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SHORT COMMUNICATION

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Distributed fusion Kalman filtering under binary sensors

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Summary

Binary sensors are special sensors that only transmit one-bit information at each time and have been widely applied to environmental awareness and medical monitoring. This paper is concerned with the distributed fusion Kalman filtering problem for a class of binary sensor systems. A novel uncertainty approach is proposed to better extract valid information from binary sensors at switching instant. By minimizing a local estimation error covariance, the local robust Kalman estimates are firstly obtained. Then, the distributed fusion Kalman filter is designed by resorting to the covariance intersection fusion criterion. Finally, a blood oxygen content model is employed to show the effectiveness of the proposed methods.

KEYWORDS

binary sensors, fusion estimation, Kalman filtering

1 | INTRODUCTION

Binary sensors compress the measurement information into two possible values as outputs and have been commonly employed in many practical applications including environmental awareness,[1] medical monitoring[2] and localization.[3] There are two most important advantages that make binary sensors potential to replace traditional sensors in some cases. They are firstly cost-effective, which means numerous binary sensors can be deployed to achieve acceptable performance at a low cost. An example is provided by target tracking,[4] where multiple binary sensors are used to acquire the position information of moving objects. Meanwhile, the communication cost of binary sensors is minimal since each sensor only transmits one bit of information each time. The bandwidth constraint problem, therefore, can be avoided naturally in network communication environments by using binary sensors. Unfortunately, the advantages of binary sensors will matter only if their effective information is extracted, which seems to be an extremely difficult process.

Recently, the estimation problem under binary sensors has attracted more attention within research community. Several results have been presented in the literature that attempt to estimate states from binary measurements. One way to extract the valid information from binary sensors is to analyze the intrinsic form of measurements when their outputs change the sign. Wang et al[5] proposed a stable state observer for continuous time-invariant systems under binary sensors, where suitable switching time sequences were required. For discrete time-invariant systems, an uncertain linear measurement model was developed in the work of Battistelli[6] and the moving horizon estimation approach was proposed to avoid dealing with uncertainty. However, the ignorance of uncertainty can lead to lower estimation accuracy. Another way is to formulate the problem using a Bayesian approach. For example, the estimation of hybrid systems was addressed under binary sensors in the work of Koutsoukos.[7] Ivanov et al[8,9] studied the context-aware filter for systems with binary sensors to improve estimation accuracy. Apparently, the Gaussian distribution approximation of posterior distribution need to be satisfied for Bayesian approach,[7-9] which is a strict condition for estimators. In most of the above-mentioned research works, the raw measurements from multiple sensors are augmented to design estimators in a centralized fusion structure. However, this centralized fusion structure is not computationally efficient and not robust to sensor faults.

Distributed fusion estimation, with better robustness and reliability, has attracted considerable research interest during the past few decades.[10] The major concern of fusion estimation is to best estimate a quantity (ie, a parameter or process) by utilizing useful information contained in multiple sets of data.[11] Briefly speaking, there are three major distributed fusion estimation methods have been developed for the systems with different kinds of noise. In terms of energy-bounded noises, Chen et al[12]proposed the distributed mixed H_2/H_∞ fusion estimator for systems with stochastic and deterministic parameter uncertainties, and Ding et al[13] studied the distributed H_∞ state estimation problem for stochastic systems with network-induced phenomena. Meantime, a novel bounded recursive optimization estimator was developed from a fusion perspective in the work of Chen[14,15] for bounded noises, where the bounds of noises are unknown. As for Gaussian noises, Talebi et al[16] developed a distributed Kalman filtering algorithm by distributing the operations of embedded average consensus information fusion filters, while Xie et al[17] proposed a reliable distributed Kalman fusion scheme over sensor networks subject to abnormal measurements and energy constraints. The work of Chen[18] was concerned with distributed fusion Kalman filtering problem for systems with missing sensor measurements, random transmission delays and packet dropouts, and the globally optimal distributed Kalman filtering fusion with singular covariances of filtering errors and measurement noises was designed by Song et al.[19] Notice that Kalman filtering is one of the most popular recursive Least Mean Square Error (LMSE) algorithm, and it is also reasonable to approximate the noises or disturbances in practical applications as Gaussian noises. Although the significant advance in the field of distributed fusion estimation has been achieved by the work presented above, the distributed fusion estimation under binary sensors remains an open problem. Especially, when take the uncertainty in the measurement information into account, most of distributed Kalman filters mentioned cannot be directly applied to binary sensor systems.

