This article has been accepted for publication in IEEE Control Systems Letters. This is the author's version which has not been fully edited and content may change prior to final publication. Citation information: DOI 10.1109/LCSYS.2024.3376458

Minimal Sensor Activation for Detectable Networked Discrete Event Systems

Anjie Mao, Yuchen Zhang, Zheming Wang, *Member, IEEE* , and Bo Chen, *Senior Member, IEEE*

***Abstract*— This paper is concerned with the minimal sensor activation problem for a class of networked discrete event systems (DESs) with channel delays. A new concept called networked detectability is proposed to characterize the detectability of networked DESs in minimal sensor activation problem. Then, an observer-based approach is proposed to verify the networked detectability. Under verified networked detectability, the feasible sensor activation policies for networked DESs are investigated. By searching for the maximum feasible sub-policy of a given activation policy, a minimal sensor activation policy for networked DESs is designed as an algorithm. Several examples are used to illustrative the proposed methods.**

***Index Terms*— Networked discrete event systems (DESs), networked detectability, minimal sensor activation, state observer.**

I. Introduction

DISCRETE Event Systems (DESs) are dynamic systems with discrete state spaces and event-triggered dynamics [1]. DES models are widely used in the study of complex automated systems where the behavior is inherently event driven [2]. An event may correspond to the arrival or departure of a customer in a queue and the completion of a task or the failure of a machine in a manufacturing system. Over the past decades, the theory of DES has been successfully applied to many real-world problems, such as the control of automated systems [3] and fault diagnosis [4].

The activation of sensors is costly in DESs, because the sensors have limited energy and lifetime. Minimal sensor activation has become an important issue in DESs because it can reduce the energy consumption of sensors by using an appropriate sensor activation policy. The existing works focus on minimizing the sensor activation while maintaining certain original characteristics of DESs or certain tasks, such as fault diagnosis [5], supervisory control [6], observability [7], and detectability [8], [9]. "Detectability" was defined to measure the state estimation performance of DESs [10]. A DES is identified as detectable iff its states can be uniquely determined after a finite number of observations. In [8], an offline algorithm was first proposed to minimize sensor activation while maintaining the detectability of DESs. Subsequently, online algorithms were also proposed in [9].

The above researches on sensor activation assumed that communications between the supervisor and the plant are instantaneous. In other words, there is no delay in communications. This assumption is not valid for networked discrete event systems (NDESs) [11]. In NDESs, supervisors and plants are deployed at distant locations to transmit information via wired or wireless networks, so the communication delays are unavoidable [12]. The delay necessitates adaptations across all methodologies to minimize sensor activation for applicability to the NDESs. On the one hand, the activation of sensors at unnecessary moments will reduce the the sensors' lifetime and consume sensors' energy [9]. On the other hand, the unwanted information from sensors will occupy the limited communication bandwidth [13]. Therefore, it is necessary to minimize sensor activation under the delays.

In this paper, we first extend the detectabilities to be networked in sensor activation problem. It is difficult to handle the observation mapping because event observation is state-dependent and the delay will change the observed event sequence. Therefore, a new observation mapping combining transmission delays and sensor activation is defined. On this basis, we define two types of networked detectabilities. Their necessary and sufficient conditions are derived using delayed observers. Subsequently, we investigate the problem of minimizing sensor activation while preserving networked detectabilities. The basic solution is removing events of maximum feasible sensor activation policy in turn to derive its sub-policies. All the sub-policies preserving networked detectabilities are compared to find the minimal one.

Contributions: (i) A new concept called networked detectability is proposed to characterize the detectability of NDESs in the sensor activation problem. Their necessary and sufficient conditions are derived based on delayed observers. Compared with the existing detectability in [8], [9], the proposed networked detectability applies to the NDESs. (ii) The minimal sensor activation policy for NDESs is investigated. An algorithm for determining the minimal sensor activation policy of a detectable NDES is proposed. Compared with the existing policy in [5], [6], the proposed minimal sensor activation policy achieves the different goal that remain the system's detectability.

Notation: The symbol $\vee$ means "or", $\wedge$ means "and", $\neg$

Manuscript received 9 December 2023; revised 17 February, 2024; accepted 4 March 2024. This work was supported in part by the National Funds of China under Grant 92367205 and 62303416; in part by the Zhejiang Provincial Science and Technology Department key research and development program under Grant 2023C01144. This article was recommended by Associate Editor XX. (Corresponding author: Bo Chen.)

The authors are with the Department of Automation, Zhejiang University of Technology, Hangzhou 310023, China, and are also with the Zhejiang Provincial United Key Laboratory of Embedded Systems, Hangzhou 310023, China (e-mail: maoanjie@aliyun.com; YuchenZhang95@163.com; wangzheming@zjut.edu.cn; bchen@aliyun.com).

means "negation". The symbol $\mathbb{N}$ represents the set of positive integers and $\emptyset$ represents an "empty set". The symbol $\circ$ represents the composition operator. For a string s, $|s|$ is its length and $\Pr(s)$ is the set of its prefixes. For a set A, $|A|$ is its cardinality and 2^A is its power set. For sets X and Y, $X \cup Y$ is their union, $X \cap Y$ is their intersection, and $X \times Y$ is their product. For a mapping M, M^{-1} is its inverse mapping. For policies Ω_1 and Ω_2, $\Omega_1 \leftarrow \Omega_2$ means "content of Ω_2 is assigned to Ω_1".

