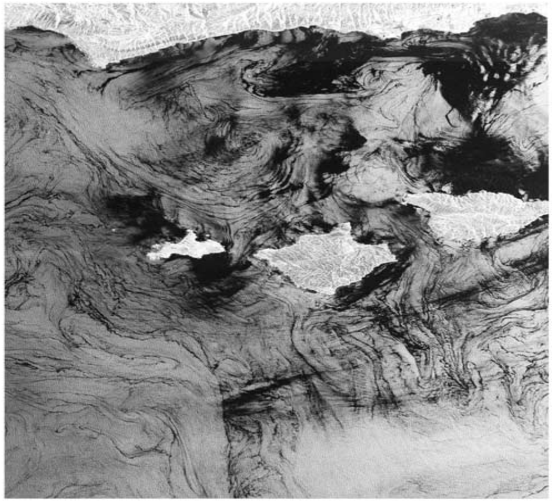
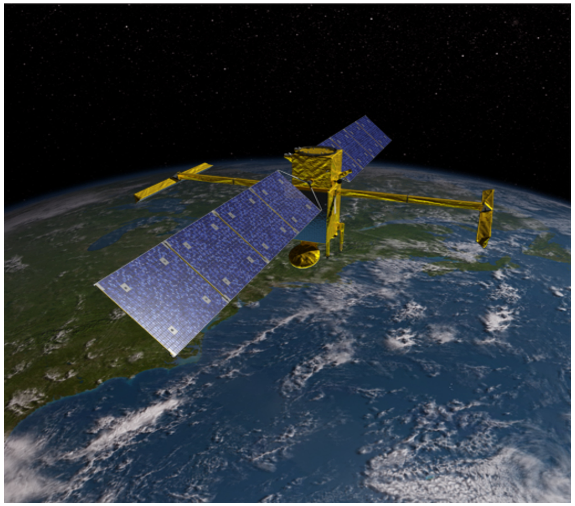
“Wave-Turbulence Interactions in the Oceans”

https://oceanturbulence.github.io

Patrice Klein (Caltech/JPL/Ifremer)

(XX) A synthesis: Internal Tides and GM spectrum revisited





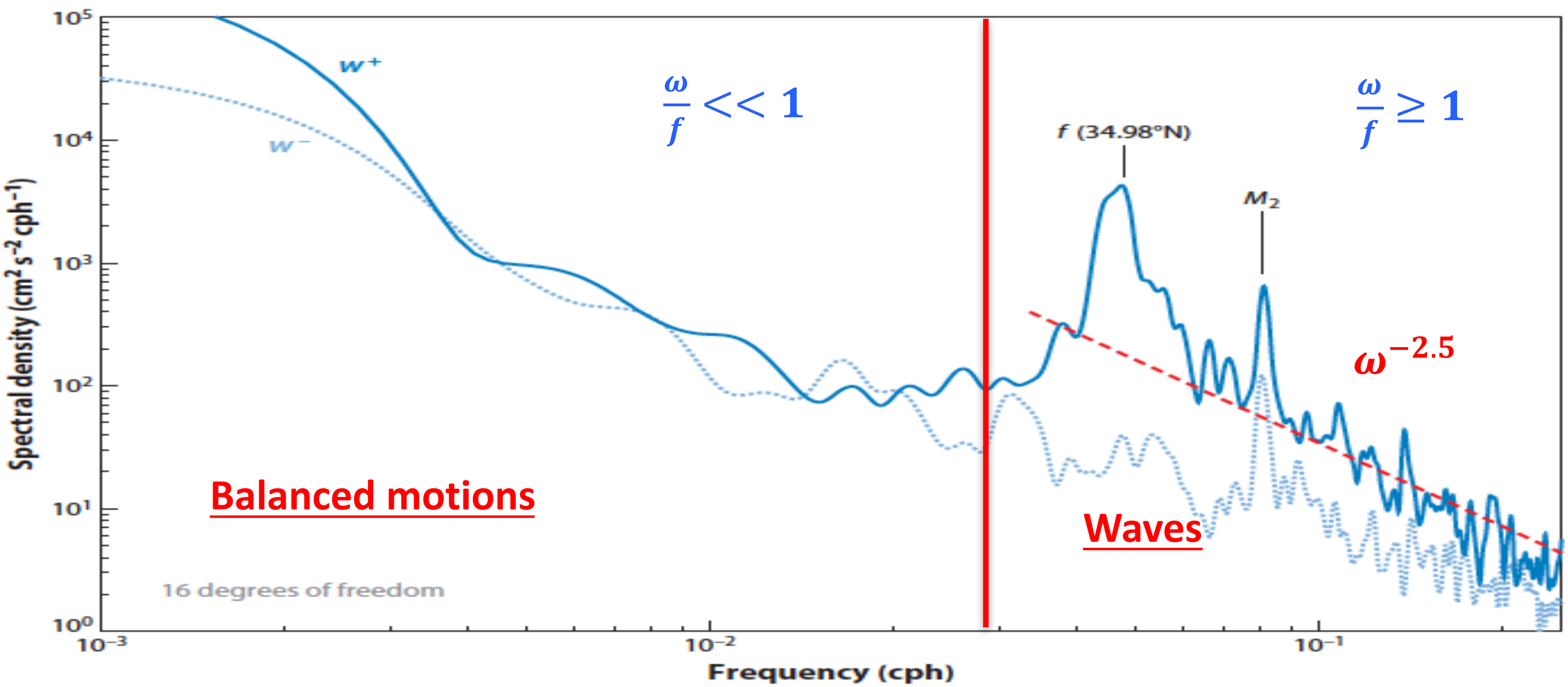


Figure 1

Rotary velocity spectrum at 261-m depth from current-meter data from the WHOI699 mooring gathered during the WESTPAC1 experiment (mooring at 6,149-m depth.) The solid blue line (w^+) is clockwise motion, and the dashed blue line (w^-) is counterclockwise motion; the differences between these emphasize the downward energy propagation that often dominates the near-inertial band. The dashed red line is the line $E_0 N \omega^{-p}$ with $N = 2.0$ cycles per hour (cph), $E_0 = 0.096$ cm^2 s^{-2} cph^{-2}, and $p = 2.25$, which is quantitatively similar to levels in the Cartesian spectra presented by Fu (1981) for station 5 of the Polygon Mid-Ocean Experiment (POLYMODE) II array.

SPECTRAL GAP AT FREQUENCIES (ω) LARGER (BUT CLOSE TO) f

Waves (near-inertial, tidal, internal gravity waves):

- Fast motions $f \geq \omega \geq N$
- assumed to explain **most of the mixing** (at small-scale) **in the ocean interior**
- strong signature in in-situ observations (moorings, gliders, ADCP, surface drifters) and satellite observations [SAR, altimetry] at high-resolution.

Geostrophic turbulence [10-500 km]:

- Slow motions
- explains **most of the kinetic energy in the oceans,** well captured by satellite observations on a global scale [SSH (> 100 km), SST, Ocean Color, …]

Open questions. see: Alford et al. ARMS 2016, Garrett and Kunze, ARFM 2007, Polzin and Lvov, Review of Geophysics 2011 ...

Another review paper: MacKinnon et al. BAMS 2017 (in press):

« Climate Process Team on Internal-Wave Driven Ocean Mixing »

« We have sufficient evidence from theory, process models, laboratory experiments, and field measurements to conclude that **away from ocean boundaries** (atmosphere, ice, or the solid ocean bottom), **diapycnal mixing is largely related to the breaking of internal gravity waves.** »

« Ocean internal gravity waves propagate through the stratified interior of the ocean. They are generated by a variety of mechanisms, with the most important being**, wind variations at the sea-surface, internal tides, and flow of ocean currents and eddies over topography leading to lee-waves** »