Motivated by above analysis, we shall study the distributed fusion Kalman filtering problem under binary sensors. The main contributions can be summarized as follows: (i) A novel estimation method that can better extract the effective information is developed from the proposed uncertain measurement model for binary sensors. (ii) From the perspective of fusion estimation, the distributed fusion Kalman filter (DFKF) is designed for binary sensor systems by resorting to the covariance intersection (CI) fusion criterion.

Notations: The superscript "T" represents the transpose, while "I" represents the identity matrix with appropriate dimensions. $X > (<)0$ denotes a positive-definite (negative-definite) matrix, while diag$\{\cdot\}$ stands for a block diagonal matrix. Tr(A) represents the trace of the matrix A, and col$\{a_1, \dots, a_n\}$ means a column vector whose elements are $a_1, \dots, a_n$. The ij-th block of matrix A is represented as A^{ij}.

2 | PROBLEM STATEMENT

Consider the following linear time-varying systems

$$x(t+1) = A(t)x(t) + B(t)w(t), \tag{1}$$

$$z_i(t) = C_i(t)x(t) + D_i(t)v_i(t), \ i = 1, \dots, L, \tag{2}$$

with binary measurements from binary sensors

$$y_i(t) = h_i(z_i(t)) = \begin{cases} +1, & \text{if } z_i(t) \ge \tau_i \\ -1, & \text{if } z_i(t) < \tau_i, \end{cases} \tag{3}$$

where $x(t) \in \mathbb{R}^n$ is the state of the process, $z_i(t) \in \mathbb{R}$ is the sensed variable, $y_i(t)$ is the binary measurement, $w(t)$ and $v_i(t)$ are uncorrelated zero-mean Gaussian white noises satisfying

$$\mathrm{E}\left\{ \left[w^{\mathrm{T}}(t) v_i^{\mathrm{T}}(t) \right]^{\mathrm{T}} \left[w^{\mathrm{T}}(t_1) v_j^{\mathrm{T}}(t_1) \right] \right\} = \delta_{t,t_1} \mathrm{diag}\left\{ Q_w, \delta_{i,j} Q_{v_i} \right\}, \tag{4}$$

where $\delta_{t,t_1} = 0$ if $t \neq t_1$ and $\delta_{t,t_1} = 1$ otherwise. $A(t), B(t), C_i(t), D_i(t)$ are time-varying matrices with appropriate dimensions. The i-th binary sensor gives a binary measurement $y_i(t) \in \{-1, +1\}$ according to whether $z_i(t)$ falls below or above the i-th binary sensor's threshold τ_i, where τ_i is assumed to be time invariant at a given time interval.

From the definition of $y_i(t)$, the binary signal seems only provide one bit of information at each time. However, when $y_i(t)$ changes the sign (i.e., $y_i(t)y_i(t-1) < 0$) at switching instant, one can deduce that the threshold τ_i must fall into the interval between $z_i(t)$ and $z_i(t-1)$. To describe this relationship, the uncertain parameter $\alpha_i(t) \in [-0.5, 0.5]$ is introduced, and thus τ_i can be expressed as a convex combination of $z_i(t)$ and $z_i(t-1)$. Thus, τ_i at switching instant is proposed to satisfy

$$(0.5 - \alpha_i(t))z_i(t-1) + (0.5 + \alpha_i(t))z_i(t) = \tau_i, \tag{5}$$

where the exact value of $\alpha_i(t)$ is unknown and unobservable from the binary measurements. Let the set $\mathfrak{J}_t$ be the indexes of sensors when encountering the switching at time t, while the set $\mathfrak{J}_t^C$ is denoted as the indexes of sensors when there is no switching at time t. Then, $\mathfrak{J}_t$ and $\mathfrak{J}_t^C$ are given by:

$$\mathfrak{J}_t \overset{\Delta}{=} \{i | y_i(t-1)y_i(t) < 0\}, \tag{6}$$

$$\mathfrak{J}_t^C \overset{\Delta}{=} \{i | y_i(t-1)y_i(t) > 0\}, \tag{7}$$

In this case, when $\mathfrak{J}_t$ is not empty, it follows from (2), (3) and (5) that the intrinsic form of measurements at switching instant can be modeled by the following uncertain equation:

$$\begin{aligned}\tau_i = {} & (0.5 + \alpha_i(t))C_i(t)x(t) + (0.5 + \alpha_i(t))D_i(t)v_i(t) + (0.5 - \alpha_i(t))C_i(t-1)x(t-1) \\ & + (0.5 - \alpha_i(t))D_i(t-1)v_i(t-1) \quad i \in \mathfrak{J}_t.\end{aligned} \tag{8}$$

Notice that the uncertainty associated with the measurement model depends on the state $x(t)$ and $x(t-1)$. Roughly speaking, the nonswitching instant of binary sensors can hardly provide valid information, while the switching instant provides uncertain measurement information that can be further used for state estimation. Consequently, based on the equality equations of measurements (ie, Equation (7)), the aim of this paper is to design a local estimator by minimizing a local estimation error covariance, and then derive the DFKF under the CI fusion criterion.

