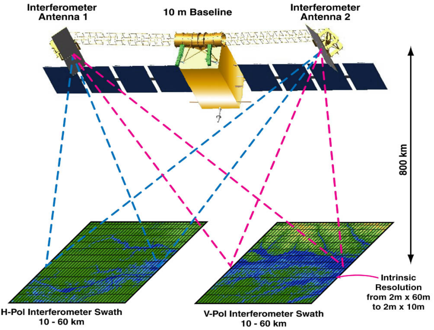
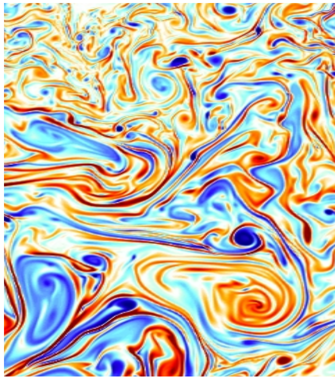
”Ocean Turbulence from SPACE”

Patrice Klein (Caltech/JPL/Ifremer)

(V) - <u>2-D Turbulence (a)</u>





webpage: http://oceanturbulence.github.io/
class email: oceanturbulencefromspace@gmail.com
dropbox: oceanturbulence (pswd: patriceklein)

Next reading classes: two papers:

- Capet et al. (2008): Surface KE transfer in Surface Quasi-Geostrophic flows; J.F.M., 604, 165-174.
- Munk et al. (2000) : Spirals on the Sea. Proc. R. Soc. London A. 456, 1217-1280

2-D Turbulence:

It is a simple framework (involving simple equations) that captures many of the main characteristics of the balanced (QG) 3-D turbulence.

Two-dimensional Turbulence : reading list :

Review papers:

- Rhines PB. 1983. Lectures in geophysical fluid dynamics. Lect. Appl. Math. 20:3–58.
- McWilliams JC. 1991. Geostrophic vortices. In Nonlinear Topics in Ocean Physics, Proceedings of the International School of Physics, "Enrico Fermi" Course CIX. ed. A.R. Osborne. Amsterdam: IOS Press
- McWilliams, J. (1984) : The Emergence of isolated coherent vortices in Turbulent Flow. J. Fluid Mech., 146, 21-43.
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Coherent (isolated) structures:

- McWilliams, J. : Submesoscale coherent vortices in the ocean. Rev. Gephys. 23 :165-182.
- Provenzale, A.. (1999) Transport by Coherent Barotropic Vortices. Ann. Rev. Fluid Mech. . 31 :55-93.
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- Shutts, G.J. 1983 : The propagation of eddies in diffluent jetstreams : eddy vorticity forcing of « blocking » flow fields. 83, 109, 737-761.
- McWilliams, J.. 1980 : An application to equivalent modons to atmospheric blocking. D.A.O. , 5, 43-66.
- Ingersoll, A.P. 1973 : Jupiter's Great Red Spot: A free atmospheric vortex? Science 182, 1346-1348.

Perturbated vortex evolution :

Axisymmetrisation or erosion of coherent structures :

- Melander et al., 1987 : Axisymmetrisation and vorticity-gradeint intensification of an isolated two-dimensional vortex through filamentation. J .F.M., 178, 137-159.
- Mariotti A. et al., 1994 : Vortex stripping and the erosion of coherent structures in two-dimensional flows. Phys. Fluids. 3954-3962.
- McWilliams, J. 1990 : Te vortices of two-dimensional turbulence. J.F.M., 219,361-385.

Vortex close interactions : vortex merging :

- Melander et al., 1988 : Symmetric vortex merger in two dimensions : causes and conditions. J .F.M., 195, 303-340.

Strongly perturbated vortex evolution : two-dimensional turbulence :

Inverse kinetic energy cascade :

- Rhines PB. 1975. Waves and turbulence on a beta-plane. J.F.M., 69, 417-443.

Textbook :

- Vallis G. (2006) : Atmospheric and Oceanic Fluid Dynamics. Cambridge University Press.

Two dimensional turbulence

Euler equations

$$\mathbf{U} = (u, v).$$

$$\nabla \cdot \mathbf{U} = 0$$

$$\frac{\partial u}{\partial t} + \mathbf{U} \cdot \nabla u - (f_0 + \beta y) v = -\frac{1}{\rho_0} \frac{\partial p}{\partial x} + [\text{mixing}].$$

$$\frac{\partial v}{\partial t} + \mathbf{U} \cdot \nabla v + (f_0 + \beta y) u = -\frac{1}{\rho_0} \frac{\partial p}{\partial y} + [\text{mixing}].$$

In 2-D flow we can define a stream function ψ:

$$u = -\psi_y \qquad v = \psi_x$$

We use $\tilde{p} = \frac{p}{\rho_0} - (f_0 + \beta y)\psi$

Note: in the QG context, ψ is the geostrophic part of the pressure and $\tilde{p}$ the ageostrophic part

$$\Rightarrow \quad \frac{\partial u}{\partial t} + \mathbf{U} \cdot \nabla u = -\frac{\partial \tilde{p}}{\partial x} + [\text{mixing}]$$

$$\frac{\partial v}{\partial t} + \mathbf{U} \cdot \nabla v = -\frac{\partial \tilde{p}}{\partial y} - \beta \psi + [\text{mixing}].$$

Vorticity eq.

$$\zeta = v_x - u_y$$

$$\frac{\partial \zeta}{\partial t} + \mathbf{U} \cdot \nabla \zeta + \beta v = 0.$$

Divergence eq.

$$\nabla \cdot [\mathbf{U} \cdot \nabla \mathbf{U}] = -\Delta \tilde{p} + \beta u.$$

Note that:

$$\boxed{\nabla \cdot [\mathbf{U} \cdot \nabla \mathbf{U}] = \frac{1}{2} W = \frac{1}{2} [S_1^2 + S_2^2 - \zeta^2]}$$

with $S_1 = u_x - v_y = 2u_x$, $S_2 = v_x + u_y$

W is the Okubo – Weiss criterion $W = [S_1^2 + S_2^2 - \zeta^2]$.

