

# **Week 2**

## **Mathematics and Computational Methods for Complex Systems, 2023**

**Dhruva V. Raman**

# How is notebook 1 going?

Too much

Too little

Too slow

Too fast



# **Are you using the padlet?**

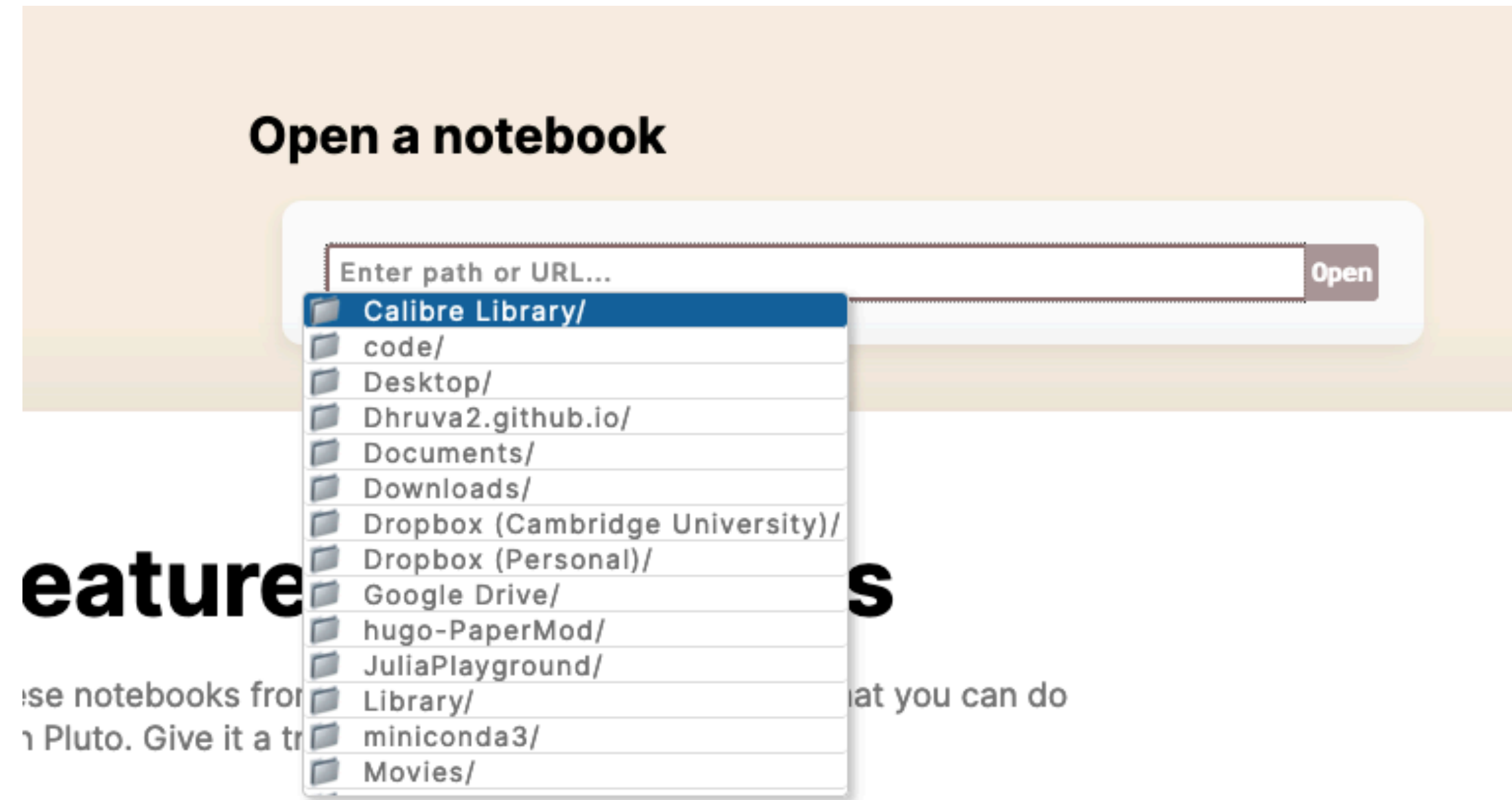
**(No)**

[https://uofsussex.padlet.org/draman2/  
mcmcs-23-24-nobpvtdw6hzblkkg](https://uofsussex.padlet.org/draman2/mcmcs-23-24-nobpvtdw6hzblkkg)

**Have you read the  
canvas/course website  
info?**

# Some week 1 issues

Pluto notebooks open in  
strange folder



# Some week 1 issues

Pluto notebooks open in  
strange folder

## Solution

```
[julia> using Pluto  
  
[shell> cd  
/Users/dr360
```

Semicolon ; to switch from Julia to shell

cd = change to home directory

Then open Pluto with Pluto.run()

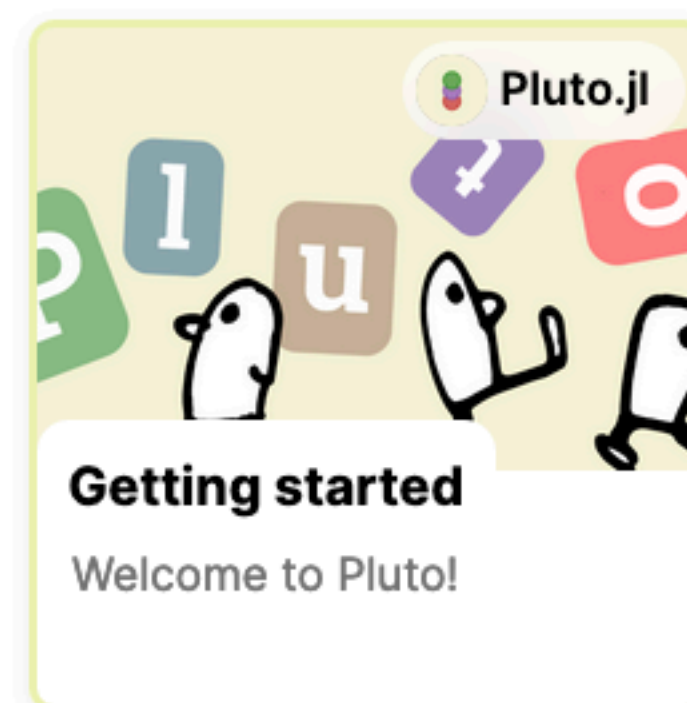
# Some week 1 issues

## Sliders not working

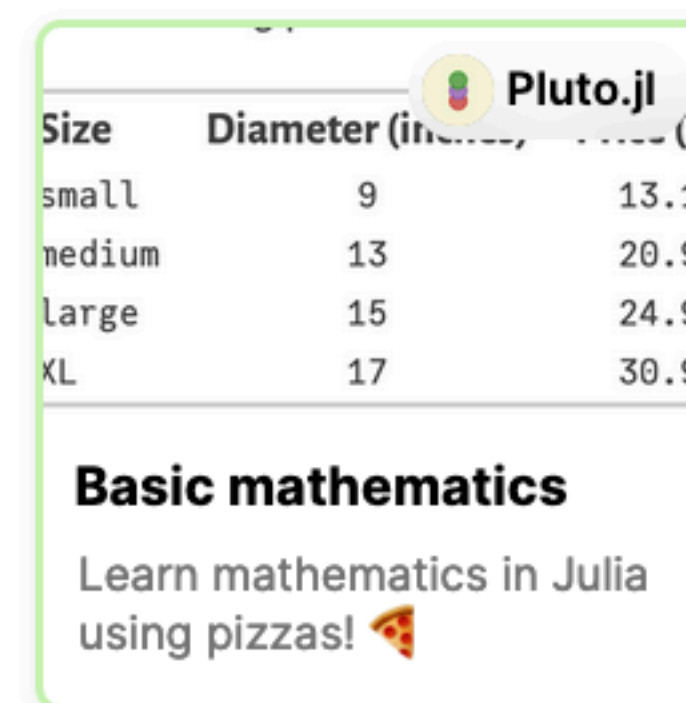


## Getting started

Learn the basics of Pluto in just 10 minutes!




**Getting started**  
Welcome to Pluto!

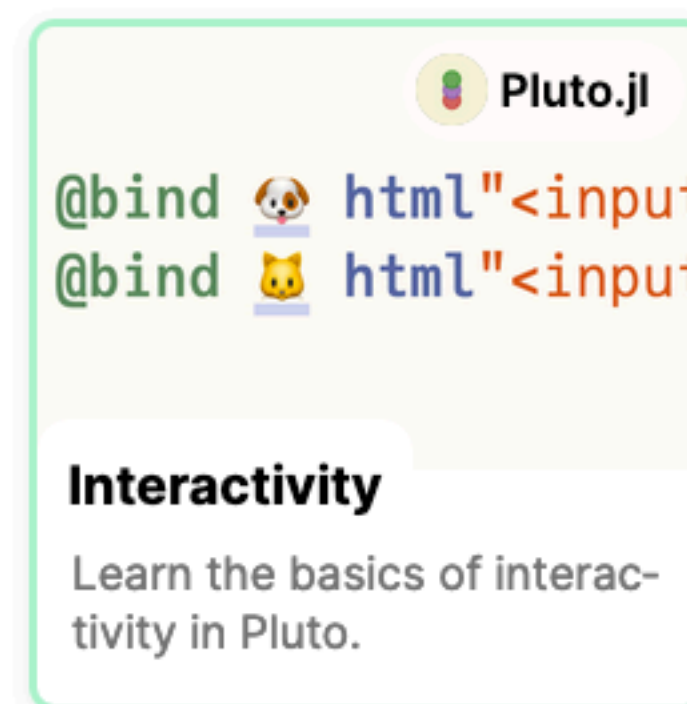


**Basic mathematics**  
Learn mathematics in Julia using pizzas! 🍕

Size	Diameter (inches)	Price (\$)
small	9	13.10
medium	13	20.99
large	15	24.90
XL	17	30.99



**PlutoUI.jl**  
Slider, buttons, dropdowns and more from PlutoUI.jl!



**Interactivity**  
Learn the basics of interactivity in Pluto.



**Tower of Hanoi**  
An interactive Julia puzzle about stacking disks.



**Plots.jl**  
An introduction to Plots.jl

# Packages

*augment languages*

- `using Random` ✓

**UndefVarError: `eigen` not defined**

1. `top-level scope` @ `Local: 1`

- `eigen`



# Packages

*augment languages*

- `using Random` ✓

**UndefVarError: `eigen` not defined**

1. `top-level scope` @ `Local: 1`

- `eigen`

+

»

- `using LinearAlgebra` ✓ , `Random` ✓

+

`eigen` (generic function with 17 methods)

- `eigen`|

# Packages

*augment languages*

eigen is **exported** by  
LinearAlgebra.jl

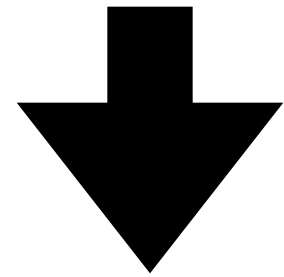
**Principle:** only load  
functions you need!

**Avoid confusion** from too  
many function names

```
+  
> • using LinearAlgebra ✓, Random ✓  
+  
  eigen (generic function with 17 methods)  
  • eigen|
```

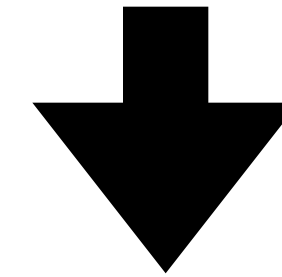
# Name conflicts?

LinearAlgebra



eigen

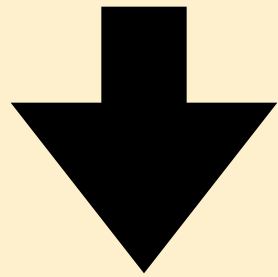
DhruvaAlgebra



eigen

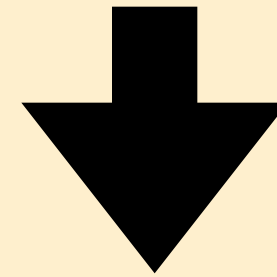
# Name conflicts?

