

SIT

$$\begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

$$x_1 l_{11} = b_1 \Rightarrow x_1 = \frac{b_1}{l_{11}}$$

$$x_1 l_{21} + x_2 l_{22} = b_2 \Rightarrow x_2 = \frac{b_2 - x_1 l_{21}}{l_{22}}$$

$$x_1 l_{31} + x_2 l_{32} + x_3 l_{33} = b_3 \Rightarrow x_3 = \frac{b_3 - x_1 l_{31} - x_2 l_{32}}{l_{33}}$$

$\Downarrow$

$$\text{for } i = 1:n$$
$$x_i = \frac{b_i - \sum_{h=1}^{i-1} x_h \cdot l_{ih}}{l_{ii}}$$

SSF

$$\begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

$$u_{33} \cdot x_3 = b_3 \Rightarrow x_3 = \frac{b_3}{u_{33}}$$

$$u_{22} \cdot x_2 + u_{23} \cdot x_3 = b_2 \Rightarrow x_2 = \frac{b_2 - u_{23}x_3}{u_{22}}$$

$$x_1 u_{11} + x_2 \cdot u_{12} + x_3 \cdot u_{13} = b_1 \Rightarrow x_1 = \frac{b_1 - x_2 \cdot u_{12} - x_3 \cdot u_{13}}{u_{11}}$$

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for  $i = n:-1:1$

$$x_i = \frac{b_i - \sum_{h=i+1}^n u_{ih} \cdot x_h}{u_{ii}}$$

Gauss

$$\begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ u_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$l_{11} = a_{11}$$

$$l_{21} = a_{21}$$

$$l_{31} = a_{31}$$

$$u_{12} l_{11} = a_{12} \Rightarrow u_{12} = \frac{a_{12}}{l_{11}}$$

$$l_{11} u_{13} = a_{13} \Rightarrow u_{13} = \frac{a_{13}}{l_{11}}$$

$$l_{21} u_{12} + l_{22} = a_{22} \Rightarrow l_{22} = a_{22} - l_{21} u_{12}$$

$$l_{31} u_{12} + l_{32} = a_{32} \Rightarrow l_{32} = a_{32} - l_{31} u_{12}$$

$$l_{21} u_{13} + l_{22} u_{23} = a_{23} \Rightarrow u_{23} = \frac{a_{23} - l_{21} u_{13}}{l_{22}}$$

$$l_{31} u_{13} + l_{32} u_{23} + l_{33} = a_{33} \Rightarrow l_{33} = a_{33} - l_{31} u_{13} - l_{32} u_{23}$$

~~for i=1:~~

for  $p=1:n$

$$l_{ip} = a_{ip} - \sum_{h=1}^{p-1} l_{ih} \cdot u_{hp}, \quad i=p:n$$

$$u_{pj} = \frac{1}{l_{pp}} \cdot \left( a_{pj} - \sum_{h=1}^{p-1} l_{ph} \cdot u_{hj} \right), \quad j=p+1:n$$



Doolittle

$$\begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$u_{11} = a_{11}$$

$$u_{12} = a_{12}$$

$$u_{13} = a_{13}$$

$$l_{21} u_{11} = a_{21} \Rightarrow l_{21} = \frac{a_{21}}{u_{11}}$$

$$l_{31} u_{11} = a_{31} \Rightarrow l_{31} = \frac{a_{31}}{u_{11}}$$

$$l_{21} \cdot u_{12} + u_{22} = a_{22} \Rightarrow u_{22} = a_{22} - l_{21} \cdot u_{12}$$

$$l_{21} \cdot u_{13} + u_{23} = a_{23} \Rightarrow u_{23} = a_{23} - l_{21} \cdot u_{13}$$

$$l_{31} \cdot u_{12} + l_{32} \cdot u_{22} = a_{32} \Rightarrow l_{32} = \frac{a_{32} - l_{31} \cdot u_{12}}{u_{22}}$$

$$\del{l_{31} \cdot u_{13} + l_{32} \cdot u_{23} + u_{33} = a_{33}}$$

$$u_{33} = a_{33} - l_{31} u_{13} - l_{32} \cdot u_{23}$$

for  $p = 1:n$

$$u_{pj} = a_{pj} - \sum_{h=1}^{p-1} l_{ph} \cdot u_{hj}, \quad j = p:n$$

$$l_{ip} = \left( a_{ip} - \sum_{h=1}^{p-1} l_{ih} \cdot u_{hp} \right) \cdot \frac{1}{u_{pp}}, \quad i = p+1:n$$

$$A = \begin{bmatrix} A_1 & A_3 \\ A_2 & A_4 \end{bmatrix}$$

Notam  $A^{-1} = X = \begin{bmatrix} x_1 & x_3 \\ x_2 & x_4 \end{bmatrix}$

$$A \cdot A^{-1} = i \Rightarrow \begin{bmatrix} A_1 & A_3 \\ A_2 & A_4 \end{bmatrix} \begin{bmatrix} x_1 & x_3 \\ x_2 & x_4 \end{bmatrix} = \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}$$

$$\begin{cases} A_1 x_1 + A_3 x_2 = i \\ A_1 x_3 + A_3 x_4 = 0 \Rightarrow A_1 x_3 = -A_3 x_4 / \cdot A_1^{-1} \Rightarrow x_3 = -A_1^{-1} \cdot A_3 \cdot x_4 \\ A_2 x_1 + A_4 x_2 = 0 \Rightarrow x_2 = -A_4^{-1} \cdot A_2 \cdot x_1 \\ A_2 x_3 + A_4 x_4 = i \end{cases}$$

$$A_1 x_1 + A_3 x_2 = i \Rightarrow A_1 x_1 - A_3 \cdot A_4^{-1} \cdot A_2 \cdot x_1 = i \Rightarrow$$

$$\Rightarrow (A_1 - A_3 \cdot A_4^{-1} \cdot A_2) \cdot x_1 = i \Rightarrow x_1 = (A_1 - A_3 \cdot A_4^{-1} \cdot A_2)^{-1} \cdot i$$

$$A_2 x_3 + A_4 x_4 = i \Rightarrow A_4 x_4 - A_2 \cdot A_1^{-1} \cdot A_3 x_4 = i \Rightarrow$$

$$\Rightarrow (A_4 - A_2 \cdot A_1^{-1} \cdot A_3) \cdot x_4 = i \Rightarrow x_4 = (A_4 - A_2 \cdot A_1^{-1} \cdot A_3)^{-1}$$

Deci:

$$\begin{cases} x_1 = (A_1 - A_3 \cdot A_4^{-1} \cdot A_2)^{-1} \\ x_2 = -A_4^{-1} \cdot A_2 \cdot x_1 \\ x_3 = -A_1^{-1} \cdot A_3 \cdot x_4 \\ x_4 = (A_4 - A_2 \cdot A_1^{-1} \cdot A_3)^{-1} \end{cases}$$