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Fusion estimation under binary sensors ☆

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ABSTRACT

Binary sensors compress the measurement information into two possible values as output and have found applications in many fields such as medical monitoring, environment monitoring and localization. This paper is concerned with the fusion estimation problem for a class of binary sensor systems with bounded noises, where the monitored physical process is described by time-varying state-space models. Since binary sensors only provide the valid information at the switching instant, a novel uncertain measurement model is proposed to characterize this feature. Then, by constructing an upper bound of fusion/estimation error at each time, convex optimization problems are established in terms of linear matrix inequalities to design stable centralized and distributed fusion estimators. An illustrative example is given to show the effectiveness of the proposed methods.

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1. Introduction

Multi-sensor fusion estimation is one of the most important focuses in information fusion and has been widely applied to practical engineering tasks such as target tracking, integrated navigation, industrial monitoring, fault-tolerant control (Sun, Lin, Ma, & Li, 2017). The aim of fusion estimation is to best estimate a quantity (i.e., a parameter or process) by utilizing useful information contained in multiple sets of data (Li, Zhu, Wang, & Han, 2003). Generally, there exist two fundamental fusion estimation structures: centralized fusion structure and distributed fusion structure. The centralized fusion estimation processes all raw sensor measurements in the fusion center(FC) to provide optimal estimation performance, while the distributed fusion estimation firstly processes each local sensor measurements to obtain local estimates and then fuses them in the FC by designing some fusion criteria. Since distributed fusion estimation has better robustness and reliability, more attention was focused on this fusion structure. In this case, different distributed fusion estimation methods have been developed (see Chen, Hu, Ho, & Yu, 2019; Noack, Sijs, Reinhardt, & Hanebeck, 2017, and the reference therein).

Recently, networked multi-sensor fusion estimation has attracted much research interest because the introduction of communication networks can bring various advantages including flexible architecture, the reduction of installation and maintenance costs, decreasing the implementation difficulties (Chen, Zhang, & Yu, 2014b). However, under the finite bandwidth capacity in network environments, the required length of data packets for regular sensors, whose outputs are combinations of state components, may be limited to certain ranges. Therefore, several methods have been developed to reduce the sizes of the data packets based on regular sensors, including quantization methods (Chen, Yu, Zhang and Wang, 2014; Ribeiro, Giannakis, & Roumeliotis, 2006) and dimensionality reduction methods (Chen, Zhang, & Yu, 2014a; Chen, Zhang, Yu, Hu, & Song, 2015). Meanwhile, binary sensors compress the measurement information into one bit automatically, and thus the bandwidth constrain problem can be directly avoided when utilizing fusion estimation algorithms based on binary sensors. However, how to design stable fusion estimators under binary sensors is not yet solved.

Binary sensors have also been commonly employed in many practical systems including exhaust gas oxygen sensors in individual cylinder air–fuel ratio control (Grizzle, Cook, & Dobbibs, 1991) and available bit rate traffic control (Schwiebert & Wang, 2001). Moreover, it has attracted more and more research attention in different fields such as system identification (Wang, Zhang, & Yin, 2003; Zhao, Chen, Tempo, & Dabbene, 2017), state construction (Wang, Li, Yin, Guo, & Xu, 2011; Wang, Xu, & Yin, 2008), target tracking and localization (Bai, 2018; Djuric, Vemula, & Bugallo, 2008). Particularly, the moving horizon estimation method with binary sensors for linear systems was developed and the corresponding stability conditions were derived in Battistelli,

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Chisci, and Gherardini (2017), while the state estimation problem was addressed in Koutsoukos (2003) for nonlinear systems with binary sensors. Notice that the assumption of Gaussian white noises was required to be satisfied in Koutsoukos (2003), while the accurate covariances may not be obtained in practical applications. Meanwhile, the work in Battistelli et al. (2017) proposed an uncertain linear measurement at the switching instant and avoided dealing with uncertainty in measurements by moving horizon estimation approach. However, the ignorance of uncertainty may lead to lower estimation accuracy.

To overcome the drawback of Gaussian white noises assumption, the energy-bounded noises (Chen, Hu, Ho, Zhang and Yu, 2017; Liang, Chen, & Pan, 2010) and bounded noises (Chen, Ho, Zhang and Yu, 2017; Chen et al., 2019) have been considered when designing fusion estimators. It should be pointed out that the bounded noises, which do not need any statistical information and bounds of noises, can be easily satisfied in practical applications. This motivates us to study the fusion estimation problem under binary sensors with bounded noises. The main contributions can be summarized as follows: (i) For linear time-varying systems, a novel uncertain measurement model is proposed to depict the effective information from binary sensors; (ii) Under bounded noises, stable centralized and distributed fusion estimators for binary sensor systems are designed by establishing different convex optimization problems that can be easily solved by standard software packages.

Notations: The superscript 'T' represents the transpose, while 'I' represents the identity matrix with appropriate dimensions. $X > 0$ denotes a positive-definite matrix, while diag$\{\cdot\}$ stands for a block diagonal matrix. $\|A\|_2$ is the 2-norm of matrix A, and col$\{a_1,\ldots,a_n\}$ means a column vector whose elements are $a_1,\ldots,a_n$. The symmetric terms in a symmetric matrix are denoted by '$*$', and Tr(A) represents the trace of the matrix A.

