CS 412 Intro. to Data Mining

Chapter 6. Mining Frequent Patterns, Association and Correlations: Basic Concepts and Methods

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Chapter 6: Mining Frequent Patterns, Association and Correlations: Basic Concepts and Methods

- Basic Concepts
- Efficient Pattern Mining Methods
- Pattern Evaluation
- Summary

Pattern Discovery: Basic Concepts

- What Is Pattern Discovery?   Why Is It Important?
- Basic Concepts: Frequent Patterns and Association Rules
- Compressed Representation: Closed Patterns and Max-Patterns

What Is Pattern Discovery?

- What are patterns?
  - Patterns: A set of items, subsequences, or substructures that occur frequently together (or strongly correlated) in a data set
  - Patterns represent intrinsic and important properties of datasets
- Pattern discovery: Uncovering patterns from massive data sets
- Motivation examples:
  - What products were often purchased together?
  - What are the subsequent purchases after buying an iPad?
  - What code segments likely contain copy-and-paste bugs?
  - What word sequences likely form phrases in this corpus?

Pattern Discovery: Why Is It Important?

- Finding inherent regularities in a data set
- Foundation for many essential data mining tasks
  - Association, correlation, and causality analysis
  - Mining sequential, structural (e.g., sub-graph) patterns
  - Pattern analysis in spatiotemporal, multimedia, time-series, and stream data
  - Classification: Discriminative pattern-based analysis
  - Cluster analysis: Pattern-based subspace clustering
- Broad applications
  - Market basket analysis, cross-marketing, catalog design, sale campaign analysis, Web log analysis, biological sequence analysis

Basic Concepts: k-Itemsets and Their Supports

- **Itemset**: A set of one or more items
  มี k ตัว
- **k-itemset**: $X = \{x_1, \ldots, x_k\}$
  - Ex. {Beer, Nuts, Diaper} is a 3-itemset
  ไม่รู้ว่าเยอะมาก/น้อยเมื่อเทียบกับข้อมูลทั้งหมด
- *(absolute) support (count)* of X, sup{X}: Frequency or the number of occurrences of an itemset X
  - Ex. sup{Beer} = 3
  - Ex. sup{Diaper} = 4
  - Ex. sup{Beer, Diaper} = 3
  - Ex. sup{Beer, Eggs} = 1

Tid	Items bought
10	Beer, Nuts, Diaper
20	Beer, Coffee, Diaper
30	Beer, Diaper, Eggs
40	Nuts, Eggs, Milk
50	Nuts, Coffee, Diaper, Eggs, Milk

- *(relative) support*, *s{X}*: The fraction of transactions that contains X (i.e., the probability that a transaction contains X)
  - Ex. s{Beer} = 3/5 = 60%
  - Ex. s{Diaper} = 4/5 = 80%
  - Ex. s{Beer, Eggs} = 1/5 = 20%

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Basic Concepts: Frequent Itemsets (Patterns)

- An itemset (or a pattern) X is *frequent* ความถี่ if the support of X is no less than a *minsup* threshold σ
- Let σ = *50%* (σ: *minsup* threshold)

  For the given เกิน 5-transaction dataset
  - All the frequent 1-itemsets:
    - Beer: 3/5 (60%); Nuts: 3/5 (60%)
    - Diaper: 4/5 (80%); Eggs: 3/5 (60%)
  - All the frequent 2-itemsets:
    - {Beer, Diaper}: 3/5 (60%)
  - All the frequent 3-itemsets?
    - None

เกิน 50%

แสดงว่าบ่อยแต่ไม่ได้

หมายความว่าจะสำคัญ

Tid	Items bought
10	Beer, Nuts, Diaper
20	Beer, Coffee, Diaper
30	Beer, Diaper, Eggs
40	Nuts, Eggs, Milk
50	Nuts, Coffee, Diaper, Eggs, Milk

- Why do these itemsets (shown on the left) form the complete set of frequent *k*-itemsets (patterns) for any *k*?
- **Observation**: We may need an efficient method to mine a complete set of frequent patterns

From Frequent Itemsets to Association Rules

- Comparing with itemsets, rules can be more telling
  - Ex. *Diaper* → *Beer*
    - *Buying diapers may likely lead to buying beers*
- How strong is this rule? (support, confidence)
  - Measuring association rules: *X* → *Y* (s, c)
    - Both *X* and *Y* are itemsets
  - Support, *s*: The probability that a transaction contains X ∪ Y
    - Ex. s{Diaper, Beer} = 3/5 = 0.6 (i.e., 60%)
  - Confidence, *c: The conditional probability* that a transaction containing X also contains *Y*
    - Calculation: *c* = sup(X ∪ Y) / sup(X)
    - Ex. *c* = sup{Diaper, Beer}/sup{Diaper} = ¾ = 0.75

