

# Statistical models and methods for human genetic data

Alejandro Ochoa

 DrAlexOchoa

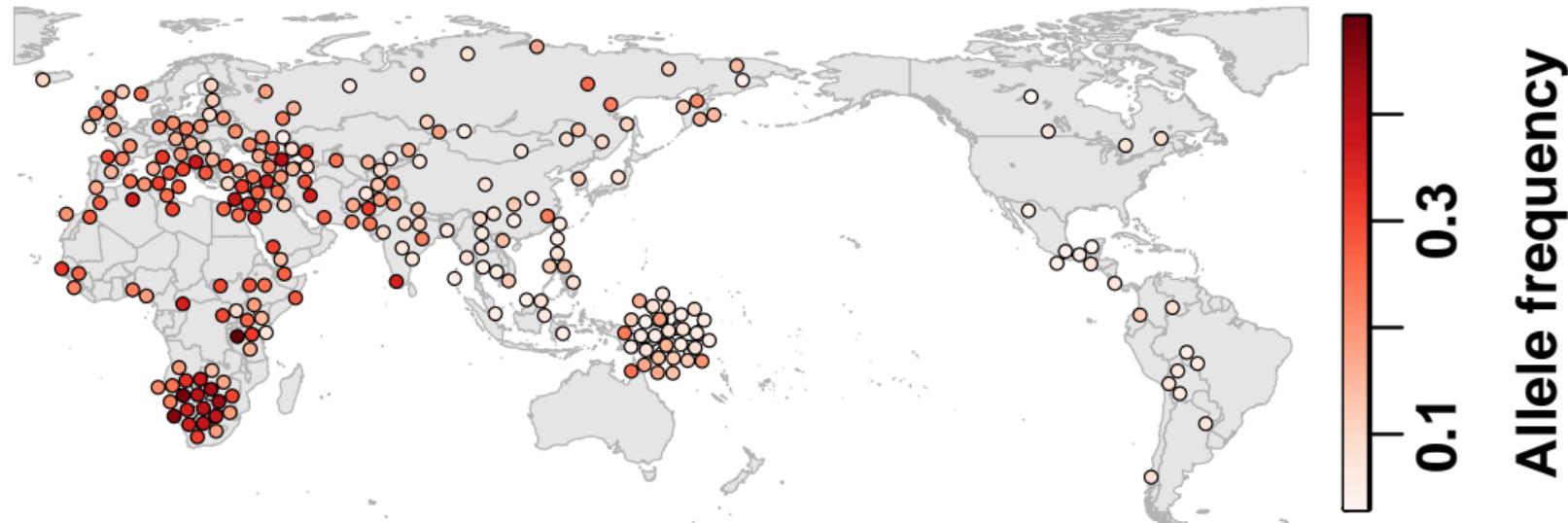
 ochoalab.github.io

 alejandro.ochoa@duke.edu

Biostatistics and Bioinformatics, StatGen — Duke University

2021-03-10 — WSU Mathematics and Statistics

# Human genetic structure



Ochoa and Storey (2019a) doi:10.1101/653279

rs17110306; median differentiation among loci with minor allele frequency  $\geq 10\%$

Why? Migration and isolation, admixture, family structure

# Overview

New population kinship and  $F_{ST}$  estimates

- ▶ Human Origins dataset
- ▶ Simulation validations

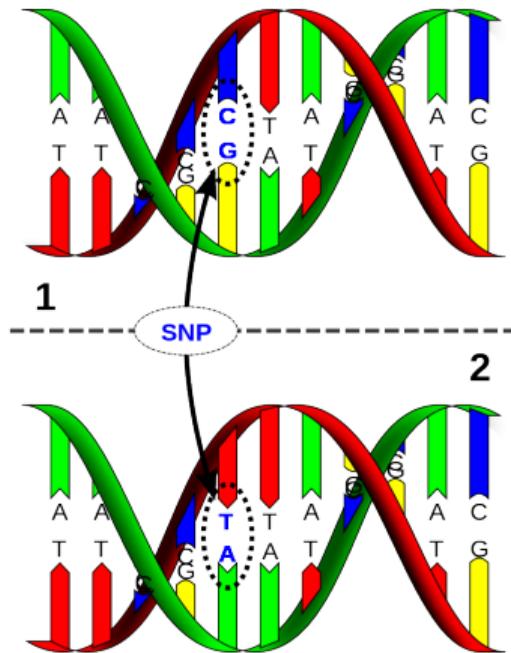
Genetic association models

- ▶ Robustness of PCA and LMM approaches
- ▶ Biases in heritability estimation
- ▶ LIGERA: Light Genetic Robust Association

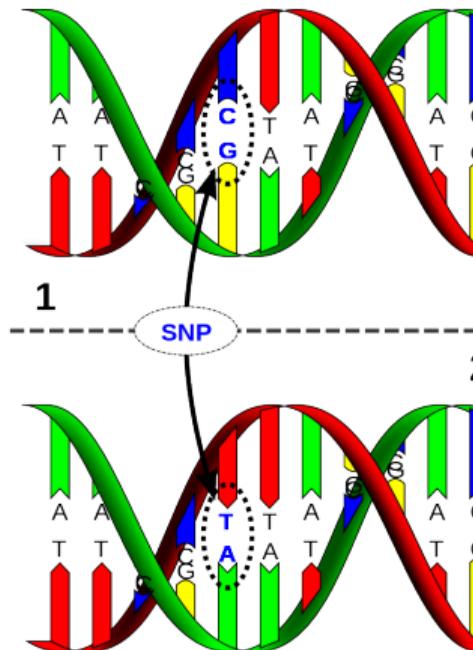
Admixture model

- ▶ Hispanics in 1000 Genomes Project
- ▶ Joint inference of admixture and population history from genetic covariance

# Single Nucleotide Polymorphism (SNP) data



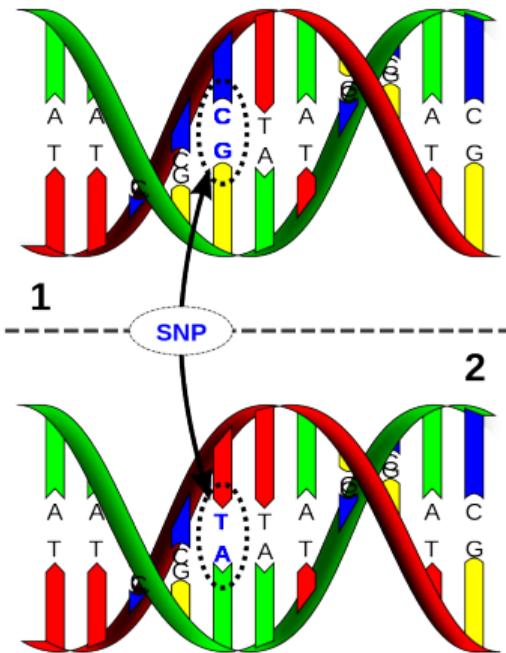
# Single Nucleotide Polymorphism (SNP) data



⇒

Genotype	$x_{ij}$
CC	0
CT	1
TT	2

# Single Nucleotide Polymorphism (SNP) data



→

Genotype	$x_{ij}$
CC	0
CT	1
TT	2

→

Loci

X

Individuals

0	2	2	1	1	0	1
0	2	1	0	1		
2	...					

## Hardy-Weinberg Equilibrium (HWE): Binomial draws

$x_{ij}$  = genotype at locus  $i$  for individual  $j$ .

$p_i$  = frequency of reference allele at locus  $i$ .

# Hardy-Weinberg Equilibrium (HWE): Binomial draws

$x_{ij}$  = genotype at locus  $i$  for individual  $j$ .

$p_i$  = frequency of reference allele at locus  $i$ .

Under HWE:

$$\Pr(x_{ij} = 2) = p_i^2,$$

$$\Pr(x_{ij} = 1) = 2p_i(1 - p_i),$$

$$\Pr(x_{ij} = 0) = (1 - p_i)^2.$$

# Hardy-Weinberg Equilibrium (HWE): Binomial draws

$x_{ij}$  = genotype at locus  $i$  for individual  $j$ .

$p_i$  = frequency of reference allele at locus  $i$ .

Under HWE:

$$\Pr(x_{ij} = 2) = p_i^2,$$

$$\Pr(x_{ij} = 1) = 2p_i(1 - p_i),$$

$$\Pr(x_{ij} = 0) = (1 - p_i)^2.$$

HWE not valid under genetic structure!

# Goal: measure dependence structure of genotype matrix columns

Individuals

0 2 2 1 1 0 1

0 2 1 0 1

2 ...

Loci

X

High-dimensional binomial data

- ▶ No general likelihood function
- ▶ My work: method of moments

Goal: measure dependence structure of genotype matrix columns

## Individuals

0 2 2 1 1 0 1

02101

2

## High-dimensional binomial data

- ▶ No general likelihood function
  - ▶ My work: method of moments

## Relatedness / Population structure

- ▶ Dependence between individuals (columns)

X

## Goal: measure dependence structure of genotype matrix columns

Loci	Individuals
0	0 2 2 1 1 0 1
1	0 2 1 0 1
2	...