NEAR-INERTIAL (f) AND TIDAL WAVES (M2) EXPLAIN A LARGE PART OF THE WAVE SPECTRUM

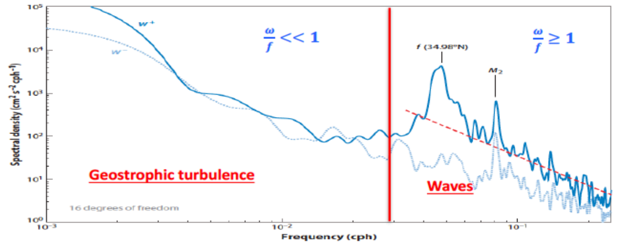


Figure 1

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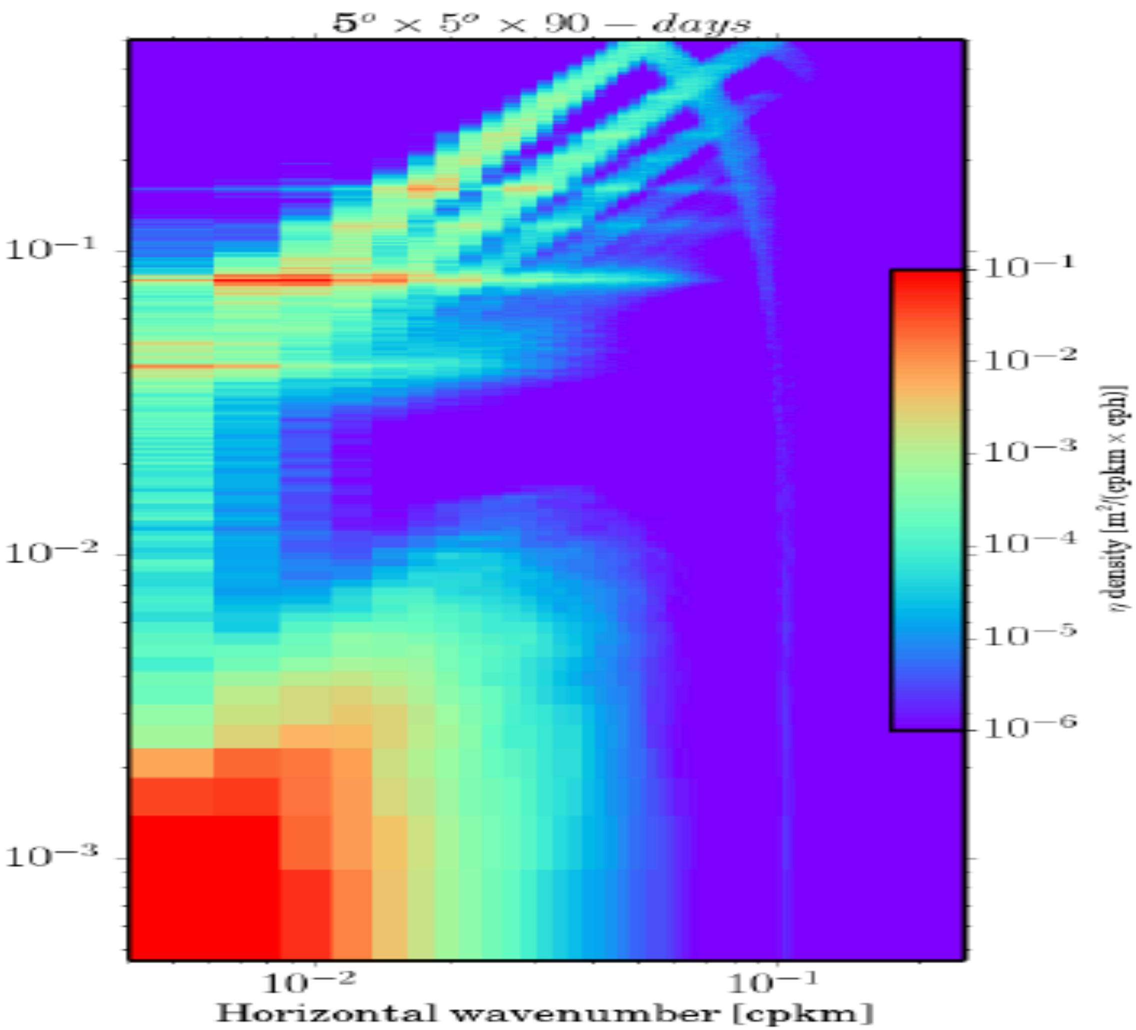


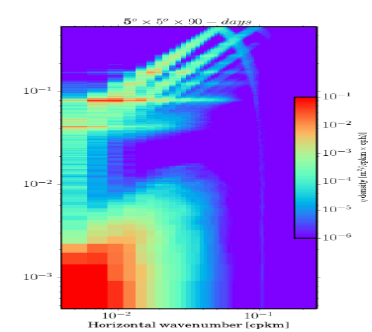
The high frequency part of the wave spectrum is characterized by discrete bands at high wavenumbers!

(wave-wave interactions are weak)

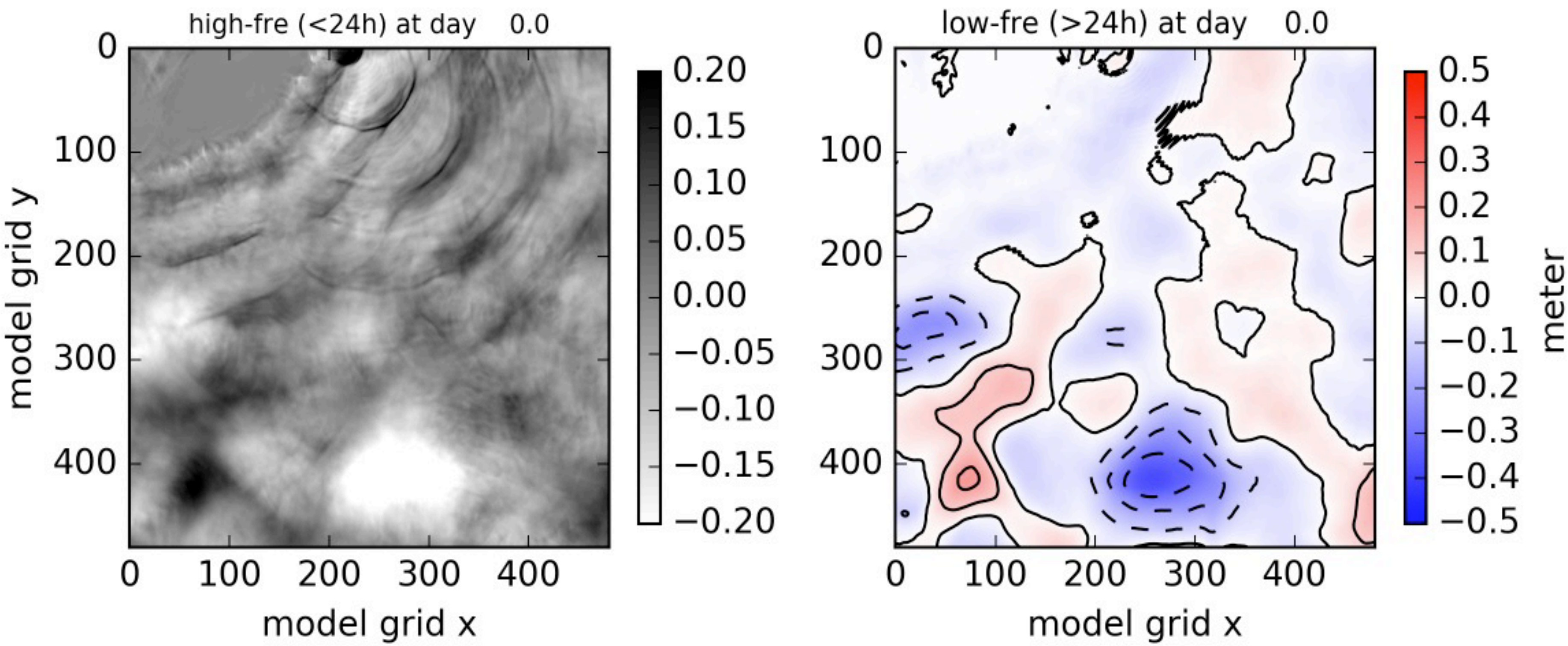
NEAR-INERTIAL WAVES

The YBJ approach (**for the near-inertial waves**):

- valids the WKB approximation (Kunze 1985) but
- better takes into account the impact of the dispersion terms (when scales of NIWs are close to the eddy scales) and
- explains the vertical propagation (as in Gill 1984).
- explains the discrete bands (see class XV)

