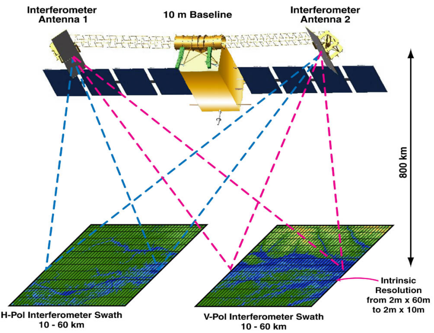
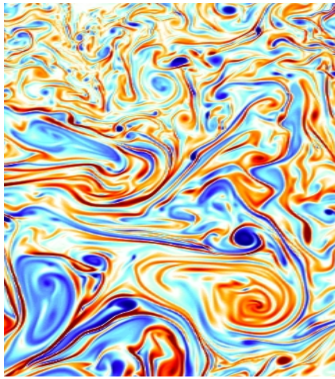
“Ocean Turbulence from SPACE”

Patrice Klein (Caltech/JPL/Ifremer)

(XIV) – Geostrophic turbulence (a)





Geostrophic turbulence

The oceanic flow is fully turbulent. Mesoscale eddies represent about 80% of the total KE of the oceans (Ferrari & Wunsch ARFM'09).

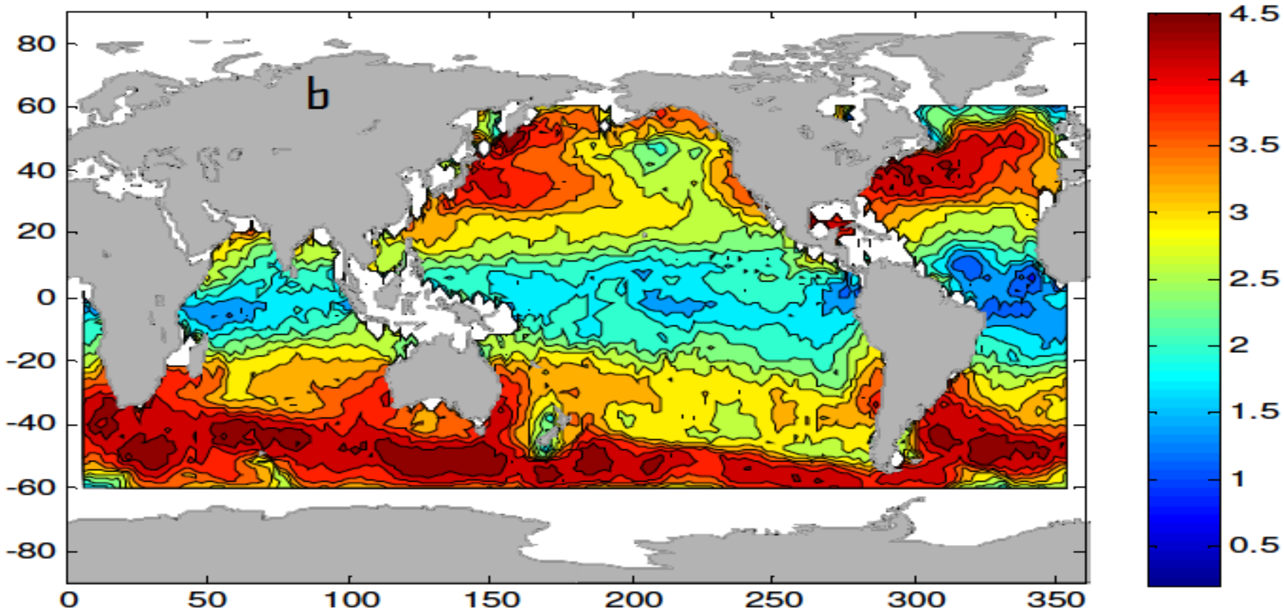
This stratified and rotating (3-D) turbulence is characterized by several non-dimensional numbers [cf 2nd class], including small Rossby number, and is called **geostrophic turbulence**. It is governed by the conservation of two quantities: the total energy (potential + kinetic) and the potential enstrophy (variance of the potential vorticity).

The **global coverage of satellite data** allows to monitor the time evolution of the ocean geostrophic turbulence and in particular to estimate **its spectral characteristics at the surface** . [cf 1st, 3rd, 4th classes].

Geostrophic turbulence

Some improvements in terms of resolution are highly needed for these satellite data. However the existing results emphasize the strong regional and seasonal variations of these spectral characteristics.

(Xu & Fu'12)



These spectral characteristics of the geostrophic (3-D) turbulence are the **signature of the mechanisms** that drive the **non-linear scale interactions**.

What is the link between these spectral characteristics and the mechanisms that drive the non-linear scale interactions?

Geostrophic and 2-D turbulence

Charney (1971) emphasized that there is **a deep similarity between the spectral properties of geostrophic turbulence and those of 2-D turbulence**. This has been confirmed by numerical simulations (see Hua & Haidvogel'86, McWilliams'89,90 and later studies).

2-D turbulence is also governed by the conservation of two quantities: kinetic energy and enstrophy (variance of the relative vorticity) [cf 5^{th} – 7^{th} class].

The mechanisms that drive the **(non-linear) scale interactions** in 2-D turbulence depend (as in geostrophic turbulence) on the **forcing, dispersive and dissipative scales**. Since the stratification is not involved, the relationship between these mechanisms and the resulting spectral characteristics is simpler.

<u>Let us examine first</u> the signature of the mechanisms that drive the **non-linear scale interactions** on the **spectral properties in 2-D turbulence** .

[*next class: additional properties linked to the non-zero stratification that characterizes geostrophic turbulence*]

Fourier transform (we need to move to the spectral space to understand the non-linear scale interactions)

It is convenient to express variables (ψ) in terms of double Fourier integral:

$$\boxed{\psi(x,y) = \frac{1}{2\pi}\iint_{k\,l} \hat{\psi}(k,l)\, e^{i(kx+ly)}\, dk\, dl.}$$

with k and l respectively the zonal and meridional wavenumbers.