X

## High-dimensional binomial data

- ▶ No general likelihood function
  - ▶ My work: method of moments

## Relatedness / Population structure

- ▶ Dependence between individuals (columns)

## Linkage disequilibrium

- ### ► Dependence between loci (rows)

## Model parameters

IBD: “Identical By Descent” (given implicit ancestral pop.) — shared coin flips

## Model parameters

IBD: “Identical By Descent” (given implicit ancestral pop.) — shared coin flips

$f_j$ : **Inbreeding coefficient**

Pr. that the two alleles at a random locus of individual  $j$  are IBD

$$\text{Var}(\textcolor{blue}{x}_{ij}) = 2p_i(1 - p_i)(1 + f_j)$$

## Model parameters

IBD: “Identical By Descent” (given implicit ancestral pop.) — shared coin flips

$f_j$ : **Inbreeding coefficient**

Pr. that the two alleles at a random locus of individual  $j$  are IBD

$$\text{Var}(\textcolor{blue}{x}_{ij}) = 2p_i(1 - p_i)(1 + f_j)$$

$\varphi_{jk}$ : **Kinship coefficient**

Pr. that two alleles, one at random from each of individuals  $j$  and  $k$ , at one random locus are IBD

$$\text{Cov}(\textcolor{blue}{x}_{ij}, \textcolor{blue}{x}_{ik}) = 4p_i(1 - p_i)\varphi_{jk}$$

## Model parameters

IBD: “Identical By Descent” (given implicit ancestral pop.) — shared coin flips

$f_j$ : **Inbreeding coefficient**

Pr. that the two alleles at a random locus of individual  $j$  are IBD

$$\text{Var}(\textcolor{blue}{x}_{ij}) = 2p_i(1 - p_i)(1 + f_j)$$

$\varphi_{jk}$ : **Kinship coefficient**

Pr. that two alleles, one at random from each of individuals  $j$  and  $k$ , at one random locus are IBD

$$\text{Cov}(\textcolor{blue}{x}_{ij}, \textcolor{blue}{x}_{ik}) = 4p_i(1 - p_i)\varphi_{jk}$$

$F_{ST}$ : **Fixation index**

Pr. that two random alleles in a **subpopulation** at a random locus are IBD

# Overview

## New population kinship and $F_{ST}$ estimates

- ▶ **Human Origins dataset**
- ▶ **Simulation validations**

## Genetic association models

- ▶ Robustness of PCA and LMM approaches
- ▶ Biases in heritability estimation
- ▶ LIGERA: Light Genetic Robust Association

## Admixture model

- ▶ Hispanics in 1000 Genomes Project
- ▶ Joint inference of admixture and population history from genetic covariance

## New kinship estimator for general relatedness

## New kinship estimator for general relatedness

Kinship model for neutral genotypes  $x_{ij} \in \{0, 1, 2\}$ :

$$E[x_{ij}] = 2p_i, \quad \text{Cov}(x_{ij}, x_{ik}) = 4p_i(1 - p_i) \varphi_{jk}.$$

## New kinship estimator for general relatedness

Kinship model for neutral genotypes  $x_{ij} \in \{0, 1, 2\}$ :

$$E[x_{ij}] = 2p_i, \quad \text{Cov}(x_{ij}, x_{ik}) = 4p_i(1 - p_i)\varphi_{jk}.$$

Standard estimator is **biased**:

$$\hat{p}_i = \frac{1}{2n} \sum_{j=1}^n x_{ij}, \quad \hat{\varphi}_{jk}^{\text{std}} = \frac{1}{m} \sum_{i=1}^m \frac{(x_{ij} - 2\hat{p}_i)(x_{ik} - 2\hat{p}_i)}{4\hat{p}_i(1 - \hat{p}_i)} \approx \frac{\varphi_{jk} - \bar{\varphi}_j - \bar{\varphi}_k + \bar{\varphi}}{1 - \bar{\varphi}}.$$

# New kinship estimator for general relatedness

Kinship model for neutral genotypes  $x_{ij} \in \{0, 1, 2\}$ :

$$E[x_{ij}] = 2p_i, \quad \text{Cov}(x_{ij}, x_{ik}) = 4p_i(1 - p_i)\varphi_{jk}.$$

Standard estimator is **biased**:

$$\hat{p}_i = \frac{1}{2n} \sum_{j=1}^n x_{ij}, \quad \hat{\varphi}_{jk}^{\text{std}} = \frac{1}{m} \sum_{i=1}^m \frac{(x_{ij} - 2\hat{p}_i)(x_{ik} - 2\hat{p}_i)}{4\hat{p}_i(1 - \hat{p}_i)} \approx \frac{\varphi_{jk} - \bar{\varphi}_j - \bar{\varphi}_k + \bar{\varphi}}{1 - \bar{\varphi}}.$$

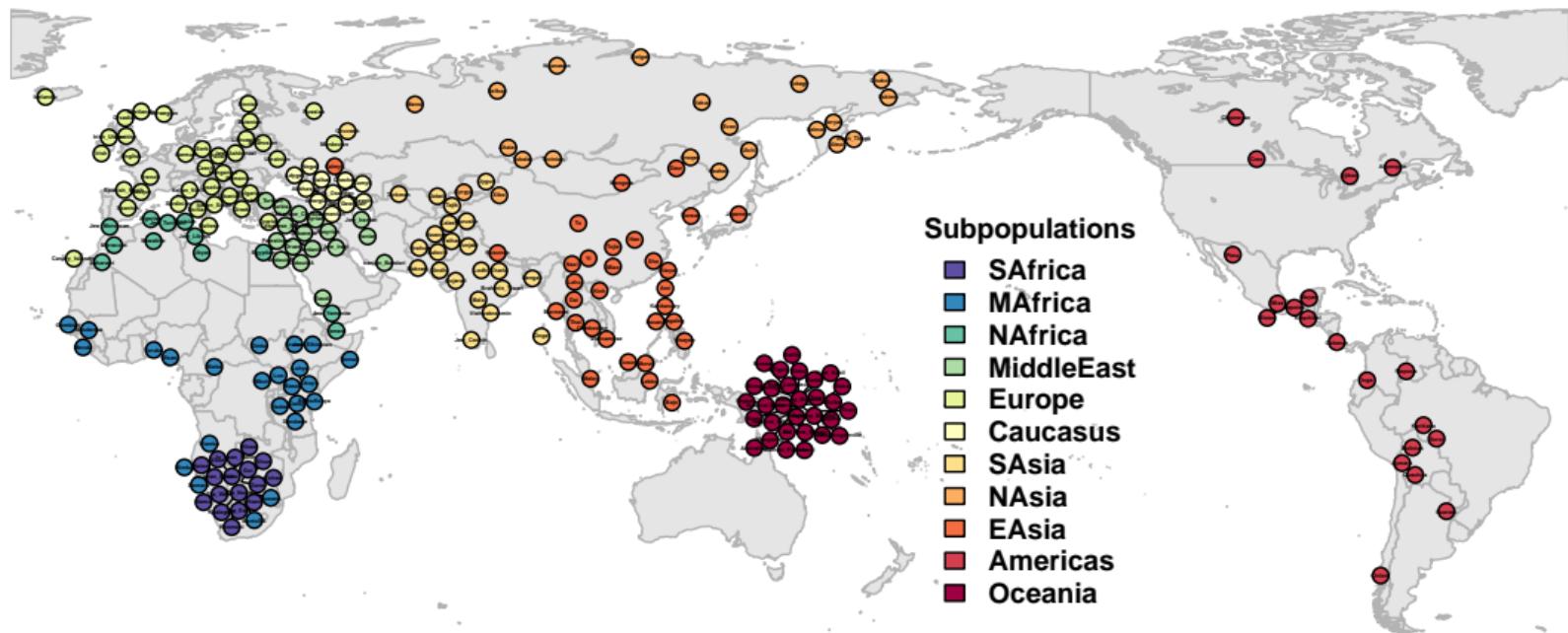
`popkin`: first unbiased kinship estimator! R package (Ochoa and Storey, 2021)

$$A_{jk} = \frac{1}{m} \sum_{i=1}^m (x_{ij} - 1)(x_{ik} - 1) - 1, \quad \hat{A}_{\min} = \min_{u \neq v} \frac{1}{|S_u||S_v|} \sum_{j \in S_u} \sum_{k \in S_v} A_{jk},$$

$$\hat{\varphi}_{jk}^{\text{new}} = 1 - \frac{A_{jk}}{\hat{A}_{\min}} \xrightarrow[m \rightarrow \infty]{\text{a.s.}} \varphi_{jk}.$$



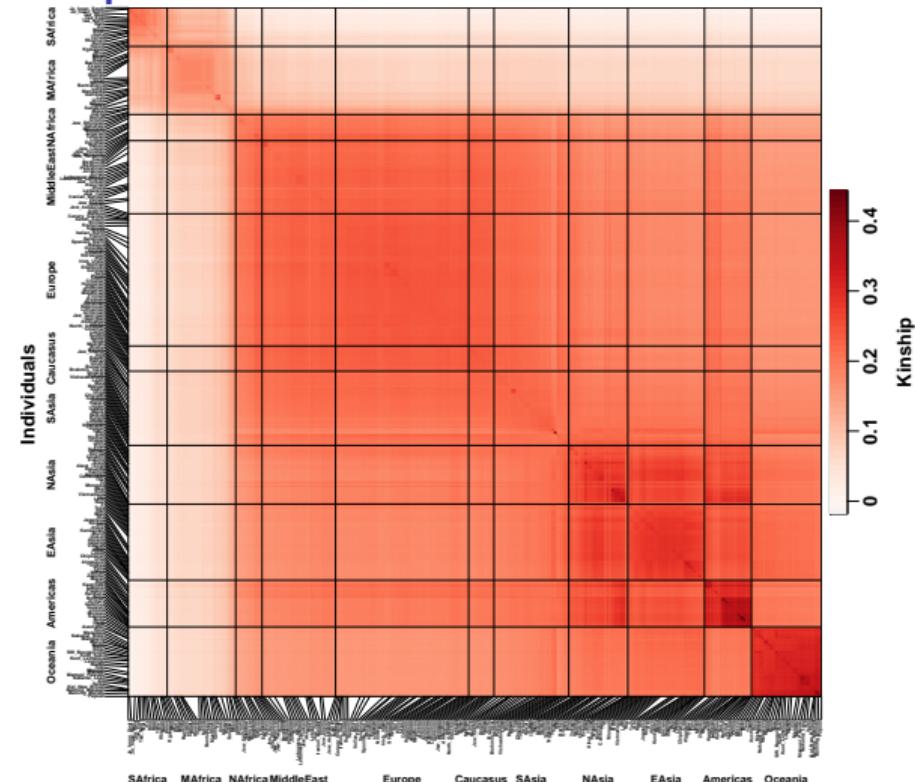
# Dataset: Human Origins



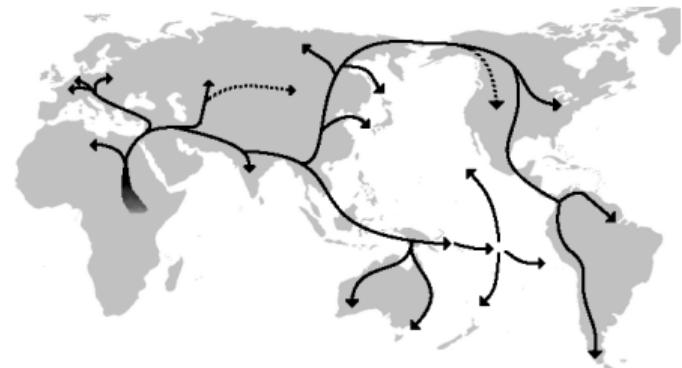
Lazaridis *et al.* (2014), (2016); Skoglund *et al.* (2016)

