“Wave-Turbulence Interactions in the Oceans”

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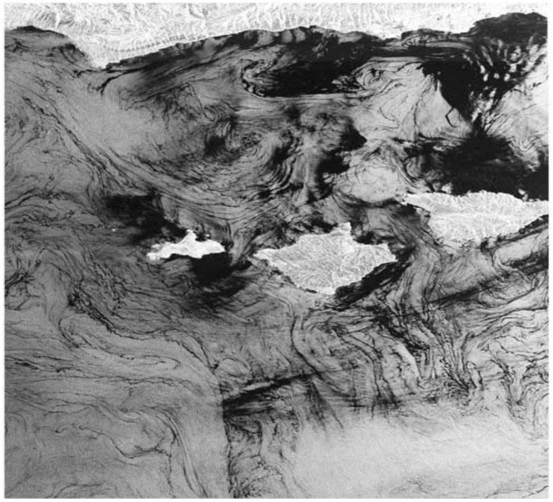
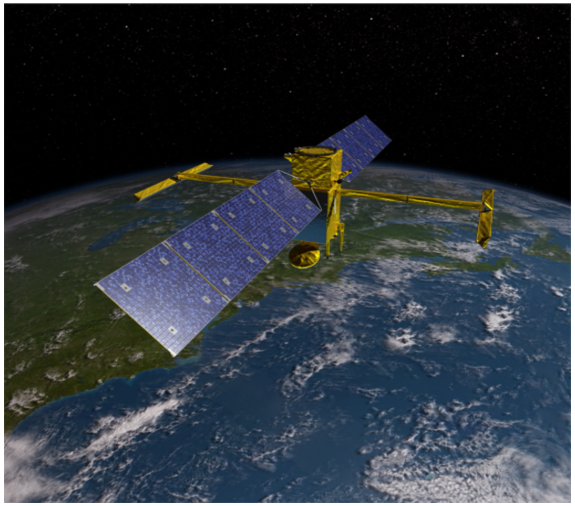
(XII) Interactions between wave and balanced motions (c)





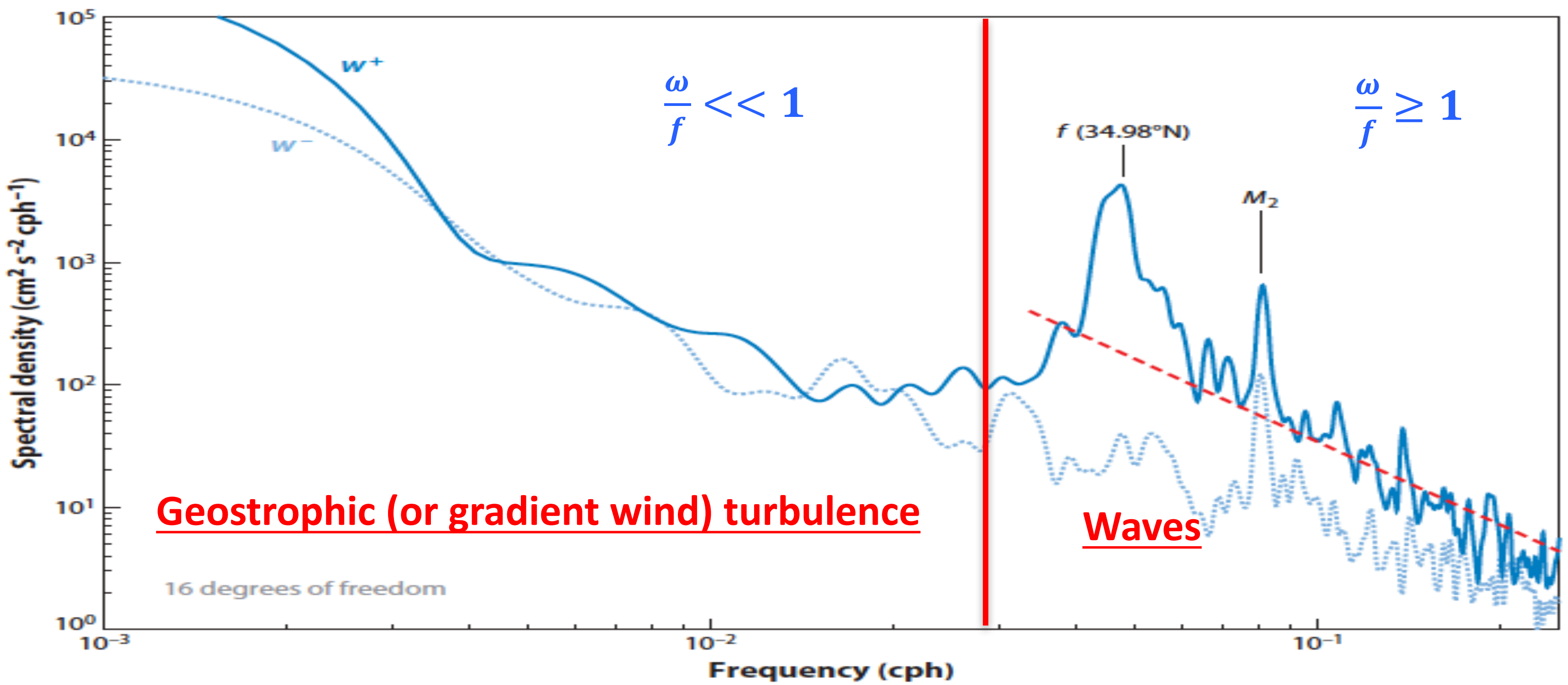
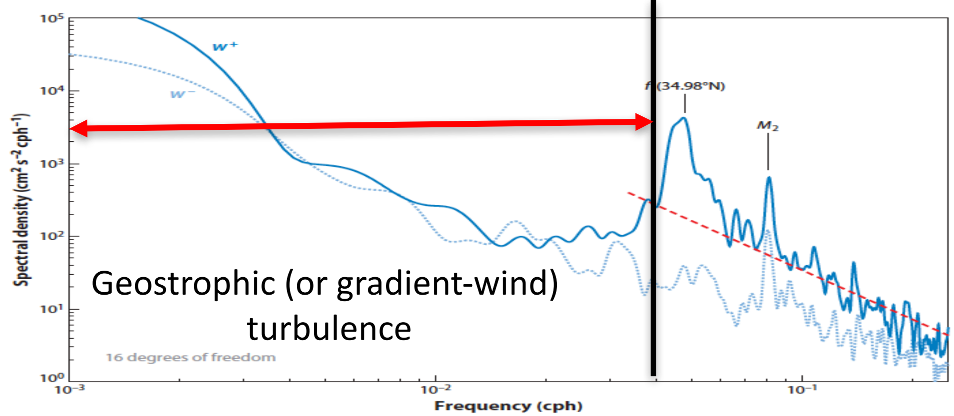


Figure 1

Rotary velocity spectrum at 261-m depth from current-meter data from the WHOI699 mooring gathered during the WESTPAC1 experiment (mooring at 6,149-m depth.) The solid blue line (w^+) is clockwise motion, and the dashed blue line (w^-) is counterclockwise motion; the differences between these emphasize the downward energy propagation that often dominates the near-inertial band. The dashed red line is the line $E_0 N\omega^{-p}$ with $N = 2.0$ cycles per hour (cph), $E_0 = 0.096$ cm^2 s^{-2} cph^{-2}, and $p = 2.25$, which is quantitatively similar to levels in the Cartesian spectra presented by Fu (1981) for station 5 of the Polygon Mid-Ocean Experiment (POLYMODE) II array.