II. Problem Formulation

A. System Model

Consider a NDES composed by a plant, a group of sensors, a networked communication channel, and a remote supervisor (see Fig. 1). The NDES is described by a nondeterministic finite automaton (NFA) [1], [14], [15]:

$$G = (Q, \Sigma, f, Q_0) \tag{1}$$

where Q is a finite set of states, Σ is a finite set of events, $f : Q \times \Sigma \to 2^Q$ is a next-state transition function, and $Q_0 \subseteq Q$ is a set of initial states. Suppose the current state of G is $q \in Q$, the next state of G will move to $q' \in f(q, \sigma)$ with the occurrence of event $\sigma \in \Sigma$. The transition function can be extended to $f : Q \times \Sigma^* \to 2^Q$ in a usual way, where "Σ^*" denotes the set of all sequences of events from Σ. The behavior of G is described by its "language" :

$$L(G) = \{s \in \Sigma^* : f(q_0, s)!\} \tag{2}$$

where s is a string, $f(q, s)!$ means that $f(q, s)$ is defined, and initial state $q_0 \in Q_0$.

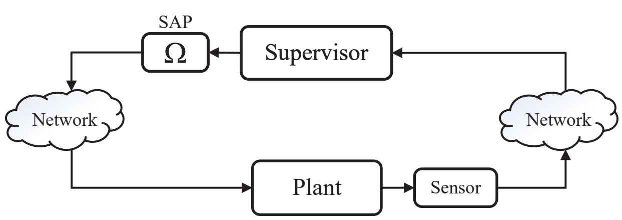


Fig. 1. The diagram of a NDES.

The plant is observed by a group of sensors. The set of events that deployed sensors is denoted by $\Sigma_o \subseteq \Sigma$, and the set of events that without deployed sensors is denoted by $\Sigma_{uo} = \Sigma \backslash \Sigma_o$. To save energy consumption, the activation of sensors is optional depending on the state of G. Only the sensor of the observable event is activated, then the event can be observed. The sensor activation policy is defined as an observation mapping [16]:

$$\Omega : Q \to 2^{\Sigma_o}. \tag{3}$$

Given a state $q \in Q$, the notation $\Omega(q)$ denotes the set of events that are observed, also known as the activated sensors when the plant is in state q. Thereafter, the mapping $\theta_A^\Omega : Q \times \Sigma^* \to \Sigma_o^*$ from a given state $q \in Q$ and a given string $s \in \Sigma^*$ to an observation sequence is defined recursively as follows:

$$\begin{aligned} \theta_A^\Omega(q, \sigma) &= \begin{cases} \sigma & \text{if } \sigma \in \Omega(q) \\ \varepsilon & \text{if } \sigma \notin \Omega(q) \end{cases} \\ \theta_A^\Omega(q, s\sigma) &= \begin{cases} \theta_A^\Omega(q, s) & \text{if } \sigma \notin \Omega(f(q, s)) \\ \theta_A^\Omega(q, s)\sigma & \text{if } \sigma \in \Omega(f(q, s)) \end{cases} \end{aligned} \tag{4}$$

If all sensors are always activated, then the mapping (4) from the initial state q_0 can be described by the natural projection $P : L(G) \to \Sigma_o^*$.

Then, the following two assumptions [17] are introduced without loss of generality.

Assumption 1: G is deadlock free, i.e., $\forall q \in Q$, $\exists \sigma \in \Sigma$ s.t. $f(q, \sigma)$ is defined.

Assumption 2: No loops in G contain only unobservable events, i.e., $\neg(\exists q \in Q, \exists s \in \Sigma_{uo}^*)$ s.t. $s \neq \varepsilon$, $q \in f(q, s)$.

According to Assumption 1, the length of a string $s \in L(G)$ can be infinite. The ω-language $L^\omega(G)$ [18] is defined as follows:

$$L^\omega(G) = \{s \in L(G) : s \text{ has an infinite length}\} \tag{5}$$

An infinite transition path [8] is denoted by $(q_0, s) \in Q_0 \times L^\omega(G)$. According to Assumptions 2, the length of the observation sequence for an infinite transition path is also infinite.

Note that the network characteristics of NDESs are not explicitly modeled in G. Instead, they are modeled as transmission delays [11], which are assumed to be bounded. The delayed observation with delays bounded by N steps (one step represents the occurrence of an event) is denoted as $\theta_D^N : \Sigma^* \to 2^{\Sigma^*}$. For a string $s \in \Sigma^*$,

$$\theta_D^N = \{s_{-i} : i = 0, 1, ..., N\} \tag{6}$$

where s_{-i} is the prefix of string s with the last i events removed. In other words, the transmission delays with N-steps will change an occurred string $s \in \Sigma^*$ to one of strings $\theta_D^N(s)$. Note that the superscript N will be removed if: $\theta_D = \theta_D^N$.

Combining the above definitions and descriptions, a new observation mapping is used to describe the composition of θ_A^Ω and θ_D, denoted by $\theta_{AD}^\Omega : Q \times \Sigma^* \to 2^{\Sigma_o^*}$, defined as

$$\theta_{AD}^\Omega = \theta_A^\Omega \circ \theta_D. \tag{7}$$

The observed language of a NDES G is denoted by:

$$\begin{aligned} L_0(G) = &\{s \in \Sigma_o^* : (\exists q \in Q_0)(\exists t \in \Sigma^*) \\ &\text{s.t. } \theta_{AD}^\Omega(q, t) = s,\ f(q, t)!\}. \end{aligned} \tag{8}$$

Remark 1: The transmission delay does not change the order of the event sequence.