If ψ is the stream function $[u = -\psi_y,\ v = \psi_x]$ we get,

$$|\hat{u}(k,l)|^2 + |\hat{v}(k,l)|^2 = (k^2+l^2)\,|\hat{\psi}(k,l)|^2$$

$$|\hat{\zeta}(k,l)|^2 = -(k^2+l^2)^2\,|\hat{\psi}(k,l)|^2 \quad \text{with } \zeta = v_x - u_y.$$

We will use:

$$\boxed{\begin{aligned} E(K) &= \int_k \int_l \left[|\hat{u}(k,l)|^2 + |\hat{v}(k,l)|^2\right] dk\, dl. \\ &\qquad\quad {\scriptstyle k^2+l^2=K^2} \\ Z(K) &= \int_k \int_l |\hat{\zeta}(k,l)|^2\, dk\, dl. \\ &\qquad\quad {\scriptstyle k^2+l^2=K^2} \end{aligned}}$$

Vorticity equation in 2-D flows

$$\xi = v_x - u_y.$$

$$\Rightarrow \quad \frac{d\xi}{dt} = F - \beta v - r\xi + \nu \Delta \xi.$$

F is the forcing	K_F
β-effect.	K_β
r : linear drag	K_r
ν : viscosity	K_ν

and $\frac{d}{dt} \cdot = \frac{\partial}{\partial t} \cdot + u \cdot \nabla \cdot$

The relative separation between K_F, K_β, K_r and K_ν determines the dynamical regime that drive the non-linear interactions. And as a consequence they set up the spectral characteristics.

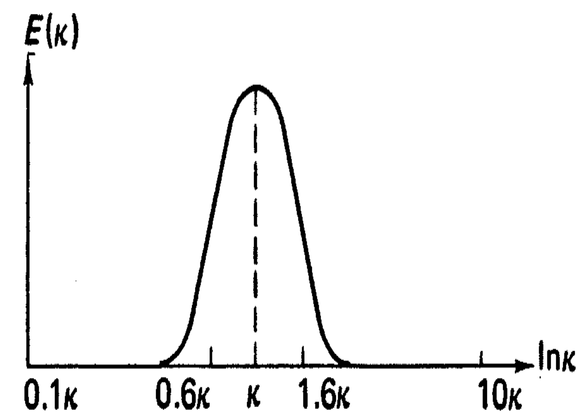
Some preliminaries …

KE energy associated with an eddy of wavenumber K

we expect eddies to lose their identity because of their interactions with others within one or two wavelengths. Therefore, the contribution of an eddy to the spectrum should be a fairly broad spike, wide enough to avoid oscillatory behavior

So it is convenient to define an eddy of wavenumber K as a disturbance containing energy between, say, $0.62\,K$ and $1.62\,K$.

$[\ln(1.62) = \ln(1/0.62) \sim 1/2]$



$$\text{So:} \quad E_E(K) = \int_{0.62K}^{1.62K} E(k)\,dk = \int_{0.62K}^{1.62K} E(k)\,k\,d(\ln k)$$

$$\Rightarrow \boxed{E_E(K) = E(K)\,K\left[\ln\left(\frac{1.62K}{0.62K}\right)\right] = E(K)\,K}$$

$$\Rightarrow \boxed{\hat{v}(K) = \left[E(K)\,K\right]^{1/2}}$$

Some preliminaries …

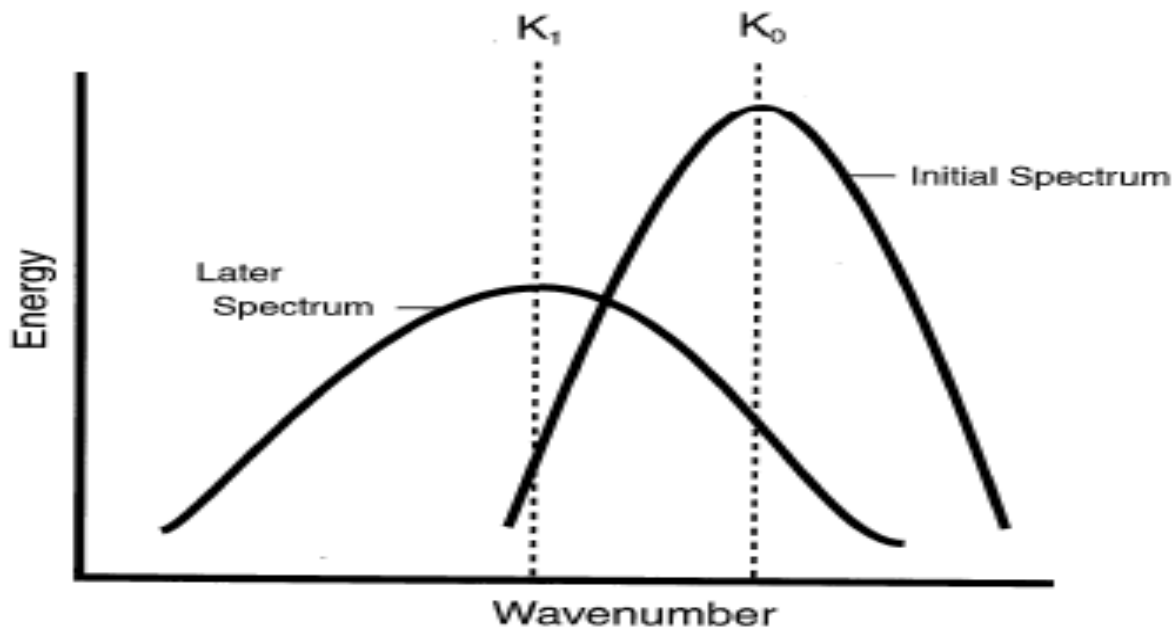
Time scale and strain rate associated with an eddy of wavenumber K

$$\hat{v}(k) = \left[E(k)\,k\right]^{1/2} \sim \frac{L}{T}$$

$$\Rightarrow \boxed{\tau(k) = k^{-1}\,\hat{v}(k)^{-1} = \left[E(k)\,k^{3}\right]^{-1/2}}$$

$$\boxed{S(k) = \left[\int_{0.62k}^{1.62k} E(k)\,k^{2}\,dk\right]^{1/2} = \left[E(k)\,k^{3}\right]^{1/2}}$$

E and Z transfers through the triad interactions (non-linear terms) [Forjtof Tellus'53]



There is a dominant E transfer to larger scales and a dominant Z transfer to smaller scales (6th class).

