Comparing BLP, MIQP, and a Genetic Algorithm

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The arm being optimized,

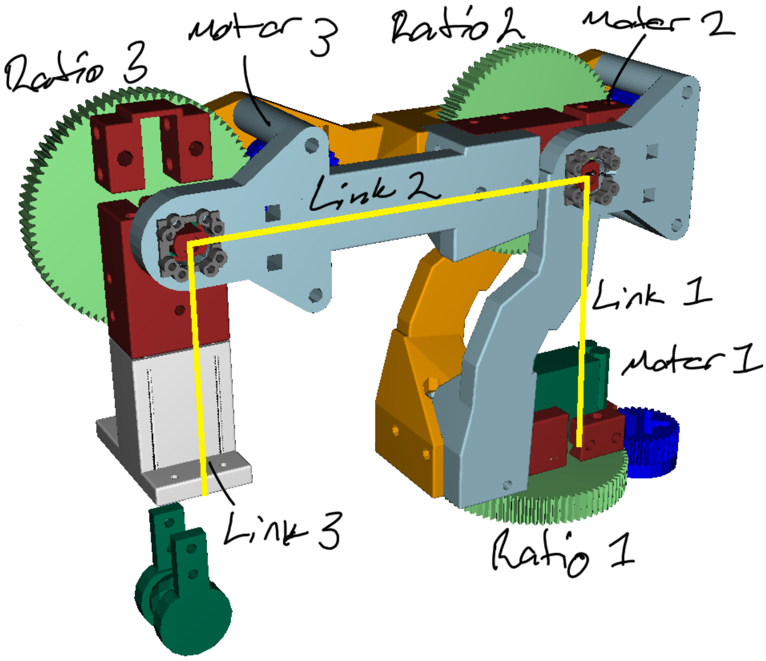

Figure 1: RBE 3001 3DOF Arm

1 BLP

1.1 Feature Matrix

The feature matrix of a motor module is given by

$$F_m = \begin{bmatrix} \frac{\tau^{(1)}}{G^{(1)}} & & \frac{\tau^{(N)}}{G^{(N)}} \\ \omega^{(1)}G^{(1)} & & \omega^{(N)}G^{(N)} \\ P^{(1)} & & P^{(N)} \\ M^{(1)} & & M^{(N)} \\ G^{(1)} & & G^{(N)} \\ \ln(\omega^{(1)}G^{(1)}) & & \ln(\omega^{(N)}G^{(N)}) \\ R_1 & & R_1 \\ R_2 & \cdots & R_2 \\ R_3 & & R_3 \\ \ln(R_1) & & \ln(R_1) \\ \ln(R_2) & & \ln(R_2) \\ \ln(R_3) & & \ln(R_3) \\ \ln(R_1+R_2+R_3) & & \ln(R_1+R_2+R_3) \\ \ln(R_2+R_3) & & \ln(R_2+R_3) \\ M^{(1)}R_1 & & M^{(N)}R_1 \\ M^{(1)}R_2 & & M^{(N)}R_2 \\ \tau_c^{(2)} & & \tau_c^{(2)} \\ \tau_c^{(3)} & & \tau_c^{(3)} \end{bmatrix} \tag{1}$$

where $\tau^{(i)}$ is the stall torque in Newton-meters for motor i, $\omega^{(i)}$ is the free speed in radians per second for motor i, $P^{(i)}$ is the price of motor i in USD, $M^{(i)}$ is the mass in kilograms of motor i, $G^{(i)}$ is the gear ratio on motor i, and R_i is the r parameter of link i. $\tau_c^{(i)}$ is the required joint torque at the target $(T_x, 0, T_z)$ for joint i given by

$$\tau_c^{(i)} = \left(J(\boldsymbol{\theta})^\top \begin{bmatrix} 0 \\ 0 \\ F_c \end{bmatrix} \right)_{(i,1)} \tag{2}$$

where $J(\boldsymbol{\theta})$ is the manipulator jacobian at configuration $\boldsymbol{\theta}$ ($\boldsymbol{\theta}$ is computed using an inverse kinematics algorithm parameterized over the link lengths with target position $(T_x, 0, T_z)$ supplied from the configuration file).

