Can we get better guarantees?

OPD: Optimistic Planning for Deterministic systems

- Introduced by [Hren and Munos 2008]
- Another optimistic algorithm
- Only for deterministic MDPs

Theorem (OPD sample complexity)

$$\mathbb{E}\, r_n = \mathcal{O}\left(n^{-\frac{\log 1/\gamma}{\log \kappa}}\right), \textit{if } \kappa > 1$$

OLOP: Open-Loop Optimistic Planning

- Introduced by [Bubeck and Munos 2010]
- Extends OPD to the stochastic setting
- Only considers open-loop policies, i.e. sequences of actions

The idea behind OLOP

A direct application of Optimism in the Face of Uncertainty

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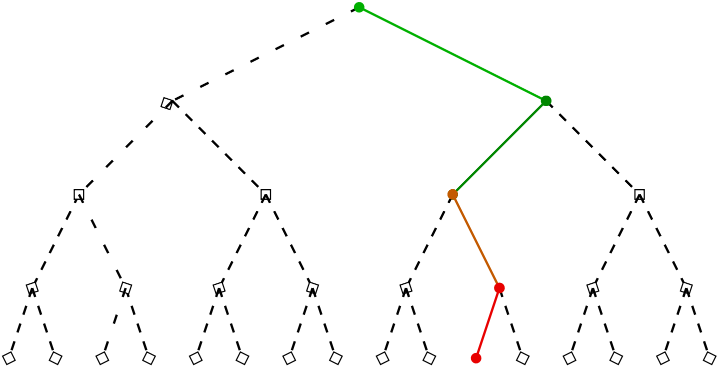
3. Sample the sequence with highest UCB:
$$\arg\max_a U_a$$

The idea behind OLOP



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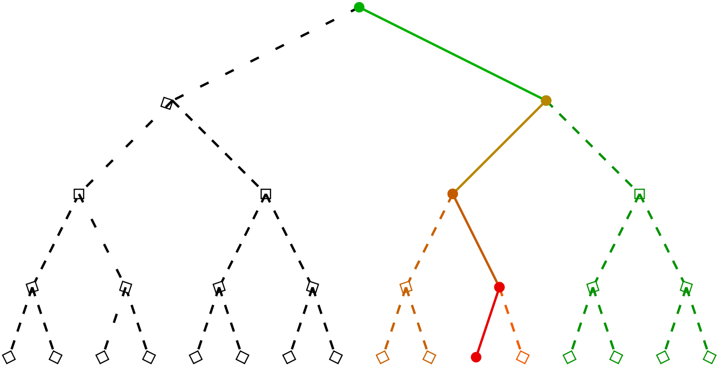

Under the hood

Upper-bounding the value of sequences

$$V(a) = \overbrace{\sum_{t=1}^{h} \gamma^t \mu_{a_{1:t}}}^{\text{follow the sequence}} + \overbrace{\sum_{t \geq h+1} \gamma^t \mu_{a^*_{1:t}}}^{\text{act optimally}}$$

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Under the hood

OLOP main tool: the Chernoff-Hoeffding deviation inequality

$$\underbrace{U_a^{\mu}(m)}_{\text{Upper bound}} \stackrel{\text{def}}{=} \underbrace{\hat{\mu}_a(m)}_{\text{Empirical mean}} + \underbrace{\sqrt{\frac{2\log M}{T_a(m)}}}_{\text{Confidence interval}}$$

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Bounds sharpening

$$B_a(m) \stackrel{\text{def}}{=} \inf_{1\leq t\leq L} U_{a_{1:t}}(m)$$

OLOP guarantees

Theorem (OLOP Sample complexity)

OLOP satisfies:

$$\mathbb{E}\, r_n = \begin{cases} \widetilde{\mathcal{O}}\left(n^{-\frac{\log 1/\gamma}{\log \kappa'}}\right), & \text{if } \gamma\sqrt{\kappa'} > 1 \\ \widetilde{\mathcal{O}}\left(n^{-\frac{1}{2}}\right), & \text{if } \gamma\sqrt{\kappa'} \leq 1 \end{cases}$$

"Remarkably, in the case $\kappa\gamma^2 > 1$, we obtain the same rate for the simple regret as Hren and Munos (2008). Thus, in this case, we can say that planning in stochastic environments is not harder than planning in deterministic environments".