SPECTRAL GAP AT FREQUENCIES (ω) LARGER (BUT CLOSE TO) f



GEOSTROPHIC (OR GRADIENT WIND) TURBULENCE: A BRIEF SUMMARY OF THE MAIN PROPERTIES

- **Mesoscale eddies (100 - 400 km) and sub-mesoscale structures(1-50km)**
- **Geostrophic balance (or gradient wind) balance. Capture most of the kinetic energy in the ocean**
- **At first order, their time evolution is driven by nonlinear interactions (leading to 3D direct tracer cascade and 3D inverse KE cascade)**

Momentum equations

Rossby number: U/fL $\leq$ 1

Slow motions: $\boldsymbol{\frac{\omega}{f} << 1}$

$$\cancel{\frac{\partial \boldsymbol{U}}{\partial t}} + \mathbf{U}.\boldsymbol{\nabla U} - \boldsymbol{f k \times U} = -\frac{\boldsymbol{\nabla p}}{\boldsymbol{\rho_o}}$$

$\Rightarrow$ **Geostrophic balance at zero order** $\boldsymbol{[c = \frac{\omega}{k} \ll U]}$

The Okubo-Weiss quantity allows to understand the inverse KE cascade and direct tracer cascade

$$\frac{\partial \vec{v}}{\partial t} + \vec{v}\cdot\nabla\vec{v} - f\,\vec{k}\times\vec{v} = -\frac{\nabla p}{\rho_0}$$

$$\rightarrow \boxed{\nabla\cdot(\vec{v}\cdot\nabla\vec{v}) - f\zeta = -\frac{\Delta p}{\rho_0}}$$

$O(R_o)$ $\quad O(1)$ $\quad O(1)$.

$\nabla\cdot(\vec{v}\cdot\nabla\vec{v})$ is the Okubo-Weiss quantity

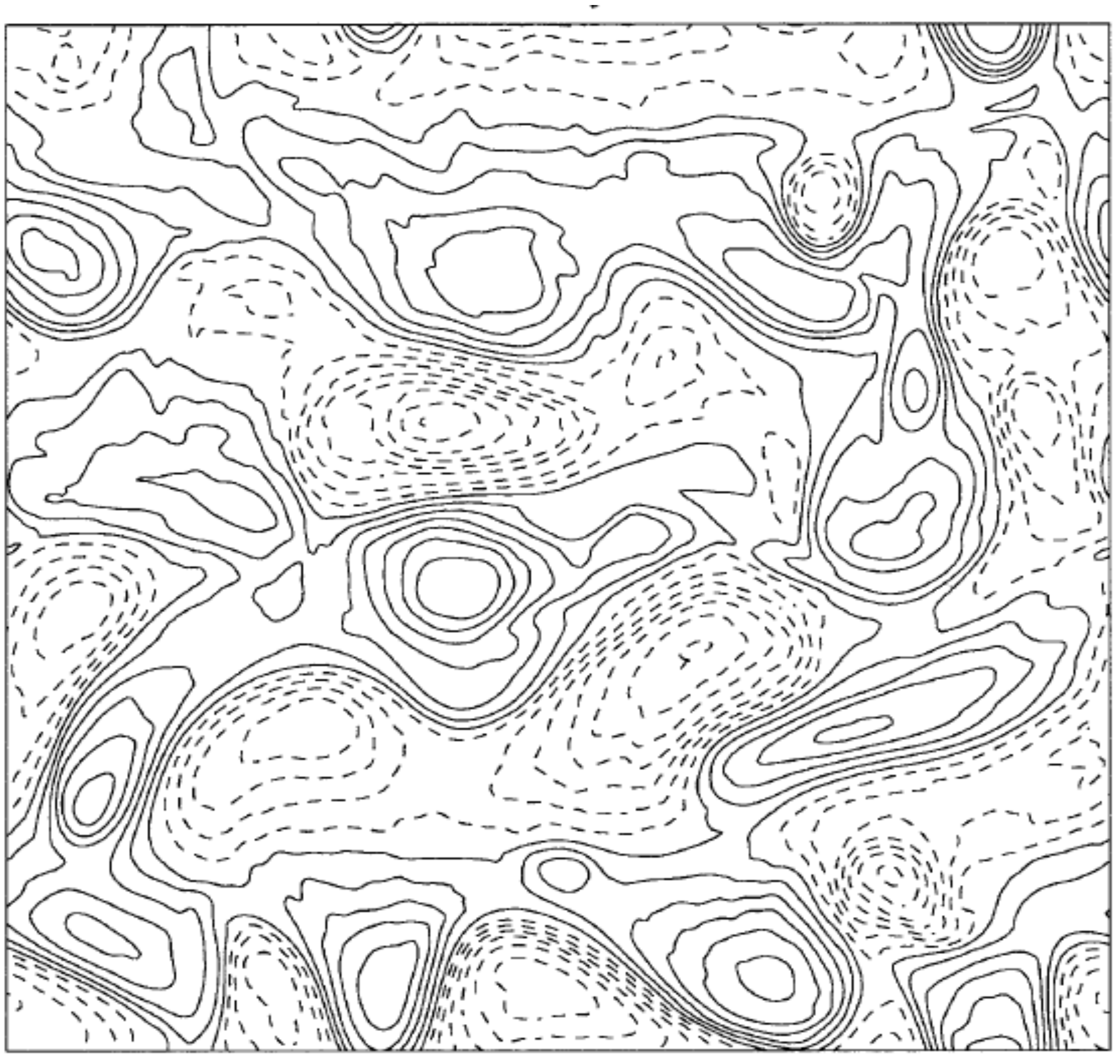
$$\rightarrow \boxed{\nabla\cdot(\vec{v}\cdot\nabla\vec{v}) = \frac{1}{2}\left[S_1^2 + S_2^2 - \zeta^2\right]}$$

$$S_1 = \left[\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}\right],\quad S_2 = \left[\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right],\quad \zeta = \left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right]$$

O.W. < 0 in vorticity regions. → Eddy tracking

O.W. > 0 in strain regions. → **Dispersion of tracers** (Lyapounov exponents)

