”Ocean Turbulence from SPACE”

(XVIII) - <u>submesoscale instability</u>

[The role of mixed-layer instabilities in submesoscale turbulence]

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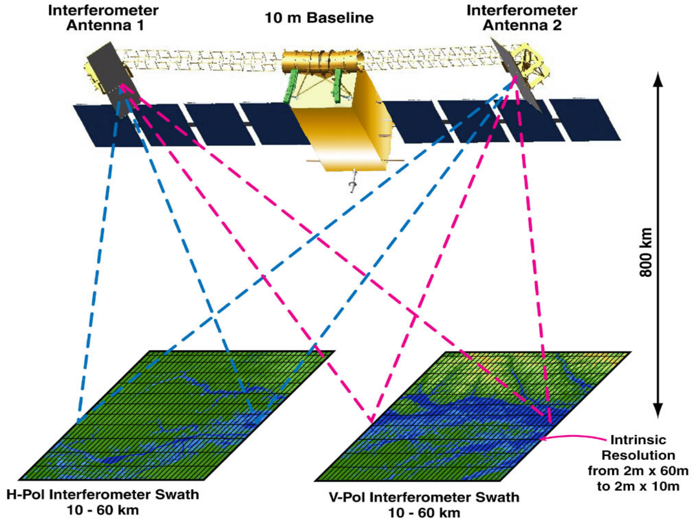
(Caltech)



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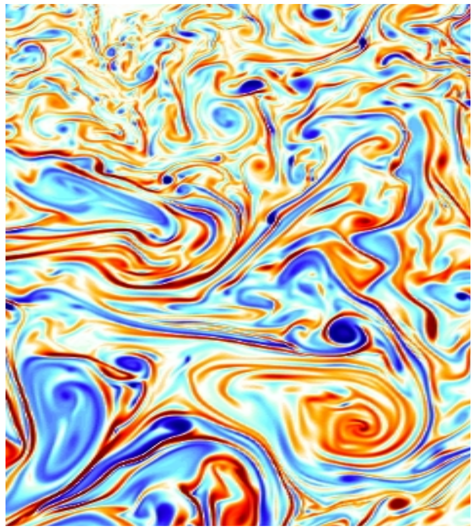


Introduction

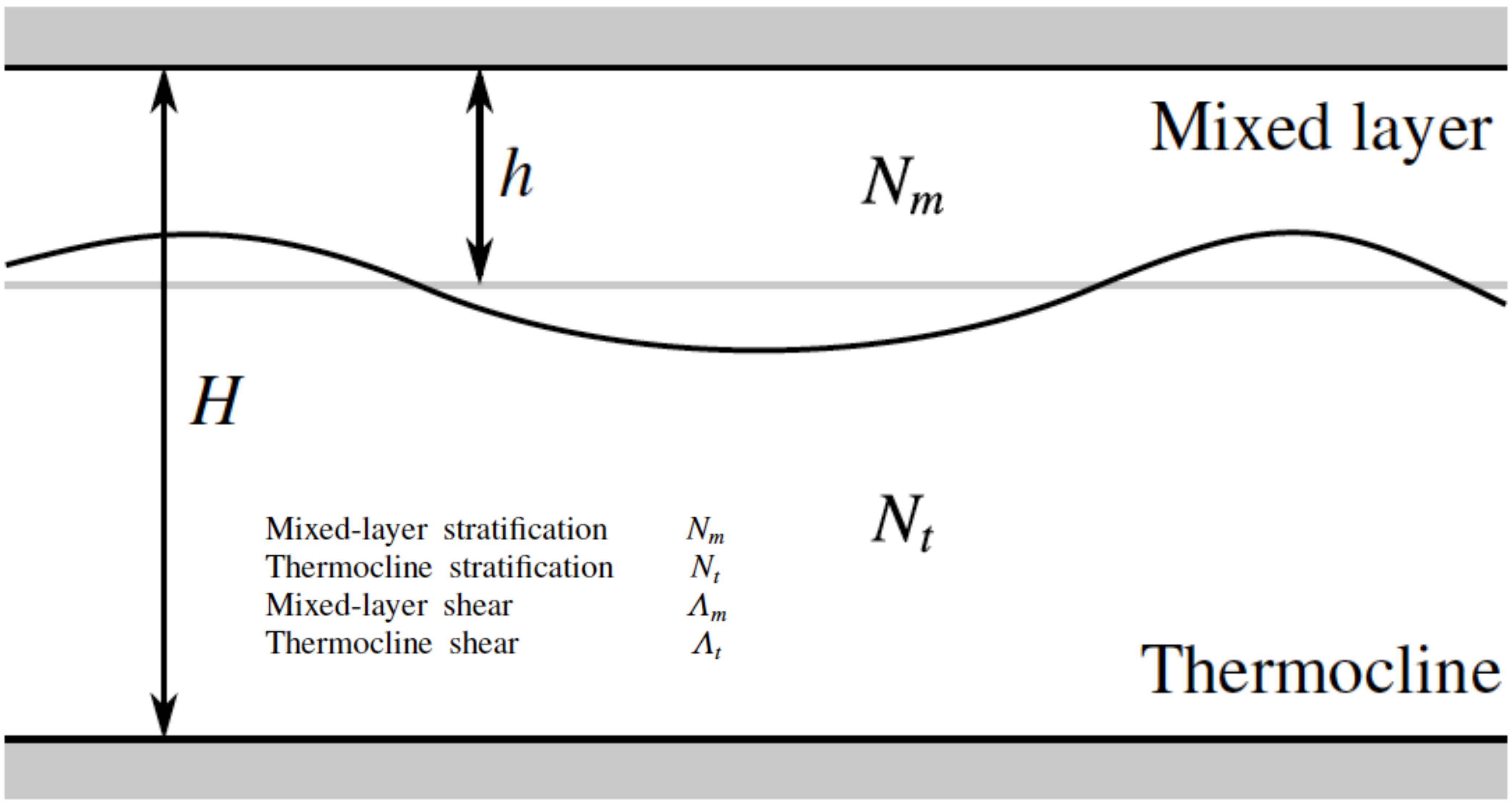
What energizes the upper ocean submesoscale turbulence?

mesoscale-driven surface frontogenesis	V.S.	baroclinic mixed-layer instability
SQG		Eady (mixed layer + thermocline)

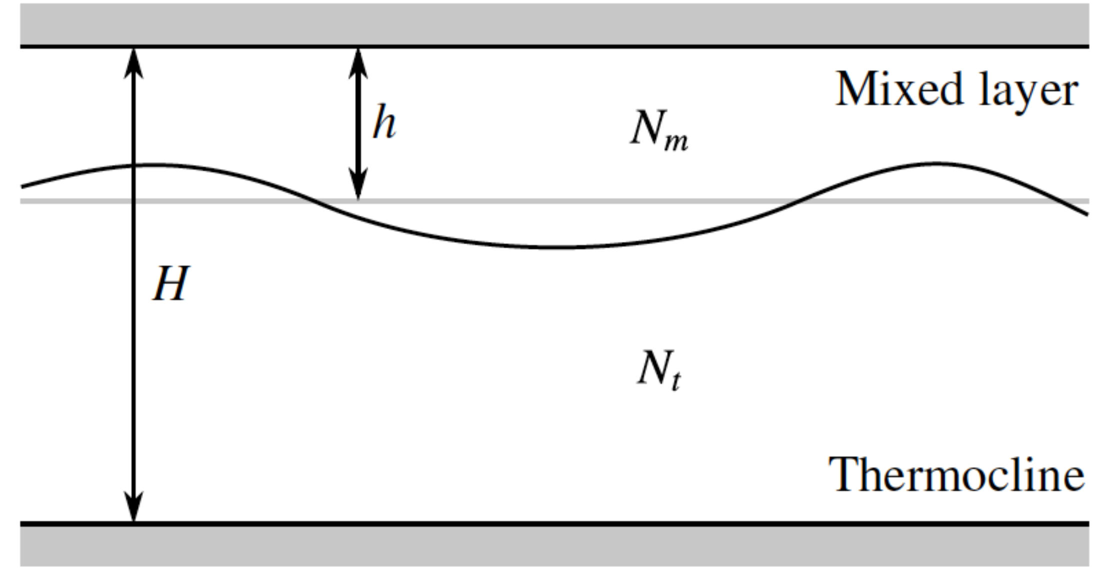
A strong seasonal cycle in submesoscale turbulence: BMI?

(Mensa et al. 2013; Sasaki et al. 2014; Callies et al.2015)

Model formulation



Two layers with constant stratification and constant mean shear on an f-plane (each layer has constant PV)



Eady problem

Flow in the interior

$$q = \nabla^2\psi + \frac{\partial}{\partial z}\left(\frac{f^2}{N^2}\frac{\partial\psi}{\partial z}\right) = 0,$$

$$b = f\partial\psi/\partial z$$

$$\frac{\partial b}{\partial t} + \mathrm{J}(\psi, b) + \cancel{wN^2} = 0.$$

b.c.

matching conditions at the interface?

just above the interface at $z = -h$,

$$\frac{\partial b^+}{\partial t} + \mathrm{J}(\psi_1, b^+) + wN_m^2 = 0, \quad b^+ = f\frac{\partial \psi}{\partial z}(-h^+),$$

eliminating w

same stream function

$$\frac{\partial \theta_1}{\partial t} + \mathrm{J}(\psi_1, \theta_1) = 0.$$

just below the interface,

$$\frac{\partial b^-}{\partial t} + \mathrm{J}(\psi_1, b^-) + wN_t^2 = 0, \quad b^- = f\frac{\partial \psi}{\partial z}(-h^-),$$

$$\theta_1 = f\left(\frac{b^+}{N_m^2} - \frac{b^-}{N_t^2}\right),$$

a conserved quantity
(integrated PV across the interface)

two-layer model —> three PV sheets

Approaches one-layer Eady model when two stratifications are the same.

$$q = \theta_0\delta(z) + \theta_1\delta(z+h) + \theta_2\delta(z+H),$$

where δ is Dirac's delta function and $\theta_0 = -fb/N_m^2$ at $z=0$, $\theta_2 = fb/N_t^2$ at $z=-H$

$$\frac{\partial \theta_j}{\partial t} + \mathrm{J}(\psi_j, \theta_j) = 0,$$

essentially the boundary conditions on buoyancy!

$$\hat{\boldsymbol{\theta}} = \boldsymbol{L}\hat{\boldsymbol{\psi}}, \quad \boldsymbol{\theta} = (\theta_0, \theta_1, \theta_2)^{\mathrm{T}}, \quad \boldsymbol{\psi} = (\psi_0, \psi_1, \psi_2)^{\mathrm{T}}.$$

Fourier transformed equations

$$-{k_h}^2\hat{\psi} + \frac{\partial}{\partial z}\left(\frac{f^2}{N^2}\frac{\partial \hat{\psi}}{\partial z}\right) = 0$$

$$\frac{\partial \hat{\boldsymbol{\theta}}}{\partial t} + \mathrm{i}k\boldsymbol{U}\hat{\boldsymbol{\theta}} + \mathrm{i}k\boldsymbol{\Gamma}\hat{\boldsymbol{\psi}} = 0,$$

The forcing term comes from the velocity shear

Parameter	Symbol	Value
Mixed-layer depth	h	100 m
Total depth	H	500 m
Mixed-layer stratification	N_m	2×10^{-3} s^{-1}
Thermocline stratification	N_t	8×10^{-3} s^{-1}
Mixed-layer shear	Λ_m	10^{-4} s^{-1}
Thermocline shear	Λ_t	10^{-4} s^{-1}
Coriolis frequency	f	10^{-4} s^{-1}
Domain size	a	500 km
Numerical resolution	Δx	$\sim$1 km

Same velocity shear in two layers. The stratification in the thermocline is 4 times larger than that in the mixed-layer!