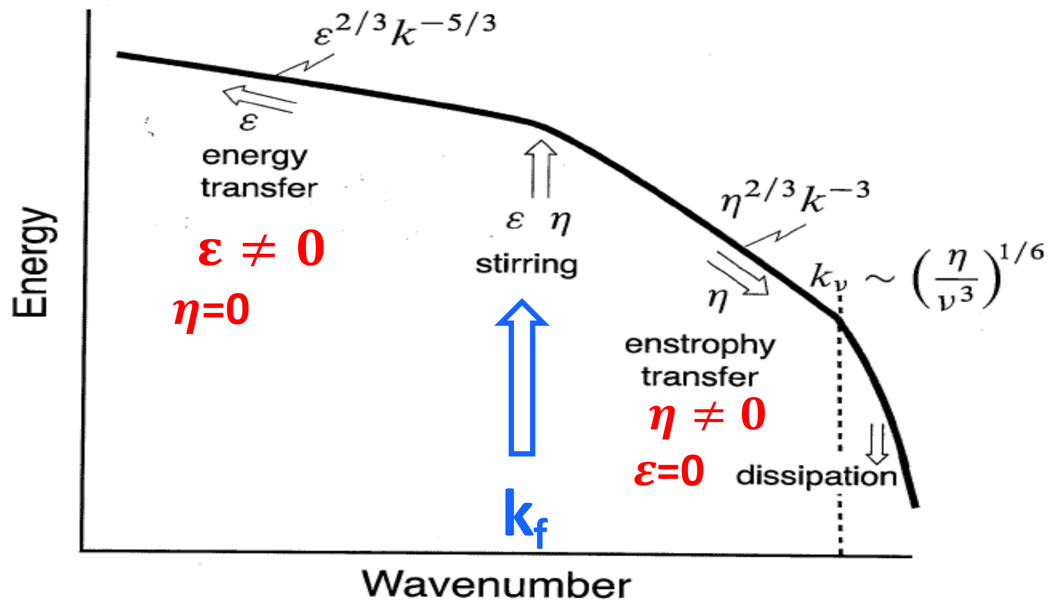
Inverse E cascade: eddy merging process (5th class)
Direct Z cascade: stirring mechanisms (7th class)

Kraichnan postulate (Kraichnan PF'67) for 2-D flows

R. Fjortoft found a dominance of KE transfer towards lower wavenumbers and a dominance of enstrophy transfer towards higher wavenumbers. This is due to the nonlinear terms (triad interactions) and the E and Z conservation.

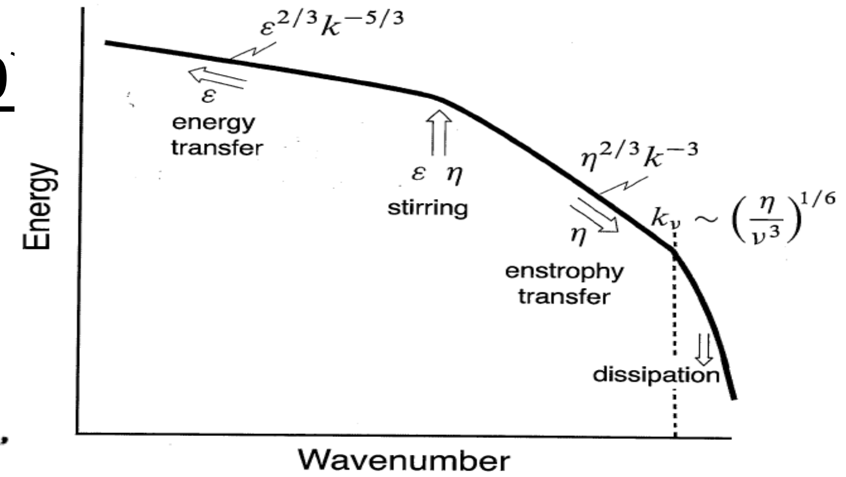
This led R. Kraichnan to postulate the existence of **two** inertial ranges, **one** in which KE injected in a given K is transferred uniformly to lower wavenumbers and a **second one** where enstrophy is transferred uniformly to higher wavenumbers. BUT the enstrophy transfer **(η)** in the first range AND KE transfer **(ε)** in the second one are zero!

Inertial range means that transfer is only due to the nonlinear terms So, in a steady state, ε and η are constant within these ranges!



Energy inertial range ($\varepsilon \neq 0, \eta = 0$)

Note that ε is constant.



We assume:

$$E(k) = g(k, k_F, k_\beta, k_R, k_\eta, \varepsilon).$$

ε is the KE transfer rate to larger scales (smaller k).