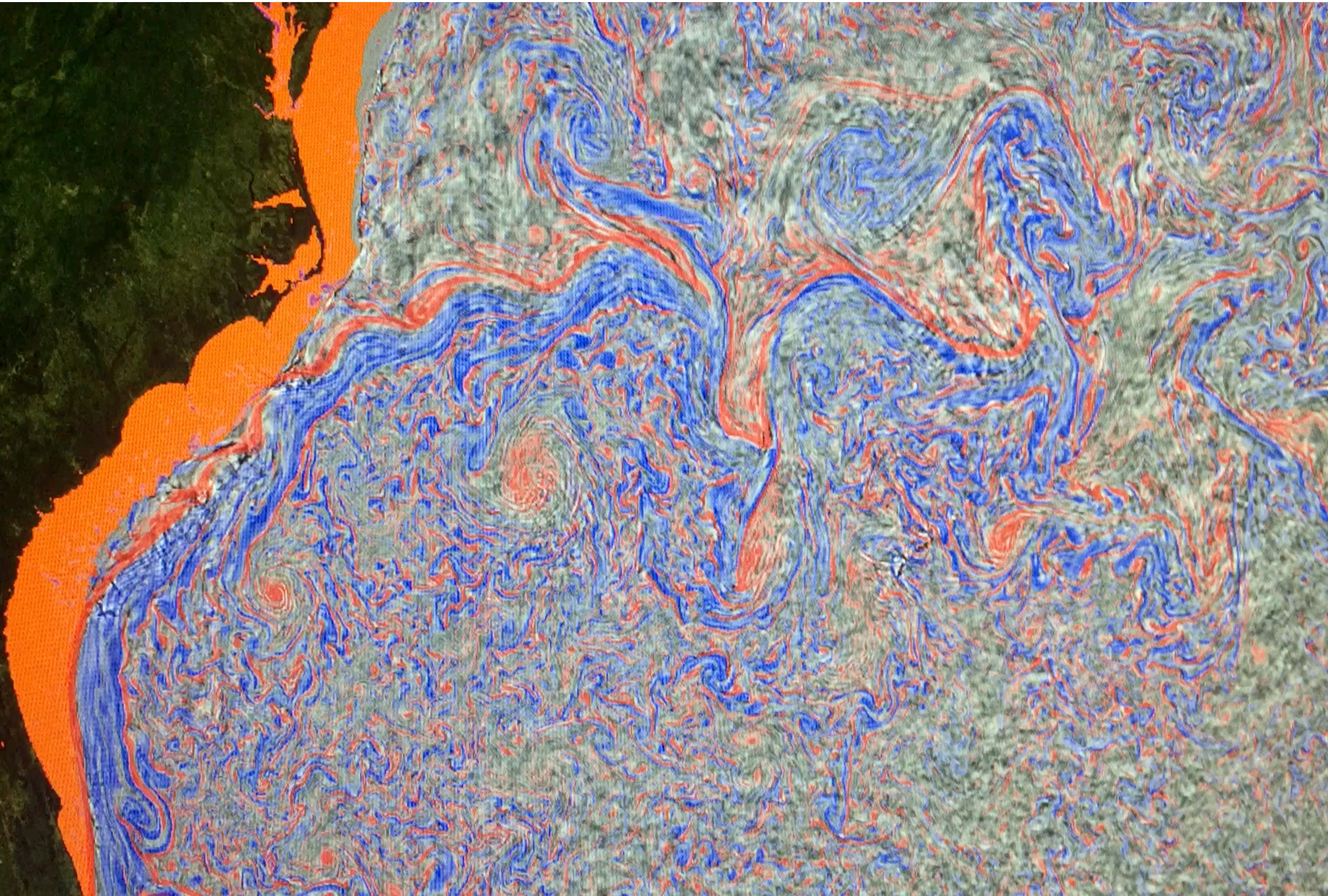


INTERNAL TIDES

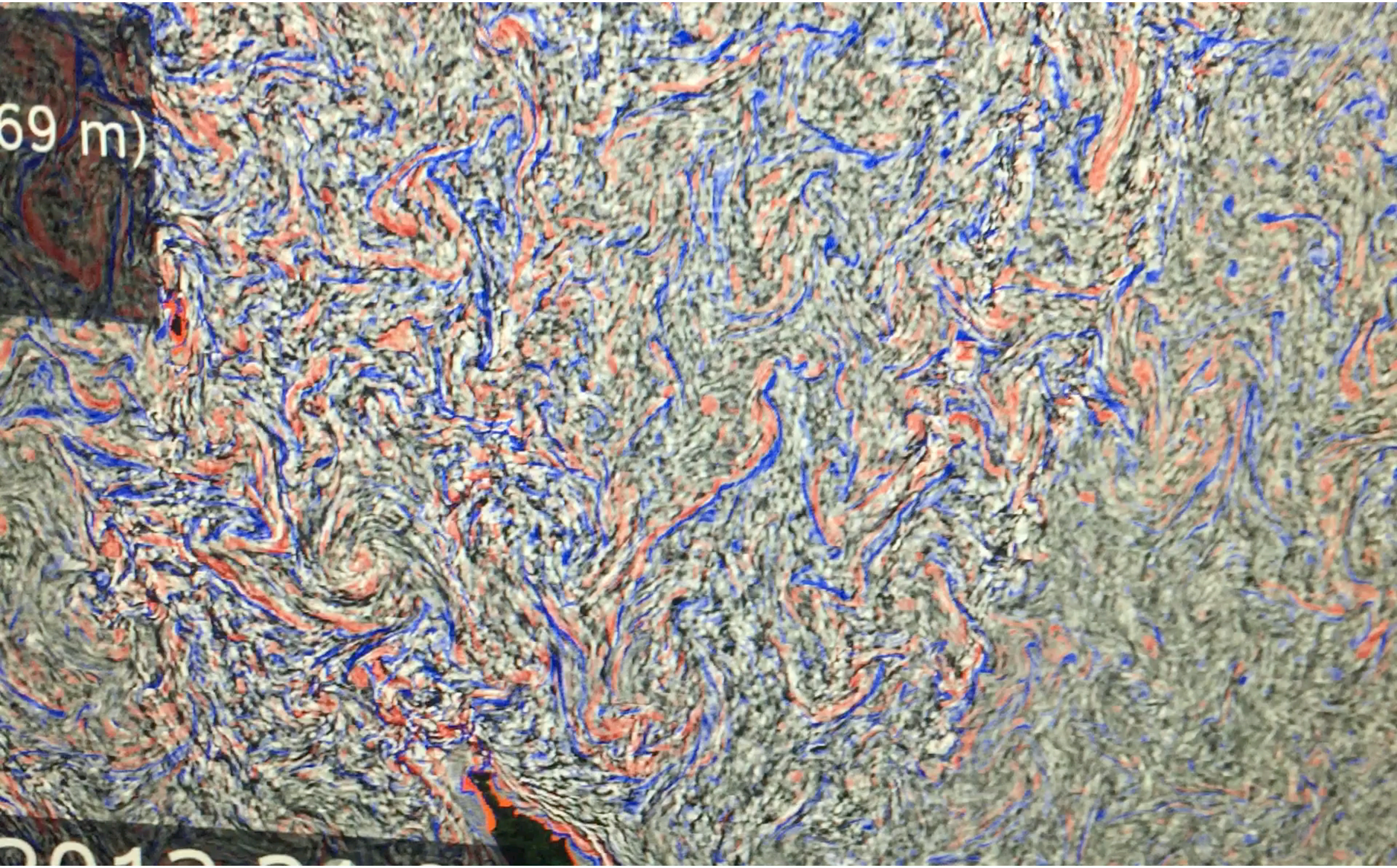


From Jinbo Wang, JPL 2017

Relative Vorticity + W (Gulf stream on the hyperwall)