B. Minimal sensor activation

The goal of this work is to design a policy, so that minimal sensor activation is achieved. Notice that the activation of a sensor usually reduces its energy and lifespan. So the activation of unnecessary sensors is wanted to reduce. It is well known that how to choose the optimal policy depends on the system itself and the task goal. The system in this paper is NDES and its observation mapping θ_{AD}^Ω differs from the related researches. Additionally, the task goal in this paper is to minimize sensor activation while maintaining system's state estimation performance (detectability [10], [13], [17], introduced in the following section).

Definition 1 (Minimal Sensor Activation Policy): Given a system G (1) and a transmission delay with upper bound, design a policy Ω (3) achieves minimal sensor activation policy Ω^* if and only if all the following conditions hold:

(i) Ω is feasible, i.e., same observed events in two paths make them indistinguishable, requiring policy Ω^* makes consistent decisions:

$$\forall q_1, q_2 \in Q_0, \forall s_1, s_2 \in \Sigma^* : \theta_{AD}^{\Omega}(q_1, s_1) = \theta_{AD}^{\Omega}(q_2, s_2) \Rightarrow \Omega(f(q_1, s_1)) = \Omega(f(q_2, s_2)).$$

(ii) Ω is valid with respect to the given detectability, i.e., the system remains its detectability after implementing Ω.

(iii) Ω is minimal [8], i.e., there is no other sub-policy of Ω that satisfies (i) and (ii).

Note that for a sensor activation policies Ω_1, another policy Ω_2 is thought to be its "sub-policy", iff $\forall q \in Q : \Omega_1(q) \supseteq \Omega_2(q)$. With this definition, the problem we solve in this paper is formally present as follows.

Problem: Given a system (1) and a transmission delay upper bound, design a policy (3) such that *minimal sensor activation policy* in Definition 1 is achieved.

III. Main results

A. Detectability Analysis

Suppose that the system G is in a set of possible states $Q' \subseteq Q$, when the observation of supervisor is $t \in \Sigma_o^*$, then the current state estimation is denoted by,

$$R(Q', t) = \{f(q', s) : (\exists q' \in Q')(\exists s \in \Sigma^*) \text{ s.t. } \theta_{AD}^{\Omega}(q', s) = t,\ f(q', s)!\}. \tag{9}$$

In particular, the unobservable range is defined as:

$$UR(Q') = R(Q', \varepsilon). \tag{10}$$

According to [10], two types detectabilities are relevant in this paper, which are defined as follows.

Definition 2 (Strong Networked Detectability): A nondeterministic NDES G is strongly networked detectable with respect to θ_{AD}^{Ω} if we can determine, after a finite number of observations, the current state and subsequent states for all trajectories of the system, i.e.,

$$\exists n \in \mathbb{N}, \forall (q, s) \in Q_0 \times L^{\omega}(G), \forall t \in \Pr(\theta_{AD}^{\Omega}(q, s)) \text{ s.t. } |t| > n \rightarrow |R(Q_0, t)| = 1. \tag{11}$$

Definition 3 (Strong Periodic Networked Detectability): A nondeterministic NDES G is strong periodic networked detectable with respect to θ_{AD}^{Ω} if we can periodically determine the current state for all trajectories of the system, i.e.,

$$\exists n \in \mathbb{N}, \forall (q, s) \in Q_0 \times L^{\omega}(G), \forall t \in \Pr(\theta_{AD}^{\Omega}(q, s)), \exists t' \in \Sigma^* \text{ s.t. } tt' \in \Pr(\theta_{AD}^{\Omega}(q, s)), |t'| < n \rightarrow |R(Q_0, tt')| = 1. \tag{12}$$

The above detectabilities are verified by constructing an "observer" [10], [17]. The observer constructed from G, denoted by G_{obs} [1]:

$$G_{obs} = (X, \Sigma_o, \delta, x_0) = AC(2^Q, \Sigma_o, \delta, UR(Q_0)) \tag{13}$$

where AC denotes the accessible part (the set of states reachable from the initial state through state transitions). Note that a state $x \in X$ is a subset of $Q (x \subseteq Q)$. For $x \subseteq Q$ and $\sigma \in \Sigma_o$, the transition function $\delta : X \times 2^{\Sigma_o} \rightarrow X$ is defined as:

$$\delta(x, \sigma) = UR(\{f(q, \sigma) : (\exists q \in x) \text{ s.t. } \sigma \in \Omega(q), f(q, \sigma)!\}).$$

Similar to [11], [13], the observers are made some developments to adapt to NDESs. The set of states reachable from $Q' \subseteq Q$ in less than or equal to N steps is denoted by:

$$D(Q') = \{q \in Q : (\exists q' \in Q')(\exists s \in \Sigma^*) \text{ s.t. } |s| \leq N, f(q', s) = q\}.$$

Then, we extend each state $x \in X$ to $D(x)$ and denote the resulting state as y. Y is the set of all such extensions. With operator D, a "delayed observer" is constructed:

$$G_{D,obs} = D(G_{obs}) = (Y, \Sigma_o, \zeta, y_0) \tag{14}$$

where $y_0 = D(x_0)$. For $y \in Y$ and $\sigma \in \Sigma_o$, the transition function ζ is defined as $\zeta = \{(y_i, \sigma, y_j) : (x_i, \sigma, x_j) \in \delta\}$.

