# Ecuații cu derivate parțiale

- $$\frac{dx}{y+z}=\frac{dy}{z+x}=\frac{dz}{x+y}=\frac{d(x+y+z)}{2(x+y+z)}$$

$$\Rightarrow \frac{d(x-y)}{-(x-y)}=\frac{d(x+y+z)}{2(x+y+z)} \Big| \cdot(-2) \Rightarrow 2\frac{d(x-y)}{x-y}=-\frac{d(x+y+z)}{x+y+z} \Rightarrow$$

$$\Rightarrow 2\int\frac{d(x-y)}{x-y}=-\int\frac{d(x+y+z)}{x+y+z} \Rightarrow 2\ln(x-y)=-\ln(x+y+z)+\ln c_1 \Rightarrow$$

$$\Rightarrow \ln(x-y)^2+\ln(x+y+z)=\ln c_1 \Rightarrow \ln(x-y)^2(x+y+z)=\ln c_1 \Rightarrow$$

$$\Rightarrow (x-y)^2(x+y+z)=c_1$$

$$\Rightarrow \varphi_1(x,y,z)=(x-y)^2(x+y+z)$$

$$\frac{dx}{y+z}=\frac{dz}{x+y}=\frac{dx-dz}{y+z-(x+y)}=\frac{d(x-z)}{z-x}$$

$$\Rightarrow \frac{d(x-z)}{-(x-z)}=\frac{d(x-y)}{-(x-y)} \Rightarrow \ln(x-z)=\ln(x-y)+\ln c_2 \Rightarrow$$

$$\Rightarrow \ln(x-z)-\ln(x-y)=\ln c_2 \Rightarrow \ln\frac{x-z}{x-y}=\ln c_2 \Rightarrow \frac{x-z}{x-y}=c_2 \Rightarrow$$

$$\Rightarrow \varphi_2(x,y,z)=\frac{x-z}{x-y}$$

$\varphi_1$ și $\varphi_2$ sunt liniar independente

$$\Rightarrow u(x,y,z)=\Phi\left((x-y)^2(x+y+z),\ \frac{x-z}{x-y}\right),\quad \Phi \text{ arbitrară}$$

- $$\begin{cases} x\dfrac{\partial u}{\partial x}+y\dfrac{\partial u}{\partial y}+xy\dfrac{\partial u}{\partial z}=0, & u=u(x,y,z) \\ u(x,y,0)=x^2+y^2 \end{cases}$$

$$\frac{dx}{x}=\frac{dy}{y}=\frac{dz}{xy}$$

$$\frac{dx}{x}=\frac{dy}{y} \Rightarrow \int\frac{dx}{x}=\int\frac{dy}{y} \Rightarrow \ln x=\ln y+\ln c_1 \Rightarrow \ln x-\ln y=\ln c_1$$

$$\Rightarrow \ln\frac{x}{y}=\ln c_1 \Rightarrow \frac{x}{y}=c_1 \Rightarrow \varphi_1(x,y,z)=\frac{x}{y}$$

$$\overset{y/}{}\frac{dx}{x}=\overset{x/}{}\frac{dy}{y}=\overset{-2/}{}\frac{dz}{xy}=\frac{y\,dx+x\,dy-2\,dz}{xy+xy-2xy} \Rightarrow y\,dx+x\,dy-2\,dz=0$$

$$\Rightarrow d(xy-2z)=0 \Rightarrow \varphi_2(x,y,z)=xy-2z$$

$$\begin{cases} \dfrac{x}{y} = c_1 \\ xy - 2z = c_2 \\ u(x,y,0) = x^2 + y^2 \end{cases} \qquad \begin{cases} \dfrac{x}{y} = c_1 \\ xy = c_2 \end{cases} \Rightarrow \frac{x}{y}\cdot xy = c_1 c_2 \Rightarrow x^2 = c_1 c_2$$

$$\Rightarrow y = \frac{c_2}{x} \Rightarrow y^2 = \frac{c_2^2}{x^2} = \frac{c_2^2}{c_1 c_2} = \frac{c_2}{c_1}$$

$$\Rightarrow u(x,y,0) = c_1 c_2 + \frac{c_2}{c_1} \Rightarrow u(x,y,z) = \frac{x}{y}(xy - 2z) + \frac{y}{x}(xy - 2z)$$

- $(x + y^2 + u^2)\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = u, \quad u = u(x,y)$

$$(x + y^2 + u^2)\frac{\partial v}{\partial x} + y\frac{\partial v}{\partial y} + u\frac{\partial v}{\partial u} = 0, \quad v = v(x,y,u)$$

$$\frac{dx}{x + y^2 + u^2} = \frac{dy}{y} = \frac{du}{u}$$

$$\frac{dy}{y} = \frac{du}{u} \Rightarrow \int \frac{dy}{y} = \int \frac{du}{u} \Rightarrow \ln y = \ln u + \ln c_1 \Rightarrow \ln y - \ln u = \ln c_1$$

$$\Rightarrow \ln \frac{y}{u} = \ln c_1 \Rightarrow \frac{y}{u} = c_1 \Rightarrow \varphi_1(x,y,u) = \frac{y}{u}$$

$$\overset{1}{\frac{dx}{x + y^2 + u^2}} = \overset{-2y}{\frac{dy}{y}} = \overset{-2u}{\frac{du}{u}} = \frac{dx - 2y\,dy - 2u\,du}{x + y^2 + u^2 - 2y^2 - 2u^2} = \frac{d(x - y^2 - u^2)}{x - y^2 - u^2}$$

