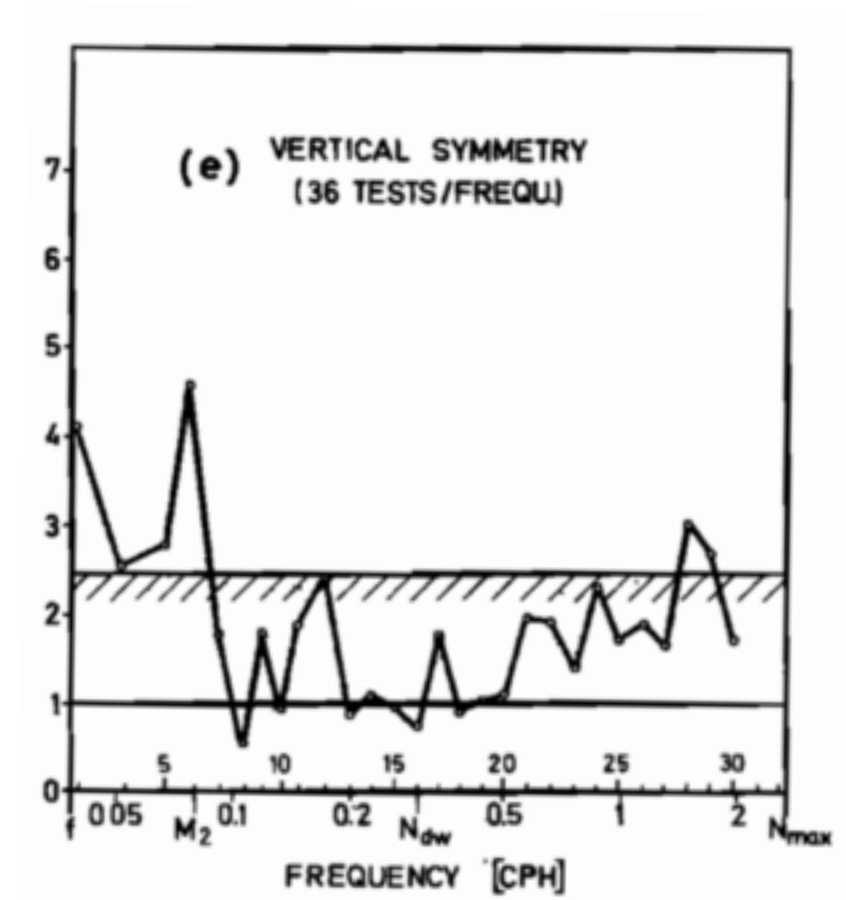
Test GM assumptions

5. horizontally isotropic



Test: coherences do not change with rotation (about vertical axis)

Test GM assumptions

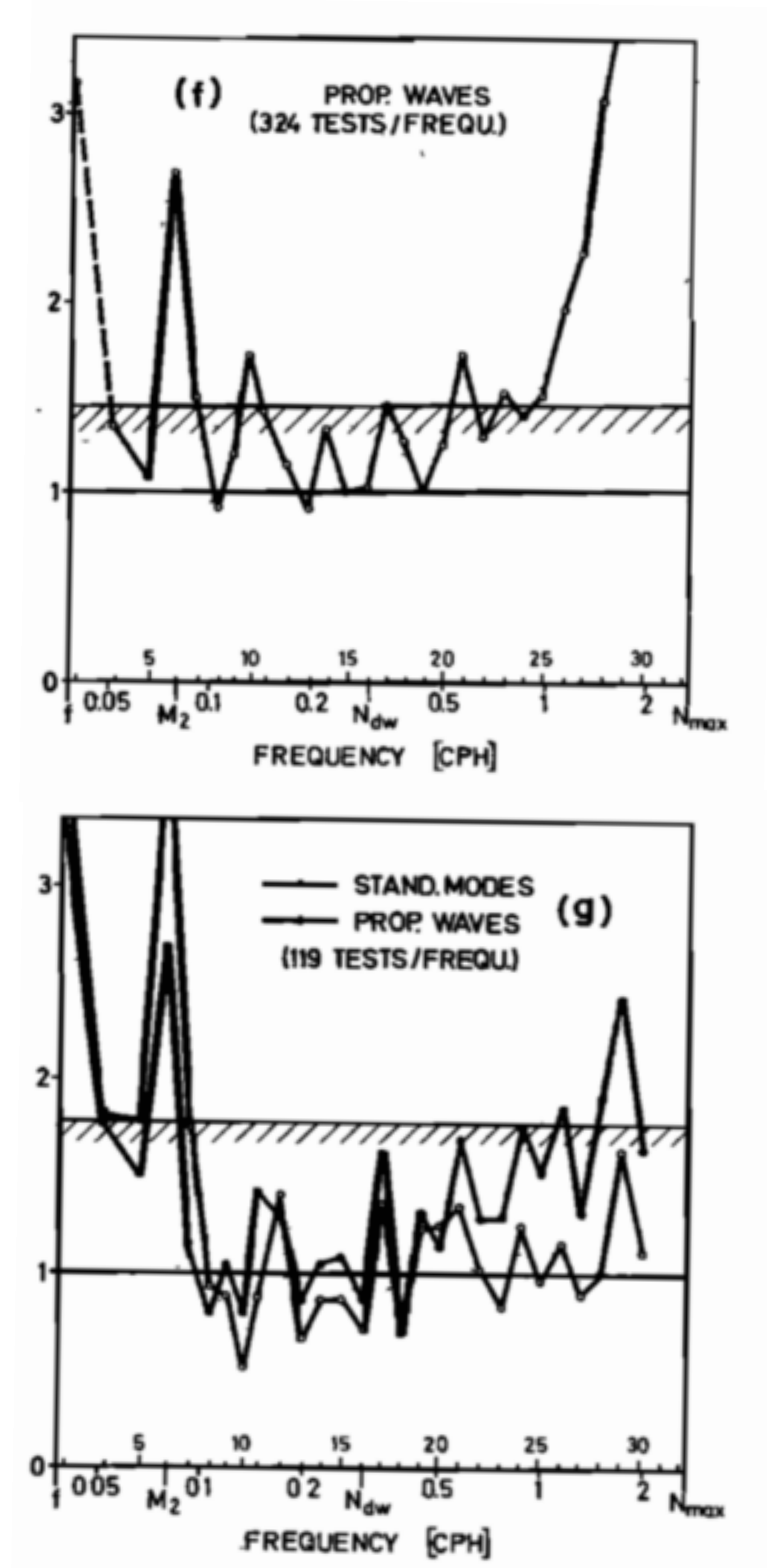


6. vertically symmetric

Test: coherences do not change with reflection (about horizontal axis)