2,922 indivs. from 243 locs. — 588,091 loci — SNP chip

# Kinship matrix of world-wide human population

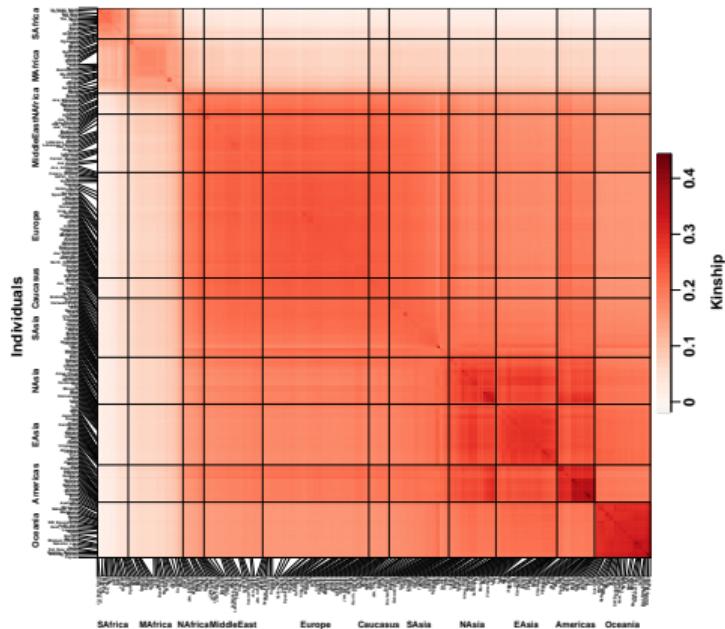


Ochoa and Storey (2019) doi:10.1101/653279

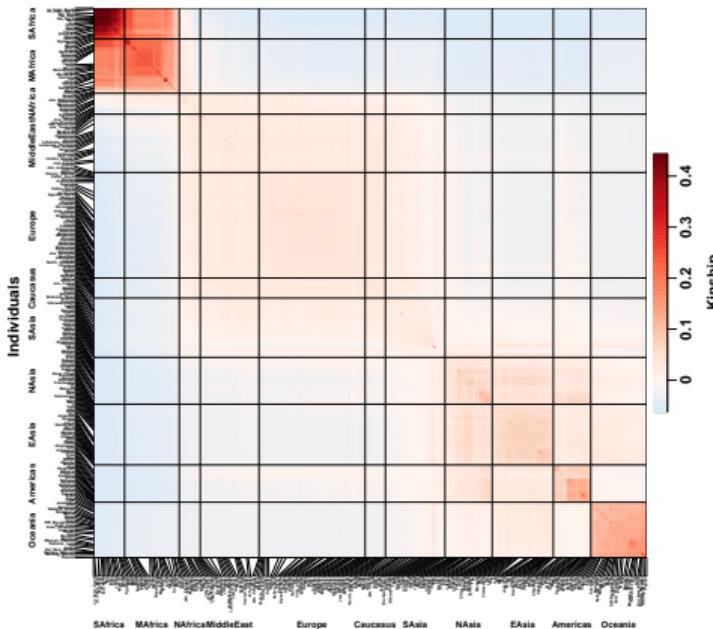


# Standard kinship estimator is severely biased

New

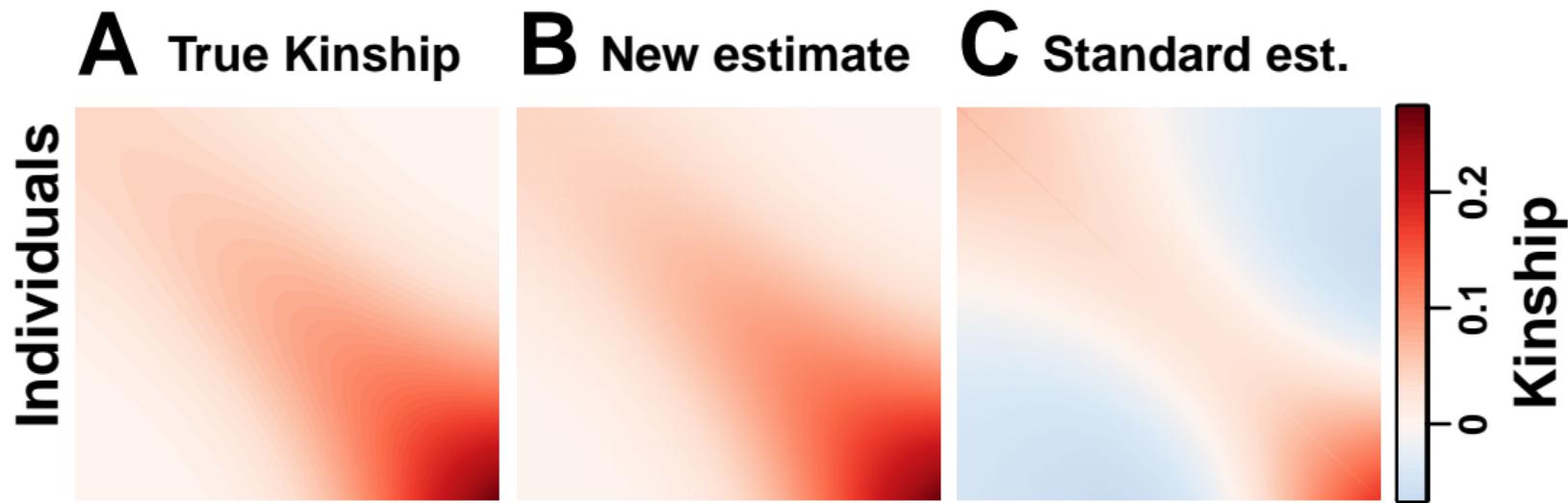


Standard



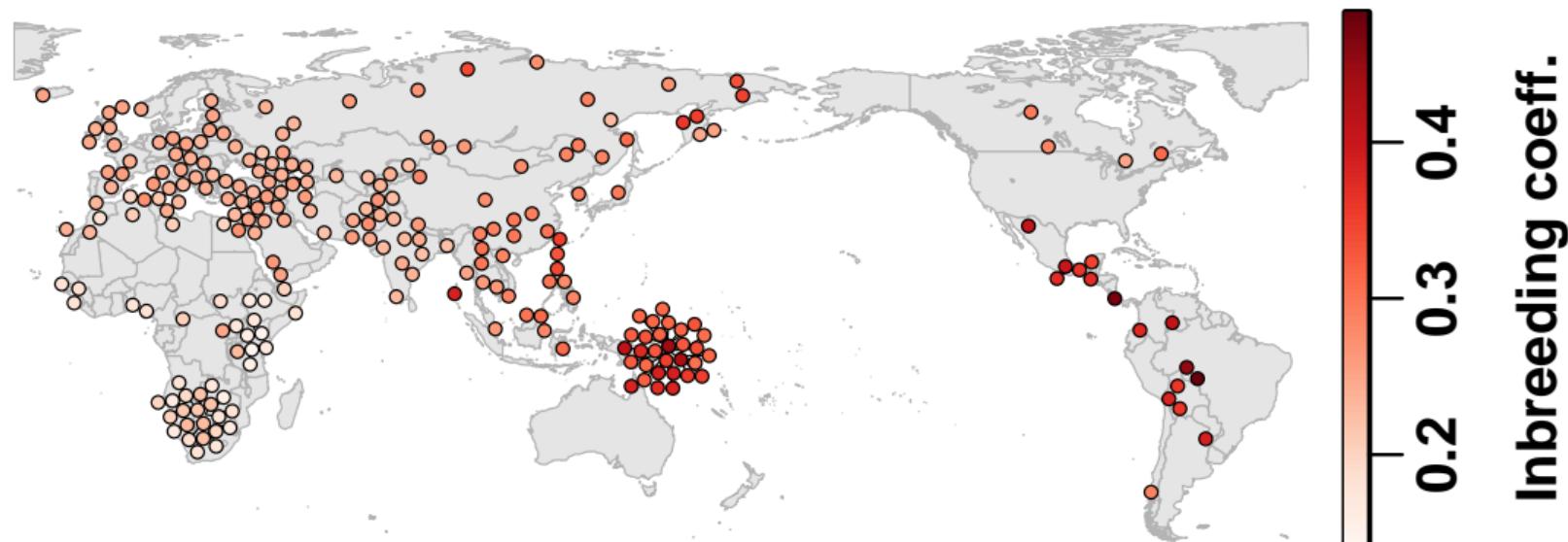
Ochoa and Storey (2019) doi:10.1101/653279

# Validation in simulation



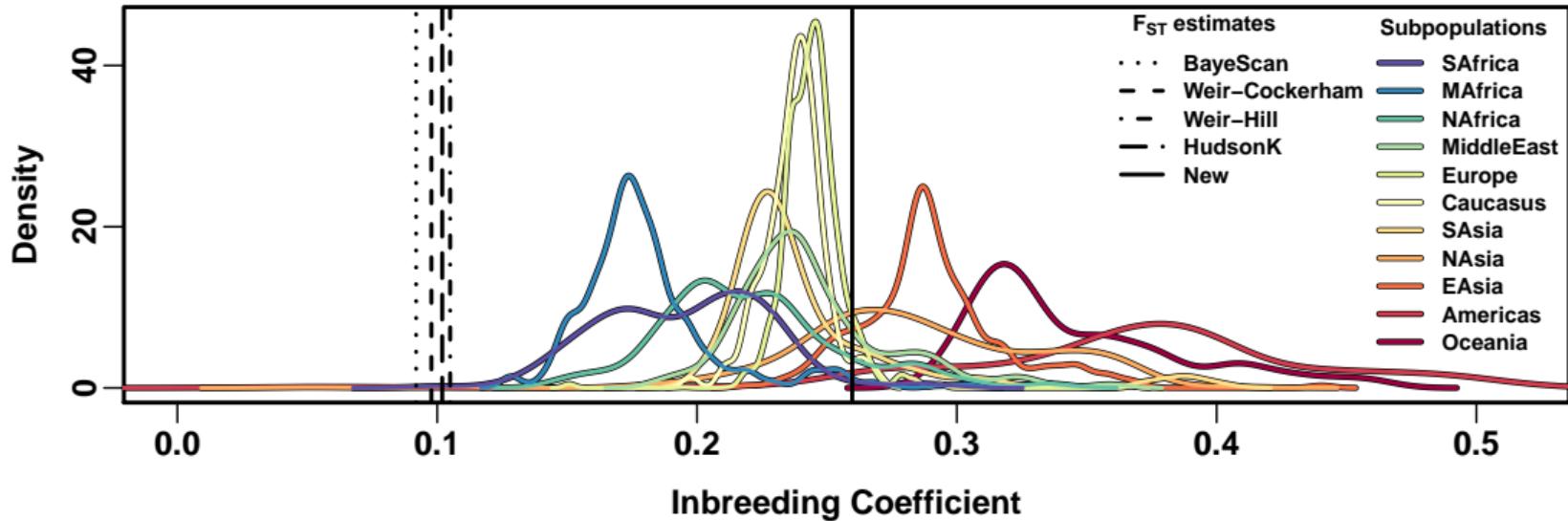
Ochoa and Storey (2021) doi:10.1371/journal.pgen.1009241