1.2 Variables

τ_j denotes the τ of slot j, ω_j denotes the ω of slot j, and R_j denotes the R of slot j. τ_j and ω_j are implemented using a binary vector for each slot j. Each R_j is integral and bounded by a minimum and maximum length parameter given in the constraints file.

1.3 Constraints

V is the tip velocity, F is the tip force. The arm is mounted $90\deg$ off vertical. In places where a quadratic term is in the feature matrix, the implementation pulls the term from the feature matrix instead of introducing a quadratic constraint (e.x., M_2R_1 is taken from the feature matrix).

$$\tau_1 \geq F\left(R_1 + R_2 + R_3\right) \tag{3}$$

$$\tau_2 \geq F\left(R_2 + R_3\right) + M_3 G R_2 \tag{4}$$

$$\tau_3 \geq F R_3 \tag{5}$$

$$\ln(\omega_1) \geq \ln(V) - \ln(R_1 + R_2 + R_3) \tag{6}$$

$$\ln(\omega_2) \geq \ln(V) - \ln(R_2 + R_3) \tag{7}$$

$$\ln(\omega_3) \geq \ln(V) - \ln(R_3) \tag{8}$$

$$R_1 + R_2 + R_3 = 0.4 \tag{9}$$

$$\tau_2 \geq \tau_c^{(2)} \tag{10}$$

$$\tau_3 \geq \tau_c^{(3)} \tag{11}$$

1.4 Results

BLP took 3.27 seconds to solve and found:

```
Optimal objective:  42.93
Optimal motors:
Motor("vexMotor-393", 1.637, 10.471, 14.99, 0.0945), ratio=0.3333333333333333,
link1=100.0, link2=166.66666666666666, link3=133.33333333333334
Motor("vexMotor-393", 1.637, 10.471, 14.99, 0.0945), ratio=0.3333333333333333,
link1=100.0, link2=166.66666666666666, link3=133.33333333333334
Motor("stepperMotor-Pololu35x26", 0.098, 139.626, 12.95, 0.12), ratio=0.14285714285714285,
link1=100.0, link2=166.66666666666666, link3=133.33333333333334
```

2 Genetic Algorithm

The genetic algorithm was implemented using a constraint handling technique from [1].

2.1 Entity

The entity is given by

$$\begin{bmatrix} M_1 & M_2 & M_3 & R_1 & R_2 & R_3 & G_1 & G_2 & G_3 \end{bmatrix} \tag{12}$$

where M_i is the integer index of motor i in the array of motors parsed from the motor options file, R_i is the real r parameter of link i, bounded by the minimum and maximum length parameter given in the constraints file, and G_i is the real gear ratio for motor i.

2.2 Constraints

V is the tip velocity, F is the tip force, τ_j is the τ for the motor in index j, and ω_j is the ω for the motor in index j. The arm is mounted $90\,\mathrm{deg}$ off vertical.

$$F\left(R_1 + R_2 + R_3\right) - \frac{\tau_1}{G_1} \leq 0 \tag{13}$$

$$F\left(R_2 + R_3\right) + M_3 G R_2 - \frac{\tau_2}{G_2} \leq 0 \tag{14}$$

$$F R_3 - \frac{\tau_3}{G_3} \leq 0 \tag{15}$$

$$\frac{V}{R_1 + R_2 + R_3} - \omega_1 G_1 \leq 0 \tag{16}$$

$$\frac{V}{R_2 + R_3} - \omega_2 G_2 \leq 0 \tag{17}$$

$$\frac{V}{R_3} - \omega_3 G_3 \leq 0 \tag{18}$$

$$|R_1 + R_2 + R_3 - 0.4| - 0.0001 \leq 0 \tag{19}$$

$$\left(\tau_c^{(2)} - \frac{\tau_2}{G_2}\right) + \left(\tau_c^{(3)} - \frac{\tau_3}{G_3}\right) \leq 0 \tag{20}$$

$\tau_c^{(i)}$ is the required joint torque at the target $(T_x, 0, T_z)$ for joint i given by

$$\tau_c^{(i)} = \left(J(\boldsymbol{\theta})^\top \begin{bmatrix} 0 \\ 0 \\ F_c \end{bmatrix}\right)_{(i,1)} \tag{21}$$

where $J(\boldsymbol{\theta})$ is the manipulator jacobian at configuration $\boldsymbol{\theta}$ ($\boldsymbol{\theta}$ is computed using an inverse kinematics algorithm parameterized over the link lengths with target position $(T_x, 0, T_z)$ supplied from the configuration file).

