BLACK-SCHOLES TUNING

Hackathon Teratec
2025 Edition

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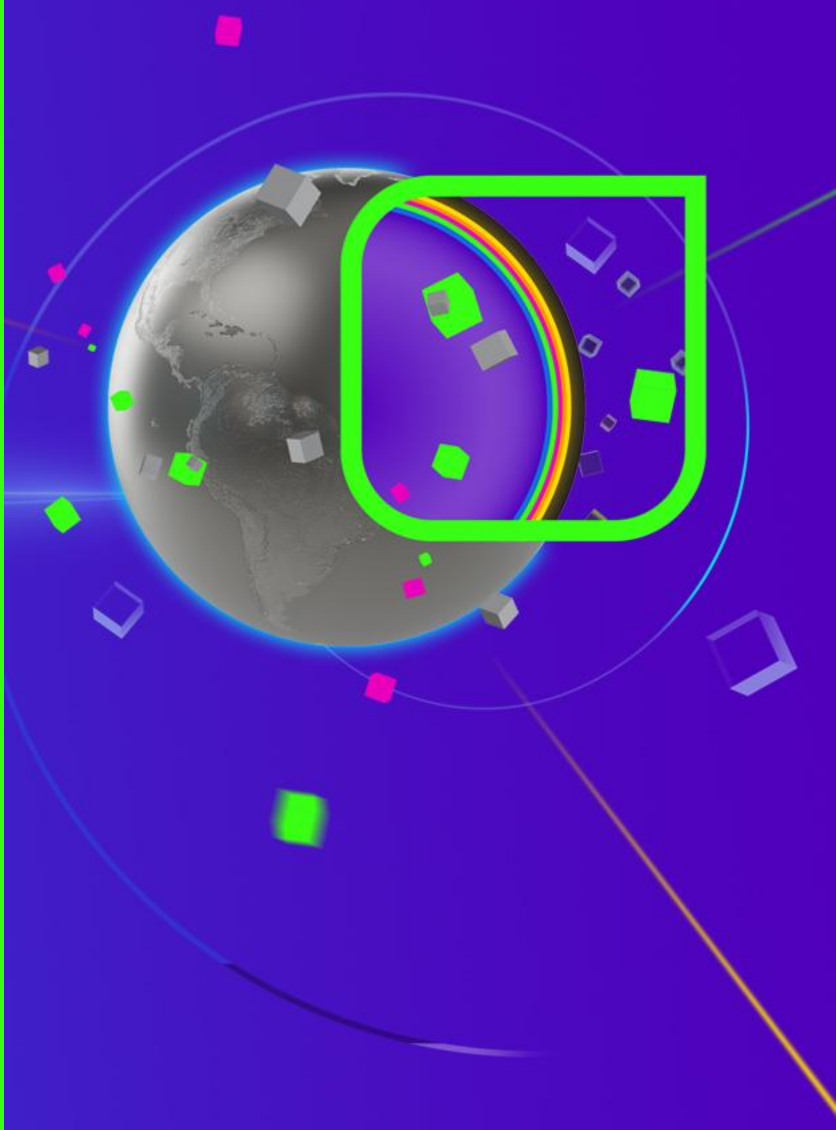


VIRIDIEN

Contents

- Covering **call and put options**
- Black-Scholes
- Hackathon objectives

Foreword

Main code classes in finance

1. Trading algorithms: Automated strategies for buying or selling on financial markets based on technical or fundamental analysis, or machine learning.

2. Microtrading : High-frequency trading with execution in fractions of a second, fast and efficient for exploiting small price variations.

3. Monte Carlo codes: Probabilistic simulations to evaluate markets, value options and manage risk.

4. Financial modeling: Tools for modeling financial assets, returns, interest rates, etc. *(e.g. Python, R, MATLAB)*.

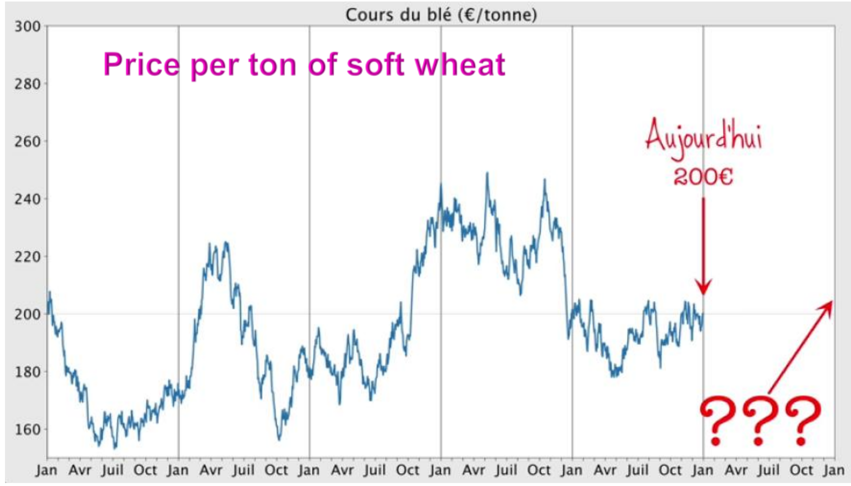
5. Quantitative calculation codes: Complex calculations linked to financial derivatives, risk management, portfolio optimization, etc.