Let us consider a 2-D (non-divergent) mesoscale eddy field

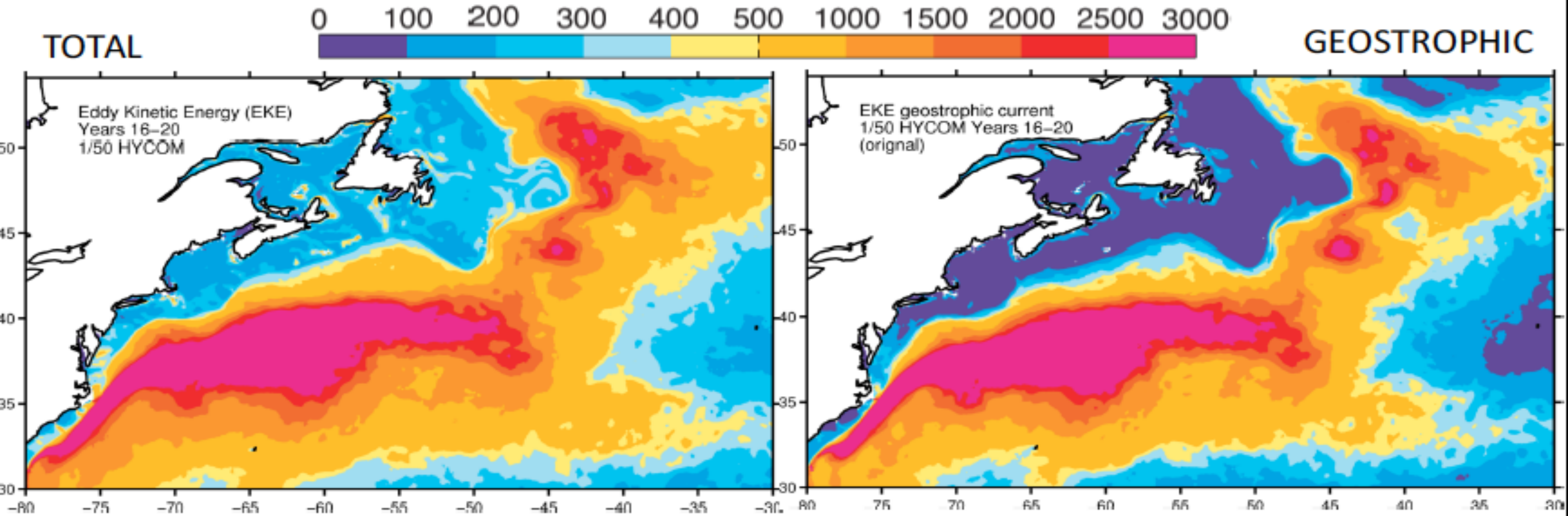


$$U = -\psi_y$$
$$V = \psi_x$$

$$P = f\psi\rho_o + \tilde{p}$$

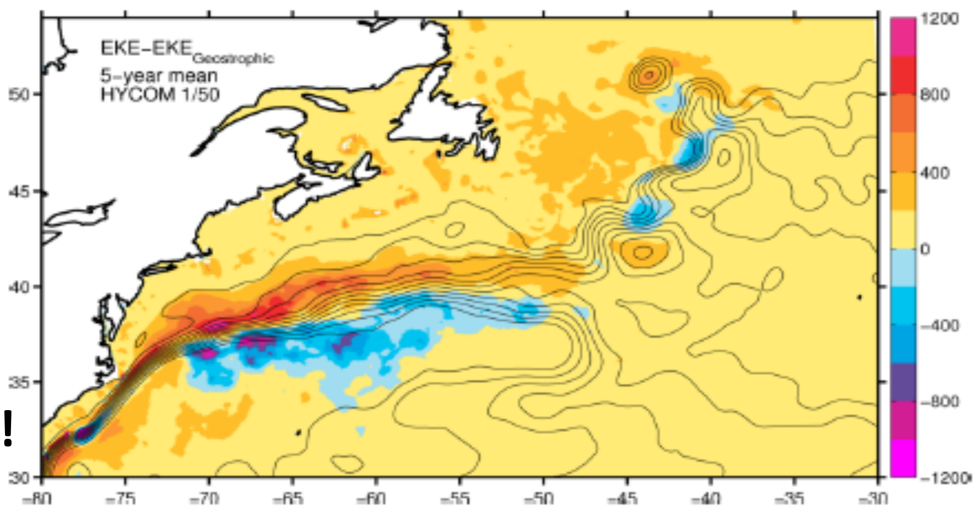
STREAM FUNCTION $\psi(x, y)$

EKE geostrophic difference (1/50°)



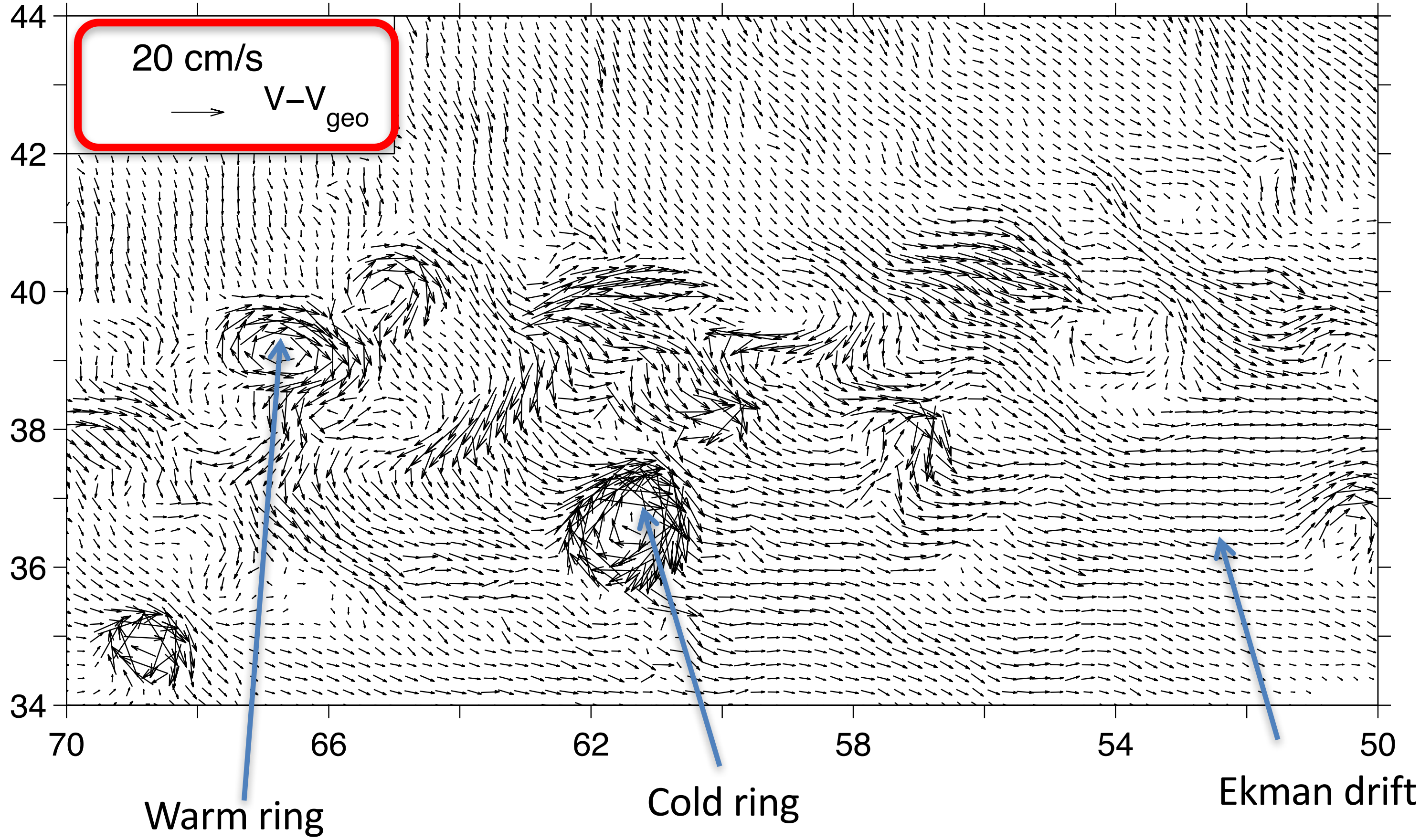
$$f.\zeta = \frac{\Delta SSH}{\rho_o} + OW$$

$\Rightarrow$ **anticyclones are more energetic than cyclones for the same SSH anomalies!**



From Chassignet & Xu JPO 2017, In press

Nonlinear terms are important !



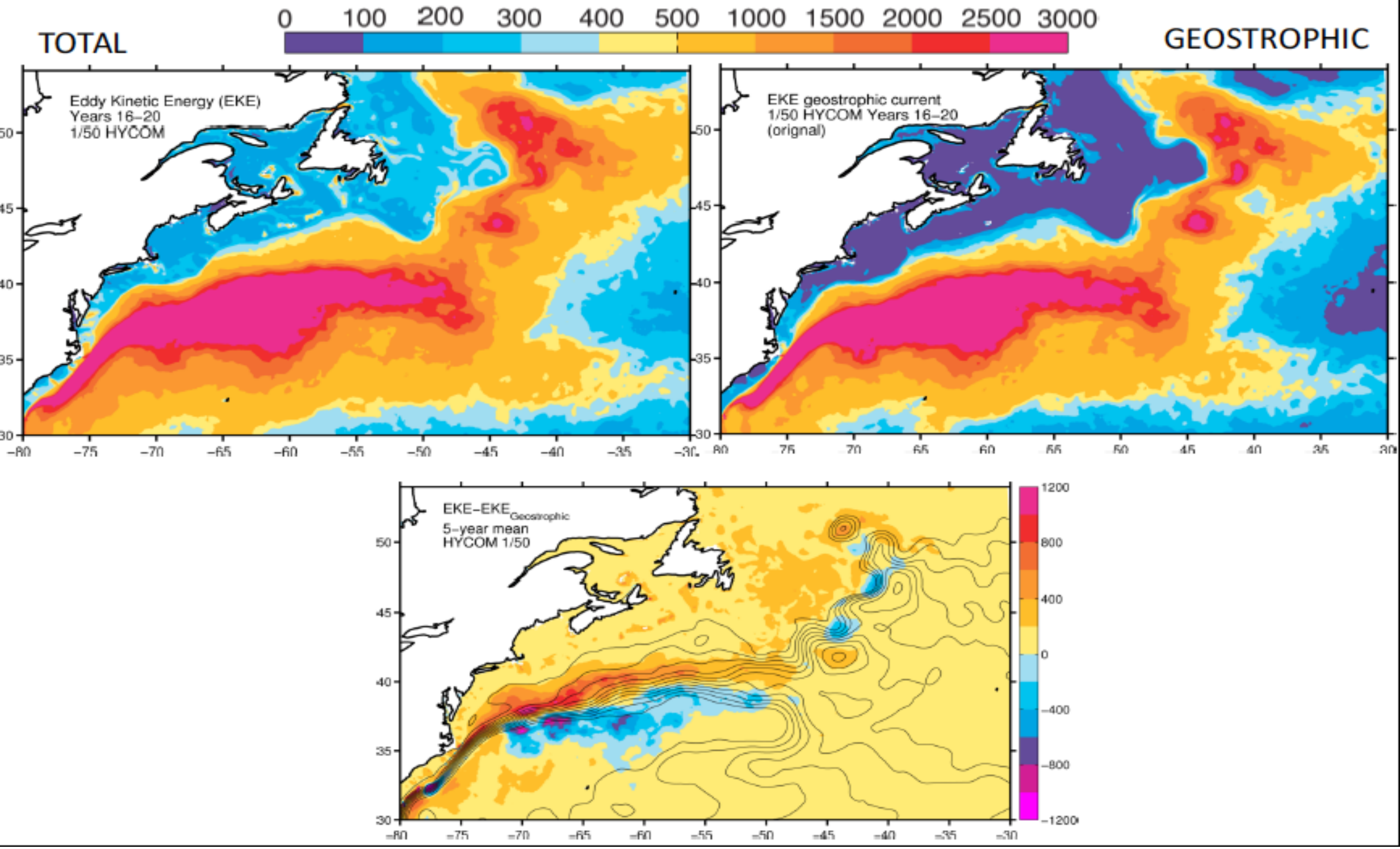
$$f\zeta = g\Delta\eta + \nabla(U.\nabla U)$$

