What Is Bayesian Classification?

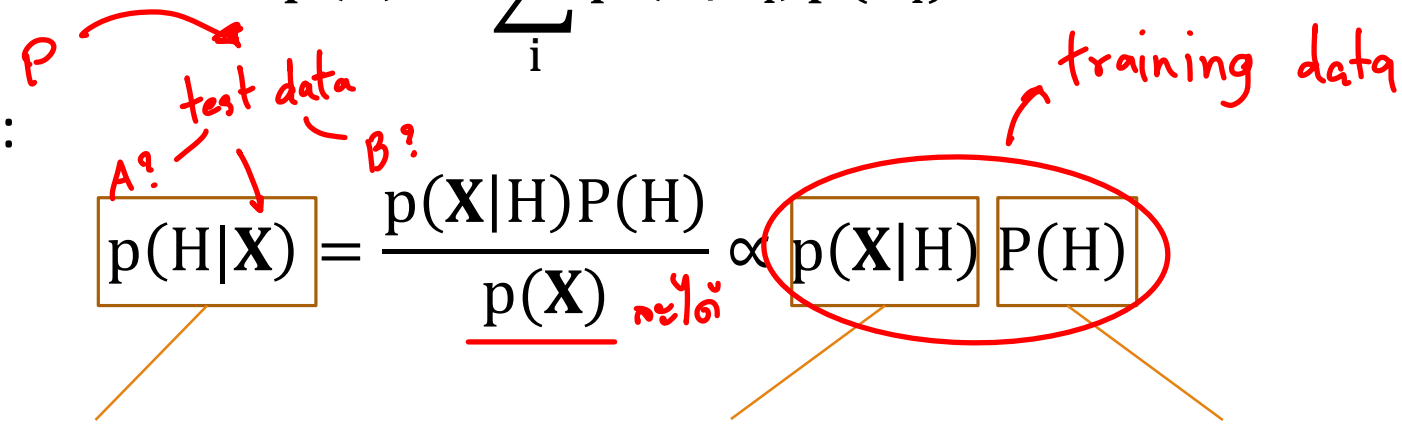
- A statistical classifier
  - Perform *probabilistic prediction* (*i.e.,* predict class membership probabilities)
- Foundation—Based on Bayes' Theorem
- Performance
  - A simple Bayesian classifier, *naïve Bayesian classifier*, has comparable performance with decision tree and selected neural network classifiers
- Incremental
  - Each training example can incrementally increase/decrease the probability that a hypothesis is correct—prior knowledge can be combined with observed data
- **Theoretical Standard**
  - Even when Bayesian methods are computationally intractable, they can provide a standard of optimal decision making against which other methods can be measured

Bayes' Theorem: Basics

- Total probability Theorem:

$$p(B) = \sum_i p(B|A_i)p(A_i)$$

- Bayes' Theorem:



- **X:** a data sample ("*evidence*")
- H: X belongs to class C

Prediction can be done based on Bayes' Theorem:

Classification is to derive the maximum posteriori

Naïve Bayes Classifier: Making a Naïve Assumption

- Practical difficulty of Naïve Bayes inference: It requires initial knowledge of many probabilities, which may not be available or involving significant computational cost
- A Naïve Special Case
  - Make an additional assumption to simplify the model, but achieve comparable performance.

attributes are conditionally independent
(i.e., no dependence relation between attributes)

$$p(X|C_i) = \prod_k p(x_k|C_i) = p(x_1|C_i) \cdot p(x_2|C_i) \cdots p(x_n|C_i)$$

  - Only need to count the class distribution w.r.t. features

Naïve Bayes Classifier: Categorical vs. Continuous Valued Features

- If feature x_k is categorical, $p(x_k = v_k|C_i)$ is the # of tuples in C_i with $x_k = v_k$, divided by $|C_{i,D}|$ (# of tuples of C_i in D)

$$p(X|C_i) = \prod_k p(x_k|C_i) = p(x_1|C_i) \cdot p(x_2|C_i) \cdots\cdots p(x_n|C_i)$$

