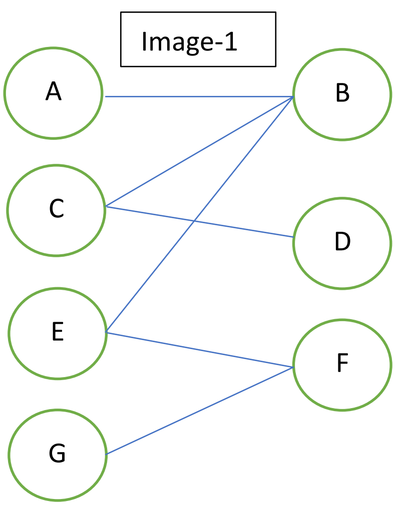
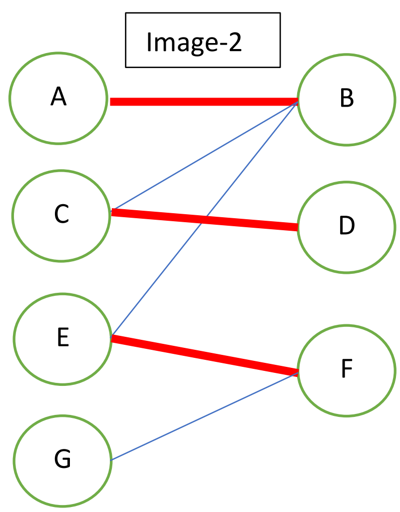
Ochirgarid Chinzorig, Assignment-12

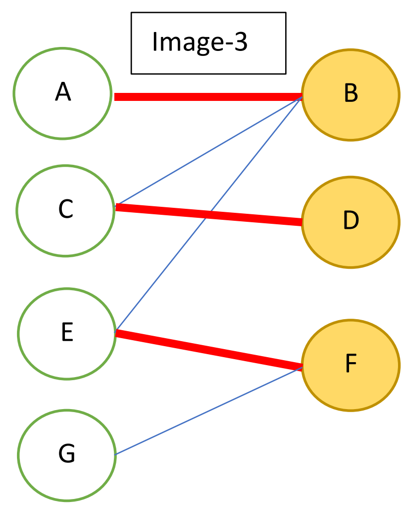
1. No, the Ore's theorem indicated that following condition should suffice.

   deg v + deg w ≥ n for every pair of distinct non-adjacent vertices v and w of G (*)

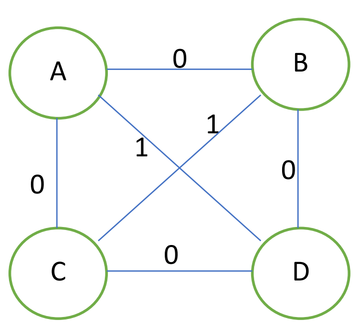
   There exists a dense graph that is not connected which clearly is not a Hamiltonian. For example, a graph G consists of 2 complete graphs with n vertices. It has a total of N*(N-1) edges. So, G is a dense graph. At the same time, By the Ore's theorem deg v + deg w = N – 1 + N – 1 = 2N – 2 >= 2N is false, and **G is not a Hamiltonian**.
2. Answers:
   a. Yes, it is
   b. Deg(A)+Deg(F) = 2+2 = 4 < 6
   c. No. The Ore's Theorem indicates that if a graph satisfies (*)-condition the graph would be a Hamiltonian. It does not say that if the graph is Hamiltonian it should satisfy (*)-condition.
3. Answers:
   a. X={A,C,E,G}, Y={B,D,F}. Presented on the image-1.
   b. Indicated one of the possible "Maximum Matchings" in red on the image-2.
   c. Highlighted one of the possible "Minimum Vertex Cover" in orange on the image-3.
   d. Yes, since each end point of the any edge belongs to different columns the minimum number of vertex cover is same as the maximum matching.







4. To transform the HC problem into Traveling salesman problem: First we need to convert our G graph into a weighted complete graph H, as for the weight we will put 1 on the new edges and 0 on the old edges. Now it converts a TSP problem with H-graph and K=0. If the TSP problem has a solution, it will only consist of the 0-weight edges which is our HC problem answer.



5. TSP is NP-complete. We have the following facts:
   (i) Since HamiltonianCycle (HC) is NP-complete, given any NP problem Q, Q is polynomial reducible to HC.
   (ii) "The problem is said to be NP-complete if it belongs to NP and every NP problem can be polynomial-reduced to it."

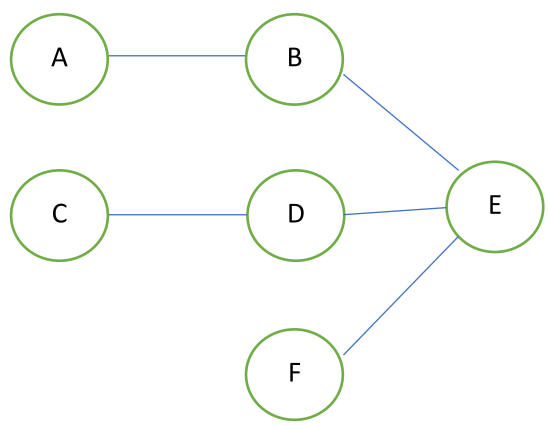
   If we can prove that HC is polynomial reducible to TSP problem, we can prove that TSP is NP-Complete problem.

   <u>Proof:</u>

   Assume we have a graph G given for a HC problem. Let us define algorithm C as follows: C: Complete the G graph to H and put a weight of 1 to the new edges, and 0 to the previous edges. Initiate value k=0. This is an algorithm with running time of $O(N^2) = O(P(y))$. Then If we solve this problem as a TSP (H, k) we can get an answer to HC(G). Hence, C and P(y) witness that HC is polynomial reducible to TSP problem.

   With this proof along with fact (i) and (ii) we proved that TSP is a NP-complete.

6. For example, on the following graph with edges={(A,B), (C,D), (B,E), (D,E),(F,E)} (with this exact order) could result the worst case. One of the "Smallest vertex cover" is {E, A, C} with size of 3. Meanwhile, the VertexCoverApprox algorithm will result with {A, B, C, D, E, F} which is size of 6.

   If we walkthrough the steps of VertexCoverApprox:
   1-We first get edge (A,B) and add (A, B) vertices to the result then we remove (B,E) edge.
   2-Now we get the edge (C,D) and add (C,D) vertices to the result then we remove (D,E) edge.
   3-Now we left with edge (F, E) and add (F,E) vertices to the result.
   4-The result would be {A, B, C, D, E, F}