Ecuații cu derivate parțiale

- $(x-z)\frac{\partial u}{\partial x}+(y-z)\frac{\partial u}{\partial y}+2z\frac{\partial u}{\partial z}=0, \quad u=u(x,y,z)$

$$\frac{dx}{x-z}=\frac{dy}{y-z}=\frac{dz}{2z}$$

$$\frac{d(x-y)}{x-y}=\frac{dz}{2z} \Rightarrow 2\ln(x-y)=\ln z+\ln c_1 \Rightarrow \ln\frac{(x-y)^2}{z}=\ln c_1 \Rightarrow$$

$$\Rightarrow \frac{(x-y)^2}{z}=c_1 \Rightarrow \varphi_1(x,y,z)=\frac{(x-y)^2}{z}$$

$$\left.\begin{aligned}\frac{dx}{x-z}=\frac{dz}{2z}=\frac{d(x+z)}{x+z}\\ \frac{dy}{y-z}=\frac{dz}{2z}=\frac{d(y+z)}{y+z}\end{aligned}\right\} \Rightarrow \frac{d(x+z)}{x+z}=\frac{d(y+z)}{y+z} \xrightarrow{\int} \ln(x+z)=\ln(y+z)+\ln c_2$$

$$\Rightarrow \varphi_2(x,y,z)=\frac{x+z}{y+z}$$

$$\begin{pmatrix}\frac{2(x-y)}{z} & \frac{-2(x-y)}{z} & -\frac{(x-y)^2}{z^2}\\ \frac{1}{y+z} & -\frac{x+z}{(y+z)^2} & \frac{y+z-x-z}{(y+z)^2}\end{pmatrix}$$

$$\Delta_1=\frac{2(x-y)}{z(y+z)}\begin{vmatrix}1 & -1\\ 1 & -\frac{x+z}{y+z}\end{vmatrix}=\frac{2(x-y)}{z(y+z)}\cdot\frac{-x-z+y+z}{y+z}=\frac{-2(x-y)^2}{z(y+z)^2}$$

$$\Delta_2=\frac{(x-y)}{z(y+z)}\begin{vmatrix}2 & \frac{y-x}{z}\\ 1 & \frac{y-x}{y+z}\end{vmatrix}=-\frac{(x-y)^2}{z(y+z)}\left(\frac{2}{y+z}-\frac{1}{z}\right)=-\frac{(x-y)^2}{z(y+z)}\,\frac{2z-y-z}{z(y+z)}=$$

$$=-\frac{(x-y)^2(z-y)}{z^2(y+z)^2}$$

$$\Delta_3=-\frac{(x-y)}{z(y+z)^2}\begin{vmatrix}-2 & \frac{x-y}{z}\\ -x-z & y-x\end{vmatrix}=-\frac{(x-y)}{z(y+z)^2}\left(-2y+2x+\frac{(x+z)(x-y)}{z}\right)$$

$$u(x,y,z)=\Phi\left(\frac{(x-y)^2}{z},\ \frac{x+z}{y+z}\right)$$

- $(y-x)\frac{\partial u}{\partial x}+(x+y+z)\frac{\partial u}{\partial y}+(x-y)\frac{\partial u}{\partial z}=0, \quad u=u(x,y,z)$

$$\frac{dx}{y-x}=\frac{dy}{x+y+z}=\frac{dz}{x-y}$$

$$\frac{dx}{y-x}=\frac{-dz}{y-x} \Big| \cdot (y-x) \Rightarrow dx=-dz \Rightarrow x=-z+c_1 \Rightarrow c_1=x+z$$

$$\left.\begin{aligned} &{}^{1/}\frac{dx}{y-x}={}^{1/}\frac{dy}{x+y+z}={}^{1/}\frac{dz}{x-y}=\frac{dx+dy+dz}{\cancel{y}\cancel{-x}+x+y+z+\cancel{x}\cancel{-y}}=\frac{d(x+y+z)}{x+y+z} \\ &{}^{-3/}\frac{dx}{y-x}=\frac{dy}{x+y+z}={}^{-1/}\frac{dz}{x-y}=\frac{-3dx+dy-dz}{-3y+3x+\cancel{x}+\cancel{y}+\cancel{z}\cancel{-x}+\cancel{y}}=\frac{-d(3x-y+z)}{3x-y+z} \end{aligned}\right\}\Rightarrow$$

$$\Rightarrow \frac{d(x+y+z)}{x+y+z}=\frac{-d(3x-y+z)}{3x-y+z}$$

$$\Rightarrow \ln(x+y+z)=-\ln(3x-y+z)+\ln c_2$$

$$\Rightarrow (3x-y+z)(x+y+z)=c_2$$

$$\Rightarrow \varphi_2(x,y,z)=(3x-y+z)(x+y+z)$$

$$\begin{pmatrix} 1 & 0 & 1 \\ 6x+2y+4z & 2x-2y & 4x+2z \end{pmatrix}$$

$$\Delta_1=2(x-y)$$

$$\Delta_2=2(y-x)$$

$$\Delta_3=4(2x+z-3x-y-2z)=4(-x-y-z)$$

$$u(x,y,z)=\Phi\big(x+z,\quad (3x-y+z)(x+y+z)\big)$$