

Lab 6

1. A red-black tree is said to be *derivable* if it is obtained from an insertion sequence of nodes, using the rules for insertions starting from an empty tree. Give an example to show that not every red-black tree is derivable. (In other words, you can build a BST that satisfies the four conditions for a red-black tree, and yet there is no way to obtain this tree by successively inserting nodes using the insertion algorithm rules.)
2. An *AVL Tree* is a BST that satisfies a different balance condition, namely:

The AVL Balance Condition For each internal node x , the height of the left child of x differs from the height of the right child of x by at most 1. (Equivalently, the heights of the left and right subtrees of x differ by at most 1.)

Create a red-black tree that does *not* satisfy the AVL Balance Condition.

3. Use the insertion algorithm for red-black trees to successively insert the following nodes, starting with an empty tree.
 - a. 1, 2, 3, 4, 5, 6, 7, 8
 - b. 3, 2, 1, 4, 5, 6

Note on Part (a): Recall that an already sorted insertion sequence is a worst case for an ordinary BST. Notice how the red-black balancing operations handle this to remain balanced.

4. *Interview Question.* Give a $o(n)$ (“little-oh”) algorithm for determining whether a sorted array A containing n distinct integers contains an element m for which $A[m] = m$. You must also provide a proof that your algorithm runs in $o(n)$ time.