# Population-level inbreeding increases with distance from Africa



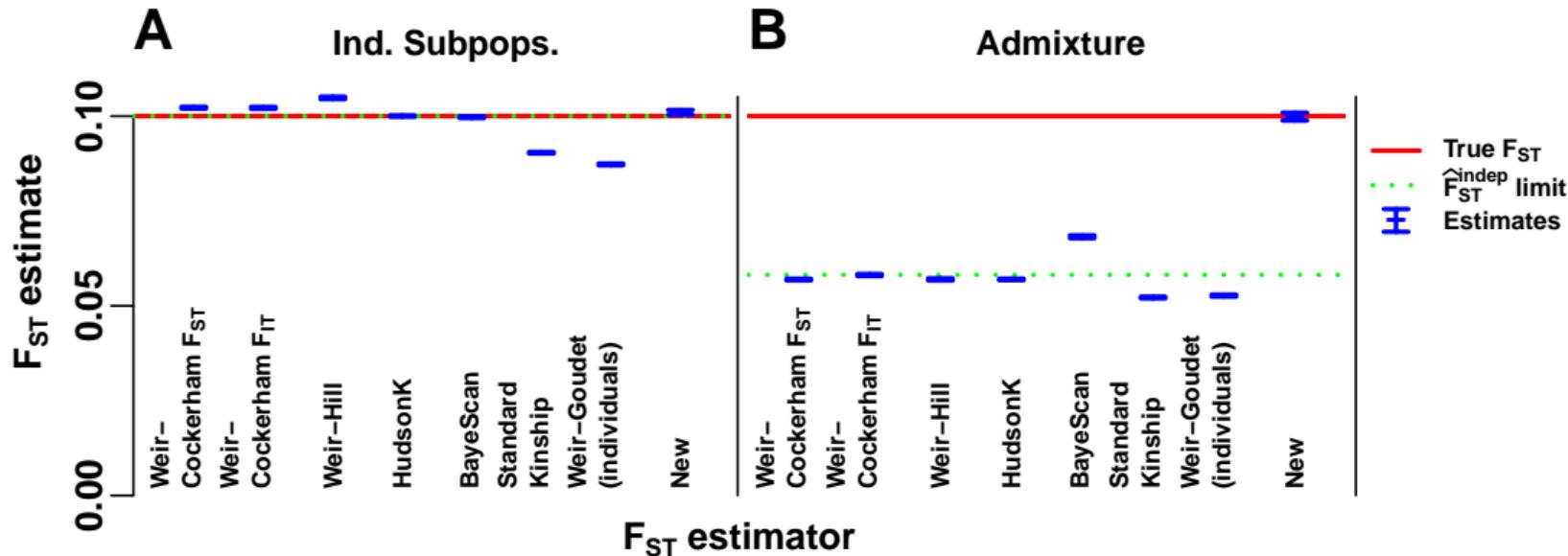
Ochoa and Storey (2019) doi:10.1101/653279

# Differentiation ( $F_{ST}$ ) previously underestimated



Ochoa and Storey (2019) doi:10.1101/653279

# Validation in simulation



Ochoa and Storey (2021) doi:10.1371/journal.pgen.1009241

# Overview

New population kinship and  $F_{ST}$  estimates

- ▶ Human Origins dataset
- ▶ Simulation validations

## Genetic association models

- ▶ Robustness of PCA and LMM approaches
- ▶ Biases in heritability estimation
- ▶ LIGERA: Light Genetic Robust Association

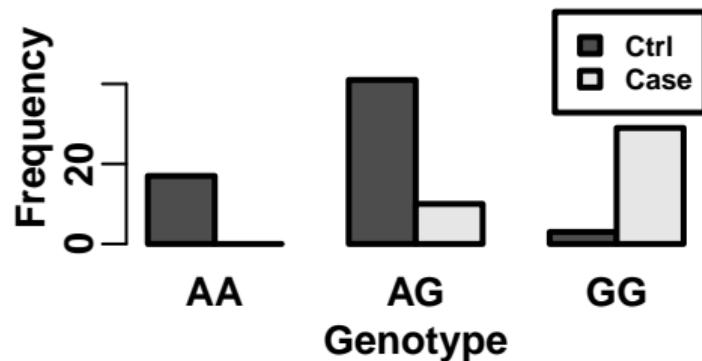
Admixture model

- ▶ Hispanics in 1000 Genomes Project
- ▶ Joint inference of admixture and population history from genetic covariance

## Genetic association study: genotype-phenotype correlation

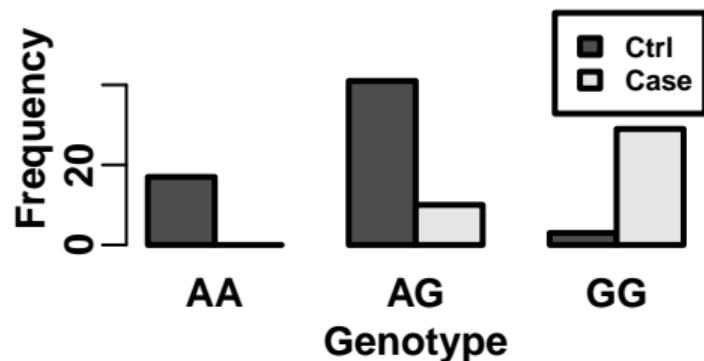
# Genetic association study: genotype-phenotype correlation