2.3 Fitness

The fitness is the negative of the sum of the price of each motor,

$$-\sum_{i=1}^{N} P_i \tag{22}$$

2.4 Crossover

Crossover is performed by taking the motors and gear ratios from one entity and the link lengths from the other entity.

2.5 Mutation

Mutation is performed by selecting a random motor index, a random link length between the minimum and maximum length parameter given in the constraints file, and a random gear ratio. A random gear ratio is selected by first selecting the range of the ratio ($50\% > 1$, $50\% < 1$) and then sampling a ratio from a uniform distribution of ratios in that range.

2.6 Stopping Criteria

The genetic algorithm is stopped after a maximum generation number is reached.

2.7 Results

With a mutation rate of 5%, an elitism ratio of 25%, and a mutation ratio of 5%, and a maximum of $200,000$ generations, the genetic algorithm took 95.83 seconds to solve and found:

```
Motor 1=Motor("vexMotor-393", 1.637, 10.471, 14.99, 0.0945)
Motor 2=Motor("vexMotor-393", 1.637, 10.471, 14.99, 0.0945)
Motor 3=Motor("stepperMotor-Pololu35x26", 0.098, 139.626, 12.95, 0.12)
Link 1=100.0
Link 2=170.0
Link 3=130.0
Gear ratio 1=0.33206607726344173
Gear ratio 2=0.8486810156880156
Gear ratio 3=0.1529117565209915
Fitness=-42.93
Constraint values=[-2.9684137354115947, -0.2578238182300667, -0.0034609136279283303,
-0.9770638950254984, -5.553205581935878, -13.658149223692268, -0.0001, -7.069767991857092]
Feasible=true
```

The algorithm was run 20 times. The best fitness per run was,

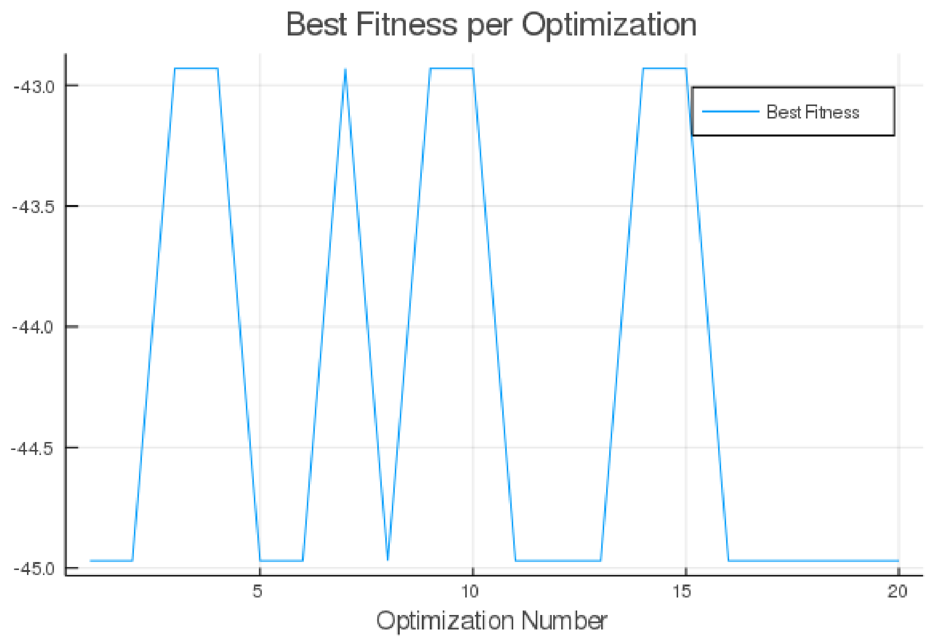

Figure 2: Best Fitness per Optimization Run

3 Summary

All algorithms found an optimal solution to the problem. BLP the fastest at 3.27 seconds and the genetic algorithm was the slowest at 95.83 seconds.

References

[1] Adam Chehouri, Rafic Younes, Jean Perron, and Adrian Ilinca. A constraint-handling technique for genetic algorithms using a violation factor. *arXiv preprint arXiv:1610.00976*, 2016.