Basic products

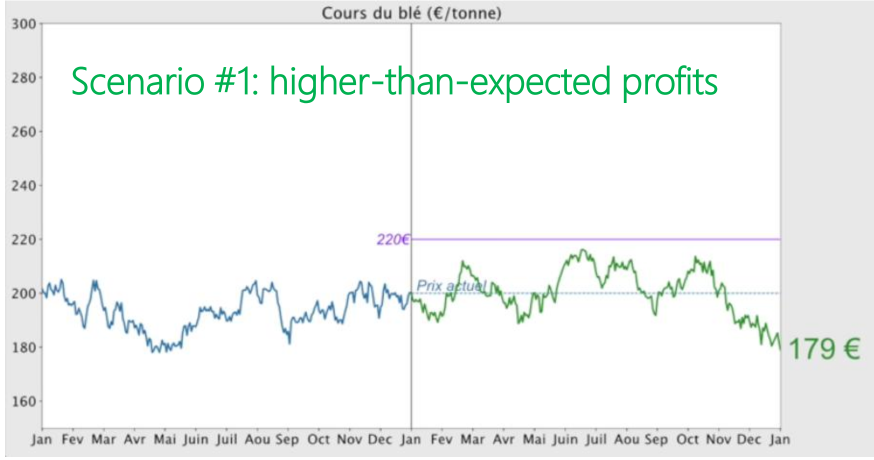
- Large fluctuations observed on commodity products

e.g: Order of 10K croissants to be delivered in 1 year for a reception.

How to guarantee a price without losing money?



Several possible scenarios



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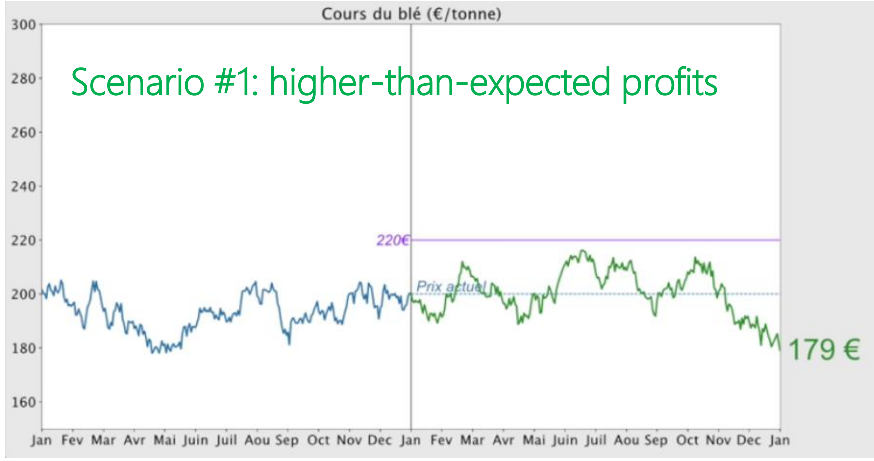

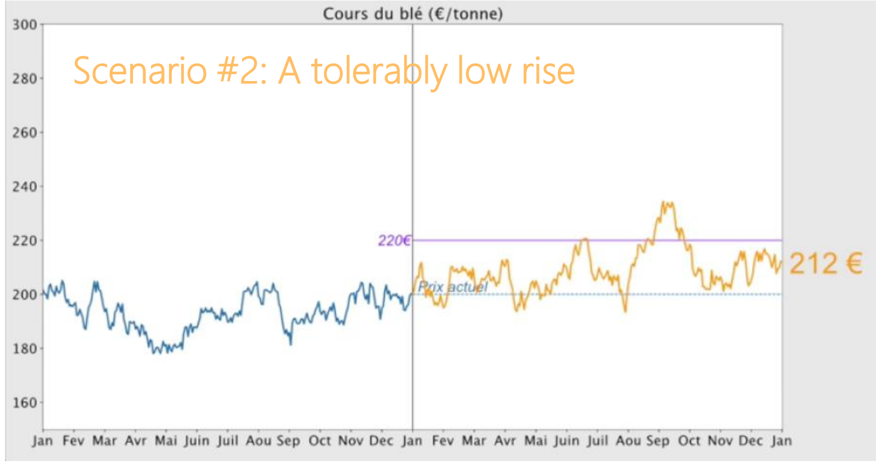

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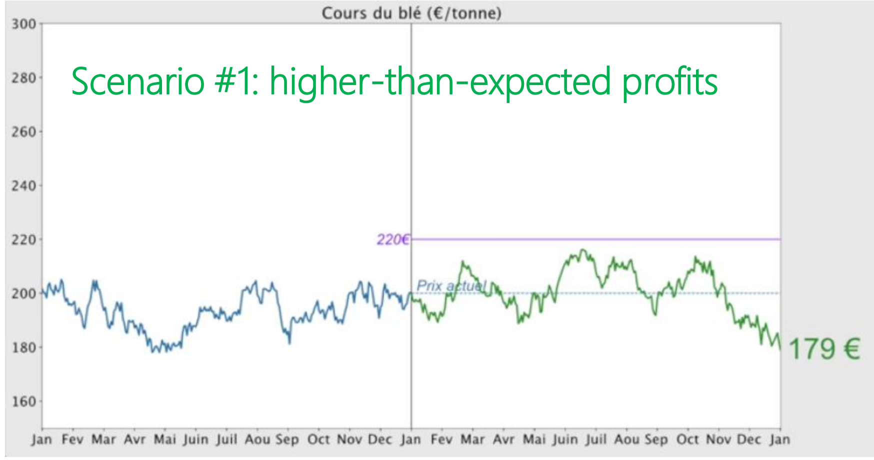