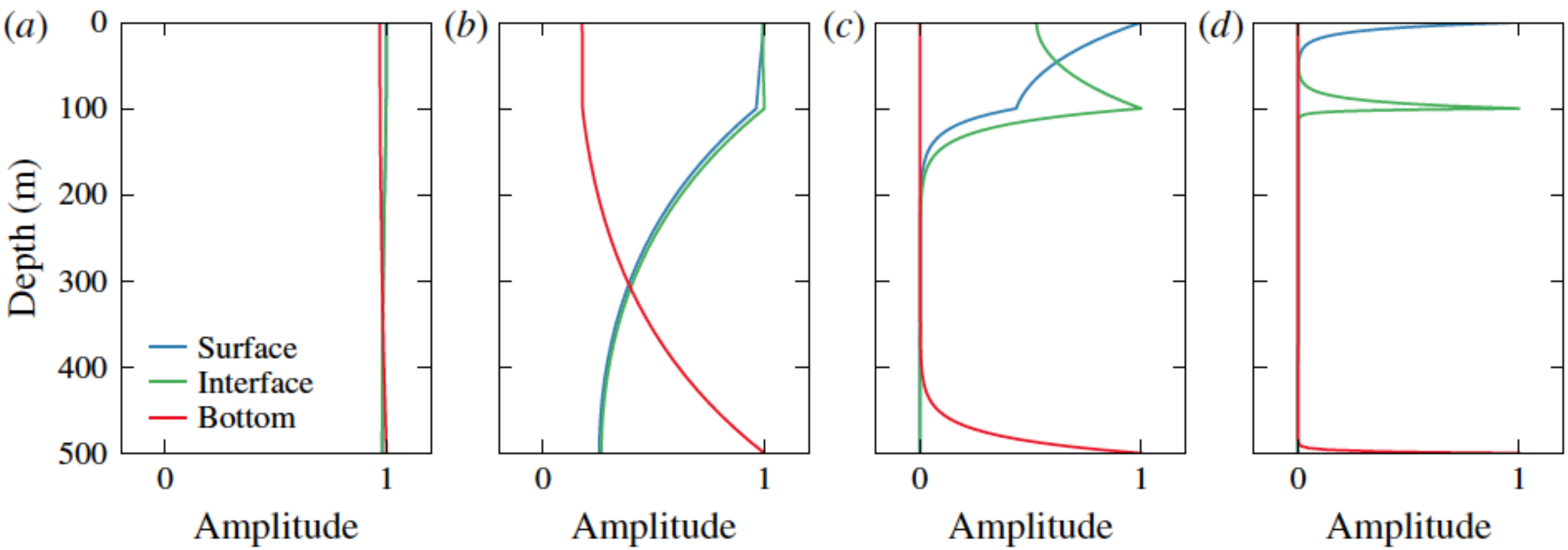
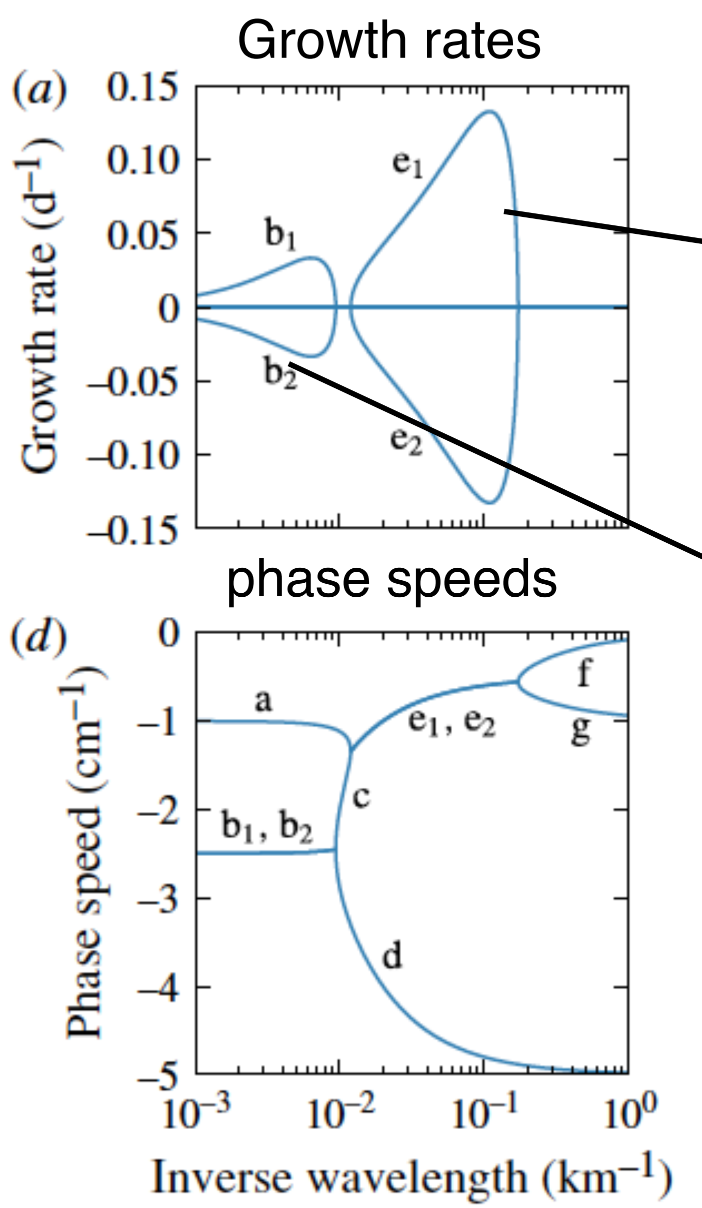
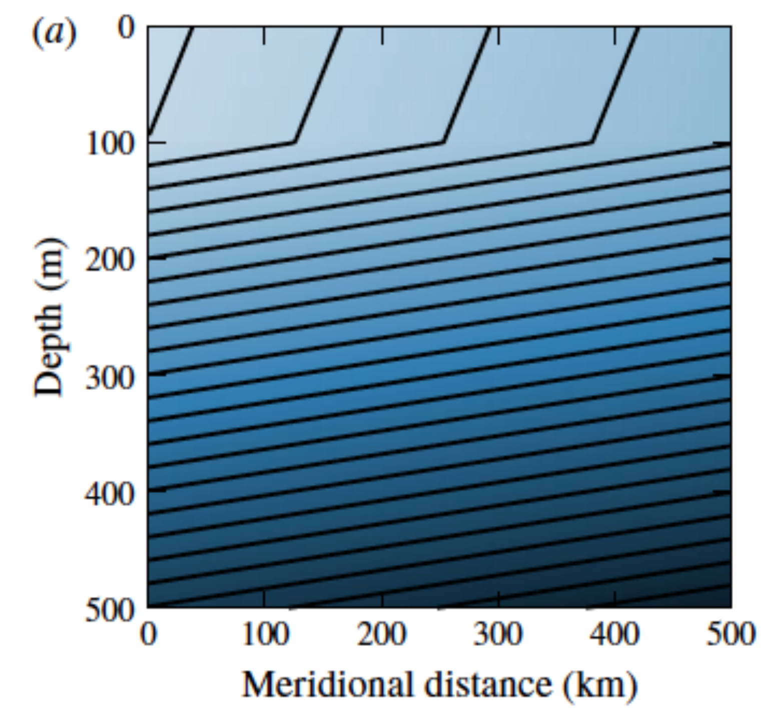


FIGURE 3. Vertical structure of streamfunction amplitude associated with anomalies of θ_0 (surface), θ_1 (interface) and θ_2 (bottom). Shown are the vertical profiles for θ_j anomalies with different horizontal wavenumbers $k_h = 2\pi/\lambda$. The wavelength λ is given in the respective panel title: (*a*) 1000 km wavelength; (*b*) 100 km wavelength; (*c*) 10 km wavelength; (*d*) 1 km wavelength.

Linear stability analysis

Full model





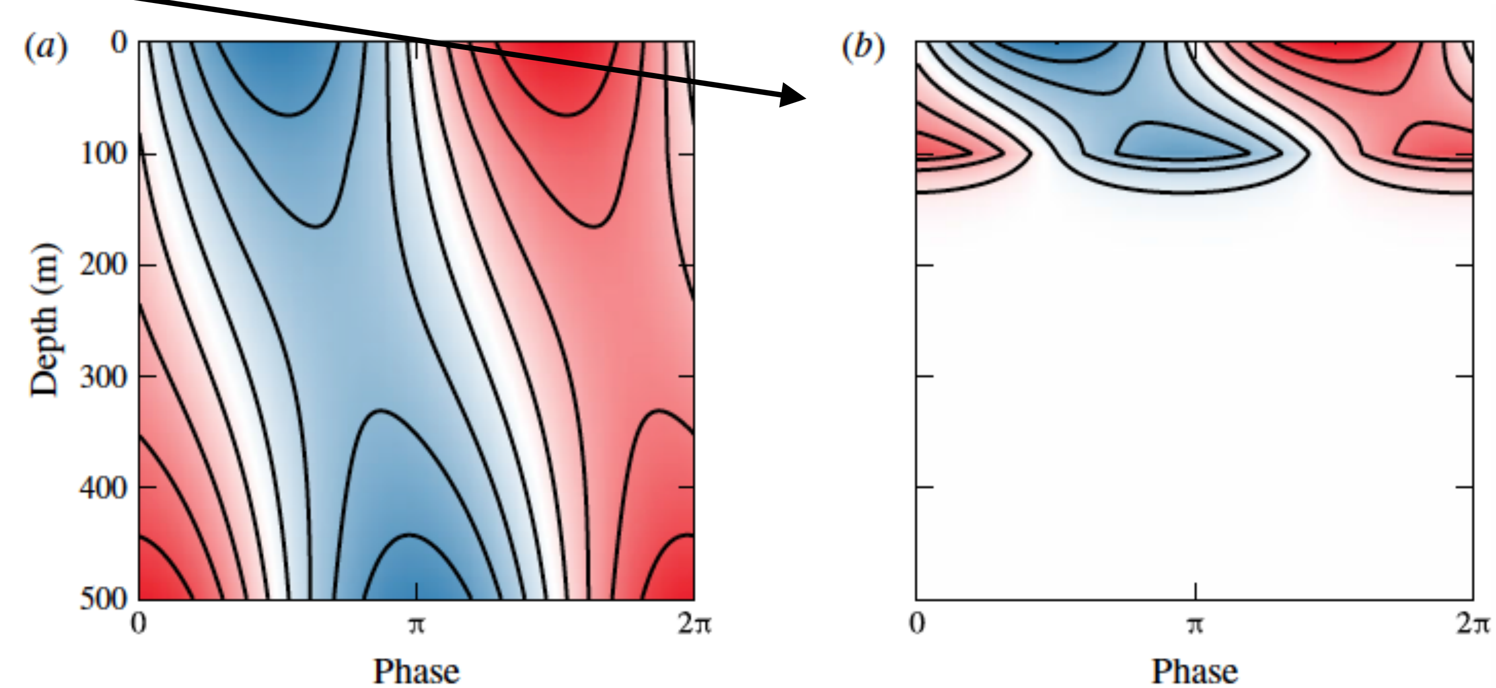


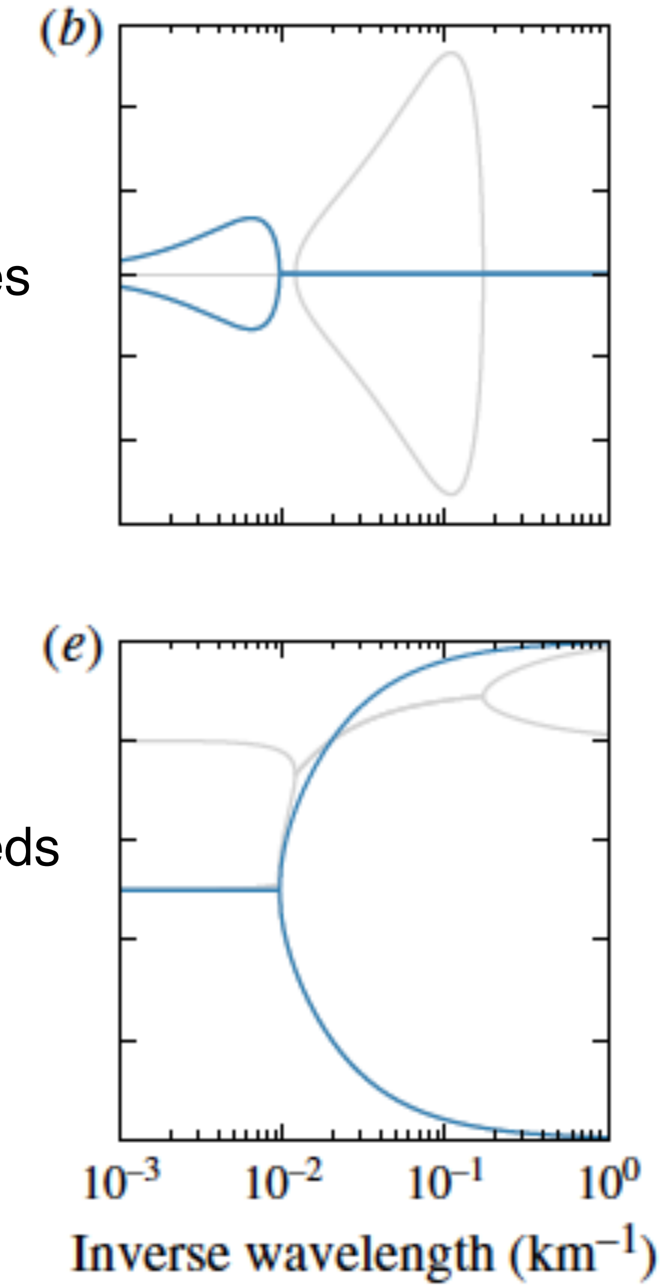
FIGURE 6. Perturbation streamfunction of the most unstable (*a*) mesoscale (160 km) and (*b*) submesoscale (9 km) modes of the full model. Red and blue shading represents positive and negative values, respectively.