LinearAlgebra



eigen

DhruvaAlgebra



eigen

eigen 

...use full name:      LinearAlgebra.eigen

# Week 1 issues

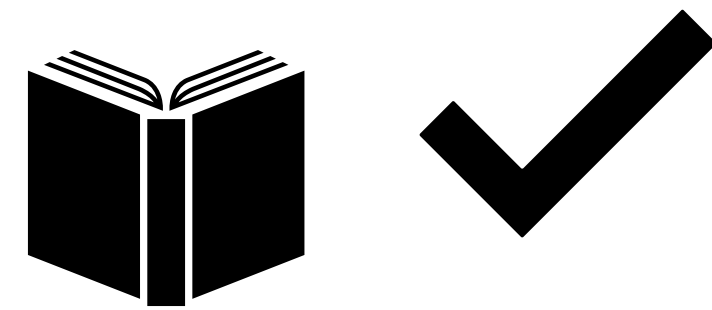
## Functions vs methods

The screenshot shows a Scala REPL session. It starts with two function definitions for a function named `concatenate`. The first function takes two arguments, `a` and `b`, and returns a list containing them. The second function takes three arguments, `a`, `b`, and `c`, and returns a list containing all three. Below the definitions, the function `concatenate` is called with arguments `1`, `2`, and `3`. The output of the call is `concatenated three things`.

```
+  
concatenate (generic function with 2 methods)  
• function concatenate(a,b)  
•   println("concatenated two things")  
•   return [a,b]  
• end  
+  
concatenate (generic function with 2 methods)  
• function concatenate(a::Number,b,c)  
•   println("concatenated three things")  
•   return [a,b,c]  
• end  
  
► [1, 2, 3]  
• concatenate(1,2,3)  
  
concatenated three things
```

# Some week 1 issues

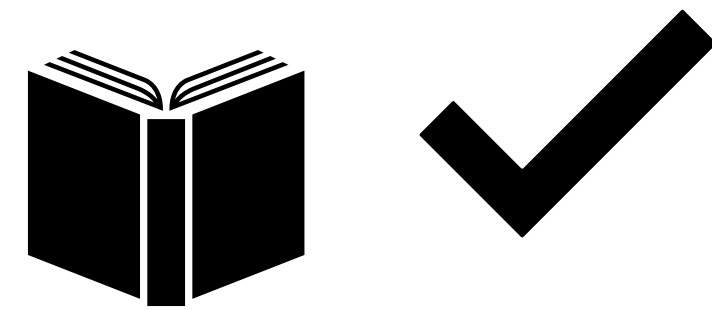
Flying planes can be dangerous!



Noam Chomsky (1965)  
*Aspects of the Theory of Syntax*

# Some week 1 issues

Try to state things twice, in complementary ways, especially when giving a definition



Donald Knuth (1987)  
*Mathematical writing*

# Language types

	NATURAL	PROGRAMMING
Ambiguous	X	
Redundant	X	
Literal		X



# Some week 1 issues

```
[julia> using pluto
ERROR: ArgumentError: Package pluto not found in current path.
- Run `import Pkg; Pkg.add("pluto")` to install the pluto package.
Stacktrace:
 [1] ...
```

```
[julia> using Pluto

julia> █
```

# Debugging

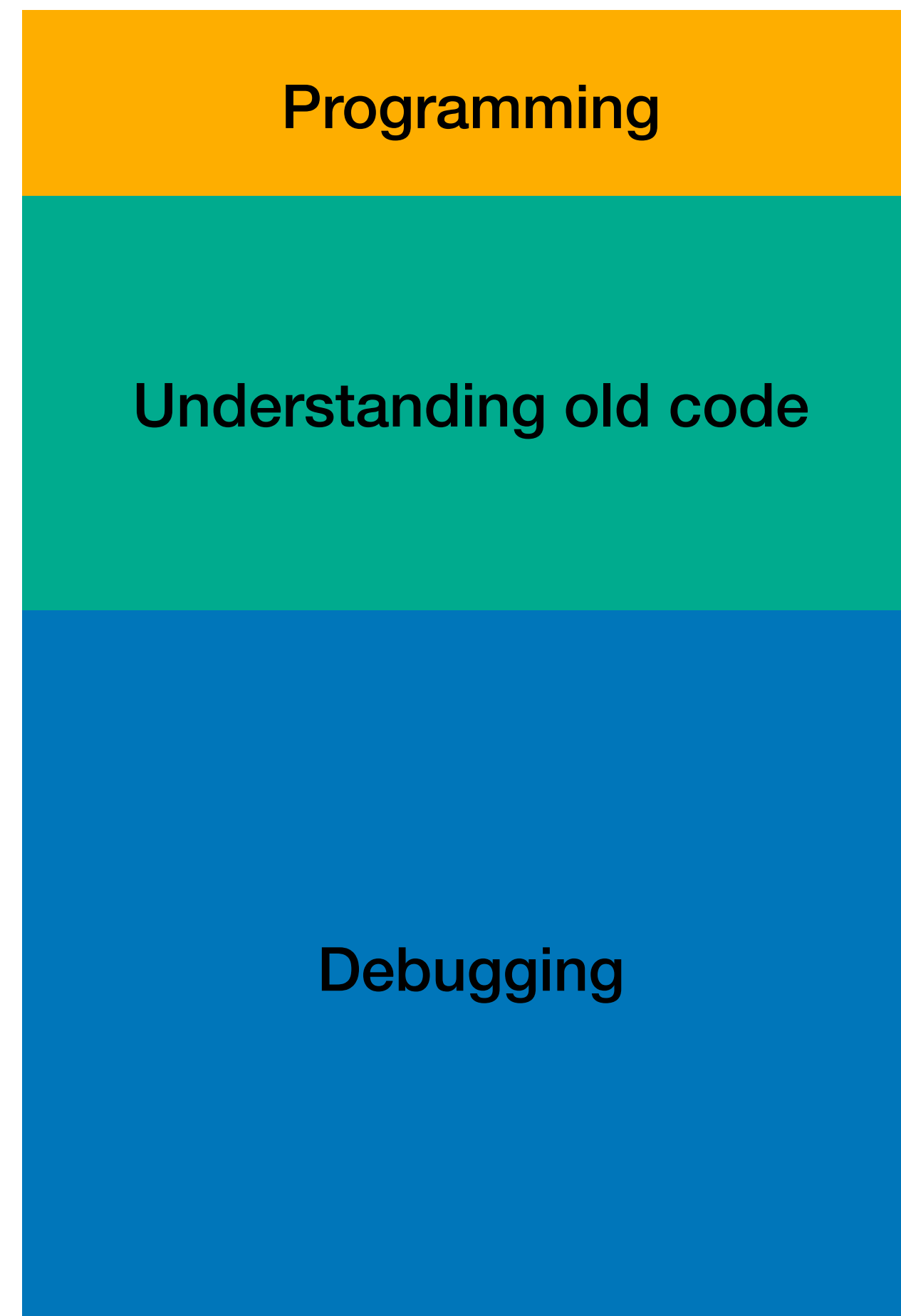


# Debugging

Good debugger

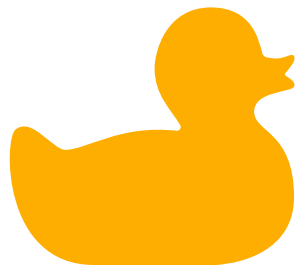
>

Good programmer





# Debugging



Search

## Rubber duck debugging

Article [Talk](#)

From Wikipedia, the free encyclopedia

 The Duck House Brighton  
<https://www.theduckhousebrighton.online>

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At The Duck House you will find over four hundred different ...

#### Special Occasions

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# Goals

## Mathematics and Computational Methods for Complex Systems, 2023

How do computers  
do maths?

How do we make sure  
computer maths is **efficient**?

# Getting the answer is **not** enough

## Mathematics and Computational Methods for **Complex Systems**, 2023

We often know **how** to  
analyse a complex system

Writing an efficient algorithm is  
the **bottleneck**

# Structure

What are programs? How  
are they run?

Interpreted vs compiled  
languages

Why Julia?

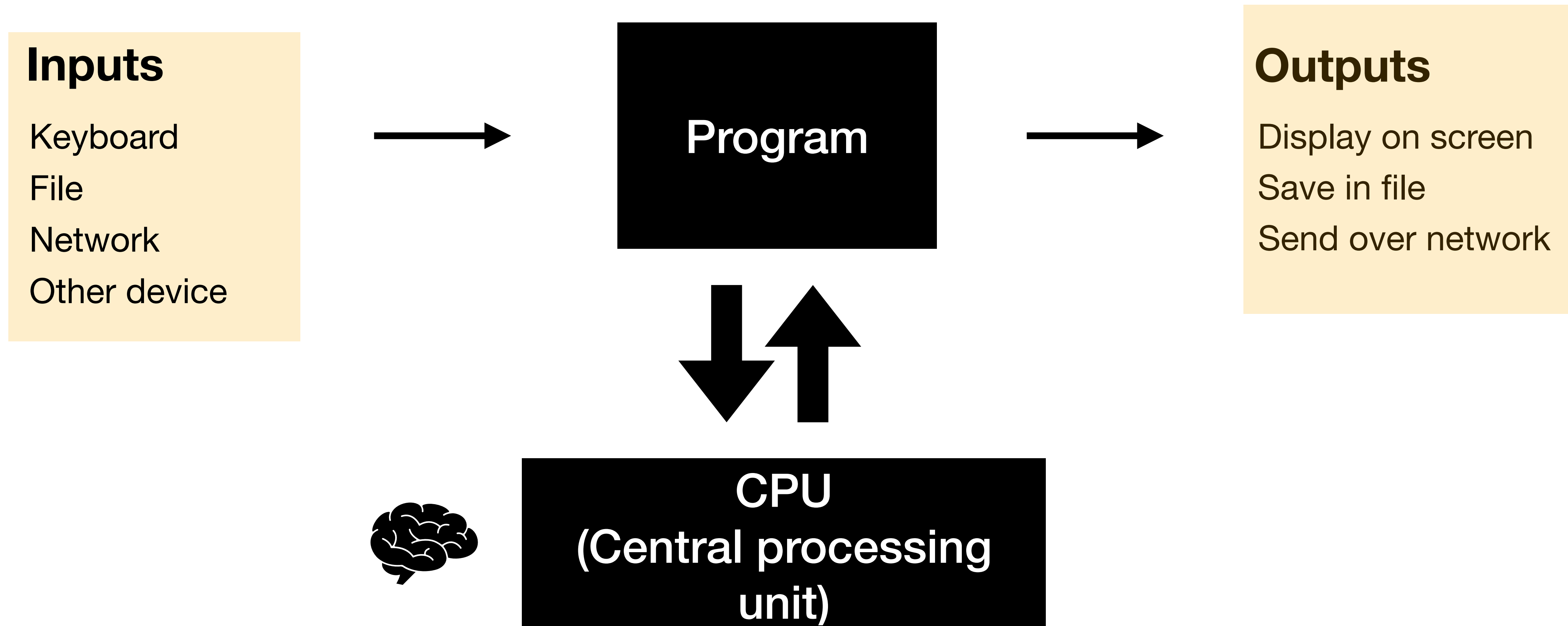
Memory models for  
programming

Allocations

Types

....next week maths!

# What is a computer program?





# What is a computer program?



## Program

**Basic maths operations**  
(+, -, ...)

**Conditional execution**  
(run code if ... is true)

**Repetition**  
(run things repeatedly,  
with variation)

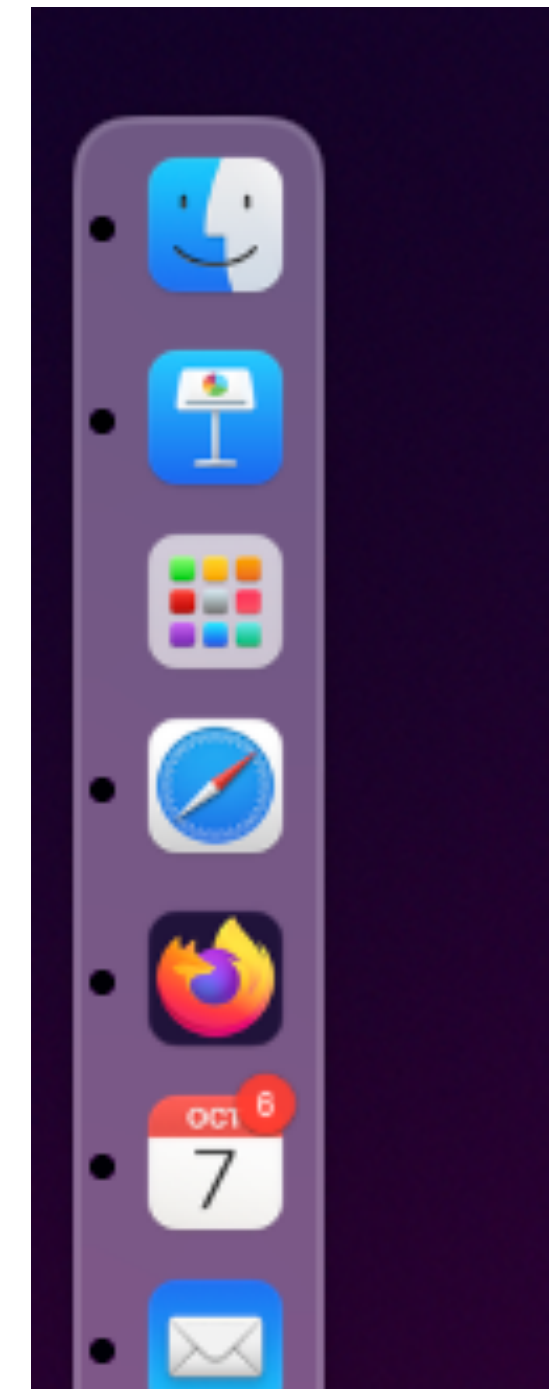


....that's it!!!

# How do we run a program?

**Graphical user interface  
(GUI)**

...you click!