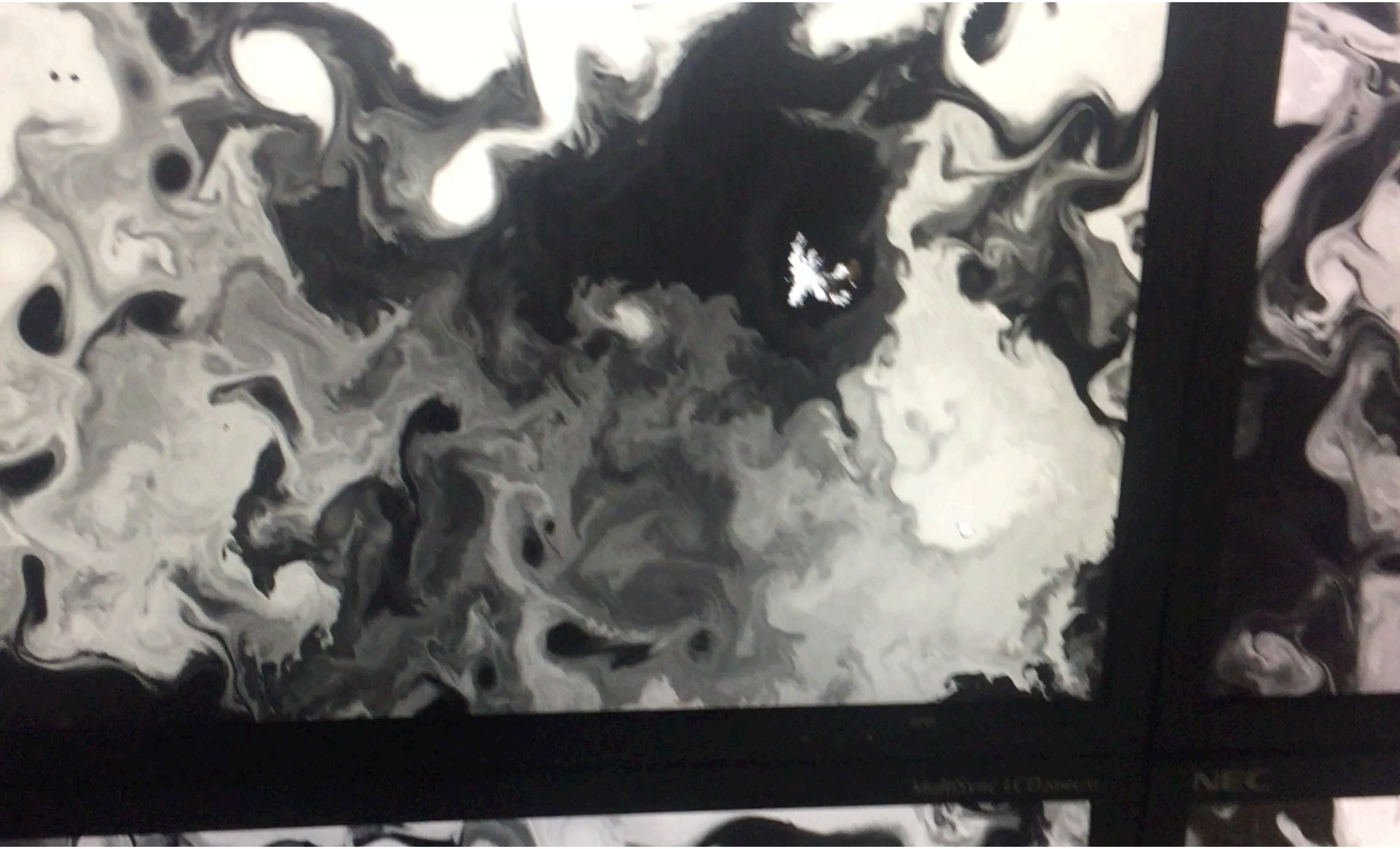


Relative Vorticity + W

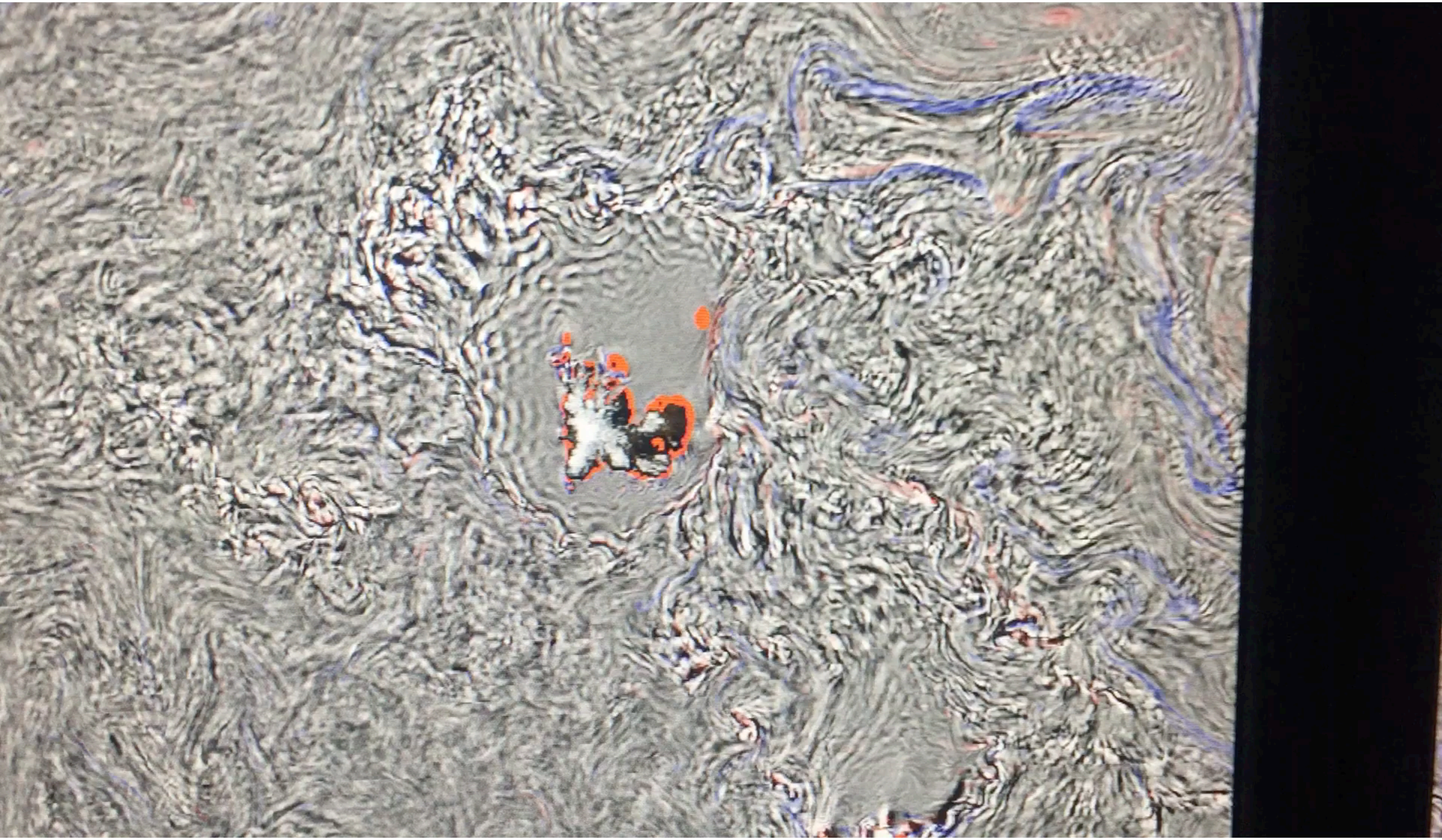


$$\left|U^2 + V^2\right|^{1/2}$$

SALINITY



Relative Vorticity + W



Impact of internal waves on PE/KE

We assume the flow is hydrostatic at first order, and R_{ol}<<1 (nonlinear terms are negligeable), and 1/fT=O(1). The resulting equations are:

$$\frac{\partial u}{\partial t} - fv = -\frac{1}{\rho_0}\frac{\partial p'}{\partial x}$$

$$\frac{\partial v}{\partial t} + fu = -\frac{1}{\rho_0}\frac{\partial p'}{\partial y}$$

$$0 = -\frac{1}{\rho_0}\frac{\partial p'}{\partial z} - \frac{g}{\rho_0}\rho' \quad \text{or} \quad \frac{1}{\rho_o}\frac{\partial p'}{\partial z} = b'$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$\frac{\partial \rho'}{\partial t} + w\frac{\mathrm{d}\bar{\rho}}{\mathrm{d}z} = 0. \quad \text{or} \quad \frac{\partial b'}{\partial t} + w.N^2 = 0$$

We use: $u = -\psi_y + \phi_x$, $\mathrm{v} = \psi_x + \phi_y$.

This leads to:

$$\zeta = v_x - u_y = \Delta\psi$$

$$w_z = -\Delta\phi$$

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This leads to:

$$\zeta = v_x - u_y = \Delta\psi$$
$$w_z = -\Delta\phi$$

$$|\hat{u}|^2 = l^2\left|\hat{\psi}\right|^2 + k^2\left|\hat{\phi}\right|^2$$

$$|\hat{v}|^2 = k^2\left|\hat{\psi}\right|^2 + l^2\left|\hat{\phi}\right|^2$$

$$\widehat{KE} = |\hat{u}|^2 + |\hat{u}|^2 = k^2[\left|\hat{\psi}\right|^2 + \left|\hat{\phi}\right|^2] = K^2\left|\hat{\psi}\right|^2 \frac{(\omega^2 + f^2)}{\omega^2}$$

Impact of internal waves on PE/KE

We use: $u = -\psi_y + \phi_x$, $\mathrm{v} = \psi_x + \phi_y$. This leads to:

$$\zeta = v_x - u_y = \Delta\psi, \qquad w_z = -\Delta\phi$$

We also use: $\omega^2 = f^2 + N^2 \frac{K^2}{m^2}$

$$\omega^2 \frac{\left|\hat{b}\right|^2}{N^2} = |\hat{w}|^2 N^2 = |\hat{w}|^2 \frac{m^2}{K^2}(\omega^2 - f^2)$$

$$\widehat{PE} = \frac{\left|\hat{b}\right|^2}{N^2} = K^2 \left|\hat{\psi}\right|^2 \frac{(\omega^2 - f^2)}{\omega^2}$$

For a single internal wave:

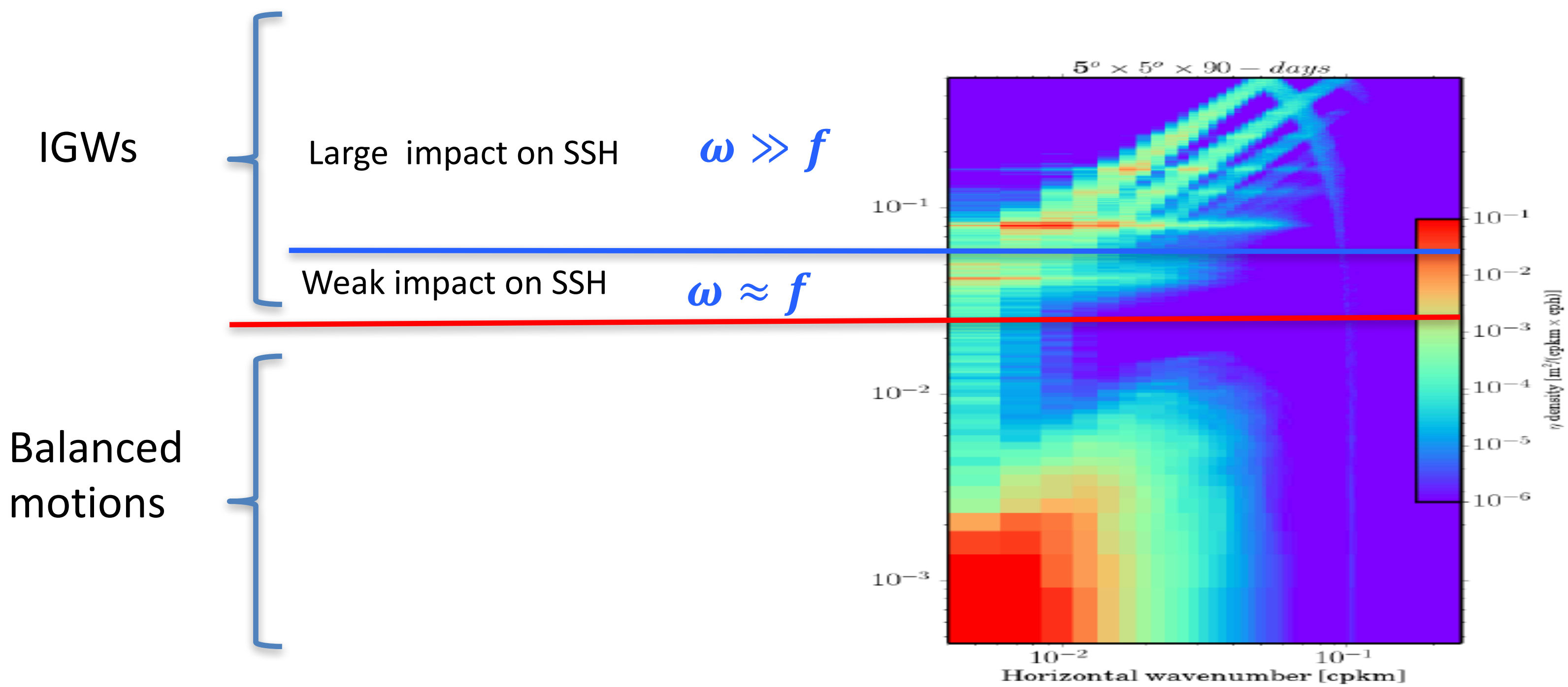
=> $$\boxed{\frac{PE}{KE} = \frac{\omega^2 - f^2}{\omega^2 + f^2}}$$

Impact of internal waves on PE/KE (and on SSH)

For a single internal wave:

=> $$\boxed{\frac{PE}{KE} = \frac{\omega^2 - f^2}{\omega^2 + f^2}}$$

- Waves with $\omega \approx f$ (as NIWs) have a weak impact on PE (and therefore on SSH)
- Waves with $\omega \gg f$ have a much larger impact on PE (and therefore on SSH)



Propagation of a wave packet in a stationary barotropic jet

Group velocity is: $\overrightarrow{C_g} = \nabla_k \omega$

$$C_{gx} = \frac{\partial \omega}{\partial k} = \frac{N^2 k}{\omega m^2} = \frac{\omega^2 - f^2}{\omega} \frac{k}{k^2 + l^2}$$

$$C_{gy} = \frac{\partial \omega}{\partial l} = \frac{N^2 l}{\omega m^2} = \frac{\omega^2 - f^2}{\omega} \frac{l}{k^2 + l^2}$$

$$C_{gz} = \frac{\partial \omega}{\partial m} = -\frac{N^2 (k^2 + l^2)}{\omega m^3} = -\frac{\omega^2 - f^2}{\omega m}$$

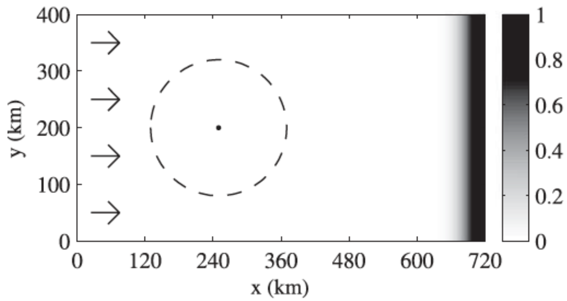
The group velocity is proportional to $\omega^2 - f^2$.

Waves with $\boldsymbol{\omega \approx f}$ propagate slowly and those with $\boldsymbol{\omega \approx f}$ propagate much faster!

INTERNAL TIDES

- Internal (M2) tides propagate much faster than near-inertial waves
- Because of their frequencies, they have a larger impact on PE and SSH
- They are associated with a larger vertical velocity
- They are much more affected by the horizontal variations of N2 (at eddy scales) than by motions shears.

DUNPHY and LAMB JPO 2014



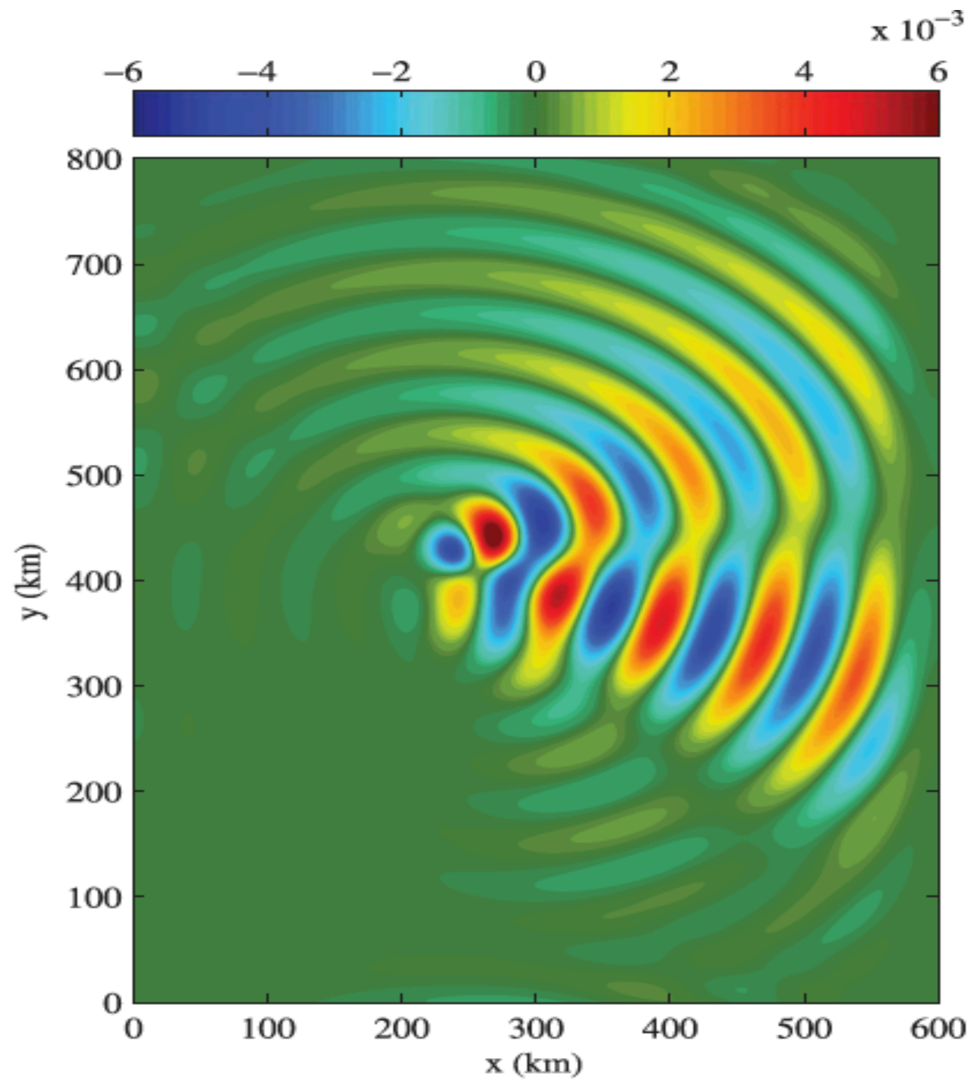


Figure 7. Difference in density perturbation (kg/m^3) between the low-latitude barotropic case where $L_E = 50$ km and $U_E = 45$ cm/s and the no-eddy case when $t = 16T$.

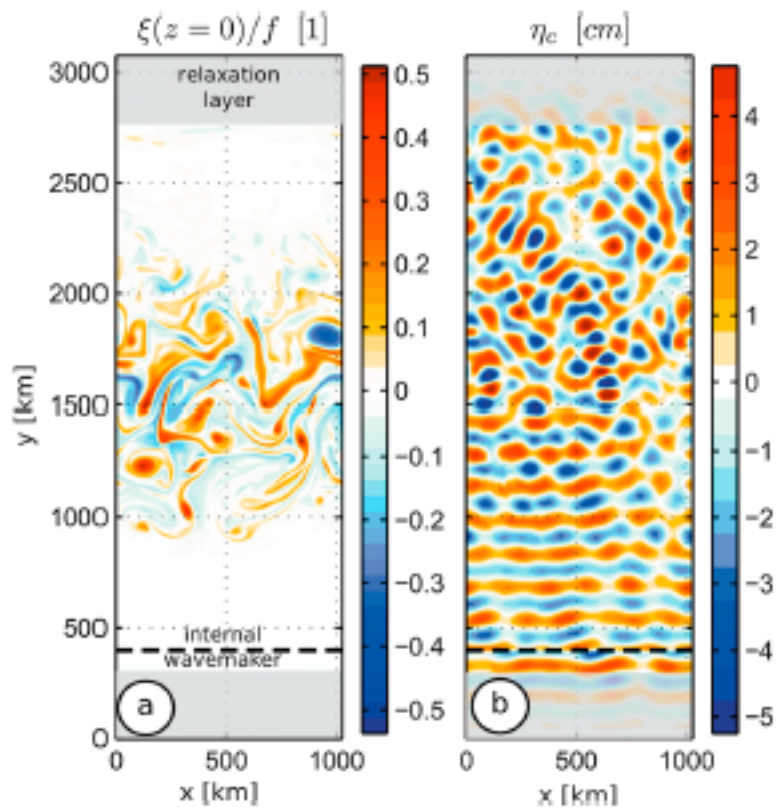


Figure 1. Overview of the numerical simulation KE4 (most intense mesoscale turbulence) at t = 500 days. (a) Detided surface relative vorticity $\xi(z = 0)$, normalized by the Coriolis frequency. (b) Tidal harmonic of sea level η_{cos}. The location of the internal tide wavemaker is represented by the horizontal dashed line. Grey shadings represent areas where all fields are strongly relaxed toward initial values.