[See Okubo (DSR 1970) and Weiss (1981 (Report Scripps) and 1991 Physica D)]

W is related to second-order derivatives of ψ and therefore to the ψ-curvature.

W partitions the stream function into hyperbolic and elliptic regions

$$\underline{\text{Two invariants:}} \quad KE = \frac{u^2+v^2}{2} \qquad Z = \frac{\zeta^2}{2}$$

Z is the enstrophy

$$(\mathrm{I}) \quad \frac{\partial \vec{U}}{\partial t} + \vec{U}\cdot\nabla\vec{U} = -\nabla\tilde{p} - \beta\psi\vec{j} \qquad \vec{j} = \begin{bmatrix}0\\1\end{bmatrix}$$

$$\Rightarrow \frac{\partial KE}{\partial t} + \nabla\cdot\left[\vec{U}\, KE\right] = -\nabla\cdot\left[\vec{U}\tilde{p}\right] - \beta\left(\frac{\psi^2}{2}\right)_x$$

$$\Rightarrow \iint_{xy} \frac{\partial KE}{\partial t}\, dx\, dy = 0. \qquad \boxed{KE = \text{cst}}$$

KE and Z conservation

$$(\mathrm{II}) \quad \frac{\partial \zeta}{\partial t} + \vec{U}\cdot\nabla\zeta + \beta v = 0$$

$$\Rightarrow \frac{\partial Z}{\partial t} + \nabla\cdot\left(\vec{U}\, Z\right) = 0$$

$$\Rightarrow \iint_{x,y} \frac{\partial Z}{\partial t}\, dx\, dy = 0 \qquad \boxed{Z = \text{cst.}}$$

<u>Comment:</u>

The vorticity eq. can also be written as:

$$\frac{\partial \zeta}{\partial t} + J(\psi, \zeta) + \beta v = 0$$

or $$\frac{\partial(\zeta + \beta y)}{\partial t} + J(\psi, \zeta + \beta y) = 0$$

with $J(a,b) = a_x b_y - b_x a_y$

<u>The Jacobian $J(a,b) = 0$ when isolines of a are // to isolines of b!</u>

In 2-D turbulence, the flow is strongly nonlinear when $U/(\beta L^2) \gg 1$

A 2-D turbulent flow can be characterized by different regimes.

(i) – Existence of quasi steady (isolated) coherent vortices

(ii) – Evolution of weakly perturbed vortices

(iii) – Evolution of strongly perturbed vortices

(i) – Existence of quasi steady coherent vortices

A coherent vortex is characterized by:

- highly nonlinear dynamics ($U/(\beta L^2) \gg 1$)

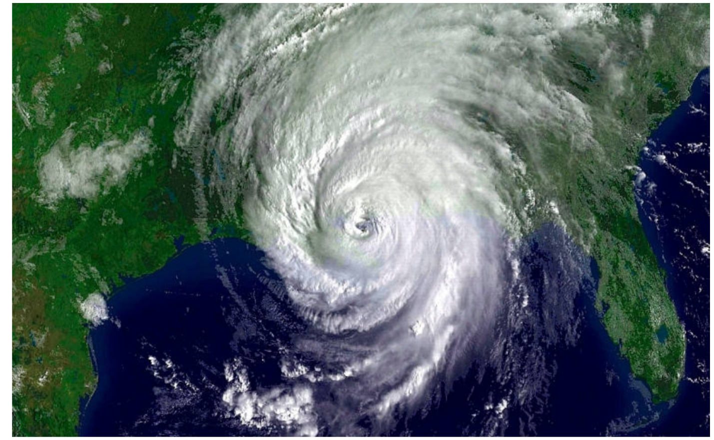
- a recurrent spatially local pattern in the ζ - field and <u>isolated from others</u> (not close to other structures)

- a long life time (and thus weakly dissipative) & capable of anomalous transport.

$$\boxed{=> J(\psi - cy, \zeta + \beta y) \approx 0}$$

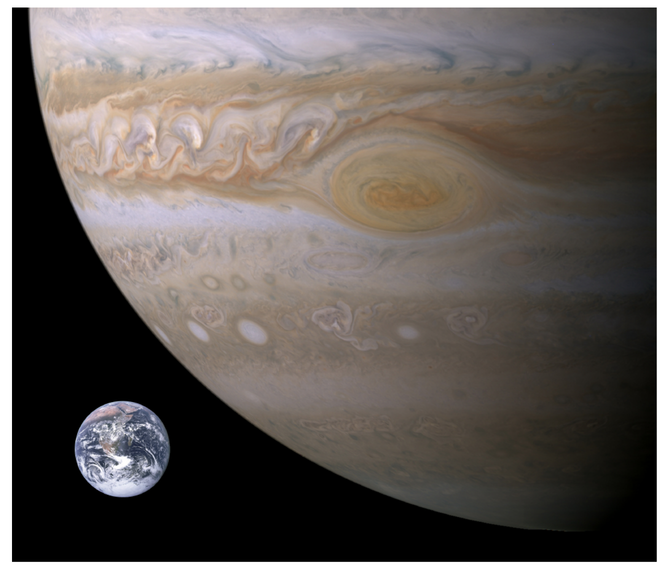
Examples of coherent vortices:

