Distributed Estimation for Interconnected Systems with Arbitrary Coupling Structures

Yuchen Zhang, *Member, IEEE*, Bo Chen, *Senior Member, IEEE*, Li Yu, *Senior Member, IEEE*, and Daniel W.C. Ho, *Life Fellow, IEEE*

***Abstract*—This paper is concerned with the problem of distributed estimation for time-varying interconnected systems with arbitrary coupling structures. A distributed stability condition requiring only the information from each subsystem and its neighbors is proposed to guarantee the stability of the designed distributed estimator. Then, a simplified condition without the real-time exchange of estimator gains is further proposed to reduce the communication burden. Under distributed stability constraints, the optimal estimator gain is designed by solving a convex optimization problem. Notice that the involved convex optimization problem can be easily solved by standard software packages because the constraints can be transformed into linear matrix inequalities. It is also shown that the designed distributed estimator is scalable for adding or subtracting subsystems. Finally, an illustrative example of a three-tank interconnected system is employed to show the effectiveness of the proposed methods.**

***Index Terms*—Time-varying interconnected system; distributed stability condition; distributed estimation; optimal estimator; convex optimization.**

I. Introduction

In recent decades, complex networks have received considerable attention in various fields, including physics, mathematics, engineering, biology, and sociology [1, 2]. One of the most essential focuses in complex networks is understanding, inferring, and intervening in complex dynamic systems' behavior. For example, the controllability and observability for complex systems have been discussed in [3–5], and the pinning control for network synchronization has been investigated in [1, 6, 7]. Interconnected systems are high-dimensional complex dynamic systems composed of numerous dispersed subsystems that can be state-coupled with their neighboring subsystems [8]. The past few decades have witnessed a surge of research interest in interconnected systems due primarily to their extensive applications in power systems [9], multi-robot systems [10], epidemic networks [11], and gene regulatory networks [12].

This work was supported in part by the National Natural Science Foundation of China under Grants 62073292 and 92367205; in part by the Key Research and Development Program of Zhejiang Province under Grant 2023C01144; in part by the Fundamental Research Funds for the Provincial Universities of Zhejiang under Grant RF-C2023007; and in part by the Research Grants Council of the Hong Kong Special Administrative Region, China, under Grants CityU 11203521 and 11213023.

Y. Zhang, B. Chen and L. Yu are with the Department of Automation, Zhejiang University of Technology, Hangzhou 310023, China (email: Yuchen-Zhang95@163.com, bchen@aliyun.com, lyu@zjut.edu.cn).

D. W. C. Ho is with the Department of Mathematics, City University of Hong Kong, Hong Kong, 999077 (email: madaniel@cityu.edu.hk).

Real-time state estimation is helpful to understand and infer interconnected systems' behavior. However, the increased complexity of system topologies and dynamics has prevented traditional estimation approaches from achieving satisfactory performance [13]. The reason is mainly attributed to the poor scalability of a centralized structure in traditional approaches. The centralized structure will suffer from heavy computational and communication burdens as well as a high field deployment cost if the number and dimension of subsystems are increased. Different distributed estimation approaches have been developed to reduce computing and communication overhead [14–30]. Most distributed estimation approaches for interconnected systems are based on special coupling structures. For example, a distributed moving horizon estimator was designed in [20] for sparse banded interconnected systems by the Chebyshev approximation of a centralized solution. Moreover, a distributed Kalman filter [21] and a consensus-based decentralized estimator [22] were designed for sparse systems by decomposing them into interconnected subsystems with coupled and overlapping states. Meanwhile, a sub-optimal distributed Kalman filtering problem was addressed in [23] for a class of sequentially interconnected systems. As for general interconnected systems without any structure constraints, the problem is more complicated due to the coupled estimation error iterations. By a completely local design, distributed estimators with a decoupling strategy were designed in [24–26] to deal with coupled estimation errors, while a moving horizon estimator was proposed in [27] with the assumption of uncorrelated local estimation errors. The distributed estimators with a plug-and-play fashion were developed in [28, 29] by exploiting the small-gain condition of the one-dimensional error norm system. Besides, a distributed zonotopic estimator with a fusion strategy was proposed in [30] for interconnected systems with overlapping states. However, it is still challenging to design distributed estimators with stability guarantees for general interconnected systems.

As important as it is in network synchronization, the stability problem for interconnected systems is also fundamental and has received much attention since the 1960s [31–35]. The traditional stability analysis for interconnected systems can be roughly divided into three categories: 1) methods based on scalar or vector Lyapunov functions [36–38]; 2) methods based on the small-gain theorem [39–41]; 3) methods based on the dissipativity theory [42, 43]. The above methods usually derive a centralized stability condition from a local analysis. For example, the M-matrix condition [38] can be derived from local Lyapunov stability conditions, and the small gain condition [41] can be obtained from local input-to-state stability conditions. The traditional analysis methods have two drawbacks that limit their application in distributed estimation and control. The first drawback is that the derived condition is more conservative than the centralized stability condition of the overall system. The second one is that these conditions are not scalable and require knowledge of overall systems' dynamics and couplings. How to design distributed stability conditions for interconnected systems is still an open question. The promising idea is to decompose the centralized stability conditions of the overall system into distributed ones by a subsystem-level analysis. A centralized dissipativity condition was decomposed into distributed conditions for individual subsystems in [44]. Note that these conditions require a considerable communication burden to exchange message matrices among subsystems and cannot be generalized to time-varying interconnected systems.

It should be pointed out that the distributed estimator designed in [25] only provides stability conditions for specific coupling structures, which are further interpreted as the structure of a directed acyclic graph in [26]. As for interconnected systems with arbitrary coupling structures, the problem of distributed estimation with stability guarantees has not yet been fully solved. Motivated by the above analysis, we shall investigate the distributed estimation problem for general interconnected systems. The problem will be addressed by introducing distributed stability conditions for designing an optimization-based distributed estimator. The main contributions of this paper can be summarized as follows:

- A distributed stability condition is proposed by a subsystem-level analysis for a centralized stability condition. Compared with existing stability conditions for interconnected systems [36–43], the proposed distributed stability condition requires only the information from each subsystem and its neighbors.
- A simplified distributed stability condition is proposed for a non-ideal communication environment. The proposed simplified condition avoids the real-time exchange of estimators' gain information, which is different from the conditions in [44] that require a considerable communication burden to exchange message matrices.
- An optimal and stable estimator is recursively designed for time-varying interconnected systems by solving an optimization problem with the proposed distributed stability constraints. Compared with the distributed estimators in [20–23], the designed estimator does not require any structural assumptions about the interconnected systems, such as sparse and sequential structures. The designed estimator also shows its advantages when interconnected systems cannot be decoupled by the approaches in [24–26].

Notation: The set of n-dimensional real vectors is denoted by $\mathbb{R}^n$. The identity matrix with appropriate dimensions is represented as '$\mathbf{I}$', and the matrix with all zero elements is denoted by '$\mathbf{0}$'. Define $\mathbb{N}_l := \{1, 2, ..., l\}$, where l is a natural number excluding zero. Given sets A and B, $A \setminus B$ represents the set of all elements of A that are not in B, and $A \cap B$ is the intersection set of A and B. The superscript 'T' represents the transpose, while the symmetric terms in a symmetric matrix are denoted by '$*$'. Meanwhile, the notations A^{-1}, $\mathrm{Tr}(A)$, $\|A\|_2$, and $\lambda_{\max}(A)$ are the inverse, trace, 2-norm, and maximum eigenvalue of matrix A, respectively. The notation $X \succ 0$ ($\prec 0$) is a positive definite (negative definite) matrix, and $X \succeq 0$ ($\preceq 0$) is a positive semi-definite (negative semi-definite) matrix. Given two matrices $X, Y \in \mathbb{R}^{n\times m}$, the notation $X \le Y$ means every element of X is not greater than the corresponding element of Y. The notation $\mathrm{col}\{a_1, ..., a_n\}$ means a column vector whose elements are $a_1, ..., a_n$, while $\mathrm{diag}\{\cdot\}$ stands for a block diagonal matrix. The (i, j)th element or block of a matrix A is represented by $A_{i,j}$, and the mathematical expectation is denoted by $\mathrm{E}\{\cdot\}$.

II. Problem Formulation

A. Time-varying Interconnected Systems

Consider a time-varying interconnected system $\mathbf{S}$ composed of l subsystems. The state and measurement dynamics of the ith subsystem $\mathbf{S}_i$ $(i \in \mathbb{N}_l)$ are described as follows:

$$\mathbf{S}_i : \begin{cases} x_i(k+1) = A_i(k)x_i(k) + \Gamma_i(k)w_i(k) \\ \qquad\qquad + \sum_{i_\omega \in \overline{\Omega}_i} A_{i,i_\omega}(k)x_{i_\omega}(k) \quad (i \in \mathbb{N}_l). \\ y_i(k) = C_i(k)x_i(k) + D_i(k)v_i(k) \end{cases} \tag{1}$$

where the vectors $x_i(k) \in \mathbb{R}^{n_i}$, $y_i(k) \in \mathbb{R}^{m_i}$, $w_i(k) \in \mathbb{R}^{n_{w_i}}$, and $v_i(k) \in \mathbb{R}^{n_{v_i}}$ are the state, measurement, system noise, and measurement noise of the subsystem $\mathbf{S}_i$, respectively. The bounded matrices $A_i(k)$, $\Gamma_i(k)$, $A_{i,i_\omega}(k)$, $C_i(k)$, and $D_i(k)$ have appropriate dimensions. Meanwhile, the sets of ith subsystem's in-neighbors, out-neighbors, and neighbors are denoted by $\overline{\Omega}_i$, $\underline{\Omega}_i$, and Ω_i, respectively. The number of elements in these sets is $\overline{\theta}_i$, $\underline{\theta}_i$, and θ_i. In addition, let us denote upper bounds $\|A_i(k)\|_2 \le \alpha_{i,i} = \alpha_i$ and $\|A_{i,i_\omega}(k)\|_2 \le \alpha_{i,i_\omega}$, and set $\alpha_{i,j} = 0$ $\left(j \notin \overline{\Omega}_i \cap \{i\}\right)$.