# How do we run a program?

## Command line interface (CLI)

Terminal / powershell

*... a program that runs programs!*

Text input



Shell



Output:  
program

# How do we run a program?

## Shell

```
(base) → ~ julia some_program.jl
```

# What is a script?

A list of instructions, with  
no defined input or output

(Output of last line is  
shown, unless suppressed  
with semicolon ; )

► [9, 10]

```
• begin
•     x = 4
•     y = [5,6]
•     z = x .+ y
• end
```

+

»

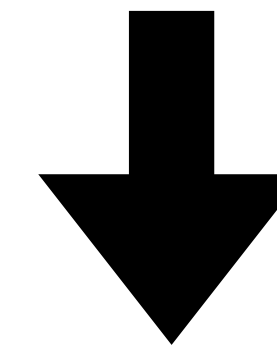
```
• begin
•     x = 4
•     y = [5,6]
•     z = x .+ y
• end;
```

+

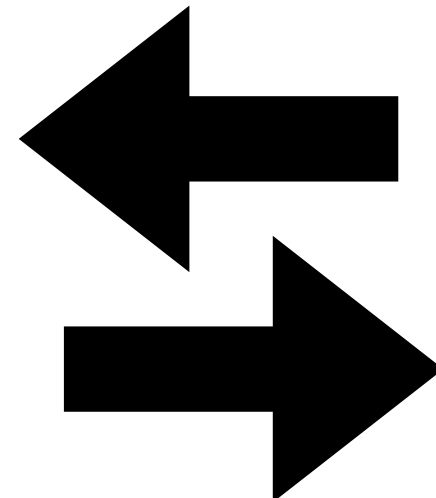
# Scripts are interpreted by a **REPL** program

(Read-eval-print loop)

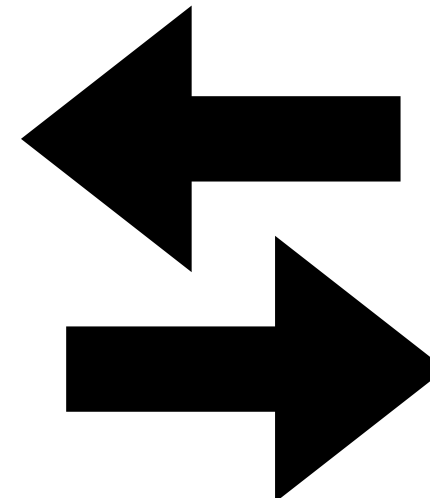
```
+  
> • begin  
•   x = 4  
•   y = [5,6]  
•   z = x.+y  
• end;|  
+
```



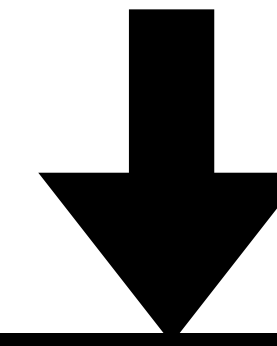
CPU



Julia



REPL  
(Pluto notebook)



Display results



# REPL scripts are for **exploratory** programming

Python

IPython3

Julia

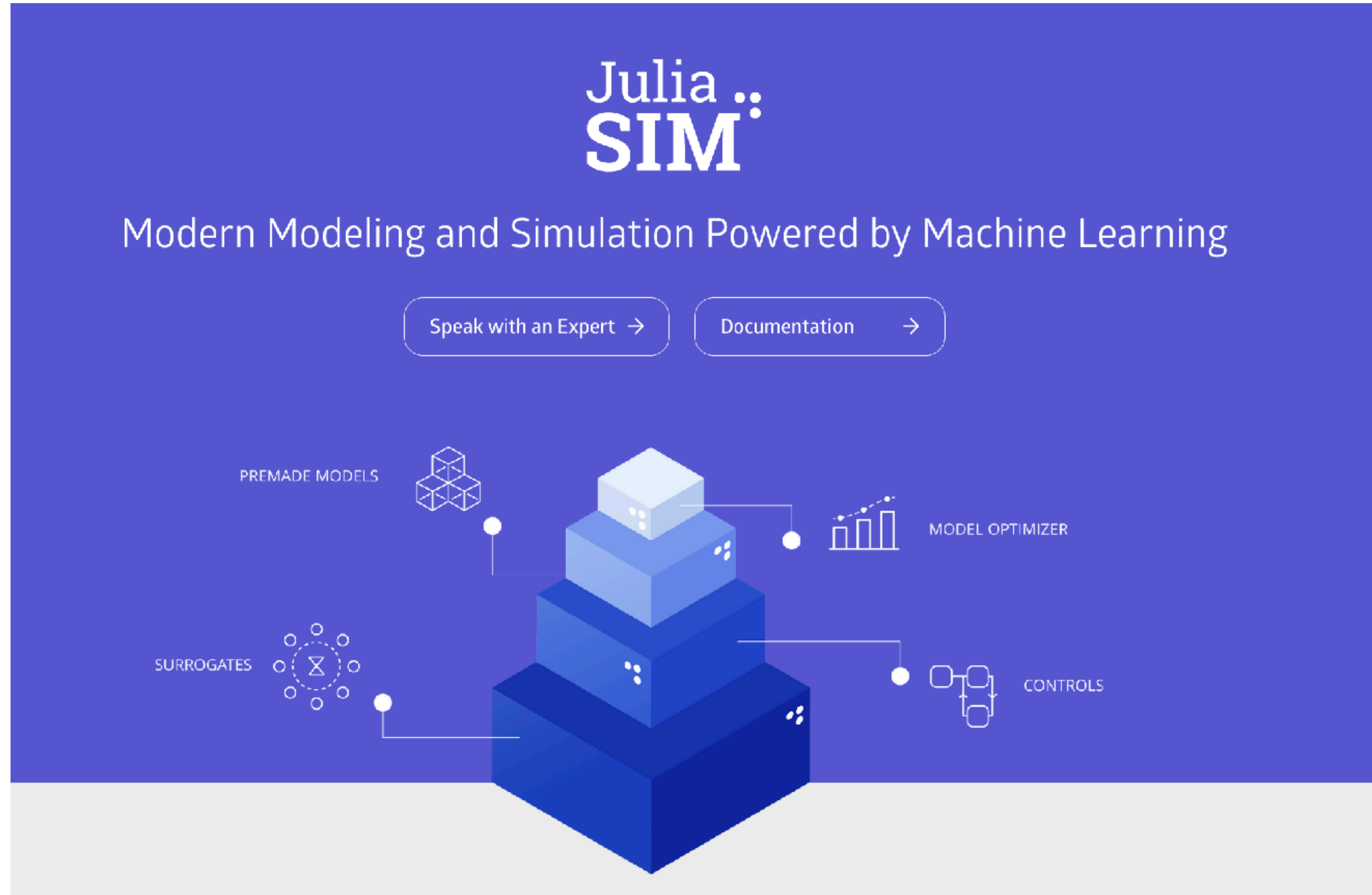
The Julia REPL / Pluto

Eg C++

Nothing

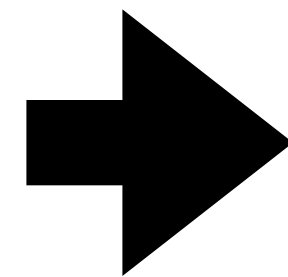
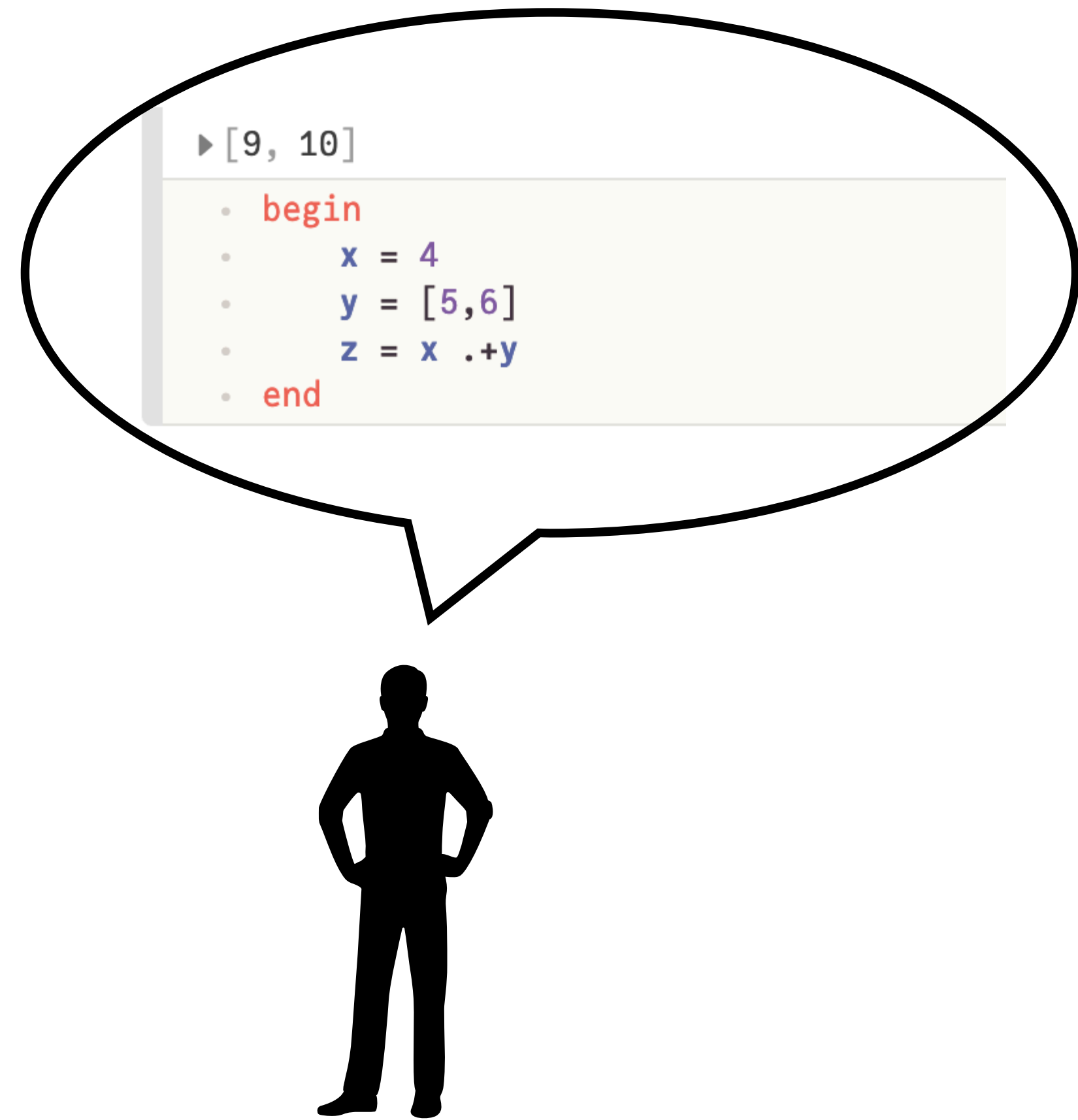
# Not all programming is in the REPL!

But that's all we'll use in MCMCS...

