

# Problem Set X

Math Course

Due: Date

## Instructions

- Write your solutions clearly and legibly.
- Show all work for full credit.
- Collaboration is allowed, but write your own solutions.
- Due at the beginning of class on the due date.

## Problems

1. **(10 points)** Let  $f(x) = x^3 - 3x^2 + 2x - 1$ . Find all critical points and determine their nature (local maximum, local minimum, or neither).
2. **(15 points)** Prove that for all  $n \geq 1$ :

$$1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$$

3. **(15 points)** Let  $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ . Find  $A^{-1}$  and verify your answer.
4. **(20 points)** Determine whether the series converges or diverges:

$$\sum_{n=1}^{\infty} \frac{n^2}{n^3 + 1}$$

Justify your answer.

5. **(20 points)** Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  be defined by  $f(x) = x^2$ .
  - (a) Show that  $f$  is continuous at every point.
  - (b) Is  $f$  uniformly continuous on  $\mathbb{R}$ ? Prove or give a counterexample.
6. **(20 points)** Evaluate the following integral:

$$\int_0^{\pi/2} \frac{\sin(x)}{1 + \cos^2(x)} dx$$

## Bonus Problem (Optional)

**(10 points)** Prove or disprove: If  $f$  and  $g$  are uniformly continuous functions on  $\mathbb{R}$ , then  $f \cdot g$  is uniformly continuous on  $\mathbb{R}$ .