Before giving the main results, the following lemma is introduced.

Lemma 1. [20] *Given matrices A, H, E and F with compatible dimensions such that $F^{\mathrm{T}}F \leq I$. Let X be a symmetric positive-defined matrix and $\alpha > 0$ be an arbitrary positive constant such that $\alpha^{-1}I - EXE^{\mathrm{T}} > 0$, then the following inequality holds:*

$$(A + HFE)X(A + HFE)^{\mathrm{T}} \leq AXA^{\mathrm{T}} + AXE^{\mathrm{T}}\left(\alpha^{-1}I - EXE^{\mathrm{T}}\right)^{-1}EXA^{\mathrm{T}} + \alpha^{-1}HH^{\mathrm{T}}.$$

Remark 1. It is known from the work of Battistelli et al[6] that the original measurement model only described the relationship of τ_i and $x(t)$. However, the uncertainty term associated with $x(t)$ and $x(t-1)$ was not included in the cost function of moving horizon estimation method. In this sense, dealing with the uncertainty is avoided at the expense of valid information reduction. Notice that the uncertainty can indeed affect the estimation performance, which means that the estimator proposed in the work of Battistelli et al[6] may lead to lower estimation accuracy. Different form the idea of Battistelli et al,[6] the intrinsic form of measurements in this paper depict the relationship of τ_i, $x(t)$ and $x(t-1)$ by introducing an uncertain model, and the whole uncertain measurement is utilized for local estimation by minimizing a local estimation error covariance.

Remark 2. The estimation problems addressed in the work of Koutsoukos et al[7] and Ivanov et al[8,9] assumed that the probability of outputs at a given state is known for binary sensors. However, this assumption is not reasonable in practical applications. To extract valid information in binary outputs without above assumption, the intrinsic form of measurements is developed in this paper from the definition of binary sensors.

3 | DISTRIBUTED FUSION KALMAN FILTERING

Under the distributed fusion structure, a Kalman-like local estimator is firstly introduced at each switching instant in a recursive form:

$$\hat{x}_i(t) = A(t-1)\hat{x}_i(t-1) + K_i(t)\left[\tau_i - 0.5C_i(t)A(t-1)\hat{x}_i(t-1) - 0.5C_i(t-1)\hat{x}_i(t-1)\right] \quad i \in \mathfrak{J}_t, \tag{9}$$

where the optimal local estimation gains $K_i(t)$ need to be further designed. The corresponding estimation error covariance $\Sigma_i(t) \overset{\Delta}{=} \mathrm{E}\left[(x(t) - \hat{x}_i(t))(x(t) - \hat{x}_i(t))^{\mathrm{T}}\right]$ cannot be determined due to the uncertainty parameters from the intrinsic form of measurements. To better fuse the local estimates, a local estimation error covariance $\overline{\Sigma}_i(t)$ satisfying trace upper bound will be given later. Note that no useful information can be obtained at nonswitching instant, and thus the local estimators will suffer dramatic performance degradation at these instants. Therefore, the following compensating strategy for local estimators at nonswitching instant is proposed:

$$\hat{x}_i(t) = A(t-1)\hat{x}_i(t-1) \quad i \in \mathfrak{J}_t^C. \tag{10}$$

Then, according to each local estimate $\hat{x}_i(t)$ that has been transmitted to the fusion center, the DFKF is given by

$$\hat{x}(t) = \sum_{i=1}^{L} W_i(t)\hat{x}_i(t), \tag{11}$$

where $\sum_{i=1}^{L} W_i(t) = I_n$. Since local estimation error cross-covariance matrix is difficult to calculate, the distributed fusion estimator cannot be designed using the optimal fusion criterion weighted by matrices. Therefore, the CI fusion criterion that does not require to know the cross-covariance matrices[21] is adopted to design the distributed estimator. In this fusion framework, the weighting matrices $W_i(t)(i = 1, 2, \dots, L)$ is given by[22]:

$$W_i(t) = P(t)\omega_i(t)\overline{\Sigma}_i^{-1}(t)(i = 1, 2, \dots, L), \tag{12}$$

where $\sum_{i=1}^{L} \omega_i(t) = 1(\omega_i(t) \geq 0)$ and $P(t) = \left(\sum_{i=1}^{L} \omega_i(t)\overline{\Sigma}_i^{-1}(t)\right)^{-1}$. Then, the optimal coefficients $\omega_i(t)(i = 1, 2, \dots, L)$ are obtained by solving the following optimization problem:

$$\begin{cases} \min_{\omega_i(t)(i=1,2,\dots,L)} \mathrm{Tr}(P(t)) \\ \text{s.t. } : \sum_{i=1}^{L} \omega_i(t) = 1, 0 \leq \omega_i(t) \leq 1. \end{cases} \tag{13}$$