As Table

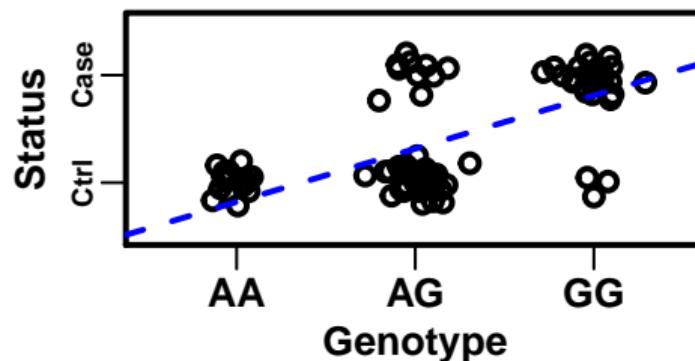


# Genetic association study: genotype-phenotype correlation

As Table

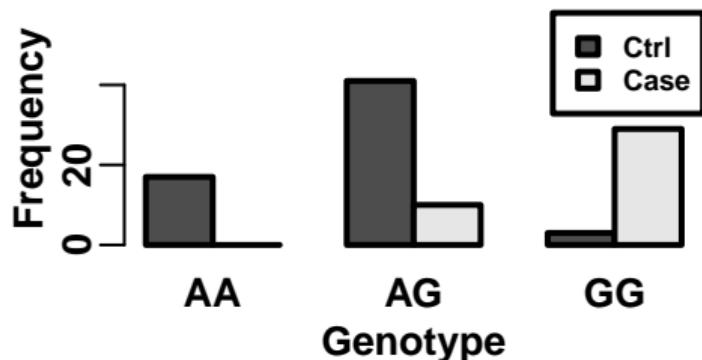


As Regression

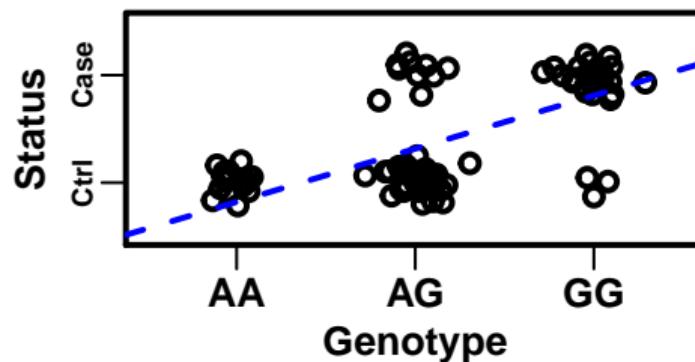


# Genetic association study: genotype-phenotype correlation

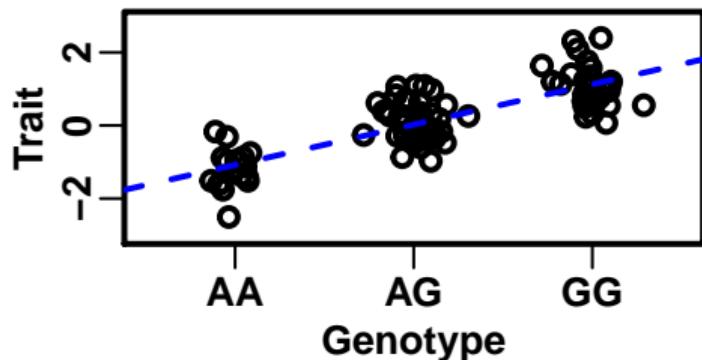
As Table



As Regression

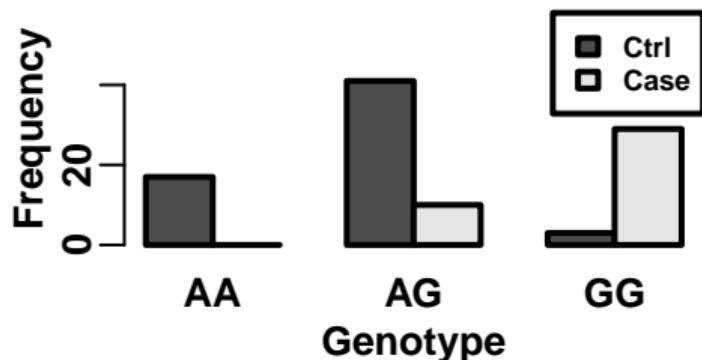


Continuous trait

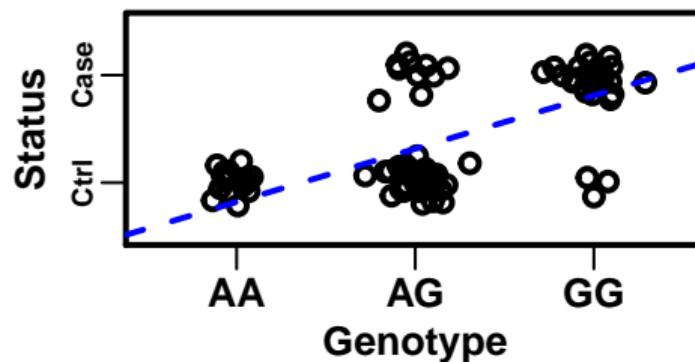


# Genetic association study: genotype-phenotype correlation

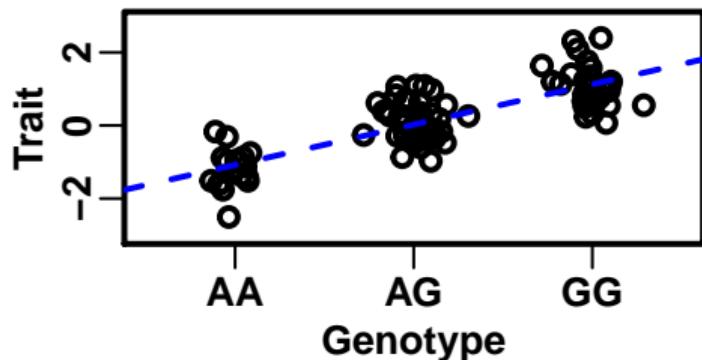
As Table



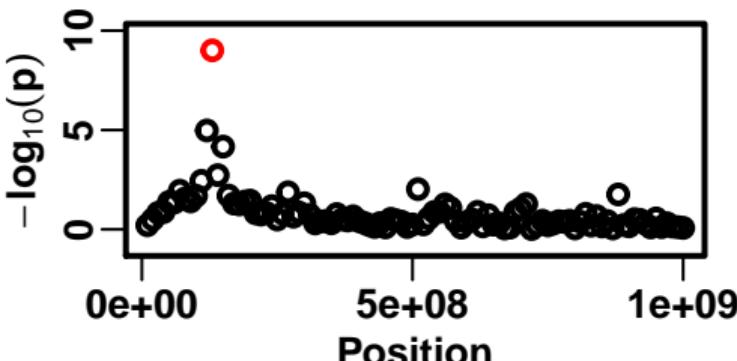
As Regression



Continuous trait



Genome Scan



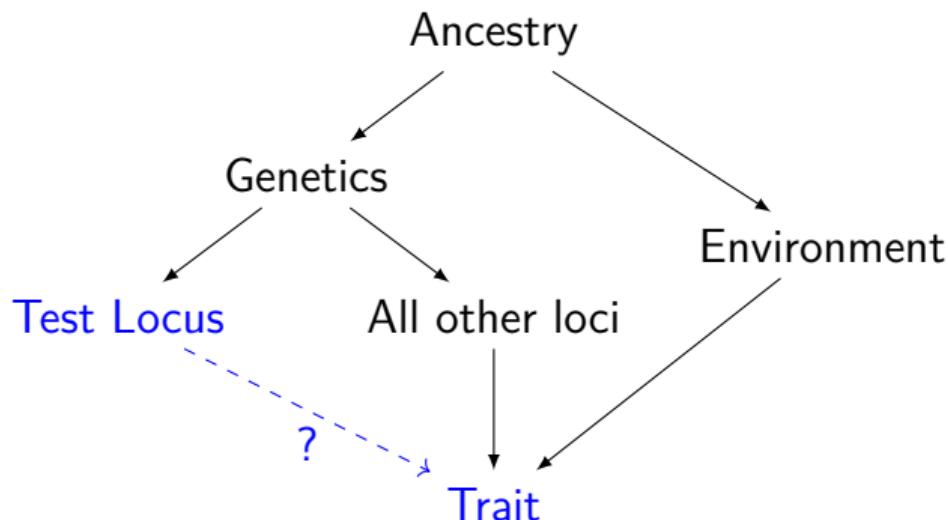
# Why is this problem so hard?

# Why is this problem so hard?

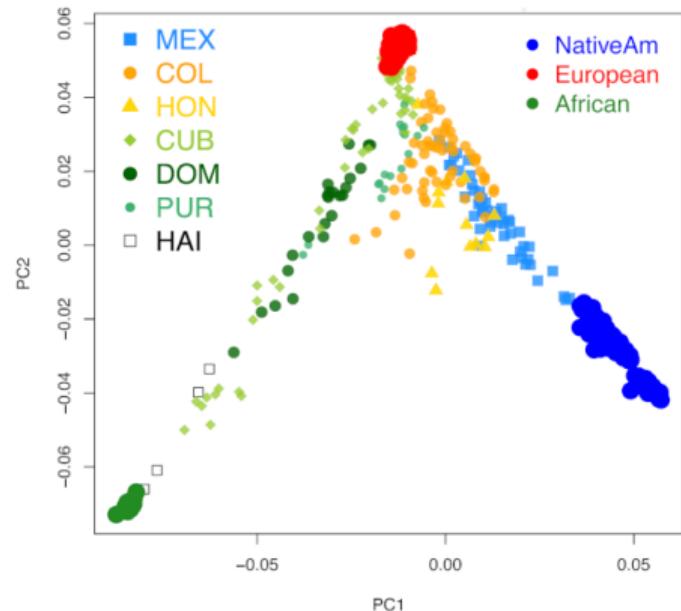
- ▶ Millions of tests
- ▶ Polygenicity
- ▶ Confounders

# Why is this problem so hard?

- ▶ Millions of tests
- ▶ Polygenicity
- ▶ Confounders

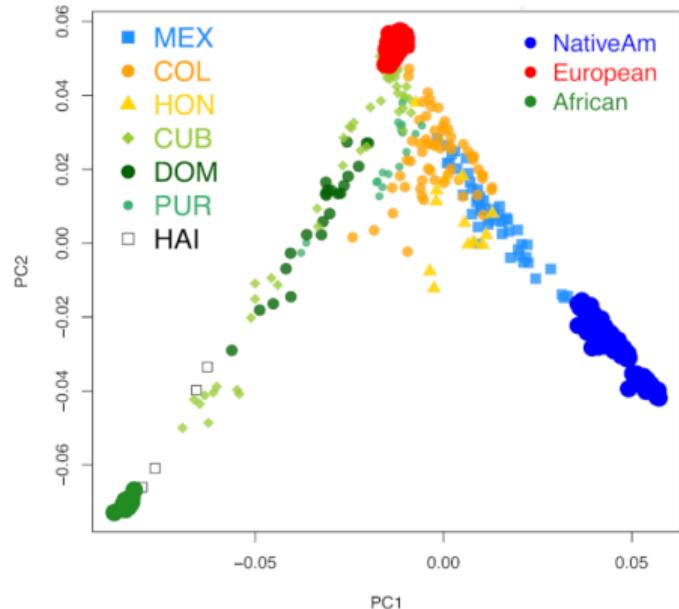


# PCA: Principal Component Analysis



Moreno-Estrada *et al.* (2013)

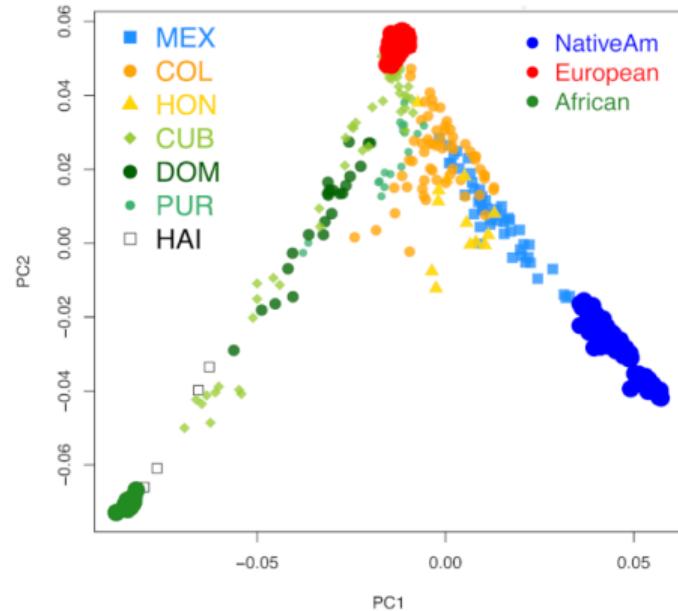
# PCA: Principal Component Analysis



PCs map to ancestry.

Moreno-Estrada *et al.* (2013)

# PCA: Principal Component Analysis

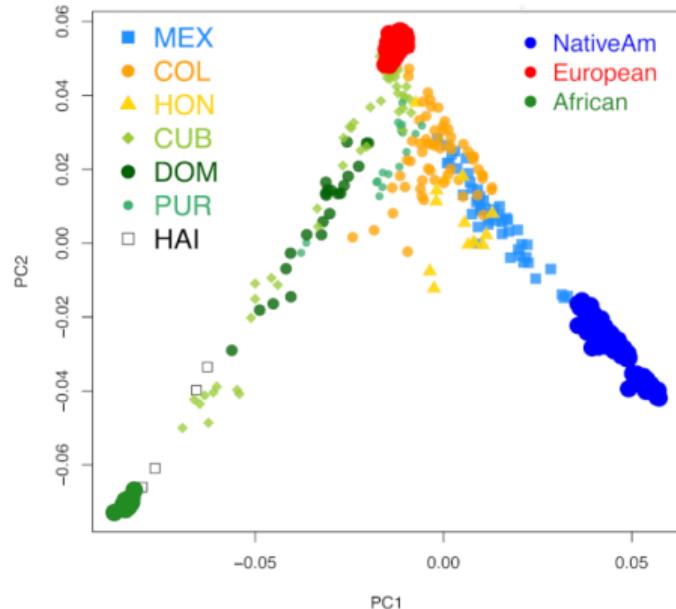


Moreno-Estrada *et al.* (2013)

PCs map to ancestry.

"PCs" are top eigenvectors of kinship matrix.

# PCA: Principal Component Analysis



PCs map to ancestry.

"PCs" are top eigenvectors of kinship matrix.

Pros: Fast!

Cons: Fails on family data.

## Genetic association methods: PCA and LMM

Principal components analysis (PCA) association model: fixed-effects regression:

$$\mathbf{y} = \mathbf{1}\alpha + \mathbf{x}_i\beta + \mathbf{U}_d\gamma_d + \epsilon.$$

- ▶  $\mathbf{U}_d$  are top  $d$  eigenvectors of kinship matrix  $\Phi$ .

## Genetic association methods: PCA and LMM

Principal components analysis (PCA) association model: fixed-effects regression:

$$\mathbf{y} = \mathbf{1}\alpha + \mathbf{x}_i\beta + \mathbf{U}_d\gamma_d + \epsilon.$$

- ▶  $\mathbf{U}_d$  are top  $d$  eigenvectors of kinship matrix  $\Phi$ .