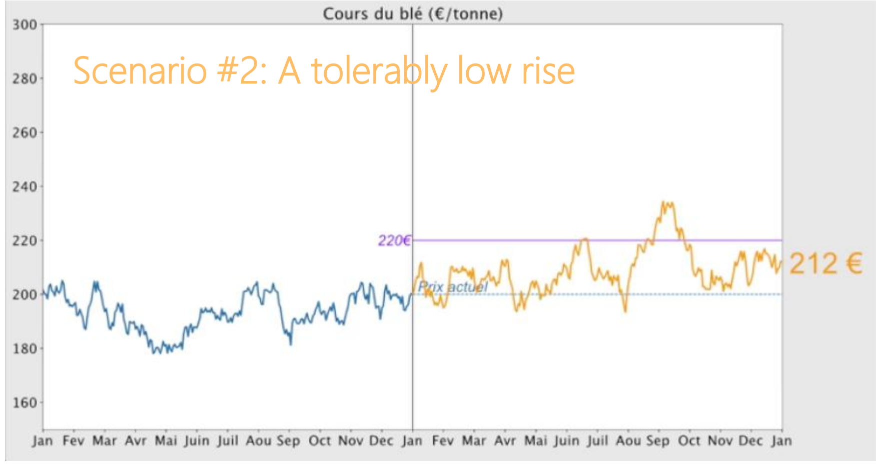

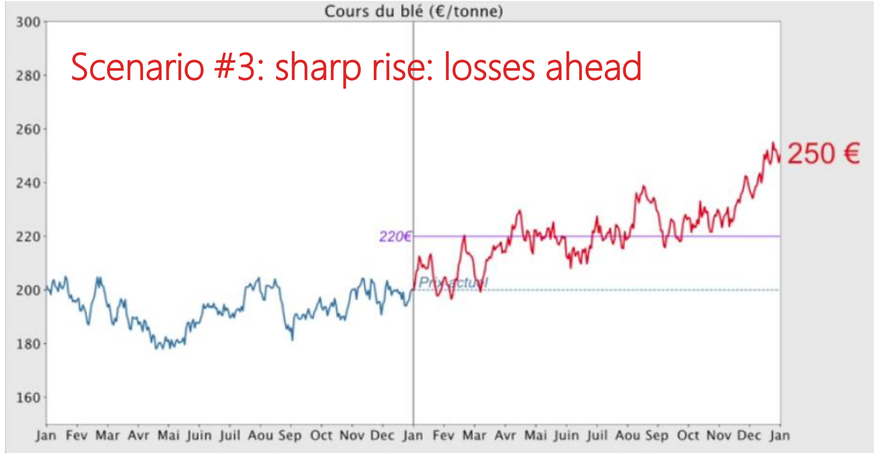

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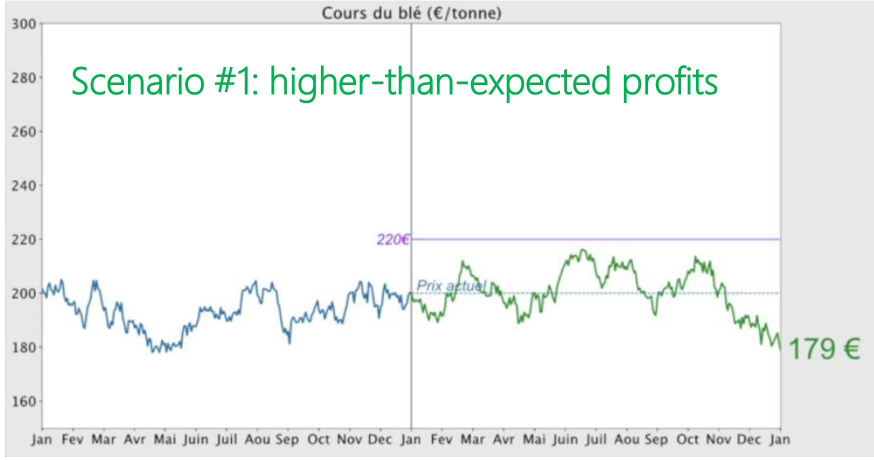