To solve the optimization problem (13), a local estimator gain $K_i(t)$ and a local estimation error covariance matrix $\overline{\Sigma}_i(t)$ need to be calculated in advance. Therefore, the design of the local estimator will be presented in the following theorem. Before deriving main results, let us define

$$\begin{cases} U_{i1}(t) \overset{\Delta}{=} C_i(t)A(t-1) + C_i(t-1), \quad U_{i2}(t) \overset{\Delta}{=} C_i(t)A(t-1) - C_i(t-1) \\ U_{i3}(t) \overset{\Delta}{=} C_i(t)B(t-1), \quad U_{i4}(t) \overset{\Delta}{=} -U_{i3}(t)Q_w U_{i3}^{\mathrm{T}}(t) - D_i(t)Q_{v_i}D_i^{\mathrm{T}}(t) + D_i(t-1)Q_{v_i}D_i^{\mathrm{T}}(t-1) \\ E_{i1}(t) \overset{\Delta}{=} \begin{bmatrix} 0 & U_{i2}(t) \end{bmatrix}, \quad E_{i2}(t) \overset{\Delta}{=} \begin{bmatrix} U_{i3}(t) & D_i(t) & -D_i(t-1) \end{bmatrix} \\ Z_{i1}(t) \overset{\Delta}{=} \left(\zeta_i(t)I - E_{i1}(t)\overline{\Psi}_i(t-1)E_{i1}^{\mathrm{T}}(t)\right)^{-1}, \quad Z_{i2}(t) \overset{\Delta}{=} \left(\eta_i(t)I - E_{i2}(t)\overline{W}E_{i2}^{\mathrm{T}}(t)\right)^{-1} \\ M_i(t) \overset{\Delta}{=} \left(\overline{\Psi}_i^{22}(t-1) - \overline{\Psi}_i^{12}(t-1)\right) U_{i2}^{\mathrm{T}} Z_{i1}(t) \times U_{i2} \left(\overline{\Psi}_i^{22}(t-1) - \overline{\Psi}_i^{21}(t-1)\right) \\ \bar{A}_i(t) \overset{\Delta}{=} \begin{bmatrix} F_i(t) & 0.5K_i(t)U_{i1}(t) \\ 0 & A(t-1) \end{bmatrix} \\ \overline{B}_i(t) \overset{\Delta}{=} \begin{bmatrix} 0.5K_i(t)U_{i3}(t) & 0.5K_i(t)D_i(t) & 0.5K_i(t)D_i(t-1) \\ B(t-1) & 0 & 0 \end{bmatrix} \\ F_i(t) \overset{\Delta}{=} A(t-1) - 0.5K_i(t)U_{i1}(t) \\ H_i(t) \overset{\Delta}{=} 0.5\mathrm{col}\{K_i(t), 0\} \\ \overline{W}_i \overset{\Delta}{=} \mathrm{diag}\{Q_w, Q_{v_i}, Q_{v_i}\} \\ \tilde{A}_i(t) \overset{\Delta}{=} \mathrm{diag}\{A(t-1), A(t-1)\}, \quad \tilde{B}_i(t) \overset{\Delta}{=} \mathrm{col}\{0, B(t-1)\}, \end{cases} \tag{14}$$

Theorem 1. *When minimizing the trace of a local estimation error covariance $\overline{\Sigma}_i(t)$, an optimal local estimator gain $K_i(t)$ in (9) is calculated by*