Assumption 1. The noises $w_i(k)$ and $v_i(k)$ are uncorrelated Gaussian white noises satisfying

$$\begin{cases} \mathrm{E}\left[w_i(k)w_j(k_1)\right] = \delta_{i,j}\delta_{k,k_1}Q_{w_i} \\ \mathrm{E}\left[v_i(k)v_j(k_1)\right] = \delta_{i,j}\delta_{k,k_1}Q_{v_i} \\ \mathrm{E}\left[w_i(k)v_j(k_1)\right] = \mathbf{0} \quad (\forall i, j \in \mathbb{N}_l, k \ge k_0, k_1 \ge k_0) \end{cases} \tag{2}$$

where Q_{w_i} and Q_{v_i} are the known covariances of $w_i(k)$ and $v_i(k)$, respectively. The notation k_0 is the initial instant, while $\delta_{k,k_1} = 0$ if $k \ne k_1$ and $\delta_{k,k_1} = 1$ otherwise.

Assumption 2. Each subsystem knows only its own state and measurement dynamics and can communicate with its neighbors.

Remark 1. The considered interconnected systems with time-varying dynamics and couplings are more general and have wider applications. Taking blocked power systems as an example, the couplings among different blocks change with real-time power dispatching [9, 45]. Another example of heavy-duty vehicle systems with aerodynamic interconnections [46] can also be modeled as an interconnected system (1). Compared with the other models [20–23, 47], the interconnected

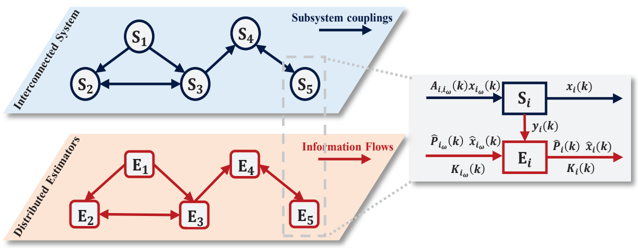


Fig. 1. The structure diagram of an interconnected system with a distributed estimator.

system (1) does not require any structural assumptions, such as sparse and sequential structures. On the other hand, the distributed communication structure in Assumption 2 is more practical than the centralized one due to the limitations of network bandwidth and device energy.

B. Problem of Interest

It is known from Assumption 2 that subsystems can communicate with their neighbors to collaboratively estimate real states. The following distributed estimator $\mathbf{E}_i$ for the ith subsystem (see Fig. 1 for its structure) is proposed:

$$\mathbf{E}_i : \begin{cases} \hat{x}_i^p(k) = A_i(k-1)\hat{x}_i(k-1) \\ \qquad\qquad + \sum_{i_\omega \in \overline{\Omega}_i} A_{i,i_\omega}(k-1)\hat{x}_{i_\omega}(k-1) \\ \hat{x}_i(k) = \hat{x}_i^p(k) + K_i(k)\left[y_i(k) - C_i(k)\hat{x}_i^p(k)\right] \end{cases} \tag{3}$$

where $\hat{x}_i^p(k)$ and $\hat{x}_i(k)$ are the one-step prediction and estimate of the state $x_i(k)$, respectively. Define $\widehat{K}_i(k) := \mathbf{I} - K_i(k)C_i(k)$, then the estimation error iteration and the error covariance iteration for the ith subsystem are derived from (1) and (3) as

$$\begin{aligned} \tilde{x}_i^p(k) =& A_i(k-1)\tilde{x}_i(k-1) + \Gamma_i(k-1)w_i(k-1) \\ &+ \sum_{i_\omega \in \overline{\Omega}_i} A_{i,i_\omega}(k-1)\tilde{x}_{i_\omega}(k-1) \end{aligned} \tag{4a}$$

$$\tilde{x}_i(k) = \widehat{K}_i(k)\tilde{x}_i^p(k) - K_i(k)D_i(k)v_i(k) \tag{4b}$$

$$\begin{aligned} P_i^p(k) =& A_i(k-1)P_i(k-1)A_i^{\mathrm{T}}(k-1) \\ &+ \Gamma_i(k-1)Q_{w_i}\Gamma_i^{\mathrm{T}}(k-1) \\ &+ \sum\nolimits_{i_\omega \in \overline{\Omega}_i} A_i(k-1)P_{i,i_\omega}(k-1)A_{i,i_\omega}^{\mathrm{T}}(k-1) \\ &+ \sum\nolimits_{i_\omega \in \overline{\Omega}_i} A_{i,i_\omega}(k-1)P_{i_\omega,i}(k-1)A_i^{\mathrm{T}}(k-1) \\ &+ \sum_{i_\omega^1 \in \overline{\Omega}_i} \sum_{i_\omega^2 \in \overline{\Omega}_i} A_{i,i_\omega^1}(k-1)P_{i_\omega^1,i_\omega^2}(k-1)A_{i,i_\omega^2}^{\mathrm{T}}(k-1) \end{aligned} \tag{4c}$$

$$\begin{aligned} P_i(k) =& \widehat{K}_i(k)P_i^p(k)\widehat{K}_i^{\mathrm{T}}(k) \\ &+ K_i(k)D_i(k)Q_{v_i}D_i^{\mathrm{T}}(k)K_i^{\mathrm{T}}(k). \end{aligned} \tag{4d}$$

Here, the vectors $\tilde{x}_i^p(k) := x_i(k) - \hat{x}_i^p(k)$ and $\tilde{x}_i(k) := x_i(k) - \hat{x}_i(k)$ are the one-step prediction error and estimation error, respectively. The matrices $P_i^p(k) := \mathrm{E}\left\{\tilde{x}_i^p(k)\left[\tilde{x}_i^p(k)\right]^{\mathrm{T}}\right\}$, $P_i(k) := \mathrm{E}\left\{\tilde{x}_i(k)\tilde{x}_i^{\mathrm{T}}(k)\right\}$, and $P_{i,j}(k) := \mathrm{E}\{\tilde{x}_i(k)\tilde{x}_j^{\mathrm{T}}(k)\}$ are the one-step prediction error covariance, estimation error covariance, and cross-covariance, respectively.

The primary concern of the distributed estimation problem is to design a suitable gain matrix $K_i(k)$ $(i \in \mathbb{N}_l)$ such that the estimation error system (4a-4b) is stable and the selected estimation performance index $J_i(k)$ about $P_i(k)$ is minimized. If the performance index for the local estimation is designed as $J_i(k) = \mathrm{Tr}\{P_i(k)\}$, then the optimal estimator gain $K_i^{\mathrm{opt}}(k)$ can be searched by solving an optimization problem:

$$K_i^{\mathrm{opt}}(k) = \arg \min_{K_i(k) \in \mathcal{K}_i(k)} \mathrm{Tr}\{P_i(k)\} \tag{5}$$

where $\mathcal{K}_i(k)$ is a subspace of stable estimator gains for subsystem $\mathbf{S}_i$. It is usually difficult for subsystems to timely obtain the cross-covariance $P_{i,j}(k)$ used in (4c) by only local communication. Therefore, an upper bound of $\mathrm{Tr}\{P_i(k)\}$ is required to improve the optimization problem (5). It is also known from (4a-4b) that the stability of each local estimator depends on the stability of its in-neighbors' estimators due to the interconnected estimation error term $\tilde{x}_{i_\omega}(k-1)$. Hence, it is difficult to determine $\mathcal{K}_i(k)$ and design a stable estimator by a local analysis. On the other hand, a centralized analysis needs the overall system's knowledge about its dynamics and couplings, which is not suitable for large-scale interconnected systems.

Consequently, the problems to be solved in this paper are described as follows:

1) Derive distributed stability conditions for the proposed estimator (3) such that only subsystem-level knowledge of dynamics and couplings is required.
2) Design a distributed, stable, and optimal estimator for time-varying interconnected systems using the proposed distributed stability conditions.

Remark 2. To design a fully distributed estimator, both the iteration form and the stability conditions for the estimator need to achieve local communication, computation, and storage. Though the estimator (3) only uses the information of local measurement and neighbors' estimates, the local estimation errors in (4b) are still interconnected. Therefore, the major difficulty of the distributed estimator design for general interconnected systems is calculating the optimal estimator gain and maintaining local stability for each subsystem without any global information about estimation errors.

III. Main Results

In this section, two distributed stability conditions will be derived by a subsystem-level analysis for a centralized condition, and then a distributed, stable, and optimal estimator will be designed by optimization-based methods.

A. Centralized Stability Condition

To introduce the overall system dynamics and overall estimator iteration, let us define $x(k) := \mathrm{col}\{x_1(k), ..., x_l(k)\}$, $y(k) := \mathrm{col}\{y_1(k), ..., y_l(k)\}$, $w(k) := \mathrm{col}\{w_1(k), ..., w_l(k)\}$, and $v(k) := \mathrm{col}\{v_1(k), ..., v_l(k)\}$. Then, the dynamics of the overall system $\mathbf{S}$ can be obtained as

$$\begin{cases} x(k+1) = A(k)x(k) + \Gamma(k)w(k) \\ y(k) = C(k)x(k) + D(k)v(k). \end{cases} \tag{6}$$

Here, $A(k)$ is a block matrix with $[A(k)]_{i,j} := A_{i,j}(k)$ $(i,j \in \mathbb{N}_l)$, where $A_{i,i}(k) := A_i(k)$ and $A_{i,j} = \mathbf{0}$ $(j \notin \overline{\Omega}_i \cap \{i\})$. Other matrices are defined by

$$\begin{cases} \Gamma(k) := \mathrm{diag}\{\Gamma_1(k), ..., \Gamma_l(k)\} \\ C(k) := \mathrm{diag}\{C_1(k), ..., C_l(k)\} \\ D(k) := \mathrm{diag}\{D_1(k), ..., D_l(k)\}. \end{cases} \tag{7}$$