- Hurricanes, tornados;



- Gaz-giant planetary vortices: stationary states in mean shear flows
  => Great Red Spot of Jupiter

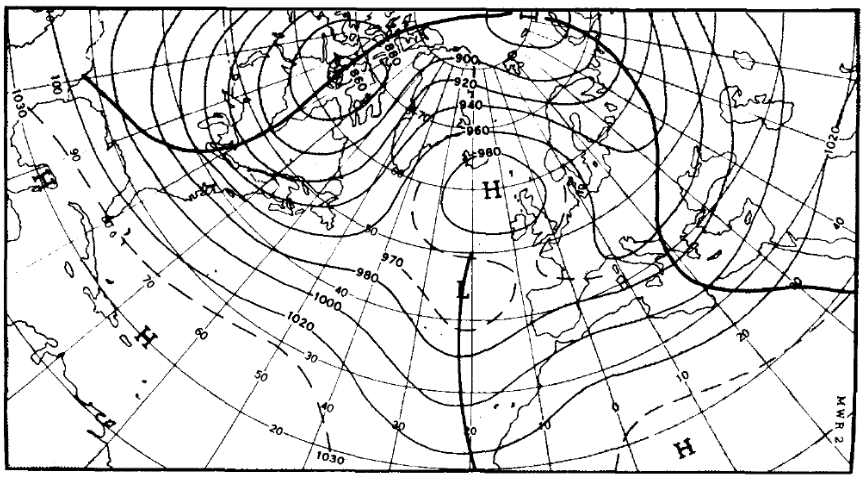
  has lasted as long as 350 years



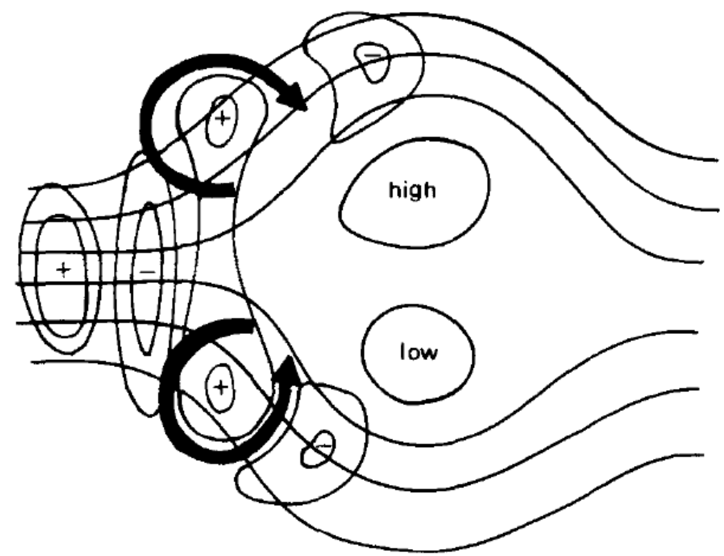
Examples of coherent vortices:

- Modons $J(\psi - cy, \zeta + \beta y) = 0$, with $\beta, c \neq 0$

Atmospheric blocking (see McWilliams J.'80; Shutts J.G.'83)