2. Problem formulation

Consider the following linear time-varying systems

$$x(k+1)=A(k)x(k)+B(k)w(k) \tag{1}$$

$$z_i(k)=C_i(k)x(k)+D_i(k)v_i(k),\quad i=1,\ldots,L \tag{2}$$

where $x(k)\in\mathbb{R}^n$ is the state of the process, $z_i(k)\in\mathbb{R}$ is the sensed variable, $A(k)$, $B(k)$, $C_i(k)$, $D_i(k)$ are time-varying matrices with appropriate dimensions. The ith binary sensor gives a binary measurement $y_i(k)\in\{-1,+1\}$ according to whether $z_i(k)$ falls below or above the ith binary sensor's threshold τ_i, i.e.,

$$y_i(k)=h_i(z_i(k))=\begin{cases}+1, & \text{if } z_i(k)\geq\tau_i\\ -1, & \text{if } z_i(k)<\tau_i\end{cases} \tag{3}$$

It is considered that the thresholds of binary sensors are time invariant. In most situations, binary sensors hardly provide valid information from outputs of one bit. However, when the binary signal $y_i(k)$ changes the sign (i.e., $y_i(k)y_i(k-1)<0$) at the switching instant, a new uncertainty measurement that describes the relationship of the thresholds and the sensed variables can be further developed. Inspired by the form of binary sensor measurement in (3), the threshold τ_i must fall into the interval between $z_i(k)$ and $z_i(k-1)$ at these instants. Then, τ_i is proposed at the switching instant in this paper to satisfy

$$\tau_i=(0.5+\alpha_i(k))z_i(k)+(0.5-\alpha_i(k))z_i(k-1) \tag{4}$$

where τ_i is expressed as a convex combination of $z_i(k)$ and $z_i(k-1)$, and the introduced parameter $\alpha_i(k)\in[-0.5,\ 0.5]$ is an uncertain parameter whose exact value is unknown. At time k, let $\Im_k$ be the set that contains sensors with switching, while $\Im_k^C$ is denoted as the set that contains sensors with non-switching. Then, $\Im_k$ and $\Im_k^C$ are given by:

$$\Im_k\triangleq\{i|y_i(k-1)y_i(k)<0\} \tag{5}$$

$$\Im_k^C\triangleq\{i|y_i(k-1)y_i(k)>0\} \tag{6}$$

In this case, when $\Im_k$ is not empty, it follows from (2)–(4) that τ_i can be expressed by the following uncertain equality equation at time k:

$$\begin{aligned}\tau_i=&(0.5+\alpha_i(k))C_i(k)x(k)+(0.5+\alpha_i(k))D_i(k)v_i(k)\\&+(0.5-\alpha_i(k))C_i(k-1)x(k-1)\\&+(0.5-\alpha_i(k))D_i(k-1)v_i(k-1)\ \ i\in\Im_k\end{aligned} \tag{7}$$

Consequently, based on the measurement information (7), the aim of this paper is to design stable centralized and distributed fusion estimators such that an upper bound of estimation error is minimized at each time and the fusion estimation errors are asymptotically bounded.

Assumption 1. $w(k)$ and $v_i(k)$ are bounded noises with unknown bounds, i.e., $w^{\mathrm{T}}(k)w(k)\leq\delta_w$, $v_i^{\mathrm{T}}(k)v_i(k)\leq\delta_{v_i}$, where δ_w and δ_{v_i} are unknown.

Remark 1. In Battistelli et al. (2017), the original uncertain linear measurement was given by "$\tau_i=C_i(k)x(k)+\delta_i(k)+n_i(k)$", where $n_i(k)$ contains all the noise terms in (7), and $\delta_i(k)\triangleq(-0.5+\alpha_i(k))C_i(k)x(k)+(0.5-\alpha_i(k))C_i(k-1)x(k-1)$ represents the uncertain term. Notice that the uncertainty can indeed affect the estimation performance, but it is difficult to deal with the uncertain term when the measurement model is described by the equality equation just mentioned. Since the cost function of the moving horizon estimation in Battistelli et al. (2017) is designed regardless of the uncertainty, it is avoided directly dealing with the uncertain term $\delta_i(k)$. Different from the idea in Battistelli et al. (2017), the uncertain term $\delta_i(k)$ is remodeled by the state-dependent parameter uncertainties in this paper, and a novel fusion estimation strategy is developed by the matrix theory and augmentation method. Moreover, the designed binary fusion algorithm in this paper does not require any statistical information of the noises, and the upper and lower bounds of the noises also do not need to be known. This means that the addressed noises can be easily satisfied in practical applications.