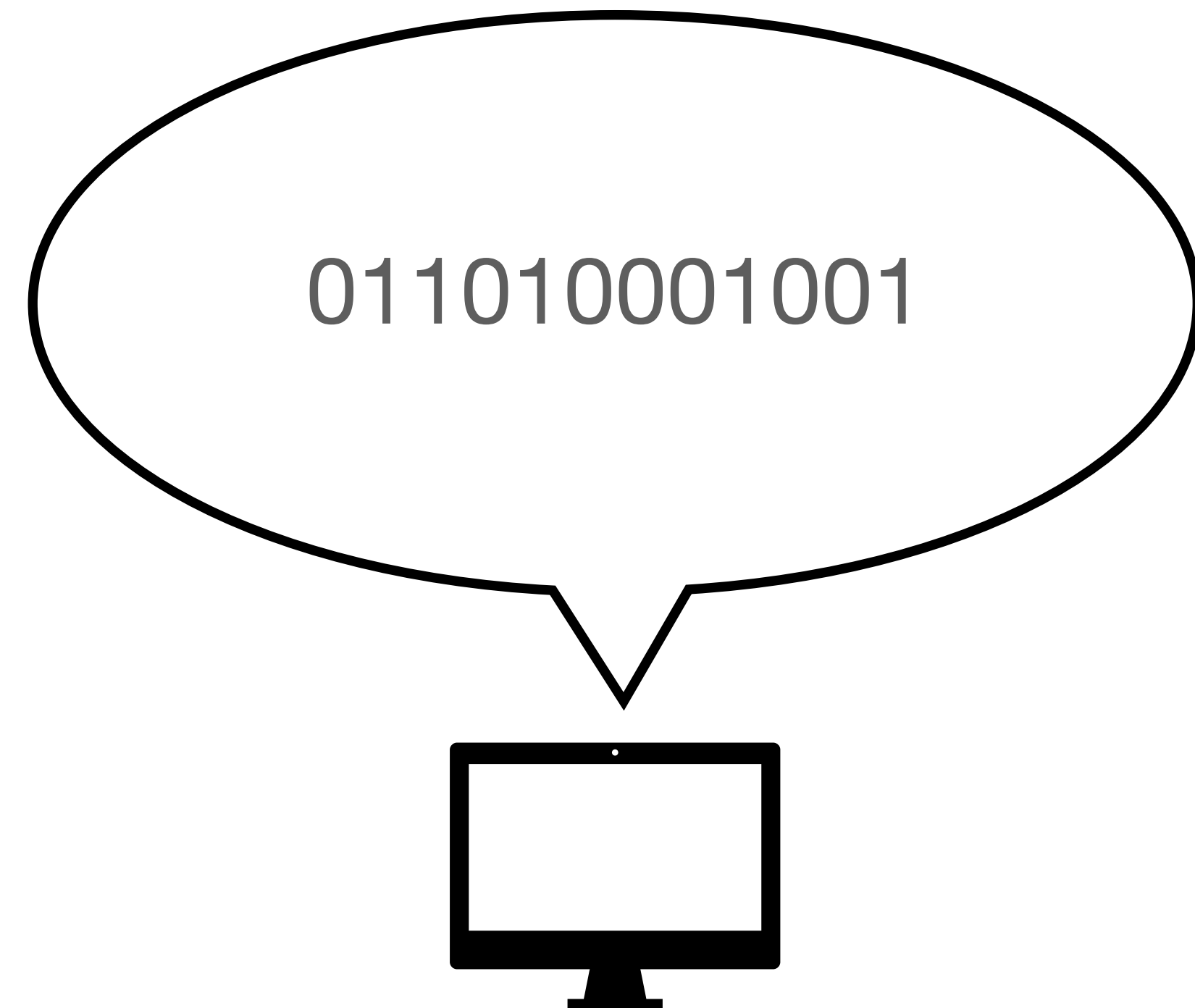




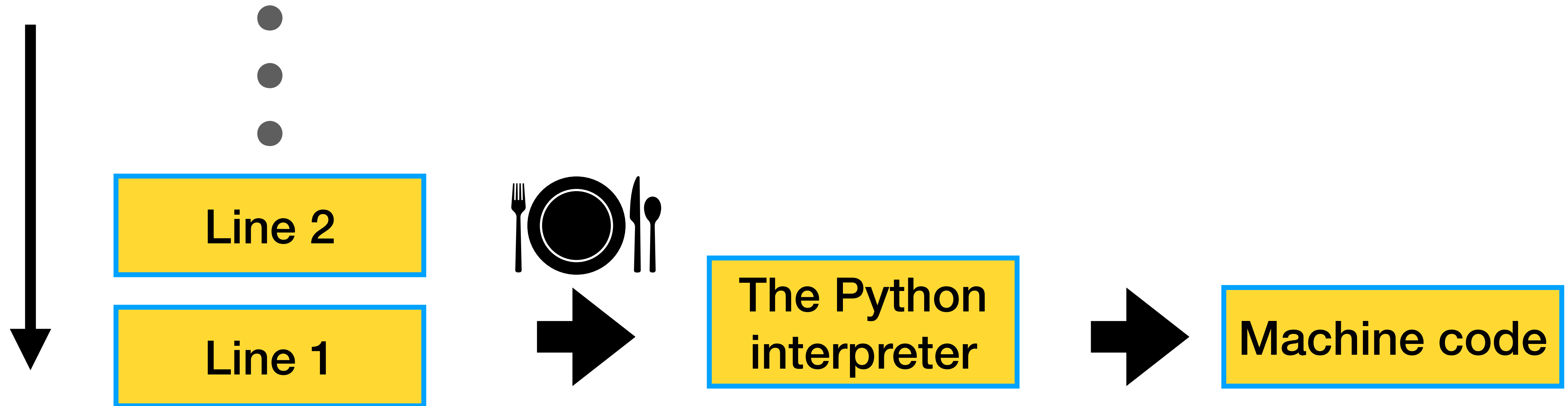
# Running programs



## Machine code



# Interpreted languages

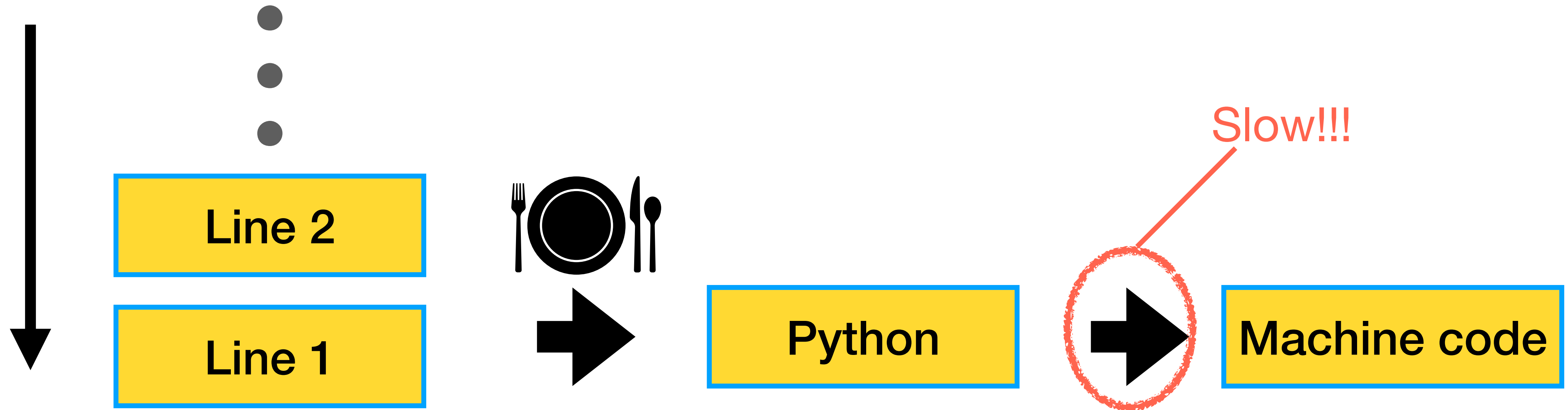


# Interpreted languages

► [9, 10]

- `begin`
- `x = 4`
- `y = [5,6]`
- `z = x .+y`
- `end`

# Interpreted languages



# Interpreted languages

*...should **never** do this!*

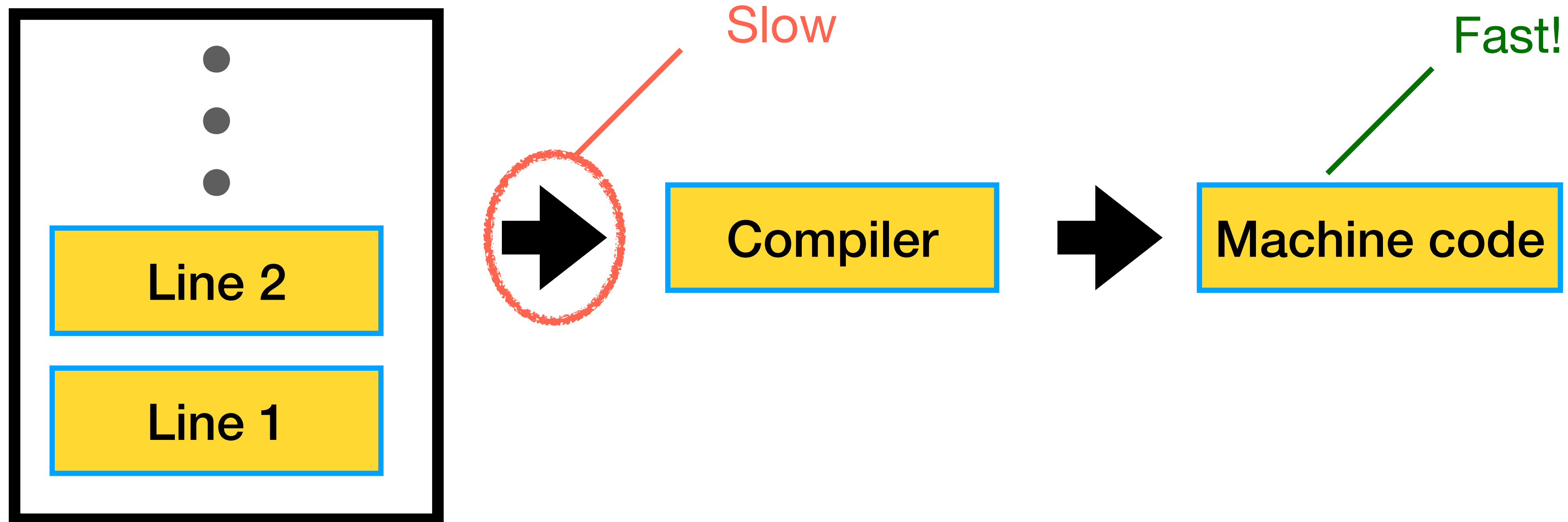
lots\_of\_lines (generic function with 1 method)

```
• function lots_of_lines(a::Int64)
•     for i = 1:100000
•         a +=1
•     end
• end
```

Executing each line  
takes time. x100000

# Compiled languages

*...are much faster*



# Compiled languages

*...will happily munch this*

lots\_of\_lines (generic function with 1 method)

```
• function lots_of_lines(a::Int64)
•     for i = 1:1000000
•         a +=1
•     end
• end
```

# Compiled language example

## C/C++

gcc is a **compiler** program that produces an **executable**

Unreadable to humans, but computer will run it

Your code

```
gcc sourcefile_name.c -o outputfile.exe
```

```
outputfile.exe
```

Machine code



# Production software is usually written in compiled languages



...but this isn't a software development course

# Always use compiled languages?

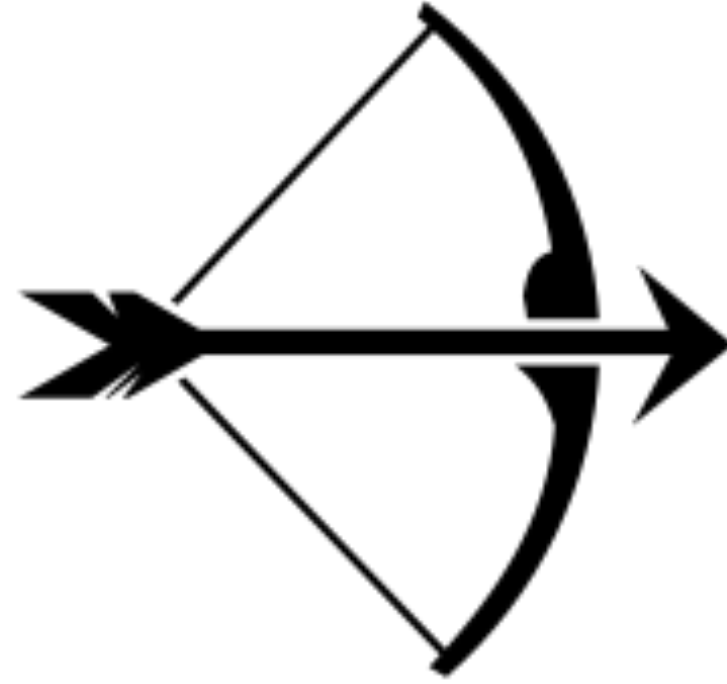
```
gcc sourcefile_name.c -o outputfile.exe
```

Every change needs  
recompilation!

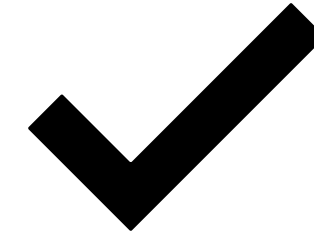
Compiling induces  
delayed feedback

Compiled languages are  
usually harder to learn

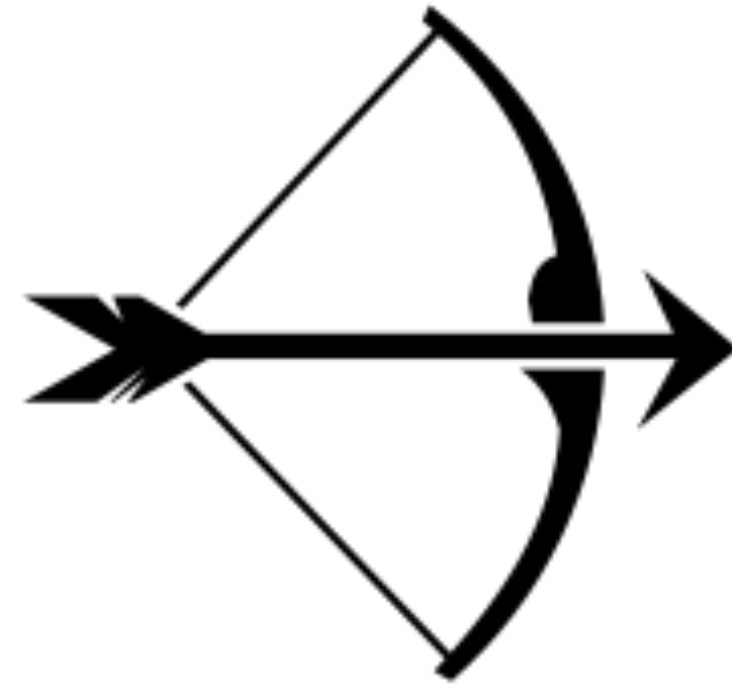
# English laws



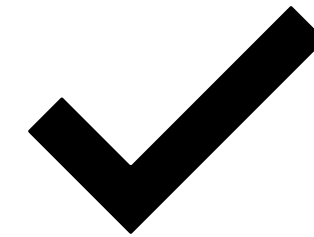
All English males over the age of 14 are to carry out two hours of longbow practice every week, supervised by the local clergy.