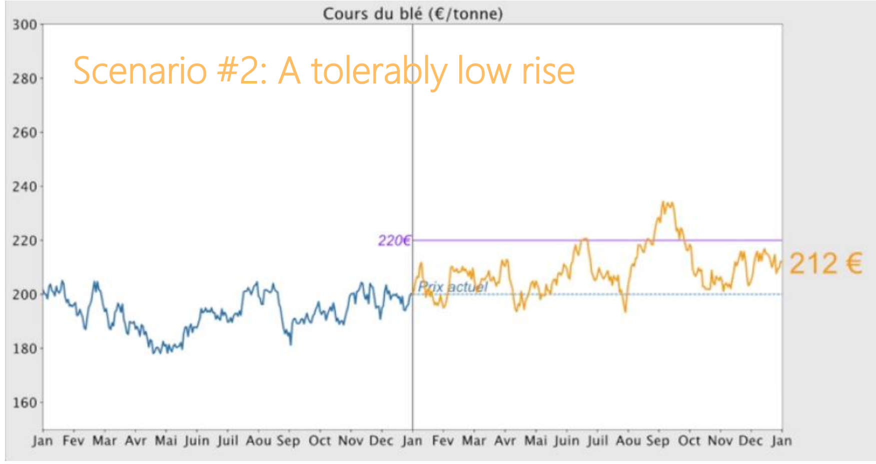
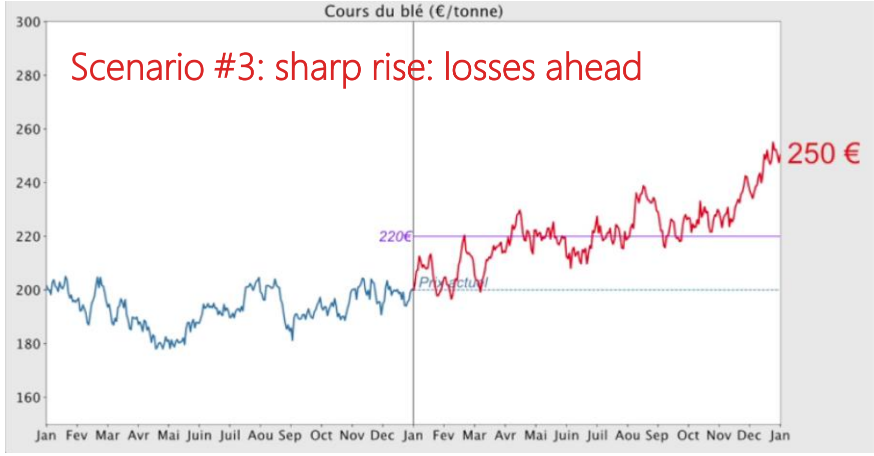
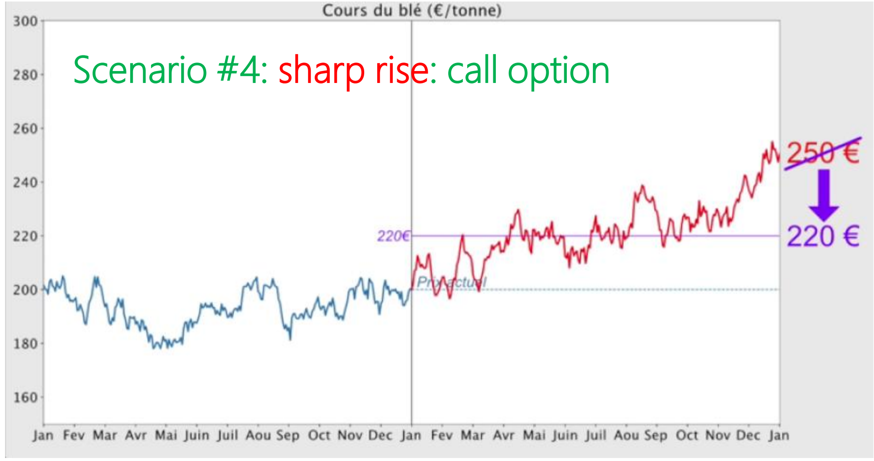
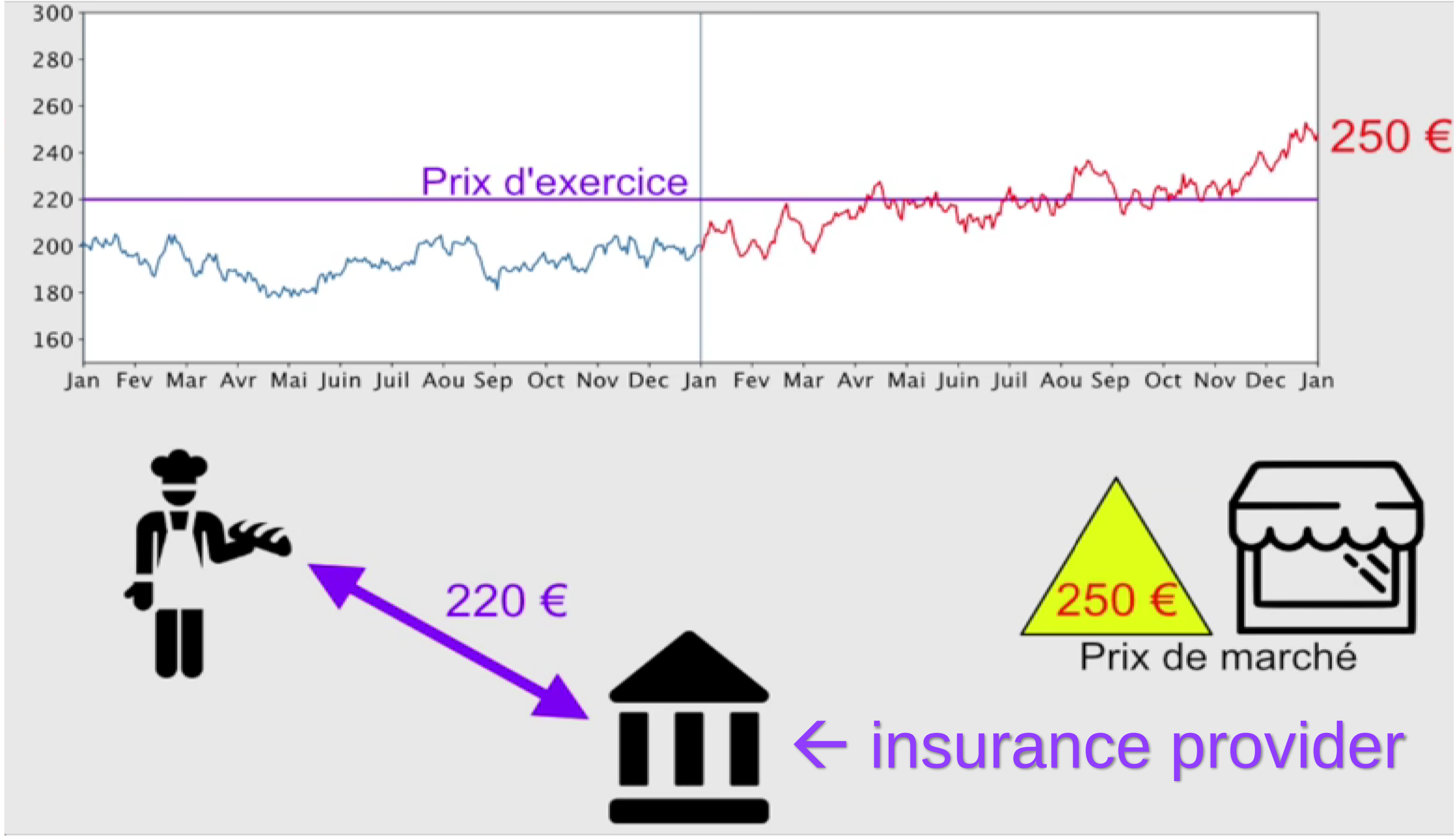