Theorem 1: For a NDES G, there exist:

(i) $L_0(G) = L(G_{D,obs})$.

(ii) $\forall s \in L_0(G)$ s.t. $\zeta(y_0, s) = D(R(Q_0, s))$.

Proof: See Appendix.

Afterwards, we mark the states in $G_{D,obs}$ with a singleton state and label the set by:

$$Y_m = \{y \in Y : |y| = 1\}. \tag{15}$$

It is obvious that we can accurately determine the states of G when the states of $G_{D,obs}$ are in Y_m. Then, the set of all loops in $G_{D,obs}$ is denoted by

$$Loop = \{(y, u) \in Y \times \Sigma_o^* : |u| \geq 1, \zeta(y, u) = y\}. \tag{16}$$

Detectability of a NDES can be verified by using delayed observer, as shown in the following theorem.

Theorem 2: A nondeterministic NDES G is identified as:

(i) Strongly networked detectable with respect to θ_{AD}^{Ω}, iff in the delayed observer $G_{D,obs}$,

$$\forall (y, u) \in Loop, \forall w \in \Sigma_o^* \text{ s.t. } \zeta(y, w) \in Y_m. \tag{17}$$

In other words, any reachable state from any loop in $G_{D,obs}$ is in Y_m.

(ii) Strongly periodically networked detectable with respect to θ_{AD}^{Ω}, iff in the delayed observer $G_{D,obs}$,

$$\forall (y, u) \in Loop, \exists w \in \Pr(u) \text{ s.t. } \zeta(y, w) \in Y_m. \tag{18}$$

In other words, each loop in $G_{D,obs}$ contains at least one state belonging to Y_m.

Proof: See Appendix.

This article has been accepted for publication in IEEE Control Systems Letters. This is the author's version which has not been fully edited and content may change prior to final publication. Citation information: DOI 10.1109/LCSYS.2024.3376458

B. Minimal sensor activation policy

Based on Definition 1, the theorems for determining the minimal sensor activation policy are proposed as follows. These theorems are made some adjustments to adapt to NDESs, such as the observer we use in this paper is delayed observer $G_{D,obs}$. The proofs of these theorems are omitted because similar proofs can be found in [8].

Theorem 3: A sensor activation policy Ω is feasible, iff the delayed observer $G_{D,obs}$ with respect to Ω satisfies:

$$\forall y \in Y_m, \forall q, q' \in y \text{ s.t. } \Omega(q) = \Omega(q'). \tag{19}$$

Theorem 4: If some finite sensor activation policies $\Omega_1, \Omega_2, ..., \Omega_n$ are all feasible, then $\bigcup_{i=1}^{n} \Omega_i$ is also feasible, where $n \geq 1$ is arbitrary.

Theorem 5: If some finite sensor activation policies $\Omega_1, \Omega_2, ..., \Omega_n$ are all valid with respect to a given networked detectability, then $\bigcup_{i=1}^{n} \Omega_i$ is also valid with respect to the given networked detectability, where $n \geq 1$ is arbitrary.

Note that the minimum feasible sensor activation policy Ω_{min} and the maximum feasible sensor activation policy Ω_{max} are given by

$$\forall q \in Q \text{ s.t. } \Omega_{min}(q) = \emptyset, \text{and } \forall q \in Q \text{ s.t. } \Omega_{max}(q) = \Sigma_o.$$

Further, the method in this paper to determine the minimal sensor activation policy Ω^* is formally stated as follows. By using enumerations, we start with the maximum feasible sensor activation policy Ω_{max} and remove events from it in turn to derive its sub-policies. Since the removal of events may result in its sub-policies becoming infeasible, additional events have to be removed to counteract it. After obtaining its feasible sub-policy, its validity can be verified with Theorem 2. The maximum feasible sub-policy is defined as follows.

Definition 4 (Maximum Feasible Sub-policy, [8]): For a given sensor activation Ω, if $\Omega_1, \Omega_2, ..., \Omega_n$ are all its feasible sub-policies, the maximum feasible sub-policy of Ω is

$$\Omega^{\uparrow f} = \bigcup_{i=1}^{n} \Omega_i \tag{20}$$

where $\Omega^{\uparrow f}$ satisfy that: (1) $\Omega^{\uparrow f} \leq \Omega$; (2) $\Omega' \leq \Omega^{\uparrow f}$ for each feasible sub-policy Ω'.

Theorem 6: The $\Omega^{\uparrow f}$ obtained by Algorithm 1 is the maximum feasible sub-policy of a given policy Ω_0.

Proof: See Appendix.

The algorithm to determine the minimal sensor activation policy of a detectable NDES is as follows. The proof is omitted because similar proof can be found in [8].

Theorem 7: The Ω^* obtained by Algorithm 2 is the minimal sensor activation policy of a given NDES G.

Remark 2: Notice that the confusing state pair set T in Algorithm 1 contains only state pairs with different elements: $(q, q') \in Q \times Q \wedge q \neq q'$, which means that the state pairs with same elements $(q, q') \in Q \times Q \wedge q = q'$ are no longer identified as in [8], [16]. It is obvious that the state pairs with same elements do not aid in determining $\Omega^{\uparrow f}$ but only inflate the inventory unnecessarily.

Remark 3: As steps 7 to 10 of Algorithm 2 require a judgement on "valid", i.e. whether or not to preserve the original detectability, Algorithm 2 is different for two systems with different detectability.

Algorithm 1 Maximum feasible sub-policy $\Omega^{\uparrow f}$.