$$\begin{aligned} K_i(t) = & \left\{A(t-1)\overline{\Psi}_i(t-1)U_{i1}^{\mathrm{T}}(t) + A(t-1)M_i(t)U_{i1}^{\mathrm{T}}(t) + B(t-1)Q_wU_{i3}^{\mathrm{T}}(t) - B(t-1)Q_wU_{i3}^{\mathrm{T}}(t)Z_{i2}(t)U_{i4}(t)\right\} \\ & \times 2\left\{U_{i1}(t)\overline{\Psi}_i(t-1)U_{i1}^{\mathrm{T}}(t) + U_{i1}(t)M_i(t)U_{i1}^{\mathrm{T}}(t) + U_{i3}(t)Q_wU_{i3}^{\mathrm{T}}(t)\left[I - Z_{i2}(t)U_{i4}(t)\right]\right. \\ & \left. + D_i(t-1)Q_{v_i}D_i^{\mathrm{T}}(t-1)\left[I + Z_{i2}(t)U_{i4}(t)\right] + D_i(t)Q_{v_i}D_i^{\mathrm{T}}(t)\left[I - Z_{i2}(t)U_{i4}(t)\right] + \zeta_i(t)I + \eta_i(t)I\right\}^{-1}, \end{aligned} \tag{15}$$

where $\zeta_i(t)$ and $\eta_i(t)$ are given time-varying parameters satisfying

$$\begin{cases} \zeta_i(t)I > E_{i1}(t)\overline{\Psi}_i(t-1)E_{i1}^{\mathrm{T}}(t) \\ \eta_i(t)I > E_{i2}(t)\overline{W}E_{i2}^{\mathrm{T}}(t). \end{cases} \tag{16}$$

Meanwhile, a local estimation error covariance matrix $\overline{\Sigma}_i(t)$ is obtained by

$$\overline{\Sigma}_i(t) = \begin{bmatrix}-I & I\end{bmatrix}\overline{\Psi}_i(t)\begin{bmatrix}-I & I\end{bmatrix}^{\mathrm{T}}, \tag{17}$$

where $\overline{\Psi}_i(t)$ is recursively calculated by

$$\overline{\Psi}_i(t) = \begin{cases} \bar{A}_i(t)\overline{\Psi}_i(t-1)\bar{A}_i^{\mathrm{T}}(t) + \bar{A}_i(t)\overline{\Psi}_i(t-1)E_{i1}^{\mathrm{T}}(t)Z_{i1}(t)E_{i1}(t)\overline{\Psi}_i(t-1)\bar{A}_i^{\mathrm{T}}(t) \\ \quad + \zeta_i(t)H_i(t)H_i^{\mathrm{T}}(t) + \overline{B}_i(t)\overline{W}_i\overline{B}_i^{\mathrm{T}}(t) + \overline{B}_i(t)\overline{W}_iE_{i2}^{\mathrm{T}}(t)Z_{i2}(t)E_{i2}(t)\overline{W}_i\overline{B}_i^{\mathrm{T}}(t) \\ \quad + \eta_i(t)H_i(t)H_i^{\mathrm{T}}(t) \qquad i \in \mathfrak{J}_t \\ \tilde{A}_i(t)\hat{\Psi}_i(t-1)\tilde{A}_i^{\mathrm{T}}(t) + \tilde{B}_i(t)Q_w\tilde{B}_i^{\mathrm{T}}(t) \qquad i \in \mathfrak{J}_t^C. \end{cases} \tag{18}$$

Proof. Define $\psi_i(t) \overset{\Delta}{=} \mathrm{col}\{\hat{x}_i(t), x(t)\}$ and $\overline{w}_i(t-1) \overset{\Delta}{=} \mathrm{col}\{w(t-1), v_i(t), v_i(t-1)\}$, the augmented system at switching instant is given by

$$\psi_i(t) = (\bar{A}_i(t) + H_i(t)\Delta_i(t)E_{i1}(t))\psi_i(t-1) + (\overline{B}_i(t) + H_i(t)\Delta_i(t)E_{i2}(t))\overline{w}_i(t-1), \tag{19}$$

where $\Delta_i(t) = 2\alpha_i(t)I$. Define $\Psi_i(t) \overset{\Delta}{=} \mathrm{E}\left[\psi_i(t)\psi_i^T(t)\right]$, one has that

$$\begin{aligned} \Psi_i(t) = & (\bar{A}_i(t) + H_i(t)\Delta_i(t)E_{i1}(t))\Psi_i(t-1)(\bar{A}_i(t) + H_i(t)\Delta_i(t)E_{i1}(t))^{\mathrm{T}} \\ & + (\overline{B}_i(t) + H_i(t)\Delta_i(t)E_{i2}(t))\overline{W}_i(\overline{B}_i(t) + H_i(t)\Delta_i(t)E_{i2}(t))^{\mathrm{T}}. \end{aligned} \tag{20}$$