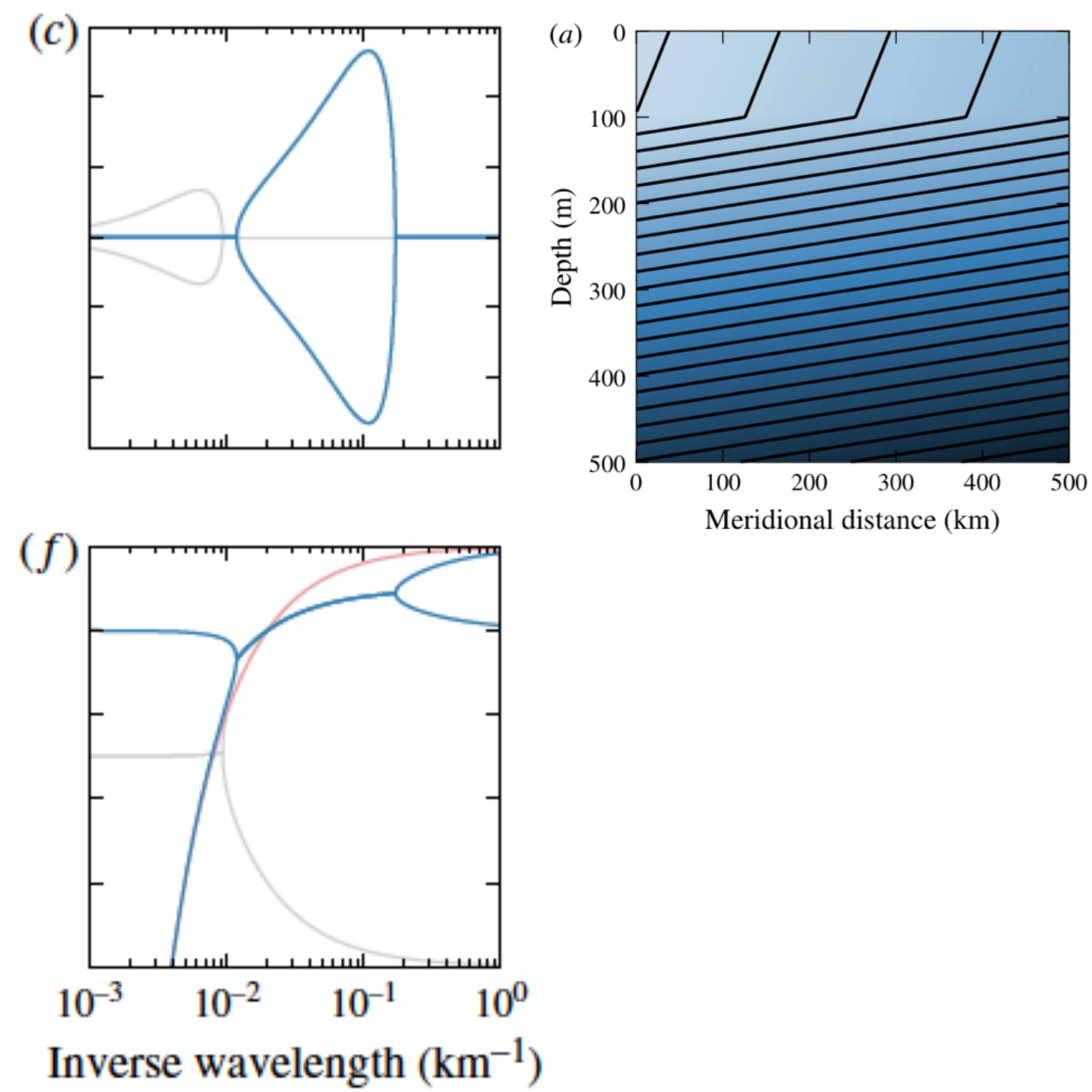
Thermocline only

Mixed layer only

Growth rates

phase speeds





$$c = -\frac{\Lambda h}{2}\left(1+\frac{\alpha}{\mu_m}\right) \pm \frac{\mathrm{i}\Lambda h}{\mu_m}\left[\frac{(1-\alpha^2)(\mu_m - \tanh\mu_m)}{\tanh\mu_m+\alpha} - \frac{1}{4}(\mu_m-\alpha)^2\right]$$

Approaches Eady model growth rate with these limits

$$\left(\alpha \ll 1 \text{ and } \alpha \ll \mu_m\right)$$

$$c = -\frac{\Lambda_t H}{2} \pm \frac{\mathrm{i}\Lambda_t H}{\mu_m}\left(\mu_t \coth\mu_t - 1 - \frac{\mu_t^2}{4}\right)^{1/2}$$

$$\sigma = 0.31 f \Lambda_t / N_t$$
$$\lambda = 3.9 N_t H / f.$$

Blumen_JAS79

Atmosphere (boundary layer + troposphere)

interface $0 \leq H \leq 1$ $\alpha = N_2/N_1 = 1.1$

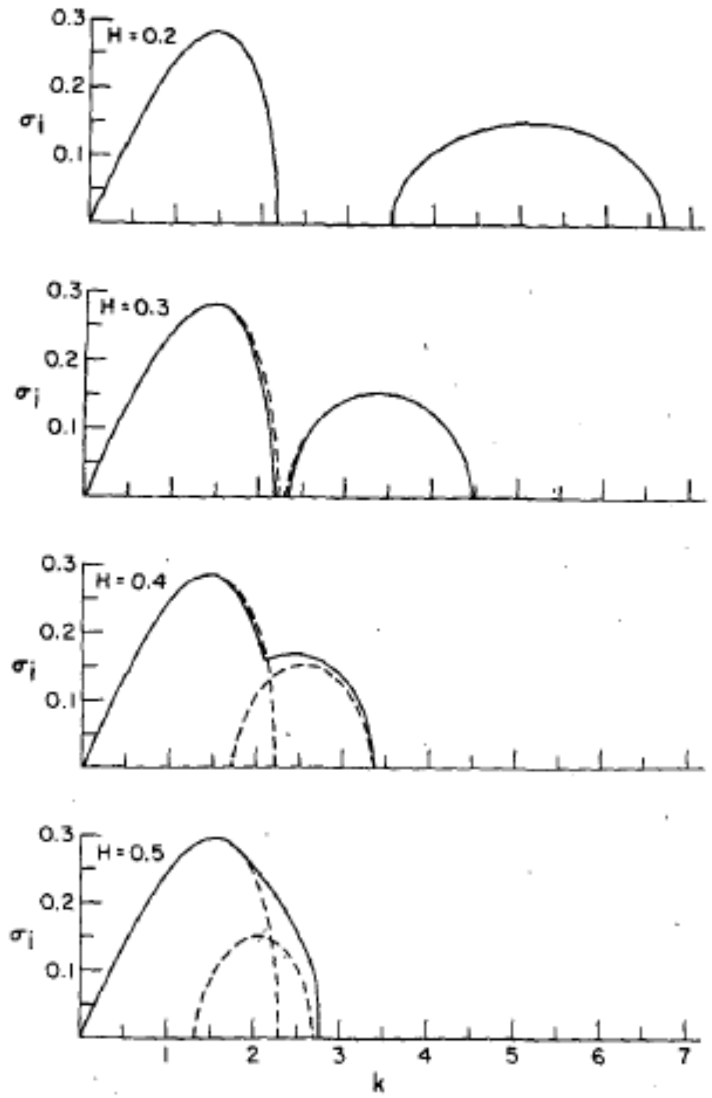


FIG. 2. Unstable growth rates $\sigma_i(k)$ for a two-layer Eady model with $\alpha = N_2/N_1 = 1.1$. H is the interface height. The dashed lines represent approximations derived in the text. Zonal wavenumbers at $\phi = \pi/4$ are as in Fig. 1.

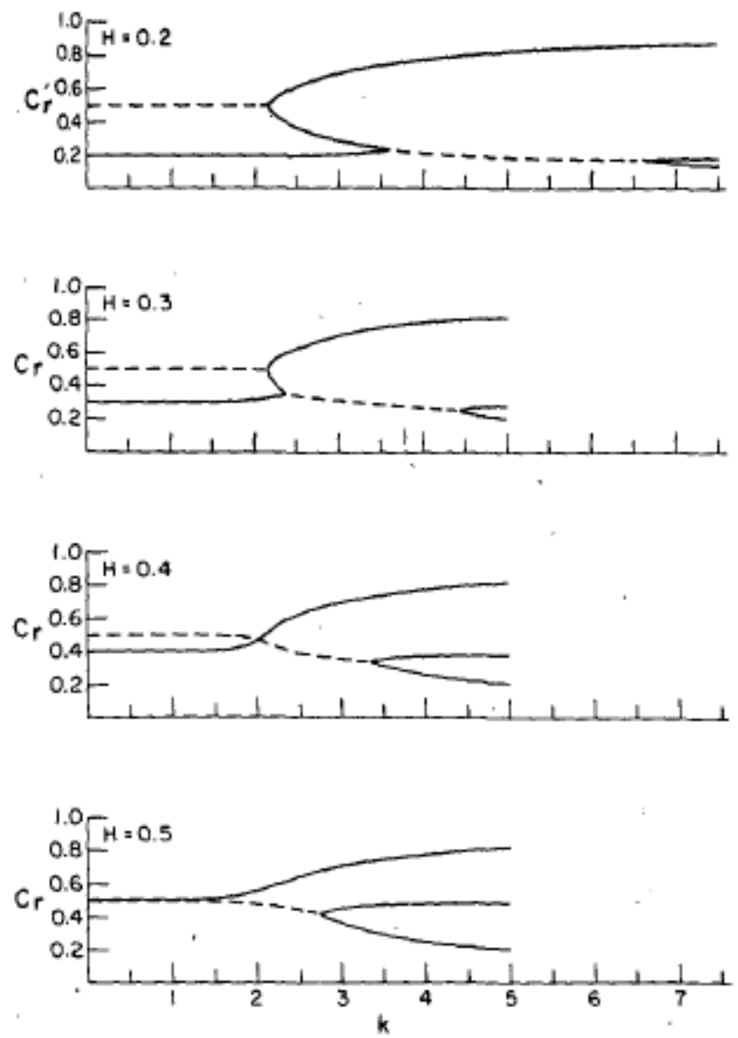


FIG. 3. Phase speeds $c_r(k)$ for a two-layer Eady model with $\alpha = N_2/N_1 = 1.1$. H is the interface height. The dashed lines represent the phase speeds of the unstable modes shown in Fig. 2. The solid lines refer to stable modes.

Ocean (boundary layer + thermocline)

$$\alpha = N_2/N_1 = 0.9$$

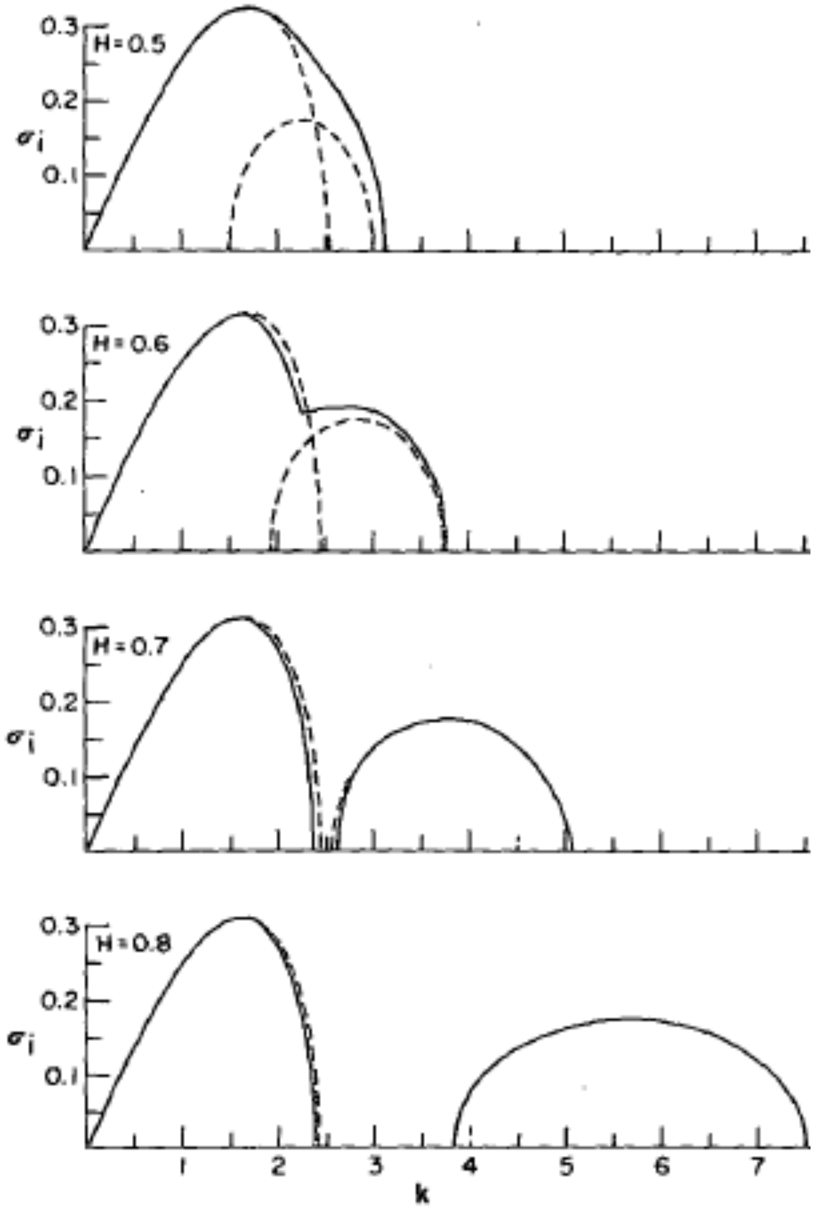


FIG. 4. As in Fig. 2 except $\alpha = N_2/N_1 = 0.9$.