Input: A NDES G, a transmission delay with upper bound, and a sensor activation policy Ω_0.
Output: The maximum feasible sub-policy $\Omega_0^{\uparrow f}$ of Ω_0.

1: Initially, set $\Omega = \Omega_0$;
2: Construct the $G_{D,obs}$ based on Ω_0 ;
3: **for all** $y \in Y$ **do**
4:   **if** $|y|>1$ **then** then set
    $T \leftarrow T \cup \{(q, q') \in Q \times Q : q, q' \in y \wedge q \neq q'\}$;
  else $T \leftarrow \emptyset$
5: **end for**
6: **for all** $(q, q') \in T$ **do**
7:   **if** $\exists \sigma \in \Sigma_o$ s.t. $\sigma \in \Omega(q), \sigma \notin \Omega(q')$ **then**
    $\Omega \leftarrow \Omega : \Omega(q) \leftarrow \Omega(q) \backslash \sigma$;
  else Ω not change.
8: **end for**
9: Set $\Omega_0^{\uparrow f} \leftarrow \Omega$.

Algorithm 2 Minimal sensor activation policy Ω^*.

Input: A detectable NDES G.
Output: The minimal sensor activation policy Ω^* of G.

1: Initially, set $\bar{\Omega} \leftarrow \Omega_{max}$ and $\tilde{\Omega} \leftarrow \Omega_{max}$;
2: **for** $i = 1, 2, ..., I (I = |Y|)$ **do**
3:   **for** $j = 1, 2, ..., J (J = |\Sigma_o|)$ **do**
4:     $\tilde{\Omega} \leftarrow \bar{\Omega}$;
5:     $\bar{\Omega} \leftarrow \bar{\Omega} : \bar{\Omega}(q_i) \leftarrow \bar{\Omega}(q_i) \backslash \sigma_j$;
6:     Calculate the maximum feasible sub-policy $\bar{\Omega}^{\uparrow f}$ of $\bar{\Omega}$ using Algorithm 1.
7:     **if** $\bar{\Omega}^{\uparrow f}$ is valid **then**
8:       $\bar{\Omega} \leftarrow \bar{\Omega}^{\uparrow f}$;
9:       $\tilde{\Omega} \leftarrow \bar{\Omega}^{\uparrow f}$;
10:     **else** $\bar{\Omega} \leftarrow \tilde{\Omega}$;
11:   **end for**
12: **end for**
13: Set $\Omega^* \leftarrow \bar{\Omega}$.

IV. Examples

Example 1: Consider a NDES in Fig. 2. Assume that the initial state $Q_0 = \{q_1\}$, $\Sigma_o = \Sigma = \{a, b, c, d\}$, and the transmission delay is bounded 1 step. A given sensor activation policy Ω: $\Omega(q_1) = \{a\}, \Omega(q_2) = \{b\}, \Omega(q_3) = \{d\}$. Let's verify the system's detectability with Theorem 2.

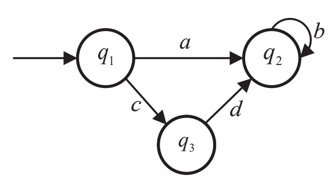
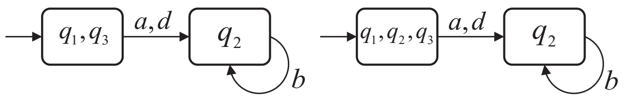


Fig. 2. System G of Example 1.

The observer G_{obs} is constructed (see Fig. 3 (a)), where event c is replaced with ε. Then, the delayed observer $G_{D,obs}$ is constructed (see Fig. 3 (b)). The system is strongly networked detectable because $\forall q \in Loop$ $(\{q_2\}, c)$ are in Y_m.

This article has been accepted for publication in IEEE Control Systems Letters. This is the author's version which has not been fully edited and content may change prior to final publication. Citation information: DOI 10.1109/LCSYS.2024.3376458



(a) (b)

Fig. 3. Example 1: (a) Observer $\boldsymbol{G_{obs}}$, and (b) delayed observer $\boldsymbol{G_{D,obs}}$.

Example 2: Consider the NDES in Fig. 2. A given sensor activation policy Ω_0: $\Omega(q_1) = \{a, c\}, \Omega(q_2) = \{a, b, c, d\}, \Omega(q_3) = \{a, b, c, d\}$. Let's determine the maximum feasible sub-policy $\Omega^{\uparrow f}$ of Ω_0 by using Algorithm 2.

Firstly, the delayed observer $G_{D,obs}$ is constructed (see Fig. 4). The confusing state pair set T contains confusing state pairs (q_1, q_2), (q_1, q_3) and (q_2, q_3). Thus, the mappings of state q_2 and q_3 are modified: $\Omega(q_2) \leftarrow \Omega(q_2)\backslash\{b, d\}$, $\Omega(q_3) \leftarrow \Omega(q_3)\backslash\{b, d\}$. Then the new policy is:

$$\Omega : \Omega(q_1) = \{a, c\},\ \Omega(q_2) = \{a, c\},\ \Omega(q_3) = \{a, c\}.$$

Ω is maximum feasible sub-policy $\Omega_0^{\uparrow f}$ because it is feasible.

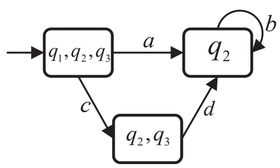


Fig. 4. Example 2: Delayed observer $\boldsymbol{G_{D,obs}}$.