The "local" hypothesis indicates that the KE flux (ε) only depends on the nonlinear terms, i.e. only on $E(k)$ and k.

$$\Rightarrow \quad E(k) = g(k, \varepsilon).$$

ε is a KE flux with dimension: $\dfrac{E(k) \cdot k}{T}$

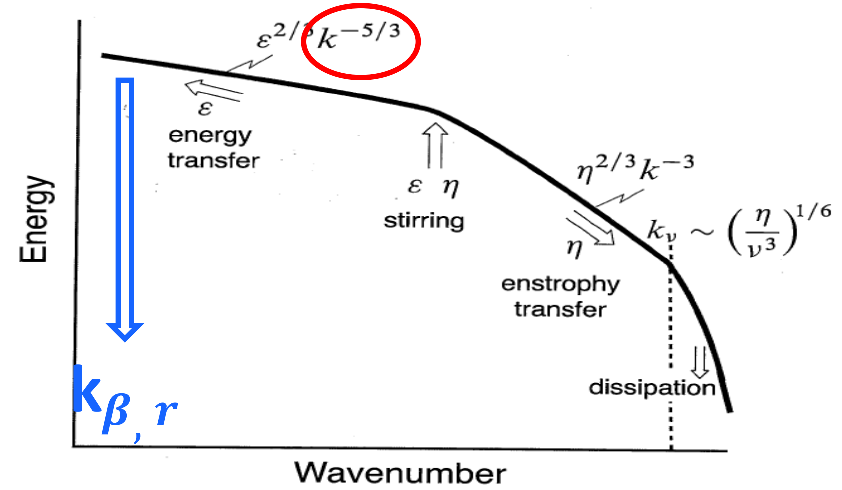
Using $\tau(k) = \left[E(k)\, k^3\right]^{-1/2}$

$$\Rightarrow \quad \varepsilon \approx \left[E^3(k)\, k^5\right]^{1/2}.$$

$$\Rightarrow \quad \boxed{E(k) = \mathcal{K}_\varepsilon\, \varepsilon^{2/3} k^{-5/3}}$$

What about K_β and K_r ?

***β*-effect** does not remove energy from the fluid and therefore does not halt the inverse KE cascade. Energy may cascade to larger scales although anisotropically (leading to zonal jets).



If $c_r = \beta/k^2$ is the phase speed of the Rossby waves, then the β-effect will affect the turbulent flow when

$c_r > \hat{v}(k)$.

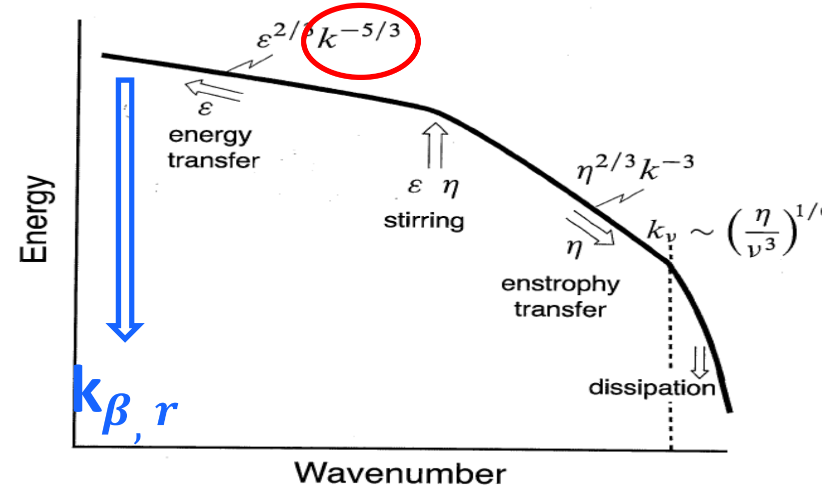
Using $\hat{v}(k) \sim [E(k) \cdot k]^{1/2}$ and $E(k) = \mathcal{K}_\varepsilon \varepsilon^{2/3} k^{-5/3}$.

The β-effect will become significant when:

$$\boxed{k < \left[\frac{\beta^3}{\mathcal{K}_\varepsilon^{3/2}\,\varepsilon}\right]^{3/5} = k_\beta.}$$

What about K_β and K_r ?

Removal of energy at large scales is usually parametrized as a linear (or quadratic) drag (Ekman): its time scale is usually much larger than the turbulence time scale.

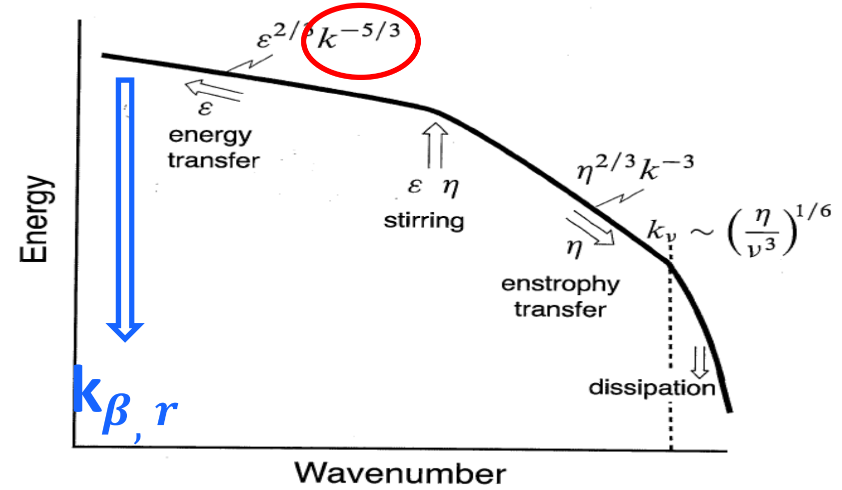


The rate of KE removal scales as T^{-1}. So the KE removal will become efficient when $r^{-1} < \tau(K)$. This occurs when:

$$\boxed{K < r^{3/2} K_\varepsilon^{-3/4} \varepsilon^{-1/2} = K_r}$$

What about $\mathbf{K}_{\boldsymbol{\beta}}$ and $\mathbf{K}_{\mathbf{r}}$?

The scale range where β-effect has an impact depends on the ratio $\mathbf{K}_{\boldsymbol{\beta}}/\mathbf{K}_{\mathbf{r}}$.

