Comparing BLP, MIQP, and a Genetic Algorithm

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The arm being optimized,

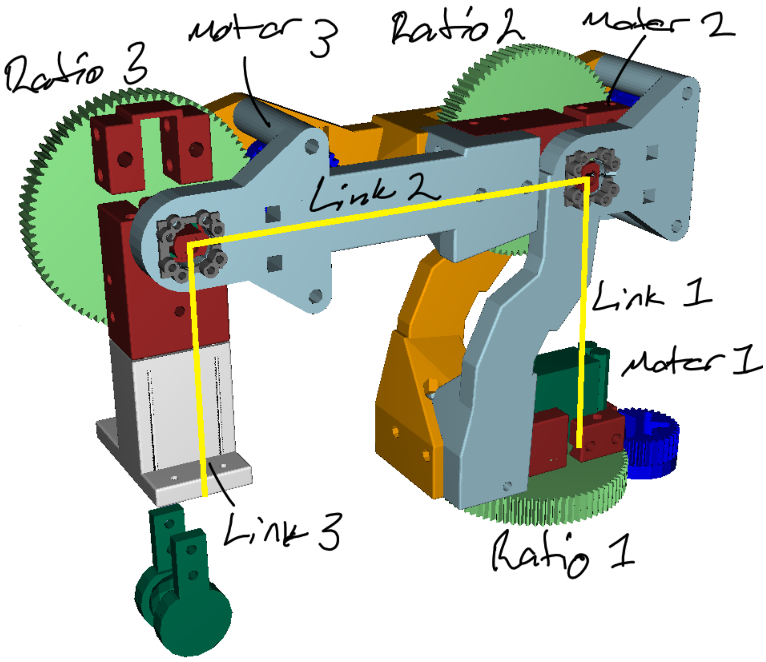

Figure 1: RBE 3001 3DOF Arm

1 MIQP

1.1 Feature Matrix

The feature matrix of a motor module is given by

$$F_m = \begin{bmatrix} \frac{\tau^{(1)}}{G^{(1)}} & & \frac{\tau^{(N)}}{G^{(N)}} \\ \omega^{(1)} G^{(1)} & & \omega^{(N)} G^{(N)} \\ P^{(1)} & \dots & P^{(N)} \\ M^{(1)} & & M^{(N)} \\ G^{(1)} & & G^{(N)} \end{bmatrix} \tag{1}$$

where $\tau^{(i)}$ is the stall torque in Newton-meters for motor i, $\omega^{(i)}$ is the free speed in radians per second for motor i, $P^{(i)}$ is the price of motor i in USD, $M^{(i)}$ is the mass in kilograms of motor i, and $G^{(i)}$ is the gear ratio on motor i.

1.2 Variables

τ_j denotes the τ of slot j, and ω_j denotes the ω of slot j. τ_j and ω_j are implemented using a binary vector for each slot j.

1.3 Constraints

V is the tip velocity, F is the tip force, and R_j is the r parameter of link j. The arm is mounted $90 \deg$ off vertical.

$$\tau_1 \geq F(R_1 + R_2 + R_3) + G(M_2 R_1 + M_3(R_1 + R_2)) \tag{2}$$

$$\tau_2 \geq F(R_2 + R_3) + M_3 G R_2 \tag{3}$$

$$\tau_3 \geq F R_3 \tag{4}$$

$$\omega_1(R_1 + R_2 + R_3) \geq V \tag{5}$$

$$\omega_2(R_2 + R_3) \geq V \tag{6}$$

$$\omega_3 R_3 \geq V \tag{7}$$

$$R_1 + R_2 + R_3 = 0.4 \tag{8}$$

1.4 Results

MIQP took 0.03 seconds to solve and found:

```
Optimal objective:  40.89
Optimal motors:
Motor("vexMotor-393", 1.637, 10.471, 14.99, 0.0945), ratio=0.3333333333333333,
link1=68.0, link2=195.0, link3=137.0
Motor("stepperMotor-GenericNEMA14", 0.098, 139.626, 12.95, 0.12), ratio=0.05263157894736842,
link1=68.0, link2=195.0, link3=137.0
Motor("stepperMotor-GenericNEMA14", 0.098, 139.626, 12.95, 0.12), ratio=0.05263157894736842,
link1=68.0, link2=195.0, link3=137.0
```