Let us denote $\hat{x}(k) := \mathrm{col}\{\hat{x}_1(k), ..., \hat{x}_l(k)\}$, $\tilde{x}(k) := \mathrm{col}\{\tilde{x}_1(k), ..., \tilde{x}_l(k)\}$, and $P(k) := \mathrm{E}\left\{\tilde{x}(k)\tilde{x}^{\mathrm{T}}(k)\right\}$. Then, the overall estimator, overall estimation error iteration, and overall estimation error covariance are obtained as

$$\begin{aligned} \hat{x}(k) =& A(k-1)\hat{x}(k-1) \\ &+ K(k)\left[y(k) - C(k)A(k-1)\hat{x}(k-1)\right] \end{aligned} \tag{8a}$$

$$\begin{aligned} \tilde{x}(k) =& \widehat{K}(k)A(k-1)\tilde{x}(k-1) - K(k)D(k)v(k) \\ &+ \widehat{K}(k)\Gamma(k-1)w(k-1) \end{aligned} \tag{8b}$$

$$P(k) = \widehat{K}(k)A(k-1)P(k-1)A^{\mathrm{T}}(k-1)\widehat{K}^{\mathrm{T}}(k) + H(k) \tag{8c}$$

where $\widehat{K}(k) := \mathbf{I} - K(k)C(k)$, $H(k) := \widehat{K}(k)\Gamma(k-1)Q_w\Gamma^{\mathrm{T}}(k-1)\widehat{K}^{\mathrm{T}}(k) + K(k)D(k)Q_vD^{\mathrm{T}}(k)K^{\mathrm{T}}(k)$, and

$$\begin{cases} K(k) := \mathrm{diag}\{K_1(k), ..., K_l(k)\} \\ Q_w := \mathrm{diag}\{Q_{w_1}, ..., Q_{w_l}\} \\ Q_v := \mathrm{diag}\{Q_{v_1}, ..., Q_{v_l}\}. \end{cases} \tag{9}$$

The following definition and lemma are provided to describe the stability property for the overall estimator in (8a).

Definition 1 [38]. For the interconnected system (1), the overall estimator (8a) is mean-square uniformly ultimately bounded (mean-square UUB) if, for arbitrarily large $\delta_0 > 0$, there is $\delta(\delta_0) > 0$ (independent of k_0) such that

$$\|P(k_0)\|_2 \le \delta_0 \quad \Rightarrow \quad \lim_{k\to\infty} \|P(k)\|_2 \le \delta. \tag{10}$$

Lemma 1. The overall estimator (8a) is mean-square UUB as (10) if the following centralized stability condition is satisfied:

$$\left\|\widehat{K}(k)A(k-1)\right\|_2 < 1. \tag{11}$$

Proof: It can be derived from (8c) that

$$\begin{aligned} P(k) =& \left(\prod_{\mu=0}^{k-k_0-1} \widetilde{K}(k-\mu)\right) P(k_0) \left(\prod_{\mu=0}^{k-k_0-1} \widetilde{K}(k-\mu)\right)^{\mathrm{T}} \\ &+ \sum_{\tau=0}^{k-k_0-1} \left\{ \left(\prod_{\mu=0}^{\tau-1} \widetilde{K}(k-\mu)\right) H(k-\mu) \right. \\ &\left. \times \left(\prod_{\mu=0}^{\tau-1} \widetilde{K}(k-\mu)\right)^{\mathrm{T}} \right\} \end{aligned} \tag{12}$$

where $\widetilde{K}(k) := \widehat{K}(k)A(k-1)$. Then, one has that

$$\begin{aligned} \|P(k)\|_2 \le& \left(\prod_{\mu=0}^{k-k_0-1} \left\|\widetilde{K}(k-\mu)\right\|_2^2\right) \|P(k_0)\|_2 \\ &+ \sum_{\tau=0}^{k-k_0-1} \left\{ \left(\prod_{\mu=0}^{\tau-1} \left\|\widetilde{K}(k-\mu)\right\|_2^2\right) \|H(k-\mu)\|_2 \right\}. \end{aligned} \tag{13}$$

If the condition (11) is satisfied, it turns out that

$$\begin{cases} \lim\limits_{k\to\infty} \prod\limits_{\mu=0}^{k-k_0-1} \left\|\widetilde{K}(k-\mu)\right\|_2^2 = 0 \\ \lim\limits_{\substack{k\to\infty \\ \mu\to\infty}} \prod\limits_{\mu=0}^{\tau-1} \left\|\widetilde{K}(k-\mu)\right\|_2^2 = 0. \end{cases} \tag{14}$$

The boundedness of $\|H(k)\|_2$ follows from (11) and the boundedness of system matrices. Therefore, $\lim_{k\to\infty} \|P(k)\|_2$ is bounded.

B. Distributed Stability Conditions

The centralized stability condition (11) needs the overall system's knowledge, so it is hard to be implemented with a distributed communication structure. The following theorem provides a distributed condition to ensure stability for local estimators.

Theorem 1. The proposed distributed estimator (3) is mean-square UUB if the following distributed stability condition is satisfied $\forall i \in \mathbb{N}_l$ and $\forall i_\rho \in \Omega_i$:

$$\left\|\widehat{K}_i(k)A_i(k-1)\right\|_2 < 1 \tag{15a}$$

$$\pi_{i,i_\rho}\pi_{i_\rho,i}N_i(k) - N_{i,i_\rho}(k)N_{i_\rho}^{-1}(k)N_{i,i_\rho}^{\mathrm{T}}(k) \prec 0 \tag{15b}$$

where parameter $\pi_{i,i_\rho} \ge 0$ satisfies $\sum_{i_\rho\in\Omega_i} \pi_{i,i_\rho} = 1$, and

$$\begin{cases} N_i(k) := \begin{bmatrix} -\mathbf{I} & \widehat{K}_i(k)A_i(k-1) \\ * & -\mathbf{I} \end{bmatrix} \\ N_{i,i_\rho}(k) := \begin{bmatrix} \mathbf{0} & \widehat{K}_i(k)A_{i,i_\rho}(k-1) \\ A_{i_\rho,i}^{\mathrm{T}}(k-1)\widehat{K}_{i_\rho}^{\mathrm{T}}(k) & \mathbf{0} \end{bmatrix}. \end{cases} \tag{16}$$

Proof: By the Schur complement lemma [48], the inequality (15a) is equivalent to $N_i(k) \prec 0$, and the condition (15) can be transformed into the following inequality:

$$M_{i,i_\rho}(k) := \begin{bmatrix} \pi_{i,i_\rho}N_i(k) & N_{i,i_\rho}(k) \\ * & \pi_{i_\rho,i}N_{i_\rho}(k) \end{bmatrix} \prec 0. \tag{17}$$

Define a permutation matrix as

$$\mathcal{Q} := \begin{bmatrix} \mathbf{I}_{n_i} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I}_{n_i} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{n_{i_\rho}} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I}_{n_{i_\rho}} \end{bmatrix}. \tag{18}$$

By left and right multiplication of $M_{i,i_\rho}(k)$ with $\mathcal{Q}$ and $\mathcal{Q}^{\mathrm{T}}$, then the following inequality is derived:

$$\widehat{M}_{i,i_\rho}(k) := \begin{bmatrix} U_{i,i_\rho} & V_{i,i_\rho}(k) \\ * & U_{i,i_\rho} \end{bmatrix} \prec 0 \tag{19}$$

where

$$\begin{aligned} U_{i,i_\rho} &:= \begin{bmatrix} -\pi_{i,i_\rho}\mathbf{I} & \mathbf{0} \\ \mathbf{0} & -\pi_{i_\rho,i}\mathbf{I} \end{bmatrix} \\ V_{i,i_\rho}(k) &:= \begin{bmatrix} \pi_{i,i_\rho}\widehat{K}_i(k)A_i(k-1) & \widehat{K}_i(k)A_{i,i_\rho}(k-1) \\ \widehat{K}_{i_\rho}(k)A_{i_\rho,i}(k-1) & \pi_{i_\rho,i}\widehat{K}_{i_\rho}(k)A_{i_\rho}(k-1) \end{bmatrix}. \end{aligned} \tag{20}$$

Notice that the set $\left\{\widehat{M}_{i,i_\rho}(k) \mid i \in \mathbb{N}_l, i_\rho \in \Omega_i\right\}$ is equivalent to the set $\left\{\widehat{M}_{i,i_\rho^o}(k) \mid i \in \mathbb{N}_l, i_\rho^o \in \Omega_i \setminus \mathbb{N}_i\right\}$, which can be augmented as

$$\begin{aligned}\widehat{M}(k) := \operatorname{diag}\Big\{&\widehat{M}_{1,1_1^o}(k), \cdots, \widehat{M}_{1,1_{\xi_1}^o}(k),\\ &\widehat{M}_{2,2_1^o}(k), \cdots, \widehat{M}_{2,2_{\xi_2}^o}(k), \cdots\Big\} \prec 0\end{aligned} \tag{21}$$

where ξ_i is the number of elements of $\Omega_i \setminus \mathbb{N}_i$. Then, define the following matrix:

$$\mathcal{R} := \operatorname{row}\left\{e_{1,1_1^o}, \cdots, e_{1,1_{\xi_1}^o}, e_{2,2_1^o}, \cdots, e_{2,2_{\xi_2}^o}, \cdots\right\} \tag{22}$$

where $e_{i,j} := \begin{bmatrix} e_i & e_j & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & e_i & e_j \end{bmatrix}$, and e_i is a matrix with dimension $n \times n_i$ that contains all zero elements, but an identity matrix of dimension n_i at rows $\sum_{j=1}^{i-1} n_j + 1$ through $\sum_{j=1}^{i} n_j$. By the property of positive definite matrix, if $\widehat{M}(k) \prec 0$, then the matrix by left and right multiplication of $\widehat{M}(k)$ with $\mathcal{R}$ and $\mathcal{R}^{\mathrm{T}}$ is also negative definite, i.e.,

$$\mathcal{R}\widehat{M}(k)\mathcal{R}^{\mathrm{T}} = \begin{bmatrix} -\mathbf{I} & \widehat{K}(k)A(k-1) \\ * & -\mathbf{I} \end{bmatrix} \prec 0. \tag{23}$$

According to Schur complement lemma, one has that

$$\left\|\widehat{K}(k)A(k-1)\right\|_2 < 1. \tag{24}$$

By Lemma 1, the inequality (24) is sufficient to ensure the ultimate boundedness of $\|P(k)\|_2$ and $\|P_i(k)\|_2$.

