Ochirgarid Chinzorig, Assignment-9

1. If the graph G is a connected then, by the Theorem from the lecture (lecture-9 p. 18 "Suppose G is a connected graph with n vertices and m edges. Then G contains a cycle if and only if $m \geq n$.") G would contain a cycle if m >= n.
   If the graph G is not connected. Let us use the Theorem from the lecture (lecture-9 p.23 "Suppose G is a graph with n vertices and m edges. Let F be a spanning forest for G (F is the union of the spanning trees inside each component of G). Let m_F denote the number of edges in F. Then G has a cycle if and only if $m > m_F$") it would contain a cycle.

   Assume each tree in forest has a $F_1, F_2, \ldots, F_k$ vertices. Then $n=F_1 + F_2 + \ldots + F_k$, and since tree each tree F_i has F_i-1 edges $mF = F_1 - 1 + F_2 - 1 + \ldots + F_k - 1 = F_1 + F_2 + \ldots + F_k - k = n - k$. Hence, the graph G has m >= n >= n – k = m_f property and it would contain a cycle by the theorem.
2. Since, S and T are a tree and we are connecting this 2 trees with any edge (x,y) in E for which x is in V_S and y is in V_T. Then the total number of the edges in $U = (V_S \cup V_T, E_S \cup E_T \cup \{(x,y)\})$ is $|E_S| + |E_T| + 1 = |V_S| - 1 + |V_T| - 1 + 1 = |V_S| + |V_T| - 1$. Now if we prove that <u>U is connected</u> then by the theorem we learned in lecture (lecture-9 p. 18 "Suppose G is a connected graph with n vertices and m edges. Then G contains a cycle if and only if $m \geq n$.") it will not contain any cycle and it would prove U is a tree.
   Let (*a, b*) be 2 distinct vertices from U. If (*a, b*) both belongs to either V_S or V_T there should exists a path between them. If (*a, b*) belongs to V_S, and V_T, we can safely assume *a* belongs to V_S and *b* belongs to V_T. Then there should a path exists *a*->x->y->b which shows U is a connected graph.
3. Code attached
4. Code attached
5. Way 1: Prove it by contradiction. Let T be a tree with all vertices have a more than 1 degree. Then the total number of edges >= 2 * N / 2 = N by the theorem it is not a valid tree, hence it is proved by contradiction.
   Way 2: Let v be any vertex in T and think of T as a rooted tree with vertex v. If the tree has more the 2 leaf vertex then the leaf vertices would have degree of 1. If it has only 1 leaf then the root should have 1 degree.