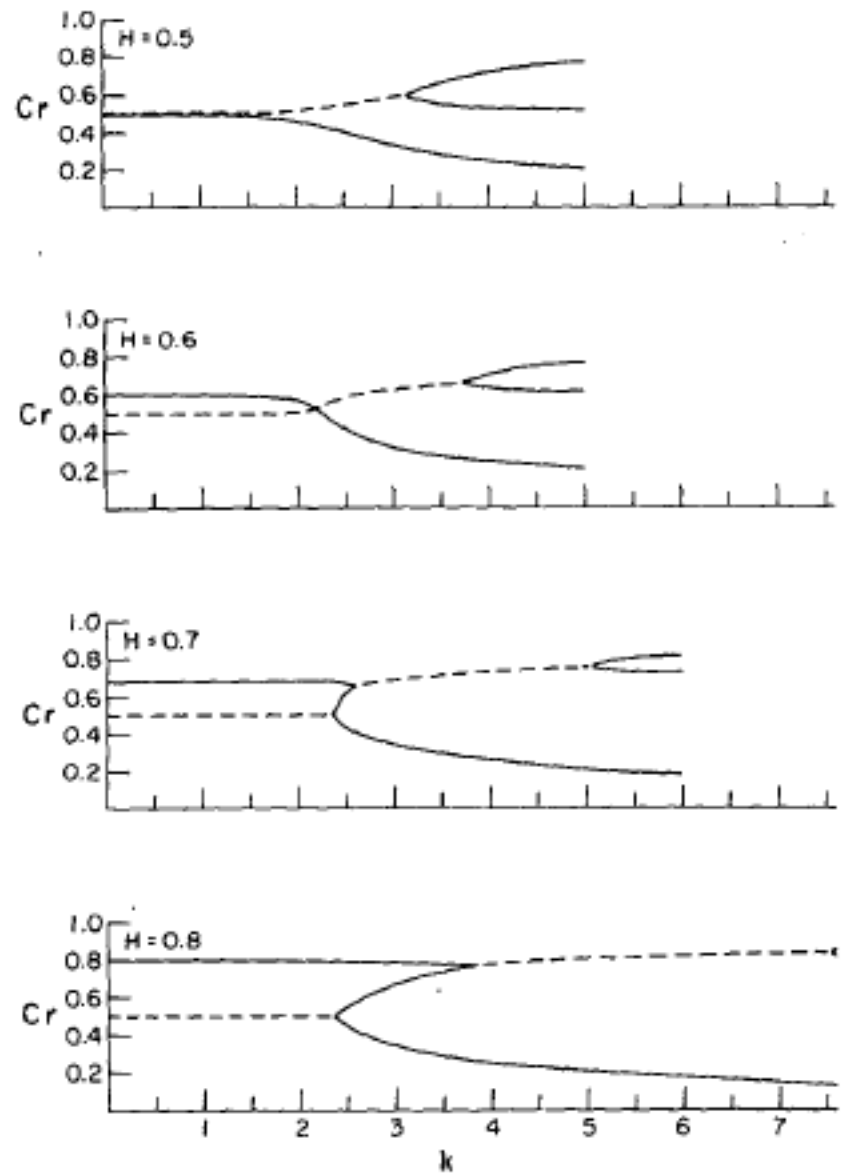
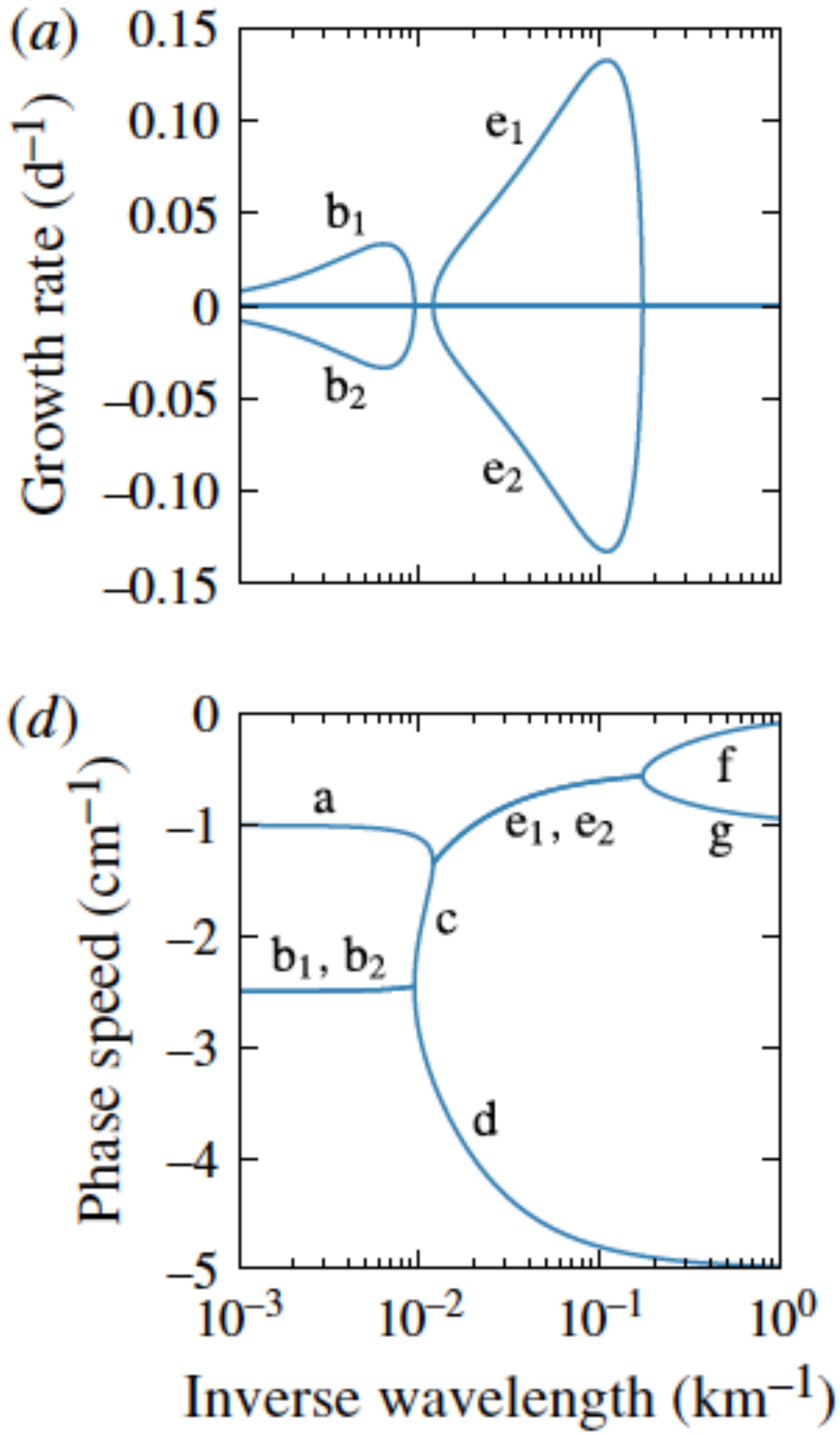


FIG. 5. As in Fig. 3 except $\alpha = N_2/N_1 = 0.9$.

summary

The long-wave unstable response is bounded by rigid horizontal surfaces. The short-wave instability is primarily confined to a layer which is bounded by a rigid lid and the surface of discontinuity z=H (a flexible lid)



a - baroclinic mode in the ML (doesn't sense the bottom)

b - conjugate Eady-like instability (thermocline)

c - baotropic mode in the ML (surface edge wave)

d - bottom edge wave

e - conjugate ML instability

f and g - edge waves on the surface and interface

Linear to nonlinear

Linear:

$$\frac{\partial \hat{\boldsymbol{\theta}}}{\partial t} + \mathrm{i}k\boldsymbol{U}\hat{\boldsymbol{\theta}} + \mathrm{i}k\boldsymbol{\Gamma}\hat{\boldsymbol{\psi}} = 0,$$

Hypoviscosity

Nonlinear:

$$\frac{\partial \boldsymbol{\theta}}{\partial t} + \boldsymbol{U}\frac{\partial \boldsymbol{\theta}}{\partial x} + \boldsymbol{\Gamma}\frac{\partial \boldsymbol{\psi}}{\partial x} + \mathrm{J}(\boldsymbol{\psi}, \boldsymbol{\theta}) = r\nabla^{-2}\boldsymbol{\theta} - \nu(-\nabla^2)^n\boldsymbol{\theta},$$

Nonlinear term

Hyperviscosity,
n=10

Thermocline-only model

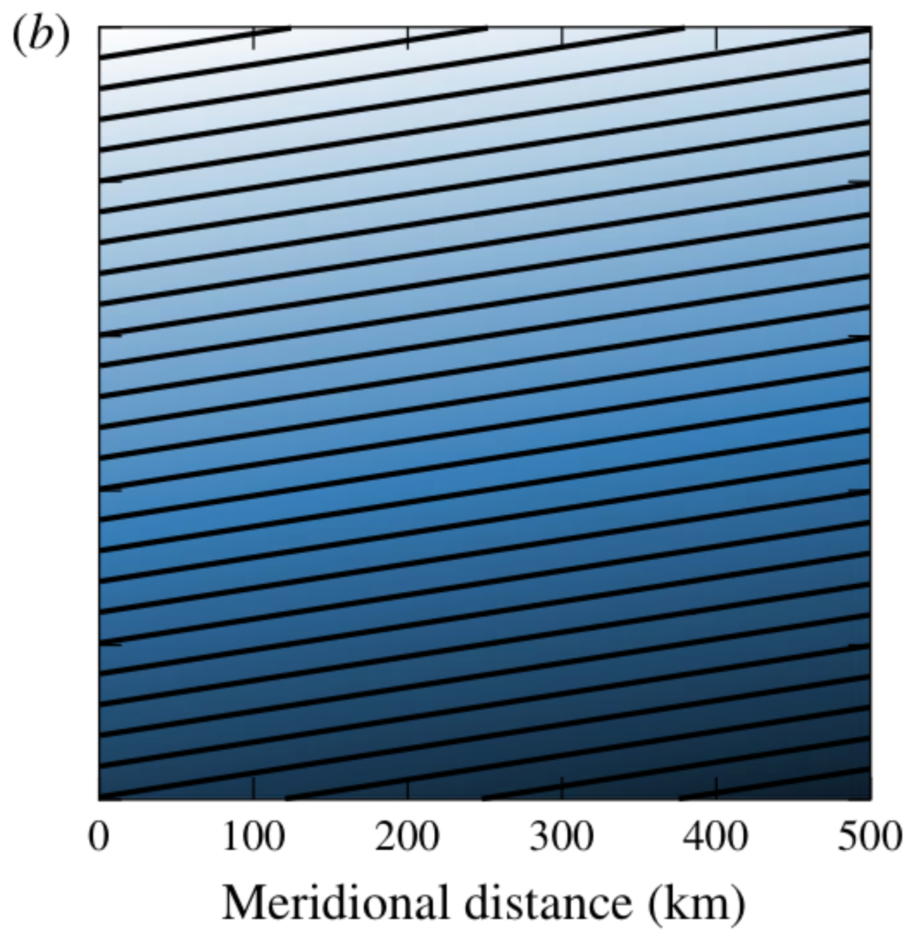


Figure 4b: Isopycnal contours, buoyancy colored.

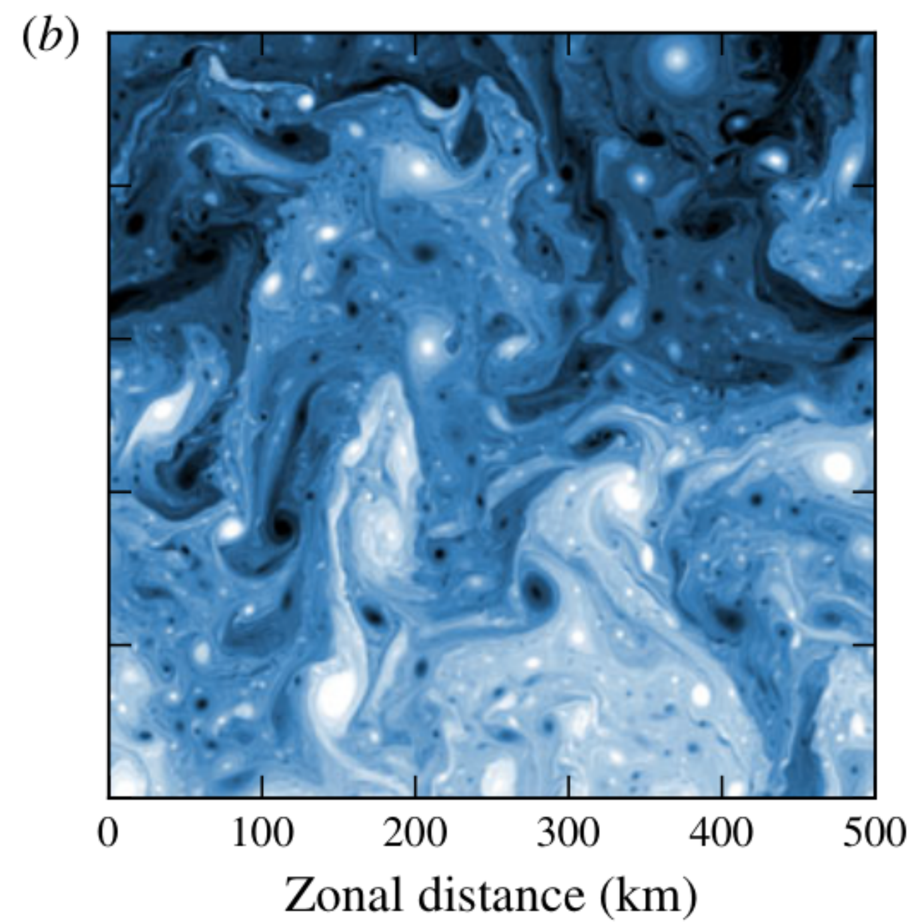


Figure 7b: Surface buoyancy snapshot, equilibrated model.

Thermocline-only model

$$K_{k,l} = \frac{1}{2}(|\hat{u}|^2 + |\hat{v}|^2), \qquad P_{k,l} = \frac{1}{2}\frac{|\hat{b}|^2}{N^2}.$$

Steeper slope at depth a result of SQG dynamics

The abrupt cut-off here is simply a product of the colorbar chosen, where all values smaller than 10^{-3} m^3/s^2 are white.

Rigid lid at bottom results in SQG dynamics at the lower boundary as well.