… can persist for more than 2 weeks

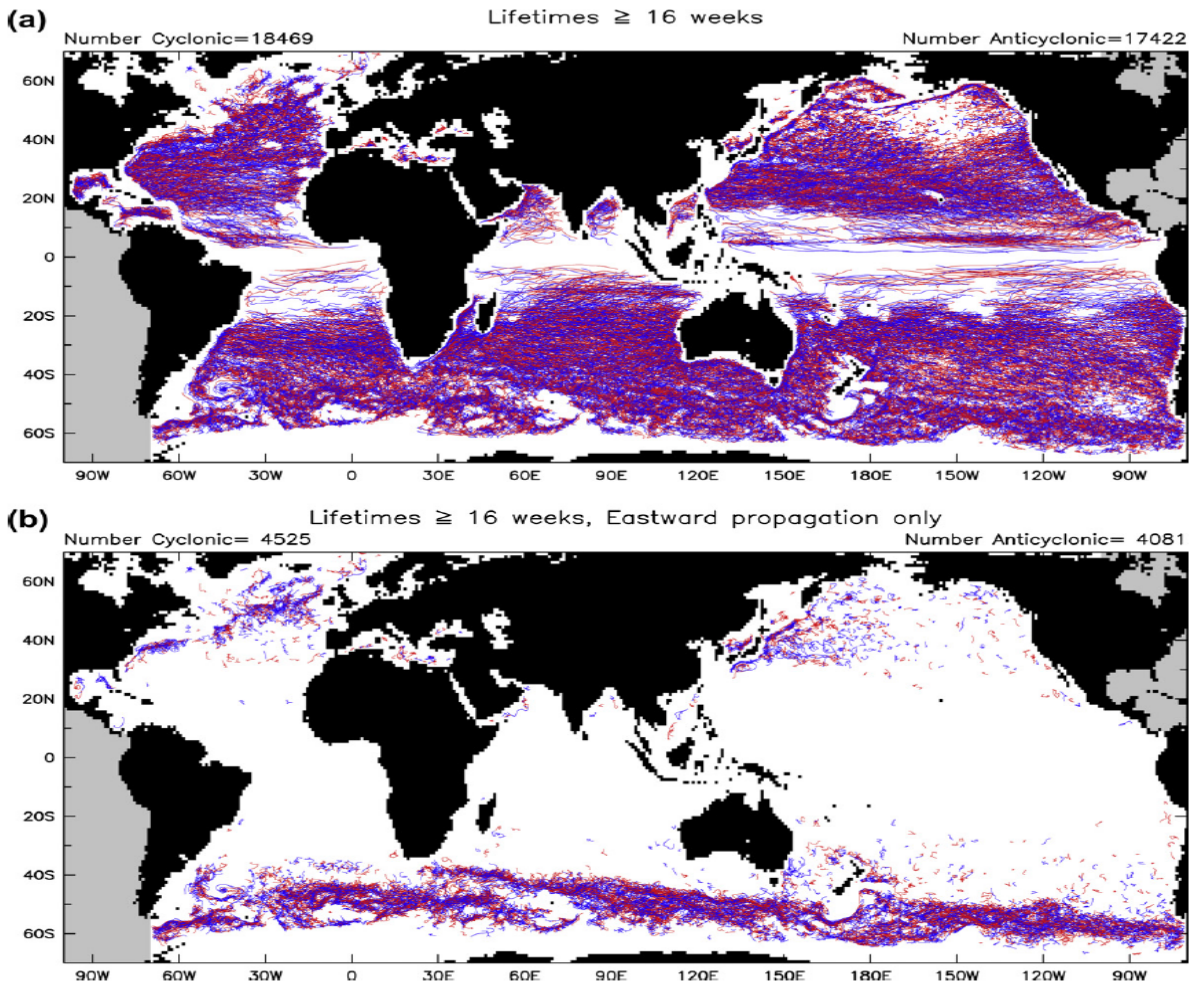


Small scale eddies are strained by the large-scale deformation field which makes their energy to be transferred to the large-scale blocking pattern. (see **Shutts J.G. QJRMS, 1983**)

Examples of coherent vortices:

Large-scale oceanic eddies (Chelton et al. 2011, PO)

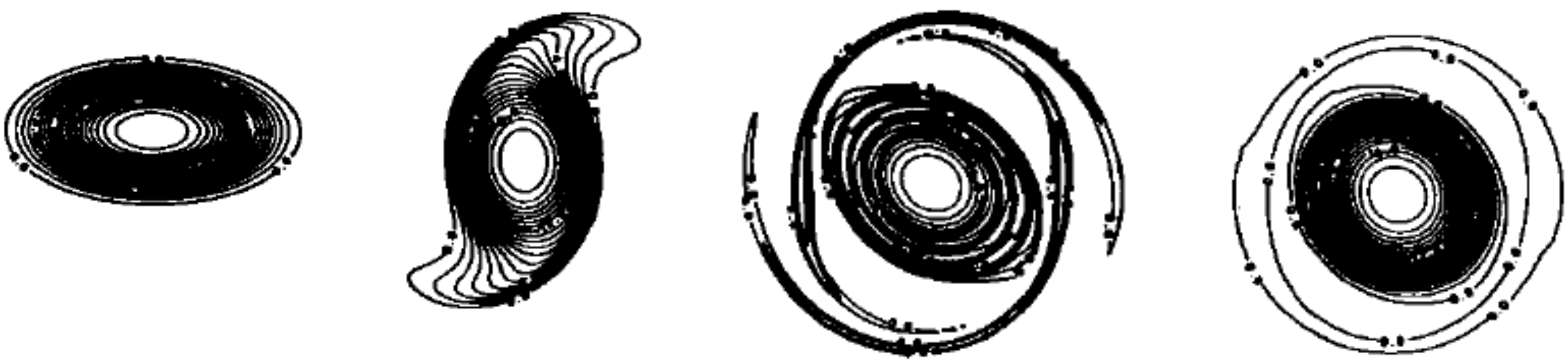
> 36000 eddies with a mean lifetime of 32 weeks



ig. 4a and b. The trajectories of cyclonic (blue lines) and anticyclonic (red lines) eddies over the 16-year period October 1992–December 2008 for (a) lifetimes ⩾16 weeks nd (b) lifetimes ⩾16 weeks for only those eddies for which the net displacement was eastward. The numbers of eddies of each polarity are labeled at the top of each panel.

(ii) – Evolution of weakly perturbed vortices

A vortex can be weakly perturbed by an external large-scale strain field. As a consequence its vorticity distribution is no more axisymmetric and becomes elliptical. Its vorticity contours are no more parallel to the streamlines and therefore $J(\psi, \zeta) \neq 0$. Its evolution is to relax towards axisymmetry on a circulation timescale as a result of filament generation (Melander et al.'87):



t=0 t=0.5 t=3 t=8

B – Evolution of weakly perturbed vortices

Axisymmetrisation of an elliptical vortex through filamentation (Melander et al.'87):