Remark 3. Intuitively, the stability of an independent subsystem is not affected by other subsystems, so it only needs to keep local mean-square UUB as in (15a). As for subsystems coupled with neighbors, the stability condition will be tighter due to the coupling terms. Therefore, the distributed stability condition in Theorem 1 contains two parts: the constraint (15a) ensures that the local estimation error system without interconnected terms is stable, while the constraint (15b) is an additional requirement for the stability of systems with coupling relationships.

Remark 4. The main idea of Theorem 1 is to derive the distributed stability condition (15) by a subsystem-level analysis for the centralized stability condition (11). It can be extended to derive other distributed stability conditions.

- For the time-varying interconnected system (1) without disturbance, the overall system dynamics can be represented as $x(k+1) = A(k)x(k)$. If there exists a sequence of diagonal positive definite matrices $P(k) = \operatorname{diag}\{P_1(k), P_2(k), \cdots, P_l(k)\}$ such that $A^T(k)P(k+1)A(k) - P(k) \prec 0$, then the system is Lyapunov stable [49, Th. 23-3]. This centralized Lyapunov stability condition leads to the following distributed condition:

$$A_i^T(k)P_i(k+1)A_i(k) - P_i(k) \prec 0 \tag{25a}$$
$$\pi_{i,i_\rho}\pi_{i_\rho,i}\hat{N}_i(k) - \hat{N}_{i,i_\rho}(k)\hat{N}_{i_\rho}^{-1}(k)\hat{N}_{i,i_\rho}^{\mathrm{T}}(k) \prec 0 \tag{25b}$$

where

$$\begin{cases} \hat{N}_i(k) := \begin{bmatrix} -P_i(k+1) & P_i(k+1)A_i(k) \\ * & -P_i(k) \end{bmatrix} \\ \hat{N}_{i,i_\rho}(k) := \begin{bmatrix} \mathbf{0} & P_i(k+1)A_{i,i_\rho}(k) \\ A_{i_\rho,i}^{\mathrm{T}}(k)P_{i_\rho}(k+1) & \mathbf{0} \end{bmatrix} \end{cases}. \tag{26}$$

- For a time-invariant interconnected system with local inputs, the overall system dynamics can be represented as $x(k+1) = Ax(k) + Bu(k)$ and $y(k) = x(k)$. If there exist diagonal matrices $Q = Q^{\mathrm{T}} = \operatorname{diag}\{Q_1, Q_2, \cdots, Q_l\}$, $S = \operatorname{diag}\{S_1, S_2, \cdots, S_l\}$, $R = R^{\mathrm{T}} = \operatorname{diag}\{R_1, R_2, \cdots, R_l\}$, and $P = \operatorname{diag}\{P_1, P_2, \cdots, P_l\} \succ 0$ such that

$$\begin{bmatrix} -P & \mathbf{0} & PA \\ * & -R + B^{\mathrm{T}}PB & B^{\mathrm{T}}PA - S^{\mathrm{T}} \\ * & * & -P - Q \end{bmatrix} \preceq 0 \tag{27}$$

then the system is (Q, S, R)-dissipative [50]. This centralized dissipativity condition leads to the following distributed condition:

$$\bar{N}_i \preceq 0 \tag{28a}$$
$$\pi_{i,i_\rho}\pi_{i_\rho,i}\bar{N}_i - \bar{N}_{i,i_\rho}\bar{N}_{i_\rho}^{-1}\bar{N}_{i,i_\rho}^{\mathrm{T}} \preceq 0 \tag{28b}$$

where

$$\begin{cases} \bar{N}_i := \begin{bmatrix} -P_i & \mathbf{0} & P_iA_i \\ * & -R_i + B_i^{\mathrm{T}}P_iB_i & B_i^{\mathrm{T}}P_iA_i - S_i^{\mathrm{T}} \\ * & * & -P_i - Q_i \end{bmatrix} \\ \bar{N}_{i,i_\rho} := \begin{bmatrix} \mathbf{0} & \mathbf{0} & P_iA_{i,i_\rho} \\ \mathbf{0} & \mathbf{0} & B_i^{\mathrm{T}}P_iA_{i,i_\rho} \\ A_{i_\rho,i}^{\mathrm{T}}P_{i_\rho} & A_{i_\rho,i}^{\mathrm{T}}P_{i_\rho}B_i & \mathbf{0} \end{bmatrix} \end{cases}. \tag{29}$$

Remark 5. The distributed stability condition (15) is slightly more conservative than the centralized stability condition (11). The conservatism is mainly attributed to the derivation step (23) from $\widehat{M}(k) \prec 0$ to $\mathcal{R}\widehat{M}(k)\mathcal{R}^{\mathrm{T}} \prec 0$. However, the adjustable parameter $\pi_{i,j}$ is introduced in (15) to reduce conservatism. By properly selecting these parameters, the conservatism caused by the above step can be mitigated. On the other hand, the conditions from traditional stability analysis of interconnected systems are more conservative than the centralized conditions of the overall system. For example, the M-matrix condition [38] and the small-gain condition [41] for a linear time-invariant system are more conservative than the centralized Lyapunov condition ($A^TPA - P \prec 0$). In this sense, the subsystem-level analysis in Theorem 1 is not as conservative as the traditional analysis [36–43].

In Theorem 1, subsystem $\mathbf{S}_i$ needs to know the gain matrix $K_{i_\rho}(k)$ from subsystem $\mathbf{S}_{i_\rho}$ timely. However, it is usually difficult to exchange information among subsystems in real time if the communication environment is non-ideal, such as a wireless communication environment. In the following lemma, a distributed stability condition without synchronously knowing neighbors' gain information is further proposed.

This article has been accepted for publication in IEEE Transactions on Network Science and Engineering. This is the author's version which has not been fully edited and content may change prior to final publication. Citation information: DOI 10.1109/TNSE.2024.3382367

Lemma 2. The distributed estimator (3) is mean-square UUB if the following inequality is satisfied:

$$\left\|\widehat{K}_i(k)\right\|_2 \le \beta_i \quad (\forall i \in \mathbb{N}_l) \tag{30}$$

where $\beta_i < \frac{1}{\alpha_i}$ is constrained by:

$$\pi_{i,i_\rho}\pi_{i_\rho,i}\left(\alpha_i\beta_i - 1\right)\left(\alpha_{i_\rho}\beta_{i_\rho} - 1\right) > \alpha_{i,i_\rho}^2\beta_i^2 \quad (\forall i_\omega \in \Omega_i). \tag{31}$$

Proof: By (30), the following upper bounds can be derived:

$$\left\|\widehat{K}_i(k)A_i(k-1)\right\|_2 \le \alpha_i\beta_i < 1 \tag{32a}$$

$$\left\|\widehat{K}_i(k)A_{i,i_\rho}(k-1)\right\|_2 \le \alpha_{i,i_\rho}\beta_i. \tag{32b}$$

According to the Schur complement lemma, the inequality (32a) can be converted into

$$\begin{bmatrix} -\alpha_i\beta_i\mathbf{I} & \widehat{K}_i(k)A_i(k-1) \\ * & -\alpha_i\beta_i\mathbf{I} \end{bmatrix} \preceq 0. \tag{33}$$

The inequality (33) means that $N_i(k)+(1-\alpha_i\beta_i)\mathbf{I} \preceq 0$. Notice that the matrix $N_i(k)$ is invertible, so it is found that

$$\lambda_{\max}(N_i(k)) \le \alpha_i\beta_i - 1 < 0 \tag{34a}$$

$$\lambda_{\min}\left(N_i^{-1}(k)\right) \ge \frac{1}{\alpha_i\beta_i - 1}. \tag{34b}$$

It follows from (32b) that

$$N_{i,i_\rho}(k)N_{i,i_\rho}^{\mathrm{T}}(k) - \begin{bmatrix} \alpha_{i,i_\rho}^2\beta_i^2\mathbf{I} & \mathbf{0} \\ \mathbf{0} & \alpha_{i_\rho,i}^2\beta_{i_\rho}^2\mathbf{I} \end{bmatrix} \preceq 0. \tag{35}$$

Then, it can be concluded that

$$\begin{aligned} &\left(\pi_{i,i_\rho}\pi_{i_\rho,i}N_i(k) - N_{i,i_\rho}(k)N_{i_\rho}^{-1}(k)N_{i,i_\rho}^{\mathrm{T}}(k)\right) - \\ &\left(\pi_{i,i_\rho}\pi_{i_\rho,i}\left(\alpha_i\beta_i - 1\right)\mathbf{I} - \begin{bmatrix} \frac{\alpha_{i,i_\rho}^2\beta_i^2}{\alpha_{i_\rho}\beta_{i_\rho}-1}\mathbf{I} & \mathbf{0} \\ \mathbf{0} & \frac{\alpha_{i_\rho,i}^2\beta_{i_\rho}^2}{\alpha_{i_\rho}\beta_{i_\rho}-1}\mathbf{I} \end{bmatrix}\right) \preceq 0. \end{aligned} \tag{36}$$

Therefore, (15a) follows from $\beta_i < \frac{1}{\alpha_i}$, and (15b) can be derived by (36) and the constraint (31). The property of stability follows from Theorem 1.

The selection of parameters $\pi_{i,j}$ and β_i should balance the stability margin of each subsystem. It is proposed to select $\pi_{i,j}$ to be proportional to the size of couplings, and one of the possible selections is given by

$$\pi_{i,j} = \frac{\alpha_{i,j} + \alpha_{j,i}}{\sum_{i_\rho\in\Omega_i}\left(\alpha_{i,i_\rho} + \alpha_{i_\rho,i}\right)}. \tag{37}$$

At the same time, the parameter β_i can be allocated offline according to (31). The distributed allocation of β_i can be implemented in Algorithm 1.