2 BLP

2.1 Feature Matrix

The feature matrix of a motor module is given by

$$F_m = \begin{bmatrix} \frac{\tau^{(1)}}{G^{(1)}} & & \frac{\tau^{(N)}}{G^{(N)}} \\ \omega^{(1)}G^{(1)} & & \omega^{(N)}G^{(N)} \\ P^{(1)} & & P^{(N)} \\ M^{(1)} & & M^{(N)} \\ G^{(1)} & & G^{(N)} \\ \ln(\omega^{(1)}G^{(1)}) & & \ln(\omega^{(N)}G^{(N)}) \\ R_1 & & R_1 \\ R_2 & \cdots & R_2 \\ R_3 & & R_3 \\ \ln(R_1) & & \ln(R_1) \\ \ln(R_2) & & \ln(R_2) \\ \ln(R_3) & & \ln(R_3) \\ \ln(R_1+R_2+R_3) & & \ln(R_1+R_2+R_3) \\ \ln(R_2+R_3) & & \ln(R_2+R_3) \\ M^{(1)}R_1 & & M^{(N)}R_1 \\ M^{(1)}R_2 & & M^{(N)}R_2 \end{bmatrix} \tag{9}$$

where $\tau^{(i)}$ is the stall torque in Newton-meters for motor i, $\omega^{(i)}$ is the free speed in radians per second for motor i, $P^{(i)}$ is the price of motor i in USD, $M^{(i)}$ is the mass in kilograms of motor i, $G^{(i)}$ is the gear ratio on motor i, and R_i is the r parameter of link i.

2.2 Variables

τ_j denotes the τ of slot j, ω_j denotes the ω of slot j, and R_j denotes the R of slot j. τ_j and ω_j are implemented using a binary vector for each slot j. Each R_j is integral and bounded by a minimum and maximum length parameter given in the constraints file.

2.3 Constraints

V is the tip velocity, F is the tip force. The arm is mounted $90\,\mathrm{deg}$ off vertical. In places where a quadratic term is in the feature matrix, the implementation pulls the term from the feature matrix instead of introducing a quadratic constraint (e.x., M_2R_1 is taken from the feature matrix).

$$\tau_1 \geq F(R_1 + R_2 + R_3) + G(M_2R_1 + M_3R_1 + M_3R_2) \tag{10}$$

$$\tau_2 \geq F(R_2 + R_3) + M_3GR_2 \tag{11}$$

$$\tau_3 \geq FR_3 \tag{12}$$

$$\ln(\omega_1) \geq \ln(V) - \ln(R_1 + R_2 + R_3) \quad (13)$$

$$\ln(\omega_2) \geq \ln(V) - \ln(R_2 + R_3) \quad (14)$$

$$\ln(\omega_3) \geq \ln(V) - \ln(R_3) \quad (15)$$

$$R_1 + R_2 + R_3 = 0.4 \quad (16)$$

2.4 Results

BLP took 0.42 seconds to solve and found:

```
Optimal objective:  40.89
Optimal motors:
Motor("vexMotor-393", 1.637, 10.471, 14.99, 0.0945), ratio=0.3333333333333333,
link1=75.0, link2=125.0, link3=200.0
Motor("stepperMotor-GenericNEMA14", 0.098, 139.626, 12.95, 0.12), ratio=0.05263157894736842,
link1=75.0, link2=125.0, link3=200.0
Motor("stepperMotor-GenericNEMA14", 0.098, 139.626, 12.95, 0.12), ratio=0.06666666666666667,
link1=75.0, link2=125.0, link3=200.0
```

3 Genetic Algorithm

The genetic algorithm was implemented using a constraint handling technique from [1].

3.1 Entity

The entity is given by

$$\begin{bmatrix} M_1 & M_2 & M_3 & R_1 & R_2 & R_3 & G_1 & G_2 & G_3 \end{bmatrix} \quad (17)$$

where M_i is the integer index of motor i in the array of motors parsed from the motor options file, R_i is the real r parameter of link i, bounded by the minimum and maximum length parameter given in the constraints file, and G_i is the real gear ratio for motor i.