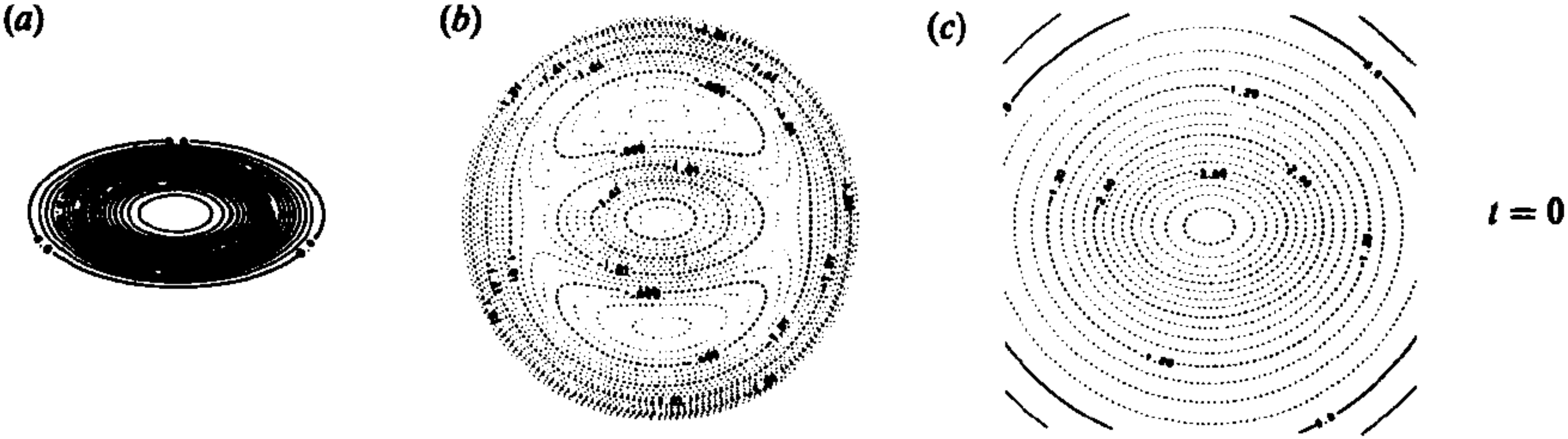
Such behavior is usually examined numerically. Analysis of the results allows to better understand the mechanisms involved.

Three variables have to be considered:

- the stream function $\psi(x, y)$
- the vorticity $\omega = -\Delta\psi$
- the **corotating stream function:** $\boldsymbol{\psi_c = = \psi + \frac{1}{2}\Omega(t)(x^2+y^2)}$

ψ_c characterizes the departure of ψ from a pure axisymmetric structure

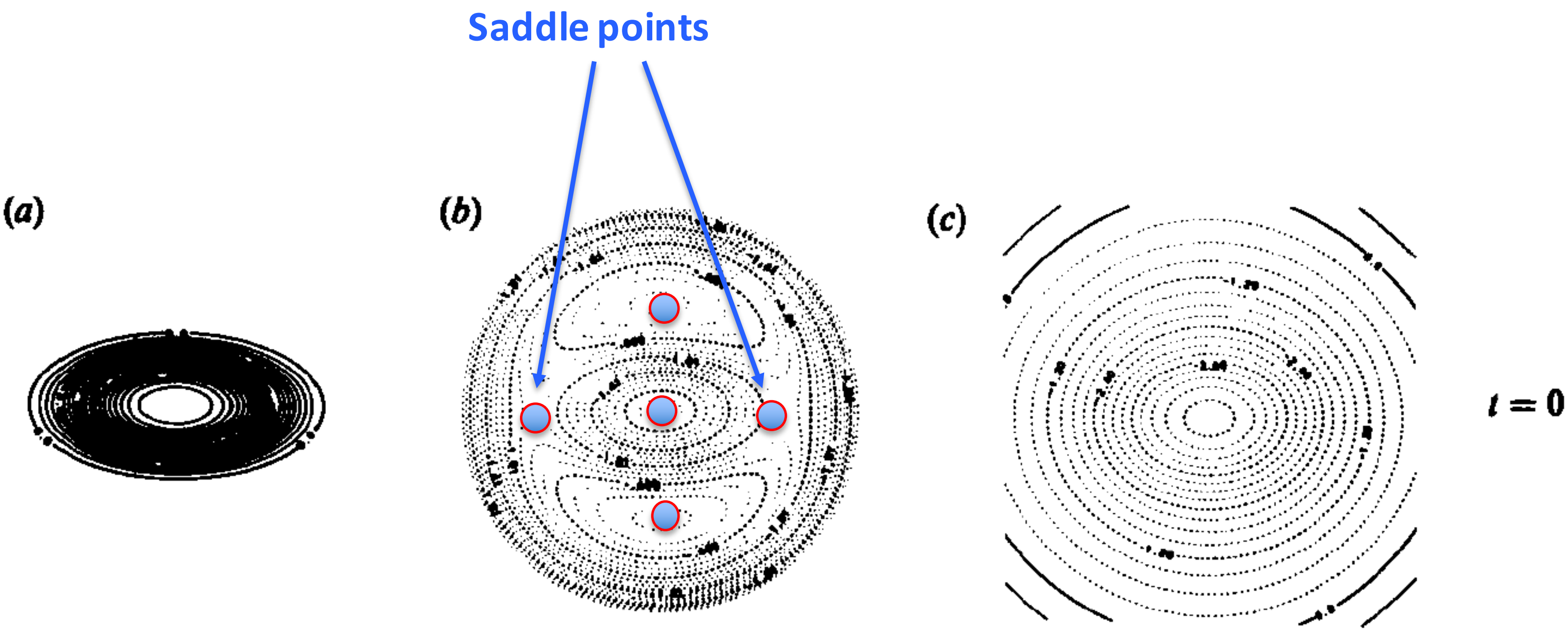
Axisymmetrisation of an elliptical vortex through filamentation (Melander et al.'87):



corotating stream function

$\omega = -\Delta\psi(x, y)$ $\psi_c(x,y) = \psi(x,y) + \frac{1}{2}\Omega(t).(x^2 + y^2)$ $\psi\ (x, y)$

