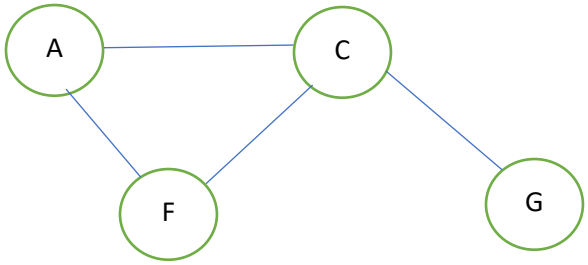
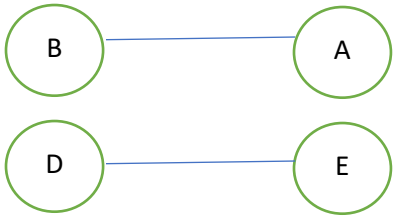
Ochirgarid Chinzorig, Assignment-8

1. A graph is given:
    a. Let U = {A, B}. Draw G[U].

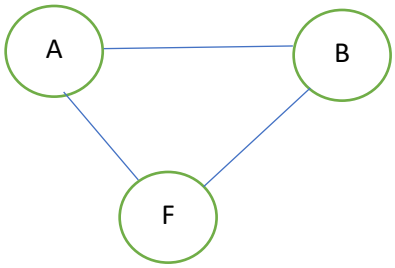
    b. W = {A, C, G, F}. Draw G[W].



    c. Y = {A, B, D, E}. Draw G[Y].



    d. No. Because X vertices {A, B, F} but G[X] has another edge.



    e. If we choose V1 = {D, E, I}, V2 = {A, B, C, F, H, G} this way it will suffice the conditions.
    G = G[v1] U G[v2] = {D, E, I, A, B, C, F, H, G}

2. The source code is attached.
3. Graph exercises:
    a. Assume there doesn't exist any edge (x,y) in E such that for some $i \neq j$, x is in V_i and y is in V_j. Then for all $i \neq j$, V_i and V_j are not connected to each other, and the G will be not connected. Thus the assumption is a false statement and there exists edge (x,y) which satisfies the condition.
    b. True. The connected graph with the fewest number of edges has a v-1 number of edges (proof on the c.). Hence following equation holds.
    $$e = v - 1 > \frac{(v-1) * (v-2)}{2}$$
    c. It is n-1. Prove it by induction. Base case is clear on n=1 (0 edge), n=2 (1 edge). Assume the hypothesis holds for all values less than n. A connected graph with n vertices could be composed by connecting 2 connected graphs with fewer vertices. Let say these 2 have k and n-k vertices, then by our hypothesis these 2 can have at minimum of (k-1) and (n-k-1) edges and we connect these using another one edge. So, the total number of edges is equal to (k-1) + (n-k-1) + 1=n-1. And the induction proved.
4. IsPrime Problem, Revisited.

    On this SecondTry algorithm I added a loop that will try f() for log(n) times which would increase the change and at the same time the running time of the algorithm won't be affected too much. It will be $O(\log^2(N))$ after this improvement. And the probability to guess wrong will decreases from 1/2 to 1/N.

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Algorithm SecondtTry:
Input: A positive integer n
Ouptut: TRUE if n is prime, FALSE if n is composite
     if n % 2 = 0 return FALSE
     for j ← 0 to log(n) do
          a ← random number in [1, n-1]
          if f(a,n) = 1
               return FALSE
     return TRUE
```