Note that $\Delta_i(t)$ satisfies $\Delta_i^T(t)\Delta_i(t) \leq I$, then an upper bound of $\Psi_i(t)$ at switching instant will be given by (18) according to Lemma 1, while $\overline{\Psi}_i(t)$ at nonswitching instant can be directly obtained from compensating estimate in (10). From (17), a local estimation error covariance $\overline{\Sigma}_i(t)$ is calculated by

$$\begin{aligned} \overline{\Sigma}_i(t) = & F_i(t)\overline{\Sigma}_i(t-1)F_i^{\mathrm{T}}(t) + F_i(t)M_i(t)F_i^{\mathrm{T}}(t) + 0.25\zeta_i(t)K_i(t)K_i^{\mathrm{T}}(t) \\ & + (B(t-1) - 0.5K_i(t)U_{i3}(t))Q_w(B(t-1) - 0.5K_i(t)U_{i3}(t))^{\mathrm{T}} \\ & + 0.25K_i(t)D_i(t)Q_{v_i}D_i^{\mathrm{T}}(t)K_i^{\mathrm{T}}(t) + 0.25K_i(t)D_i(t-1)Q_{v_i}D_i^{\mathrm{T}}(t-1)K_i^{\mathrm{T}}(t) \\ & + 0.25\eta_i(t)K_i(t)K_i^{\mathrm{T}}(t) + N_i(t)Z_{i2}(t)N_i^{\mathrm{T}}(t), \end{aligned} \tag{21}$$

where $N_i(t) = (N_{i1}(t) + N_{i2}(t) + N_{i3}(t))$ and

$$\begin{cases} N_{i1}(t) \stackrel{\Delta}{=} (B(t-1) - 0.5K_i(t)U_{i3}(t))\, Q_w U_{i3}^{\mathrm{T}}(t) \\ N_{i2}(t) \stackrel{\Delta}{=} -0.5K_i(t)D_i(t)Q_{v_i}D_i^{\mathrm{T}}(t) \\ N_{i3}(t) \stackrel{\Delta}{=} 0.5K_i(t)D_i(t-1)Q_{v_i}D_i^{\mathrm{T}}(t-1). \end{cases} \tag{22}$$

At the same time, with the fact that $\overline{\Psi}_i(t) \geq \Psi_i(t)$ for all $\Delta_i(t)$, the local estimation error covariance $\overline{\Sigma}_i(t)$ obtained from (17) satisfying:

$$\begin{aligned} \operatorname{Tr}\left(\overline{\Sigma}_i(t)\right) &= \operatorname{Tr}\left(\left[-I \;\; I\right] \overline{\Psi}_i(t) \left[-I \;\; I\right]^{\mathrm{T}}\right) \\ &\geq \operatorname{Tr}\left(\left[-I \;\; I\right] \Psi_i(t) \left[-I \;\; I\right]^{\mathrm{T}}\right) = \operatorname{Tr}\left(\mathrm{E}\left[(x(t) - \hat{x}_i(t))\,(x(t) - \hat{x}_i(t))^{\mathrm{T}}\right]\right) = \operatorname{Tr}(\Sigma_i(t)). \end{aligned} \tag{23}$$

Therefore, $\operatorname{Tr}(\overline{\Sigma}_i(t))$ is an upper bound of all the possible $\operatorname{Tr}(\Sigma_i(t))$ with different uncertain parameters. In this case, our optimization objective is $\operatorname{Tr}(\overline{\Sigma}_i(t))$, and $\partial \operatorname{Tr}\left\{\overline{\Sigma}_i(t)\right\} / \partial K_i(t)$ is calculated by

$$\begin{aligned} \frac{\partial \operatorname{Tr}\left\{\overline{\Sigma}_i(t)\right\}}{\partial K_i(t)} = &- F_i(t)\overline{\Sigma}_i(t-1)U_{i1}^{\mathrm{T}}(t) - F_i(t)M_i(t)U_{i1}^{\mathrm{T}}(t) + 0.5\zeta_i(t)K_i(t) \\ &- N_{i1}(t) - N_{i2}(t) + N_{i3}(t) + N_i(t)Z_{i2}(t)U_{i4}(t) + 0.5\eta_i(t)K_i(t). \end{aligned} \tag{24}$$

Let $\partial \operatorname{Tr}\left\{\overline{\Sigma}_i(t)\right\} / \partial K_i(t)$ equals to 0, the optimal $K_i(t)$ is obtained from (15). Then, $\overline{\Sigma}_i(t)$ at switching instant and nonswitching instant are derived by (17) and (18). ∎

From Theorem 1, the computation procedures for the DFKF can be summarized by Algorithm 1.