Test GM assumptions

3. linear (superposition of waves)



IWEX model

$$E(k,\omega,\phi) = B(\omega)A(k,k^*)S(\phi,\omega)$$

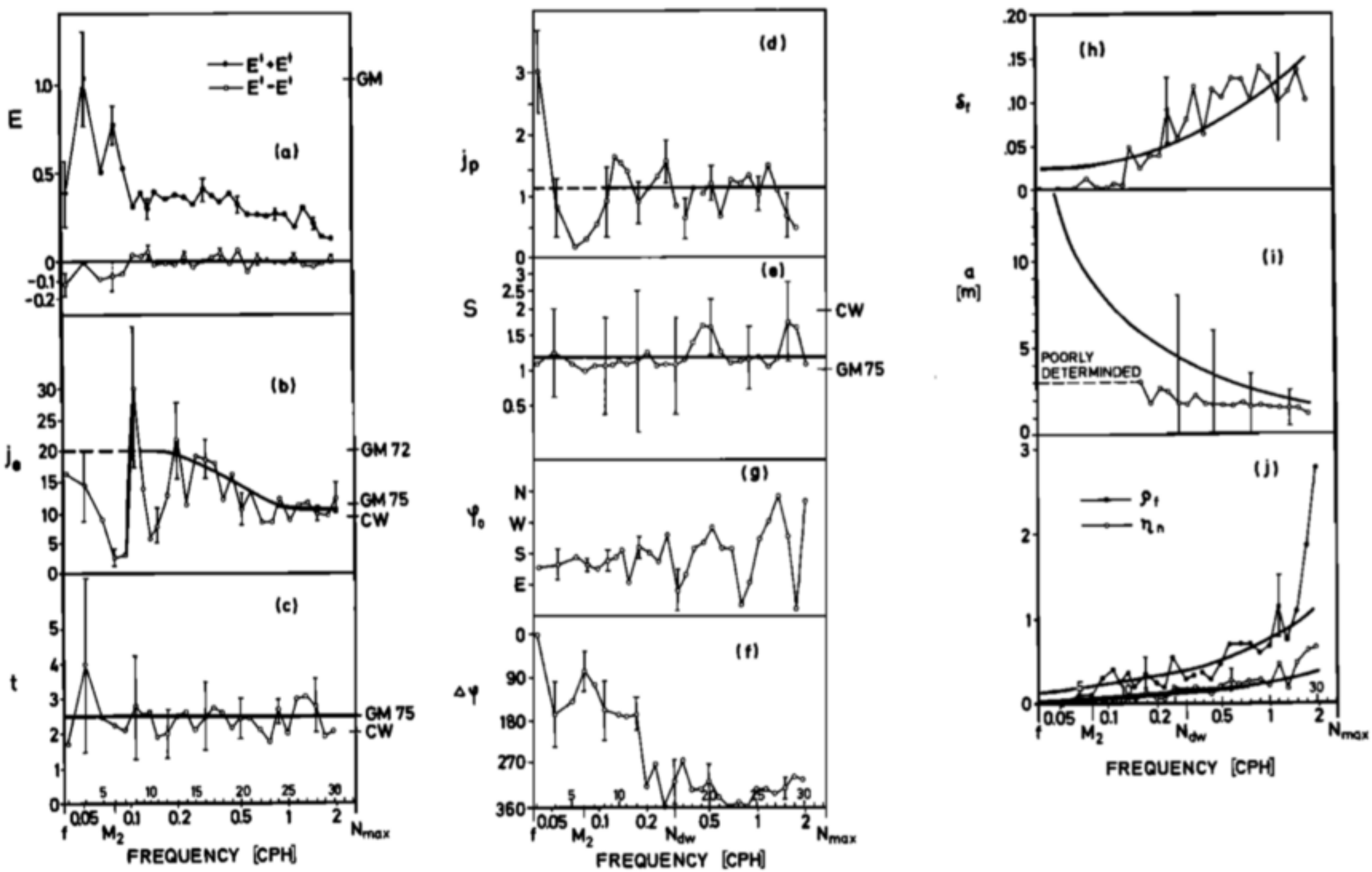
IWEX model

$$E(k,\omega,\phi) = B(\omega)A(k,k^*)S(\phi,\omega)$$

$$A(k,k^*) = I(t,s)\left\{1+\left(\frac{k-k_P}{k^*}\right)^s\right\}^{-t/s}, \text{ for } k \geq k_P$$