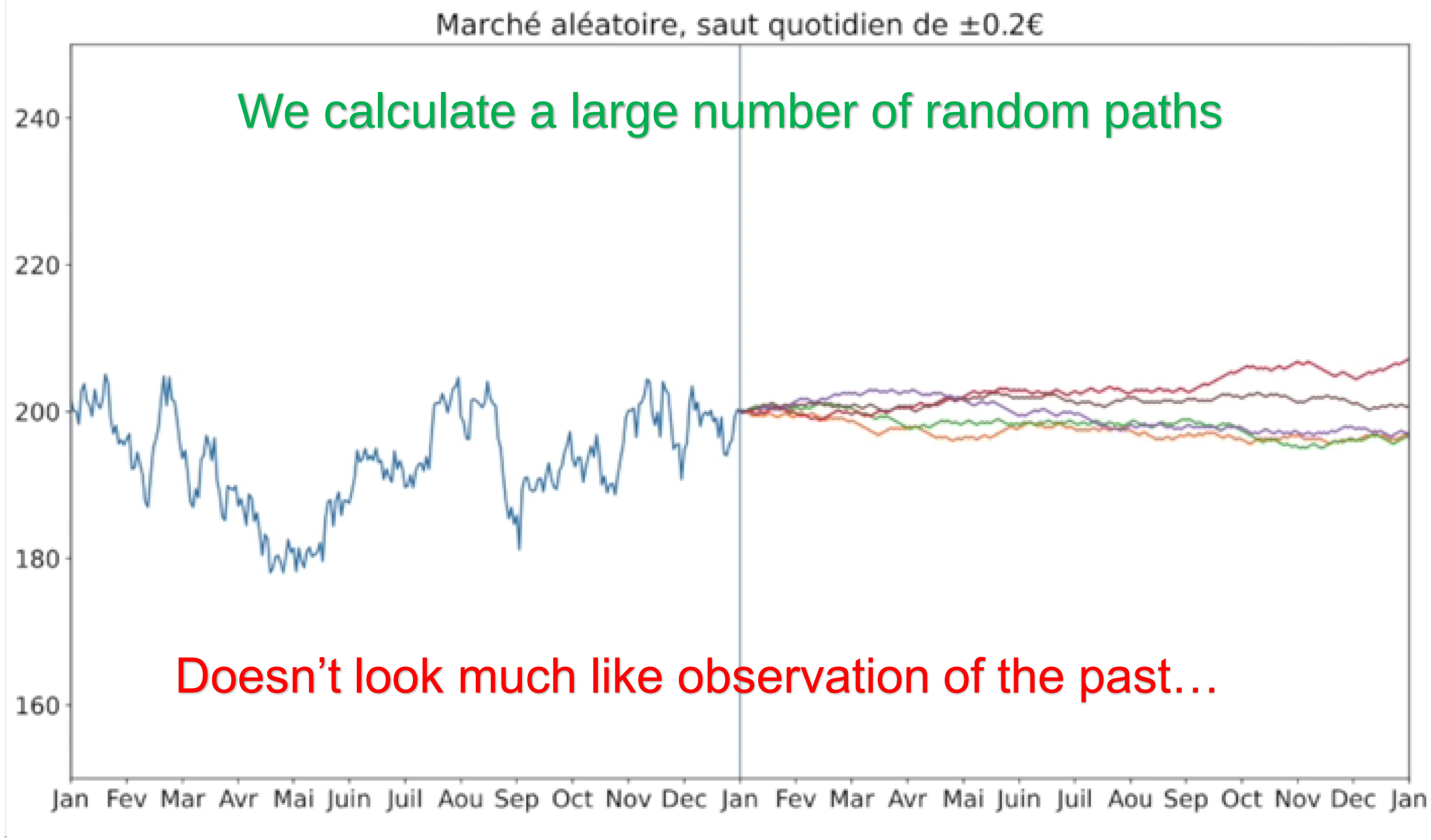


Call option exercised: price paid €220

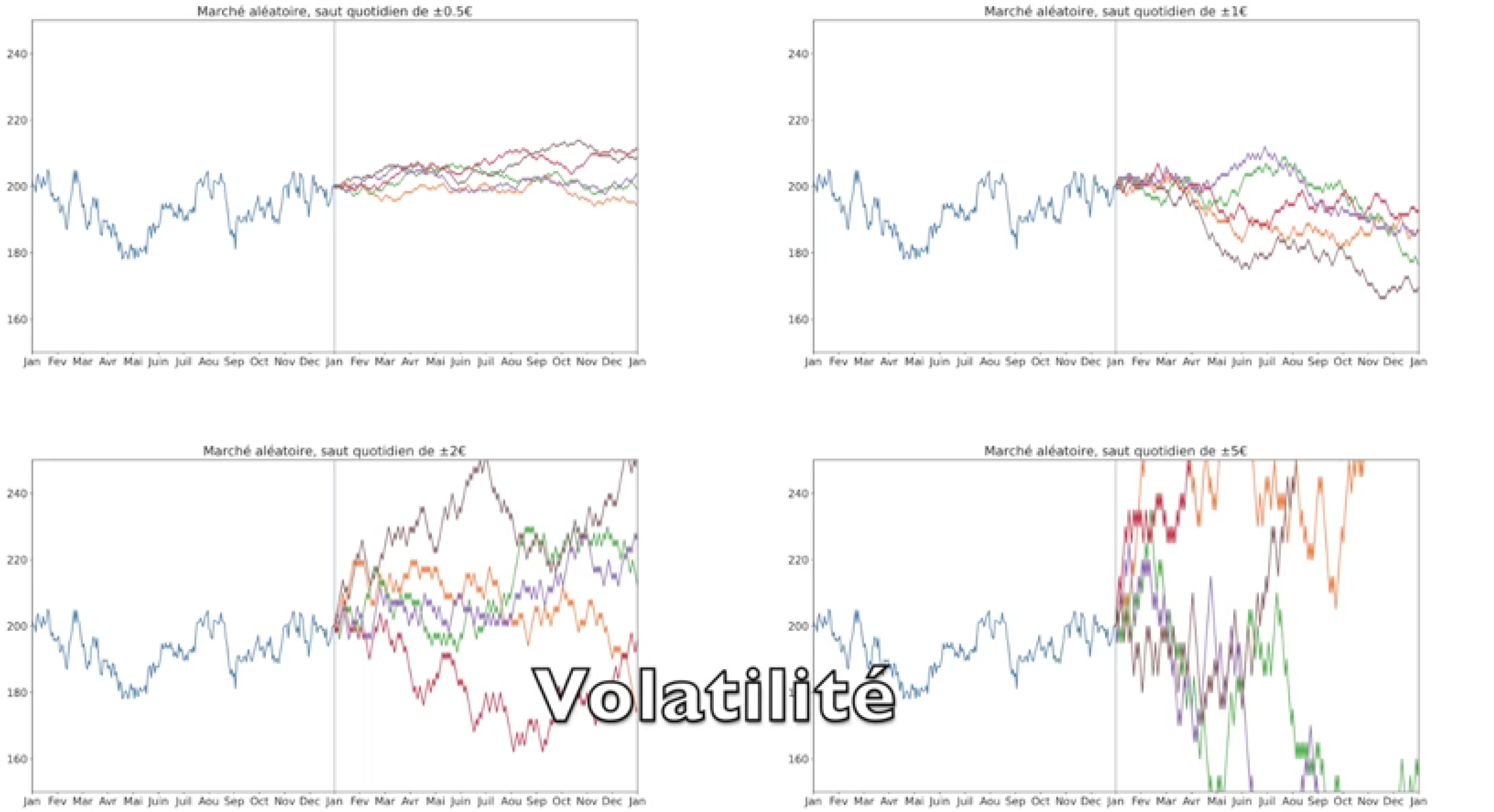