Algorithm 1. For given $\hat{x}_i(0)$, $\hat{x}(0)$, $\zeta_i(t)$ and $\eta_i(t)$

1: **for** $i := 1$ **to** L **do**
2:     **if** $y_i(t)y_i(t-1) < 0$ **then**
3:         Calculate estimator gain $K_i(t)$ by (15);
4:         Calculate local estimate $\hat{x}_i(t)$ by (9);
5:     **else**
6:         Calculate local estimate $\hat{x}_i(t)$ by (10);
7:     **end if**
8:     Calculate $\overline{\Psi}_i(t)$ by (18);
9:     Calculate a local estimation error $\overline{\Sigma}_i(t)$ by (17);
10: **end for**
11: Calculate the optimal coefficients $\omega_i(t)$ by (13);
12: Calculate fusion weight matrices $W_i(t)$ by (12);
13: Calculate distributed fusion estimate $\hat{x}(t)$ by (11);
14: Return to Step 1 and implement Steps 1–13 for calculating $\hat{x}_c(t+1)$.

Remark 3. Notice that the local estimate $\hat{x}_i(t)$ and local estimation error covariance $\overline{\Sigma}_i(t)$ are calculated by a recursive form. Compared with the moving horizon approach in the work of Battistelli et al,[6] the proposed local estimator does not need to run in a large window size. On the other hand, the optimization problem (13) is a constrained nonlinear optimization, and it can be directly solved by the function "fmincon" of MATLAB. Meanwhile, the calculation of $\overline{\Psi}_i(t)$ and $\overline{\Sigma}_i(t)$ are independent of the sequence of measurements, which implies that they can be obtained independently at the fusion center.

4 | SIMULATION RESULTS

Consider O_2 content estimation in arteries using only the noninvasive binary pulmonary measurements, where the framework of blood dynamic in the lung[9] is shown in Figure 1. According to the diffusion process described in this figure, the dynamic physiological model for the arterial O_2 content can be derived from the oxygen content equation and the alveolar gas equation as[8]:

$$x(t+1) = f\,x(t) + w(t) + U(t), \tag{25}$$

where $U(t) = (1-f)\,(1.34Hb + 0.003\,(c_1u(t) + c_2(t)e(t))) - f\mu$, and $c_2(t) = [1 - u(t)(1-RQ)]/RQ$. The state $x(t)$ is the arterial O_2 content, while $u(t)$ is the percent of O_2 in inhaled air (as input by clinicians). f represents the fraction of shunted blood, and is a patient condition related parameter. $e(t)$ is the partial pressure of exhaled CO_2 and can be measured directly. Meanwhile, the constant parameters are taken as population average values:[23] $c_1 = 714$ mmHg, $RQ = 0.8$, $\mu = 5\ mL/dL$, $Hb = 12\ g/dL$. Then, the patient condition related constant parameter is selected as $f = 0.2$. The sensed variable $z_i(t)$ is constructed by three other inputs: tidal volume, respiratory rate and peak inspiratory,[23] and it is proportional to the state:

$$y_i(t) = C_i^s x(t) + v_i(t), \tag{26}$$

where the constant coefficients are take as $C_i^s = 1, i = 1, 2, \ldots, L$. The system noises $w(t)$ and $v_i(t)$ are the Gaussian white noises with covariances 0.2 and 0.01, respectively. The O_2 content change process is monitored by 6 binary sensors, where the thresholds of these sensors are dependent on $u(t)$. Here, $u(t)$ is firstly set as 60% and all the thresholds are 134, 134.5, 135, 135.5, 136, and 136.5, then $u(t)$ is set as 90% and all the thresholds are 197.5, 198, 198.5, 199, 199.5, and 200.

In the simulation, the parameters $\zeta_i(t)$ and $\eta_i(t)$ are taken as the sum of a constant and the maximum diagonal element of the right terms in (16). By implementing Algorithm 1, the trajectories of state $x(t)$ and fusion estimation $\hat{x}(t)$ are plotted in Figure 2A, which shows that the proposed DFKF can estimate the O_2 content well. On the other hand, due to the random noises, the estimation performance is assessed by the mean square errors (MSEs) that are calculated by Monte Carlo Methods with an average of 200 runs. Then, the MSEs of the DFKF and the local estimators are plotted in Figure 2B. It is shown that the performance of the DFKF is much better than its local estimators. This is as expected for the fusion methods. Meanwhile, the MSEs of the DFKF, centralized fusion Kalman filter and moving horizon estimator are plotted in Figure 2C. The DFKF with a distributed fusion structure can largely reduce the communication burden and computational complexity with a slight performance degradation. It can be seen from Figure 2C that the DFKF, as expected, can achieve satisfactory performance as compared with its centralized form. Besides, it is also shown that the performance of the centralized fusion Kalman filter is extremely close to the centralized moving horizon estimation approach in the work of Battistelli et al.[6] Notice that the moving horizon estimation is a smoothing process that utilizes measurement data after the current instant, and the performance index and sliding window size of moving horizon estimation need to be selected to ensure the stability of the estimator. Here, the sliding window size is taken as 100, and the comparison of MSEs is started after 120 time step. On the contrary, the proposed centralized method only requires the measurement information at current instant and does not need performance index and sliding window when designing the estimator. Since the proposed centralized method is recursively calculated without solving the optimization problem (13), it also takes less computation cost as compared with the centralized moving horizon estimation method.