3. Main results

3.1. Centralized fusion estimators

Let $m(k)$ elements of $\Im_k$ be indexed from 1 to $m(k)$, where $m(k)$ is the number of sensors in $\Im_k$. Define $\tau(k)\triangleq\text{col}\left\{\tau_1,\ \ldots,\ \tau_{m(k)}\right\}$. Then, one has by (7) that the augmented uncertain equality equation can be written as:

$$\begin{aligned}\tau(k)=&\left(\overline{C}(k)+\Delta\alpha(k)\overline{C}(k)\right)x(k)+\left(\overline{D}(k)+\Delta\alpha(k)\overline{D}(k)\right)\overline{v}(k)\\&+\ \left(\underline{C}(k)-\Delta\alpha(k)\underline{C}(k)\right)x(k-1)\\&+\ \left(\underline{D}(k)-\Delta\alpha(k)\underline{D}(k)\right)\underline{v}(k)\end{aligned} \tag{8}$$

where

$$\begin{cases}\overline{v}(k)\triangleq\text{col}\left\{v_1(k),\ \ldots,\ v_{m(k)}(k)\right\}\\ \underline{v}(k)\triangleq\text{col}\left\{v_1(k-1),\ \ldots,\ v_{m(k)}(k-1)\right\}\\ \Delta\alpha(k)\triangleq2\text{diag}\left\{\alpha_1(k),\ \ldots,\ \alpha_{m(k)}(k)\right\}\\ \overline{C}(k)\triangleq0.5\text{col}\left\{C_1(k),\ \ldots,\ C_{m(k)}(k)\right\}\\ \underline{C}(k)\triangleq0.5\text{col}\left\{C_1(k-1),\ \ldots,\ C_{m(k)}(k-1)\right\}\\ \overline{D}(k)\triangleq0.5\text{diag}\left\{D_1(k),\ \ldots,\ D_{m(k)}(k)\right\}\\ \underline{D}(k)\triangleq0.5\text{diag}\left\{D_1(k-1),\ \ldots,\ D_{m(k)}(k-1)\right\}\end{cases} \tag{9}$$

When $m(k) > 0$, the centralized fusion estimator(CFE) is designed by the following recursive form:

$$\begin{aligned}\hat{x}_c(k) =& A(k-1)\hat{x}_c(k-1) + K(k) \\ &\times \left[\tau(k) - \overline{C}(k)A(k-1)\hat{x}_c(k-1) - \underline{C}(k)\hat{x}_c(k-1)\right]\end{aligned} \tag{10}$$

Then, the estimation error $e_c(k) \triangleq x(k) - \hat{x}_c(k)$ is given by

$$e_c(k) = A_c(k)e_c(k-1) + B_c(k)\xi(k-1) \tag{11}$$

where

$$\begin{cases} \xi(k-1) \triangleq \mathrm{col}\left\{x(k-1),\ w(k-1),\ \overline{v}(k),\ \underline{v}(k)\right\} \\ A_c(k) \triangleq A(k-1) - K(k)\overline{C}(k)A(k-1) - K(k)\underline{C}(k) \\ B_c(k) \triangleq \left[K(k)\Delta\alpha(k)\left(\underline{C}(k) - \overline{C}(k)A(k-1)\right),\right. \\ \qquad B(k-1) - K(k)\left(I + \Delta\alpha(k)\right)\overline{C}(k)B(k-1), \\ \qquad \left. -K(k)\left(I + \Delta\alpha(k)\right)\overline{D}(k),\ -K(k)\left(I - \Delta\alpha(k)\right)\underline{D}(k)\right] \end{cases} \tag{12}$$

When $m(k) = 0$, to prevent the rapid performance degradation, the compensating state estimate of $x(k)$ and the corresponding estimation error is proposed by

$$\hat{x}_c(k) = A(k-1)\hat{x}_c(k-1) \tag{13}$$

$$e_c(k) = A(k-1)e_c(k-1) + B(k-1)w(k-1) \tag{14}$$

Theorem 1. *When $m(k) > 0$, an optimal fusion estimator gain $K(k)$ in (10) that minimizes the upper bound of square errors of the fusion estimator can be obtained by solving the following convex optimization problem:*

$$\min_{\vartheta_c(k)>0,\Theta_c(k)>0,P_c(k)>0,\varepsilon(k)>0} \mathrm{Tr}\{\Theta_c(k)\}$$

$$\text{s.t.}: \begin{cases} \begin{bmatrix} -\varepsilon(k)I & K^{\mathrm{T}}(k) & 0 & 0 & 0 \\ * & -I & A_c(k) & \overline{B}_c(k) & 0 \\ * & * & -P_c(k) & 0 & 0 \\ * & * & * & -\Theta_c(k) & \varepsilon(k)E_c^{\mathrm{T}}(k) \\ * & * & * & * & -\varepsilon(k)I \end{bmatrix} < 0 \\ P_c(k) - \vartheta_c(k)I < 0 \\ \vartheta_c(k) < 1 \end{cases} \tag{15}$$

where

$$\begin{cases} \overline{B}_c(k) \triangleq \left[0,\ B(k-1) - K(k)\overline{C}(k)B(k-1),\ -K(k)\overline{D}(k),\right. \\ \qquad \left. -K(k)\underline{D}(k)\right] \\ E_c(k) \triangleq \left[\underline{C}(k) - \overline{C}(k)A(k-1),\ -\overline{C}(k)B(k-1),\ -\overline{D}(k),\right. \\ \qquad \left. \underline{D}(k)\right] \end{cases} \tag{16}$$

Under this case, if $\|A(k)\|_2 < 1$, the estimation error of $\hat{x}_c(k)$ will be asymptotically bounded, i.e., there must exist a scalar $p_c > 0$ such that $\lim_{t\to\infty} e_c^{\mathrm{T}}(k)e_c(k) < p_c$.