GM72 GM75 GM76 **IWEX78** GK91

IWEX model



IWEX model

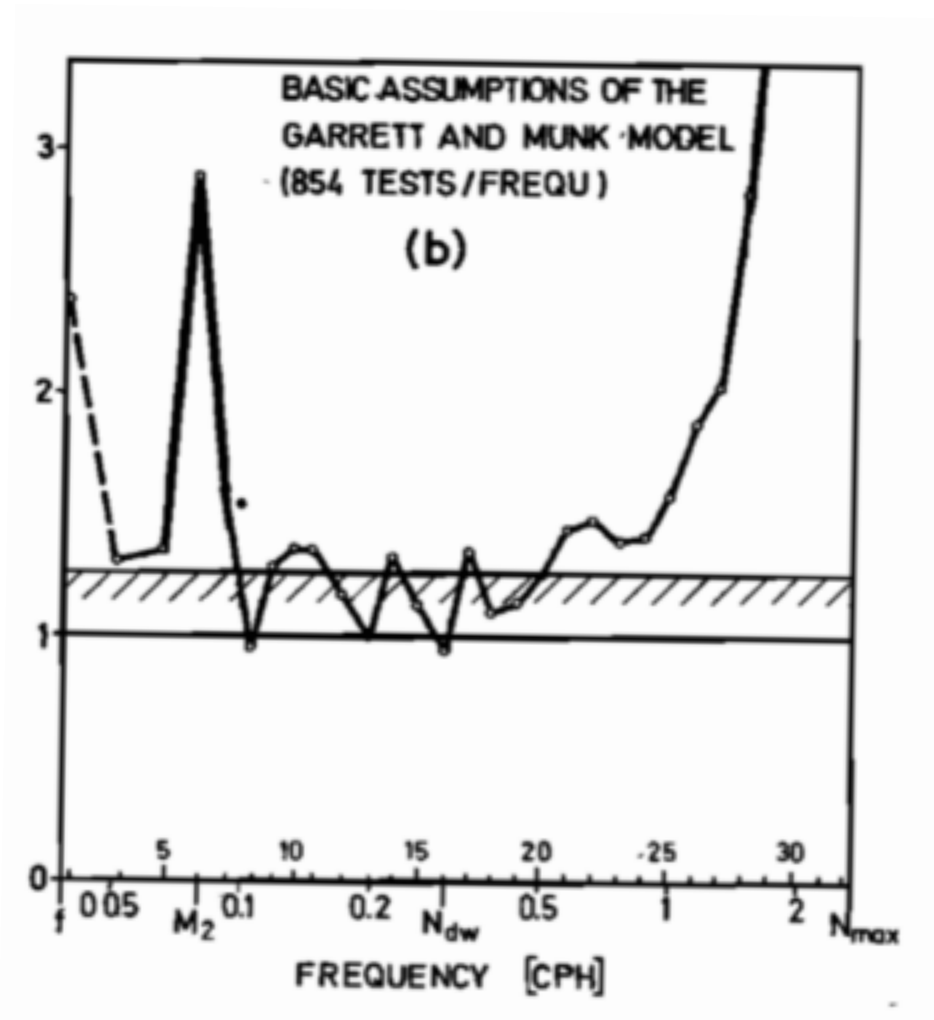
$$E(k,\omega,\phi) = B(\omega)A(k,k^*)S(\phi,\omega)$$

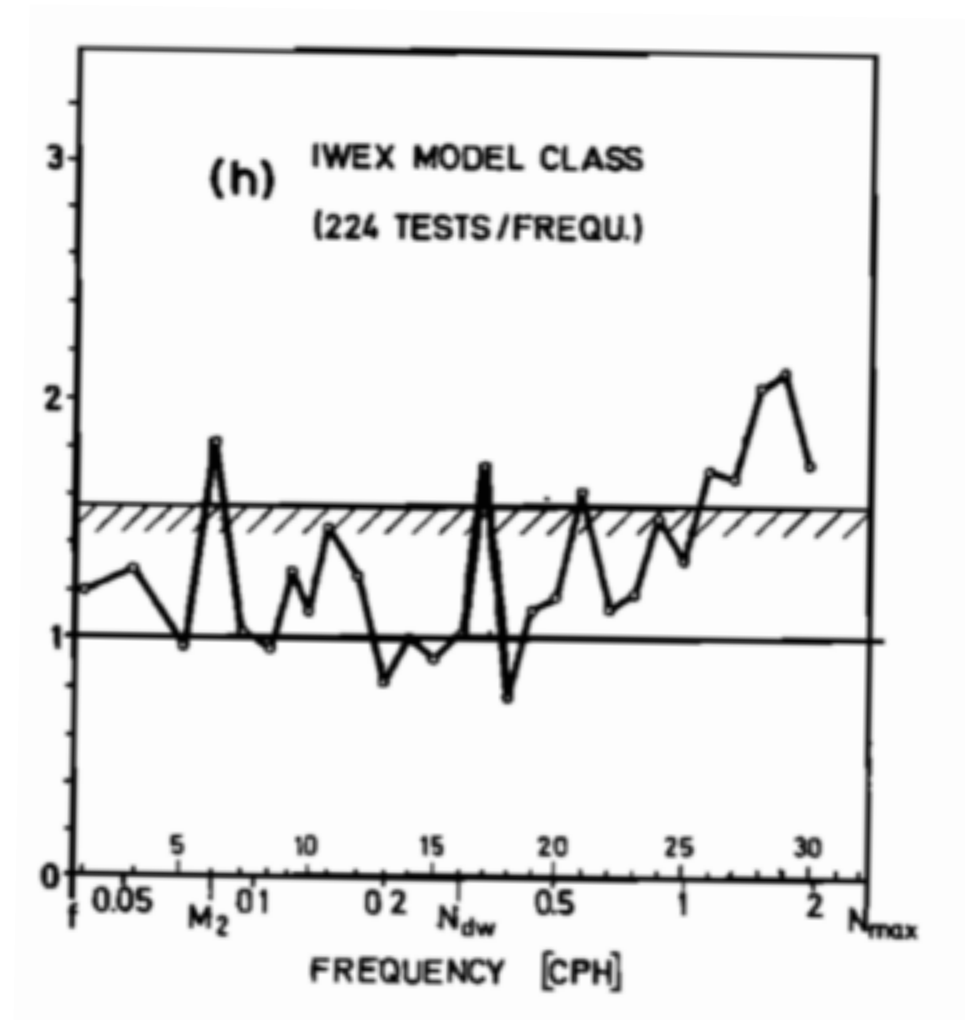
$$A(k,k^*) = I(t,s)\left\{1+\left(\frac{k-k_P}{k^*}\right)^s\right\}^{-t/s}, \text{ for } k \geq k_P$$

$$S(\phi,\omega) = \frac{\Gamma^2(p+1)2^{2p}}{2\pi\Gamma(2p+1)}\cos^{2p}\left(\frac{\phi-\phi_0}{2}\right)$$