Axisymmetrisation of an elliptical vortex through filamentation (Melander et al.’87):

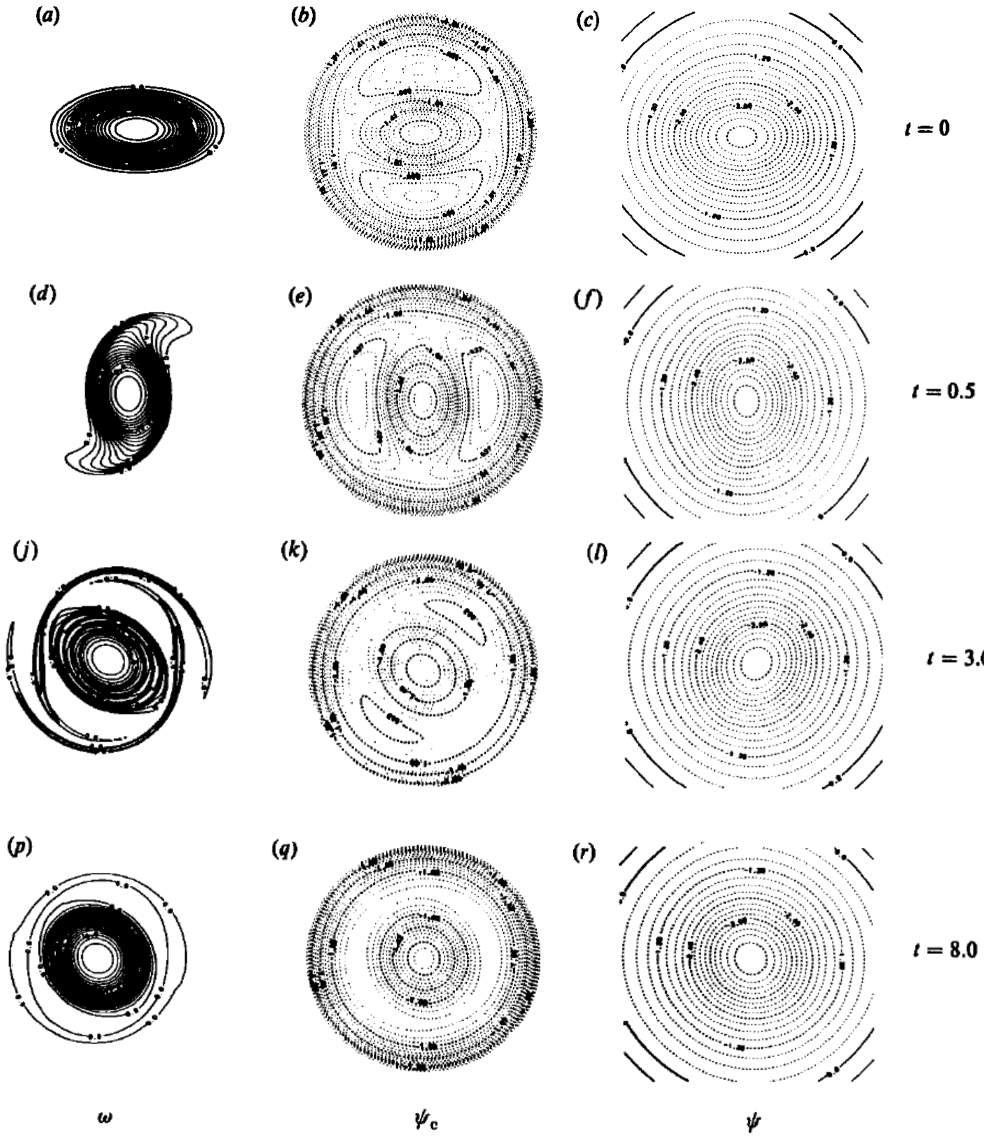


corotating stream function

$$\omega = -\Delta\psi(x,y) \qquad \psi_c(x,y) = \psi(x,y) + \frac{1}{2}\Omega(t).(x^2+y^2) \qquad \psi\,(x,y)$$

Characteristics of ψ_c allow to understand the differential deformation of the vortex:
Three mechanisms are involved …

Axisymmetrisation of an elliptical vortex through filamentation (Melander et al.'87):