Ponte and Klein JPO 2015

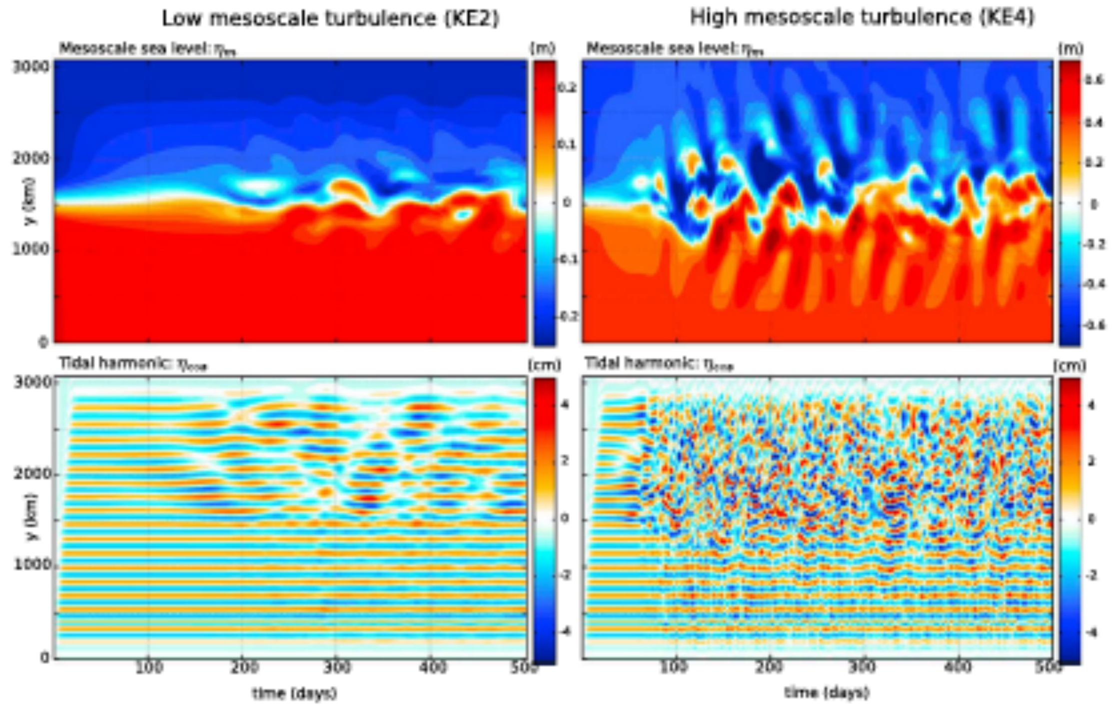


Figure 3. (top row) Mesoscale (η_m) and (bottom row) harmonic tidal (η_{cos}) contributions to sea level as a function of time and space (y) for KE2 (left) and KE4 (right) at $x = 512$ km. The colorbar for η_{cos} and KE4 is saturated, as amplitudes actually reach values up to 10 cm.

Ponte and Klein JPO 2015

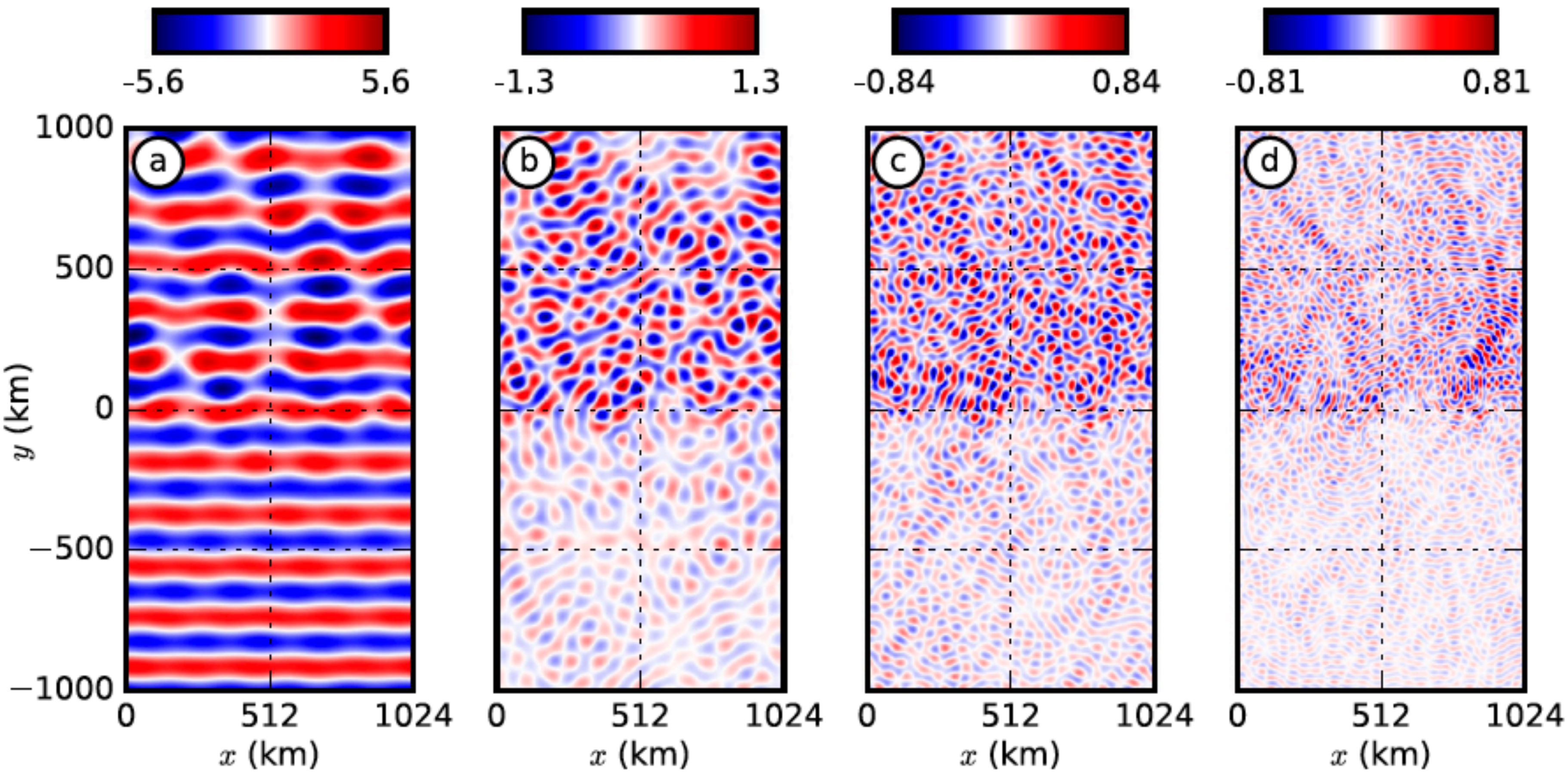
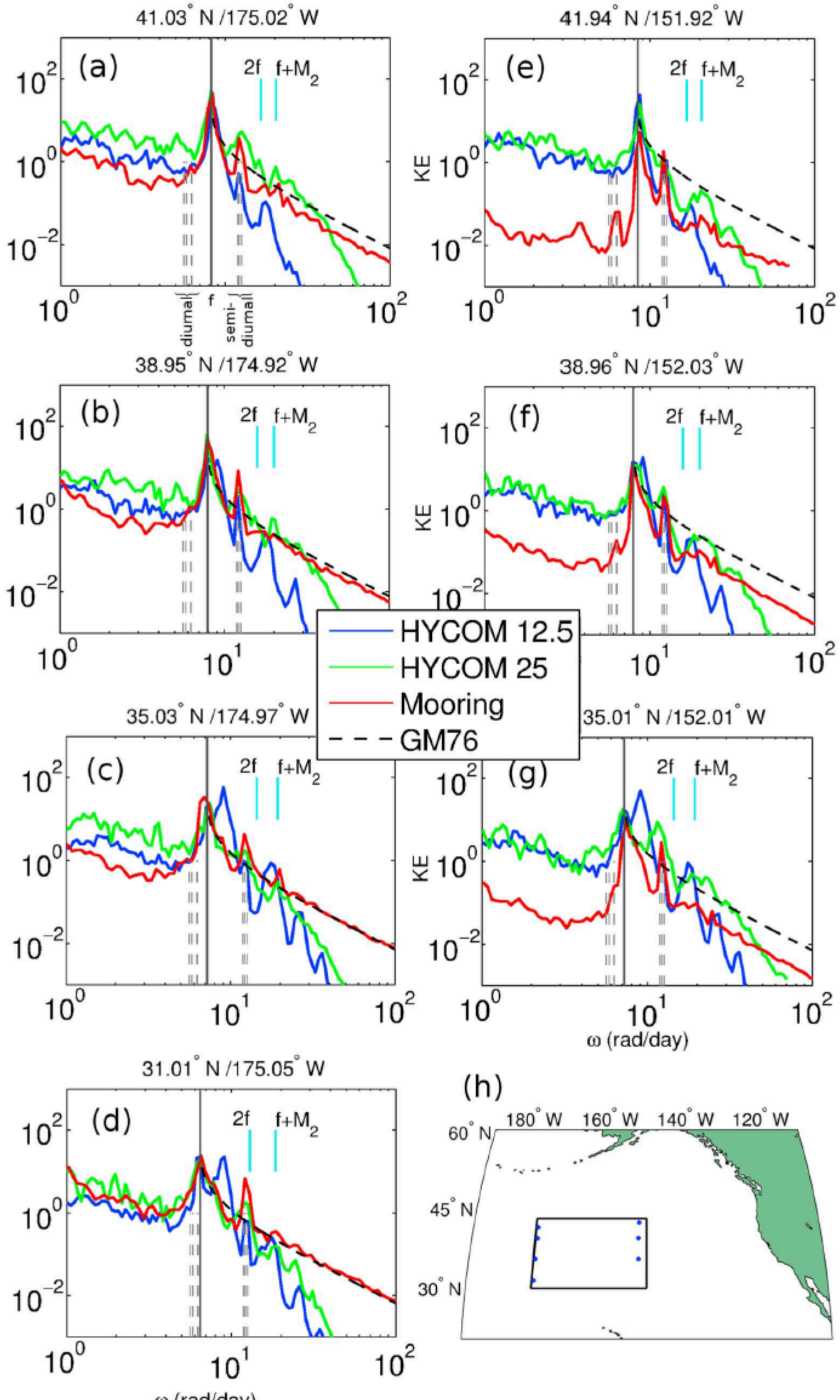