Note that $\nabla(U.\nabla U)$ is always <0 within mesoscale eddies

=> Anticyclones are intensified by the non-linear terms

From Chassignet & Xu JPO 2017, In press

EKE geostrophic difference (1/50°)



From Chassignet & Xu JPO 2017, In press

The Okubo-Weiss quantity allows to understand the inverse KE cascade and direct tracer cascade

$\nabla\cdot(\mathbf{v}\cdot\nabla\mathbf{v})$ is the Okubo-Weiss quantity

$$\Rightarrow \boxed{\nabla\cdot(\mathbf{v}\cdot\nabla\mathbf{v}) = \frac{1}{2}\left[S_1^2 + S_2^2 - \zeta^2\right]}$$

$$S_1 = \left[\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}\right],\quad S_2 = \left[\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right],\quad \zeta = \left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right]$$

O.W. < 0 in vorticity regions. Weak dispersion of tracers and particles

O.W. > 0 in strain regions. Strong dispersion of tracers and particles

$\longrightarrow$ **Dispersion of tracers**

$$\vec{U} = (u, v) = (-\psi_y, \psi_x)$$

$$\frac{\partial \zeta}{\partial t} + \vec{U} \cdot \nabla \zeta + \cancel{\beta v} = 0 \qquad \text{with } \zeta = v_x - u_y$$

Let us consider a tracer (as the vorticity) conserved on a Lagrangian trajectory

$$\frac{\partial C}{\partial t} + \vec{U} \cdot \nabla C = 0 \qquad \text{or} \qquad \frac{d}{dt} C = 0$$

Time evolution for the tracer gradient, ∇C, is:

$$\frac{d \nabla C}{dt} = -[\nabla U]^T \nabla C \qquad \text{with } [\nabla U]^T = \begin{bmatrix} u_x & v_x \\ u_y & v_y \end{bmatrix} \text{ and } \nabla C = \begin{bmatrix} C_x \\ C_y \end{bmatrix}$$

Note that $[\nabla U]^T = \frac{1}{2} \begin{bmatrix} S_1 & S_2 + \zeta \\ S_2 - \zeta & -S_1 \end{bmatrix}$ with $S_1 = u_x - v_y = 2u_x$, $S_2 = v_x + u_y$

The eigenvalues of $[\nabla U]^T$ are:

$$\lambda_0 = \pm \frac{1}{2} \left[S_1^2 + S_2^2 - \zeta^2 \right]^{1/2} = \pm \frac{1}{2} [W]^{1/2}$$

Assuming the flow is slowly varying with respect to the tracer gradients, the solution becomes:

$$\nabla C(t) \approx \nabla C(t=0) \exp\left[\pm \lambda_0 t\right]$$

Solutions:

1) If $\lambda_0^2 \left(= \frac{W}{4}\right) > 0$ the strain dominates and λ_0 is real.

solutions are:

$$\boxed{\nabla C(t) \approx \begin{bmatrix} e^{\pm\lambda_0 t} & 0 \\ 0 & e^{\mp\lambda_0 t} \end{bmatrix} \cdot \nabla C(t=0)}$$

An exponential growth in one direction and exponential decay in the other. The directions are given by the eigenvectors.

2) If $\lambda_0^2 \left(= \frac{W}{4}\right) < 0$ the vorticity dominates and λ_0 is pure imaginary.

$$\rightarrow \quad \boxed{\nabla C(t) \approx \exp\left[\pm i|\lambda_0|t\right] \cdot \nabla C(t=0).}$$

No growth. Just a rotation

3) If $\lambda_0^2 \left(= \frac{W}{4}\right) = 0$. This corresponds to regions such as in a current shear (for example $U_y \neq 0$ $V_x = U_x = 0$).

For this example solutions are:

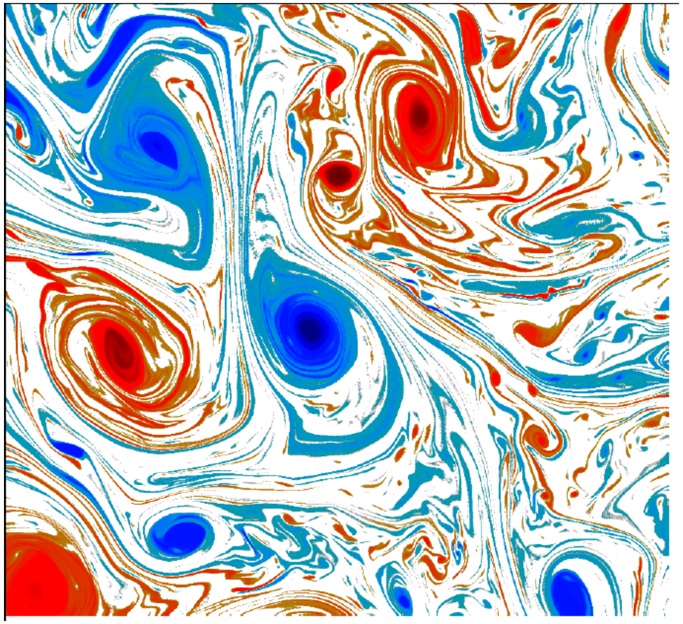
$$\boxed{\nabla C(t) = \begin{bmatrix} C_x(t) \\ C_y(t) \end{bmatrix} = \begin{bmatrix} C_x(t=0) \\ U_y t \cdot C_y(t=0) \end{bmatrix}}$$

There is a linear growth.

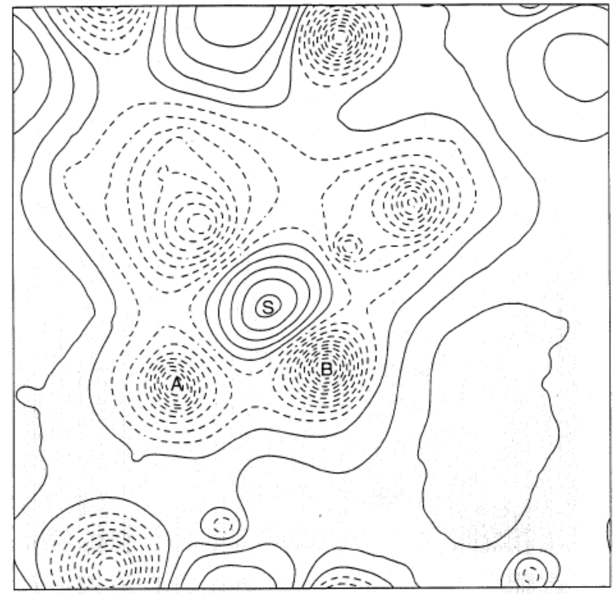
So tracer gradients either grow (exponentially or linearly) or rotate.