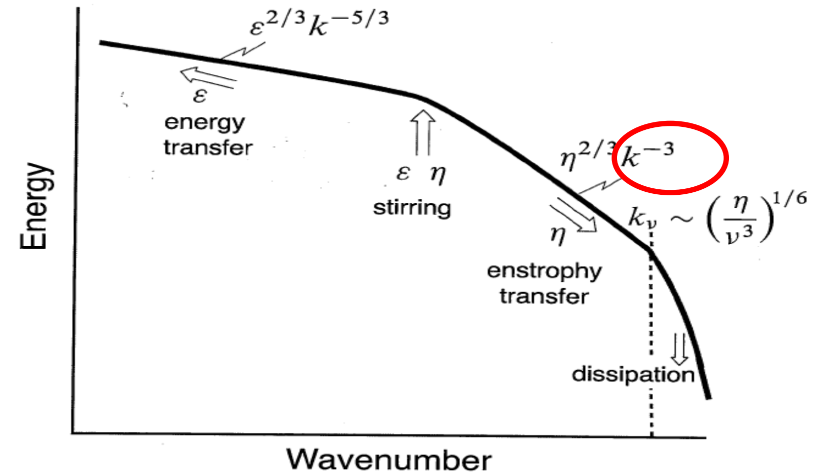


$$\boxed{K_\beta / K_r = \gamma \approx \left[\frac{\beta\, \varepsilon^{1/2}}{r^{5/2}}\right]^{3/5}}$$

When $\boldsymbol{\gamma}$ is large, the $\boldsymbol{\beta}$ - effect will be felt, but when $\boldsymbol{\gamma}$ is small, frictional effects will dominate. Extension of the scale range [$\mathbf{K}_{\boldsymbol{\beta}}$ - $\mathbf{K}_{\mathbf{r}}$] depends on **r** but also on $\boldsymbol{\varepsilon}^{\mathbf{1/2}}$ and therefore on the KE source.

Enstrophy inertial range $\eta \neq 0, \epsilon = 0$)

Note that η is constant.



By analogy with the formalism used for the K6 inertial range we assume that the enstrophy cascade rate, η, is given by:

$$\eta \approx \frac{E(k)\, k^3}{\tau(k)}.$$

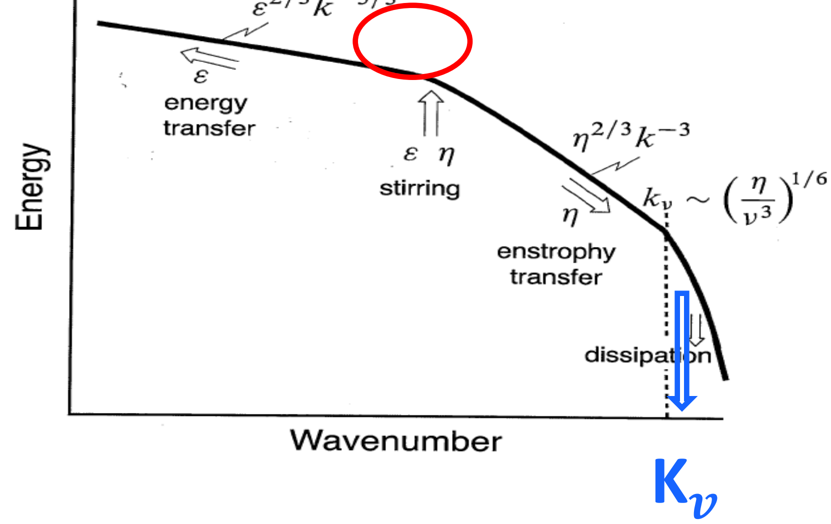
This leads to:

$$\boxed{E(k) = \mathcal{K}_\eta\, \eta^{2/3}\, k^{-3}}$$

with $\mathcal{K}_\eta$ again a universal constant analog to the Kolmogorov constant.

What about K_ν ?

Removal of enstrophy at small scales is usually parametrized as a Laplacian operator. its time scale
is usually much larger than the turbulence time scale.



The dissipation time scale is $(\nu K^2)^{-1}$

Dissipation occurs when $(\nu K^2)^{-1} < \tau(K)$.

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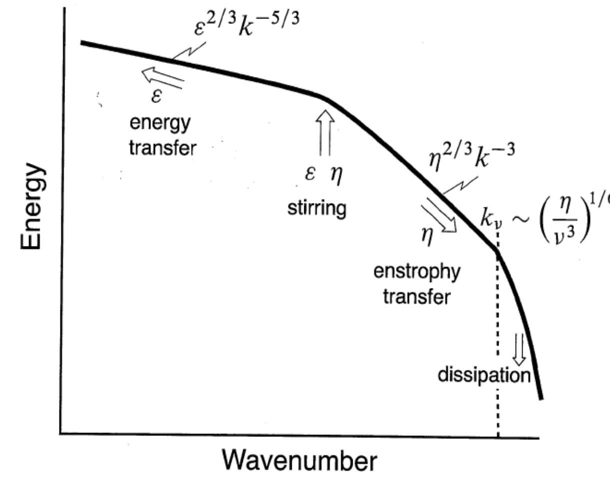
$$\Rightarrow \boxed{K > \nu^{-1/2} J_\eta^{1/4} \eta^{1/6} = k_\nu}$$

« Local » and « non-local » dynamics



Stirring mechanisms (through the strain rate S(K)) govern the direct cascade of any tracers (including vorticity)

The stirring mechanisms (and therefore the strain rate $S(K)$) drive the tracer cascade (and so the enstrophy cascade).