Example 3: Consider the NDES in Fig. 2. It is obvious that the system is strongly networked detectable after implementing Ω_{max}. Let's determine Ω^* by using Algorithm 2.

Firstly, we pick $q_1 \in Q$ and $a \in \Sigma_o$. By using Algorithm 1, its maximum feasible sub-policy $\bar{\Omega}^{\uparrow f}$ is obtainaed as follows.

$$\bar{\Omega}^{\uparrow f} : \bar{\Omega}(q_1) = \{b, c, d\},\ \bar{\Omega}(q_2) = \{b, c, d\},\ \bar{\Omega}(q_3) = \{b, c, d\}.$$

The delayed observer is constructed (see Fig. 5 (a)), from which we can see that the system G is still strongly networked detectable. Therefore, the policy is valid.

Next, we pick $q_1 \in Q$ and $b \in \Sigma_o$. Its maximum feasible sub-policy $\bar{\Omega}^{\uparrow f}$ is obtained by using Algorithm 1 as follows.

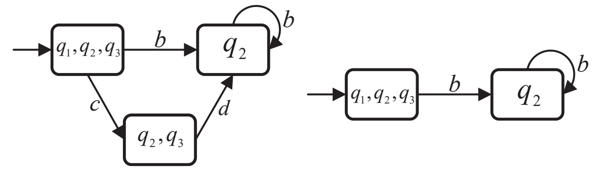
$$\bar{\Omega}^{\uparrow f} : \bar{\Omega}(q_1) = \{c, d\},\ \bar{\Omega}(q_2) = \{c, d\},\ \bar{\Omega}(q_3) = \{c, d\}.$$

Since there is a loop in the system containing only unobservable events, Assumption 2 is contradicted. Therefore, the new sensor activation policy is not valid.

The minimal sensor activation policy Ω^* is obtained as follows. The delayed observer of it is shown in Fig. 5 (b).

$$\Omega^* : \Omega(q_1) = \{b\}, \Omega(q_2) = \{b\}, \Omega(q_3) = \{b\}.$$

Compared with maximum feasible sensor activation policy $\Omega_{max} : \Omega_{max}(q_1) = \Omega_{max}(q_2) = \Omega_{max}(q_3) = \Omega_{max}(q_4) = \{a, b, c, d\}$, Ω^* significantly reduce sensor activation.



(a) (b)

Fig. 5. Example 3: (a) Delayed observer $\boldsymbol{G_{D,obs}}$ after first iteration, and (b) delayed observer $\boldsymbol{G_{D,obs}}$ with respect to $\boldsymbol{\Omega^*}$.

V. Conclusions

In this paper we considered the problem of sensor activation in the NDESs, where communication between the plant and the supervisor is via a network that is affected by delays. Two types of new detectabilities were defined to describe the state estimation performance of a NDES with a given sensor activation policy. The necessary and sufficient conditions to check above detectabilities were derived. Subsequently, minimizing sensor activation while maintaining the detectabilities of NDESs was investigated and the algorithms to obtain the minimal sensor activation policy were proposed.

VI. Appendix

A. Proof of Theorem 1

(i) Base: For $|s| = 0 (s = \varepsilon)$, both $\varepsilon \in L_0(G)$ and $\varepsilon \in L(G_{D,obs})$ are true. That is, $\varepsilon \in L_0(G) \Leftrightarrow \varepsilon \in L(G_{D,obs})$.

Induction hypothesis: Suppose that $\forall s \in \Sigma^*$ s.t. $|s| \leq n$, it satisfies $s \in L_0(G) \Leftrightarrow s \in L(G_{D,obs})$.

Induction step ([8]): Prove that $\forall s\sigma \in \Sigma^*$ s.t. $|s\sigma| = n+1$, it satisfies $s\sigma \in L_0(G) \Leftrightarrow s\sigma \in L(G_{D,obs})$. Indeed,

$$\begin{aligned} s\sigma \in L_0(G) &\Leftrightarrow s \in L_0(G) \wedge s\sigma \in L_0(G) \\ &\Leftrightarrow s \in L(G_{D,obs}) \wedge s\sigma \in L_0(G) \\ &\Leftrightarrow \zeta(y_0, s)! \wedge \exists q \in Q_0, \exists t\sigma \in \Sigma^* : \theta_{AD}^{\Omega}(q, t) = s \\ &\quad \wedge \zeta(y_0, s\sigma)! \Leftrightarrow \zeta(y_0, s\sigma)! \Leftrightarrow s\sigma \in L(G_{D,obs}). \end{aligned}$$

(ii) Base: For $|s| = 0 (s = \varepsilon)$, we have, by definition $\zeta(y_0, \varepsilon) = y_0 = D(x_0) = D(UR(Q_0)) = D(R(Q_0, \varepsilon))$.

Induction hypothesis: Suppose that $\forall s \in Y_0(G)$ s.t. $|s| \leq n$, it satisfies $\zeta(y_0, s) = D(R(Q_0, s))$.