$$B(\omega) = B(E^+,E^-,\delta_f,a,\rho_f,\eta_n)$$

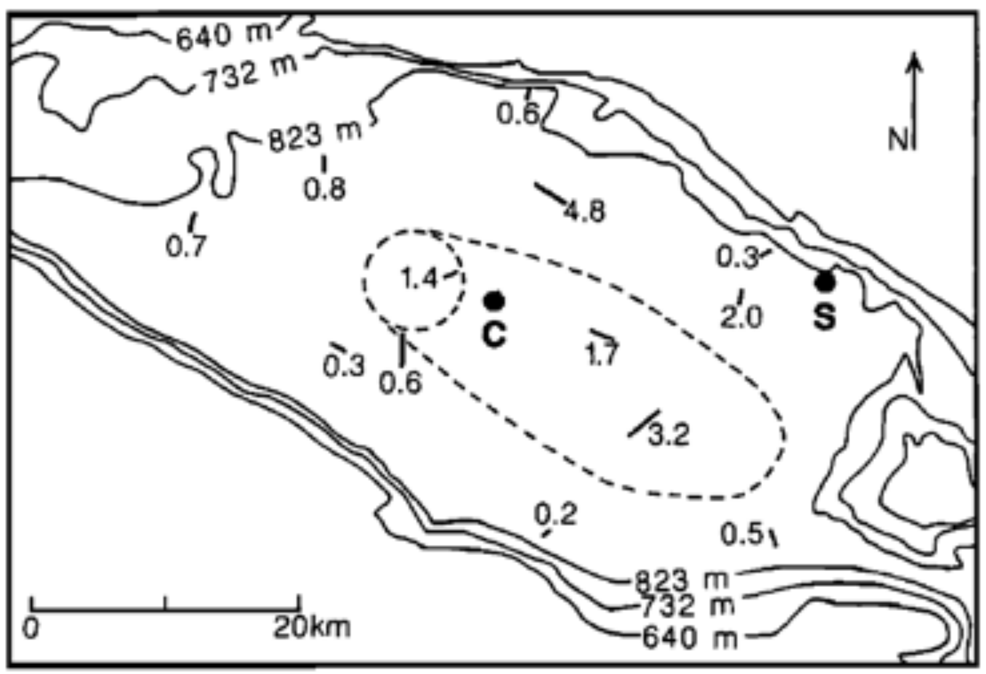
IWEX model

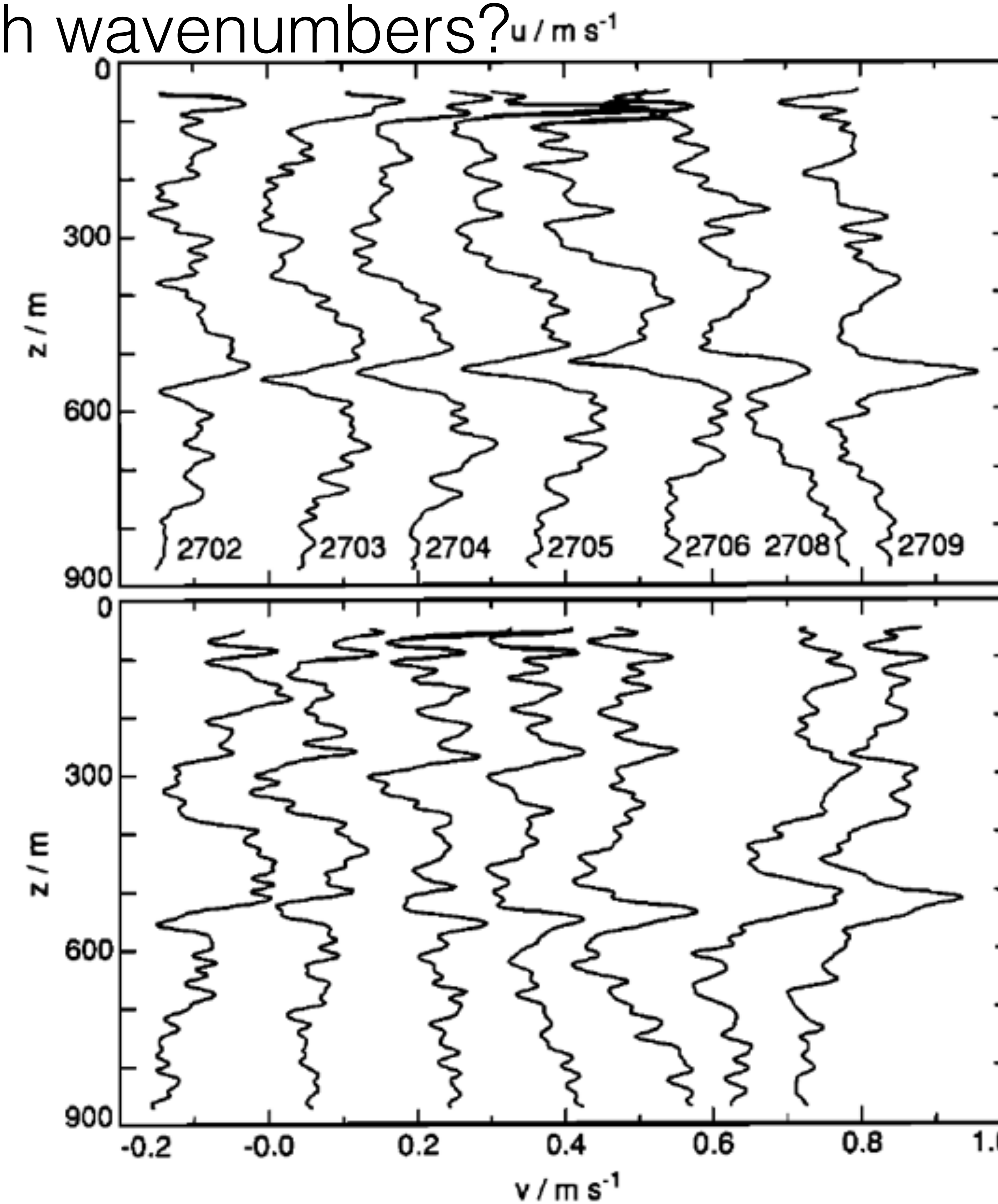

BASIC ASSUMPTIONS OF THE
GARRETT AND MUNK MODEL
(854 TESTS/FREQU)
(b)
FREQUENCY [CPH]


(h)
IWEX MODEL CLASS
(224 TESTS/FREQU.)
FREQUENCY [CPH]