$$S(K) = \left[E(K)\, K^3\right]^{1/2}$$

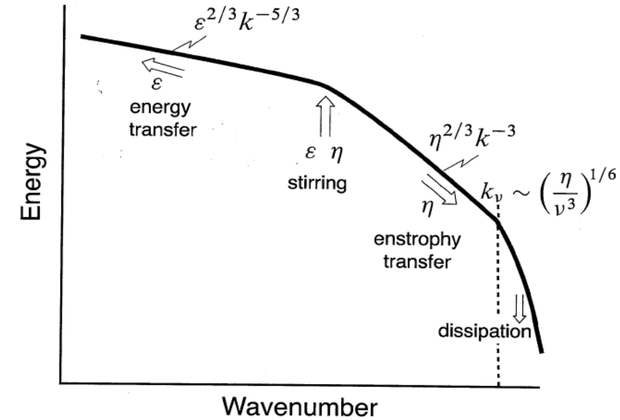
In the energy inertial range ($K^{-5/3}$).

$$\boxed{S(K) = \left[K_\varepsilon\, \varepsilon^{2/3} K^{4/3}\right]^{1/2}}$$

The wavenumber just below a given one contributes to the tracer cascade. The dynamics (a tracer advection) is "LOCAL"

« Local » and « non-local » dynamics

Stirring mechanisms (through the strain rate S(K)) govern the direct cascade of any tracers (including vorticity)



The stirring mechanisms (and therefore the strain rate S(K)) drive the tracer cascade (and so the enstrophy cascade).

$$S(K) = \left[E(K)\, K^3\right]^{1/2}$$

In the enstrophy inertial range (K^{-3}).

$$\boxed{S(K) = \left[\int K_\eta \, \eta^{2/3}\right]^{1/2}}$$

All wavenumbers below a given one, and not only the one close to this wavenumber, contribute to the tracer cascade, The dynamics is "NON-LOCAL".

« LOCAL » and « NON LOCAL » dynamics lead to very different spatial distribution of tracers (see below)

TRACER DISTRIBUTION

We consider a tracer driven by:

$$\frac{d\Phi}{dt} = F[\Phi] + k\,\Delta\Phi$$

Because of the stirring mechanisms, the tracer variance should be transfered to scales smaller than the scale of the tracer injection.

By analogy with the formalism used for the energy and enstrophy cascade we assume that:

$$\chi \approx \frac{\Gamma(k)\,k}{\tau(k)}$$

with χ the cascade rate of the tracer variance and $\Gamma(k)$ the tracer spectrum.

Assuming that χ is constant and that the velocity spectrum is $E(k) = A\,k^{-n}$ $(n = 5/3$ or $3)$, we get:

$$\Gamma(k) = K_\chi\, A^{-1/2}\, \chi\, k^{(n-5)/2}$$

$$\Gamma(k) = K_\chi \cdot \varepsilon^{-1/3} \cdot \chi \cdot k^{-5/3}$$ when n=5/3

$$\Gamma(k) = K_\chi \cdot \eta^{-1/3} \cdot \chi \cdot k^{-1}$$ when n=3

Next slides show some examples of the tracer spatial distribution when the velocity spectrum involves a slope in $K^{-5/3}$, K^{-3} or both.

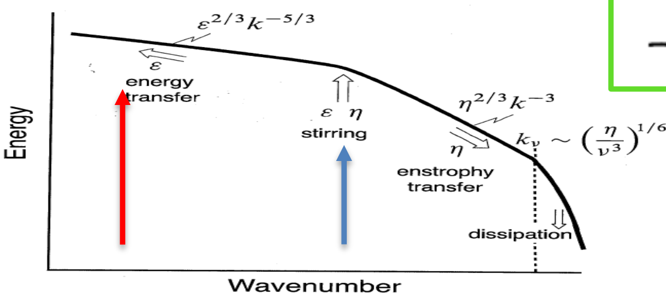
These examples come from Scott 2006: « Local and nonlocal advection of a passive scalar ». Physics of Fluid.

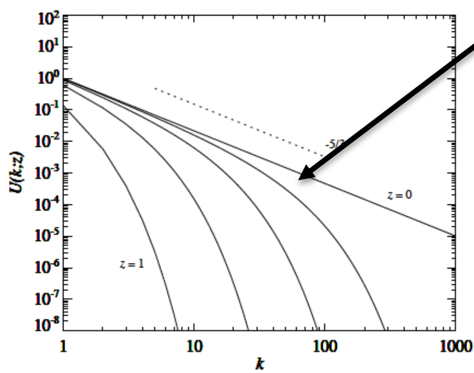
Scott used the SQG solution at different levels.
At the surface the velocity spectrum has a slope in $K^{-5/3}$.
At large depth the velocity spectrum slope is in K^{-3} or steeper.
At intermediate depth, both slopes are observed.

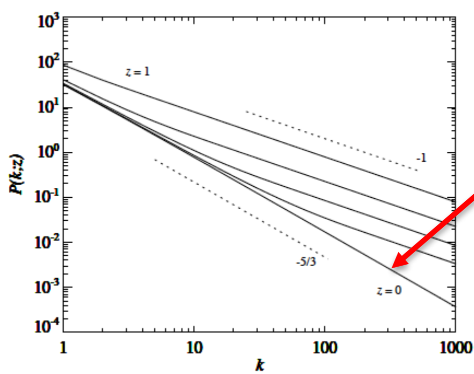
See his figures 1 and 2.

