Cholesky

$$\begin{pmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{pmatrix} \begin{pmatrix} l_{11} & l_{21} & l_{31} \\ 0 & l_{22} & l_{32} \\ 0 & 0 & l_{33} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{21} & a_{31} \\ a_{21} & a_{22} & a_{32} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

$$l_{11}^2 = a_{11} \Rightarrow l_{11} = \sqrt{a_{11}}$$

$$l_{21} l_{11} = a_{21} \Rightarrow l_{21} = \frac{a_{21}}{l_{11}}$$

$$l_{21}^2 + l_{22}^2 = a_{22} \Rightarrow l_{22} = \sqrt{a_{22} - l_{21}^2}$$

$$l_{31} l_{11} = a_{31} \Rightarrow l_{31} = \frac{a_{31}}{l_{11}}$$

$$l_{31} l_{21} + l_{32} l_{22} = a_{32} \Rightarrow l_{32} = \frac{a_{32} - l_{31} l_{21}}{l_{22}}$$

$$l_{31}^2 + l_{32}^2 + l_{33}^2 = a_{33} \Rightarrow l_{33} = \sqrt{a_{33} - l_{31}^2 - l_{32}^2}$$

for $i = 1 : n$

  for $j = 1 : i-1$

$$l_{ij} = \frac{a_{ij} - \sum_{k=1}^{j-1} l_{ik} \cdot l_{jk}}{l_{jj}}$$

$$l_{ii} = \sqrt{a_{ii} - \sum_{k=1}^{i-1} l_{ik}^2}$$

Ionescu Andrei Ionut
375CB

27.04.2021

Parțial

N=7

Gram - Schmidt

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$\begin{array}{ccc} \uparrow & \uparrow & \uparrow \\ a_1 & a_2 & a_3 \end{array}$$

Notăm $a_j \to$ coloana j din A

$q_j \to$ coloana j din Q

$$u_1 = a_1$$

$$q_1 = \frac{u_1}{\|u_1\|}$$

$$u_2 = a_2 - \langle q_1, a_2 \rangle \cdot q_1$$

$$q_2 = \frac{u_2}{\|u_2\|}$$

$$u_3 = a_3 - \langle q_1, a_3 \rangle \cdot q_1 - \langle q_2, a_3 \rangle \cdot q_2$$

$$q_3 = \frac{u_3}{\|u_3\|}$$

$\vdots$

$$u_j = a_j - \sum_{i=1}^{j-1} \langle q_i, a_j \rangle \cdot q_i$$

$$q_j = \frac{u_j}{\|u_j\|}$$

$$R = \begin{bmatrix} \|u_1\| & \langle q_1, a_2 \rangle & \langle q_1, a_3 \rangle \\ 0 & \|u_2\| & \langle q_2, a_3 \rangle \\ 0 & 0 & \|u_3\| \end{bmatrix}$$

$\boxed{\text{exemplu}}$

$$A = \begin{bmatrix} 1 & +1 & 1 \\ -1 & 0 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$
$$\begin{matrix} \uparrow & \uparrow & \uparrow \\ a_1 & a_2 & a_3 \end{matrix}$$

$$u_1 = a_1 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

$$q_1 = \frac{u_1}{\|u_1\|} = \frac{1}{\sqrt3} \cdot \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

$$u_2 = a_2 - \langle q_1, a_2 \rangle q_1 = \begin{bmatrix} +1 \\ 0 \\ 1 \end{bmatrix} - \left[\frac{1}{\sqrt3} \; \frac{-1}{\sqrt3} \; \frac{1}{\sqrt3} \right] \cdot \begin{bmatrix} +1 \\ 0 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{\sqrt3} \\ -\frac{1}{\sqrt3} \\ \frac{1}{\sqrt3} \end{bmatrix} =$$

$$= \begin{bmatrix} +1 \\ 0 \\ 1 \end{bmatrix} - \frac{1}{\sqrt3}\left(\frac{2}{\sqrt3}\right) \cdot \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} +1 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} -\frac{2}{3} \\ \frac{2}{3} \\ -\frac{2}{3} \end{bmatrix} = \begin{bmatrix} \frac{1}{3} \\ \frac{2}{3} \\ \frac{1}{3} \end{bmatrix}$$