Algorithm 1 Distributed Allocation of β_i

for $i := 1$ **to** l **do**
  if $i \neq 1$ **then**
    Subsystem $\mathbf{S}_i$ receives $\beta_{i_\rho^s}$ and $\alpha_{i_\rho^s}$ from subsystem $\mathbf{S}_{i_\rho^s}$ ($i_\rho^s \in \Omega_i \cap \mathbb{N}_{i-1}$);
  end if
  Subsystem $\mathbf{S}_i$ determines β_i that satisfies (31) $\forall i_\rho^s \in \Omega_i \cap \mathbb{N}_{i-1}$;
  Subsystem $\mathbf{S}_i$ sends α_i and the allocated β_i to subsystem $\mathbf{S}_{i_\rho^o}$ ($i_\rho^o \in \Omega_i \setminus \mathbb{N}_i$).
end for

Remark 6. The distributed stability condition (15) is simplified into finding an appropriate time-invariant parameter β_i for each subsystem in Lemma 2. The determination of β_i in Algorithm 1 only needs subsystems to communicate with their neighbors to exchange the knowledge of β_{i_ρ} and α_{i_ρ}. Compared with the results in [44] that require a considerable communication burden to exchange message matrices among subsystems, the proposed condition (30) can be achieved offline with less communication overhead. Therefore, the stability condition (30) with less communication burden is more suitable for interconnected systems with time-varying couplings and dynamics.

C. Optimization-based Distributed Estimator

Then, an optimization-based distributed estimator will be designed. Let us define the following matrices:

$$\begin{cases} \mathcal{D}_{P_i}(k) := \mathrm{col}\left\{\sqrt{[P_i(k)]_1}, \ldots, \sqrt{[P_i(k)]_{n_i}}\right\} \\ \mathcal{D}_{\hat{P}_i}(k) := \mathrm{col}\left\{\sqrt{\left[\hat{P}_i(k)\right]_1}, \ldots, \sqrt{\left[\hat{P}_i(k)\right]_{n_i}}\right\}. \end{cases} \tag{38}$$

where $[P_i(k)]_\tau$ and $\left[\hat{P}_i(k)\right]_\tau$ are the τth diagonal element of $P_i(k)$ and $\hat{P}_i(k)$, respectively. The optimal estimator gain is designed in the following theorem.

Theorem 2. For the distributed estimator (3), the optimal gain matrix $K_i^{\mathrm{opt}}(k)$ can be determined by solving the following optimization problem:

$$\begin{aligned} &\min_{K_i(k)} \mathrm{Tr}\left\{\hat{G}_i(k)\right\} \\ &\text{s.t.} \begin{cases} \begin{bmatrix} -\hat{G}_i(k) & \widehat{K}_i(k)\hat{P}_i^p(k) & K_i(k)D_i(k)Q_{v_i} \\ * & -\hat{P}_i^p(k) & \mathbf{0} \\ * & * & -Q_{v_i} \end{bmatrix} \prec 0 \\ (15) \ \text{ or } \ (30) \end{cases} \end{aligned} \tag{39}$$

where $\mathrm{Tr}\left\{\hat{G}_i(k)\right\}$ is an upper bound of the trace of estimation error covariance, and (15) and (30) are the stability constraints for ideal and non-ideal communication environments, respectively. Let us denote $\widehat{K}_i^{\mathrm{opt}}(k-1) := \mathbf{I} - K_i^{\mathrm{opt}}(k-1)C_i(k-1)$, then the matrices $\hat{P}_i^p(k)$ and $\hat{P}_i(k-1)$ are recursively calculated by

$$\begin{aligned} \hat{P}_i^p(k) = {} & A_i(k-1)\hat{P}_i(k-1)A_i^{\mathrm{T}}(k-1) \\ & + \Gamma_i(k-1)Q_{w_i}\Gamma_i^{\mathrm{T}}(k-1) \\ & + \sum_{i_\omega\in\overline{\Omega}_i} A_i(k-1)\mathcal{D}_{\hat{P}_i}(k-1)\mathcal{D}_{\hat{P}_{i_\omega}}^{\mathrm{T}}(k-1)A_{i,i_\omega}^{\mathrm{T}}(k-1) \\ & + \sum_{i_\omega\in\overline{\Omega}_i} A_{i,i_\omega}(k-1)\mathcal{D}_{\hat{P}_{i_\omega}}(k-1)\mathcal{D}_{\hat{P}_i}^{\mathrm{T}}(k-1)A_i^{\mathrm{T}}(k-1) \\ & + \sum_{i_\omega^1\in\overline{\Omega}_i}\sum_{i_\omega^2\in\overline{\Omega}_i}\Big\{A_{i,i_\omega^1}(k-1)\mathcal{D}_{\hat{P}_{i_\rho^1}}(k-1) \\ & \qquad \times \mathcal{D}_{\hat{P}_{i_\omega^2}}^{\mathrm{T}}(k-1)A_{i,i_\omega^2}^{\mathrm{T}}(k-1)\Big\} \end{aligned} \tag{40a}$$

$$\begin{aligned} \hat{P}_i(k-1) = {} & \widehat{K}_i^{\mathrm{opt}}(k-1)\hat{P}_i^p(k-2)\left[\widehat{K}_i^{\mathrm{opt}}(k-1)\right]^{\mathrm{T}} \\ & + K_i^{\mathrm{opt}}(k-1)D_i(k-1)Q_{v_i}D_i^{\mathrm{T}}(k-1)\left[K_i^{\mathrm{opt}}(k-1)\right]^{\mathrm{T}}. \end{aligned} \tag{40b}$$

Proof: It is infeasible to calculate the one-step prediction error covariance $P_i^p(k)$ in (4c) directly because the cross-covariance $P_{i,j}(k)$ is hard to compute by local communication. Therefore, a matrix $\hat{P}_i(k)$ satisfying $P_i(k) \leq \hat{P}_i(k)$ is constructed for designing the estimator gain matrix. Let $[\tilde{x}_i(k)]_{\tau_1}$ be the τ_1th component of $\tilde{x}_i(k)$ and $[\tilde{x}_j(k)]_{\tau_2}$ be the τ_2th component of $\tilde{x}_j(k)$. By resorting to the Hölder inequality, one has that

$$\begin{aligned}[P_{i,j}(k)]_{\tau_1,\tau_2} &= \mathrm{E}\left\{[\tilde{x}_i(k)]_{\tau_1}[\tilde{x}_j(k)]_{\tau_2}\right\} \\ &\leq \mathrm{E}\left\{\left|[\tilde{x}_i(k)]_{\tau_1}[\tilde{x}_j(k)]_{\tau_2}\right|\right\} \\ &\leq \sqrt{\mathrm{E}\left\{[\tilde{x}_i(k)]_{\tau_1}^2\right\}}\sqrt{\mathrm{E}\left\{[\tilde{x}_j(k)]_{\tau_2}^2\right\}} \\ &= \sqrt{[P_i(k)]_{\tau_1}}\sqrt{[P_j(k)]_{\tau_2}}.\end{aligned} \tag{41}$$

It is known from (41) that

$$P_{i,j}(k) \leq \mathcal{D}_{P_i}(k)\mathcal{D}_{P_j}^{\mathrm{T}}(k) \leq \mathcal{D}_{\hat{P}_i}(k)\mathcal{D}_{\hat{P}_j}^{\mathrm{T}}(k). \tag{42}$$

By applying the inequality (42) to (4c), it is found that $P_i^p(k) \leq \hat{P}_i^p(k)$. Then, the matrix $\hat{P}_i(k)$ can be constructed from (4d) as

$$\begin{aligned}P_i(k) \leq \hat{P}_i(k) :=& \widehat{K}_i(k)\hat{P}_i^p(k)\widehat{K}_i^{\mathrm{T}}(k) \\ &+ K_i(k)D_i(k)Q_{v_i}D_i^{\mathrm{T}}(k)K_i^{\mathrm{T}}(k).\end{aligned} \tag{43}$$

At the same time, it is proposed to construct a matrix $\hat{G}_i(k)$ satisfying $\hat{P}_i(k) - \hat{G}_i(k) \prec 0$, so one has that

$$\mathrm{Tr}\left\{P_i(k)\right\} \leq \mathrm{Tr}\left\{\hat{P}_i(k)\right\} < \mathrm{Tr}\left\{\hat{G}_i(k)\right\}. \tag{44}$$

The optimal estimator gain is obtained by minimizing the upper bound $\mathrm{Tr}\left\{\hat{G}_i(k)\right\}$, which turns out to be an optimization problem. The constraint $\hat{P}_i(k) - \hat{G}_i(k) \prec 0$ can be converted by the Schur complement lemma as follows:

$$\begin{bmatrix} K_i(k)D_i(k)Q_{v_i}D_i^{\mathrm{T}}(k)K_i^{\mathrm{T}}(k) - \hat{G}_i(k) & \widehat{K}_i(k)\hat{P}_i^p(k) \\ * & -\hat{P}_i^p(k)\end{bmatrix} \prec 0. \tag{45}$$

Then, the first inequality constraint in (39) is further derived using the Schur complement lemma. By adding the distributed stability constraints in Theorem 1 or Lemma 2, the optimization problem (39) is formulated.

Then, the computation procedures of the distributed estimation for time-varying interconnected systems with ideal and non-ideal communication environments can be summarized by Algorithm 2. The shared information among subsystems includes $\hat{x}_i(k)$ and $\hat{P}_i(k)$ for the estimator iteration and $K_i(k)$ for the stability guarantee in an ideal communication environment.

Remark 7. The inequality constraints (15) and (30) can be rewritten as linear matrix inequalities, so the optimization problem in Theorems 2 can be directly solved by the function "mincx" of the MATLAB LMI toolbox [48]. Notice that the first inequality constraint in (39) is converted from $\hat{P}_i(k) - \hat{G}_i(k) \prec 0$ for constructing the matrix $\hat{G}_i(k)$ and is a feasible constraint. Therefore, the solvability of the optimization problem (39) can be guaranteed by allocating the parameters in the stability constraints (15) and (30) in advance according to subsystems' dynamics and couplings. These adjustable parameters make the derivation of stability conditions less conservative than the derivation in [44], and one possible parameter allocation can be found in (37) and Algorithm 1. On the other hand, the computational complexity of (40) is $O\left(\bar{\theta}_i^2 n_{i,\max}^3\right)$, where $n_{i,\max} := \max\{n_i, n_{i_1}, ..., n_{i_{\bar{\theta}_i}}\}$. The computational complexity of (39) is also mainly determined by $\bar{\theta}_i$ and $n_{i,\max}$. For a large-scale interconnected system with a sparse coupling structure, the computational burden of (39) in the proposed distributed estimator will be much less than that of a centralized estimator.