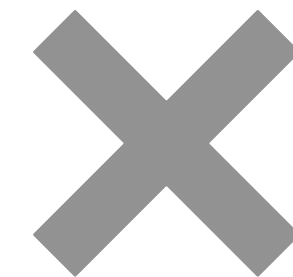
# English laws

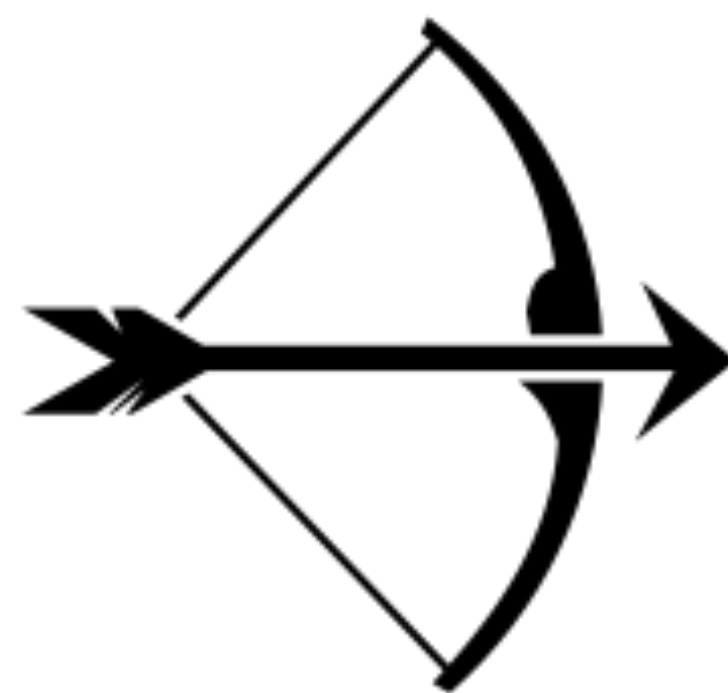


All English males over the age of 14 are to carry out two hours of longbow practice every week, supervised by the local clergy.



It is legal to shoot a Welshman with a longbow in Hereford, if he is within the city walls after midnight



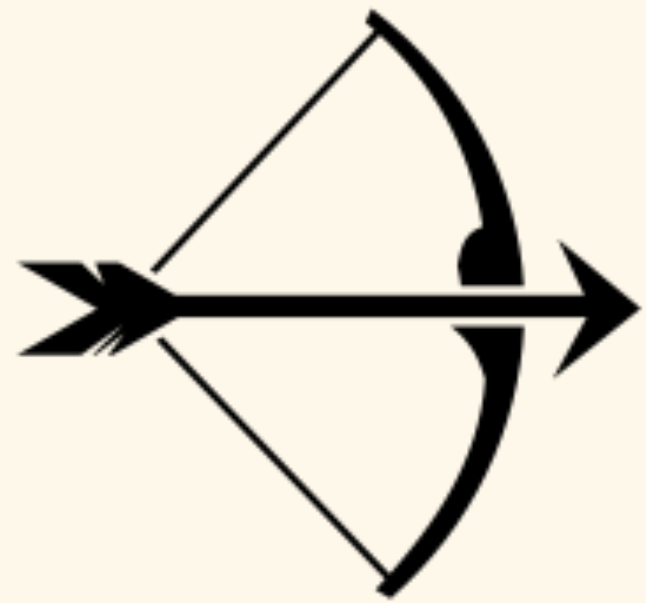


≥



?

# Programming vs shooting



## Compiled languages

C, C++, Fortran, Rust...



## Interpreted languages

Python, Matlab, PHP,  
Ruby, ...

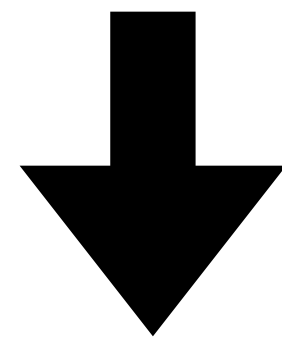
**Buying computing power is cheaper  
than training a programmer**

# New laws?

All English residents over the age of 14 are to carry out two hours of **coding** practice every week, supervised by the local TA's?

# New laws?

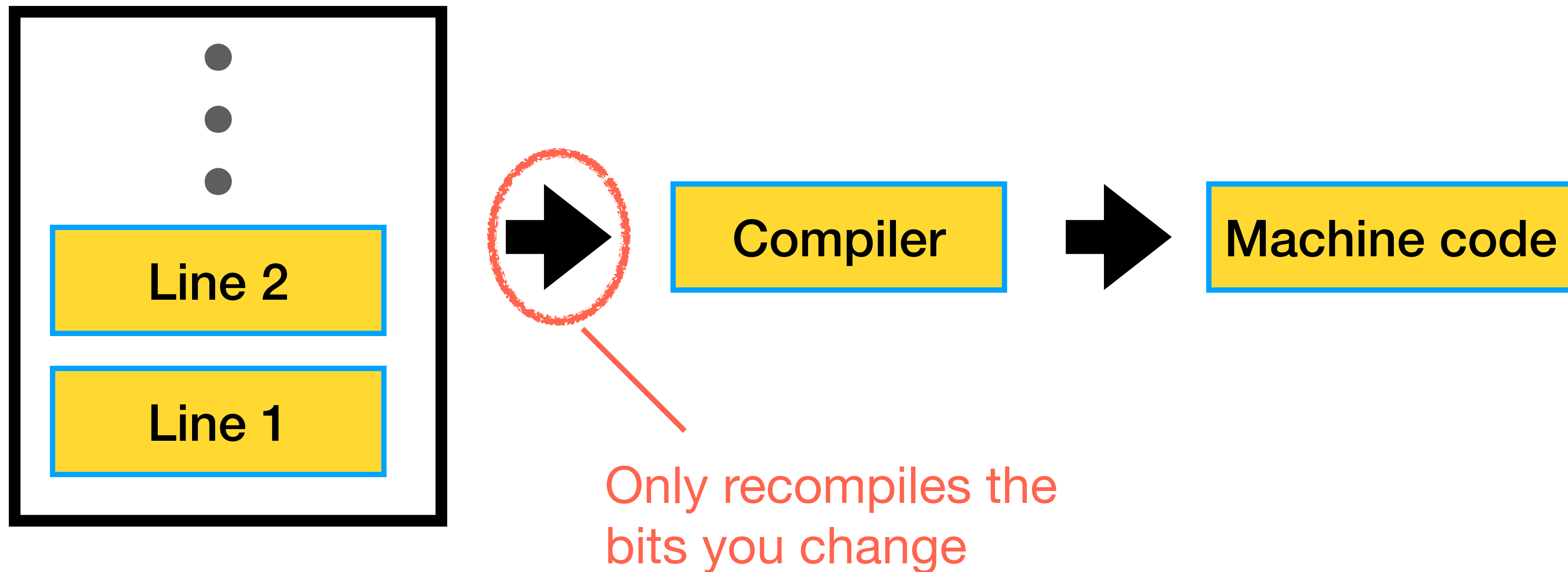
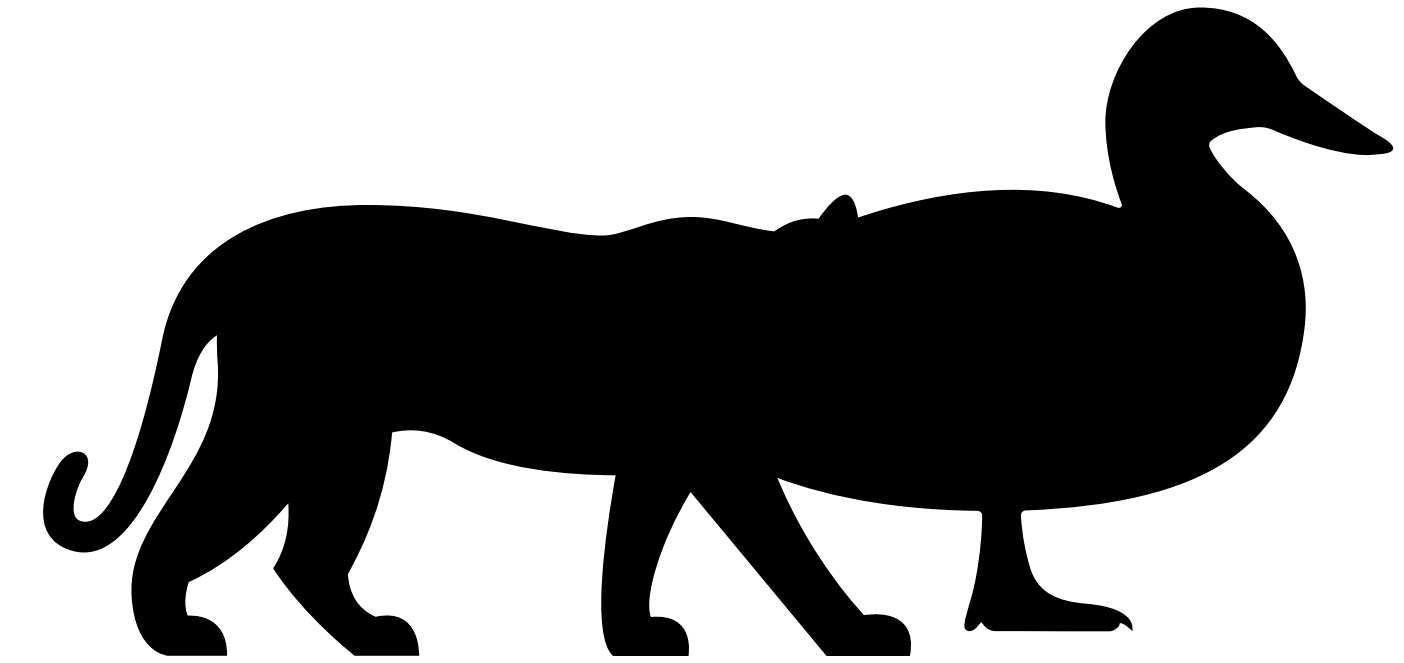
All English residents over the age of 14 are to carry out two hours of **coding** practice every week, supervised by the local TA's?



