Numerical Weather Prediction - Sheet 2, 03.05.19

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For this week's exercise consider the one-dimensional linear advection equation for some quantety ϕ, i.e.

$$\partial_t \phi + u_0 \, \partial_x \phi = 0, \tag{1}$$

where u_0 is the constant zonal wind. As spatial grid use the 45°-latitude circle of the earth with a resolution of 1°. On that grid define as initial condition a pulse-like shape of the quantity ϕ, located in the western part of the domain. Use a Gaussian bell-curve to define $\phi(t = 0)$:

$$\phi(x, t = 0) = \phi_0 \exp -\frac{(x - x_0)^2}{2\sigma^2}. \tag{2}$$

For this simple problem you know the exact solution. Plot this exact solution together with the initial condition for $u_0 = 10m/s$ and $t = 10d$.
After that generate numerical forecasts of the pulse using different discretized version of equation (1). Evaluate the following statements.

1. An Euler-upstream-scheme (foreward in time, backward in space) is stable for Courant-numbers smaller than one.
2. This scheme is unstable for Courant-numbers greater than one.
3. This scheme is more diffusive for smaller Courant-numbers.
4. An Euler-downstream-scheme (foreward in time, foreward in space) is always unstable.
5. An Euler-centered-scheme (foreward in time, centered in space) is always unstable.
6. A Leapfrog-scheme (centered in time, centered in space) is stable for Courant-numbers less than one.
7. This scheme is unstable for Courant-numbers larger than one.
8. This scheme is less diffusive than an upstream-scheme, even for small Courant-numbers.

Hint: The assessment of the stability of a numerical scheme becomes easier if you add a bit of tiny random noise to the initial conditions. Use e.g. `0.001*phi0*random.rand(nlon)`. Such noise is always present in real weather models.