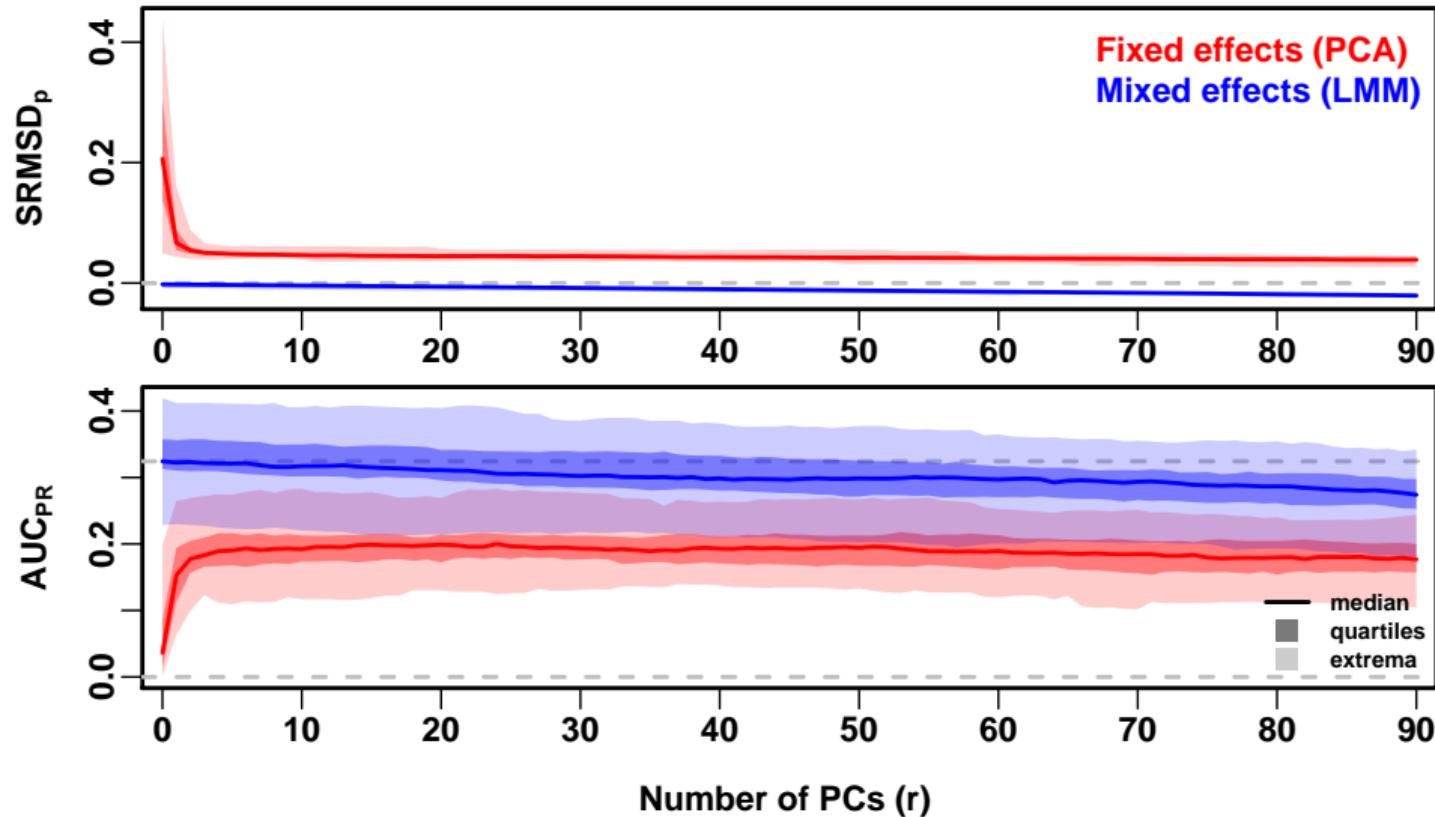
Linear mixed-effects model (LMM):

$$\mathbf{y} = \mathbf{1}\alpha + \mathbf{x}_i\beta + \mathbf{s} + \epsilon.$$

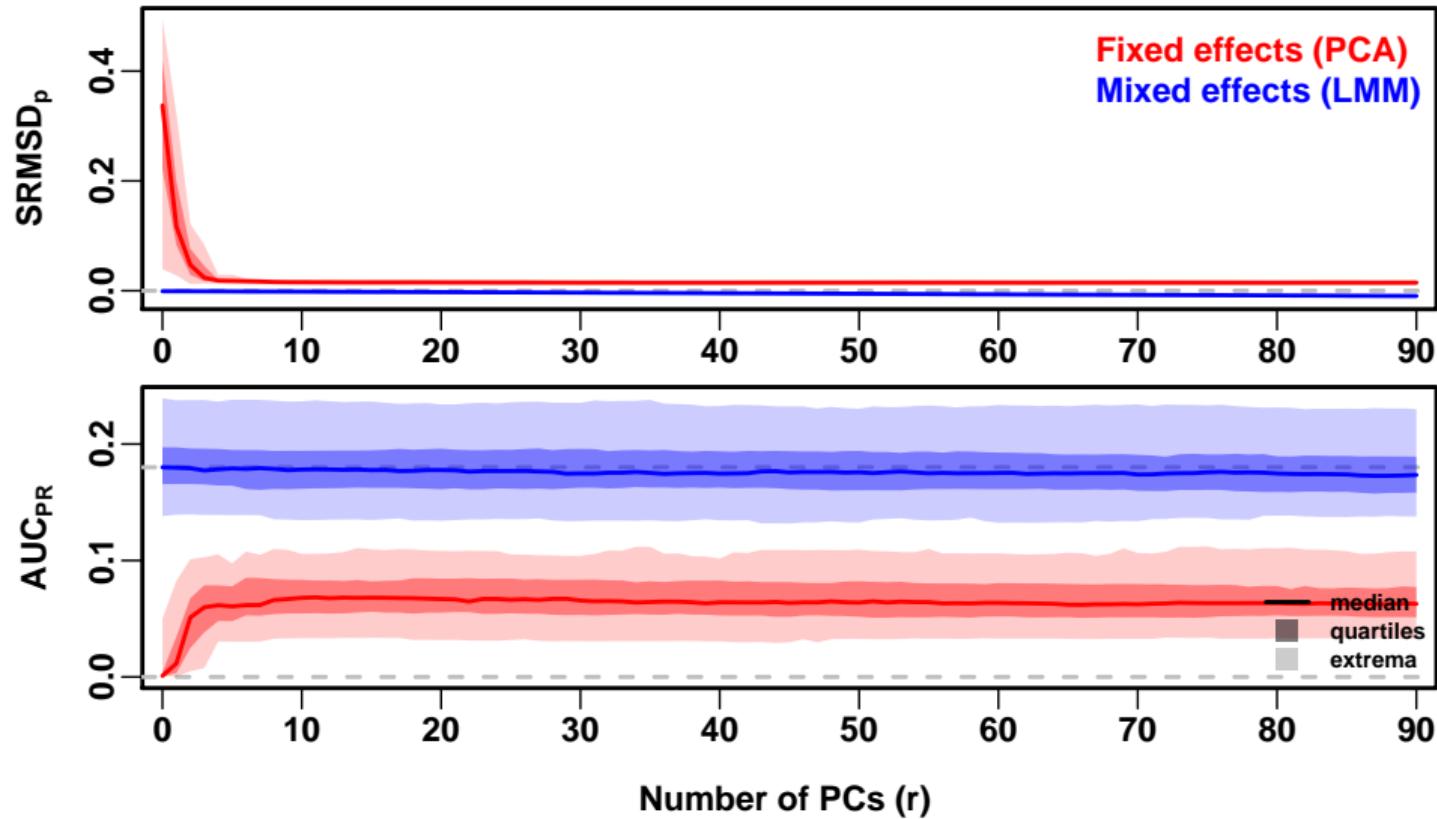
- ▶ Random effect has covariance structure from kinship matrix  $\Phi$ :

$$\mathbf{s} \sim \text{Normal}(\mathbf{0}, \sigma^2 \Phi).$$

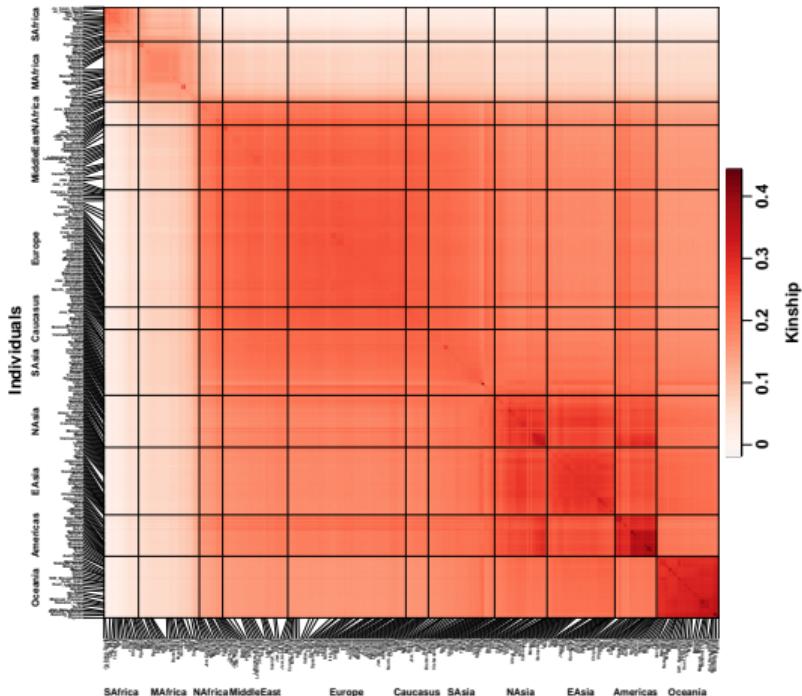
# LMM outperforms PCA: Simulated admixture + family structure