Unexercised call option: market price paid €179

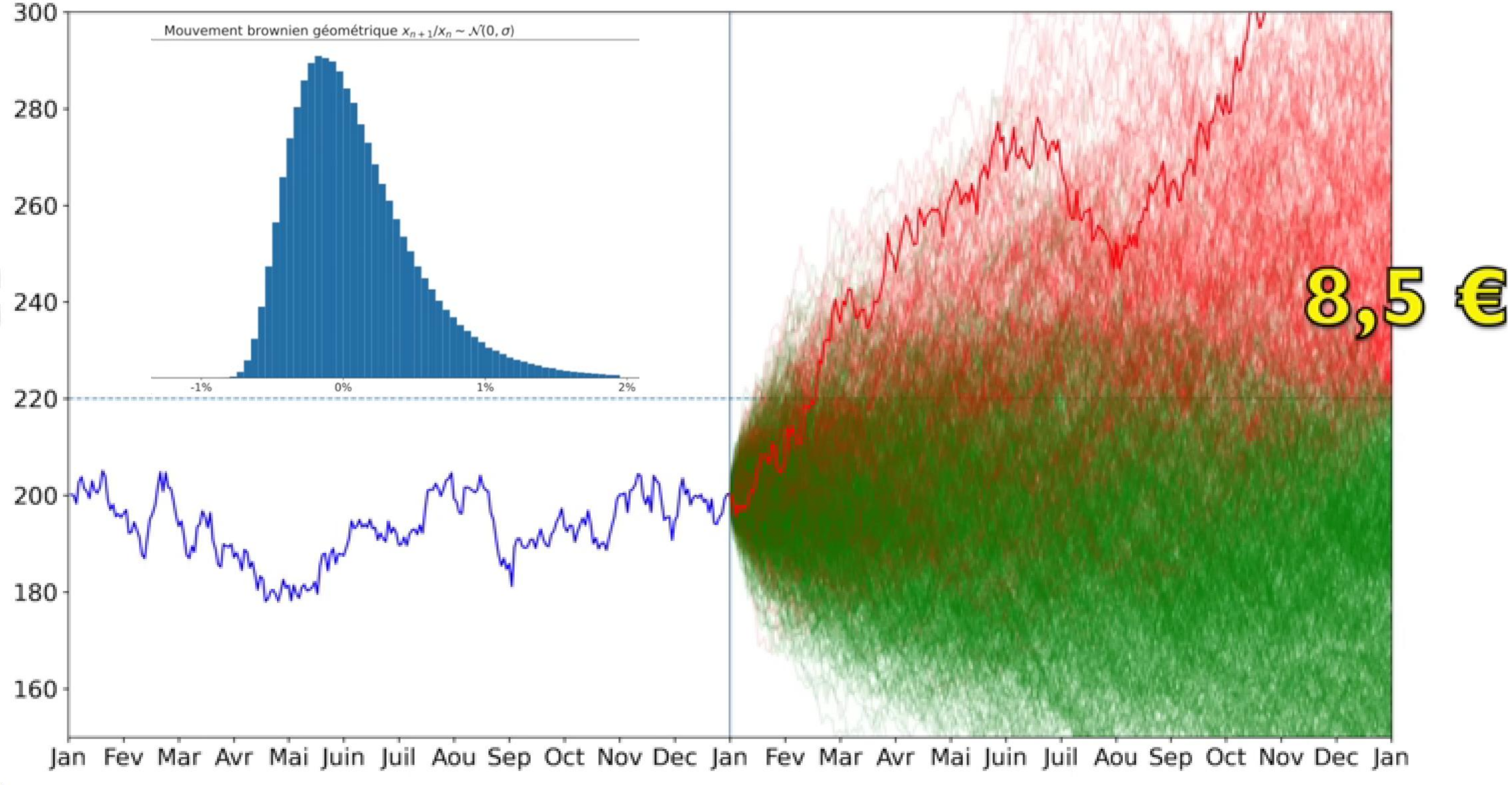
How to calculate the correct option price?