Algorithm 2 Distributed Estimation for Time-varying Interconnected Systems

1: **Offline Setup**: If the communication environment is non-ideal, then offline allocate β_i by Algorithm 1;
2: **for** $i := 1$ **to** l **do**
3: Subsystem $\mathbf{S}_i$ collects the local measurement $y_i(k)$ and the estimation results from its neighbors, including $\hat{x}_{i_\omega}(k-1)$, $\hat{P}_{i_\omega}(k-1)$, and $K_{i_\omega}(k)$ $(i_\omega \in \overline{\Omega}_i)$;
4: Subsystem $\mathbf{S}_i$ calculates $\hat{P}_i^p(k)$ by (40a);
5: Subsystem $\mathbf{S}_i$ determines the estimator gain $K_i(k)$ by solving the optimization problem (39), constrained by (30) if the communication environment is non-ideal, and (15) if the communication environment is ideal;
6: Subsystem $\mathbf{S}_i$ calculates $\hat{P}_i(k)$ by (40b);
7: Subsystem $\mathbf{S}_i$ obtains the estimate $\hat{x}_i(k)$ by (3);
8: Subsystem $\mathbf{S}_i$ sends the calculated $\hat{x}_i(k)$, $\hat{P}_i(k)$, and $K_i(k)$ to its out-neighbors;
9: **end for**
10: Return to Step 2 and implement Steps 2-9 for calculating $\hat{x}_i(k+1)$ $(i = 1, 2, ..., l)$.

Remark 8. The design methodology in Theorem 2 takes the optimality and stability of the distributed estimator into account by constructing an optimization problem. It can overcome the difficulty in stability assurance of completely local design methods, which means the proposed distributed estimator is more appealing when compared with the work in [25, 26] that only provides stability conditions for specific coupling structures. Moreover, the designed estimator enables plug-and-play operations due to the flexibility of the proposed distributed stability conditions. If one subsystem is removed, then the stability conditions and the estimator design for other subsystems do not require any changes. If a new subsystem is added, then only the stability condition and estimator design for the added subsystem need to be redesigned. The plug-and-play property is helpful for the large-scale deployment of distributed estimators for interconnected systems.

IV. Numerical Examples

To illustrate the effectiveness of the proposed distributed estimators, we shall report the numerical study results for a three-tank system in this section. The three-tank system consists of three cylindrical tanks, T1, T2, and T3, with an equivalent cross-section S_A. These tanks are connected serially

$$A_c = \begin{bmatrix} \frac{-\mu_{13}(t)gS_n}{S_A\sqrt{2g(h_1^\star - h_3^\star)}} & 0 & \frac{\mu_{13}(t)gS_n}{S_A\sqrt{2g(h_1^\star - h_3^\star)}} \\ 0 & \frac{-\mu_{32}(t)gS_n}{S_A\sqrt{2g(h_3^\star - h_2^\star)}} - \frac{\mu_{20}(t)gS_n}{S_A\sqrt{2gh_2^\star}} & \frac{\mu_{32}(t)gS_n}{S_A\sqrt{2g(h_3^\star - h_2^\star)}} \\ \frac{\mu_{13}(t)gS_n}{S_A\sqrt{2g(h_1^\star - h_3^\star)}} & \frac{\mu_{32}(t)gS_n}{S_A\sqrt{2g(h_3^\star - h_2^\star)}} & \frac{-\mu_{13}(t)gS_n}{S_A\sqrt{2g(h_1^\star - h_3^\star)}} - \frac{\mu_{32}(t)gS_n}{S_A\sqrt{2g(h_3^\star - h_2^\star)}} \end{bmatrix} \quad B_c = \frac{1}{S_A}\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \tag{46}$$

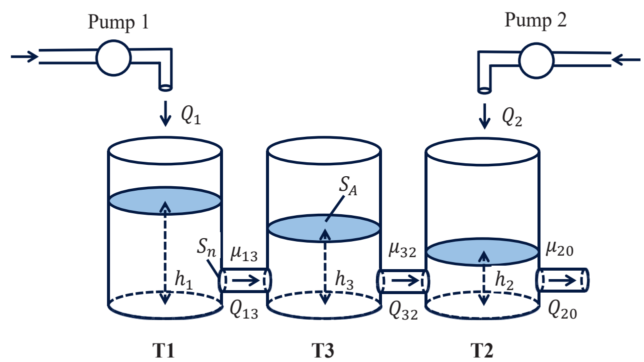


Fig. 2. The layout of a three-tank system.

by cylindrical pipes with the cross-section S_n, and the layout is shown in Fig. 2. There are two pumps configured for T1 and T2 to supply water, and one outflow valve with the cross-section S_n is located at the bottom of T2. The pump flow rates Q_1 and Q_2 are viewed as the input signals for the system, while the flow rates Q_{13}, Q_{32}, and Q_{20} can be determined with outflow coefficients $\mu_{13}(t)$, $\mu_{32}(t)$, and $\mu_{20}(t)$ by the general Torricelli rule. Let the levels of three tanks be the system state $x := [h_1\ h_3\ h_2]^{\mathrm{T}}$ and assume they satisfy $h_1 > h_3 > h_2$ during the whole experiment. Then, the nonlinear continuous-time model [52] obtained from the balance equation can be linearized at the equilibrium point $x_\star := [h_1^\star\ h_2^\star\ h_3^\star]^{\mathrm{T}}$ as:

$$\dot{x}(t) = A_c(t)x(t) + B_c u(t) \tag{47}$$

where $u := [Q_1\ Q_2]^{\mathrm{T}}$ is the input signal, and the system matrices are defined in (46). The continuous-time model (47) should be discretized into a discrete-time model: $x(k+1) = A_d(k)x(k) + B_d u(k)$. To maintain $A_d(k)$ the same coupling structure as $A_c(t)$, the block-wise discretization [53] with a sampling period T_s is applied. Thus, the parameters of the discrete-time model can be obtained by $A_d(k) = \mathbf{I} + F(T_s)A_c(kT_s)$, $B_d(k) = F(T_s)B_c$, and $F(T_s) := \mathrm{diag}\left\{\int_0^{T_s} e^{[A_c(kT_s)]_{1,1}t}dt, ..., \int_0^{T_s} e^{[A_c(kT_s)]_{3,3}t}dt\right\}$. Then, the overall system is decomposed into three subsystems $\mathbf{S}_i$ $(i = 1, 2, 3)$ with the coupling structure in Fig. 3, where

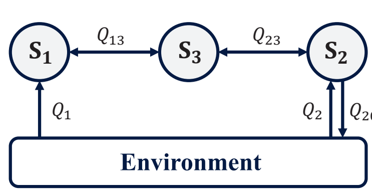


Fig. 3. The coupling structure of the three-tank interconnected system.

the state dynamics of these subsystems with system noises and leakage faults can be described as follows:

$$\begin{cases} x_1(k+1) = A_1(k)x_1(k) + A_{13}(k)x_3(k) + B_1u_1(k) \\ \qquad\qquad + f_1(k) + w_1(k) \\ x_2(k+1) = A_2(k)x_2(k) + A_{23}(k)x_3(k) + B_2u_2(k) \\ \qquad\qquad + f_2(k) + w_2(k) \\ x_3(k+1) = A_3(k)x_3(k) + A_{31}(k)x_1(k) + A_{32}(k)x_2(k) \\ \qquad\qquad + f_3(k) + w_3(k) \end{cases} \tag{48}$$

where $x_i := h_i$, $u_i := Q_i$, f_i, and w_i are the state, the input, the possible leakage fault, and the system noise of each subsystem, respectively. The matrices $A_i(k)$, $A_{ij}(k)$, and B_i are the corresponding submatrices (scalars for this example) in $A_d(k)$ and B_d. Moreover, the liquid level of the ith tank is monitored by a sensor with the measurement dynamics $y_i(k) = x_i(k) + v_i(k)$.

In the simulation, the system parameters are set as follows: $T_s = 1s$, $S_a = 0.0154\ m^2$, $S_n = 5\times10^{-5}\ m^2$, $g = 9.8\ m/s^2$, μ_{13} is randomly selected from 0.42 to 0.50 at each instant, μ_{32} is randomly selected from 0.44 to 0.52 at each instant, $\mu_{20} = 0.58$, and $x_\star = [0.4890\ 0.2332\ 0.3611]^{\mathrm{T}}$ (m). The leakage faults only happen over a certain period of time, as $f_1(k) = -2\times10^{-3}$ $(100 \le k \le 110)$ and $f_2(k) = -2\times10^{-3}$ $(50 \le k \le 60)$. The system noises $w_i(k)$ and the measurement noises $v_i(k)$ are uncorrelated Gaussian noises with covariances $Q_i = 0.0001$ and $R_i = 0.0005$. To control the liquid levels of T1 and T2 at the equilibrium point, two PID controllers with the same parameters, $K_P = 0.02$, $K_I = 5\times10^{-6}$, and $P_D = 2\times10^{-6}$, are employed in subsystems. Our primary concern in this numerical study is

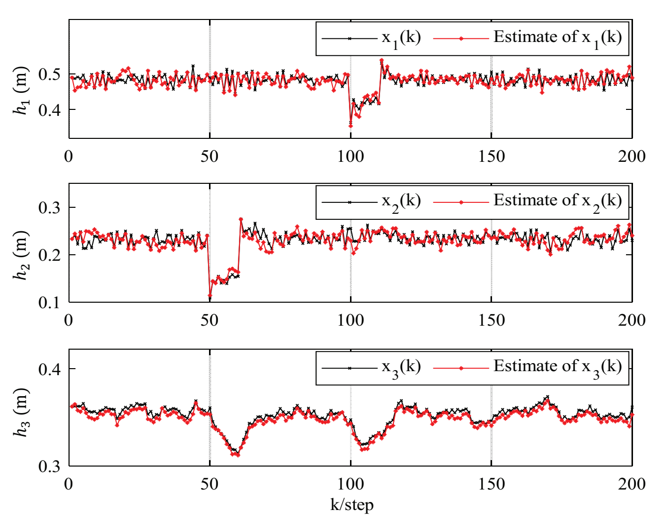


Fig. 4. The liquid levels of three tanks and their estimated values.