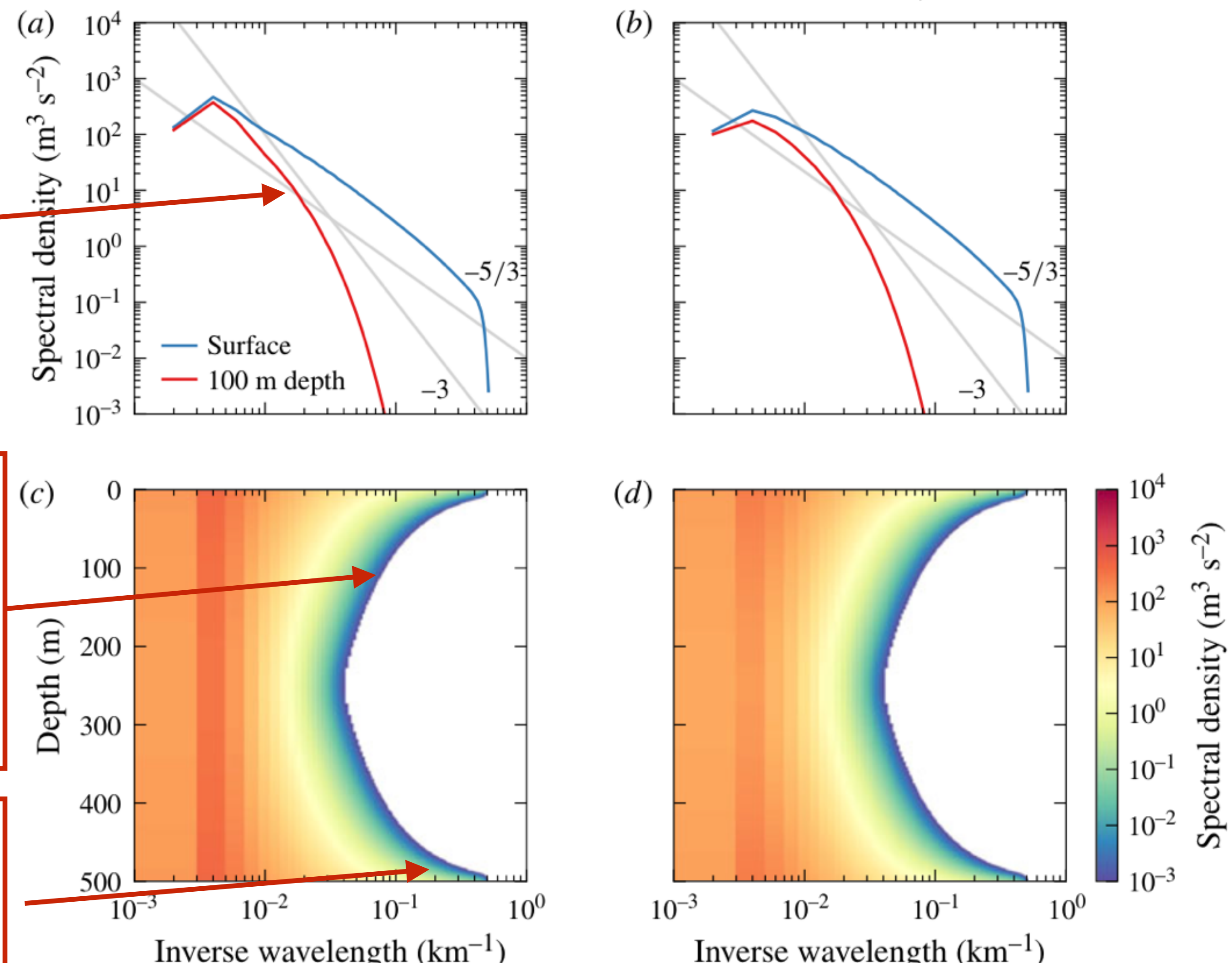


FIGURE 8. Wavenumber spectra of kinetic and potential energy from the thermocline-only simulation. (*a*) Kinetic and (*b*) potential energy spectra at the surface and 100 m depth, spectral density of (*c*) kinetic and (*d*) potential energy in the wavenumber–depth plane. In (*c*,*d*), no values below 10^{-3} m^3 s^{-2} are shown. Reference lines with slopes -3 and $-5/3$ are shown in grey.

Thermocline-only model

$$K_{k,l} = \frac{1}{2}(|\hat{u}|^2 + |\hat{v}|^2), \qquad P_{k,l} = \frac{1}{2}\frac{|\hat{b}|^2}{N^2}.$$

Terms in blue only redistribute energy

$$\frac{\partial P_{k,l}}{\partial t} = \mathrm{Re}\left[\frac{f\Lambda}{N^2}\hat{v}^*\hat{b} - \hat{w}^*\hat{b} - \frac{1}{N^2}\hat{b}^*\hat{\mathrm{J}}(\psi,\, b)\right] - (rk_h{}^{-2} + \nu k_h{}^{2n})P_{k,l}$$

Extraction of PE by mean flow

Hypoviscosity

PE → KE

Triad interactions

Vertical transfer of KE

Hyperviscosity

$$\frac{\partial K_{k,l}}{\partial t} = \mathrm{Re}\left[-f\frac{\partial}{\partial z}(\hat{w}^*\hat{\psi}) + \hat{w}^*\hat{b} + \hat{\psi}^*\hat{\mathrm{J}}(\psi,\, \nabla^2\psi)\right] - (rk_h{}^{-2} + \nu k_h{}^{2n})K_{k,l},$$

Thermocline-only model

$$K_{k,l} = \frac{1}{2}(|\hat{u}|^2 + |\hat{v}|^2), \qquad P_{k,l} = \frac{1}{2}\frac{|\hat{b}|^2}{N^2}.$$

Extraction of PE by mean flow

∇•P (triad interactions)

PE → KE

Dynamics in the interior due to baroclinic instability

Energy at smaller scales a result of SQG

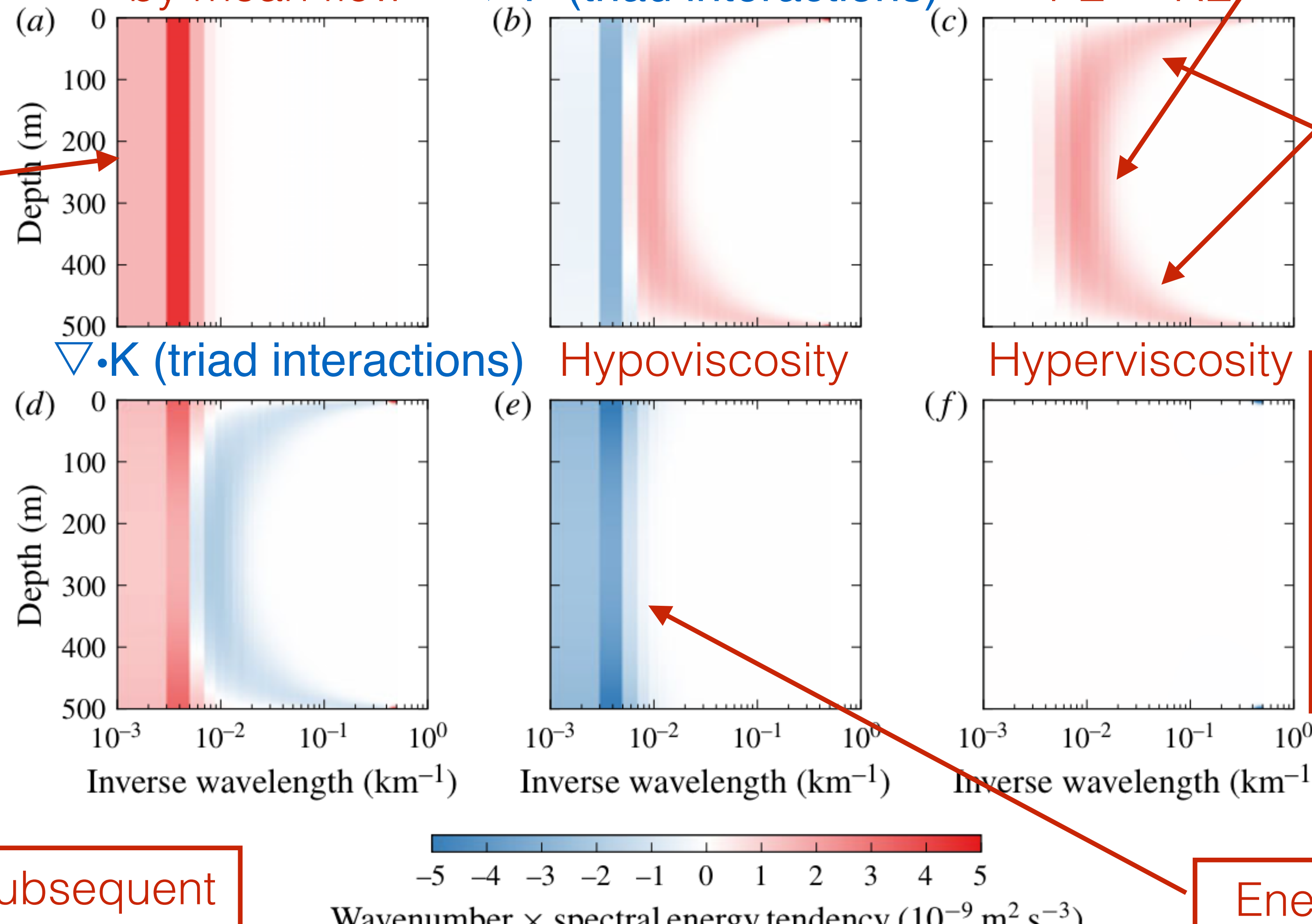
Energy enters the system here

∇•K (triad interactions)

Hypoviscosity

Hyperviscosity

This shows a direct cascade of potential energy, transfer of energy at small scales, and an inverse cascade of kinetic energy.



Energy leaves the system here

Note: Units in this and subsequent figures multiply energy by wavenumber, and therefore emphasize smaller scales.

Mixed Layer-only model

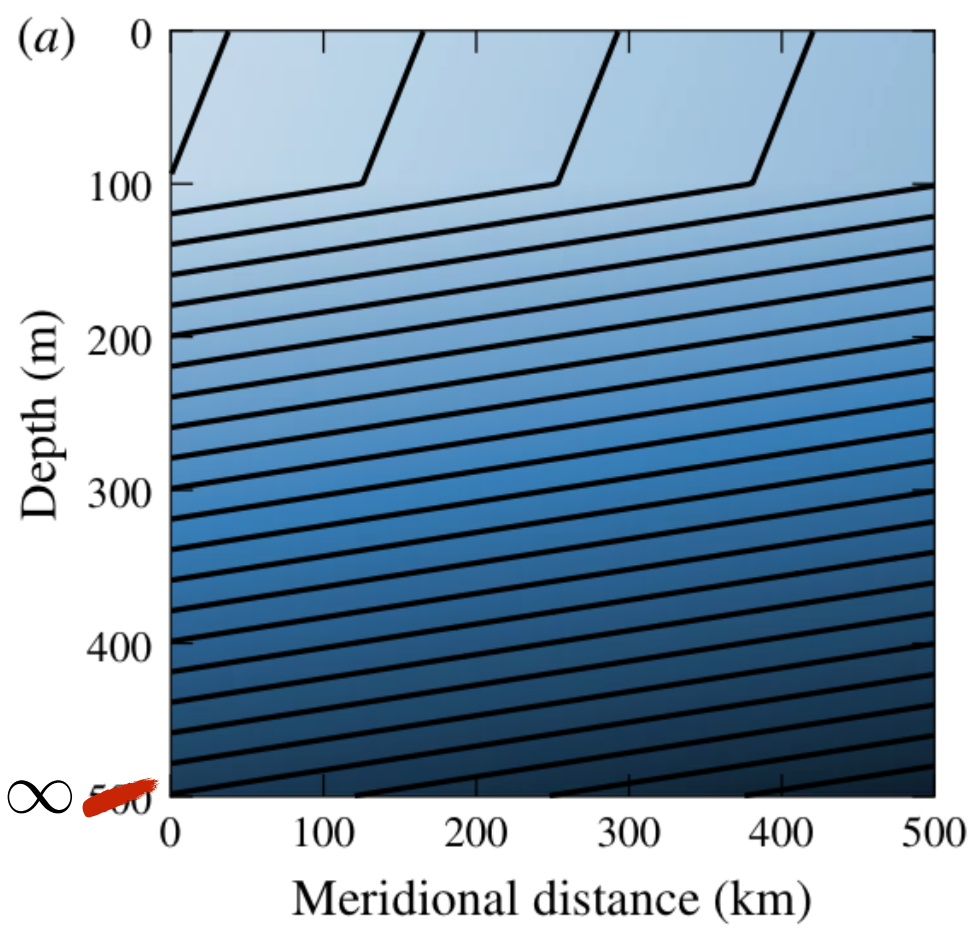


Figure 4a: Isopycnal contours, buoyancy colored.