- If feature x_k is continuous-valued, $p(x_k = v_k|C_i)$ is usually computed based on Gaussian distribution with a mean μ and standard deviation σ

$$p(x_k = v_k|C_i) = N\left(x_k|\mu_{C_i}, \sigma_{C_i}\right) = \frac{1}{\sqrt{2\pi}\sigma_{C_i}} e^{-\frac{\left(x-\mu_{C_i}\right)^2}{2\sigma^2}}$$

Naïve Bayes Classifier: Training Dataset

Class:
C1:buys_computer = ‘yes’
C2:buys_computer = ‘no’

Data to be classified:
X = (age <=30, Income = medium,
Student = yes, Credit_rating = Fair)

age	income	student	credit_rating	buys_computer
<=30	high	no	fair	no
<=30	high	no	excellent	no
31…40	high	no	fair	yes
>40	medium	no	fair	yes
>40	low	yes	fair	yes
>40	low	yes	excellent	no
31…40	low	yes	excellent	yes
<=30	medium	no	fair	no
<=30	low	yes	fair	yes
>40	medium	yes	fair	yes
<=30	medium	yes	excellent	yes
31…40	medium	no	excellent	yes
31…40	high	yes	fair	yes
>40	medium	no	excellent	no

Training data

$P(H^{-y} \mid X) = ?$ prop rai yes

$P(H^{-\sim} \mid X) = ?$ prop rai No

training Data

$= P(x \mid H^{'y})\ P(H^{-y})$ $-\left(\frac{9}{14}\right)$

Naïve Bayes Classifier: An Example

- $P(C_i)$: P(buys_computer = "yes") = 9/14 = 0.643
  P(buys_computer = "no") = 5/14= 0.357
- Compute $P(X|C_i)$ for each class
  P(age = "<=30" | buys_computer = "yes") = 2/9 = 0.222
  P(age = "<= 30" | buys_computer = "no") = 3/5 = 0.6
  P(income = "medium" | buys_computer = "yes") = 4/9 = 0.444
  P(income = "medium" | buys_computer = "no") = 2/5 = 0.4
  P(student = "yes" | buys_computer = "yes) = 6/9 = 0.667
  P(student = "yes" | buys_computer = "no") = 1/5 = 0.2
  P(credit_rating = "fair" | buys_computer = "yes") = 6/9 = 0.667
  P(credit_rating = "fair" | buys_computer = "no") = 2/5 = 0.4

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age	income	student	credit_rating	buys_computer
<=30	high	no	fair	no
<=30	high	no	excellent	no
31…40	high	no	fair	yes
>40	medium	no	fair	yes
>40	low	yes	fair	yes
>40	low	yes	excellent	no
31…40	low	yes	excellent	yes
<=30	medium	no	fair	no
<=30	low	yes	fair	yes
>40	medium	yes	fair	yes
<=30	medium	yes	excellent	yes
31…40	medium	no	excellent	yes
31…40	high	yes	fair	yes
>40	medium	no	excellent	no

- **X = (age <= 30 , income = medium, student = yes, credit_rating = fair)**

$P(X|C_i)$: P(X|buys_computer = "yes") = 0.222 x 0.444 x 0.667 x 0.667 = 0.044
P(X|buys_computer = "no") = 0.6 x 0.4 x 0.2 x 0.4 = 0.019

$P(X|C_i)*P(C_i)$: P(X|buys_computer = "yes") * P(buys_computer = "yes") = 0.028 = 0.044 × 0.643
P(X|buys_computer = "no") * P(buys_computer = "no") = 0.007

Therefore, X belongs to class ("buys_computer = yes")

X = age = 42 , student = yes ?

$P(H \mid \hat{X}) = ?$ (H: y / N)

$P(H_{buy=y} \mid (age = 42, student = y)) = P(age = 42 \mid y)\, P(student = yes \mid buy\ y)\, P(buy\ y)$

$= \frac{3}{9} \times \frac{6}{9} \times \frac{9}{14}$

$P(H\ Buy = N \mid (age = 42, student = yes)) =$