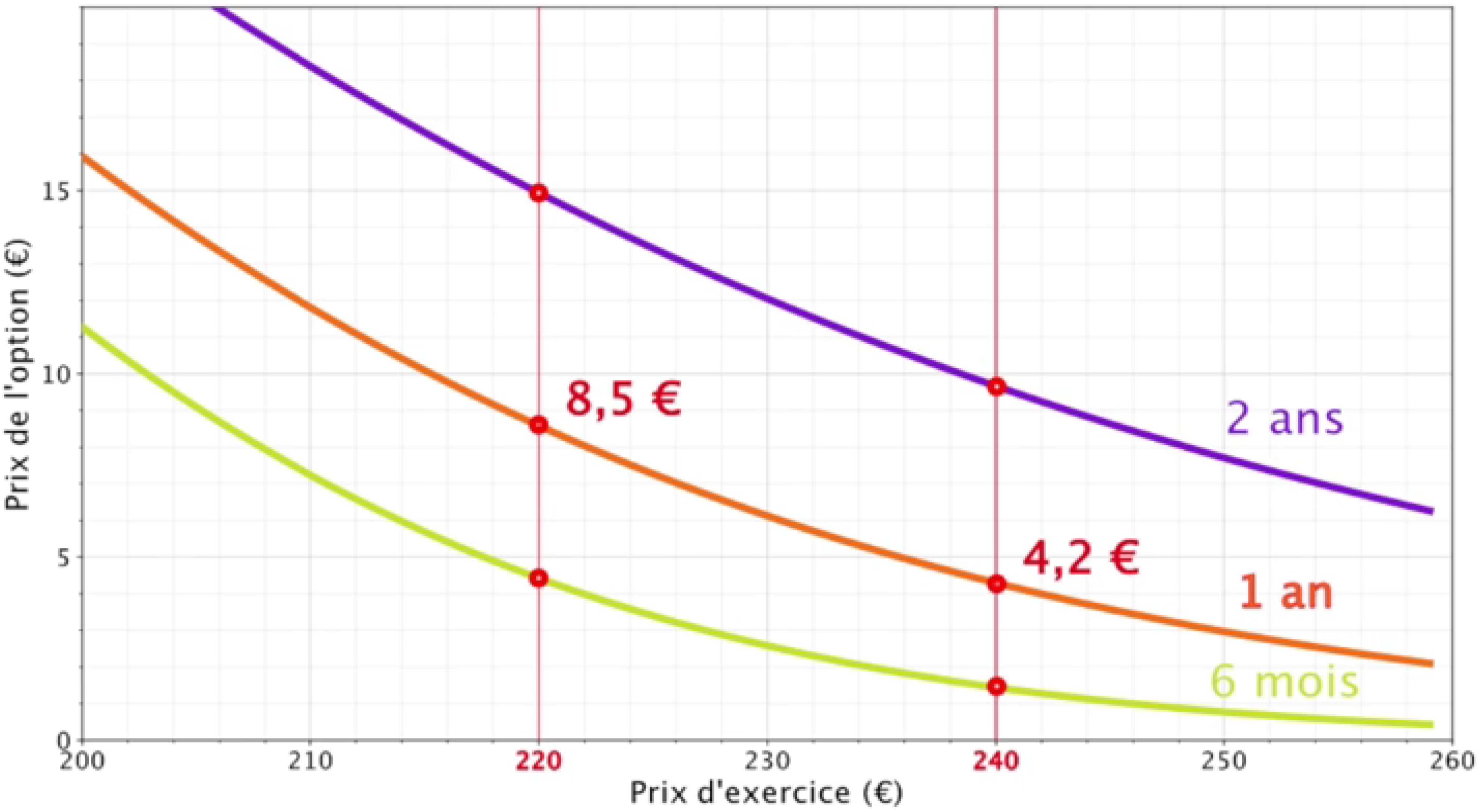
Volatility Adjustment



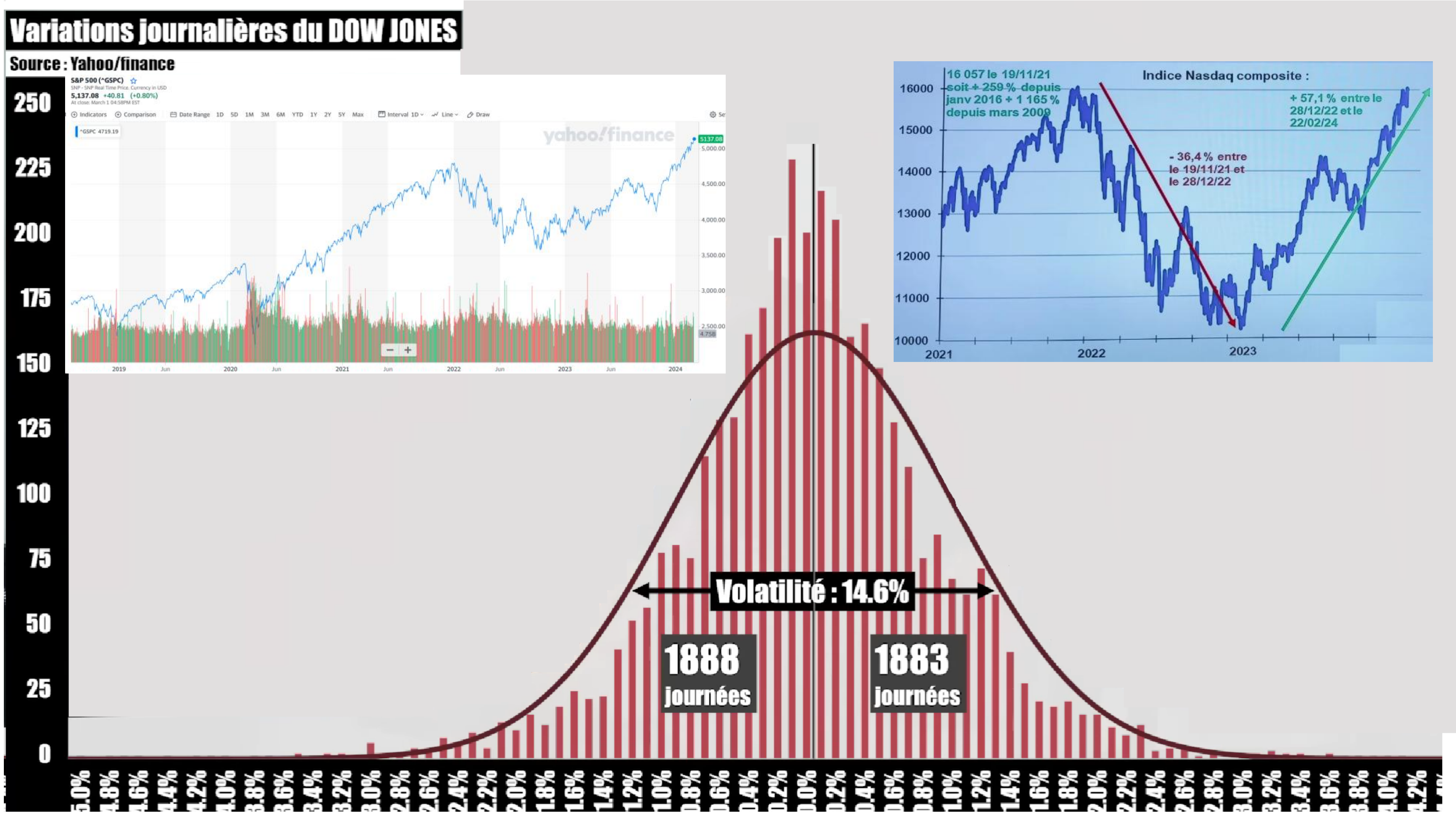
Example: option price for 220 euros



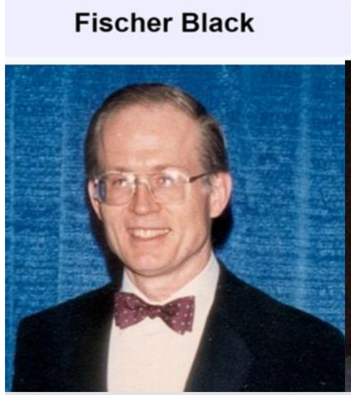
Prix de l'option en fonction du prix d'exercice



Is the movement **random**, with constant jumps?



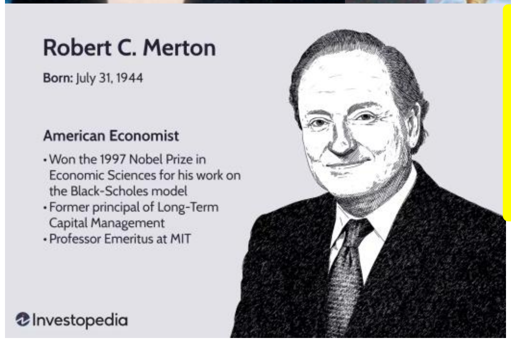
Can we eliminate the risk of waiting for the deadline?



$$C = Se^{-qt}N(d_1) - Ke^{-rt}N(d_2)$$

The Black Scholes Equations

$$P = Ke^{-rt}N(d_2) - Se^{-qt}N(d_1)$$



S = Underlying price ($)
K = Strike price ($)
σ = Volatlity (% p.a.)
r = Continuously compounded interest rate (% p.a.)
q = Continuously compounded dividend yield (% p.a.)
t = Time to expiration (% of year)

$$N(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{\frac{-t^2}{2}} dt$$

$$d_1 = \frac{ln\frac{S}{K} + t(r - q + \frac{\sigma^2}{2})}{\sigma\sqrt{t}}$$

$$d_2 = d_1 - \sigma\sqrt{t}$$

In 1997, Robert Merton and Myron Scholes were awarded the Nobel Prize in Economics for their work, carried out with Fisher Black (who died in 1995), on the valuation of derivatives.

Finance code **optimization** opportunities

Many financial codes behave in a similar way:

- The code is dominated by a **very small mathematical kernel**
- The kernel is usually a **long iteration** representing a **sum**, dominated by :
  - **Random** number calculations
  - **Mathematical functions** such as log, exp, div, sqrt, etc.

