# Problem Set X

Math Course
Due: Date

## Instructions

- Write your solutions clearly and legibly.
- Show all work for full credit.
- Collaboration is allowed, but write your own solutions.
- Due at the beginning of class on the due date.

## Problems

1. **(10 points)** Let $f(x) = x^3 - 3x^2 + 2x - 1$. Find all critical points and determine their nature (local maximum, local minimum, or neither).

2. **(15 points)** Prove that for all $n \geq 1$:
$$1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$$

3. **(15 points)** Let $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$. Find $A^{-1}$ and verify your answer.

4. **(20 points)** Determine whether the series converges or diverges:
$$\sum_{n=1}^{\infty} \frac{n^2}{n^3 + 1}$$
Justify your answer.

5. **(20 points)** Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = x^2$.
   (a) Show that $f$ is continuous at every point.
   (b) Is $f$ uniformly continuous on $\mathbb{R}$? Prove or give a counterexample.

6. **(20 points)** Evaluate the following integral:
$$\int_0^{\pi/2} \frac{\sin(x)}{1 + \cos^2(x)} dx$$

# Bonus Problem (Optional)

(**10 points**) Prove or disprove: If $f$ and $g$ are uniformly continuous functions on $\mathbb{R}$, then $f \cdot g$ is uniformly continuous on $\mathbb{R}$.