Tid	Items bought
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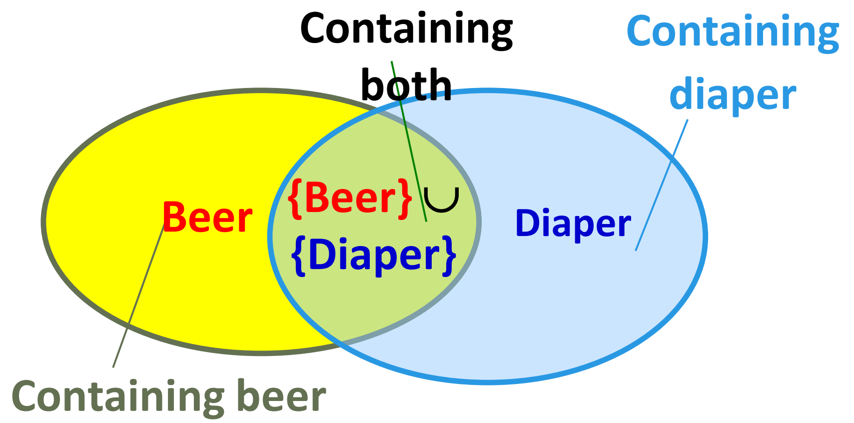

{Beer} ∪ {Diaper} = {Beer, Diaper}

Note: X ∪ Y: the union of two itemsets
- The set contains both X and Y

Mining Frequent Itemsets and Association Rules

- **Association rule mining**
  - Given two thresholds: *minsup, minconf*
  - Find all of the rules, $X \rightarrow Y$ (s, c)
    - such that, s ≥ *minsup* and *c* ≥ *minconf*
- Let *minsup = 50%*
  - Freq. 1-itemsets: Beer: 3, Nuts: 3, Diaper: 4, Eggs: 3
  - Freq. 2-itemsets: {Beer, Diaper}: 3
- Let *minconf = 50%*
  - *Beer* → *Diaper* (60%, 100%)
  - *Diaper* → *Beer* (60%, 75%)

(Q: Are these all rules?)

Tid	Items bought
10	Beer, Nuts, Diaper
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30	Beer, Diaper, Eggs
40	Nuts, Eggs, Milk
50	Nuts, Coffee, Diaper, Eggs, Milk

- **Observations:**
  - Mining association rules and mining frequent patterns are very close problems
  - Scalable methods are needed for mining large datasets

Challenge: There Are Too Many Frequent Patterns!

- A long pattern contains a combinatorial number of sub-patterns
- How many frequent itemsets does the following TDB_1 contain?
  - $TDB_{1:}$ T_1: $\{a_1, \ldots, a_{50}\}$; T_2: $\{a_1, \ldots, a_{100}\}$
  - Assuming (absolute) *minsup* = 1
  - Let's have a try

1-itemsets: $\{a_1\}$: 2, $\{a_2\}$: 2, …, $\{a_{50}\}$: 2, $\{a_{51}\}$: 1, …, $\{a_{100}\}$: 1,

2-itemsets: $\{a_1, a_2\}$: 2, …, $\{a_1, a_{50}\}$: 2, $\{a_1, a_{51}\}$: 1 …, …, $\{a_{99}, a_{100}\}$: 1,

…, …, …, …

99-itemsets: $\{a_1, a_2, \ldots, a_{99}\}$: 1, …, $\{a_2, a_3, \ldots, a_{100}\}$: 1

100-itemset: $\{a_1, a_2, \ldots, a_{100}\}$: 1

- The total number of frequent itemsets:

A too huge set for any one to compute or store!

$$\binom{100}{1} + \binom{100}{2} + \binom{100}{3} + \cdots + \binom{100}{100} = 2^{100} - 1$$

Expressing Patterns in Compressed Form: Closed Patterns

- How to handle such a challenge?
- Solution 1: **Closed patterns**: A pattern (itemset) X is closed if X is *frequent,* and there exists *no super-pattern* Y ↄ X, *with the same support* as X
  - Let Transaction DB TDB_1: T_1: $\{a_1, \ldots, a_{50}\}$; T_2: $\{a_1, \ldots, a_{100}\}$
  - Suppose *minsup* = 1. How many closed patterns does TDB_1 contain?
    - Two: P_1: "$\{a_1, \ldots, a_{50}\}$: 2"; P_2: "$\{a_1, \ldots, a_{100}\}$: 1"
- Closed pattern is a lossless compression of frequent patterns
  - Reduces the # of patterns but does not lose the support information!
  - You will still be able to say: "$\{a_2, \ldots, a_{40}\}$: 2", "$\{a_5, a_{51}\}$: 1"