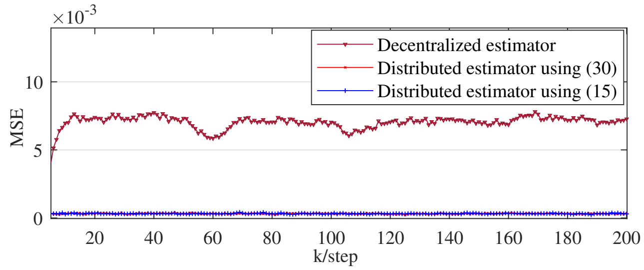


Fig. 5. The MSE comparison for the decentralized estimator in [51] and the distributed estimators in Algorithm 2.

to estimate the real states of each subsystem using only local communication. The proposed distributed estimators have been successfully applied to this interconnected three-tank system. In a non-ideal communication environment, the liquid levels of three tanks and their estimated values by Theorem 2 are plotted in Fig. 4. It is found that the leakage faults in T1 and T2 also cause a sudden drop in the liquid level of T3. At the same time, the estimators can track these real liquid levels well based on only local information. The result also shows that the estimation error in each subsystem can converge to a relatively small interval. To better reflect the estimation results, a Monte Carlo simulation with $N = 100$ runs has been performed by randomly varying the realization of process and measurement noises. The mean square error (MSE) is introduced to evaluate estimators' performance, where

$$\mathrm{MSE}(k) = \sum_{s=1}^{N} \frac{\|e_s(k)\|^2}{N} \tag{49}$$

with $e_s(k)$ being the state estimation error at the instant k in the sth simulation. The MSE comparison for the decentralized estimator [51] and the distributed estimators in Algorithm 2 using (15) and (30) is plotted in Fig. 5. It is shown in Fig. 5 that the MSE performance of the proposed distributed estimators is much better than the decentralized estimator. Moreover, the average MSE (AMSE) calculated from the mean of MSEs at all instants is also introduced to show the accuracy of estimators. The AMSE comparison under different measurement noise realizations for the distributed estimators in Algorithm 2 using (15) and (30), the distributed estimator with the assumption of uncorrelated errors in [27], and the centralized estimator is plotted in Fig. 6. It can be seen from Fig. 6 that the designed distributed estimators show their performance superiority over the distributed estimator that completely ignores the estimation errors correlations. It means that the distributed estimation method by constructing an upper bound of the trace of estimation error covariance is better than the estimation by ignoring the estimation error correlations. In addition, the performance of the distributed estimators does not degrade too much when compared with the centralized estimator.

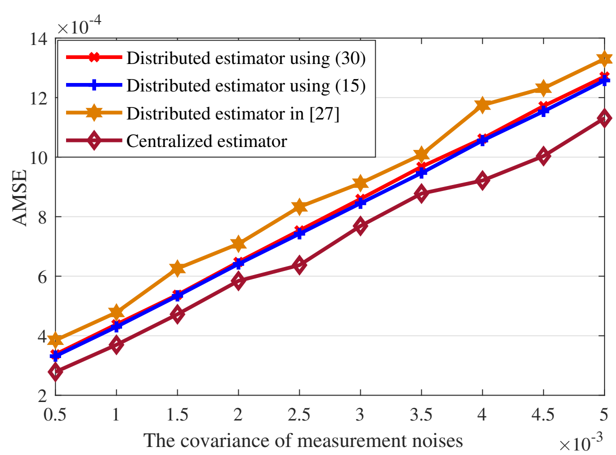


Fig. 6. The AMSE comparison for the distributed estimators in Algorithm 2, the distributed estimator with the assumption of uncorrelated errors in [27], and the centralized estimator.

V. Conclusion

In this paper, we presented new results of distributed estimator design for time-varying interconnected systems with arbitrary coupling structures. A distributed stability condition was proposed in Theorem 1 by a subsystem-level analysis for a centralized stability condition. The proposed distributed condition can keep the distributed estimator mean-square UUB without requiring the overall system's knowledge of dynamics and couplings. By further considering a non-ideal communication environment, a simplified distributed condition was proposed to avoid the real-time exchange of local estimator gains. These distributed conditions enable plug-and-play operations due to their distributed nature. Moreover, the distributed estimators were designed by applying distributed conditions to an optimization-based estimator design approach. The designed estimators are fully distributed because the estimator form and the stability conditions only use information from each subsystem and its neighbors. Finally, an illustrative example of a three-tank interconnected system was employed to show the effectiveness of the proposed methods.

Several topics for future research are left open. It will be important to extend the presented distributed stability conditions to nonlinear interconnected systems and apply them to distributed control problems. Another interesting extension is the development of secure distributed estimators for interconnected systems. Notice that interconnected systems are more vulnerable to various attacks due to the distributed communication structure. If one subsystem is compromised, then the security risk may propagate to other subsystems. How to design anti-eavesdropping mechanisms to prevent more sophisticated attacks and how to design attack detection mechanisms through subsystem cooperation are the two important problems to be solved.

References

[1] W. Yu, G. Chen, and J. Lü, "On pinning synchronization of complex dynamical networks," *Automatica*, vol. 45, no. 2, pp. 429–435, 2009.

[2] S. Majhi, M. Perc, and D. Ghosh, "Dynamics on higher-order networks: A review," *Journal of The Royal Society Interface*, vol. 19, no. 188, p. 20220043, 2022.

[3] Y. Y. Liu, J. J. Slotine, and A. L. Barabási, "Controllability of complex networks," *Nature*, vol. 473, no. 7346, pp. 167–173, 2011.

[4] Y. Y. Liu, J. J. Slotine, and A. L. Barabási, "Observability of complex systems," *Proceedings of the National Academy of Sciences*, vol. 110, no. 7, pp. 2460–2465, 2013.

[5] T. Zhou, "On the controllability and observability of networked dynamic systems," *Automatica*, vol. 52, pp. 63–75, 2015.

[6] A. Arenas, A. Díaz-Guilera, J. Kurths, Y. Moreno, and C. Zhou, "Synchronization in complex networks," *Physics Reports*, vol. 469, no. 3, pp. 93–153, 2008.

[7] L. H. Zhao, S. Wen, C. Li, K. Shi, and T. Huang, "A recent survey on control for synchronization and passivity of complex networks," *IEEE Transactions on Network Science and Engineering*, vol. 9, no. 6, pp. 4235–4254, 2022.

[8] D. D. Šiljak, *Decentralized Control of Complex Systems*. Courier Corporation, 2011.

[9] J. Yang, W. A. Zhang, and F. Guo, "Dynamic state estimation for power networks by distributed unscented information filter," *IEEE Transactions on Smart Grid*, vol. 11, no. 3, pp. 2162–2171, 2020.

[10] Z. Feng, G. Hu, Y. Sun, and J. Soon, "An overview of collaborative robotic manipulation in multi-robot systems," *Annual Reviews in Control*, vol. 49, pp. 113–127, 2020.

[11] M. Ye and B. D. O. Anderson, "Competitive Epidemic Spreading Over Networks," *IEEE Control Systems Letters*, vol. 7, pp. 545–552, 2023.

[12] T. Hillerton, D. Seçilmiş, S. Nelander, and E. L. L. Sonnhammer, "Fast and accurate gene regulatory network inference by normalized least squares regression," *Bioinformatics*, vol. 38, no. 8, pp. 2263–2268, 2022.

[13] J. Lian, "Special section on control of complex networked systems (CCNS): Recent results and future trends," *Annual Reviews in Control*, vol. 47, pp. 275–277, 2019.

[14] C. Kwon and I. Hwang, "Sensing-based distributed state estimation for cooperative multiagent systems," *IEEE Transactions on Automatic Control*, vol. 64, no. 6, pp. 2368–2382, 2019.

[15] Y. Liu, Z. Wang, L. Zou, J. Hu, and H. Dong, "Distributed filtering for complex networks under multiple event-triggered transmissions within node-wise communications," *IEEE Transactions on Network Science and Engineering*, vol. 9, no. 4, pp. 2521–2534, 2022.

[16] R. Olfati-Saber, "Distributed Kalman filtering for sensor networks," in *2007 46th IEEE Conference on Decision and Control*, (New Orleans, LA, USA), pp. 5492–5498, IEEE, Dec. 2007.

[17] B. Chen, W. A. Zhang, and L. Yu, "Distributed finite-horizon fusion Kalman filtering for bandwidth and energy constrained wireless sensor networks," *IEEE Transactions on Signal Processing*, vol. 62, no. 4, pp. 797–812, 2014.

[18] W. A. Zhang and L. Shi, "Sequential fusion estimation for clustered sensor networks," *Automatica*, vol. 89, pp. 358–363, 2018.

[19] Y. Xu, Y. Deng, Z. Huang, M. Lin, and P. Shi, "Distributed state estimation over sensor networks with substate decomposition approach," *IEEE Transactions on Network Science and Engineering*, vol. 10, no. 1, pp. 527–537, 2022.

[20] A. Haber and M. Verhaegen, "Moving horizon estimation for large-scale interconnected systems," *IEEE Transactions on Automatic Control*, vol. 58, no. 11, pp. 2834–2847, 2013.

[21] U. A. Khan and J. M. F. Moura, "Distributing the Kalman filter for large-scale systems," *IEEE Transactions on Signal Processing*, vol. 56, no. 10, pp. 4919–4935, 2008.

[22] S. S. Stanković, M. S. Stanković, and D. M. Stipanović, "Consensus based overlapping decentralized estimation with missing observations and communication faults," *Automatica*, vol. 45, no. 6, pp. 1397–1406, 2009.

[23] B. Chen, G. Hu, D. W. C. Ho, and L. Yu, "Distributed Kalman filtering for time-varying discrete sequential systems," *Automatica*, vol. 99, pp. 228–236, 2019.