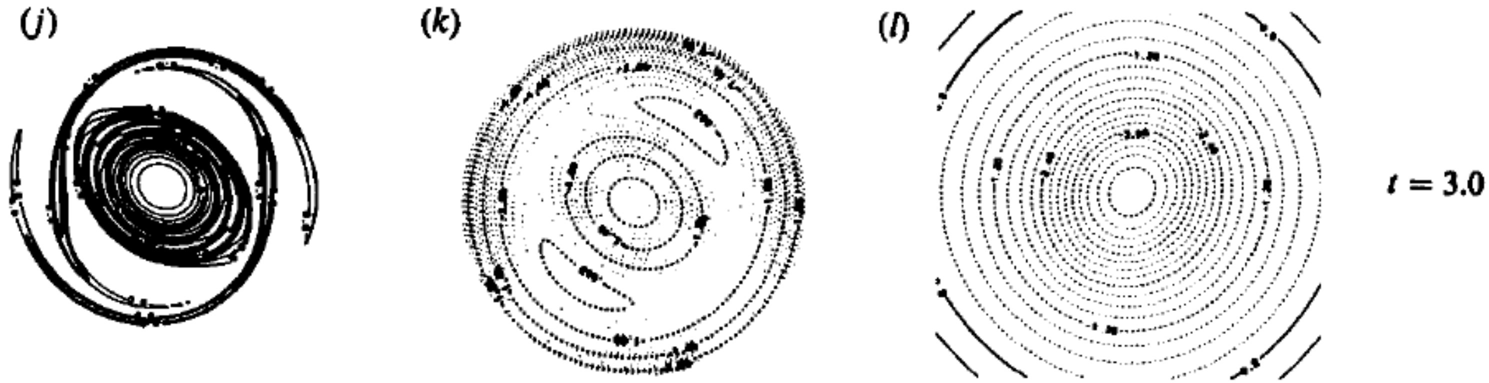


(1) The vorticity close to the saddle points is stretched and forms filaments. In this way the vortex is shedding filaments and a smaller core is forming.

(2) Filaments are wrapping around the « ghost » vortices (because of the streamlines of ψ_c). Filaments contribute to the axisymmetrisation.

(3) ψ_c (because of its spatial extension) does not change much and rotates slowly compared with the vortex. The major axis of ψ_c lags behind that of the higher vorticity contours….

Axisymmetrisation of an elliptical vortex through filamentation (Melander et al.'87):



(3) The major axis of ψ_c lags behind that of the higher vorticity contours… which further accelerates the axisymmetrisation ...

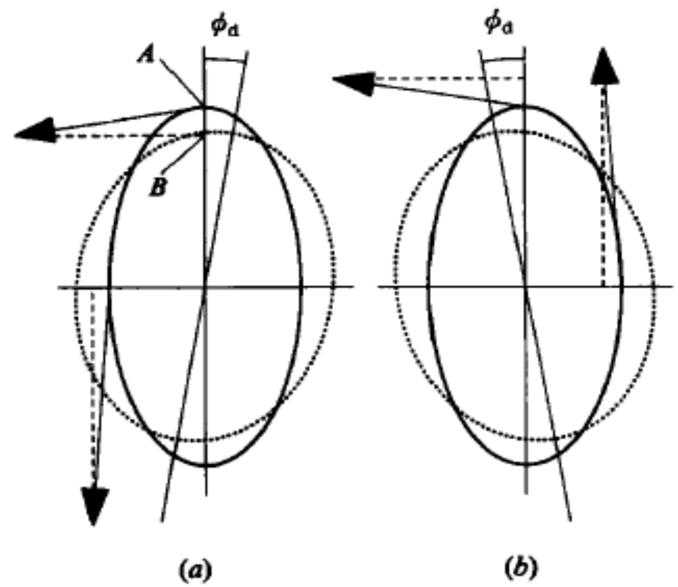


FIGURE 2. Sketch showing ——, a vorticity contour and ·····, a nearby streamline. (a) $\phi_d > 0$, (b) $\phi_d < 0$. The arrows indicate the velocity field as obtained from the nearby streamline.

All these mechanisms strongly depend on the initial conditions (saddle points inside or outside the vortex core …).

Other scenarii are possible such as when the vortex is entirely stretched into filaments that are ultimately dissipated.

A criterion that indicates **whether a vortex will survive or will be destroyed**: $\frac{S}{\zeta_{max}} < critical\ value$ (see Mariotti et al. 1994).

(iii) – Evolution of strongly perturbed vortices

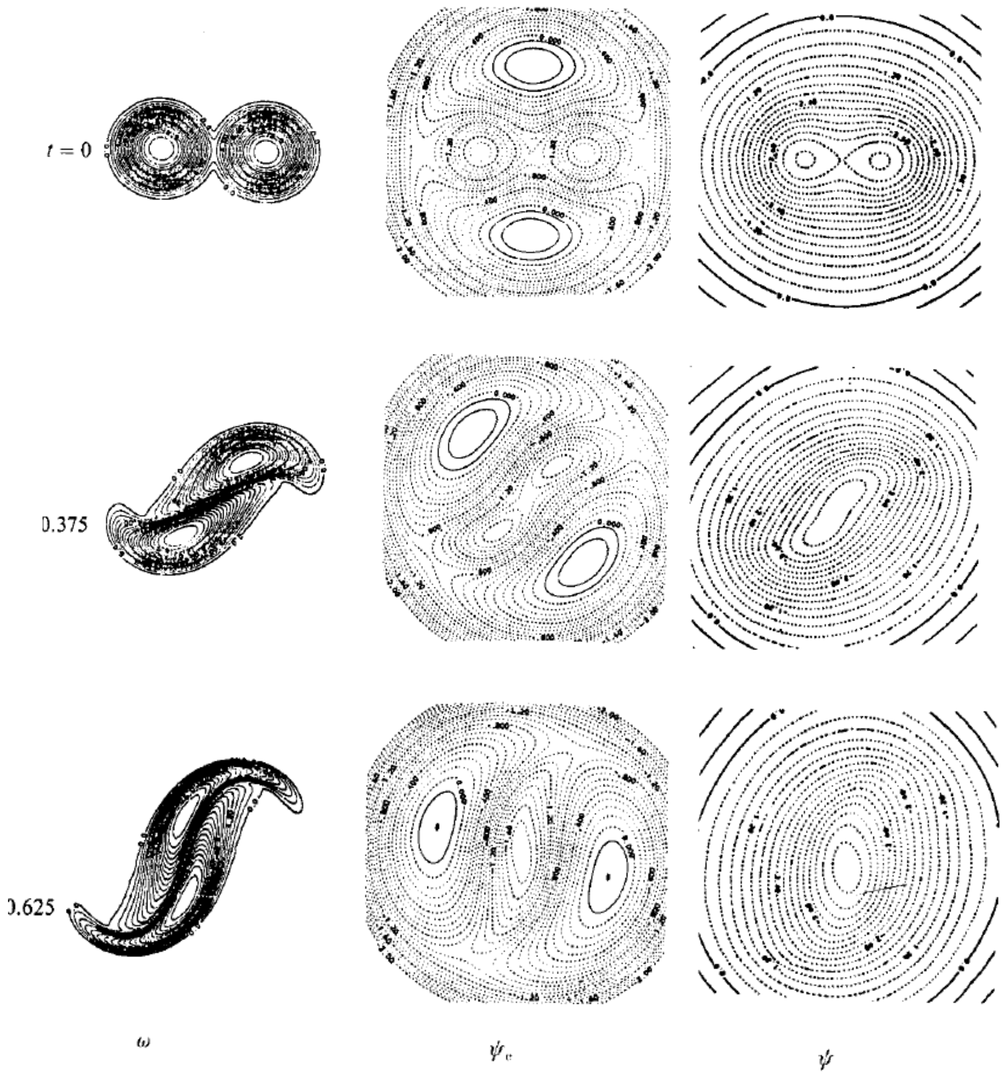
When vortices are close to each other, they strongly interact.