Proof. To construct an upper bound of square errors of the fusion estimator at each time, let us define the following index function (Chen et al., 2019):

$$\begin{aligned} J_c(k) \triangleq& e_c^{\mathrm{T}}(k)e_c(k) - e_c^{\mathrm{T}}(k-1)P_c(k)e_c(k-1) \\ &- \xi^{\mathrm{T}}(k-1)\Theta_c(k)\xi(k-1) \end{aligned} \tag{17}$$

where $P_c(k) > 0$ and $\Theta_c(k) > 0$. Then, it follows from (11) that

$$J_c(k) = \begin{bmatrix} e_c(k-1) \\ \xi(k-1) \end{bmatrix}^{\mathrm{T}} \underbrace{\begin{bmatrix} \Sigma_{11}(k) & \Sigma_{12}(k) \\ * & \Sigma_{22}(k) \end{bmatrix}}_{\Sigma(k)} \begin{bmatrix} e_c(k-1) \\ \xi(k-1) \end{bmatrix} \tag{18}$$

where $\Sigma_{11}(k) = A_c^{\mathrm{T}}(k)A_c(k) - P_c(k)$, $\Sigma_{12}(k) = A_c^{\mathrm{T}}(k)B_c(k)$ and $\Sigma_{22}(k) = B_c^{\mathrm{T}}(k)B_c(k) - \Theta_c(k)$. The condition $J_c(k) < 0$ or its equivalent condition $\Sigma(k) < 0$ must be satisfied to ensure the term $e_c^{\mathrm{T}}(k-1)P_c(k)e_c(k-1) + \xi^{\mathrm{T}}(k-1)\Theta_c(k)\xi(k-1)$ is an upper bound of square errors of the fusion estimator. According to the Schur complement lemma (Boyd, El Ghaoui, Feron, & Balakrishnan, 1994), $\Sigma(k) < 0$ is equivalent to

$$\begin{bmatrix} -I & A_c(k) & B_c(k) \\ * & -P_c(k) & 0 \\ * & * & -\Theta_c(k) \end{bmatrix} < 0 \tag{19}$$

Notice that $B_c(k) = \overline{B}_c(k) + K(k)\Delta\alpha(k)E_c(k)$ with $\|\Delta\alpha(k)\|_2 \le 1$, then the first inequality in (15) will be derived by Lemma 2 in Chen, Hu et al. (2017).

When the remaining two inequalities $P_c(k) - \vartheta_c(k)I < 0$ and $\vartheta_c(k) < 1$ in (15) are satisfied, one has by $\Sigma_{11}(k) < 0$ that

$$A_c^{\mathrm{T}}(k)A_c(k) < P_c(k) < \vartheta_c(k)I < I \tag{20}$$

In this case, the upper bound of square errors of the fusion estimator can be scaled to

$$e_c^{\mathrm{T}}(k)e_c(k) < e_c^{\mathrm{T}}(k-1)e_c(k-1) + \xi^{\mathrm{T}}(k-1)\Theta_c(k)\xi(k-1) \tag{21}$$

To design an optimal gain $K(k)$ that minimizes this upper bound at each time step, "min Tr$\{\Theta_c(k)\}$" is proposed to be the optimization objective.

Let $A_c^u(k) \in \{A(k-1),\ A_c(k)\}$ and $B_c^u(k) \in \{B_o(k),\ B_c(k)\}$, where $B_o(k) \triangleq [0,\ B(k-1),\ 0,\ 0]$. Then, the estimation errors (11) and (14) can be written as:

$$e_c(k) = A_c^u(k)e_c(k-1) + B_c^u(k)\xi(k-1) \tag{22}$$

which means that

$$\begin{aligned} e_c(k) =& \left(\prod_{\kappa=0}^{t-1} A_c^u(k-\kappa)\right) e_c(0) \\ &+ \sum_{\kappa=0}^{t-1}\left\{\left(\prod_{\phi=0}^{\kappa-1} A_c^u(k-\phi)\right) B_c^u(k-\kappa)\xi(k-\kappa-1)\right\} \end{aligned} \tag{23}$$

Since $\|A_c(k)\|_2 < 1$ and $\|A(k)\|_2 < 1$, it can be concluded from (23) that the estimation error $e_c(k)$ is bounded. This completes the proof.

From Theorem 1, the computation procedures for the CFE $\hat{x}_c(k)$ can be summarized by Algorithm 1.

Algorithm 1 For given $\hat{x}_c(k-1)$

1: **for** $i := 1$ **to** L **do**
2: Determine the sensors at the switching instant by comparing $y_i(k)y_i(k-1)$ with 0;
3: **end for**
4: Determine the element number $m(k)$ of the set $\Im_k$;
5: **if** $m(k) > 0$ **then**
6: Calculate $K(k)$ by (15);
7: Calculate $\hat{x}_c(k)$ by (10);
8: **else**
9: Calculate $\hat{x}_c(k)$ by (13);
10: **end if**