These solutions assume that $[\nabla U]^T$ is slowly varying.

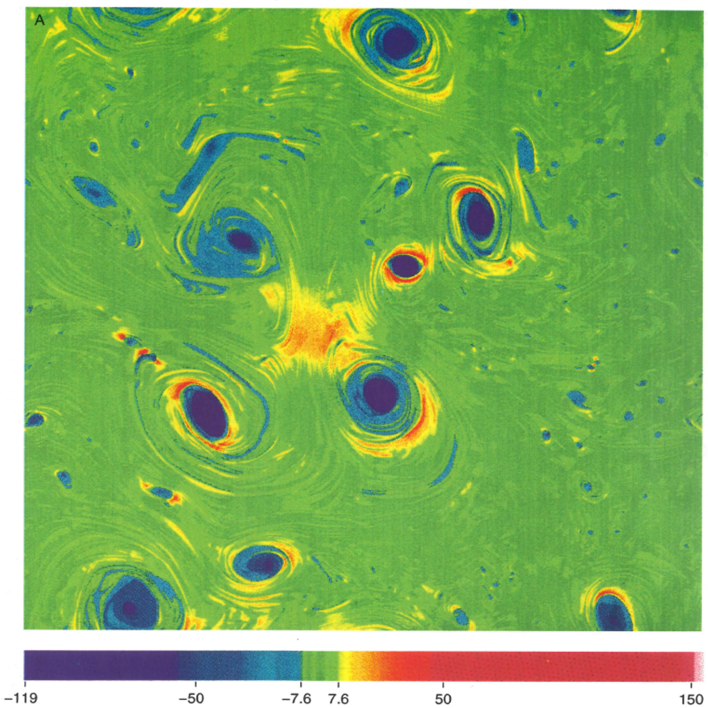
From Hua et al.(Physica D 1998)



ζ



$\tilde{p}$



$$\lambda_o^2 = \frac{OW}{4} = -\frac{\Delta\tilde{p}}{2}$$

OW is directly related to the Laplacian of the ageostrophic pressure. The total area of regions with significant positive values (stirring regions) of W is small compared to the total area of the vortices (no tracer gradient growth). **OW allows to get the ageostrophic pressure field.**

The regions $\tilde{p}_{xx} + \tilde{p}_{yy}$ and $\tilde{p}_{xy}$ further affect the stirring

One way to visualize the tracer cascade is to integrate the tracer equation. Another way is to consider the particle dispersion evolution to get the **Lyapunov exponents**. We just need to know the surface velocity field and its evolution in time.

It is now common for many applied problems (such as oil spills, dispersion of pollutants and biogeochemical transport) to make use of Lyapunov exponents

First it can be shown that the **particle dispersion is driven by the same mechanisms as those that affect the tracer gradients**.

The argument is the following …

Particle dispersion: a dual problem

[see Lapeyre, Chaos 2002]

Using $X = \begin{bmatrix} x \\ y \end{bmatrix}$ and $\frac{dX}{dt} = U(X)$ with $U = (u, v)$,

we consider $\delta X = \begin{bmatrix} \delta x \\ \delta y \end{bmatrix}$ the vector separating two particles initially close. From a Taylor expansion we get:

$$\boxed{\frac{d\,\delta X}{dt} = \left[\nabla U\right]\delta X} \quad (\mathrm{II}-1) \quad \text{with } [\nabla U] = \begin{bmatrix} u_x & u_y \\ v_x & v_y \end{bmatrix}$$

Properties of the particle dispersion can be directly related to the properties of the tracer cascade:

If C_1 and C_2 are the tracer magnitudes for two particles separated by the vector δX, then a Taylor series expansion leads to:

$$\delta C = C_1 - C_2 \approx \nabla C^T \cdot \delta X$$

Since C is conserved on a Lagrangian trajectory, we have:

$$\frac{d}{dt}\left[\nabla C^T \cdot \delta X\right] = 0 \quad (\mathrm{II}-2)$$

which leads (using II-1) to:

$$\frac{d\nabla C^T}{dt} = -\nabla C^T \cdot [\nabla U] \quad \text{or} \quad \underline{\frac{d\nabla C}{dt} = -[\nabla U]^T \cdot \nabla C} \quad (\mathrm{II}-3)$$

The OW quantity is close to 0.1-0.2 f^2 and, therefore, λ_o is statistically close to 0.3-0.4 f

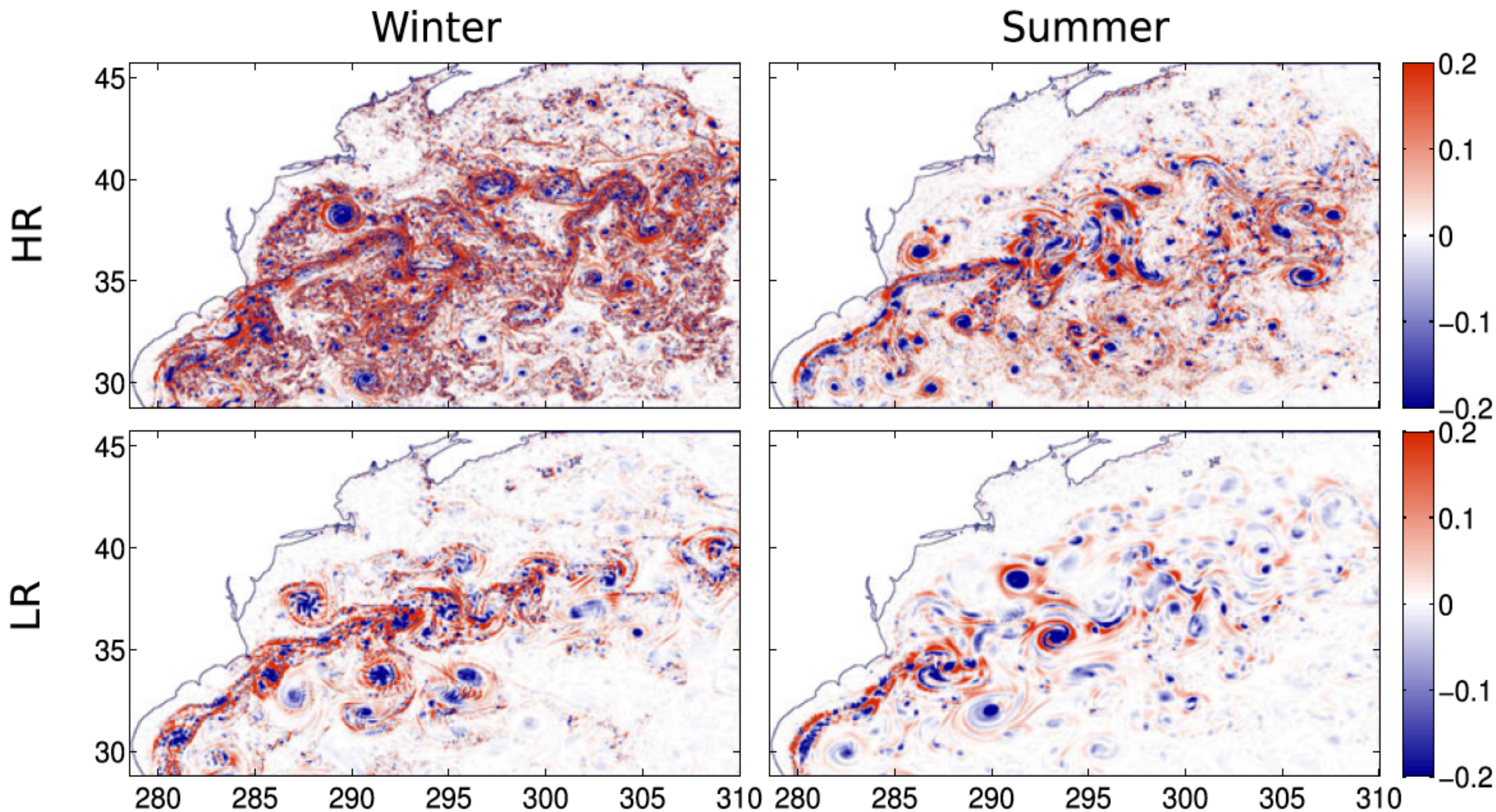


Fig. 14 Okubo–Weiss parameter normalized by f_0^2 computed at the surface (5 m) for winter (*left column*), summer (*right column*), HR (*top row*), and LR (*bottom row*)