$$q_2 = \frac{u_2}{\|u_2\|} = \frac{1}{\sqrt{\frac{6}{9}}} \cdot u_2 = \frac{3}{\sqrt6} \cdot \begin{bmatrix} \frac{1}{3} \\ \frac{2}{3} \\ \frac{1}{3} \end{bmatrix} = \begin{bmatrix} 1/\sqrt6 \\ 2/\sqrt6 \\ 1/\sqrt6 \end{bmatrix}$$

$$u_3 = a_3 - \langle q_1, a_3 \rangle q_1 - \langle q_2, a_3 \rangle q_2 =$$

$$= \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} - \left(\frac{1}{\sqrt3}, -\frac{1}{\sqrt3}, \frac{1}{\sqrt3} \right) \cdot (1, 1, 2) \cdot \begin{bmatrix} 1/\sqrt3 \\ -1/\sqrt3 \\ 1/\sqrt3 \end{bmatrix} - \left(\frac{\sqrt6}{6}, \frac{\sqrt6}{3}, \frac{\sqrt6}{6} \right) \cdot (1, 1, 2) \begin{bmatrix} \frac{\sqrt6}{6} \\ \frac{\sqrt6}{3} \\ \frac{\sqrt6}{6} \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} - \frac{2}{\sqrt{3}} \cdot \begin{bmatrix} 1/\sqrt{3} \\ -1/\sqrt{3} \\ 1/\sqrt{3} \end{bmatrix} - \frac{5\sqrt{6}}{6} \begin{bmatrix} \sqrt{6}/6 \\ \sqrt{6}/3 \\ \sqrt{6}/6 \end{bmatrix} =$$

$$= \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} + \begin{bmatrix} -\frac{2}{3} \\ \frac{2}{3} \\ -\frac{2}{3} \end{bmatrix} + \begin{bmatrix} -\frac{5}{6} \\ -\frac{5}{3} \\ -\frac{5}{6} \end{bmatrix} = \begin{bmatrix} 1/3 \\ 5/3 \\ 4/3 \end{bmatrix} + \begin{bmatrix} -5/6 \\ -5/3 \\ -5/6 \end{bmatrix} =$$

$$= \begin{bmatrix} -\frac{1}{2} \\ 0 \\ \frac{1}{2} \end{bmatrix}$$

$$q_3 = \frac{u_3}{\|u_3\|} = \frac{1}{\sqrt{\frac{1}{2}}} \begin{bmatrix} -\frac{1}{2} \\ 0 \\ \frac{1}{2} \end{bmatrix} = \sqrt{2} \begin{bmatrix} -\frac{1}{2} \\ 0 \\ \frac{1}{2} \end{bmatrix}$$

$$\| \quad q = [q_1, q_2, q_3] = \begin{bmatrix} 1/\sqrt{3} & \sqrt{6}/6 & -\sqrt{2}/2 \\ -1/\sqrt{3} & \sqrt{6}/3 & 0 \\ 1/\sqrt{3} & \sqrt{6}/6 & \sqrt{2}/2 \end{bmatrix}$$

$$R = \begin{bmatrix} \|u_1\| & \langle q_1, a_2 \rangle & \langle q_1, a_3 \rangle \\ 0 & \|u_2\| & \langle q_2, a_3 \rangle \\ 0 & 0 & \|u_3\| \end{bmatrix} = \begin{bmatrix} \sqrt{3} & 2/\sqrt{3} & 2/\sqrt{3} \\ 0 & \sqrt{2}/\sqrt{3} & 5\sqrt{6}/6 \\ 0 & 0 & \sqrt{\frac{1}{2}} \end{bmatrix}$$

Gram-Schmidt modificat

for $i = 1:n$

$r_{ii} = \|a_i\|$

$q_i = \dfrac{a_i}{r_{ii}}$

for $j = i+1:n$

$r_{ij} = \langle q_i, a_j \rangle$

$a_j = a_j - r_{ij} \cdot q_i$

Q și R vor fi aceleași, diferența este ordinea în care se ~~tre~~ calculează elementele, iar matricea A se modifică treptat.