Induction step: We need to prove that $\forall s\sigma \in L_0(G)$ s.t. $|s\sigma| = n + 1$, it satisfies $\zeta(y_0, s\sigma) = D(R(Q_0, s\sigma))$. Indeed,

$$\begin{aligned} \zeta(y_0, s\sigma) =& \zeta(\zeta(y_0, s), \sigma) = \zeta(D(R(Q_0, s), \sigma)) \\ =& D(R(\{f(q, \sigma) : (\exists q \in R(Q_0, s)) f(q, \sigma)! \\ & \wedge \sigma \in \Omega(q)\}, \varepsilon)) = D(R(Q_0, s\sigma)). \end{aligned}$$

B. Proof of Theorem 2

(i) Sufficiency: Suppose that the NDES G is not strongly networked dectable with respect to θ_{AD}^{Ω}, that is:

$$\forall n \in \mathbb{N}, \exists (q, s) \in Q_0 \times L^{\omega}(G), \exists t \in \Pr\left(\theta_{AD}^{\Omega}(q, s)\right) \\ \text{s.t. } |t| > n \rightarrow |R(Q_0, t)| \neq 1.$$

Let n be sufficiently large, the string t must go through at least one loop in the delayed observer $G_{D,obs}$. Define the first loop as $(y,u) \in Loop$. Clearly, t will pass y first: $\exists w \in \Sigma_o^*, \exists v \in \Sigma_o^*$ s.t. $t = vw \wedge \zeta(y_0, v) = y$. $\forall t : \zeta(y_0,t) = \zeta(y_0, vw) = \zeta(y,w)$. Moreover, $|R(Q_0,t)| \neq 1 \Rightarrow |\zeta(y,w)| \neq 1 \Rightarrow \zeta(y,w) \notin Y_m$. Hence,

$$\exists (y,u) \in Loop, \exists w \in \Sigma_o^* \text{ s.t. } \zeta(y,w) \notin Y_m.$$

Necessity: Assume that $\forall (y,u) \in Loop, \forall w \in \Sigma_o^*$ s.t. $\zeta(y, w) \in Y_m$ is not true, that is,

$$\exists (y,u) \in Loop, \exists w \in \Sigma_o^* \text{ s.t. } \zeta(y,w) \notin Y_m.$$

Let v to be any string heading to y from the initial state: $\zeta(y_0,v) = y$. $\forall n \in \mathbb{N}$, there exist $s \in \theta_{AD}^{-1}(vu^n w \ldots) \cap L^\omega(G)$ and $t = vu^n w \in \Pr(\theta_{AD}^\Omega(q,s))$ s.t. $\zeta(y_0,t) = \zeta(y_0, vu^n w) = \zeta(y, u^n w) = \zeta(y,w) \notin Y_m$. Hence,

$$\begin{aligned}
&\forall n \in \mathbb{N}, \exists (q,s) \in Q_0 \times L^\omega(G), \exists t \in \Pr(\theta_{AD}^\Omega(q,s)) \\
&\quad \text{s.t. } |t| > n \rightarrow \zeta(y_0,t) \notin Y_m. \\
\Rightarrow &\forall n \in \mathbb{N}, \exists (q,s) \in Q_0 \times L^\omega(G), \exists t \in \Pr(\theta_{AD}^\Omega(q,s)) \\
&\quad \text{s.t. } |t| > n \rightarrow |R(Q_0,t)| \neq 1.
\end{aligned}$$

(ii) Sufficiency: Suppose that the NDES G is not strongly periodically networked detectable with respect to θ_{AD}^Ω, then,

$$\begin{aligned}
&\forall n \in \mathbb{N}, \exists (q,s) \in Q_0 \times L^\omega(G), \exists t \in \Pr(\theta_{AD}^\Omega(q,s)), \\
&\exists t' \in \Sigma^* \text{ s.t. } tt' \in \Pr(\theta_{AD}^\Omega(q,s))) \\
&\wedge |P(t')| < n \rightarrow |R(Q_0, tt')| \neq 1.
\end{aligned}$$

Assume that $n = |Y_m| + 1$, it satisfies,

$$\begin{aligned}
&\exists (q,s) \in Q_0 \times L^\omega(G), \exists t \in \Pr(\theta_{AD}^\Omega(q,s)), \exists t' \in \Sigma^* \\
&\text{s.t. } tt' \in \Pr(\theta_{AD}^\Omega(q,s))) \wedge |P(t')| < |Y_m| + 1 \\
&\rightarrow |R(Q_0, tt')| \neq 1.
\end{aligned}$$

Consider the next $n = |Y_m|+1$ states after t in $G_{D,obs}$ on the path of t', since $|R(Q_0,t)| \neq 1$ all these states do not belong to Y_m. Since the path t' is greater than $|Y_m|$, it must contain a loop. Denote this loop by $(y,u) \in loop$. Since all visited by (y,u) do not belong to Y_m,

$$\exists (y,u) \in Loop, \forall w \in \Pr(u) : \zeta(y,w) \notin Y_m.$$

Necessity: Consider $\forall (y,u) \in Loop, \exists w \in \Pr(u)$ s.t. $\zeta(y, w) \in Y_m$ is not true. Let v to be any string heading to y from the initial state: $\zeta(y_0,v) = y$. In this case, $\forall n \in \mathbb{N}$, $\exists s \in \theta_{AD}^{-1}(vuuu\ldots) \cap L^\omega(G)$, and $t = v$ s.t. we can let t' to travel the loop (y,u) sufficient number of times so that the following is true:

$$\begin{aligned}
&\forall n \in \mathbb{N}, \exists (q,s) \in Q_0 \times L^\omega(G), \exists t \in \Pr(\theta_{AD}^\Omega(q,s)), \\
&\exists t' \in \Sigma^* \text{ s.t. } tt' \in Pr(\theta_{AD}^\Omega(q,s)) \wedge |P(t')| < n \\
&\rightarrow \zeta(y_0, tt') = \zeta(y,t') \notin Y_m.
\end{aligned}$$