$$\frac{d(x - y^2 - u^2)}{x - y^2 - u^2} = \frac{dy}{y} \Rightarrow \int \frac{d(x - y^2 - u^2)}{x - y^2 - u^2} = \int \frac{dy}{y} \Rightarrow \ln(x - y^2 - u^2) = \ln y + \ln c_2$$

$$\Rightarrow \ln(x - y^2 - u^2) - \ln y = \ln c_2 \Rightarrow \ln \frac{x - y^2 - u^2}{y} = \ln c_2 \Rightarrow \frac{x - y^2 - u^2}{y} = c_2$$

$$\Rightarrow \varphi_2(x,y,u) = \frac{x - y^2 - u^2}{y}$$

$$v(x,y,u) = \Phi\left(\frac{y}{u}, \frac{x - y^2 - u^2}{y}\right)$$

$$v(x,y,u) = 0$$

$$\Rightarrow \Phi\left(\frac{y}{u}, \frac{x - y^2 - u^2}{y}\right) = 0$$

• $\begin{cases} x\dfrac{\partial u}{\partial x} - y\dfrac{\partial u}{\partial y} = u^2(x-3y), & u=u(x,y) \\ u(1,y) = -\dfrac{1}{y} \end{cases}$

$$x\frac{\partial v}{\partial x} - y\frac{\partial v}{\partial y} + u^2(x-3y)\frac{\partial v}{\partial u} = 0$$

$$\frac{dx}{x} = \frac{dy}{-y} = \frac{du}{u^2(x-3y)}$$

$$\frac{dx}{x} = \frac{dy}{-y} \Rightarrow \int\frac{dx}{x} = -\int\frac{dy}{y} \Rightarrow \ln x = -\ln y + \ln c_1 \Rightarrow \ln x + \ln y = \ln c_1$$

$$\Rightarrow \ln(xy) = \ln c_1 \Rightarrow xy = c_1 \Rightarrow \varphi_1(x,y,u) = xy$$

$$\overset{1/}{\frac{dx}{x}} = \overset{3/}{\frac{dy}{-y}} = \frac{du}{u^2(x-3y)} = \overset{-1}{\frac{\frac{du}{u^2}}{x-3y}} = \frac{dx+3dy-\frac{du}{u^2}}{x-3y-x+3y}$$

$$\Rightarrow dx+3dy-\frac{du}{u^2} = 0 \quad \Rightarrow d\left(x+3y+\frac{1}{u}\right) = 0 \quad \Rightarrow x+3y+\frac{1}{u} = c_2$$

$$\Rightarrow \varphi_2(x,y,u) = x+3y+\frac{1}{u}$$

$$\begin{cases} xy = c_1 & \Rightarrow y = c_1 \\ x+3y+\frac{1}{u} = c_2 & \Rightarrow 1+3c_1+\frac{1}{u} = c_2 \\ u(1,y) = -\frac{1}{y} & \Rightarrow 1+3c_1-c_1 = c_2 \Rightarrow 1+2c_1 = c_2 \end{cases}$$

$$\Rightarrow 1+2xy = x+3y+\frac{1}{u} \qquad \Rightarrow u(x,y) = \frac{1}{1+2xy-x-3y}$$

• $\begin{cases} x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = u-x^2-y^2, & u=u(x,y) \\ u(x,-2) = x-x^2 \end{cases}$

$$x\frac{\partial v}{\partial x} + y\frac{\partial v}{\partial y} + (u-x^2-y^2)\frac{\partial v}{\partial u} = 0$$

$$\frac{dx}{x} = \frac{dy}{y} = \frac{du}{u-x^2-y^2}$$

$$\frac{dx}{x} = \frac{dy}{y} \Rightarrow \int\frac{dx}{x} = \int\frac{dy}{y} \Rightarrow \ln x = \ln y + \ln c_1 \Rightarrow \ln x - \ln y = \ln c_1 \Rightarrow$$

$$\Rightarrow \ln\frac{x}{y} = \ln c_1 \Rightarrow \frac{x}{y} = c_1$$

$$\overset{2x/}{\frac{dx}{x}} = \overset{2y/}{\frac{dy}{y}} = \frac{du}{u-x^2-y^2} = \frac{2x\,dx+2y\,dy+du}{2x^2+2y^2+u-x^2-y^2} = \frac{d(x^2+y^2+u)}{x^2+y^2+u}$$

$$\frac{d(x^2+y^2+u)}{x^2+y^2+u} = \frac{dy}{y} \Rightarrow \ln(u+x^2+y^2) = \ln y + \ln c_2$$

$$\Rightarrow \frac{u+x^2+y^2}{y} = c_2$$

$$\Rightarrow \begin{cases} \dfrac{u+x^2+y^2}{y} = c_2 \\ \dfrac{x}{y} = c_1 \\ u(x,-2) = x-x^2 \end{cases} \qquad \begin{cases} \dfrac{(\cancel{x}-\cancel{x^2})+\cancel{x^2}+4}{-2} = c_2 \\ x = -2c_1 \end{cases} \Rightarrow \begin{matrix} -2c_1+4 = -2c_2 \\ c_2 - c_1 + 2 = 0 \end{matrix}$$

$$\Rightarrow v(x,y,u) = \frac{u+x^2+y^2}{y} - \frac{x}{y} + 2 = 0 \quad \Rightarrow u(x,y) = -2y + x - x^2 - y^2$$