From numerical simulations [Mensa et al., OD 2013]

From Mensa et al., OD 2013

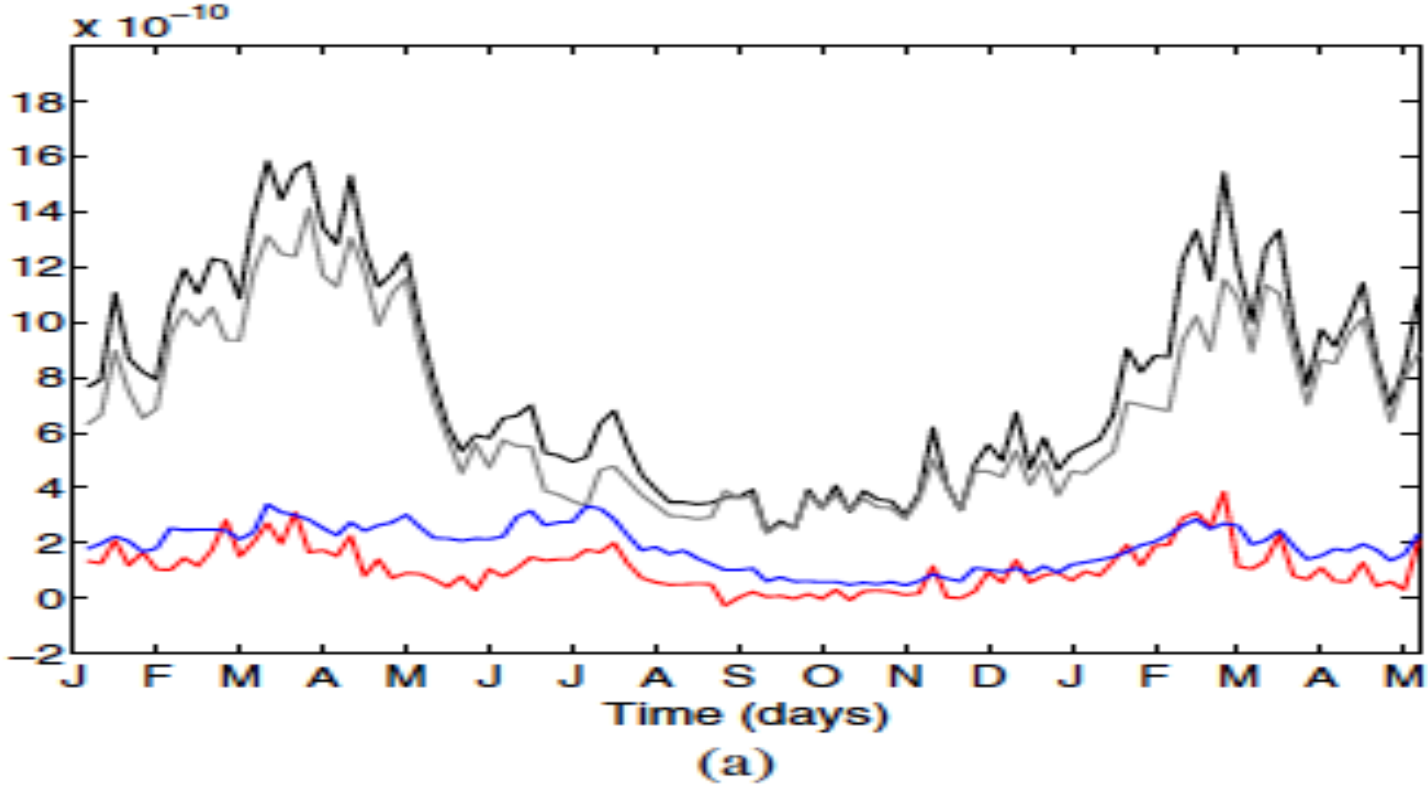


Fig. 15 Temporal evolution of the components of the Okubo–Weiss parameter (per square second, from Eq. 9) integrated over region A at a 5-m depth for **a** HR and **b** LR: OW parameter (*red line*), S^2 (strain rate, *black line*), ξ^2 (relative vorticity squared, *gray line*), and δ^2 (divergence squared, *blue line*)

S_1, S_2 and ζ are <u>statistically</u> of the order 0.4 f

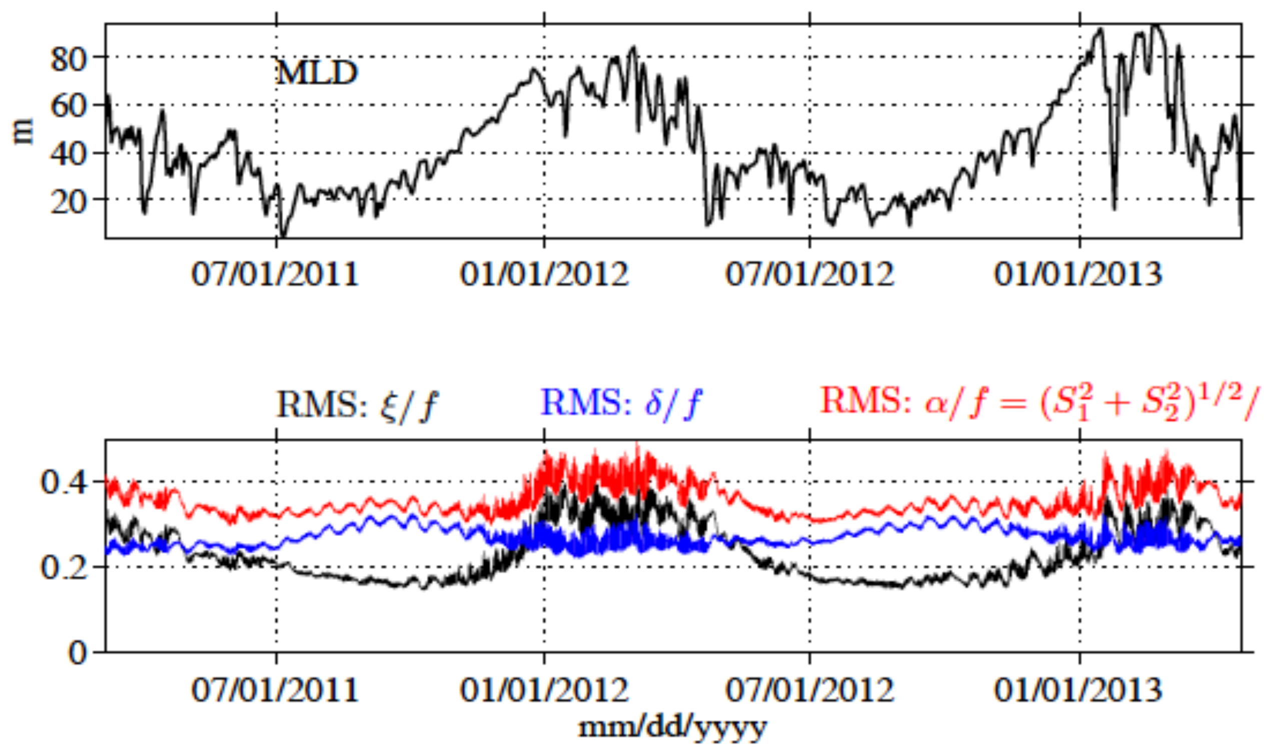
Dynamical fields (waves + balanced motions) display a strong seasonality!

Impacts the waves ($\omega > f$)

Square of the relative vorticity:
$\xi^2 = (\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y})^2$

Square of the horizontal flow divergence:
$\delta^2 = (\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y})^2$

Square of the horizontal strain:
$S^2 = S_1^2 + S_2^2 = (\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y})^2 + (\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y})^2$



Relative vorticity is larger in winter (0.4) and divergence larger in summer (0.3)!

