1. No, the Ore’s theorem indicated that following condition should suffice.

*deg v + deg w ≥ n for every pair of distinct non-adjacent vertices v and w of G (\*)*

There exists a dense graph that is not connected which clearly is not a Hamiltonian. For example, a graph G consists of 2 complete graphs with n vertices. It has a total of N\*(N-1) edges. So, G is a dense graph. At the same time, By the Ore’s theorem deg v + deg w = N – 1 + N – 1 = 2N – 2 >= 2N is false, and **G is not a Hamiltonian**.

1. Answers:
   1. Yes, it is
   2. Deg(A)+Deg(F) = 2+2 = 4 < 6
   3. No. The Ore’s Theorem indicates that if a graph satisfies (\*)-condition the graph would be a Hamiltonian. It does not say that if the graph is Hamiltonian it should satisfy (\*)-condition.
2. Answers:
   1. X={A,C,E,G}, Y={B,D,F}. Presented on the image-1.
   2. Indicated one of the possible “Maximum Matchings” in red on the image-2.
   3. Highlighted one of the possible “Minimum Vertex Cover” in orange on the image-3.
   4. Yes, since each end point of the any edge belongs to different columns the minimum number of vertex cover is same as the maximum matching.

Image-1

Image-2

Image-3

1. To transform the HC problem into Traveling salesman problem: First we need to convert our G graph into a weighted complete graph H, as for the weight we will put 1 on the new edges and 0 on the old edges. Now it converts a TSP problem with H-graph and K=0. If the TSP problem has a solution, it will only consist of the 0-weight edges which is our HC problem answer.

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1. TSP is NP-complete. We have the following facts:
2. Since HamiltonianCycle (HC) is NP-complete, given any NP problem Q, Q is polynomial reducible to HC.
3. “The problem is said to be NP-complete if it belongs to NP and every NP problem can be polynomial-reduced to it.”

If we can prove that HC is polynomial reducible to TSP problem, we can prove that TSP is NP-Complete problem.

Proof:

Assume we have a graph G given for a HC problem. Let us define algorithm C as follows:

C: Complete the G graph to H and put a weight of 1 to the new edges, and 0 to the previous edges. Initiate value k=0. This is an algorithm with running time of O(N2) = O(P(y)). Then If we solve this problem as a TSP (H, k) we can get an answer to HC(G). Hence, C and P(y) witness that HC is polynomial reducible to TSP problem.

With this proof along with fact (i) and (ii) we proved that TSP is a NP-complete.

1. For example, on the following graph with edges={(A,B), (C,D), (B,E), (D,E),(F,E)} (with this exact order) could result the worst case. One of the “Smallest vertex cover” is {E, A, C} with size of 3. Meanwhile, the VertexCoverApprox algorithm will result with {A, B, C, D, E, F} which is size of 6.

If we walkthrough the steps of VertexCoverApprox:

1-We first get edge (A,B) and add (A, B) vertices to the

result then we remove (B,E) edge.

2-Now we get the edge (C,D) and add (C,D) vertices to

the result then we remove (D,E) edge.

3-Now we left with edge (F, E) and add (F,E) vertices to

the result.

4-The result would be {A, B, C, D, E, F}