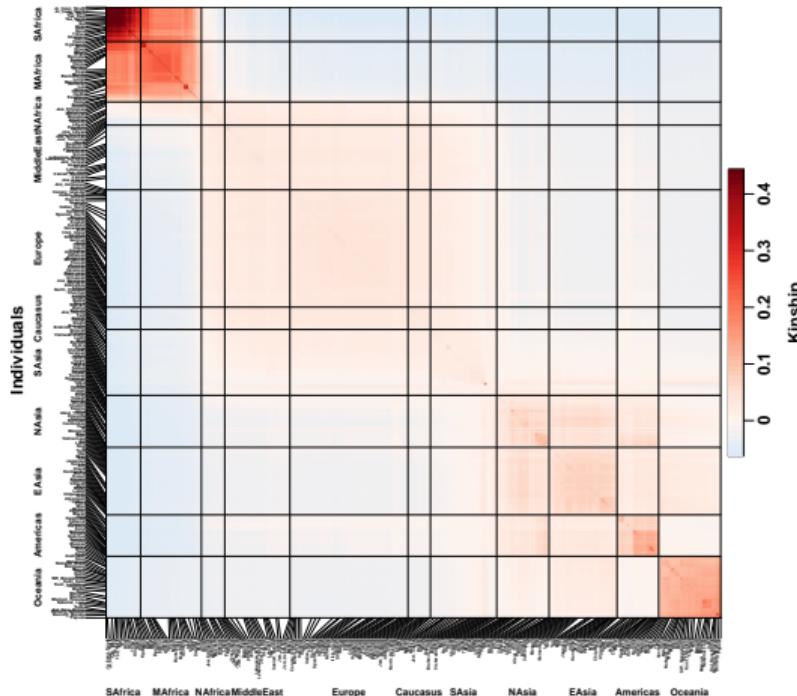
# LMM outperforms PCA: 1000 Genomes Project + sim trait



# Kinship bias does not affect genetic associations



New popkin  
kinship estimator



Standard  
kinship estimator

## Kinship bias does not affect genetic associations

Centering matrix is key to understanding kinship bias algebraically:

$$\mathbf{C} = \mathbf{I} - \frac{1}{n}\mathbf{J}.$$

## Kinship bias does not affect genetic associations

Centering matrix is key to understanding kinship bias algebraically:

$$\mathbf{C} = \mathbf{I} - \frac{1}{n}\mathbf{J}.$$

Standard kinship bias as a transformation of true kinship by centering:

$$\boldsymbol{\Phi}' = \frac{1}{1 - \bar{\varphi}} \mathbf{C} \boldsymbol{\Phi} \mathbf{C}.$$

## Kinship bias does not affect genetic associations

Centering matrix is key to understanding kinship bias algebraically:

$$\mathbf{C} = \mathbf{I} - \frac{1}{n}\mathbf{J}.$$

Standard kinship bias as a transformation of true kinship by centering:

$$\boldsymbol{\Phi}' = \frac{1}{1 - \bar{\varphi}} \mathbf{C} \boldsymbol{\Phi} \mathbf{C}.$$

Matrix square root also centered:

$$(\boldsymbol{\Phi}')^{\frac{1}{2}} = \frac{1}{\sqrt{1 - \bar{\varphi}}} \mathbf{C} \boldsymbol{\Phi}^{\frac{1}{2}}.$$

# Kinship bias does not affect genetic associations

LMM equivalent models:

$$\mathbf{y} = \mathbf{1}\alpha + \mathbf{x}_i\beta + \mathbf{s} + \epsilon, \quad \mathbf{s} \sim \text{Normal}(\mathbf{0}, \sigma^2 \boldsymbol{\Phi}),$$

$$\mathbf{y} = \mathbf{1}\alpha + \mathbf{x}_i\beta + \sigma \boldsymbol{\Phi}^{\frac{1}{2}} \mathbf{r} + \epsilon, \quad \mathbf{r} \sim \text{Normal}(\mathbf{0}, \mathbf{I}).$$

# Kinship bias does not affect genetic associations

LMM equivalent models:

$$\mathbf{y} = \mathbf{1}\alpha + \mathbf{x}_i\beta + \mathbf{s} + \epsilon, \quad \mathbf{s} \sim \text{Normal}(\mathbf{0}, \sigma^2 \boldsymbol{\Phi}),$$

$$\mathbf{y} = \mathbf{1}\alpha + \mathbf{x}_i\beta + \sigma \boldsymbol{\Phi}^{\frac{1}{2}} \mathbf{r} + \epsilon, \quad \mathbf{r} \sim \text{Normal}(\mathbf{0}, \mathbf{I}).$$

Fit under true kinship ( $\boldsymbol{\Phi}$ ) vs biased limit ( $\boldsymbol{\Phi}'$ ) is equally good  
(algebra depends on centering matrix properties):

$$\begin{aligned} \mathbf{y} &= \mathbf{1}\alpha + \mathbf{x}_i\beta + \sigma \boldsymbol{\Phi}^{\frac{1}{2}} \mathbf{r} + \epsilon \\ &= \mathbf{1}\alpha' + \mathbf{x}_i\beta' + \sigma' (\boldsymbol{\Phi}')^{\frac{1}{2}} \mathbf{r}' + \epsilon', \end{aligned}$$

$$\beta' = \beta, \quad \epsilon' = \epsilon, \quad \mathbf{r}' = \mathbf{r}, \quad \sigma' = \sigma \sqrt{1 - \bar{\varphi}}, \quad \alpha' = \alpha + \sigma \frac{1}{n} \mathbf{1}^\top \boldsymbol{\Phi}^{\frac{1}{2}} \mathbf{r}.$$

# Kinship bias does not affect genetic associations

LMM equivalent models:

$$\mathbf{y} = \mathbf{1}\alpha + \mathbf{x}_i\beta + \mathbf{s} + \epsilon, \quad \mathbf{s} \sim \text{Normal}(\mathbf{0}, \sigma^2 \boldsymbol{\Phi}),$$

$$\mathbf{y} = \mathbf{1}\alpha + \mathbf{x}_i\beta + \sigma \boldsymbol{\Phi}^{\frac{1}{2}} \mathbf{r} + \epsilon, \quad \mathbf{r} \sim \text{Normal}(\mathbf{0}, \mathbf{I}).$$

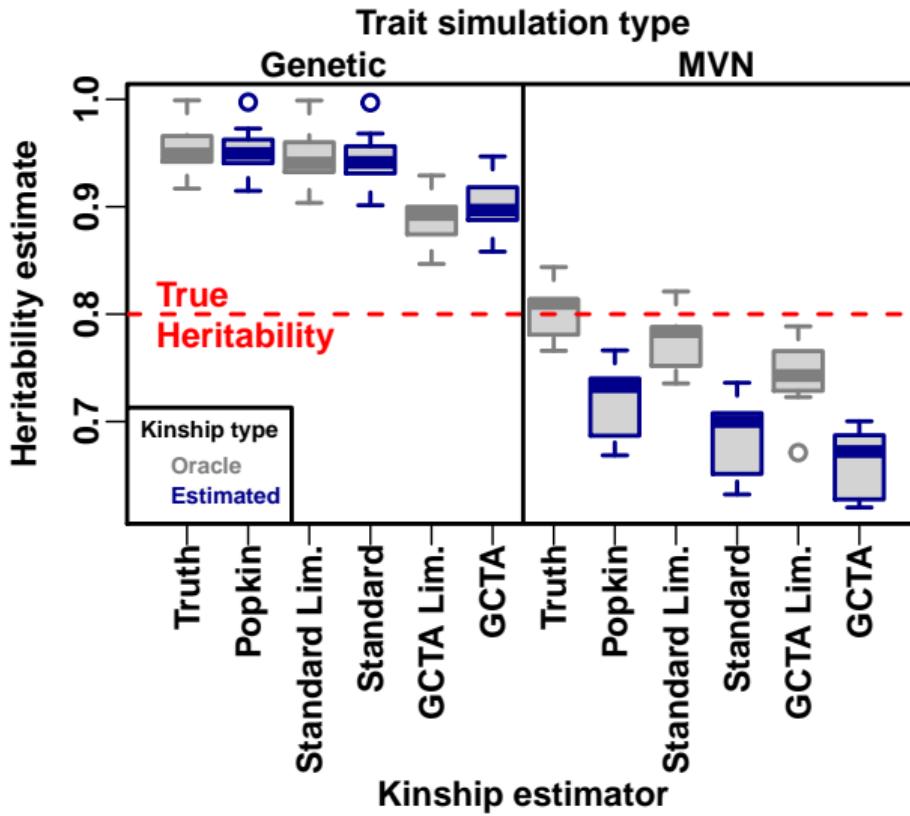
Fit under true kinship ( $\boldsymbol{\Phi}$ ) vs biased limit ( $\boldsymbol{\Phi}'$ ) is equally good  
(algebra depends on centering matrix properties):

$$\begin{aligned} \mathbf{y} &= \mathbf{1}\alpha + \mathbf{x}_i\beta + \sigma \boldsymbol{\Phi}^{\frac{1}{2}} \mathbf{r} + \epsilon \\ &= \mathbf{1}\alpha' + \mathbf{x}_i\beta' + \sigma' (\boldsymbol{\Phi}')^{\frac{1}{2}} \mathbf{r}' + \epsilon', \end{aligned}$$

$$\beta' = \beta, \quad \epsilon' = \epsilon, \quad \mathbf{r}' = \mathbf{r}, \quad \sigma' = \sigma \sqrt{1 - \bar{\varphi}}, \quad \alpha' = \alpha + \sigma \frac{1}{n} \mathbf{1}^\top \boldsymbol{\Phi}^{\frac{1}{2}} \mathbf{r}.$$

Similar argument holds approximately for PCA regression.

# Kinship bias affects heritability estimation



## Future work: Tuning elastic nets for genetic association

$$\hat{\beta} \equiv \underset{\beta}{\operatorname{argmin}} (\|y - X\beta\|^2 + \lambda_2 \|\beta\|^2 + \lambda_1 \|\beta\|_1).$$

- ▶ Validate existing PCA extension
- ▶ How to model higher-dimensional relatedness?

## Genetic association models: forward vs reversed

---

(Forward) linear mixed-effects model

Reverse model

---

## Genetic association models: forward vs reversed

---

(Forward) linear mixed-effects model

Reverse model

---

$$\mathbf{y} = \alpha_i + \mathbf{x}_i \beta_i + \mathbf{F} \gamma_i + \mathbf{r}_i$$

## Genetic association models: forward vs reversed

---

(Forward) linear mixed-effects model

$$\mathbf{y} = \alpha_i + \mathbf{x}_i \beta_i + \mathbf{F} \gamma_i + \mathbf{r}_i$$

Reverse model

$$\mathbf{x}_i = \alpha'_i + \mathbf{y} \beta'_i + \epsilon'_i$$

## Genetic association models: forward vs reversed

---

(Forward) linear mixed-effects model

Reverse model

---

$$\mathbf{y} = \alpha_i + \mathbf{x}_i \beta_i + \mathbf{F} \gamma_i + \mathbf{r}_i$$

$$\mathbf{x}_i = \alpha'_i + \mathbf{y} \beta'_i + \epsilon'_i$$

Models trait (complicated, unknowns)

## Genetic association models: forward vs reversed

---

(Forward) linear mixed-effects model

Reverse model

---

$$\mathbf{y} = \alpha_i + \mathbf{x}_i \beta_i + \mathbf{F} \gamma_i + \mathbf{r}_i$$

$$\mathbf{x}_i = \alpha'_i + \mathbf{y} \beta'_i + \epsilon'_i$$

Models trait (complicated, unknowns)

Models genotype (kinship).