FIG. 5. Projected tidal frequency vertical velocity $\hat{w}_n$ (mm s^{-1}) at $t = 3000$ days for (a)–(d) modes one through four. Each field is multiplied by the maximum of $|d/dz[\phi_n(z)]/N^2(z)|$, such that the maps represent the peak contribution to $\hat{w}$ via (9).

TOWARD REGIONAL CHARACTERIZATIONS OF THE OCEANIC INTERNAL WAVEFIELD

GM SPECTRUM revisited by POLZIN AND LVOV Rev. Geophys., 2011…

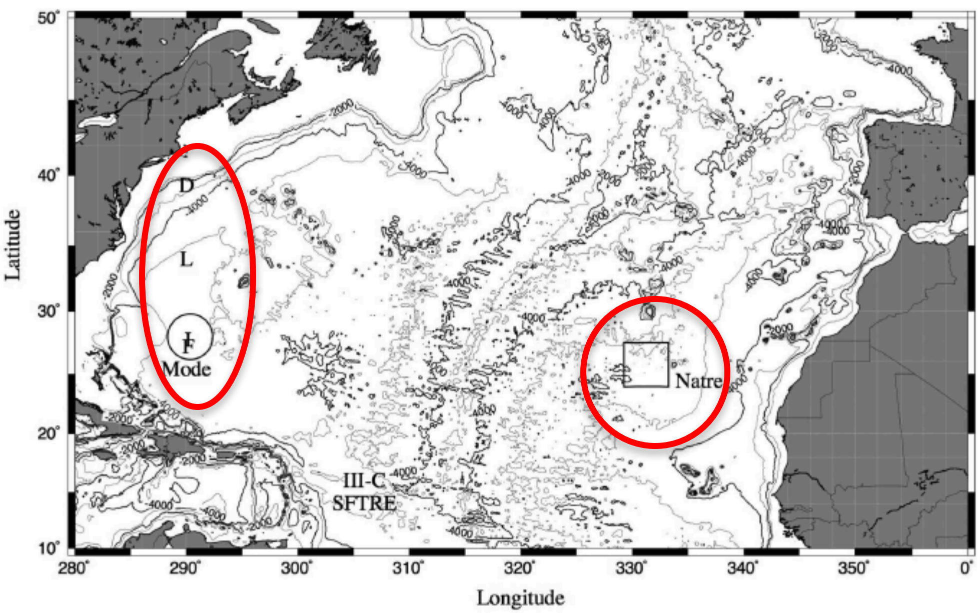


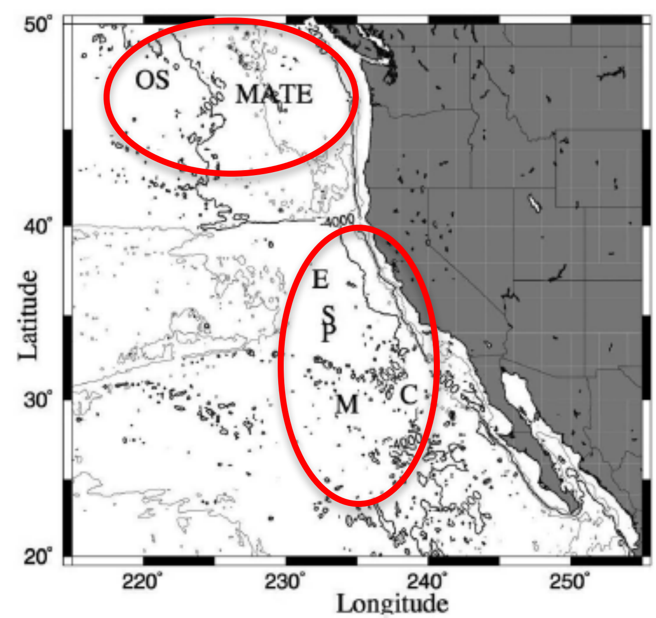
Muller et al. JGR 2015

TOWARD REGIONAL CHARACTERIZATIONS OF THE OCEANIC INTERNAL WAVEFIELD

K. L. Polzin and Y. V. Lvov

Review of Geophysics **2011**





« Observations of the last 4 decades are revisited
In the context of the universal GM spectral model »

« Much of the original GM72 paper is a demonstration of how observations could be synthesized into a combined wavenumber frequency spectrum consistent with linear internal wave kinematics. »

Furthermore GM proposed that the spectral energy density can be represented as a separable function:

$$E(\omega, m) = E_o \frac{N}{N_o} . A\left(\frac{m}{m^*}\right) . B(\omega)$$

with *m* the vertical wavenumber. and:

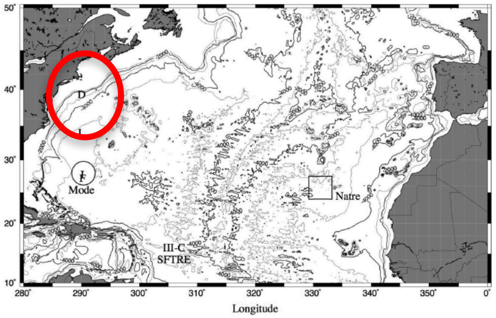
$$\boldsymbol{B(\omega) \approx \frac{\omega^{-2r+1}}{(\omega^2 - f^2)^{-\frac{1}{2}}}}$$

Note that $r = 1$ in all GM versions.

In the different GM model « incarnations », $\mathrm{A}(\boldsymbol{m}) \approx \boldsymbol{m}^{-2}$ **and** $\boldsymbol{B(\omega) \approx \omega^{-2}}$.

And since $E(\omega, m).dm = \mathrm{d}E(\omega, k_h).dk_h$: $\boldsymbol{A(\omega, k_h) \approx k_h^{-2}}$

Observations of internal wave spectra reveal that *r* can vary geographically!