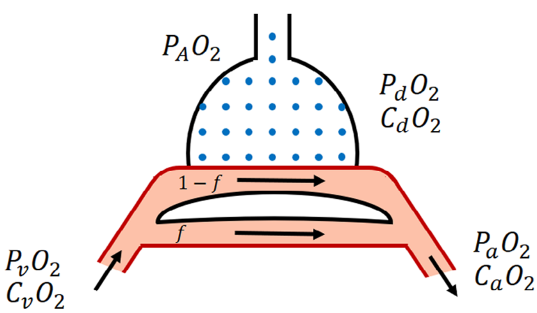

FIGURE 1 Framework of blood dynamic in the lung, where $P_AO_2, P_dO_2, P_vO_2, P_aO_2$ are respectively the partial pressures of Oxygen in alveoli, pulmonary capillaries, veins and arteries, while C_dO_2, C_vO_2, C_aO_2 are the corresponding concentration of Oxygen [Color figure can be viewed at wileyonlinelibrary.com]

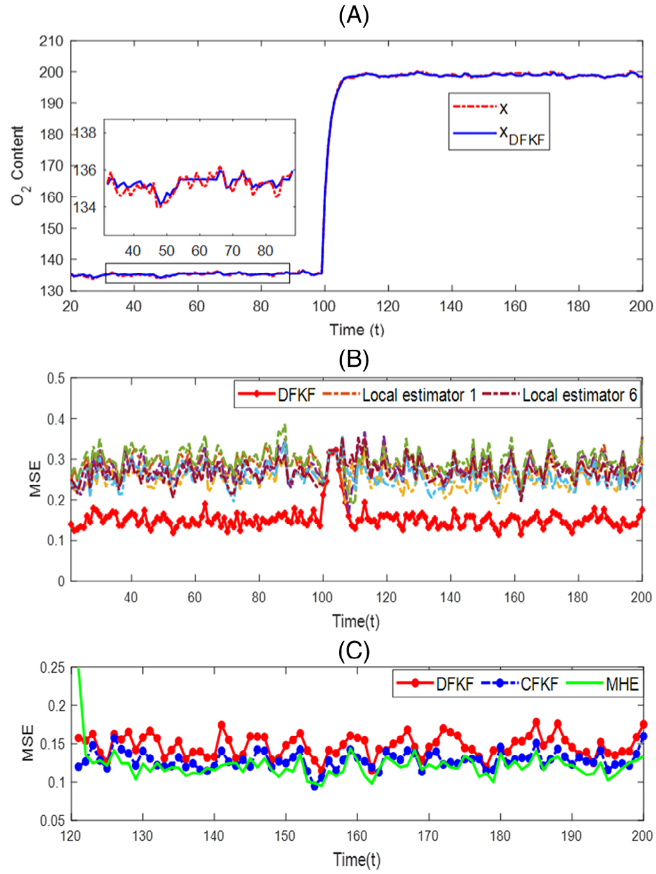

FIGURE 2 (A) Trajectories of $x(t)$ and $\hat{x}(t)$; (B) MSEs of the local estimators and the DFKF; (C) MSEs of the DFKF, centralized fusion Kalman filter (CFKF) and moving horizon estimator (MHE) [Color figure can be viewed at wileyonlinelibrary.com]

5 | CONCLUSIONS

In this paper, the distributed fusion Kalman filtering problem has been investigated for binary sensor systems. A novel uncertainty measurement model was constructed to better extract the valid information from binary outputs. By minimizing a local estimation error covariance and resorting to the CI fusion criterion, the recursive DFKF was designed for the binary sensor systems. Finally, the blood O_2 content estimation was used to demonstrate the effectiveness of the proposed methods. On the other hand, extending the proposed methods for nonlinear systems will be one of our future works. Meanwhile, how to design reliable fusion-based controllers under binary sensors is also interesting and challenging.

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SUPPORTING INFORMATION

Additional supporting information may be found online in the Supporting Information section at the end of this article.

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