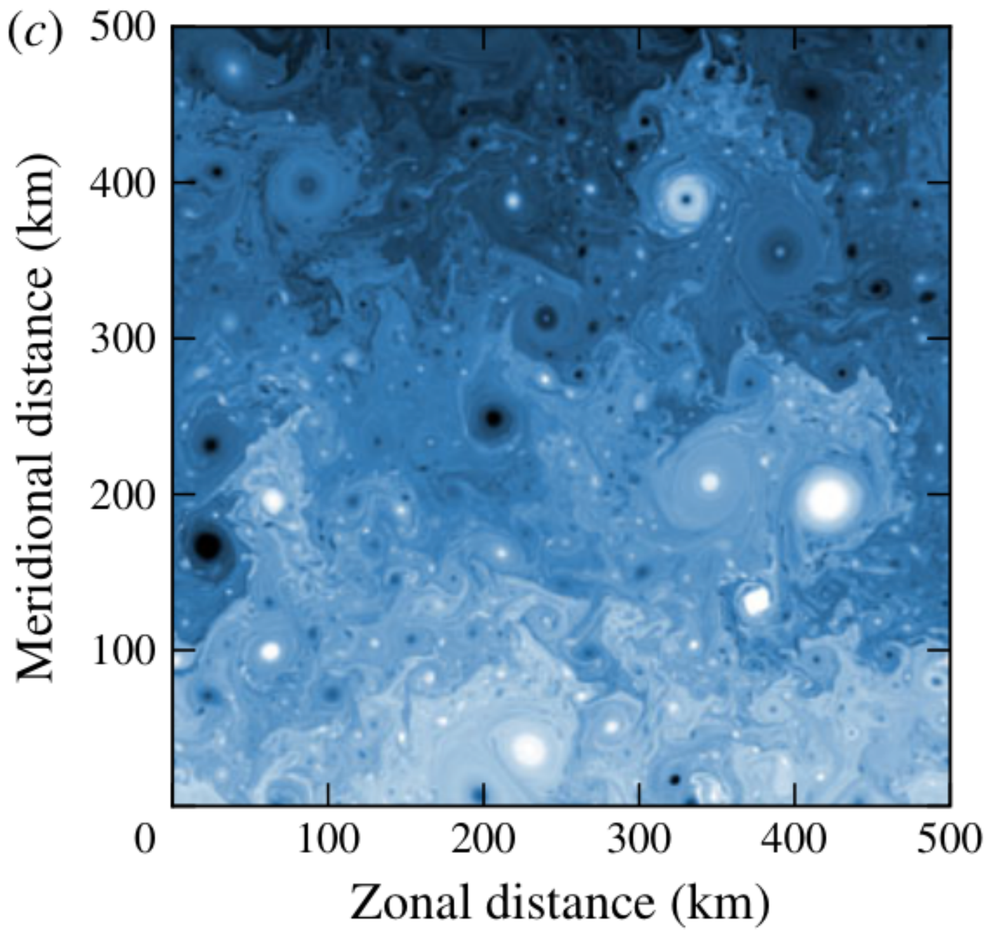


Figure 7c: Surface buoyancy snapshot, equilibrated model.

Note: This isn't REALLY a mixed-layer only model. It is more like a one-and-a-half layer model — different from the full model only in the absence of a rigid lid at the bottom boundary.

Mixed Layer-only model

Note: The choice of 100 m depth for the red line is at a discontinuity, so it is hard to know how to interpret this figure. Probably if a different depth (80 m, or 120 m, say) were chosen for the red line, the spectrum would be much steeper, and would probably look a lot like the red line of Figure 8!

$$K_{k,l} = \frac{1}{2}(|\hat{u}|^2 + |\hat{v}|^2), \qquad P_{k,l} = \frac{1}{2}\frac{|\hat{b}|^2}{N^2}.$$

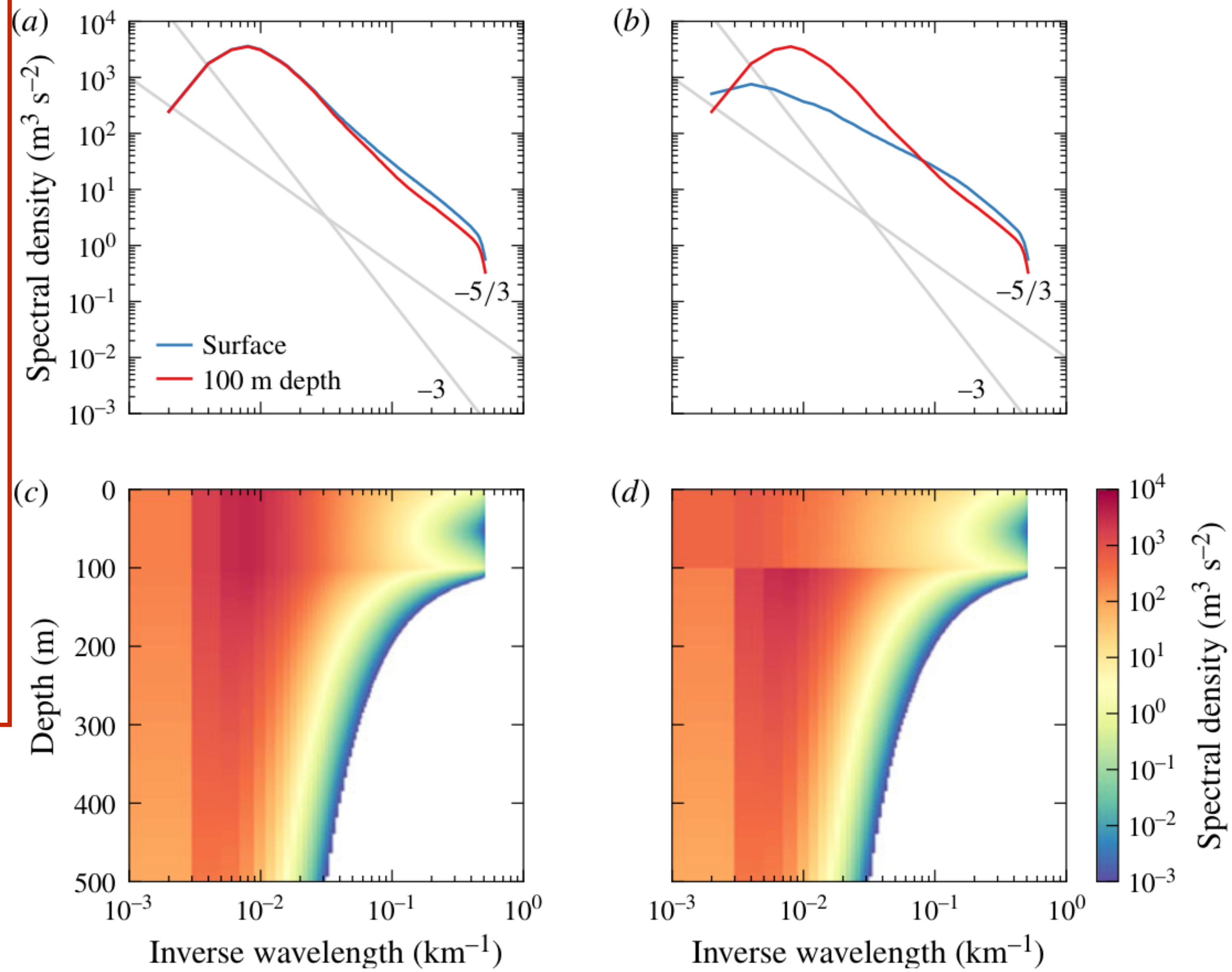
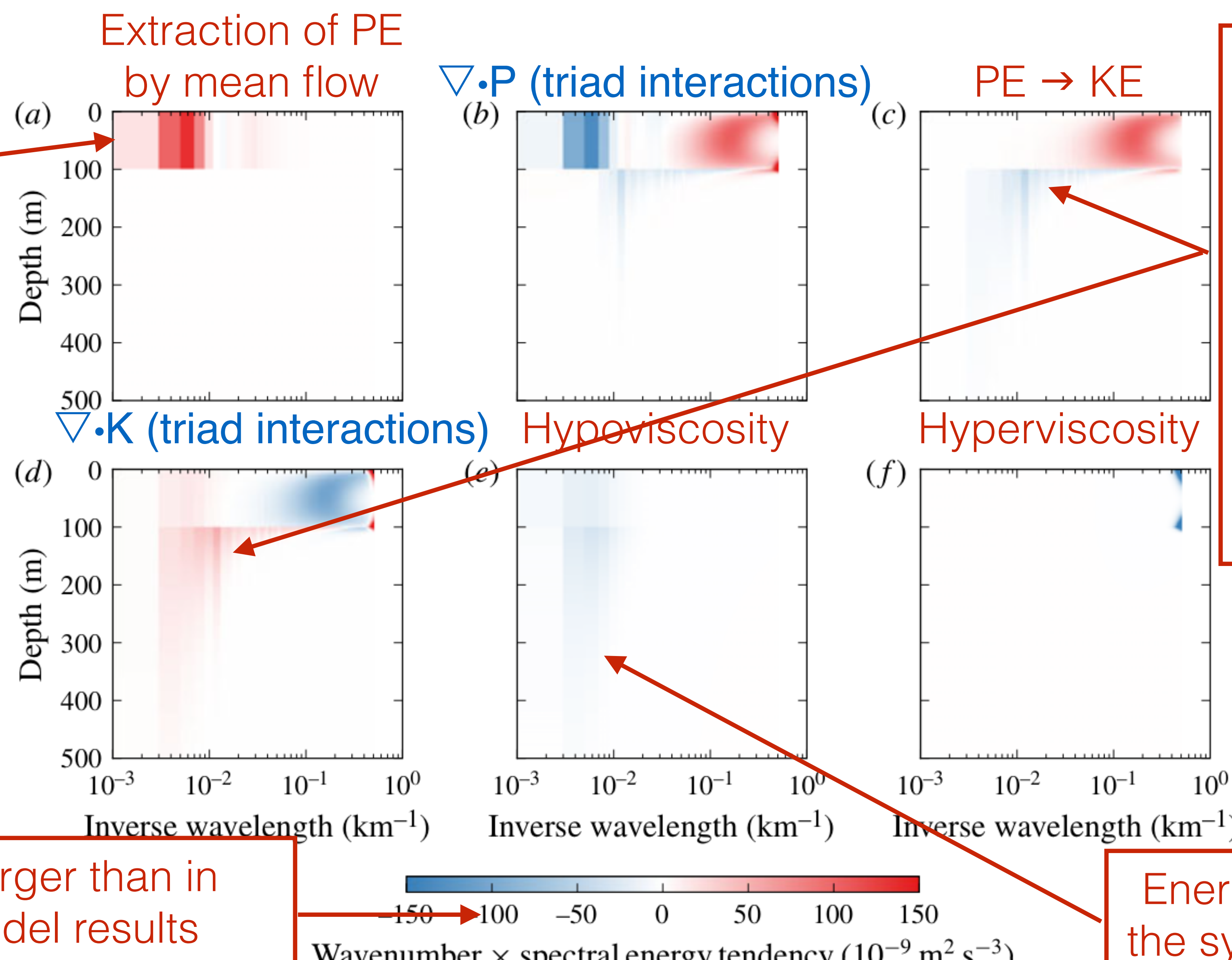


FIGURE 10. Wavenumber spectra of kinetic and potential energy from the mixed-layer-only simulation. (*a*) Kinetic and (*b*) potential energy spectra at the surface and 100 m depth (just below mixed-layer base), spectral density of (*c*) kinetic and (*d*) potential energy in the wavenumber–depth plane. In (*c*,*d*), no values below 10^{-3} m^3 s^{-2} are shown. Reference lines with slopes -3 and $-5/3$ are shown in grey.

Mixed Layer-only model

$$K_{k,l} = \frac{1}{2}(|\hat{u}|^2 + |\hat{v}|^2), \qquad P_{k,l} = \frac{1}{2}\frac{|\hat{b}|^2}{N^2}.$$

Energy enters the system here (only in mixed layer)



Transfer of energy to different depths a result of the long wave cutoff in the linear stability analysis; these long waves are more transparent to the mixed layer interface and therefore can propagate energy into the interior.

Note: Colorbar 30x larger than in thermocline-only model results (compare with the "only" 4x increase in growth rates in the linearized equations).

Energy leaves the system here (no longer only in mixed layer)

Full model

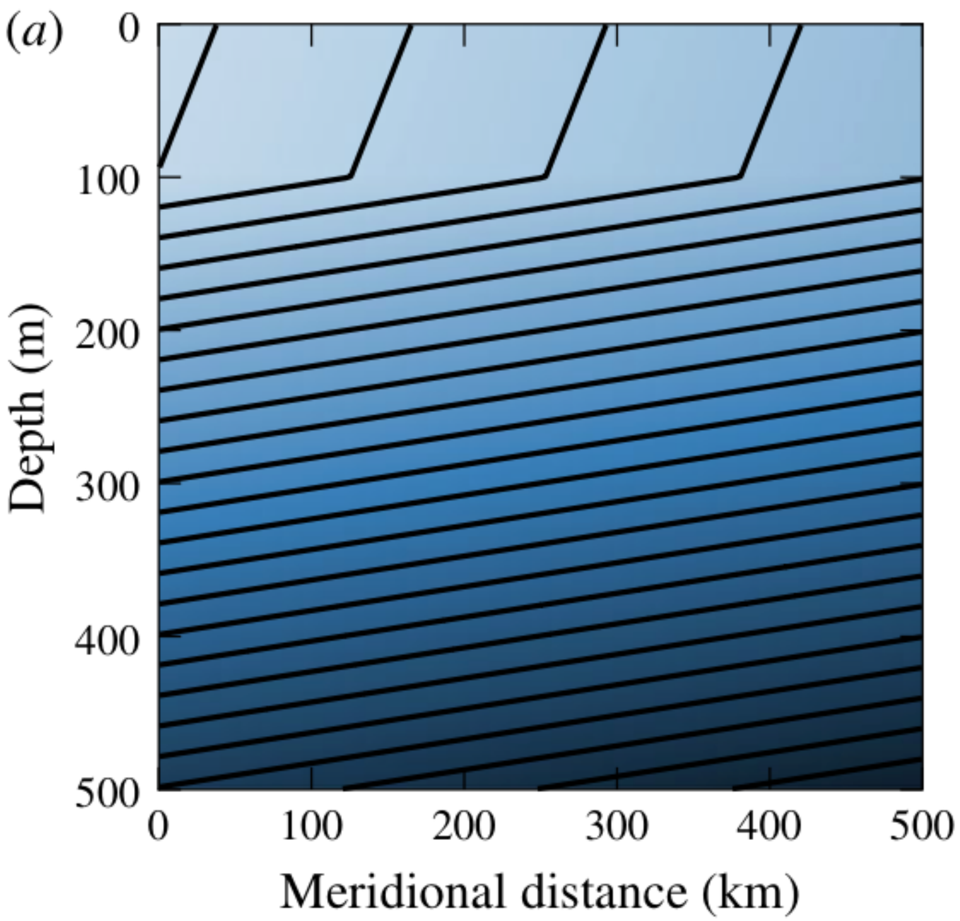


Figure 4a: Isopycnal contours, buoyancy colored.