Example : Estimation of the share price at maturity using the Black-Scholes model.

$$Value_{present} = \frac{\sum_{j=1}^{M} Value_{call}(S,T)}{M} \bullet e^{-rT}$$

$$Value_{call}(S,T) = MAX(S_T - X, 0) \qquad S_T = S_0 \bullet \exp((r - 0.5v^2 T + v\sqrt{T} N(0,1))$$

Expected performances for error reduction

Main goal: Analyze and tune the Black-Scholes application given to get the **best precision**.

Average error	Time expected (Arm)
0.001	10 hours
0.01	10 minutes

Estimated Time Based on Computed Error for Optimized Code

For safety, we propose two error thresholds to aim for:

- **0.005**
- **0.002**

The best result would be to achieve **0.001**. Points will be awarded based on the average error obtained. You may also set your own goals and include them in the submitted report for evaluation.

Evaluation

Expected outcome

- Evaluation based on error reduction, as described in the previous slide, as well as on **your** tuning ideas
- Document regularly your ideas
- Document everything that you tried or noticed, ***even if it did not work !***
- **Recommendations**: give intermediate results, make good use of all the computational time and resources offered, you have the freedom to change the random generator for example, use tools and explain the whole process
- Be **creative** in your work and its documentation!

We expect a document describing your work on the Black-Scholes tuning.

Thank you !

Good luck, have fun