3.2 Constraints

V is the tip velocity, F is the tip force, τ_j is the τ for the motor in index j, and ω_j is the ω for the motor in index j. The arm is mounted $90\,\deg$ off vertical.

$$F(R_1 + R_2 + R_3) + G(M_2 R_1 + M_3(R_1 + R_2)) - \frac{\tau_1}{G_1} \leq 0 \quad (18)$$

$$F(R_2 + R_3) + M_3 G R_2 - \frac{\tau_2}{G_2} \leq 0 \quad (19)$$

$$F R_3 - \frac{\tau_3}{G_3} \leq 0 \quad (20)$$

$$\frac{V}{R_1 + R_2 + R_3} - \omega_1 G_1 \leq 0 \quad (21)$$

$$\frac{V}{R_2 + R_3} - \omega_2 G_2 \leq 0 \quad (22)$$

$$\frac{V}{R_3} - \omega_3 G_3 \leq 0 \tag{23}$$

$$|R_1 + R_2 + R_3 - 0.4| - 0.0001 \leq 0 \tag{24}$$

3.3 Fitness

The fitness is the negative of the sum of the price of each motor,

$$-\sum_{i=1}^{N} P_i \tag{25}$$

3.4 Crossover

Crossover is performed by taking the motors and gear ratios from one entity and the link lengths from the other entity.

3.5 Mutation

Mutation is performed by selecting a random motor index, a random link length between the minimum and maximum length parameter given in the constraints file, and a random gear ratio.

3.6 Stopping Criteria

The genetic algorithm is stopped after a maximum generation number is reached.

3.7 Results

With a mutation rate of 5%, an elitism ratio of 25%, and a mutation ratio of 5%, the genetic algorithm took 20.47 seconds to solve and found:

```
Motor 1=Motor("vexMotor-393", 1.637, 10.471, 14.99, 0.0945)
Motor 2=Motor("stepperMotor-GenericNEMA14", 0.098, 139.626, 12.95, 0.12)
Motor 3=Motor("stepperMotor-GenericNEMA14", 0.098, 139.626, 12.95, 0.12)
Link 1=96.0
Link 2=134.0
Link 3=170.0
Gear ratio 1=0.3333333333333333
Gear ratio 2=0.058823529411764705
Gear ratio 3=0.09090909090909091
Fitness=-40.89
Constraint values:  [-2.56604, -0.0176998, -0.244436, -0.990333, -4.92382, -6.81092,
-0.0001]
Feasible.
```

The average fitness per generation was,

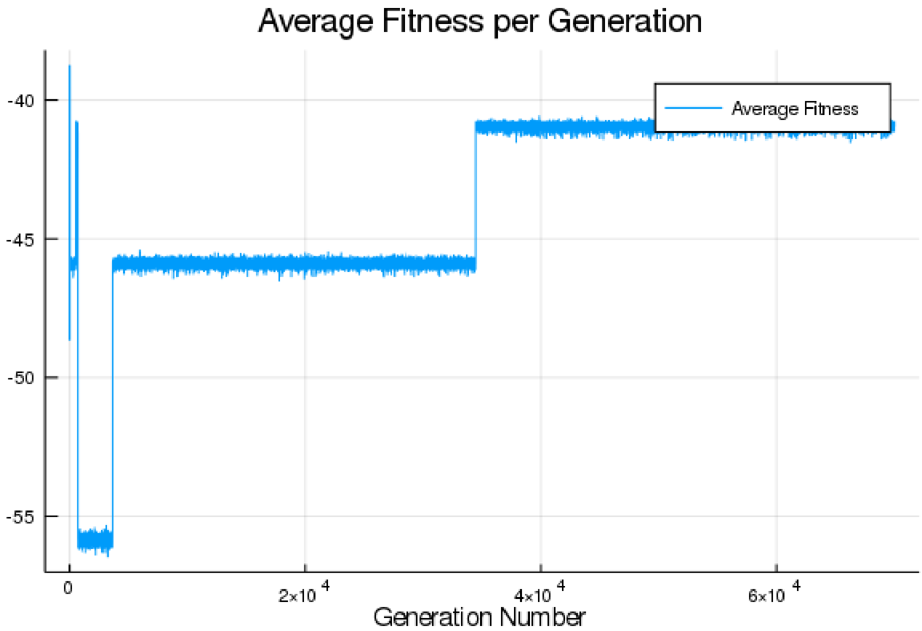

Figure 2: Average Fitness per Generation

4 Summary

All three algorithms found an optimal solution to the problem. MIQP was the fastest at 0.03 seconds, BLP was the second fastest at 0.42 seconds, at the genetic algorithm was the slowest at 20.47 seconds.

References

[1] Adam Chehouri, Rafic Younes, Jean Perron, and Adrian Ilinca. A constraint-handling technique for genetic algorithms using a violation factor. *arXiv preprint arXiv:1610.00976*, 2016.