Hector Torres: JPL 2016 … **in the Eastern North Pacific** (10°N-30°N, -130°W—110°W)

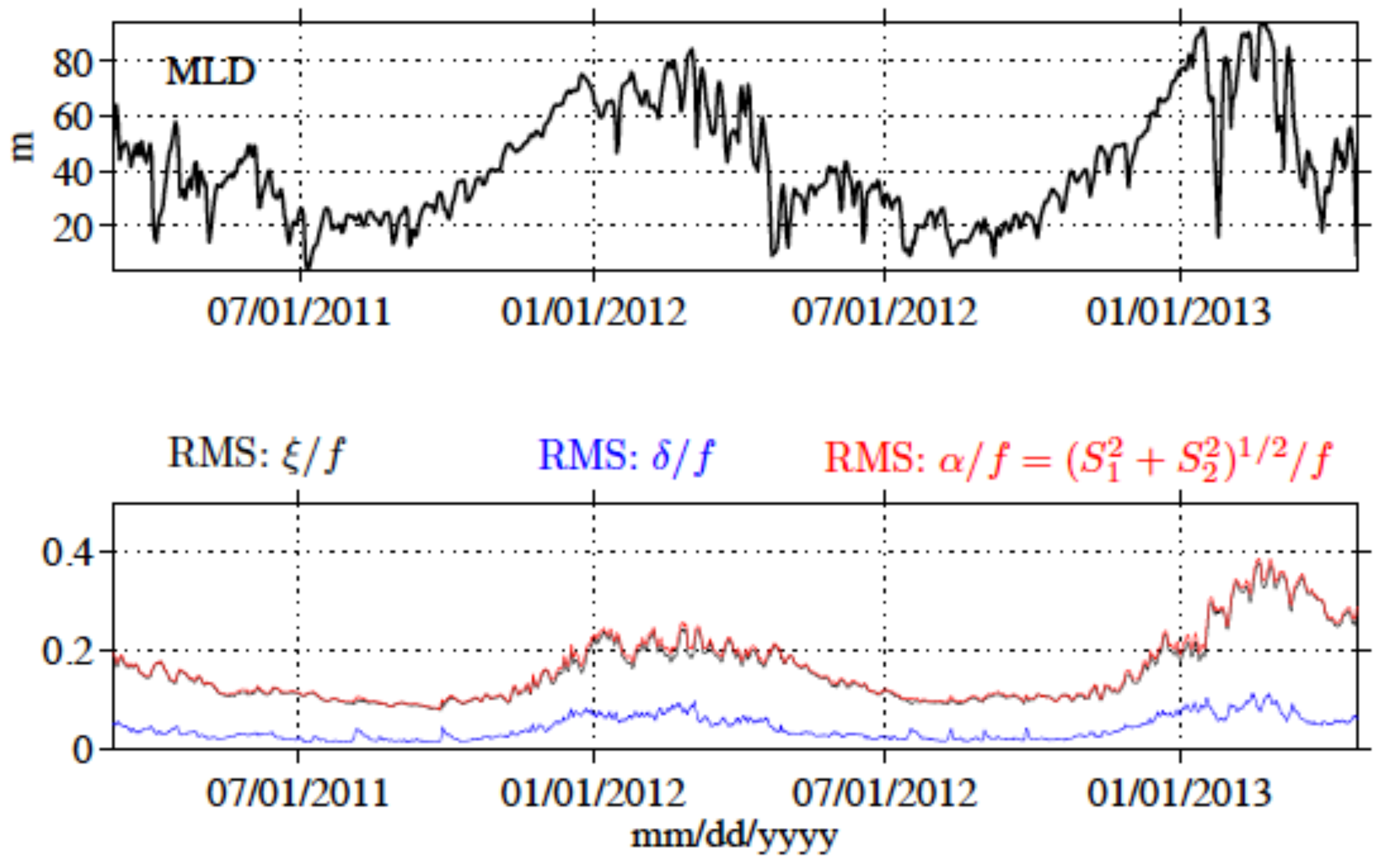
Seasonality is different when the wave part is removed!

and kinematic properties: $\boxed{\text{daily-averaged}}$

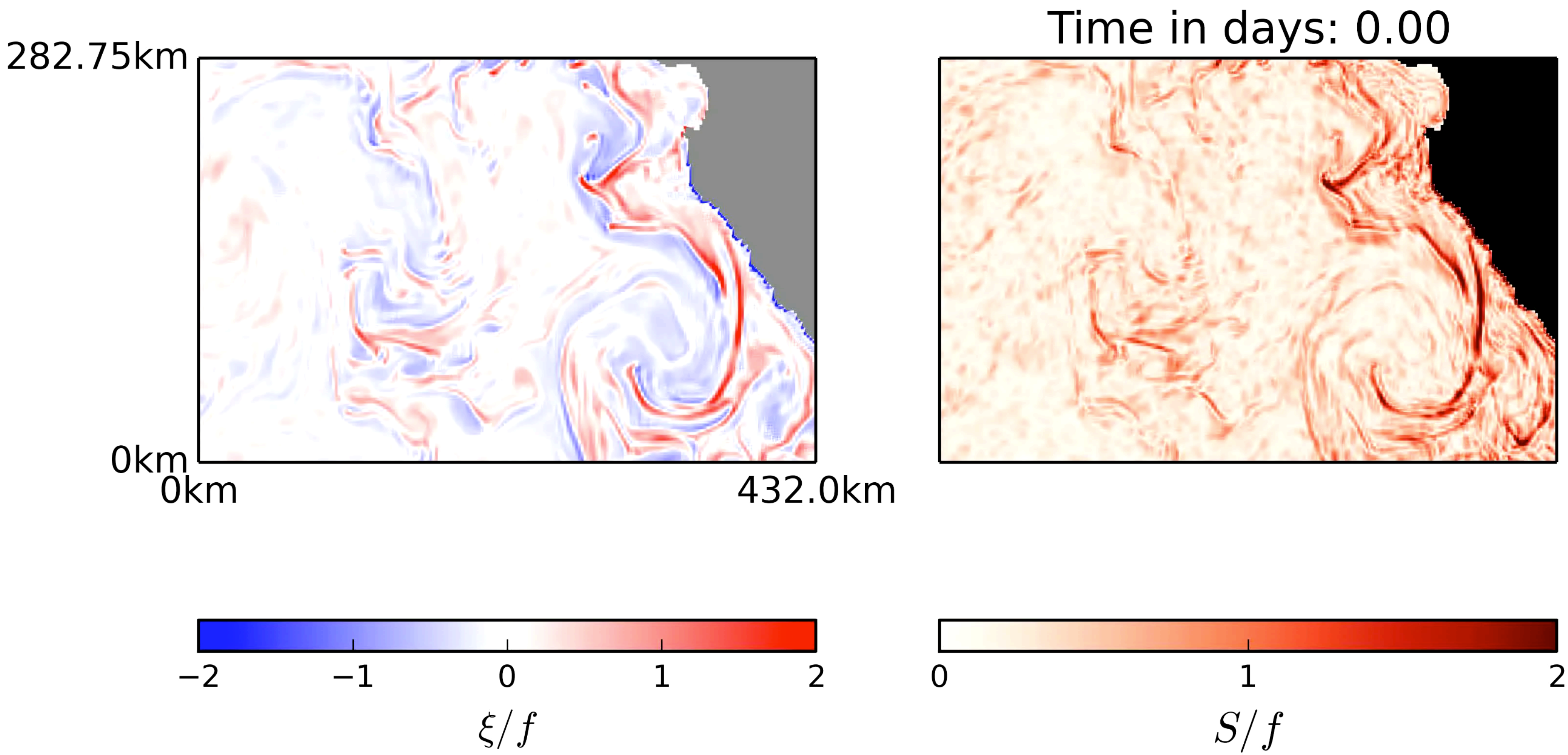
Square of the relative vorticity:
$\xi^2 = (\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y})^2$

Square of the horizontal flow divergence:
$\delta^2 = (\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y})^2$

Square of the horizontal strain:
$S^2 = S_1^2 + S_2^2 = (\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y})^2 + (\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y})^2$