[24] B. Chen, G. Hu, D. W. Ho, and L. Yu, "Distributed estimation for discrete-time interconnected systems," in *2019 Chinese Control Conference (CCC)*, (Guangzhou, China), pp. 3708–3714, IEEE, July 2019.

[25] B. Chen, G. Hu, D. W. Ho, and L. Yu, "Distributed estimation and control for discrete time-varying interconnected systems," *IEEE Transactions on Automatic Control*, vol. 67, no. 5, pp. 2192–2207, 2022.

[26] Y. Zhang, B. Chen, L. Yu, and D. W. C. Ho, "Distributed Kalman filtering for interconnected dynamic systems," *IEEE Transactions on Cybernetics*, vol. 52, no. 11, pp. 11571–11580, 2022.

[27] M. Farina, G. Ferrari-Trecate, and R. Scattolini, "Moving-horizon partition-based state estimation of large-scale systems," *Automatica*, vol. 46, no. 5, pp. 910–918, 2010.

[28] S. Riverso, M. Farina, R. Scattolini, and G. Ferrari-Trecate, "Plug-and-play distributed state estimation for linear systems," in *52nd IEEE Conference on Decision and Control*, (Florence, Italy), pp. 4889–4894, IEEE, Oct. 2013.

[29] S. Riverso, D. Rubini, and G. Ferrari-Trecate, "Distributed bounded-error state estimation based on practical robust positive invariance," *International Journal of Control*, vol. 88, no. 11, pp. 2277–2290, 2015.

[30] Y. Zhang, B. Chen, and L. Yu, "Distributed zonotopic estimation for interconnected systems: A fusing overlapping states strategy," *Automatica*, vol. 155, p. 111144, 2023.

[31] N. R. Sandell, P. Varaiya, M. Athans, and M. G. Safonov, "Survey of decentralized control methods for large scale systems," *IEEE Transactions on Automatic Control*, vol. 23, no. 2, pp. 108–128, 1978.

[32] A. N. Michel, "On the status of stability of interconnected systems," *IEEE Transactions on Systems, Man, and Cybernetics*, vol. 13, no. 4, pp. 439–453, 1983.

[33] H. Ito, "A geometrical formulation to unify construction of Lyapunov functions for interconnected iISS systems," *Annual Reviews in Control*, vol. 48, pp. 195–208, 2019.

[34] M. Vidyasagar, ed., *Input-Output Analysis of Large-Scale Interconnected Systems. Decomposition, Well-Posedness and Stability*. Berlin/Heidelberg: Springer-Verlag, 1981.

[35] W. M. Haddad and S. G. Nersesov, *Stability and Control of Large-Scale Dynamical Systems: A Vector Dissipative Systems Approach*. Princeton: Princeton University Press, 2011.

[36] F. N. Bailey, "The application of Lyapunov's second method to interconnected systems," *Journal of the Society for Industrial and Applied Mathematics, Series A: Control*, vol. 3, no. 3, pp. 443–462, 1965.

[37] A. N. Michel, "Stability analysis of interconnected systems," *SIAM Journal on Control*, vol. 12, no. 3, pp. 554–579, 1974.

[38] H. K. Khalil, *Nonlinear Systems*. New York, NY, USA: Pearson Education, 2015.

[39] S. N. Dashkovskiy, B. S. Rüffer, and F. R. Wirth, "Small gain theorems for large scale systems and construction of ISS Lyapunov functions," *SIAM Journal on Control and Optimization*, vol. 48, no. 6, pp. 4089–4118, 2010.

[40] H. Ito, "State-dependent scaling problems and stability of interconnected iISS and ISS systems," *IEEE Transactions on Automatic Control*, vol. 51, no. 10, pp. 1626–1643, 2006.

[41] Z. P. Jiang, I. M. Y. Mareels, and Y. Wang, "A Lyapunov formulation of the nonlinear small-gain theorem for interconnected ISS systems," *Automatica*, vol. 32, no. 8, pp. 1211–1215, 1996.

[42] P. Moylan and D. Hill, "Stability criteria for large-scale systems," *IEEE Transactions on Automatic Control*, vol. 23, no. 2, pp. 143–149, 1978.

[43] K. S. Schweidel and M. Arcak, "Compositional analysis of interconnected systems using delta dissipativity," *IEEE Control Systems Letters*, vol. 6, pp. 662–667, 2022.

[44] E. Agarwal, S. Sivaranjani, V. Gupta, and P. J. Antsaklis, "Distributed synthesis of local controllers for networked systems

with arbitrary interconnection topologies," *IEEE Transactions on Automatic Control*, vol. 66, no. 2, pp. 683–698, 2021.

[45] D. Liang, Y. Yang, R. Li, and Z. Zhao, "Plug-and-play robust distributed fault estimation for interconnected systems," *IEEE Transactions on Network Science and Engineering*, vol. 9, no. 5, pp. 3385–3395, 2022.

[46] A. A. Alam, A. Gattami, and K. H. Johansson, "An experimental study on the fuel reduction potential of heavy duty vehicle platooning," in *13th International IEEE Conference on Intelligent Transportation Systems*, (Madeira Island, Portugal), pp. 306–311, Sept. 2010.

[47] C. W. Sanders, E. C. Tacker, T. D. Linton, and R. Y. S. Ling, "Specific structures for large-scale state estimation algorithms having information exchange," *IEEE Transactions on Automatic Control*, vol. 23, no. 2, pp. 255–261, 1978.

[48] S. Boyd, L. El Ghaoui, E. Feron, and V. Balakrishnan, eds., *Linear Matrix Inequalities in System and Control Theory*. Philadelphia, PA, USA: Society for Industrial and Applied Mathematics, 1994.

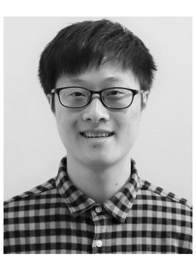
[49] W. J. Rugh, *Linear System Theory*. Prentice Hall Information and System Sciences Series, Upper Saddle River, N.J: Prentice Hall, 2nd ed ed., 1996.

[50] N. Kottenstette, M. J. McCourt, M. Xia, V. Gupta, and P. J. Antsaklis, "On relationships among passivity, positive realness, and dissipativity in linear systems," *Automatica*, vol. 50, no. 4, pp. 1003–1016, 2014.

[51] C. W. Sanders, E. C. Tacker, and T. D. Linton, "A new class of decentralized filters for interconnected systems," *IEEE Transactions on Automatic Control*, vol. 19, no. 3, pp. 259–262, 1974.

[52] D. H. Zhou, X. He, Z. Wang, G. P. Liu, and Y. D. Ji, "Leakage fault diagnosis for an internet-based three-tank system: An experimental study," *IEEE Transactions on Control Systems Technology*, vol. 20, no. 4, pp. 857–870, 2012.

[53] M. Farina, P. Colaneri, and R. Scattolini, "Block-wise discretization accounting for structural constraints," *Automatica*, vol. 49, no. 11, pp. 3411–3417, 2013.



Yuchen Zhang (Member, IEEE) received the B.E. degree in electrical engineering and automation and the Ph.D. degree in control theory and control engineering from the Zhejiang University of Technology, Hangzhou, China, in 2017 and 2024, respectively.

His current research interests include multi-sensor fusion estimation, distributed estimation and control of interconnected systems, privacy-preserving estimation, and set-based estimation and optimization.

Bo Chen (Senior Member, IEEE) received the B.S. degree in information and computing science from Jiangxi University of Science and Technology, Ganzhou, China, in 2008, and the Ph.D degree in Control Theory and Control Engineering from Zhejiang University of Technology, Hangzhou, China, in 2014.

He jointed the Department of Automation, Zhejiang University of Technology in 2018, where he is currently a Professor. He was a Research Fellow with the School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore, from 2014 to 2015 and from 2017 to 2018. He was also a Postdoctoral Research Fellow with the Department of Mathematics, City University of Hong Kong, Hong Kong, from 2015 to 2017. His current research interests include information fusion, distributed estimation and control, networked fusion systems, and secure estimation of cyber-physical systems.

Prof. Chen was a recipient of the Outstanding Thesis Award of Chinese Association of Automation in 2015 and also was a recipient of the First Prize of Natural Science of Ministry of Education in 2020. He serves as Associate Editor for IET Control Theory and Applications, Journal of the Franklin Institute, and Frontiers in Control Engineering, and also serves as Guest Editor for IEEE Transactions on Industrial Cyber-Physical Systems.

Li Yu (Senior Member, IEEE) received the B.S. degree in control theory from Nankai University, Tianjin, China, in 1982, and the M.S. and Ph.D. degrees from Zhejiang University, Hangzhou, China, in 1988 and 1999, respectively.

He is currently a Professor with the College of Information Engineering, Zhejiang University of Technology, Hangzhou, China. He has successively presided over 20 research projects. He has published five academic monographs, one textbook, and over 300 journal papers. He has also been authorized over 100 patents for invention and granted five scientific and technological awards. His current research interests include robust control, networked control systems, cyber–physical systems security, and information fusion.

Daniel W.C. Ho (Life Fellow, IEEE) received the B.S., M.S., and Ph.D. degrees in mathematics from the University of Salford, Salford, U.K., in 1980, 1982, and 1986, respectively.

From 1985 to 1988, he was a Research Fellow with the Industrial Control Unit, University of Strathclyde, Glasgow, U.K. In 1989, he joined the City University of Hong Kong, Hong Kong, where he is currently a Chair Professor of Applied Mathematics, and an Associate Dean of the College of Science. He has authored or coauthored more than 260 publications in scientific journals. His current research interests include control and estimation theory, complex dynamical distributed networks, multiagent systems, and stochastic systems.

Prof. Ho was the recipient of the Chang Jiang Chair Professor awarded by the Ministry of Education, China, in 2012, and the ISI Highly Cited Researchers Award in Engineering by Clarivate Analytics from 2014 to 2022. He has been on the editorial board of a number of journals, including IEEE Transactions on Neural Networks and Learning Systems, IET Control Theory and Applications, Journal of The Franklin Institute, and Asian Journal of Control.