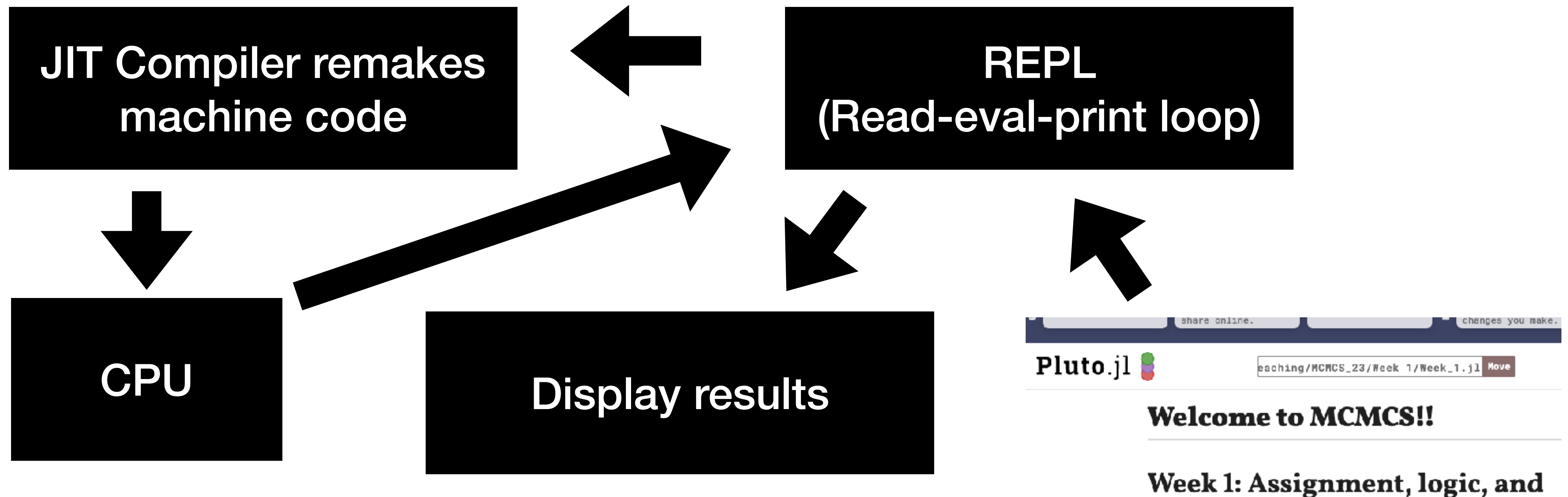
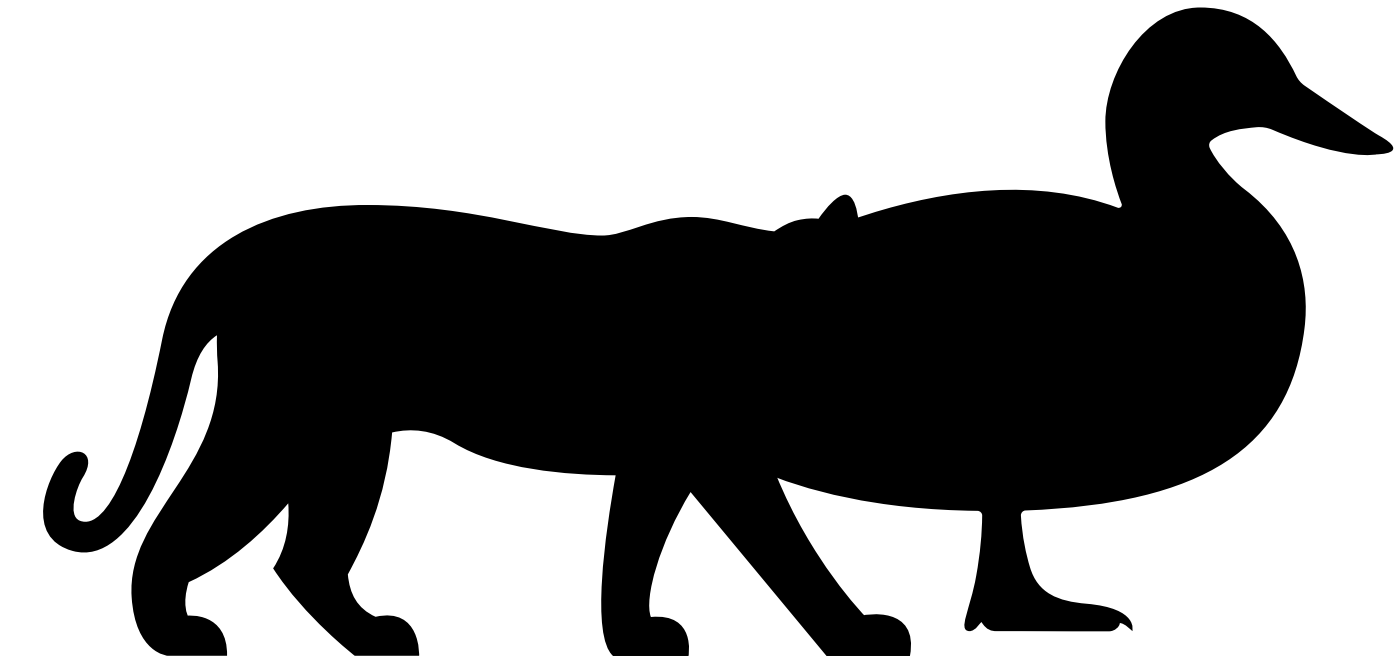
*Compiled languages would be more popular!*



# Julia is a **just-in-time** (JIT) compiled language



# Julia is a **just-in-time** (JIT) compiled language



# See the machine code

*..which is processor dependent*

```
@show @code_native lots_of_lines(0)
```

```
.section    __TEXT,__text,regular,pure_instructions
.build_version macos, 13, 0
.globl     _julia_lots_of_lines_4521      ## -- Begin function julia_lots_of_lines_4521
.p2align   4, 0x90
_julia_lots_of_lines_4521:                ## @julia_lots_of_lines_4521
; | @ /Users/dr360/.julia/pluto_notebooks/wild_report.jl#=#43425e91-760d-4f5a-8386-140a7c692e3b:1 within 'lots_of_lines'
.cfi_startproc
## %bb.0:                                ## %top
; | @ /Users/dr360/.julia/pluto_notebooks/wild_report.jl#=#43425e91-760d-4f5a-8386-140a7c692e3b:2 within 'lots_of_lines'
    leaq    100000(%rdi), %rax
; | @ /Users/dr360/.julia/pluto_notebooks/wild_report.jl#=#43425e91-760d-4f5a-8386-140a7c692e3b:5 within 'lots_of_lines'
    retq
.cfi_endproc

                                ## -- End function

.subsections_via_symbols
#= /Users/dr360/.julia/pluto_notebooks/Wild_report.jl#=#617452bc-ee7b-4f91-9e40-2ba28813ec83:1 =# @code_native(lots_of_lines(0)) = nothing
```

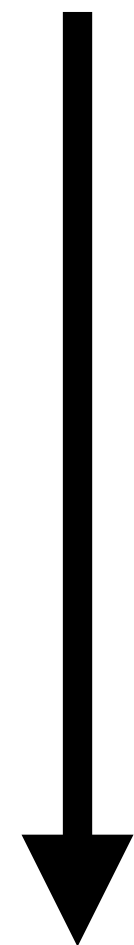
...and thank god we're not in the



# But why do ... use Python?

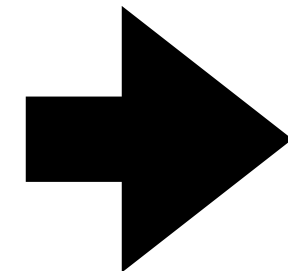
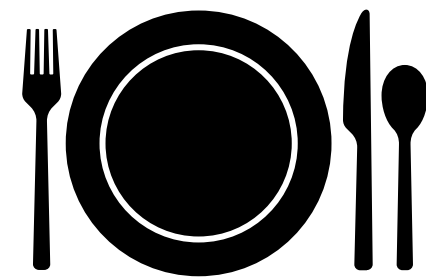
Lots of lines of code needed

$$\begin{bmatrix} 1 & 3 & 4 & \dots \\ 2 & 7 & 6 & \dots \\ \vdots & \vdots & \vdots & \vdots \end{bmatrix} \times \begin{bmatrix} 5 & 7 & 2 & \dots \\ 2 & 5 & 1 & \dots \\ \vdots & \vdots & \vdots & \vdots \end{bmatrix}$$

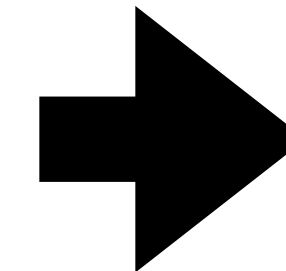


Line 2

Line 1



The Python  
interpreter



Machine code

# But why do ... use Python?

Lots of lines of code needed

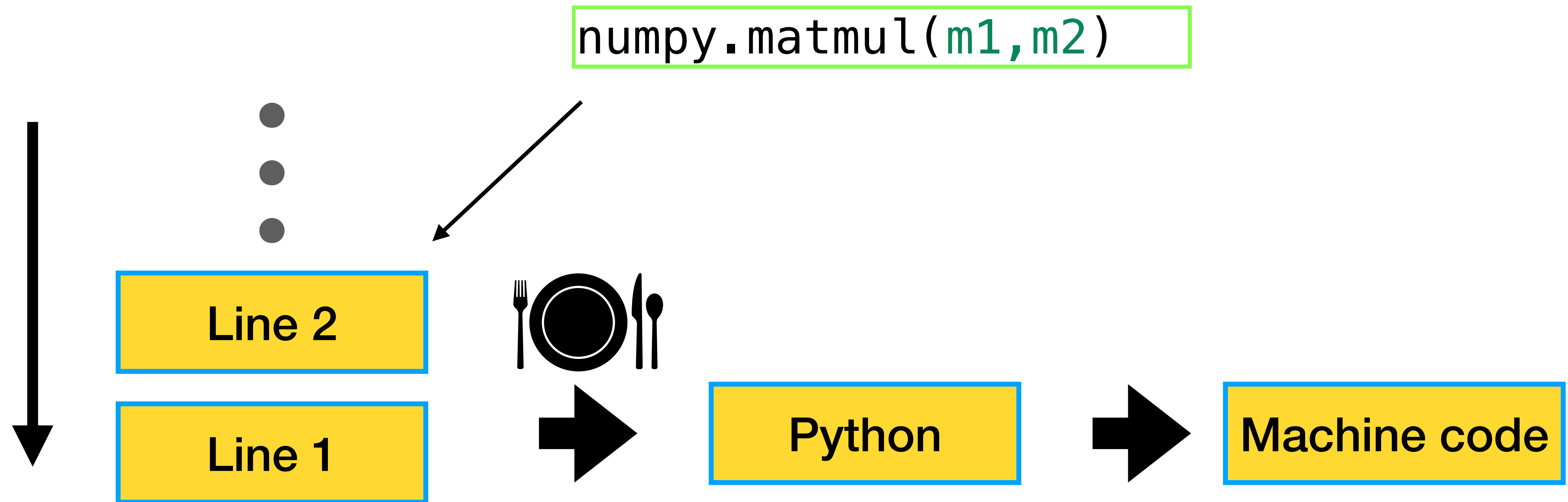
$$\begin{bmatrix} 1 & 3 & 4 & \dots \\ 2 & 7 & 6 & \dots \\ \vdots & \vdots & \vdots & \vdots \end{bmatrix} \times \begin{bmatrix} 5 & 7 & 2 & \dots \\ 2 & 5 & 1 & \dots \\ \vdots & \vdots & \vdots & \vdots \end{bmatrix}$$

```
numpy.matmul(m1, m2)
```

This is a **program**  
(written in fast language C++)



# Python is a **glue** language



Ideally, **each line is a specialised program** written in a fast language and doing lots of work

# Writing fast code

## **JIT/Compiled languages**

Write the algorithm  
yourself

Help the compiler:  
give it lots of info

# Writing fast code

## JIT/Compiled languages

Write the algorithm  
yourself

Help the compiler:  
give it lots of info

## Interpreted languages

Build algorithm by gluing  
specialised programs

Minimise lines of code



# Writing fast code

## JIT/Compiled languages

Write the algorithm yourself

Help the compiler:  
give it lots of info

## Interpreted languages

Build algorithm by gluing specialised programs

Minimise lines of code

*Future? Python is turning into a JIT compiled language!!! (Mojo, Jax etc)*

# Other reasons we are using Julia

## Transferable

Most concepts we learn have a direct analogue in Python

## Closer to the metal

Easier for maths

**Multilinguality is important!!**

# An annoying difference...

## Zero-indexed languages

Python

PHP

Java

C

Ruby

## One-indexed languages

Julia

Fortran

MATLAB

# An annoying difference...

## One-indexed languages

```
a = ► [2, 4, 7, 9]
```

- a = [2, 4, 7, 9]

4

- a[2]