GM72 GM75 GM76 IWEX78 **GK91**

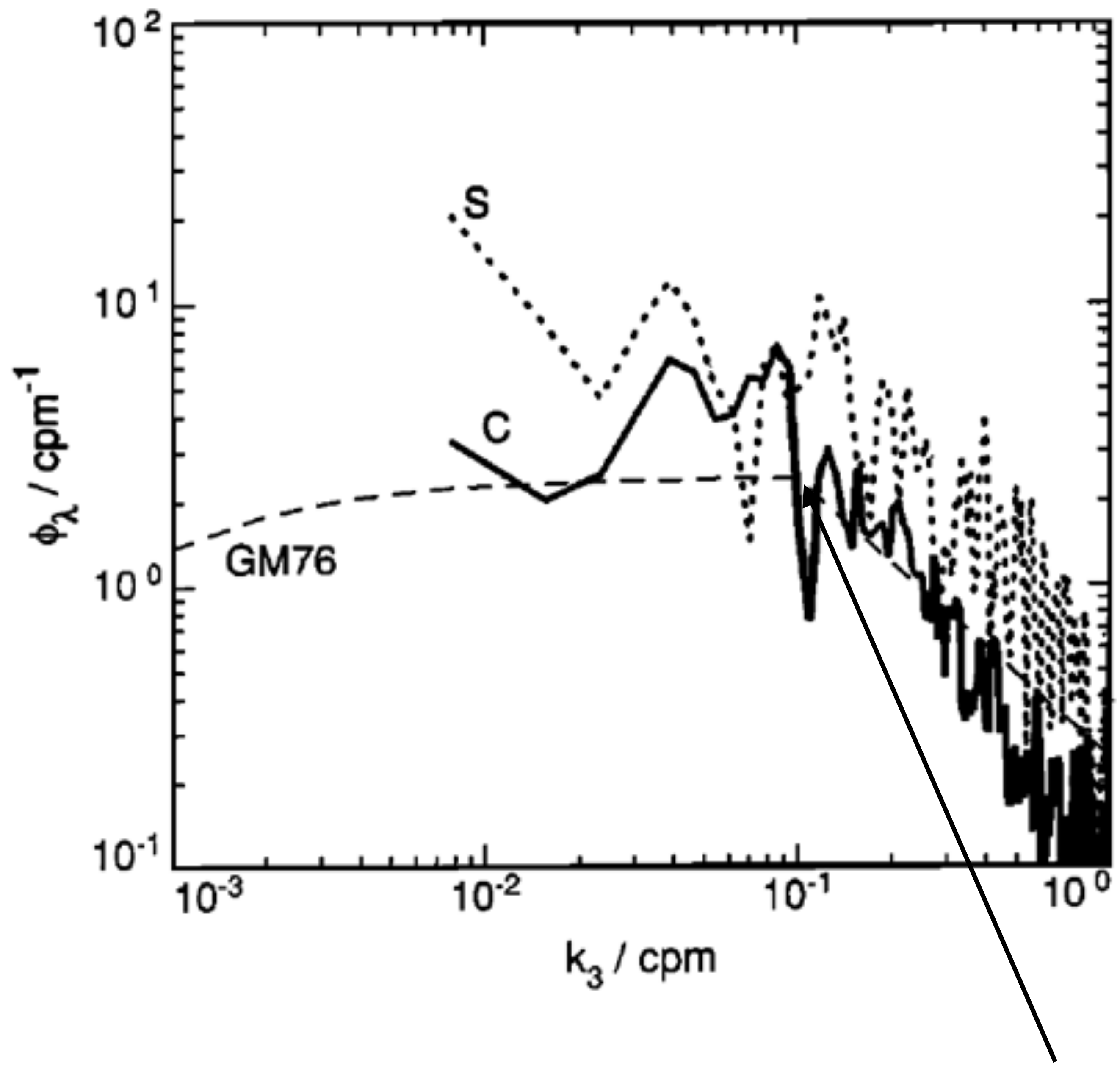
What about extremely high wavenumbers?





(Gregg and Kunze, JGR, 1991)