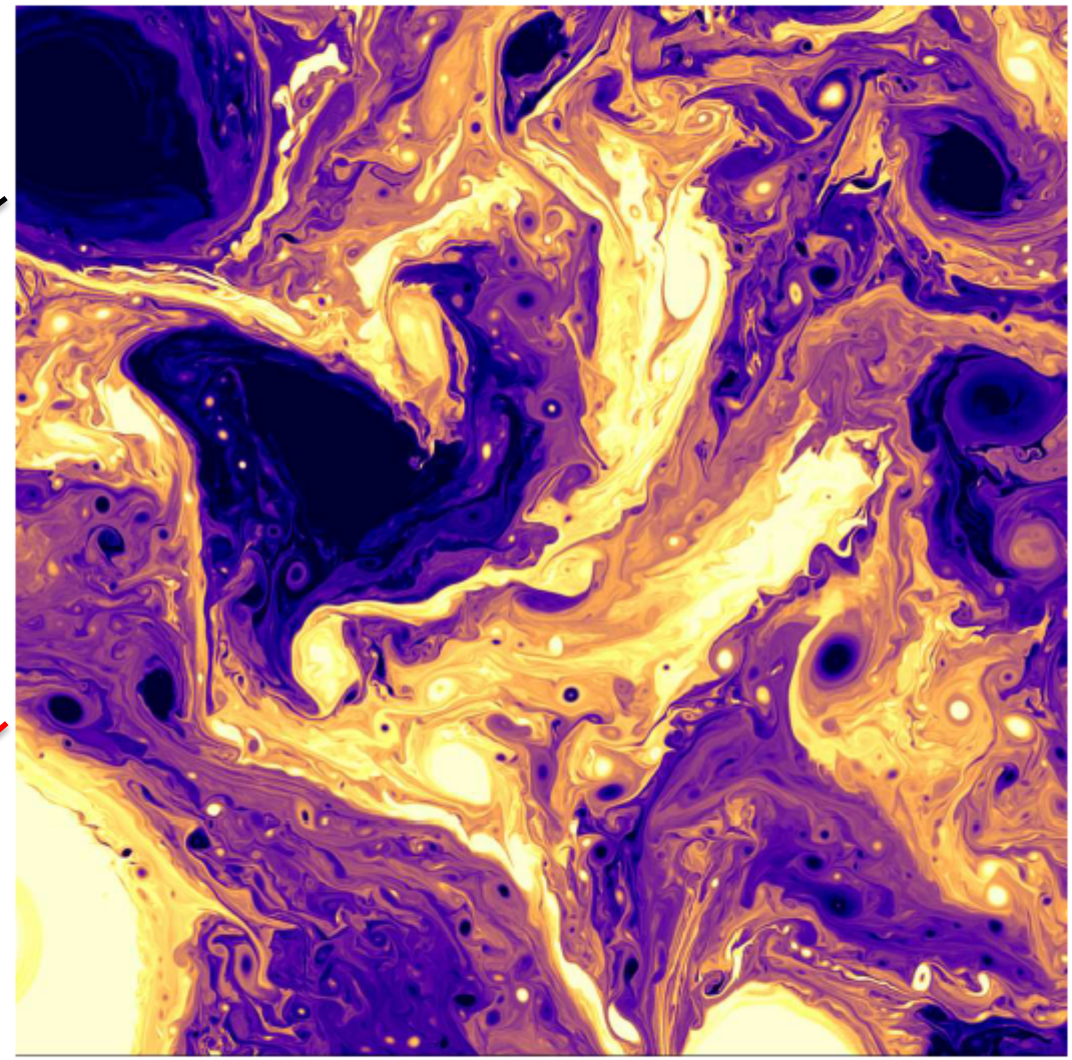
Passive tracer when the tracer injection (red) occurs at a much larger scale than the energy injection (blue):

$$P(k) = K_\chi \cdot \varepsilon^{-1/3} \cdot \chi \cdot k^{-5/3}$$



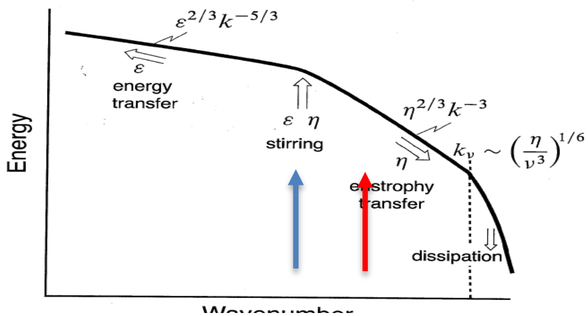


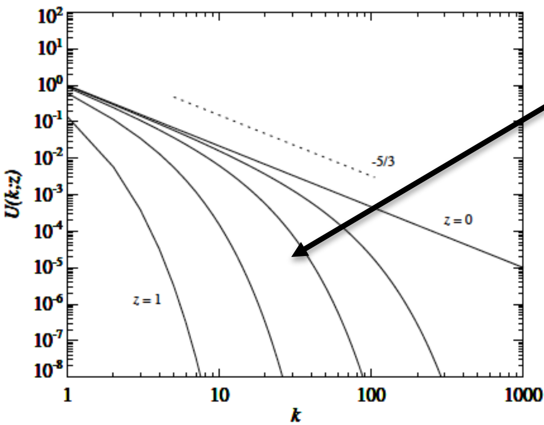


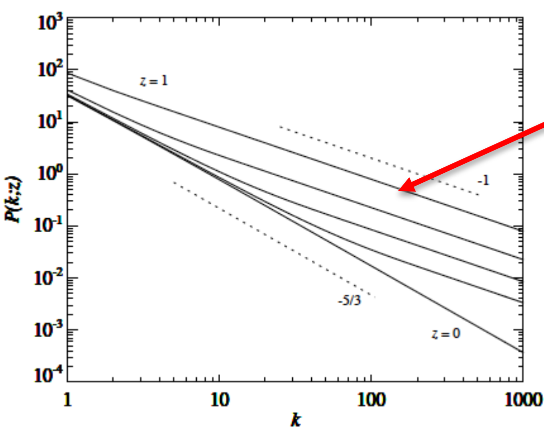


Passive tracer when the tracer injection (red) occurs at a smaller scale than the energy injection (blue):