## Genetic association models: forward vs reversed

---

(Forward) linear mixed-effects model

$$\mathbf{y} = \alpha_i + \mathbf{x}_i \beta_i + \mathbf{F} \gamma_i + \mathbf{r}_i$$

Models trait (complicated, unknowns)

Reverse model

$$\mathbf{x}_i = \alpha'_i + \mathbf{y} \beta'_i + \epsilon'_i$$

Models genotype (kinship).

Environment can be absent (Song, Hao, Storey 2015)

## Genetic association models: forward vs reversed

---

(Forward) linear mixed-effects model

$$\mathbf{y} = \alpha_i + \mathbf{x}_i \beta_i + \mathbf{F} \gamma_i + \mathbf{r}_i$$

Models trait (complicated, unknowns)

Reverse model

$$\mathbf{x}_i = \alpha'_i + \mathbf{y} \beta'_i + \epsilon'_i$$

Models genotype (kinship).

Environment can be absent (Song, Hao, Storey 2015)

Random effects are slow!

## Genetic association models: forward vs reversed

---

(Forward) linear mixed-effects model

$$\mathbf{y} = \alpha_i + \mathbf{x}_i \beta_i + \mathbf{F} \gamma_i + \mathbf{r}_i$$

Models trait (complicated, unknowns)

Random effects are slow!

Reverse model

$$\mathbf{x}_i = \alpha'_i + \mathbf{y} \beta'_i + \epsilon'_i$$

Models genotype (kinship).

Environment can be absent (Song, Hao, Storey 2015)

Fast!

---

# LIGERA: light genetic robust association

Objective function: move genetic structure to residuals:

$$G = (\mathbf{Y}\boldsymbol{\beta}_i - \mathbf{x}_i)^\top \boldsymbol{\Phi}^{-1} (\mathbf{Y}\boldsymbol{\beta}_i - \mathbf{x}_i).$$

# LIGERA: light genetic robust association

Objective function: move genetic structure to residuals:

$$G = (\mathbf{Y}\beta_i - \mathbf{x}_i)^\top \boldsymbol{\Phi}^{-1} (\mathbf{Y}\beta_i - \mathbf{x}_i).$$

Effect size estimator is matrix product of data:

$$\hat{\beta}_i = \mathbf{H}^\top \mathbf{x}_i, \quad \mathbf{H} = \boldsymbol{\Phi}^{-1} \mathbf{Y} \left( \mathbf{Y}^\top \boldsymbol{\Phi}^{-1} \mathbf{Y} \right)^{-1}.$$

## LIGERA: light genetic robust association

Objective function: move genetic structure to residuals:

$$G = (\mathbf{Y}\beta_i - \mathbf{x}_i)^\top \boldsymbol{\Phi}^{-1} (\mathbf{Y}\beta_i - \mathbf{x}_i).$$

Effect size estimator is matrix product of data:

$$\hat{\beta}_i = \mathbf{H}^\top \mathbf{x}_i, \quad \mathbf{H} = \boldsymbol{\Phi}^{-1} \mathbf{Y} \left( \mathbf{Y}^\top \boldsymbol{\Phi}^{-1} \mathbf{Y} \right)^{-1}.$$

Variance under null hypothesis has closed form:

$$\text{Cov}(\hat{\beta}_i | \mathbf{Y}) = 4p_i(1-p_i)(\mathbf{H}^\top \boldsymbol{\Phi} \mathbf{H}), \quad (\mathbf{H}^\top \boldsymbol{\Phi} \mathbf{H}) = \left( \mathbf{Y}^\top \boldsymbol{\Phi}^{-1} \mathbf{Y} \right)^{-1}.$$

# LIGERA: light genetic robust association

Objective function: move genetic structure to residuals:

$$G = (\mathbf{Y}\beta_i - \mathbf{x}_i)^\top \boldsymbol{\Phi}^{-1} (\mathbf{Y}\beta_i - \mathbf{x}_i).$$

Effect size estimator is matrix product of data:

$$\hat{\beta}_i = \mathbf{H}^\top \mathbf{x}_i, \quad \mathbf{H} = \boldsymbol{\Phi}^{-1} \mathbf{Y} \left( \mathbf{Y}^\top \boldsymbol{\Phi}^{-1} \mathbf{Y} \right)^{-1}.$$

Variance under null hypothesis has closed form:

$$\text{Cov}(\hat{\beta}_i | \mathbf{Y}) = 4p_i(1-p_i)(\mathbf{H}^\top \boldsymbol{\Phi} \mathbf{H}), \quad (\mathbf{H}^\top \boldsymbol{\Phi} \mathbf{H}) = \left( \mathbf{Y}^\top \boldsymbol{\Phi}^{-1} \mathbf{Y} \right)^{-1}.$$

This is fast! Bottleneck is calculating  $\boldsymbol{\Phi}^{-1} \mathbf{Y}$ .

Solve efficiently with “conjugate gradient” algorithm!

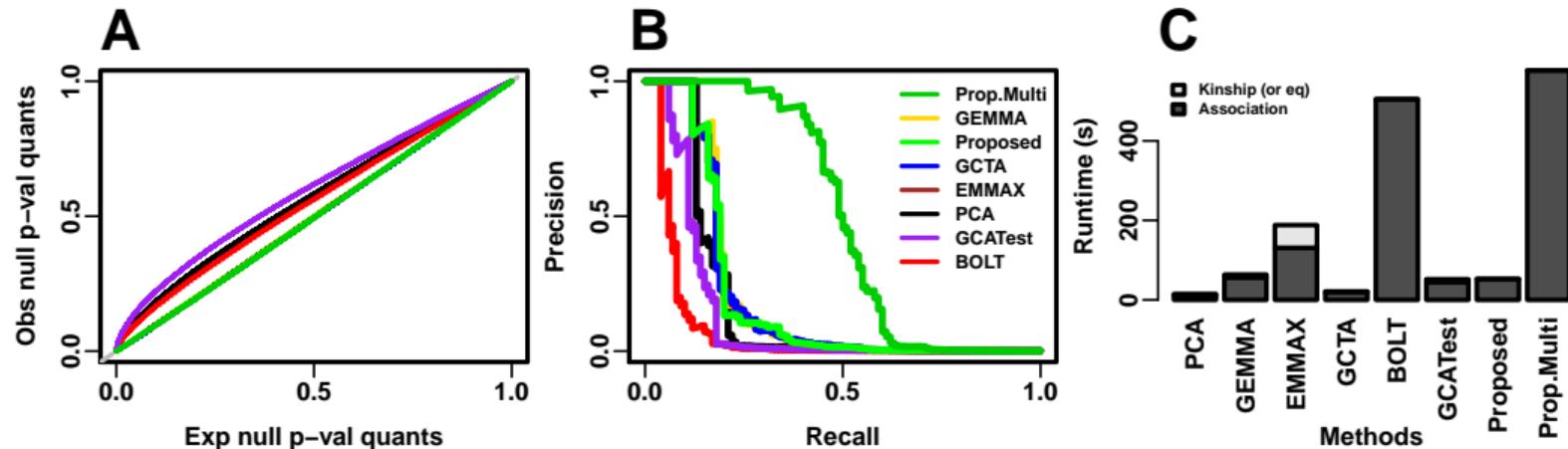
# LIGERA: light genetic robust association

“Multiscan”: forward variable selection.

At each iteration:

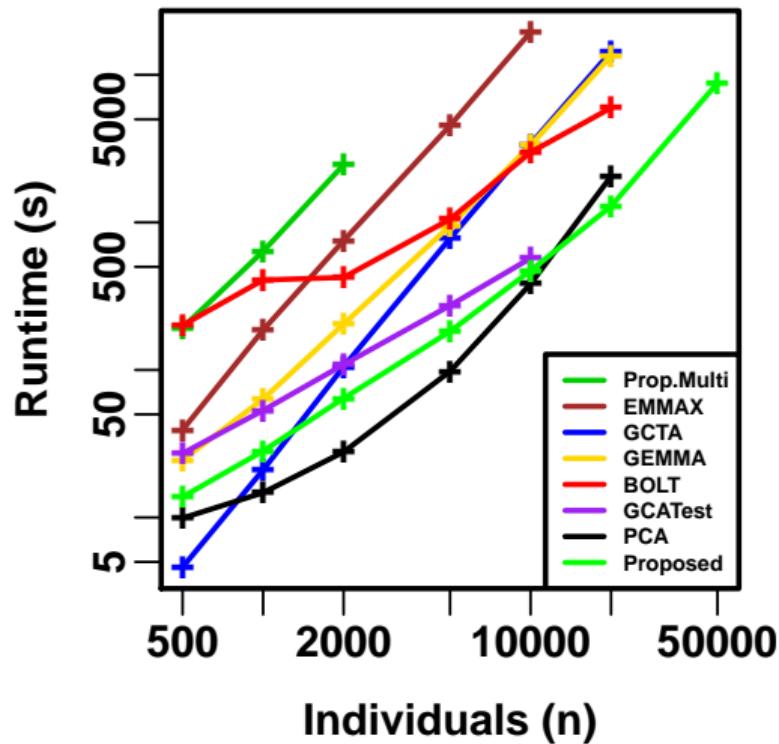
- ▶ Calculate p-values
- ▶ Estimate q-values
- ▶ Select all loci with  $q < 0.05$ , add as covariates in following iteration
- ▶ Stop if no new loci are selected.

# LIGERA: light genetic robust association



- ▶ Control of type-I error
- ▶ Increased power with multiscan
- ▶ Great runtime for single scan (enables multiscan)

# LIGERA: light genetic robust association: scalability



# Overview

New population kinship and  $F_{ST}$  estimates

- ▶ Human Origins dataset
- ▶ Simulation validations

Genetic association models

- ▶ Robustness of PCA and LMM approaches
- ▶ Biases in heritability estimation
- ▶ LIGERA: Light Genetic Robust Association

Admixture model

- ▶ Hispanics in 1000 Genomes Project
- ▶ Joint inference of admixture and population history from genetic covariance

# Acknowledgments

## Ochoa Lab

Amika Sood

Zhuoran Hou

Jiajie Shen

Yiqi Yao (now at  
IQVIA Beijing)

## Duke University

Beth Hauser

Yi-Ju Li

Andrew Allen

Amy Goldberg

Rasheed Gbadegesin

**Princeton**  
**University**  
John D. Storey

## Funding

NIH

Duke Whitehead  
Scholars



DrAlexOchoa  
 ochoalab.github.io

alejandro.ochoa@duke.edu