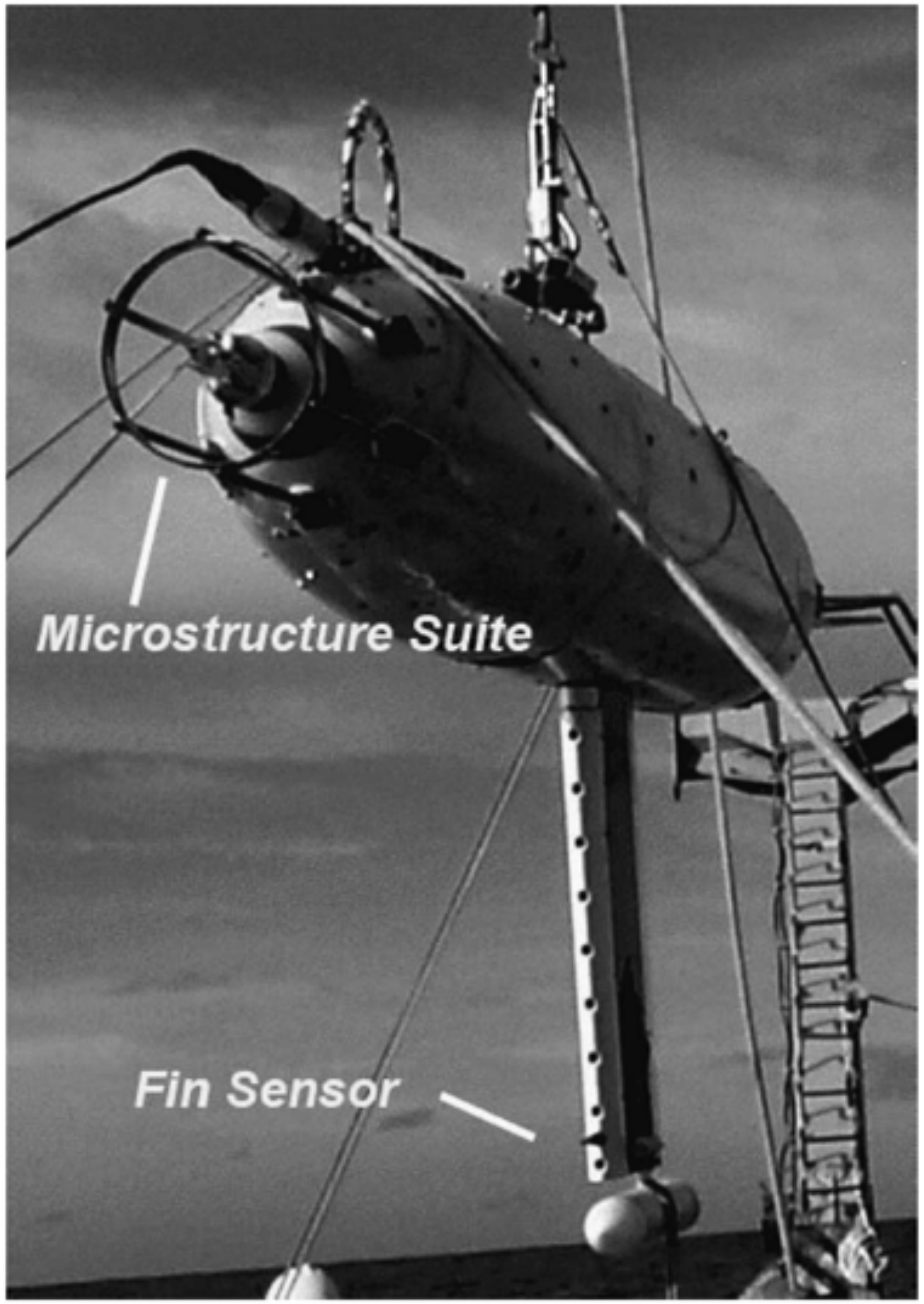
GM72 GM75 GM76 IWEX78 **GK91**

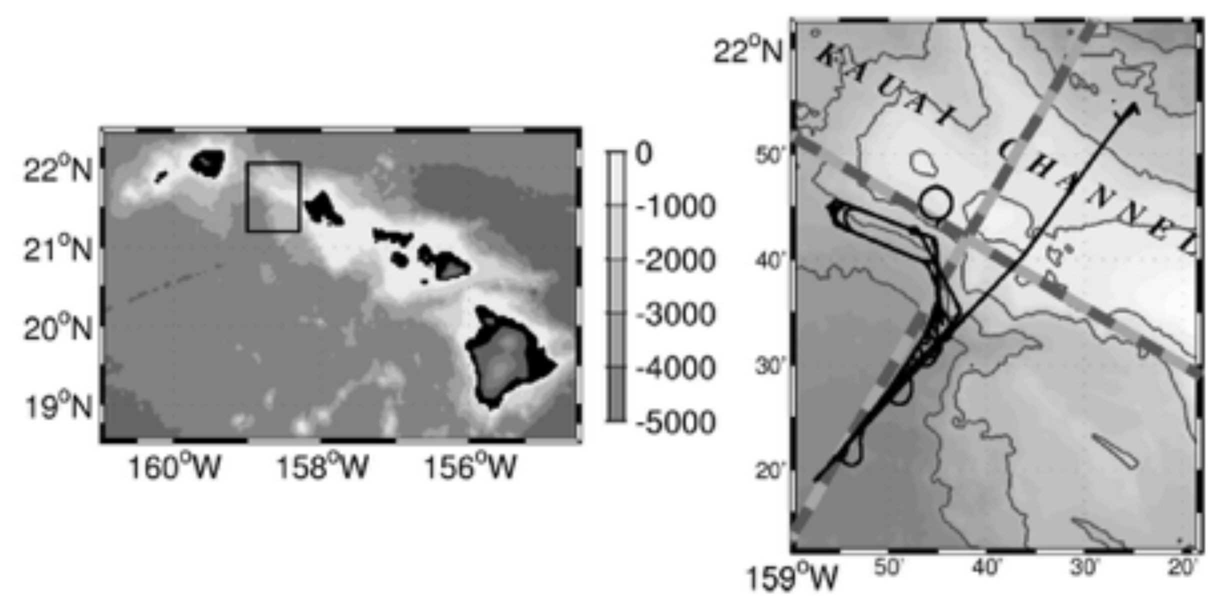


cut-off point set by Gargett et al., JPO, 1981.

(Gregg and Kunze, JGR, 1991)