## Zero-indexed languages

```
a[2] = 7
```

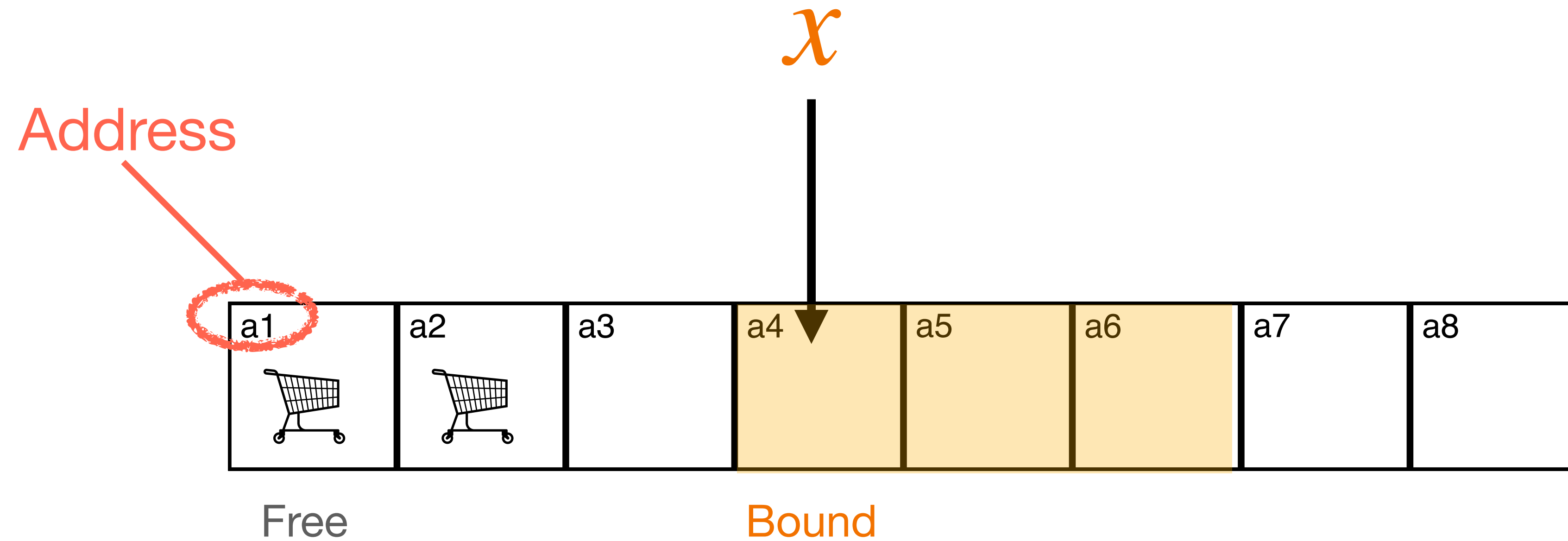
# Memory models in programming



• `x = 4`

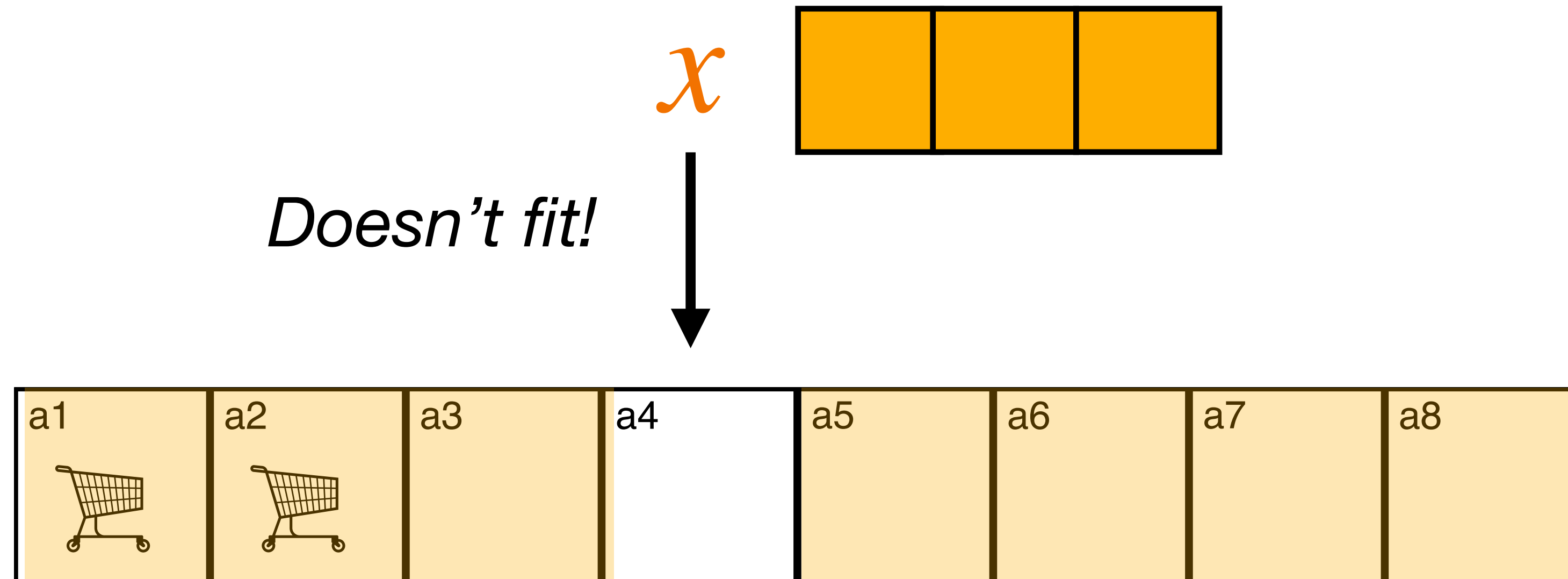
What's going on  
under the hood?

# Computer storage



Computer stores each variable  
Like a website address

# Lots of variables?



Where do I put them!

Takes **time** for computer  
to figure out

Compiler does some  
organisation **during compilation**

*(As much as possible)*

## Redefining x 5

```
• @time begin  
•     bb = [4,5,2,6]  
•     for i = 1:5  
•         bb = 2bb  
•     end  
• end
```



```
0.000004 seconds (6 allocations: 576 bytes)
```

Important to run twice  
(*first run includes compilation*)

```
► [128, 160, 64, 192]
```

```
• bb
```



## Redefining x 5

```
• @time begin  
•     bb = [4,5,2,6]  
•     for i = 1:5  
•         bb = 2*bb  
•     end  
• end
```

>

0.000004 seconds (6 allocations: 576 bytes)

## No redefining

*Why would this be  
slow in Python?*

```
• @time begin  
•     aa = [4,5,2,6]  
•     for i = 1:5  
•         for j in 1:length(aa)  
•             aa[j] = 2*aa[j]  
•         end  
•     end  
• end
```

>

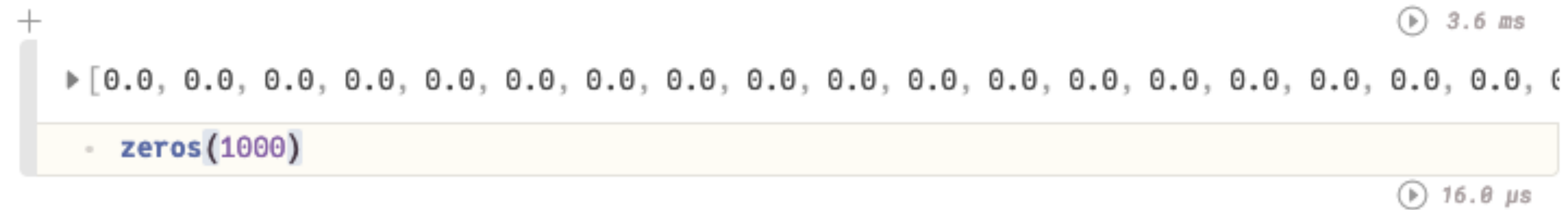
0.000004 seconds (1 allocation: 96 bytes)

# Preallocation is good

## Goal

`aa = [1,2,3,4,5,6,7,..., 1000]`

# Preallocation is good



- `@time begin`
- `aa = zeros(1000)`
- `for i = 1:1000`
- `aa[i]=i`
- `end`
- `end`



```
0.000005 seconds (1 allocation: 7.938 KiB)
```

# Changing variable size is **slow**

## Previous (preallocation)

```
• @time begin  
•     aa = zeros(1000)  
•     for i = 1:1000  
•         aa[i]=i  
•     end  
• end
```



0.000005 seconds (1 allocation: 7.938 KiB)

## New (dynamic allocation)

```
• @time begin  
•     bb = []  
•     for i = 1:1000  
•         push!(bb,i)  
•     end  
• end
```



0.000023 seconds (495 allocations: 29.484 KiB)

# Changing variable size is **slow**

## Previous (preallocation)

```
• @time begin  
•     aa = zeros(1000)  
•     for i = 1:1000  
•         aa[i]=i  
•     end  
• end
```



```
0.000005 seconds (1 allocation: 7.938 KiB)
```

## Best = preallocation



```
► [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16]
```

```
1 @time collect(1:1000)
```



```
0.000005 seconds (1 allocation: 7.938 KiB)
```



# Pass by reference

Both variables access the **same block** of memory

Think of a,b as **shortcuts** to the same website

*Changing website changes output of both links*

```
7
• begin
•   a = [3,4]
•   b = a
•   a[1] = 7;
• end

▶ [7, 4]
• a

+
> ▶ [7, 4]
• b

+
```

# Pass by value

b is a **physical copy** of the contents of a

Think of b as **printing a new website**

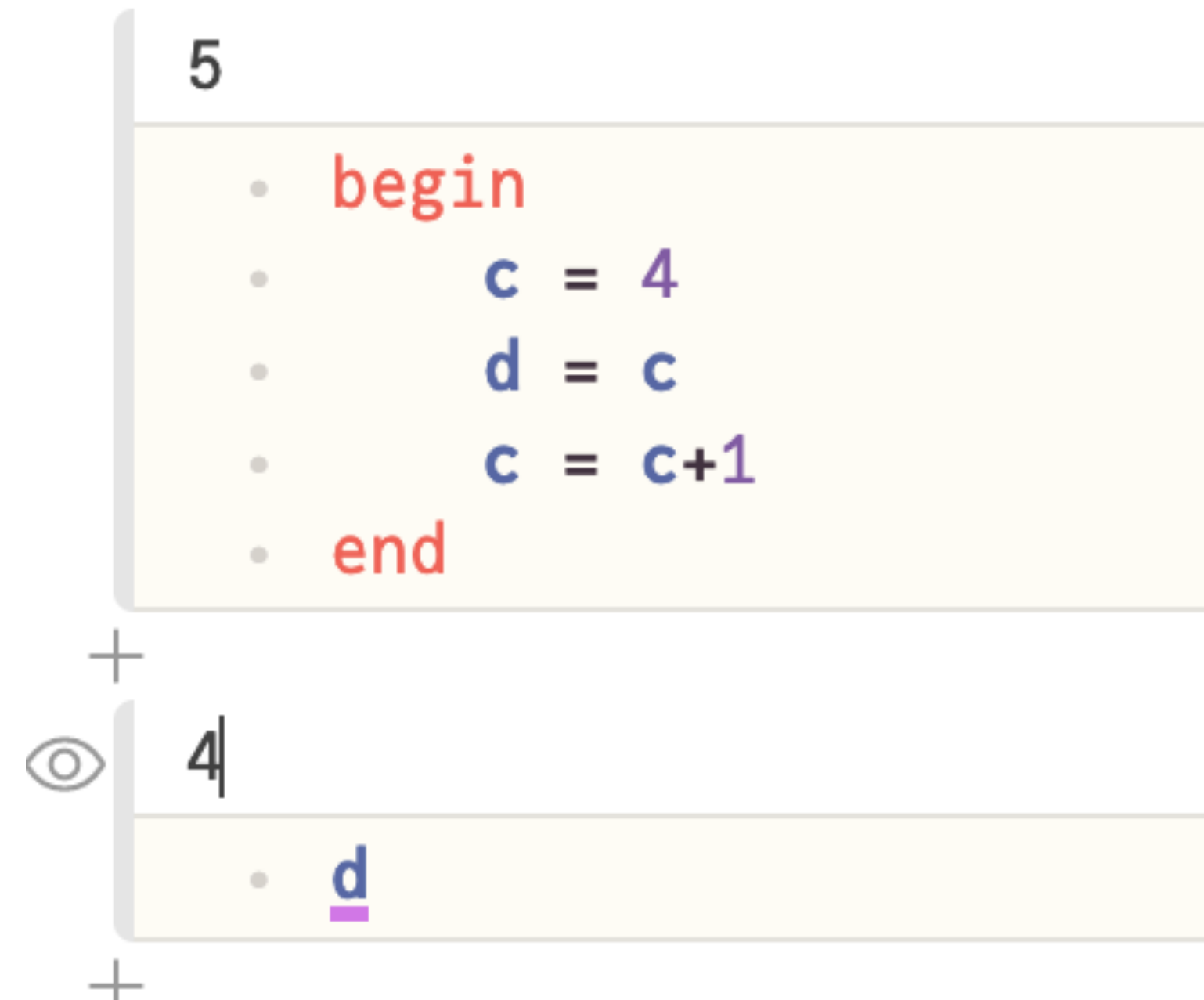
*...with the same information as a*

```
+
> 7
  • begin
  •   a = [3,4]
  •   b = deepcopy(a)
  •   a[1] = 7;
  • end

+
  ▶ [7, 4]
  • a

  ▶ [3, 4]
  • b
```

# Pass by what?





# Pass by value

*Like Python*

Immutable types are  
pass-by-value

```
+  
false  
• immutable(d)
```

```
5  
• begin  
•   c = 4  
•   d = c  
•   c = c+1  
• end  
+  
👁 4  
• d  
+
```

# Pass by reference

*Like Python*

Mutable types are  
pass-by-reference

```
⌕ true
  • ismutable(a)
```

7

```
• begin
  •   a = [3,4]
  •   b = a
  •   a[1] = 7;
  • end
```

▶ [7, 4]

• a

+

▶ [7, 4]

• b

+

# Type annotation

Every variable, in every language, has a **type**

Int64

- `typeof(1)`

Float64

- `typeof(1.)`

Float64

- `typeof(1.0)`

# Type annotation

`x::Type` is a **type assertion**

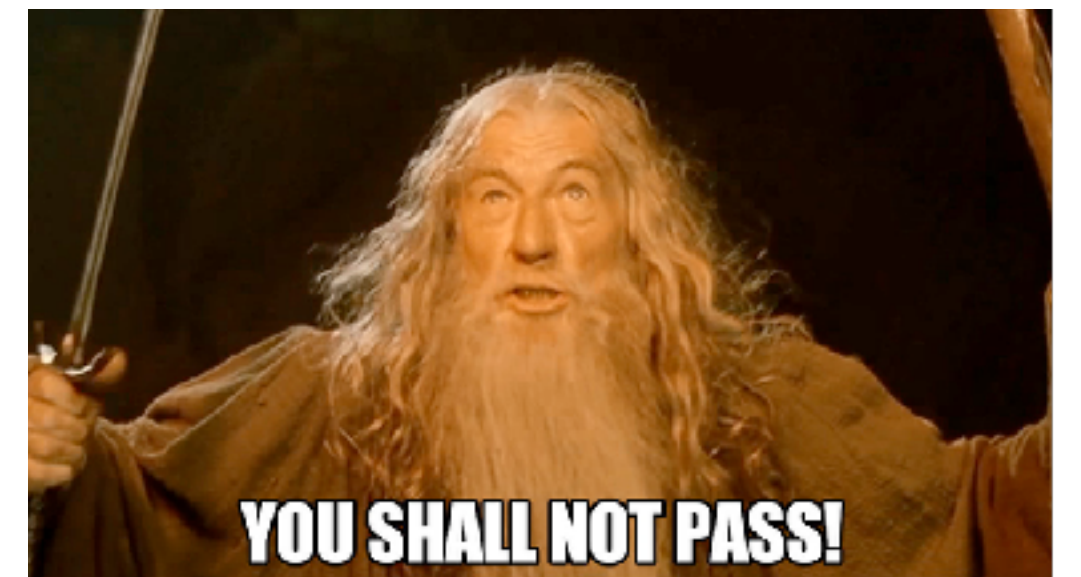
```
1
```

```
• 1::Int64
```

```
TypeError: in typeassert, expected Float64, got a value of type Int64
```

```
1. top-level scope @ [Local: 1] [inlined]
```

```
• 1::Float64
```