3.2. Distributed fusion estimators

Under the distributed fusion structure, each local estimate needs to be calculated first. In this case, based on the measurements at the switching instant, the local estimator is designed by:

$$\begin{aligned} \hat{x}_i(k) =& A(k-1)\hat{x}_i(k-1) \\ &+ K_i(k)\left[\tau_i - 0.5C_i(k)A(k-1)\hat{x}_i(k-1)\right. \\ &\left. -0.5C_i(k-1)\hat{x}_i(k-1)\right] \quad i \in \Im_k \end{aligned} \tag{24}$$

where the estimation gains $K_i(k)$ will be determined later. Then, it follows from (24) that the local estimation error $e_i(k) \triangleq x(k) - \hat{x}_i(k)$ at the switching instant is given by

$$e_i(k) = A_{f_i}(k)e_i(k-1) + B_{f_i}(k)\xi_i(k-1) \tag{25}$$

where

$$\begin{cases} \xi_i(k-1) \triangleq \text{col}\{x(k-1),\ w(k-1),\ v_i(k),\ v_i(k-1)\} \\ A_{f_i}(k) \triangleq A(k-1) - 0.5K_i(k)C_i(k)A(k-1) \\ \qquad - 0.5K_i(k)C_i(k-1) \\ B_{f_i}(k) \triangleq [\alpha_i(k)K_i(k)\left(C_i(k-1) - C_i(k)A(k-1)\right), \\ \qquad B(k-1) - (0.5+\alpha_i(k))K_i(k)C_i(k)B(k-1), \\ \qquad -(0.5+\alpha_i(k))K_i(k)D_i(k), \\ \qquad -(0.5-\alpha_i(k))K_i(k)D_i(k-1)] \end{cases} \tag{26}$$

Define $e_F(k) \triangleq \text{col}\left\{e_1(k),\ \ldots,\ e_{m(k)}(k)\right\}$. Then, one has by (25) that

$$e_F(k) = A_F(k)e_F(k-1) + B_F(k)\xi(k-1) \tag{27}$$

where

$$\begin{cases} A_F(k) \triangleq \text{diag}\left\{A_{f_1}(k),\ \ldots,\ A_{f_{m(k)}}(k)\right\} \\ K_M(k) \triangleq \text{diag}\left\{K_1(k),\ \ldots,\ K_{m(k)}(k)\right\} \\ B_M(k-1) \triangleq \text{col}\{\underbrace{B(k-1),\ \ldots,\ B(k-1)}_{m(k)\ \text{elements}}\} \\ B_F(k) \triangleq \left[K_M(k)\Delta\alpha(k)\left(\underline{C}(k) - \overline{C}(k)A(k-1)\right),\right. \\ \qquad B_M(k-1) - K_M(k)\left(I+\Delta\alpha(k)\right)\overline{C}(k)B(k-1), \\ \qquad -K_M(k)\left(I+\Delta\alpha(k)\right)\overline{D}(k), \\ \qquad \left. K_M(k)\left(\Delta\alpha(k) - I\right)\underline{D}(k)\right] \end{cases} \tag{28}$$

Moreover, the local compensating state estimate of $x(k)$ is modeled by

$$\hat{x}_i(k) = A(k-1)\hat{x}_i(k-1) \quad i \in \Im_k^C \tag{29}$$

When $\Im_k = \emptyset$, all sensors cannot provide effective information at time k. To model this case, let us introduce the following binary indicator function:

$$\eta(k) = \begin{cases} 1 & \Im_k \neq \emptyset \\ 0 & \Im_k = \emptyset \end{cases} \tag{30}$$

Then, the distributed fusion estimator(DFE) is given by

$$\hat{x}_d(k) = \eta(k)\sum_{i=1}^{m(k)} W_i(k)\hat{x}_i(k) + (1-\eta(k))A(k-1)\hat{x}_d(k-1) \tag{31}$$

where $\sum_{i=1}^{m(k)} W_i(k) = I$, and each weighting matrix $W_i(k)$ is required to be designed for $m(k) > 0$ in this paper. Then, it follows from (27) and (31) that the fusion estimation error $e(k) \triangleq x(k) - \hat{x}_d(k)$ can be described by

$$e(k) = W(k)e_F(k)(m(k) > 0) \tag{32}$$

where $W(k) \triangleq [W_1(k),\ \cdots,\ W_{m(k)}(k)]$.

Theorem 2. *An optimal local estimator gain $K_i(k)$ in (24) that minimizes the upper bound of square errors of the local estimator can be obtained by solving the following convex optimization problem:*

$$\min_{\vartheta_i(k)>0,\Theta_i(k)>0,P_i(k)>0,K_i(k),\epsilon_i(k)>0} \text{Tr}\{\Theta_i(k)\}$$
$$\text{s.t.}: \begin{cases} \begin{bmatrix} -\epsilon_i(k)I & K_i^{\mathrm{T}}(k) & 0 & 0 & 0 \\ * & -I & A_{f_i}(k) & \overline{B}_{f_i}(k) & 0 \\ * & * & -P_i(k) & 0 & 0 \\ * & * & * & -\Theta_i(k) & \epsilon_i(k)E_i^{\mathrm{T}}(k) \\ * & * & * & * & -\epsilon_i(k)I \end{bmatrix} < 0 \\ P_i(k) - \vartheta_i(k)I < 0 \\ \vartheta_i(k) < 1 \end{cases} \tag{33}$$

where

$$\begin{cases} \overline{B}_{f_i}(k) \triangleq [0,\ B(k-1) - 0.5K_i(k)C_i(k)B(k-1), \\ \qquad -0.5K_i(k)D_i(k),\ -0.5K_i(k)D_i(k-1)] \\ E_i(k) \triangleq \dfrac{1}{2}[C_i(k-1) - C_i(k)A(k-1),\ C_i(k)B(k-1), \\ \qquad -D_i(k),\ D_i(k-1)] \end{cases} \tag{34}$$