A classical consequence is the vortex merging process: two vortices with the same sign merge to produce a larger vortex. The evolution is again driven by the nonlinear interactions ($J(\psi, \zeta) \neq 0$).

Mechanisms involved in the vortex merging process are similar to those described before (see also Melander et al.'88).

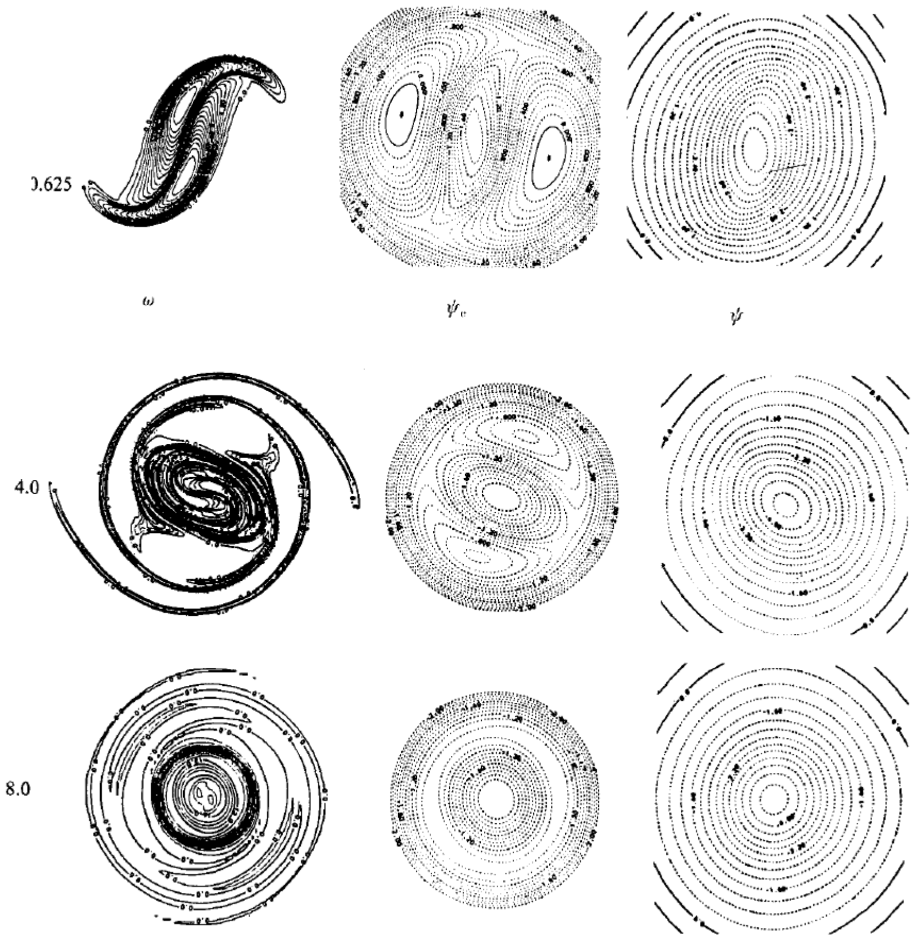
C – Evolution of strongly perturbed vortices

Vortex merging process: mechanisms involved are similar to those described before (see Melander et al.'88).



C – Evolution of strongly perturbed vortices

Vortex merging process: mechanisms involved are similar to those described before (see Melander et al.'88).



Other scenarii are possible depending again on the initial conditions. Thus the two vortices can circle around each other without merging. h

There exists a large number of situations in which vortices merge to produce larger vortices, but also in which one vortex is stretched into small-scale filaments that are ultimately dissipated.

So, through the nonlinear interactions, there is a transfer of KE that can be from small to larger scales or from large to smaller scales.

In a fully turbulent 2-D flow, is there a KE transfer direction that dominates?

The answer …

next class !