3 Polzin and Lvov: REGIONAL CHARACTERIZATIONS RG4

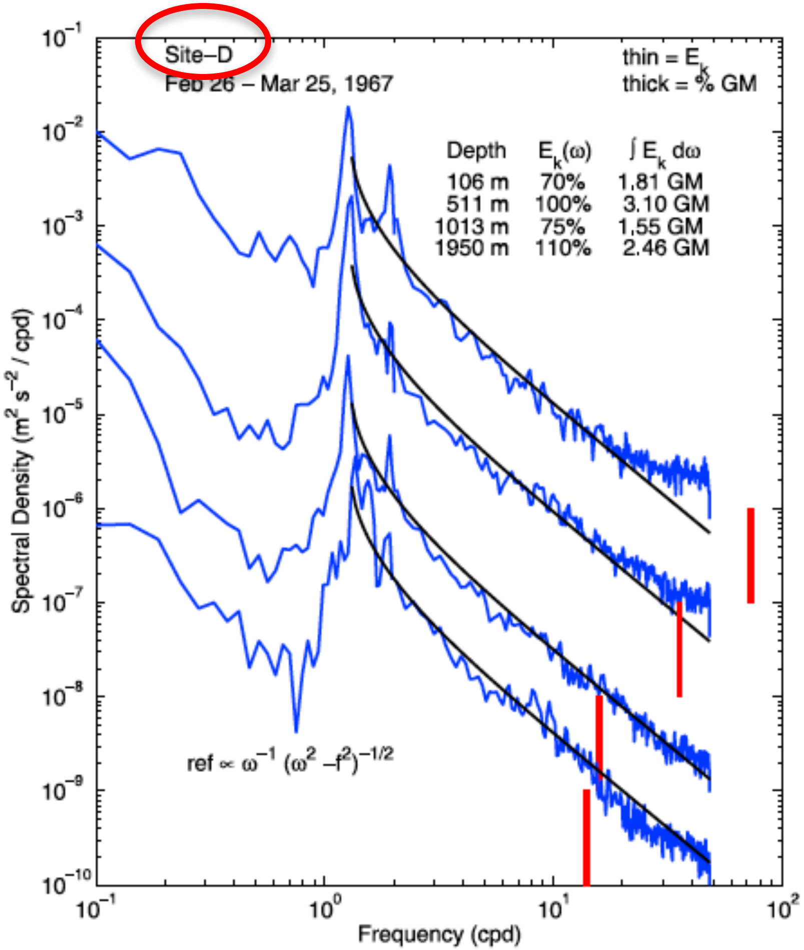
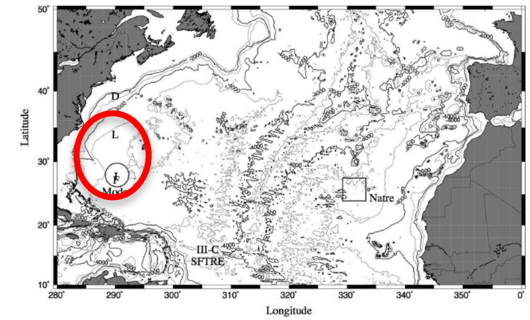


Figure 2. Site D frequency spectra of horizontal kinetic energy (blue lines). These are the Site D data that appeared in the original GM72 paper. Black curves represent fits of (21) with $r = 2$. The thick vertical lines represent the buoyancy frequency cutoff. The spectra have been offset by 1 decade for clarity.

In site D, ***r=1*** but much of these data have been used in the GM72 study.



In site LOTUS and IWEX, ***r=0.875***

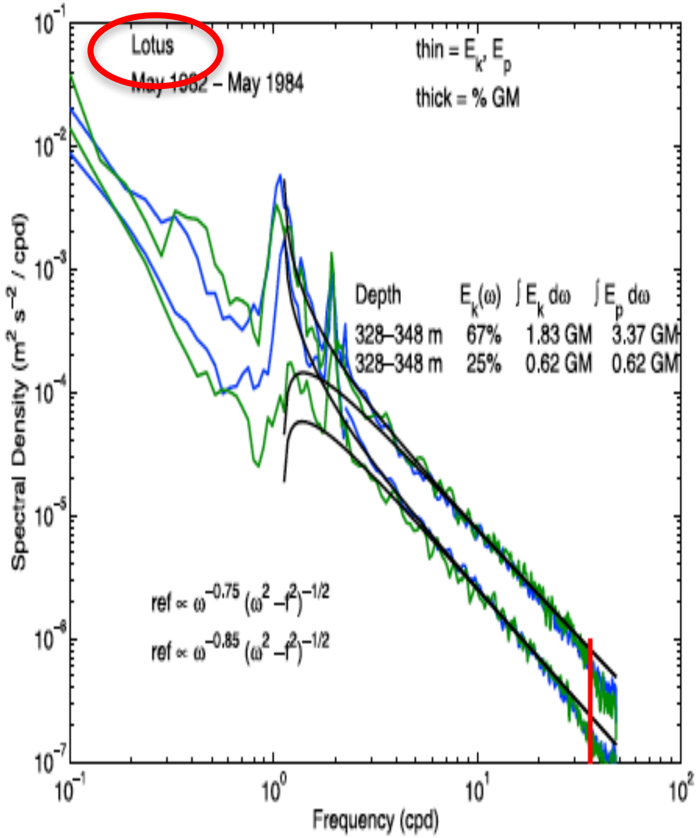
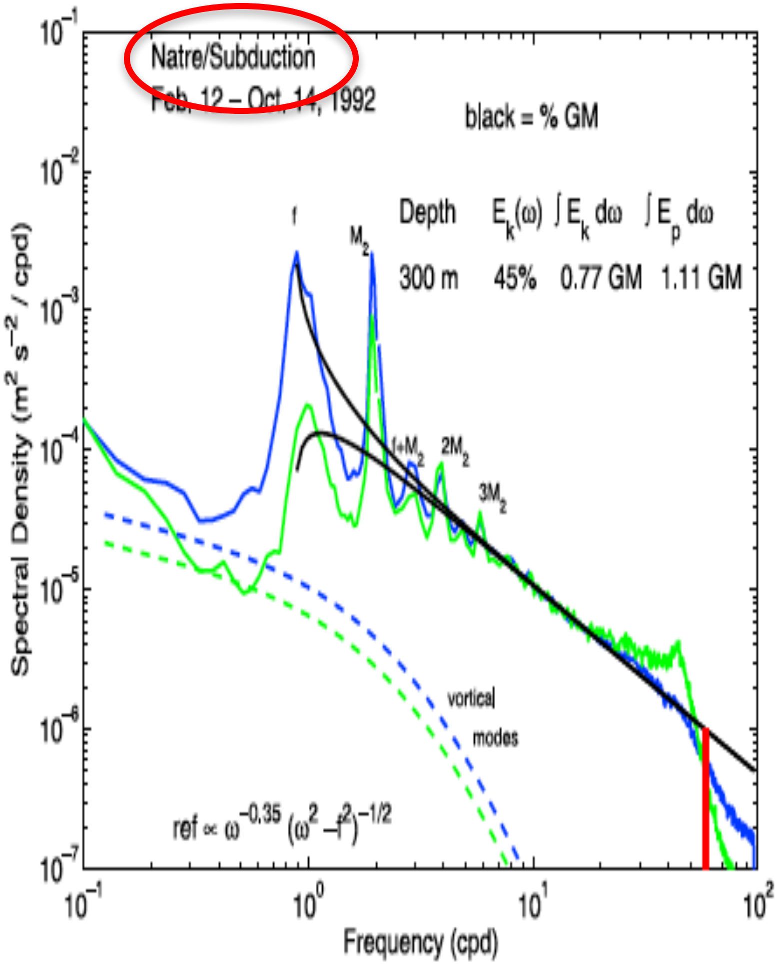
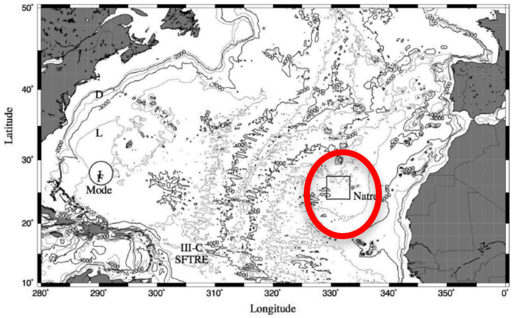


Figure 9. LOTUS frequency spectra of horizontal kinetic energy and potential energy (blue and green lines) for the high- and low-energy states depicted in Figure 8. Black curves represent fits of (21) with r = 1.75 and r = 1.85. The thick vertical line represents the buoyancy frequency cutoff. Temporal variability is dominated by variability in the overall amplitude of the spectra rather than the shape (power law).





In NATRE, ***r=0.675***

Figure 17. NATRE frequency spectra (blue and green lines) of horizontal kinetic energy and potential energy from the main thermocline (300 m). Observed spectra (solid lines) with vortical mode spectra (dashed lines) superimposed. The nonpropagating vortical fluctuations are assumed to be passively advected by the mesoscale field in this representation. See *Polzin et al.* [2003]. Inertial, semidiurnal, and harmonic peaks are noted. Black curves represent fits of (21) with r = 1.35. The thick vertical line represents the buoyancy frequency cutoff.

TOWARD REGIONAL CHARACTERIZATIONS OF THE OCEANIC INTERNAL WAVEFIELD

K. L. Polzin and Y. V. Lvov

Review of Geophysics **2011**

« We now believe that near-inertial and tidal sources dominate the high frequency sources of internal wave energy. »

« Regional and seasonality of the internal wave spectrum can be explained by different factors:

- **Roles of the seasonal variations of the near-inertial wind forcings and interactions with the mesoscale eddy field (mostly in the *atmospheric storm track regions*);**
- **PSI (parametric subharmonic instability) decay of the low-mode internal tide represents the major forcing of the internal wavefield equatorward of the *critical latitude (28.9°);***
- **Nonlinearity is clearly an organizing principle in other regions *(Eastern NP).* »**