Which implies,

$$\begin{aligned}
&\forall n \in \mathbb{N}, \exists (q,s) \in Q_0 \times L^\omega(G), \exists t \in \Pr(\theta_{AD}^\Omega(q,s)), \\
&\exists t' \in \Sigma^* \text{ s.t. } tt' \in \Pr(\theta_{AD}^\Omega(q,s)) \wedge |P(t')| < n \\
&\rightarrow |R(Q_0, tt')| \neq 1.
\end{aligned}$$

C. Proof of Theorem 6

Once Algorithm 1 convergence, the resultant Ω will be $\Omega_0^{\uparrow f}$. Its corresponding T will satisfy following requirements.

$$\begin{aligned}
&T \leftarrow T \cup \{(q,q') \in Q \times Q : q, q' \in y \wedge q \neq q'\}. \\
&\forall (q,q') \in T, \forall \sigma \in \Sigma_o \text{ s.t. } \sigma \in \Omega(q), \Omega \in \Omega(q').
\end{aligned}$$

From the above equations, $\forall q, q' \in Q_0, \forall s, s' \in \Sigma^*$, we have

$$\begin{aligned}
&\theta_{AD}^\Omega(q,s) = \theta_{AD}^\Omega(q',s') \Rightarrow (f(q,s), f(q',s')) \in T \\
\Rightarrow &\forall \sigma \in \Sigma_o \text{ s.t. } \sigma \in \Omega(f(q,s)) \Leftrightarrow \sigma \in \Omega(f(q',s')) \\
\Rightarrow &\Omega(f(q,s)) = \Omega(f(q',s')).
\end{aligned}$$

That is, Definition 1 is satisfied and $\Omega = \Omega_0^{\uparrow f}$ is the maximum sub-policy because Algorithm 2 starts with $\Omega = \Omega_0$ and remove transitions from Ω only when it is necessary.

References

[1] C. G. Cassandras and S. Lafortune, "Introduction to discrete event systems," in *The Kluwer International Series on Discrete Event Dynamic Systems*, 1999.
[2] X. Yin, "Estimation and verification of partially-observed discrete-event systems," *arXiv preprint arXiv:1903.11413*, 2019.
[3] B. J. McCarragher and H. Asada, "A discrete event approach to the control of robotic assembly tasks," in *Proceedings IEEE International Conference on Robotics and Automation*. IEEE, 1993, pp. 331–336.
[4] F. Cassez and S. Tripakis, "Fault diagnosis with static and dynamic observers," *Fundamenta informaticae*, vol. 88, no. 4, pp. 497–540, 2008.
[5] W. Wang, S. Lafortune, A. R. Girard, and F. Lin, "Optimal sensor activation for diagnosing discrete event systems," *Automatica*, vol. 46, no. 7, pp. 1165–1175, 2010.
[6] W. Wang, S. Lafortune, F. Lin, and A. R. Girard, "Minimization of dynamic sensor activation in discrete event systems for the purpose of control," *IEEE Transactions on Automatic Control*, vol. 55, no. 11, pp. 2447–2461, 2010.
[7] J. Xin, Y. Jiang, and S. Shu, "Minimal k-step event observation policy for on-line observability of discrete event systems," in *Proceedings of the 29th Chinese control conference*. IEEE, 2010, pp. 1476–1482.
[8] S. Shu and F. Lin, "Detectability of discrete event systems with dynamic event observation," *Systems & control letters*, vol. 59, no. 1, pp. 9–17, 2010.
[9] S. Shu, Z. Huang, and F. Lin, "Online sensor activation for detectability of discrete event systems," *IEEE Transactions on Automation Science and Engineering*, vol. 10, no. 2, pp. 457–461, 2013.
[10] S. Shu, F. Lin, and H. Ying, "Detectability of discrete event systems," *IEEE Transactions on Automatic Control*, vol. 52, no. 12, pp. 2356–2359, 2007.
[11] F. Lin, "Control of networked discrete event systems: Dealing with communication delays and losses," *SIAM Journal on Control and Optimization*, vol. 52, no. 2, pp. 1276–1298, 2014.
[12] Z.-M. Li, X.-H. Chang, and J. H. Park, "Quantized static output feedback fuzzy tracking control for discrete-time nonlinear networked systems with asynchronous event-triggered constraints," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 51, no. 6, pp. 3820–3831, 2019.
[13] Y. Sasi and F. Lin, "Detectability of networked discrete event systems," *Discrete Event Dynamic Systems*, vol. 28, pp. 449–470, 2018.
[14] F. Lin and W. M. Wonham, "On observability of discrete-event systems," *Inf. Sci.*, vol. 44, pp. 173–198, 1988.
[15] P. J. Ramadge and W. M. Wonham, "Supervisory control of a class of discrete event processes," *Siam Journal on Control and Optimization*, vol. 25, pp. 206–230, 1987.
[16] W. Wang, S. Lafortune, and F. Lin, "Optimal sensor activation in controlled discrete event systems," in *2008 47th IEEE Conference on Decision and Control*. IEEE, 2008, pp. 877–882.
[17] Shu, Shaolong and Lin, Feng, "Generalized detectability for discrete event systems," *Systems & control letters*, vol. 60, no. 5, pp. 310–317, 2011.
[18] J. G. Thistle and W. M. Wonham, "Control of infinite behavior of finite automata," *SIAM Journal on Control and Optimization*, vol. 32, no. 4, pp. 1075–1097, 1994.