Expressing Patterns in Compressed Form: Max-Patterns

- Solution 2: **Max-patterns**: A pattern X is a max-pattern if X is frequent and there exists no frequent super-pattern Y ɔ X
- Difference from close-patterns?
  - Do not care the real support of the sub-patterns of a max-pattern
  - Let Transaction DB TDB_1: T_1: $\{a_1, …, a_{50}\}$; T_2: $\{a_1, …, a_{100}\}$
  - Suppose *minsup* = 1. How many max-patterns does TDB_1 contain?
    - One: P: "$\{a_1, …, a_{100}\}$: 1"
- Max-pattern is a lossy compression!
  - We only know $\{a_1, …, a_{40}\}$ is frequent
  - But we do not know the real support of $\{a_1, …, a_{40}\}$, …, any more!
- Thus in many applications, mining close-patterns is more desirable than mining max-patterns

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Efficient Pattern Mining Methods

- The Downward Closure Property of Frequent Patterns
- The Apriori Algorithm
- Extensions or Improvements of Apriori
- Mining Frequent Patterns by Exploring Vertical Data Format
- FPGrowth: A Frequent Pattern-Growth Approach
- Mining Closed Patterns

The Downward Closure Property of Frequent Patterns

- Observation: From $TDB_{1:}$ T_1: $\{a_1, \ldots, a_{50}\}$; T_2: $\{a_1, \ldots, a_{100}\}$
  - We get a frequent itemset: $\{a_1, \ldots, a_{50}\}$
  - Also, its subsets are all frequent: $\{a_1\}, \{a_2\}, \ldots, \{a_{50}\}, \{a_1, a_2\}, \ldots, \{a_1, \ldots, a_{49}\}, \ldots$
  - There must be some hidden relationships among frequent patterns!
- The downward closure (also called "Apriori") property of frequent patterns
  - If **{beer, diaper, nuts}** is frequent, so is **{beer, diaper}**
  - Every transaction containing {beer, diaper, nuts} also contains {beer, diaper}
  - Apriori: Any subset of a frequent itemset must be frequent
- Efficient mining methodology
  - If any subset of an itemset S is infrequent, then there is no chance for S to be frequent—why do we even have to consider S!?

A sharp knife for pruning!

Apriori Pruning and Scalable Mining Methods

ตัดแต่ง

- Apriori pruning principle: If there is any itemset which is infrequent, its superset should not even be generated! (Agrawal & Srikant @VLDB'94, Mannila, et al. @ KDD' 94)
- Scalable mining Methods: Three major approaches
  - Level-wise, join-based approach: Apriori (Agrawal & Srikant@VLDB'94)
  - Vertical data format approach: Eclat (Zaki, Parthasarathy, Ogihara, Li @KDD'97)
  - Frequent pattern projection and growth: FPgrowth (Han, Pei, Yin @SIGMOD'00)

Apriori: A Candidate Generation & Test Approach

- Outline of Apriori (level-wise, candidate generation and test)
  - Initially, scan DB once to get frequent 1-itemset
  - Repeat
    - Generate length-(k+1) candidate itemsets from length-k frequent itemsets
    - Test the candidates against DB to find frequent (k+1)-itemsets
    - Set k := k +1
  - Until no frequent or candidate set can be generated
  - Return all the frequent itemsets derived

The Apriori Algorithm (Pseudo-Code)

C_k: Candidate itemset of size k

F_k : Frequent itemset of size k

K := 1;

F_k := {frequent items};   // frequent 1-itemset

While (F_k != ∅) **do {**   // when F_k is non-empty

  C_{k+1} := candidates generated from F_k; // candidate generation

  Derive F_{k+1} by counting candidates in C_{k+1} with respect to *TDB* at minsup;

  k := k + 1

  }

return $\cup_k F_k$ // return F_k generated at each level

The Apriori Algorithm—An Example

minsup = 2

Database TDB

Tid	Items
10	A, C, D
20	B, C, E
30	A, B, C, E
40	B, E

1^{st} scan

C_1

Itemset	sup
{A}	2
{B}	3
{C}	3
{D}	1
{E}	3

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F_1

Itemset	sup
{A}	2
{B}	3
{C}	3
{E}	3

C_2

Itemset
{A, B}
{A, C}
{A, E}
{B, C}
{B, E}
{C, E}

2^{nd} scan

C_2

Itemset	sup
{A, B}	1
{A, C}	2
{A, E}	1
{B, C}	2
{B, E}	3
{C, E}	2

F_2

Itemset	sup
{A, C}	2
{B, C}	2
{B, E}	3
{C, E}	2

C_3

Itemset
{B, C, E}

3^{rd} scan

F_3

Itemset	sup
{B, C, E}	2