Hector Torres: JPL 2016 … **in the Eastern North Pacific** (10°N-30°N, -130°W—110°W)



Time in days: 0.00
282.75km
0km
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432.0km
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−1
0
1
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ξ/f
0
1
2
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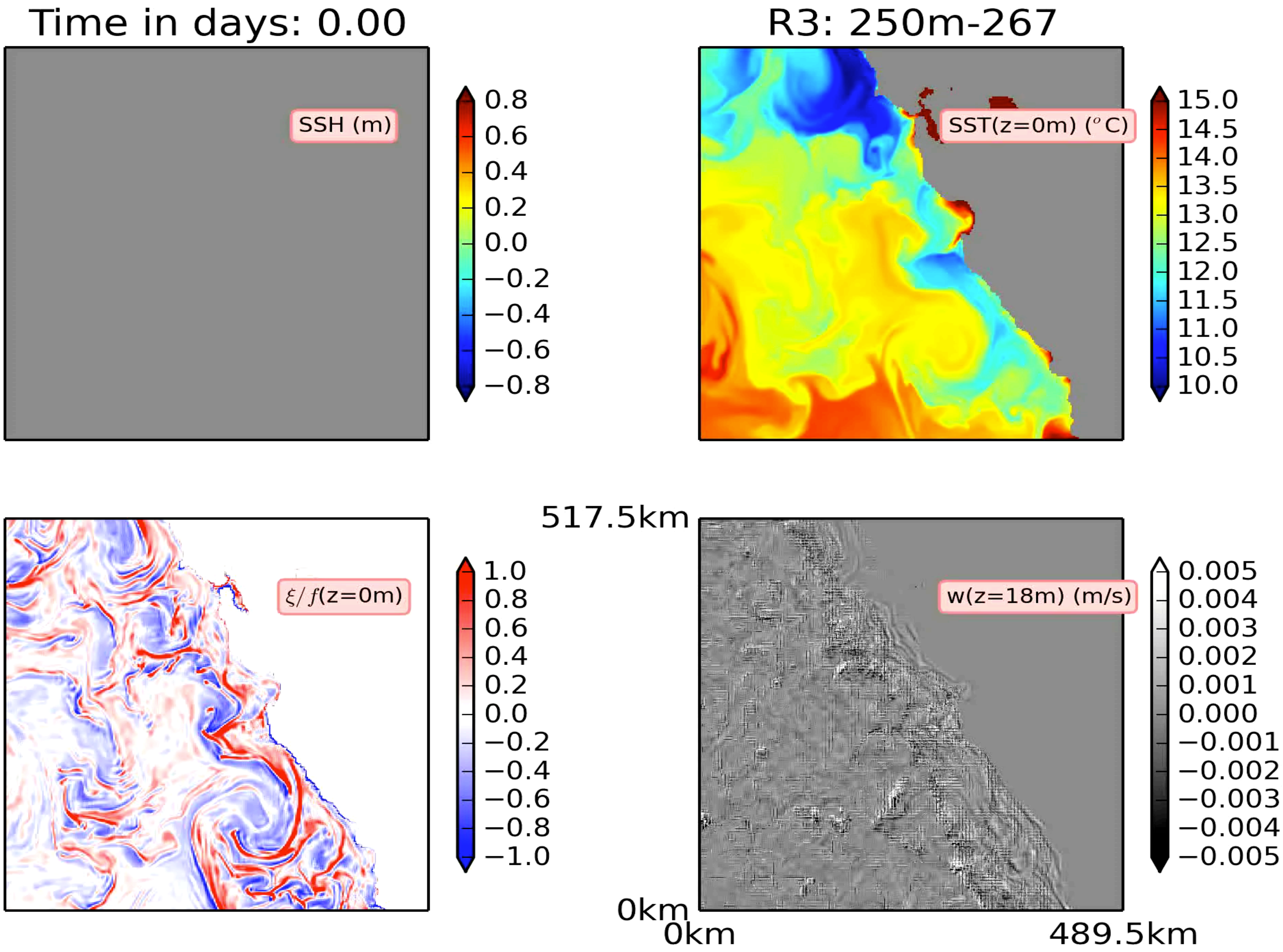

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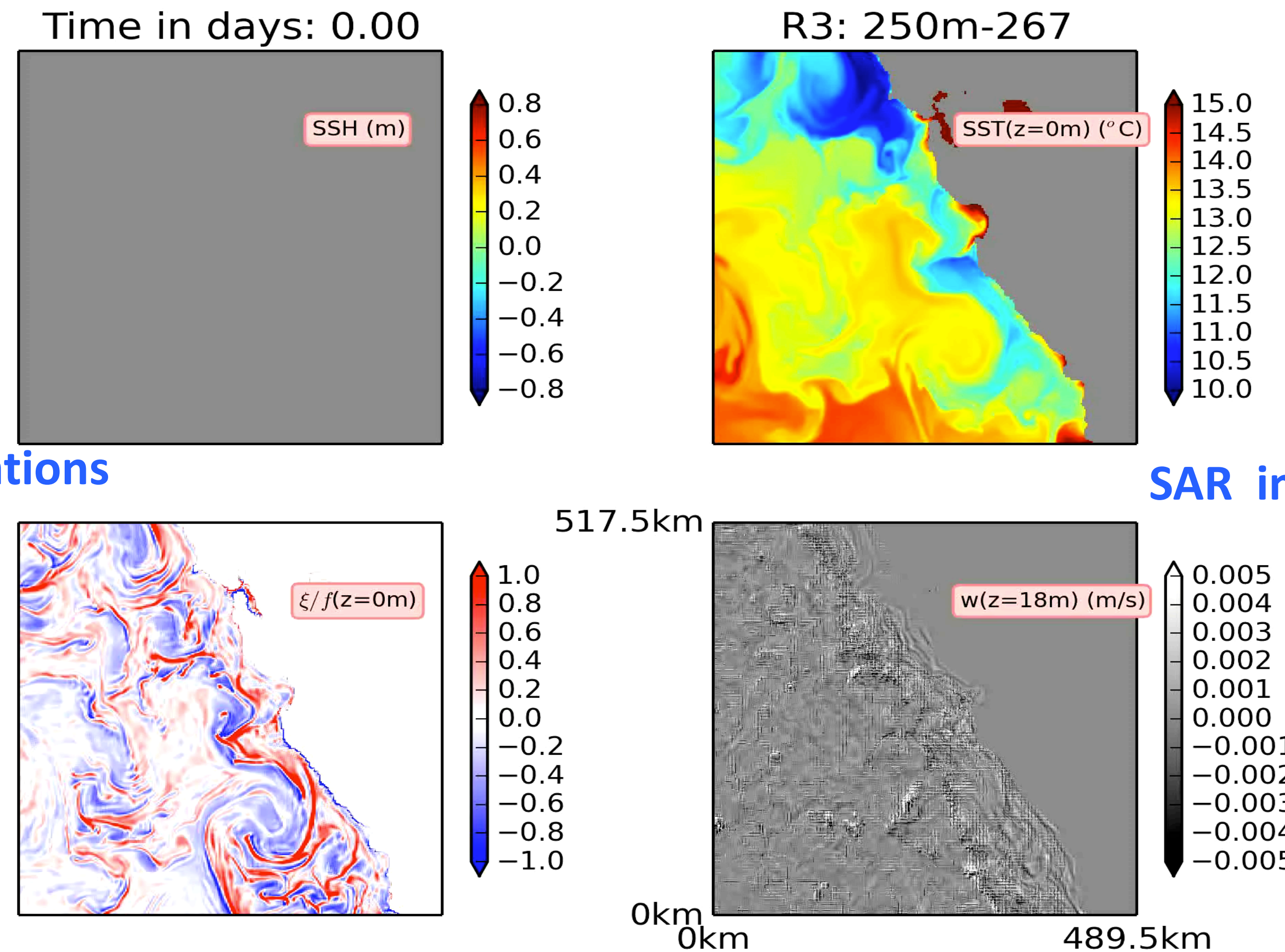
Why waves and geostrophic turbulence have different signatures on SSH, SST, … ?

Altimeter observations

Infrared images

U,V observations

SAR images



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Waves and geostrophic turbulence have different signatures on SSH, SST, … ?