# Operations on different types

*Empathising with the compiler*

Often necessary in **'untyped'**  
languages like Python

```
1::Int64 + 2.0::Float64
```

1. Shit, different types!
2. Promote to a **common** type
3. Do the addition

# Operations on different types

*Empathising with the compiler*

Often necessary in **'untyped'** languages like Python

```
1::Int64 + 2.0::Float64
```

1. Shit, different types!
2. Promote to a **common** type
3. Do the addition

```
1.0::Float64 + 2.0::Float64
```

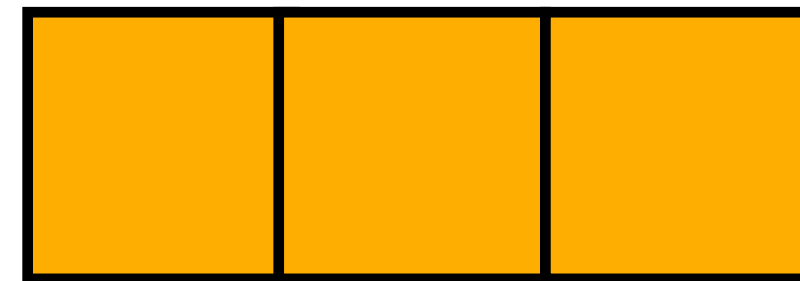
1. Guaranteed same type :)
2. Do the addition

# Type annotation

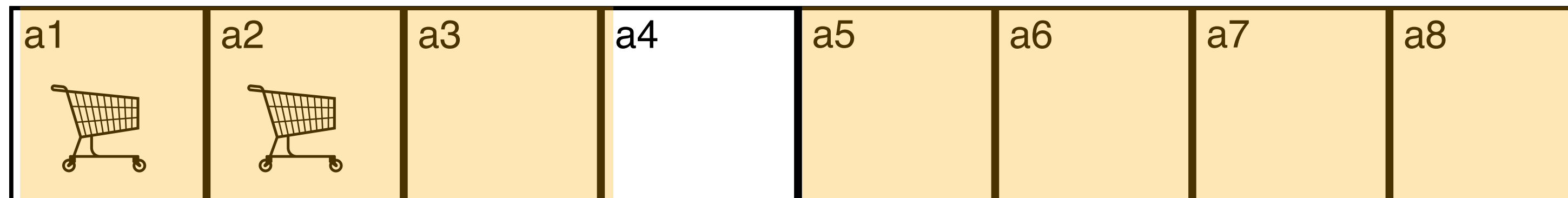
```
+  
• struct Fungus <: Organism  
•   weight  
• end  
+
```

*How big? Don't know!*

Fungus

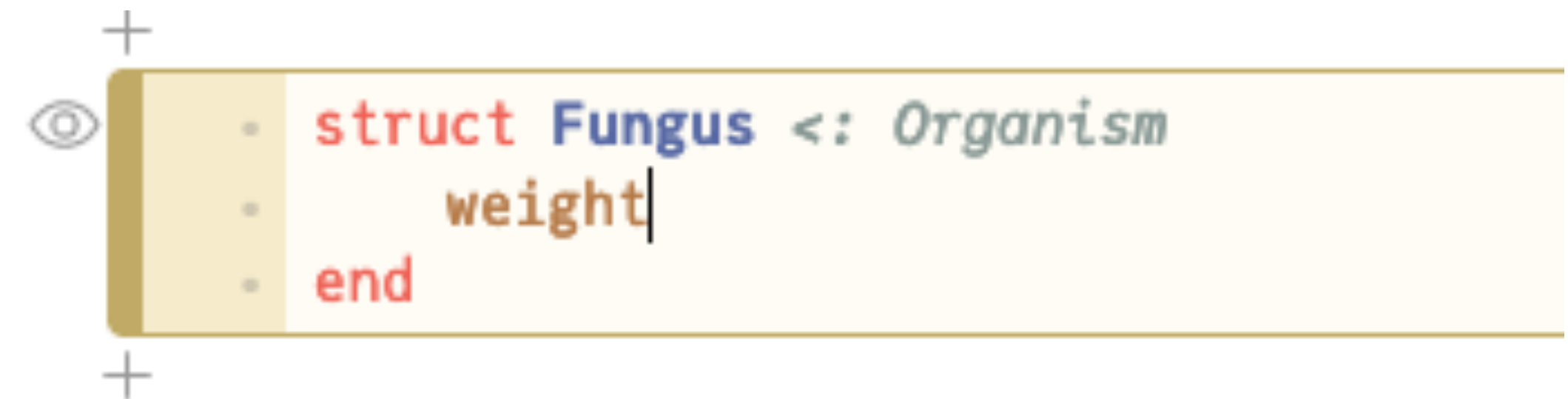


*Doesn't fit!*



# Type annotation

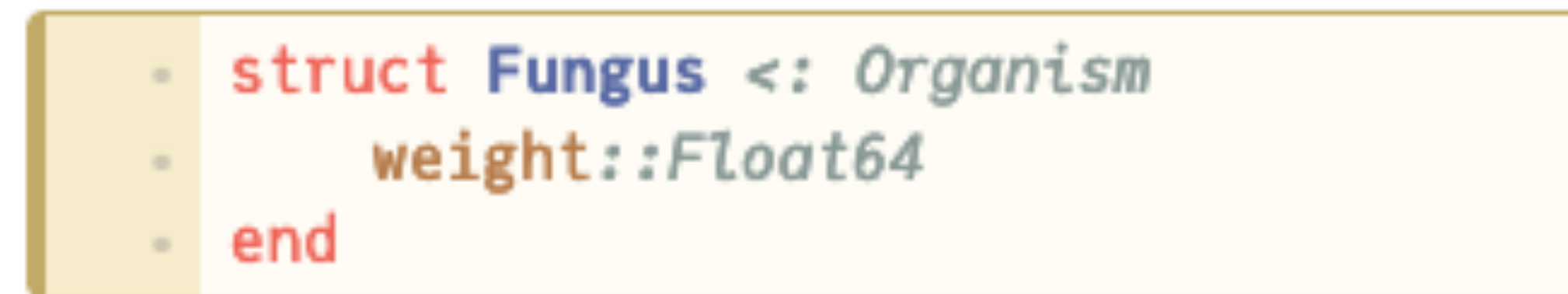
Compiler knows how much storage to allocate each Fungus



A code editor snippet showing a struct definition. The text is: `struct Fungus <: Organism`, `weight`, and `end`. The word `weight` is followed by a vertical cursor line. The editor has a light yellow background and a dark yellow border. There are small icons on the left: a plus sign at the top and an eye icon below it.

```
• struct Fungus <: Organism  
•     weight  
• end
```

Vs



A code editor snippet showing a struct definition with a type annotation. The text is: `struct Fungus <: Organism`, `weight::Float64`, and `end`. The editor has a light yellow background and a dark yellow border.

```
• struct Fungus <: Organism  
•     weight::Float64  
• end
```



# Type annotation

Compiler knows how much storage to allocate each Fungus

Can do better **compile-time** memory management

**Run-time** is faster, leaner

+  
👁

```
• struct Fungus <: Organism  
•   weight|  
• end
```

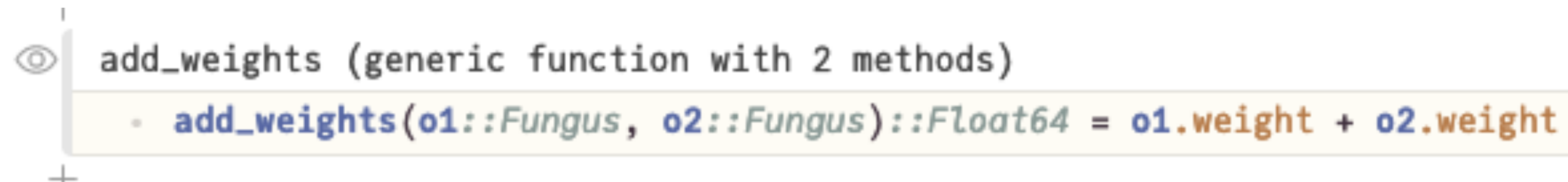
+  
Vs

```
• struct Fungus <: Organism  
•   weight::Float64  
• end
```

# Type annotation

...can be done anywhere!

## Type assertion of function output



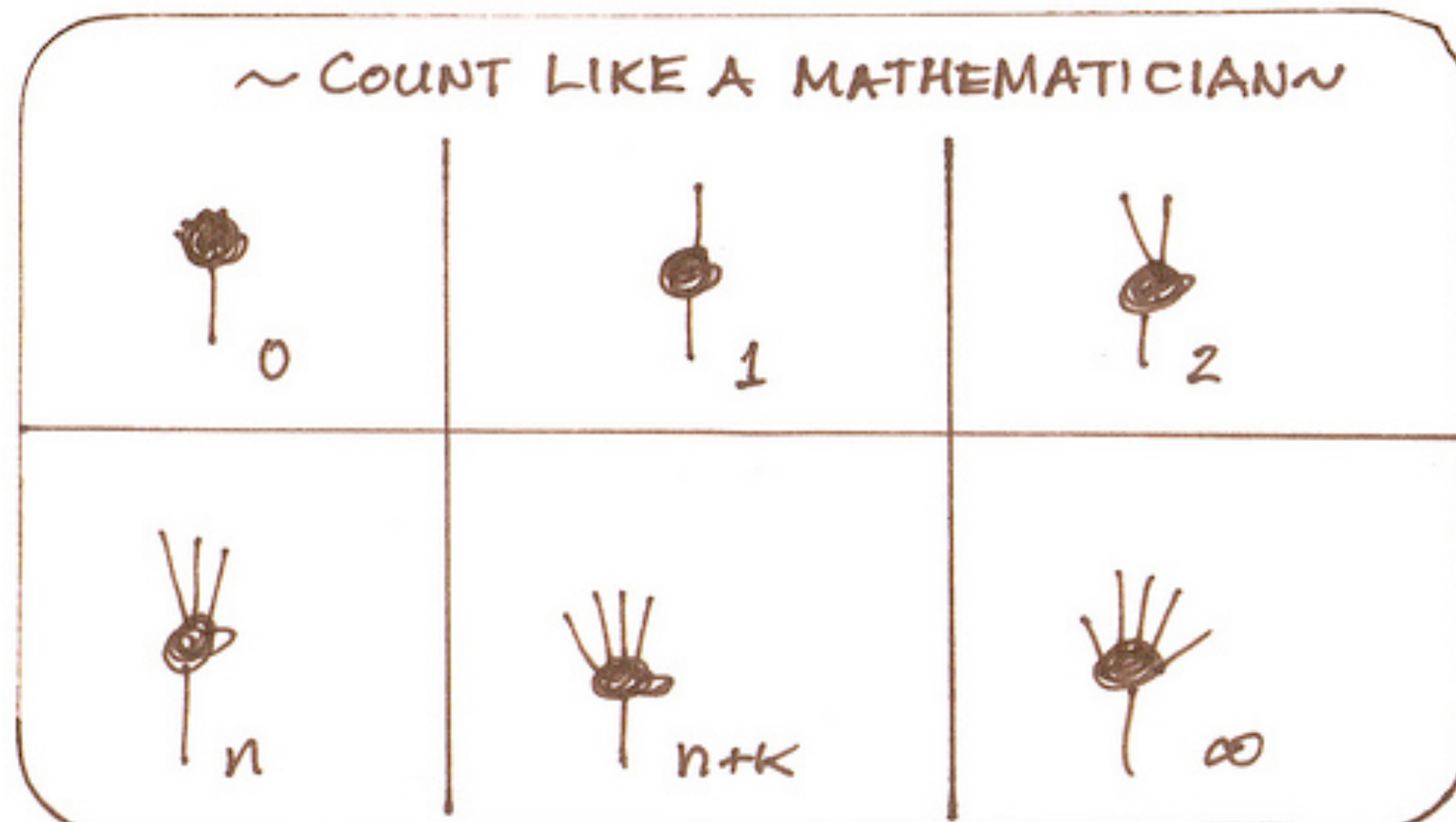
The screenshot shows a code editor with a light gray background. On the left, there is a vertical scrollbar and a small icon of an eye. The code is as follows:

```
add_weights (generic function with 2 methods)
  • add_weights(o1::Fungus, o2::Fungus)::Float64 = o1.weight + o2.weight
```

NB compilers are clever and can often infer types by detective work







- `no_inputs_or_outputs()`

>\_

hi|