Then, an optimal fusion weighting matrix $W(k)$ that minimizes the upper bound of square errors of the fusion estimator can be obtained by solving the following convex optimization problem:

$$\min_{\Theta(k)>0,P(k)>0,\Upsilon(k),W(k),\epsilon(k)>0} \text{Tr}\{P(k)\} + \text{Tr}\{\Theta(k)\}$$
$$\text{s.t.}: \begin{bmatrix} -\epsilon(k)I & K_W(k) & 0 & 0 & 0 \\ * & -I & W(k)A_F(k) & \overline{B}_W(k) & 0 \\ * & * & -P(k) & -\Upsilon(k) & 0 \\ * & * & * & -\Theta(k) & \epsilon(k)E^{\mathrm{T}}(k) \\ * & * & * & * & -\epsilon(k)I \end{bmatrix} < 0 \tag{35}$$

where

$$\begin{cases} E(k) \triangleq \left[\underline{C}(k) - \overline{C}(k)A(k-1),\ -\overline{C}(k)B(k-1),\right. \\ \qquad \left. -\overline{D}(k),\ \underline{D}(k)\right] \\ \overline{B}_{F_i}(k) \triangleq \left[0,\ B_M(k-1) - K_M(k)\overline{C}(k)B(k-1),\right. \\ \qquad \left. -K_M(k)\overline{D}(k),\ -K_M(k)\underline{D}(k)\right] \\ K_W(k) \triangleq [W(k)K_M(k)]^{\mathrm{T}} \quad \overline{B}_W(k) \triangleq W(k)\overline{B}_{F_i}(k) \end{cases} \tag{36}$$

Moreover, if $\|A(k)\|_2 < 1$, the fusion estimation errors of the DFE $\hat{x}_d(k)$ designed by (33) and (35) will be asymptotically bounded.

Proof. Each local estimator gain designed by (33) can be derived from the similar proof of Theorem 1, and the detailed derivation process is omitted here due to the page limitation.

When $m(k) > 0$, let us define $\overline{\xi}(k) = \text{col}\{e_F(k-1),\ \xi(k-1)\}$, then we introduce $\Upsilon(k)$, $P(k) > 0$ and $\Theta(k) > 0$ to construct an upper bound of square errors of the fusion estimator

$$e^{\mathrm{T}}(k)e(k) < \overline{\xi}^{\mathrm{T}}(k)\begin{bmatrix} P(k) & \Upsilon(k) \\ * & \Theta(k) \end{bmatrix}\overline{\xi}(k) \tag{37}$$

From (27) and (32), the following inequality must be satisfied to ensure (37) is true:

$$\overline{\xi}^{\mathrm{T}}(k)\Lambda(k)\overline{\xi}(k) = \overline{\xi}^{\mathrm{T}}(k)\begin{bmatrix} \Lambda_{11}(k) & \Lambda_{12}(k) \\ * & \Lambda_{22}(k) \end{bmatrix}\overline{\xi}(k) < 0 \tag{38}$$

where $\Lambda_{11}(k) = [W(k)A_F(k)]^{\mathrm{T}}W(k)A_F(k) - P(k)$, $\Lambda_{12}(k) = [W(k)A_F(k)]^{\mathrm{T}}W(k)B_F(k) - \Upsilon(k)$ and $\Lambda_{22}(k) = [W(k)B_F(k)]^{\mathrm{T}}W(k)B_F(k) - \Theta(k)$. According to the Schur complement lemma (Boyd

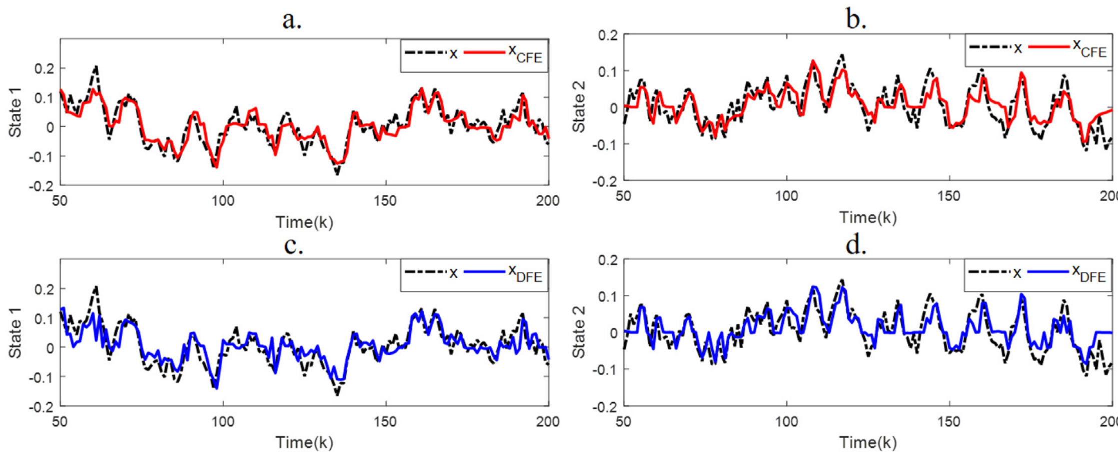

Fig. 1. (a)–(b): Trajectories of $x(k)$ and $\hat{x}_c(k)$; (c)–(d): Trajectories of $x(k)$ and $\hat{x}_d(k)$.