GM in the 21st century

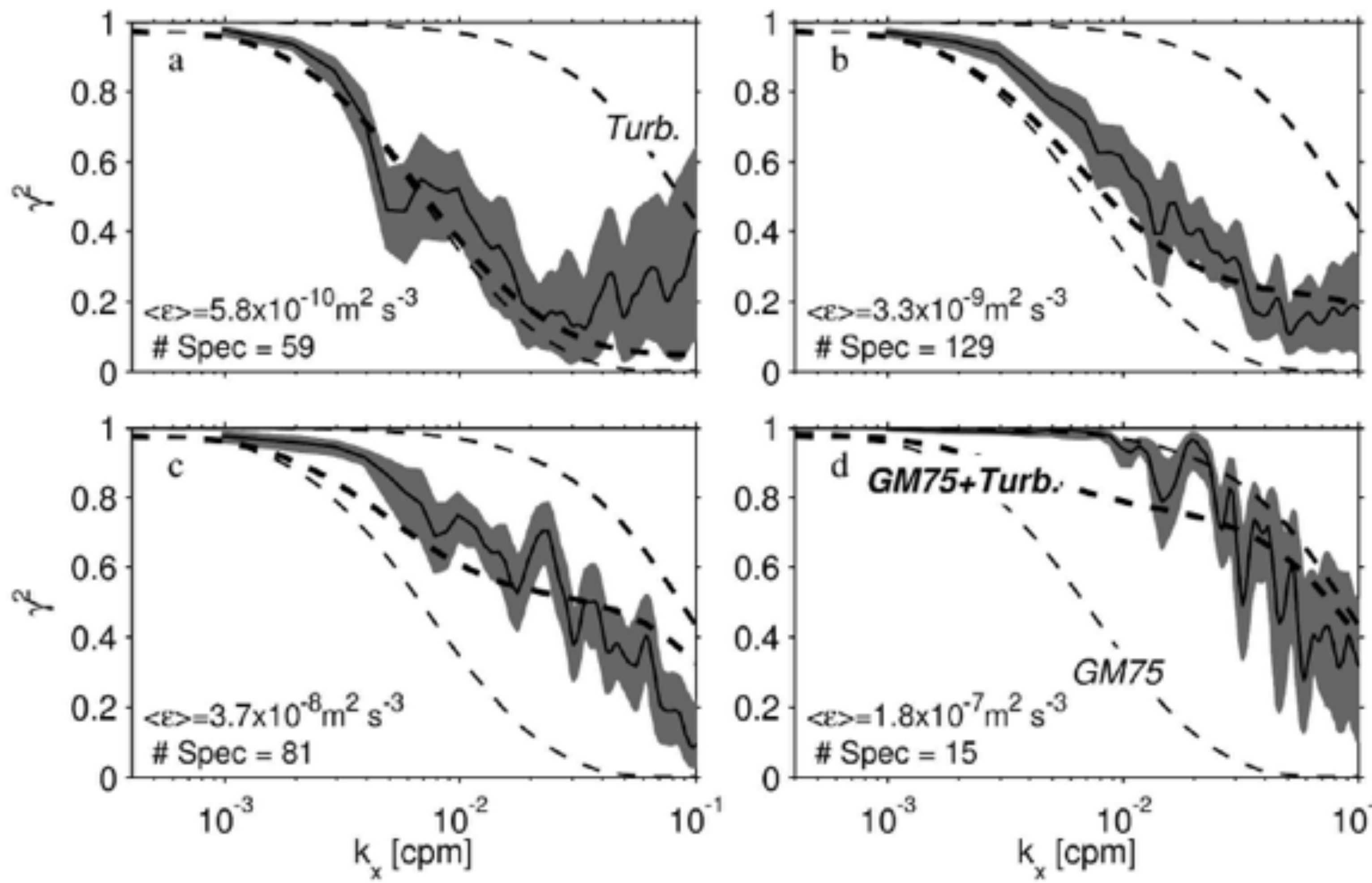




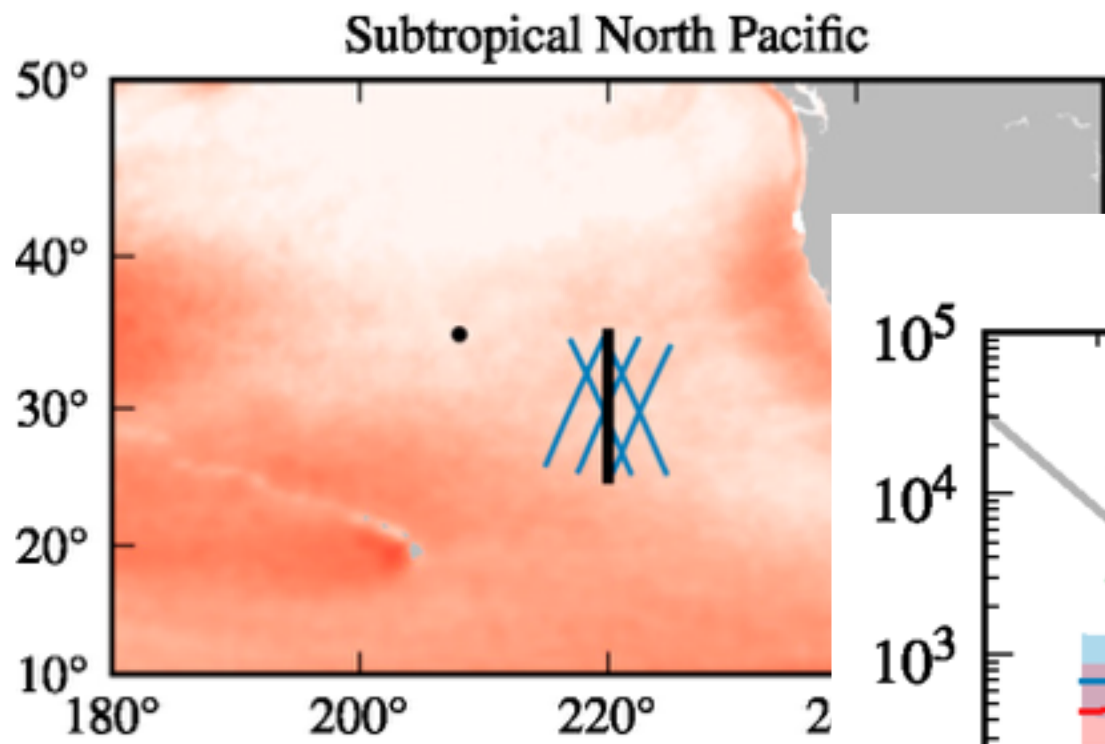
(Klymak and Moum, JPO, 2007a)