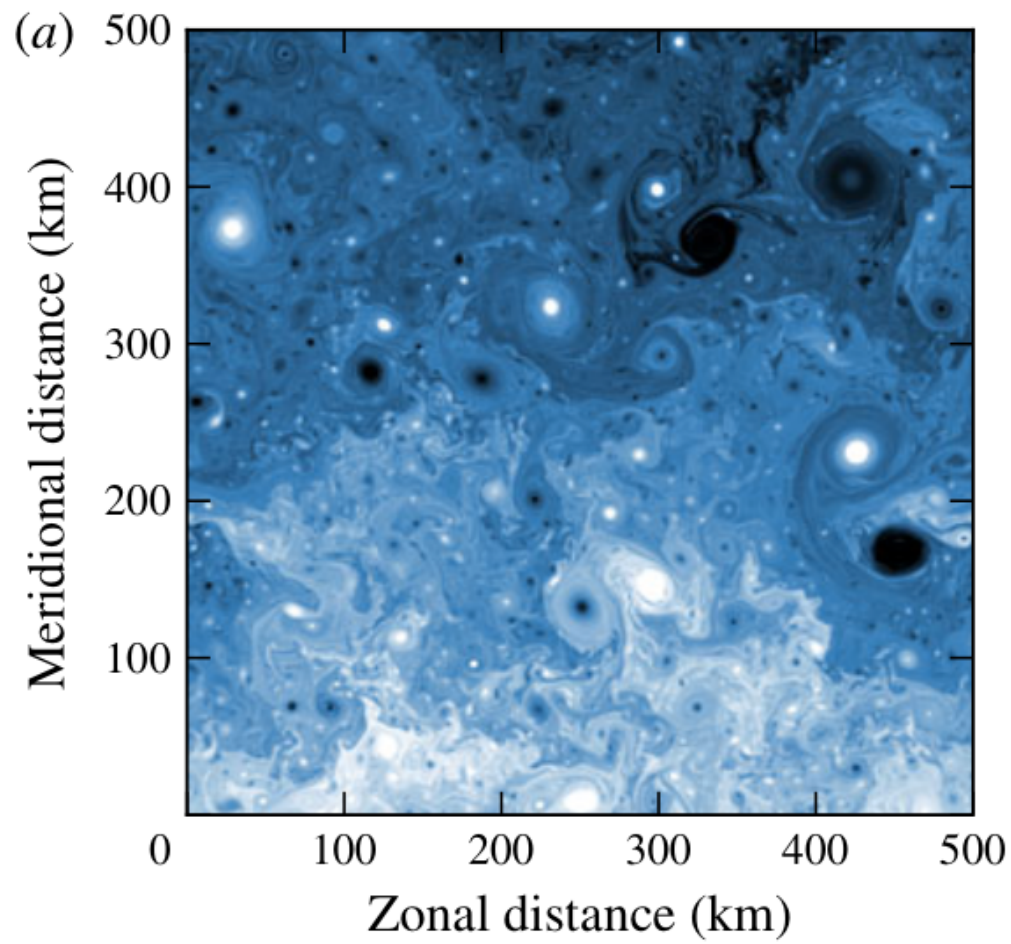


Figure 7a: Surface buoyancy snapshot, equilibrated model.

Full model

Looks very similar to the mixed layer only model, except for the SQG-like dynamics at the bottom boundary.

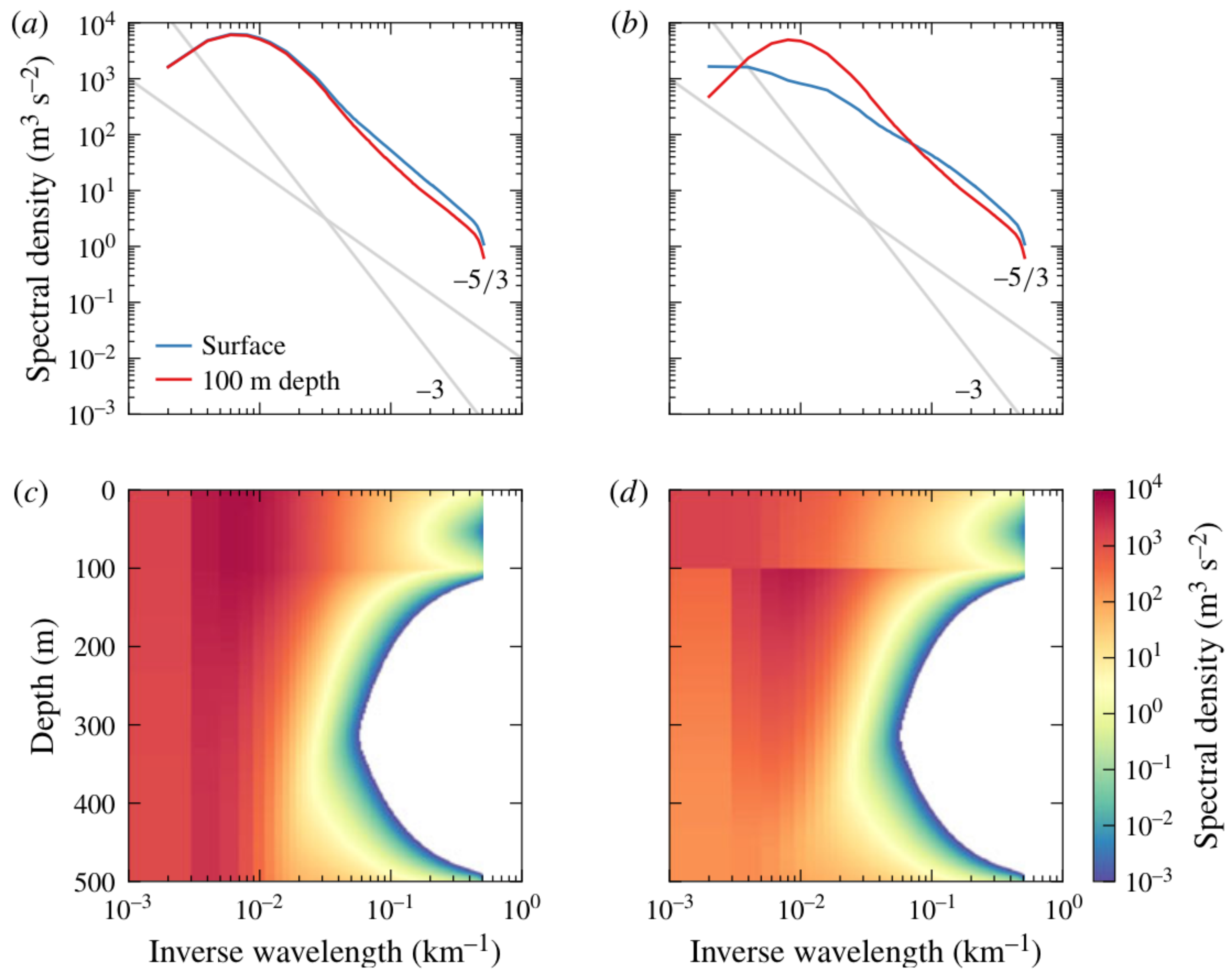
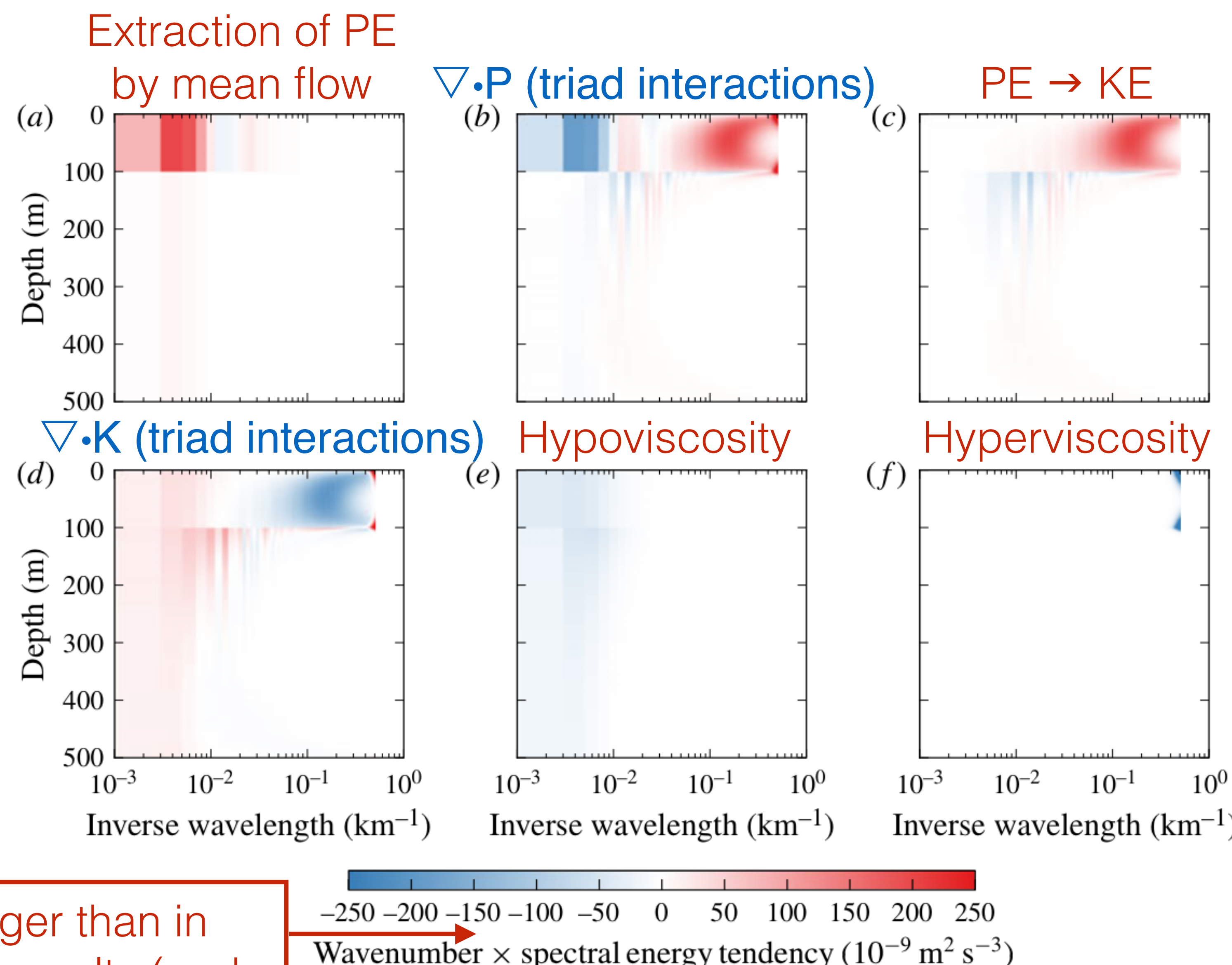


FIGURE 14. Wavenumber spectra of kinetic and potential energy from the full model simulation. (*a*) Kinetic and (*b*) potential energy spectra at the surface and 100 m depth (just below mixed-layer base), spectral density of (*c*) kinetic and (*d*) potential energy in the wavenumber–depth plane. In (*c*,*d*), no values below 10^{-3} m^3 s^{-2} are shown. Reference lines with slopes −3 and −5/3 are shown in grey.

Full model

$$K_{k,l} = \frac{1}{2}(|\hat{u}|^2 + |\hat{v}|^2), \qquad P_{k,l} = \frac{1}{2}\frac{|\hat{b}|^2}{N^2}.$$

Looks very similar to the mixed layer only model.



Note: Colorbar 50x larger than in thermocline-only model results (and also significantly larger than in mixed layer-only model results).

Mixed Layer model: Modal decomposition

$$E_{k,l} = -\frac{1}{2}(\boldsymbol{S}\hat{\boldsymbol{\psi}})^{\dagger}\boldsymbol{D}(\boldsymbol{S}\hat{\boldsymbol{\psi}}) = -\frac{1}{2}\sum_{i}\lambda_j|(\boldsymbol{S}\hat{\boldsymbol{\psi}})_j|^2.$$

Blue = ψ^0 (barotropic-ish)

Red = ψ^1(baroclinic-ish)

In the mixed layer, at least.