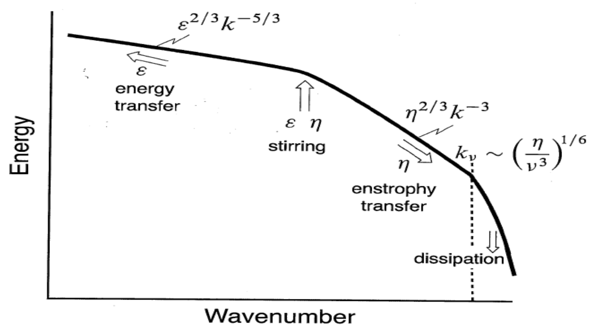
$$P(k) = K_\chi \cdot \eta^{-1/3} \cdot \chi \cdot k^{-1}$$

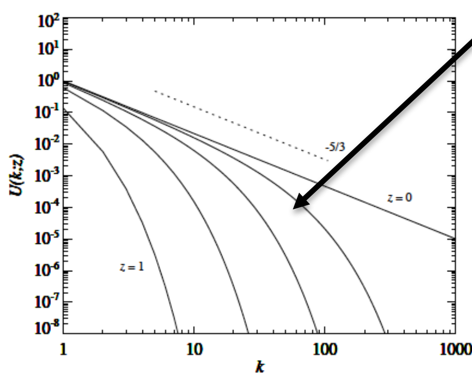


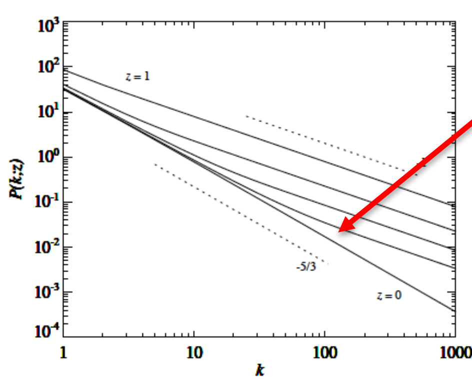


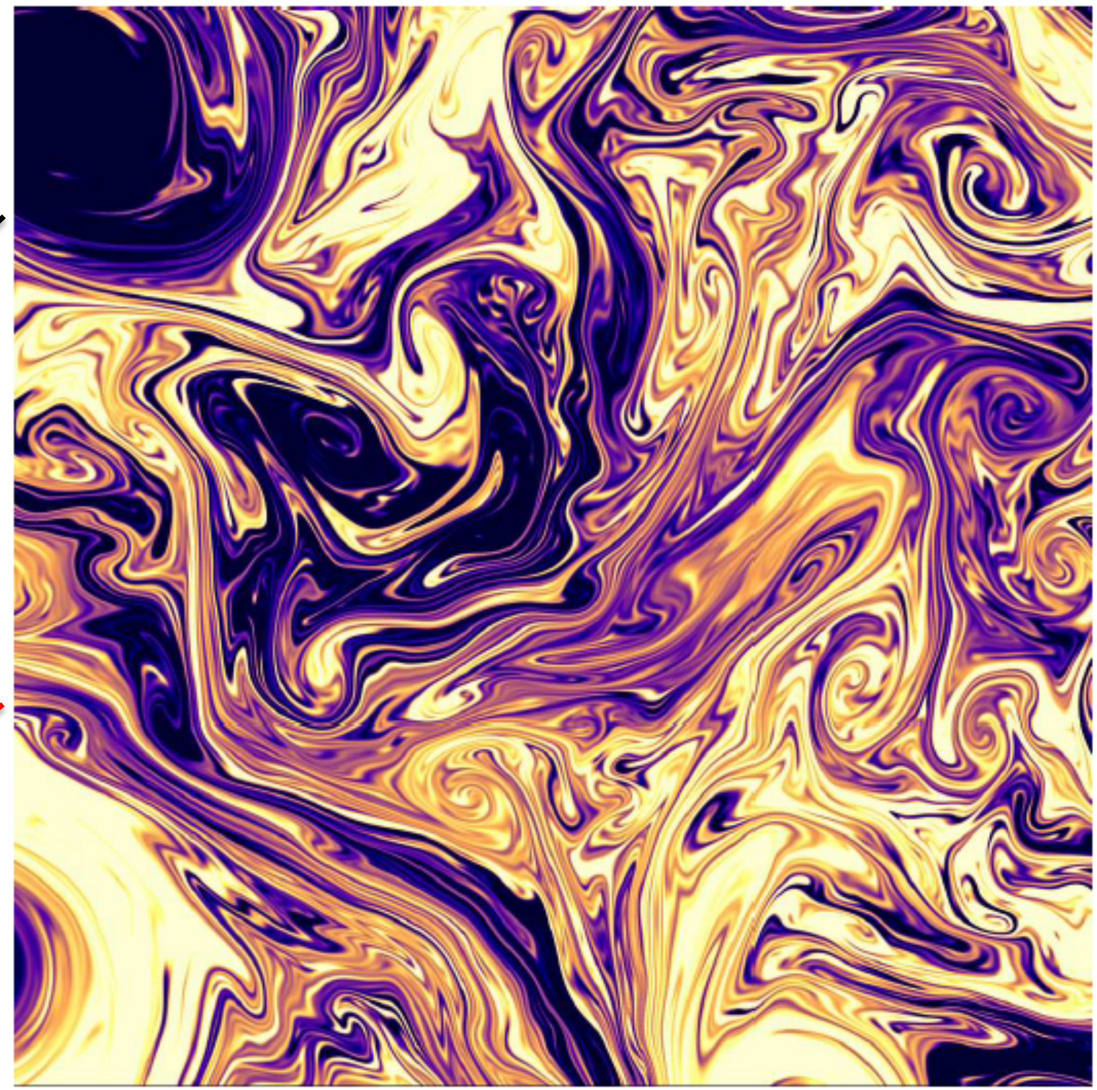


Passive tracer when the energy injection occurs at a scale slightly smaller than the tracer injection:









N*ext class:*

additional properties linked to the non-zero stratification that characterizes geostrophic (3-D) turbulence …