et al., 1994), $\Lambda(k) < 0$ is equivalent to

$$\begin{bmatrix} -I & W(k)A_F(k) & W(k)B_F(k) \\ * & -P(k) & -\Upsilon(k) \\ * & * & -\Theta(k) \end{bmatrix} < 0 \tag{39}$$

Notice that $W(k)B_F(k) = \overline{B}_W(k) + K_W^{\mathrm{T}}(k)\Delta\alpha(k)E(k)$ with $\|\Delta\alpha(k)\|_2 \leq 1$, then the first inequality in (35) will be derived by Lemma 2 in Chen, Hu et al. (2017).

To design an optimal weighting matrix $W(k)$ that minimizes an upper bound of $e^{\mathrm{T}}(k)e(k)$ at each time step, "$\min \mathrm{Tr}\{P(k)\} + \mathrm{Tr}\{\Theta(k)\}$" is proposed to be the optimization objective.

Moreover, when $\|A(k)\|_2 < 1$, it is concluded from Theorem 1 that the local estimation error $e_i(k)$ is asymptotically bounded. Then, when simultaneously considering the cases "$m(k) = 0$" and "$m(k) > 0$", it follows from (1) and (31) that the fusion estimation error $e(k)$ can be written as the following unified form:

$$\begin{aligned} e(k) &= (1-\eta(k))A(k-1)e(k-1) \\ &\quad + (1-\eta(k))B(k-1)w(k-1) + \eta(k)\sum_{i=1}^{m(k)} W_i(k)e_i(k) \end{aligned} \tag{40}$$

Since $e_i(k)$ and $w(k)$ are bounded at each time and the condition "$\|A(k)\|_2 < 1$" holds, it is concluded from (40) that the fusion estimation error $e(k)$ is asymptotically bounded. This completes the proof.

From Theorem 2, the computation procedures for the DFE $\hat{x}_d(k)$ can be summarized by Algorithm 2.

Algorithm 2 For given $\hat{x}_i(k-1)$ and $\hat{x}_d(k-1)$

for $i := 1$ **to** L **do**
  if $y_i(k)y_i(k-1) < 0$ **then**
    Calculate $K_i(k)$ by (33) and $\hat{x}_i(k)$ by (24);
  else
    Calculate $\hat{x}_i(k)$ by (29);
  end if
end for
Determine parameters $m(k)$ by (5) and $\eta(k)$ by (30);
if $\eta(k) = 1$ **then**
  Calculate $W(k)$ by (35);
end if
Calculate $\hat{x}_d(k)$ by (31).

Remark 2. The convex optimization problems (15), (33) and (35) are established in terms of linear matrix inequalities (LMIs), and thus they can be directly solved by the function "mincx" of MATLAB LMI Toolbox (Boyd et al., 1994). On the other hand, it should be pointed out that the centralized and the distributed fusion estimators have their own advantages and disadvantages by using the estimation strategy in this paper. For example, though the CFE can provide globally optimal estimation performance, the dimension of the first inequality in (15) becomes large when the number of sensors increases. This means that the solvability and computational complexity are seriously dependent on the number of sensors, while lots of binary sensors are indeed required when dealing with the addressed problem in this paper. In contrast, under the distributed fusion structure, the L optimization problems (33) with lower dimensions have less computational complexity, and the optimization problem (35) with more searching variables has better solvability.

4. Simulation examples

Consider the following linear time-varying binary sensor system

$$\begin{cases} x(k+1) = \underbrace{\begin{bmatrix} 0.85+0.1\sin k & 0.1 \\ 0.01 & 0.75+0.1\sin k \end{bmatrix}}_{A_u(k)} x(k) + \\ \qquad\qquad \begin{bmatrix} 0.3+0.01\sin k & 0 \\ 0 & 0.3+0.01\sin k \end{bmatrix} w(k) \\ z_i(k) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(k) + 0.2v_i(k), \ \ i = 1, 2, \ldots, 9 \\ z_i(k) = \begin{bmatrix} 0 & 1 \end{bmatrix} x(k) + 0.2v_i(k), \ \ i = 10, 11, \ldots, 18 \end{cases} \tag{41}$$

The system noises $w(k)$ and $v_i(k)$ are the bounded noises given by $w(k) = 0.8\varphi_w(k) - 0.4$ and $v_i(k) = 0.6\varphi_{v_i}(k) - 0.3$, where $\varphi_w(k)$ and $\varphi_{v_i}(k)$ are random variables that can be generated by the function "rand" of MATLAB. The time-varying system is monitored by two groups of binary sensors, and each group has 9 binary sensors with their thresholds chosen as -0.15, -0.12, -0.09, -0.05, 0, 0.05, 0.09, 0.12 and 0.15.