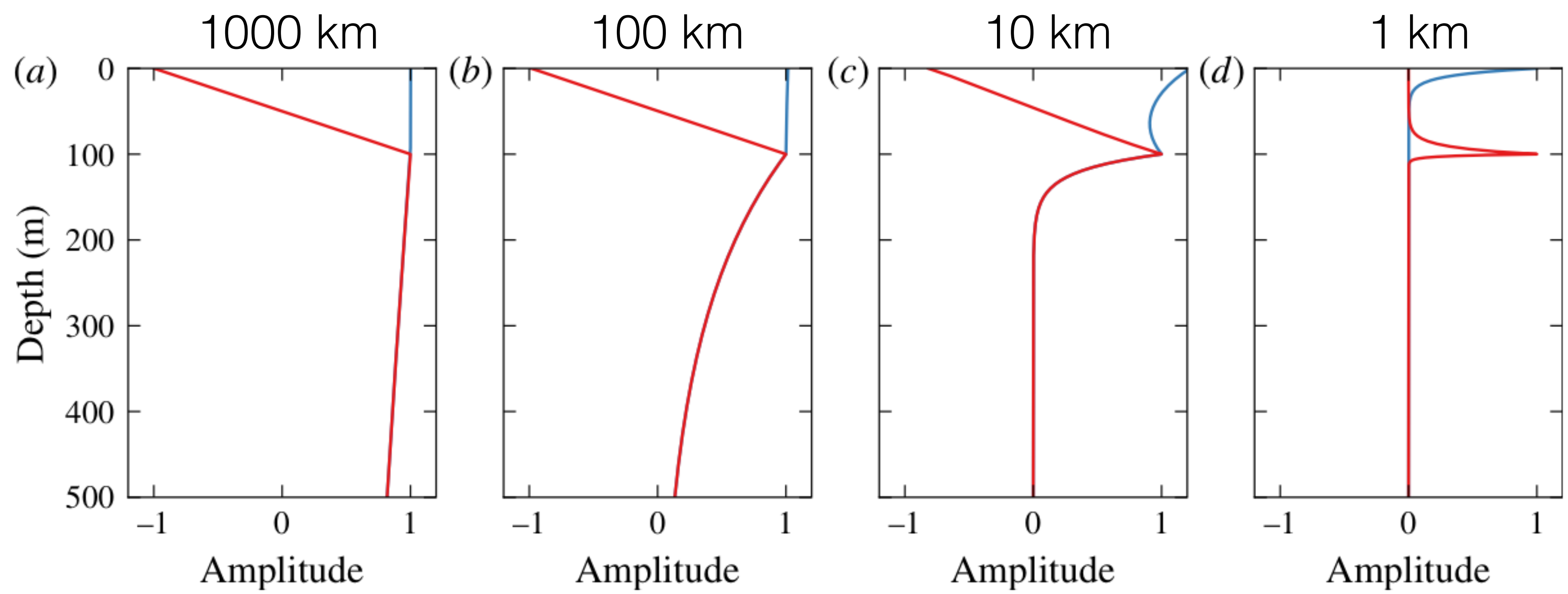


FIGURE 12. Vertical structure of the streamfunction corresponding to orthogonal modes in the mixed-layer-only case for different wavenumbers $k_h = 2\pi/\lambda$, with the wavelength λ given in the panel titles. For (*a*–*c*), the modes are normalized to unity at the interface at 100 m depth; for panel (*d*), the modes are normalized to have a maximum value of unity. Mode 0 is shown in blue, mode 1 in red. In (*a*–*c*), the two modes coincide below the interface. (*a*) 1000 km wavelength; (*b*) 100 km wavelength; (*c*) 10 km wavelength; (*d*) 1 km wavelength.

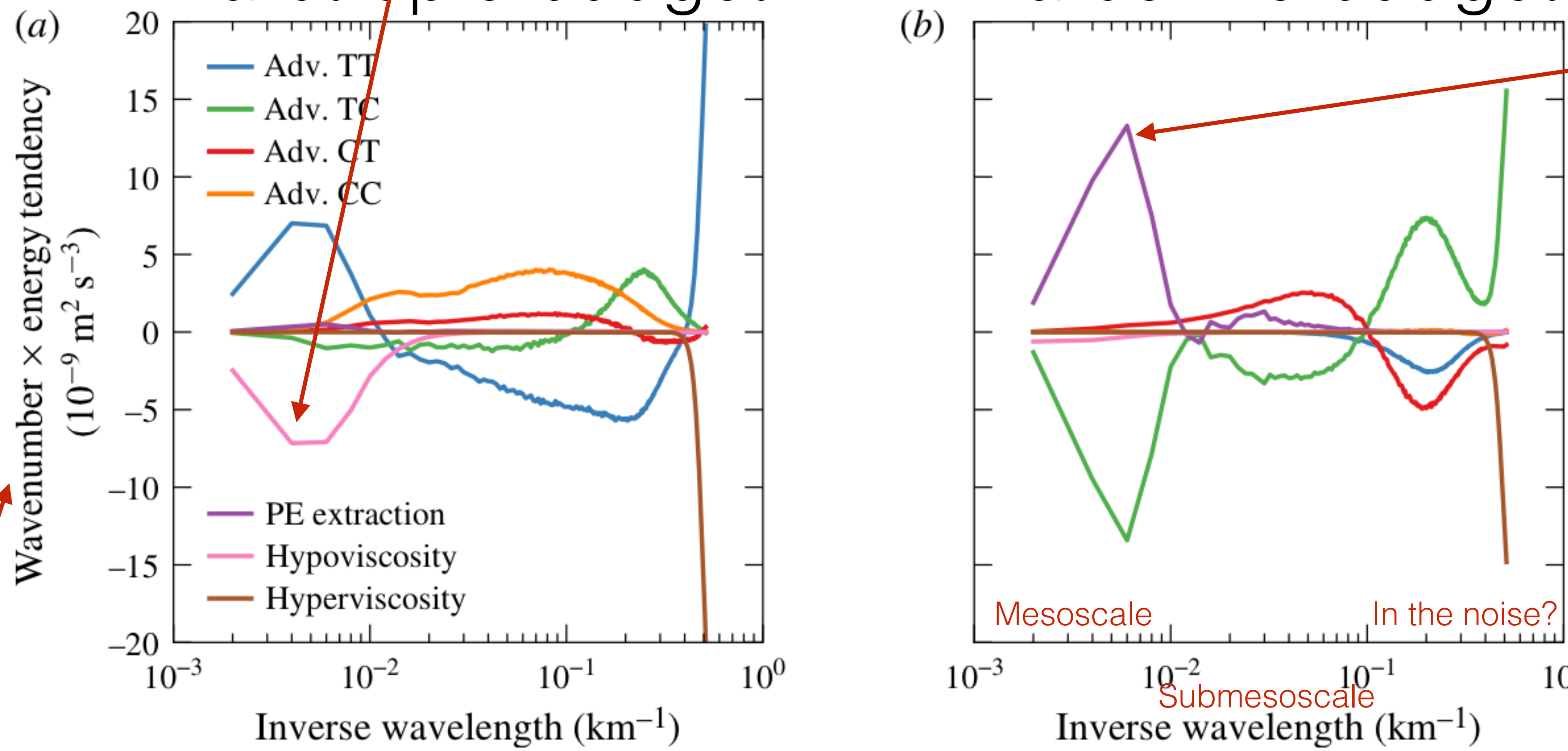
Mixed Layer model: Modal energy budget

Primary sink of energy out of barotropic mode at large scales

Barotropic budget

Baroclinic budget

Primary input of energy into baroclinic mode at large scales



Remember that this axis intensifies small scales!

FIGURE 13. Modal energy budget for the mixed-layer-only case. The advective terms correspond to the contributions from the four terms in (4.19). The energy tendencies are multiplied by wavenumber to compensate for logarithmic shrinking. (*a*) Barotropic budget; (*b*) baroclinic budget.

Enhancement of vertical velocities in full model

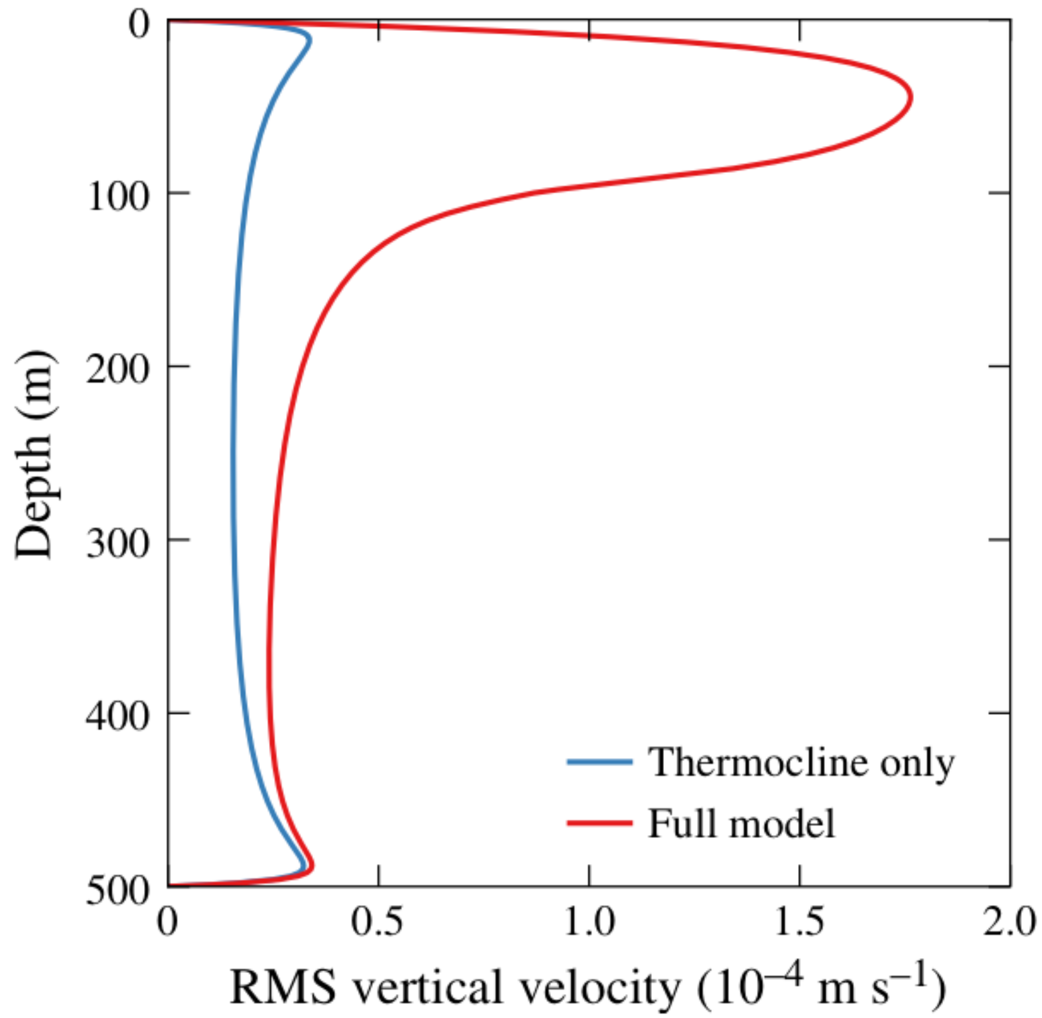


FIGURE 16. Profiles of root-mean-square vertical velocity for the thermocline-only simulation and the full model simulation with reduced mixed-layer shear.

Vertical velocities aligned to fronts

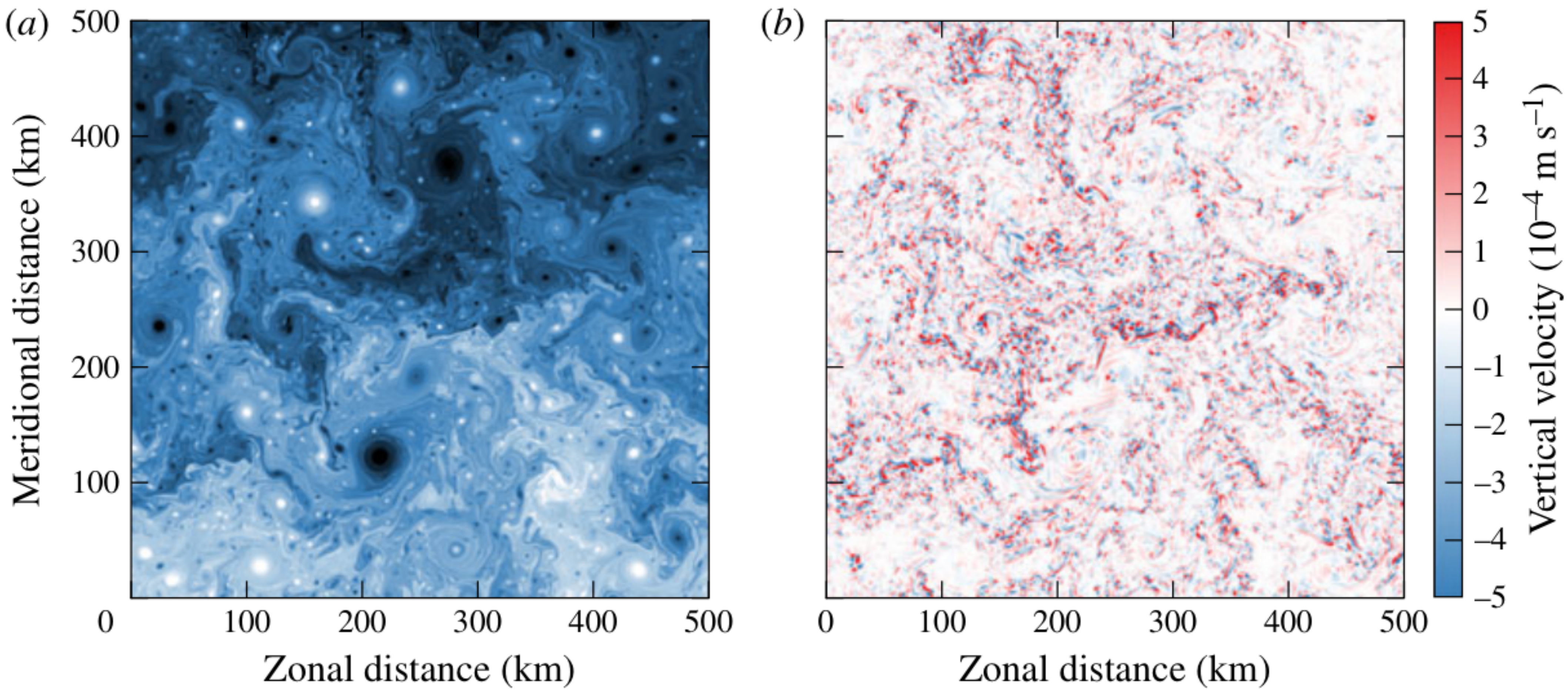


FIGURE 17. Concurrent surface buoyancy and vertical velocity snapshots from the full model simulation with reduced mixed-layer shear: (*a*) surface buoyancy equivalent to figure 7 and (*b*) vertical velocity at 47 m, the depth of the maximum root mean square vertical velocity.