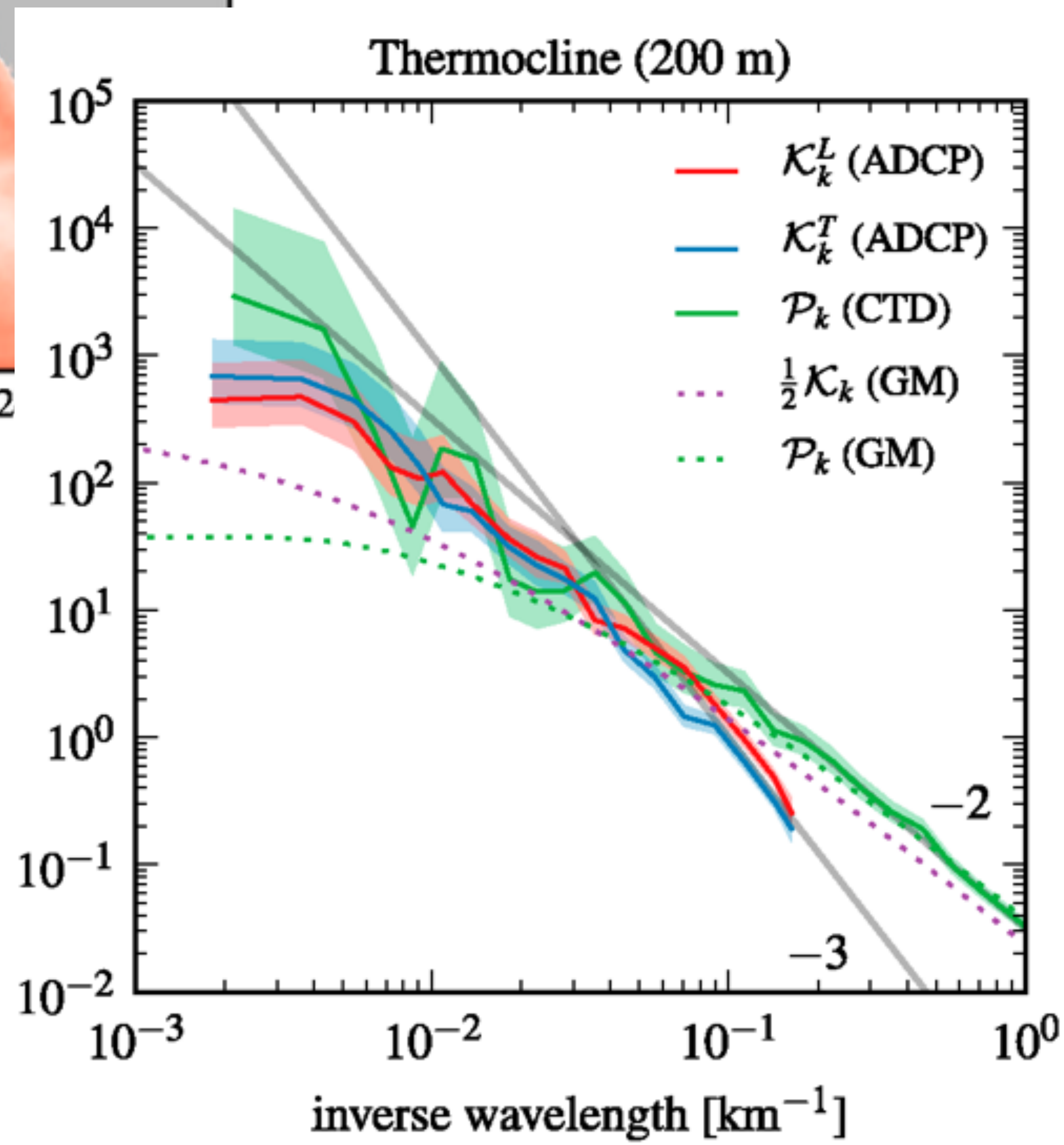
GM in the 21st century

Towed vertical coherence

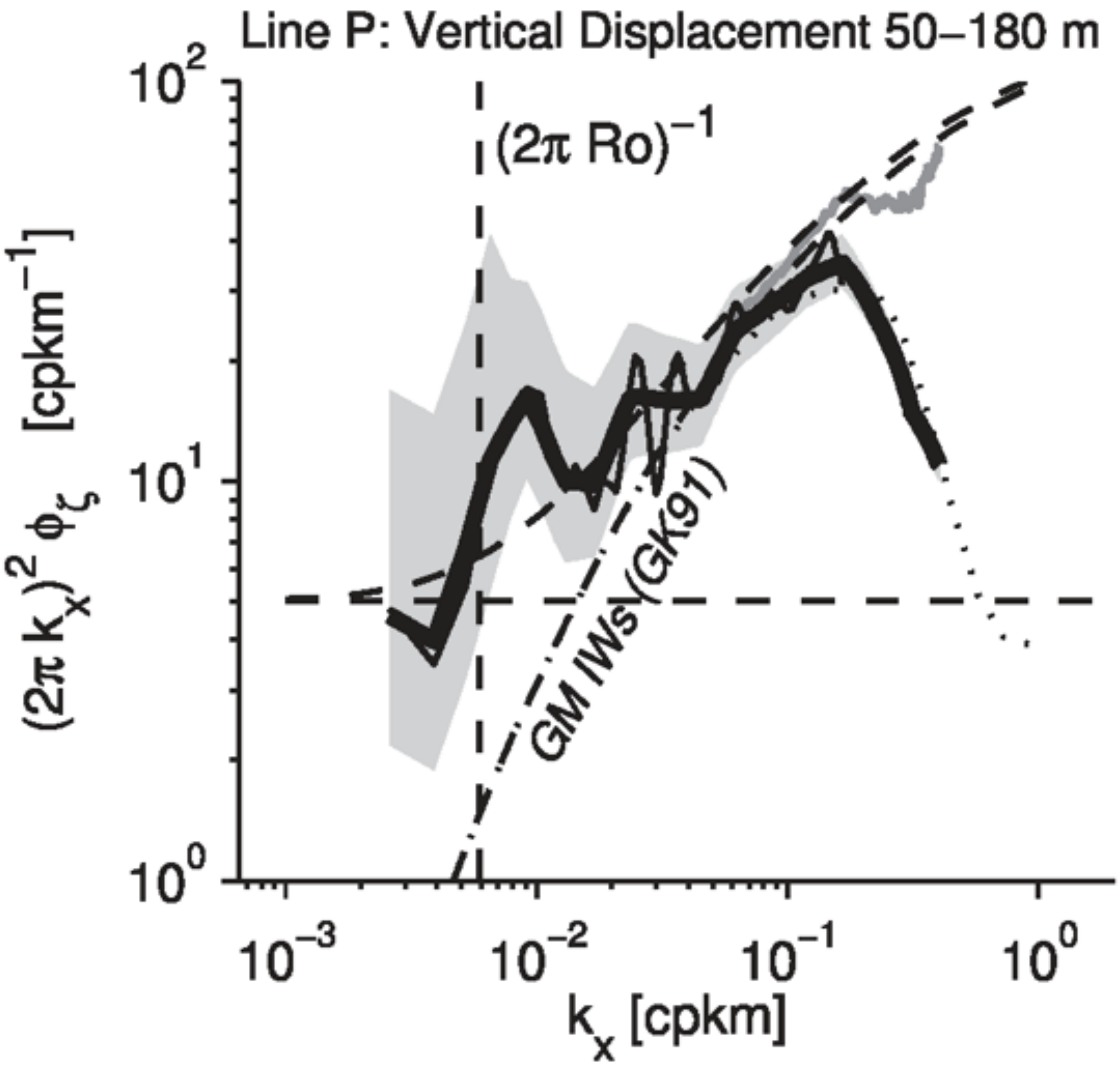


(Klymak and Moum, JPO, 2007b)





(Callies and Ferrari, JPO, 2013)



(Klymak et al., JGR, 2015)

http://jklymak.github.io/GarrettMunkMatlab/