To verify the effectiveness of the proposed fusion estimators, the CFE $\hat{x}_c(k)$ calculated by Theorem 1 and the DFE $\hat{x}_d(k)$ calculated by Theorem 2 are obtained and plotted in Fig. 1. Compared with the trajectory of the state $x(k)$, it can be seen that the designed fusion estimators can estimate real state well. On the other hand, due to the random noises, the estimation performance is assessed by mean square errors (MSEs) that are calculated by Monte Carlo method with an average of 100 runs. Fig. 2(a)–(b) shows the results of the MSEs of the CFE, the DFE and its local estimators. As can be observed, the performance of the DFE is almost close to that of the CFE, which means the distributed fusion structure does not dramatically degrade the fusion performance. Moreover, the performance of the DFE is much better

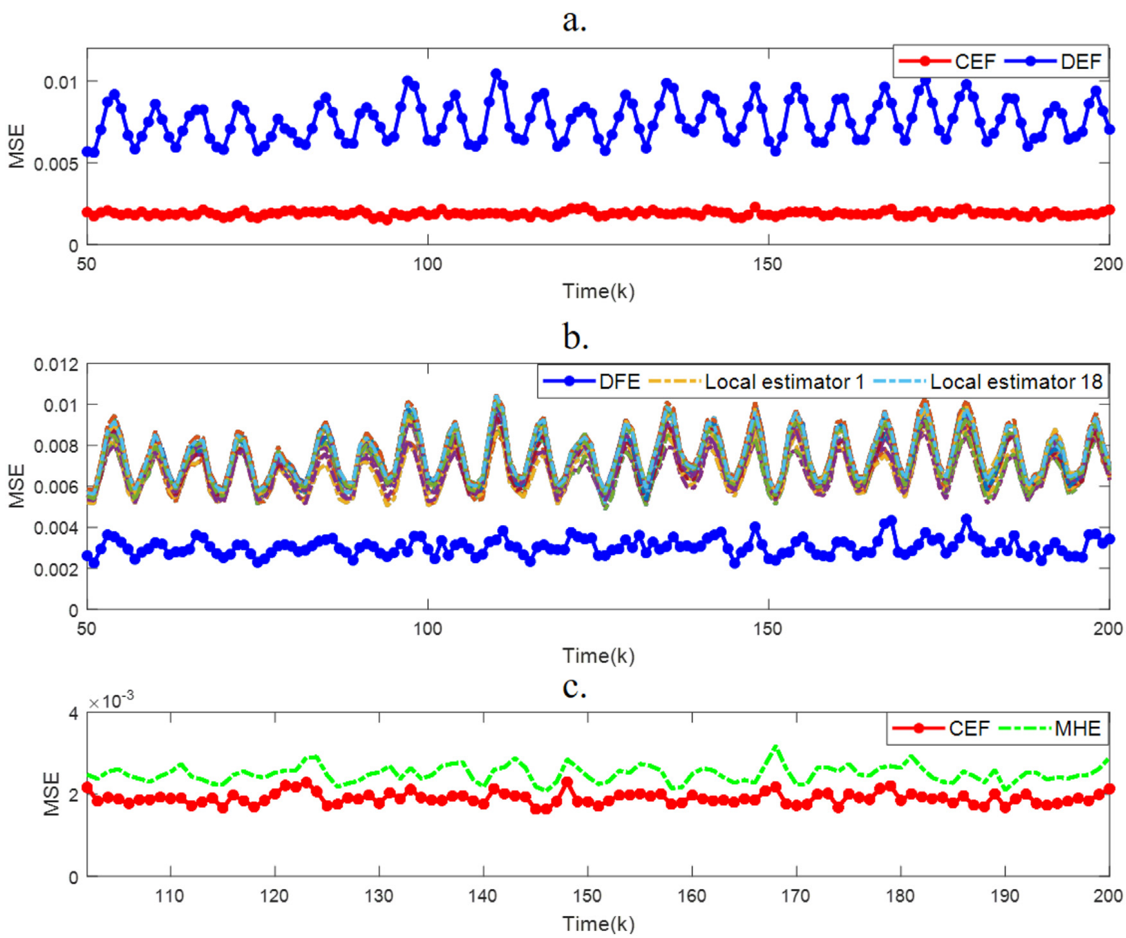

Fig. 2. (a): MSEs of the CFE and the DFE; (b): MSEs of the DFE and the 18 local estimators; (c): MSEs of the CFE and the MHE (moving horizon estimator) in Battistelli et al. (2017).

than its all local estimators, and this is as expected for the fusion methods. More importantly, under the condition "$\|A_u(k)\|_2 < 1$" and the observation that the optimization problems (15) and (33) are solvable, the stability of fusion estimators can be ensured, which is consistent with the results that the MSEs of fusion estimators are bounded. Meanwhile, the performance of CFE is compared with the moving horizon estimation method proposed in Battistelli et al. (2017) by Fig. 2(c). Notice that the performance index and sliding window size of moving horizon estimator need to be selected to ensure the stability. Here, the sliding window size is chosen as 100, and thus the comparison of MSEs is started after 100 time steps. The result shows that the proposed method in this paper outperforms the moving horizon estimation method in Battistelli et al. (2017).

5. Conclusions

In this paper, the fusion estimation problem has been investigated for discrete time-varying systems with bounded noises under multiple binary sensors. An uncertainty model was proposed to depict the limited information from binary sensors. By using matrix analysis theory, stable centralized and distributed fusion estimators were designed by establishing different convex optimization problems. Notice that these convex optimization problems can be directly solved by the MATLAB LMI Toolbox. Finally, an illustrative example was given to show the effectiveness and advantages of the proposed methods.

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