Numerical Weather Prediction - Sheet 5, 31.05.19

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This week you will write a program that computes the inversion of the Poisson equation, i.e. getting the streamfunction ψ from the vorticity ζ with

$$\nabla^2 \psi = \zeta. \tag{1}$$

As boundary conditions we will use periodic boundaries in x-direction and constant boundaries in y direction. From the streamfunction you will get the wind field back via

$$u = -\partial_y \psi, \quad v = \partial_x \psi. \tag{2}$$

Thus, the difference between the northern and southern boundary value of ψ is related to the mean zonal wind. The inversion will be done in spectral space, in which the Laplace operator reduces to a simple division:

$$\tilde{\psi} = -\frac{1}{k^2}\tilde{\zeta}, \tag{3}$$

where $\tilde{\psi}$ and $\tilde{\zeta}$ are the Fourier transfoms of the streamfunction and the vorticity respectively.

1. To test the inversion operator read the analysis file `eraint_2019020100.nc` and calculate the vorticity from the wind like in the previous sheet.

2. Start with a foreward and inverse Fourier-transform using `fft.fft2` and `fft.ifft2` on ζ from `numpy`. You should get back the original field almost exactly. Now devide the Fourier modes by $-k^2$. To get k use the helper function `fft.fftfreq` with $d = dx/(2\pi)$.

3. This inversion operator is valid for double periodic boundary conditions, but in y-direction we want a constant boundary condition. To achive this enlarge the ζ-field before the Fourier transformation in y direction by adding a copy of the inner part, flipping it over and multipying it by -1. Also set the original y-boundaries to 0. Thus the enlarged vorticity field ζ' equals:

$$\begin{aligned} \zeta'_{ij} &= \zeta_{ij} \quad j = 1, \ldots, m-2 \\ \zeta'_{i0} &= \zeta'_{im-1} = 0 \\ \zeta'_{ij} &= -\zeta_{i2m-j-2} \quad j = m, \ldots, 2m-3. \end{aligned} \tag{4}$$

   Now you should have a zero streamfunction at the y-boundaries and thus no v-flow across the y-boundary.

4. Finally you need to add a mean y-gradient to get the mean zonal wind back. So far it is aproximately zero.

5. Now test your inversion operator by doing the inversion and rederiving the wind fields u and v. Qualetatively compare them to the initial wind by plotting. Quantitatively compare them using the correlation method from the last sheet. The correlations should be larger than 0.99 in the 30°N-60°N region.