Week 6

Mathematics and Computational Methods for Complex Systems, 2023

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Some ongoing issues

Reading mathematics ➡ *End of lecture optional practice*

Reading carefully ➡ *Life skill!*

Goals today

Statistics of random variables

Expected value
Variance
Functions of RVs
Central limit theorem

Conditional probability

Independent events
Bayes' law

Maths reading practice

Optional

Probability Space
Recap

$$(\Omega, \mathscr{F}, \mathbb{P})$$

Sample space:
Every possible seating combination

Event space:
Sets of seating combinations

Example set:
All seating combinations where the back row is filled

Probability function:
Assigns a probability to every single event

Random variables
Recap

are quantitative questions
about the experiment

are functions that map from
outcomes to numbers

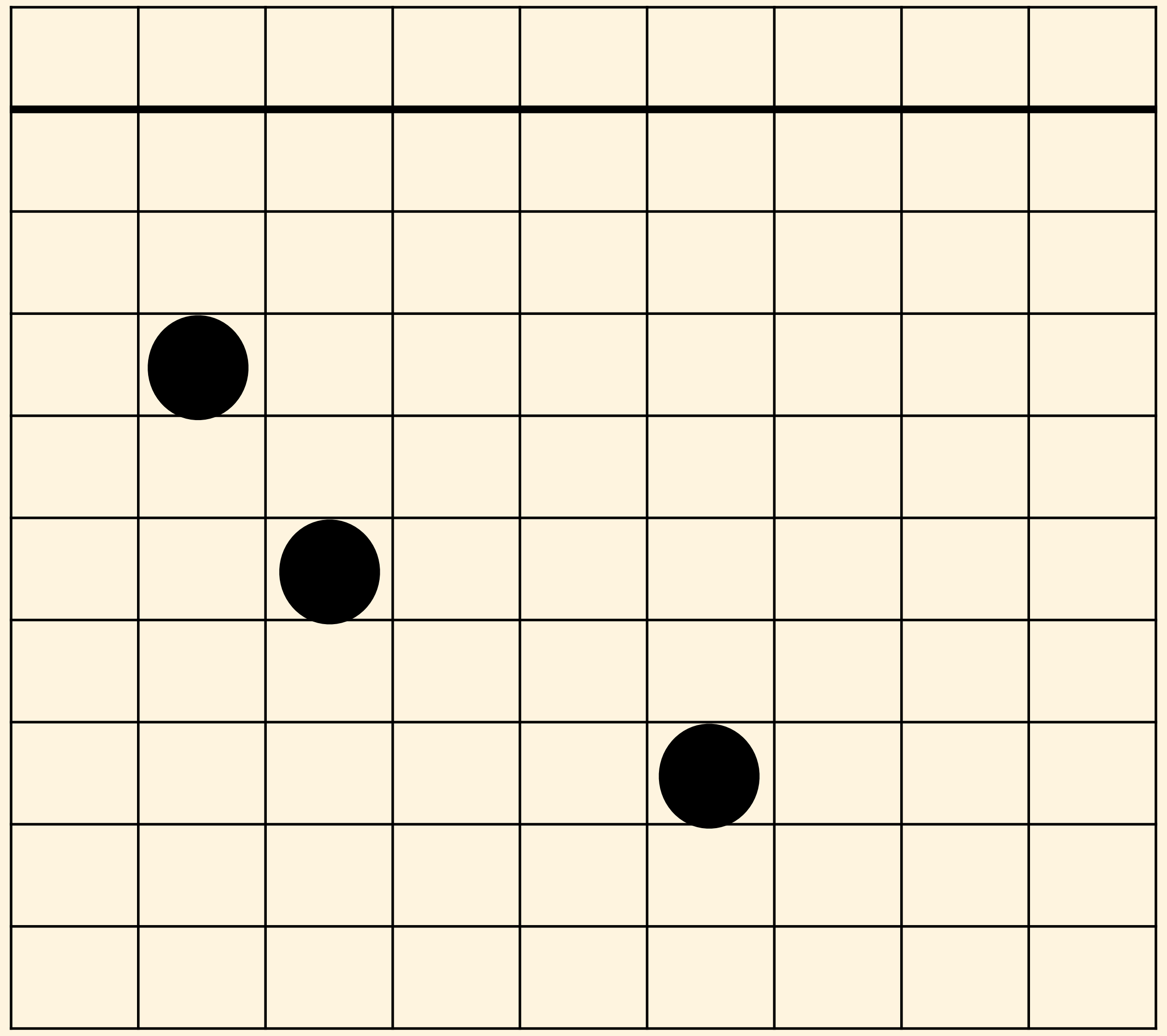
(or to any “measurable space”)

Random variables example

Recap

What was the number of unfilled seats?

Outcome ω



RV is a function:

$X(\omega)$

Number

87

(Or any integer between 0 and *number of seats*)

Support

Purpose of a random variable

Say something quantitative about a situation we can't model fully

(e.g. lecture seating next week)

↕

Quantify a hypothesis across unaccounted situations

(all matrices are invertible)

Practice

$$\begin{bmatrix} \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \end{bmatrix}$$

$$\bullet \sim U[-1,1]$$

Outcome space?

What type of RV is:

"Is the matrix invertible?"

Probability of event?

"Matrix is invertible"

Statistics

Summaries of a random variable

Statistical function:

Random variable

↓

Number

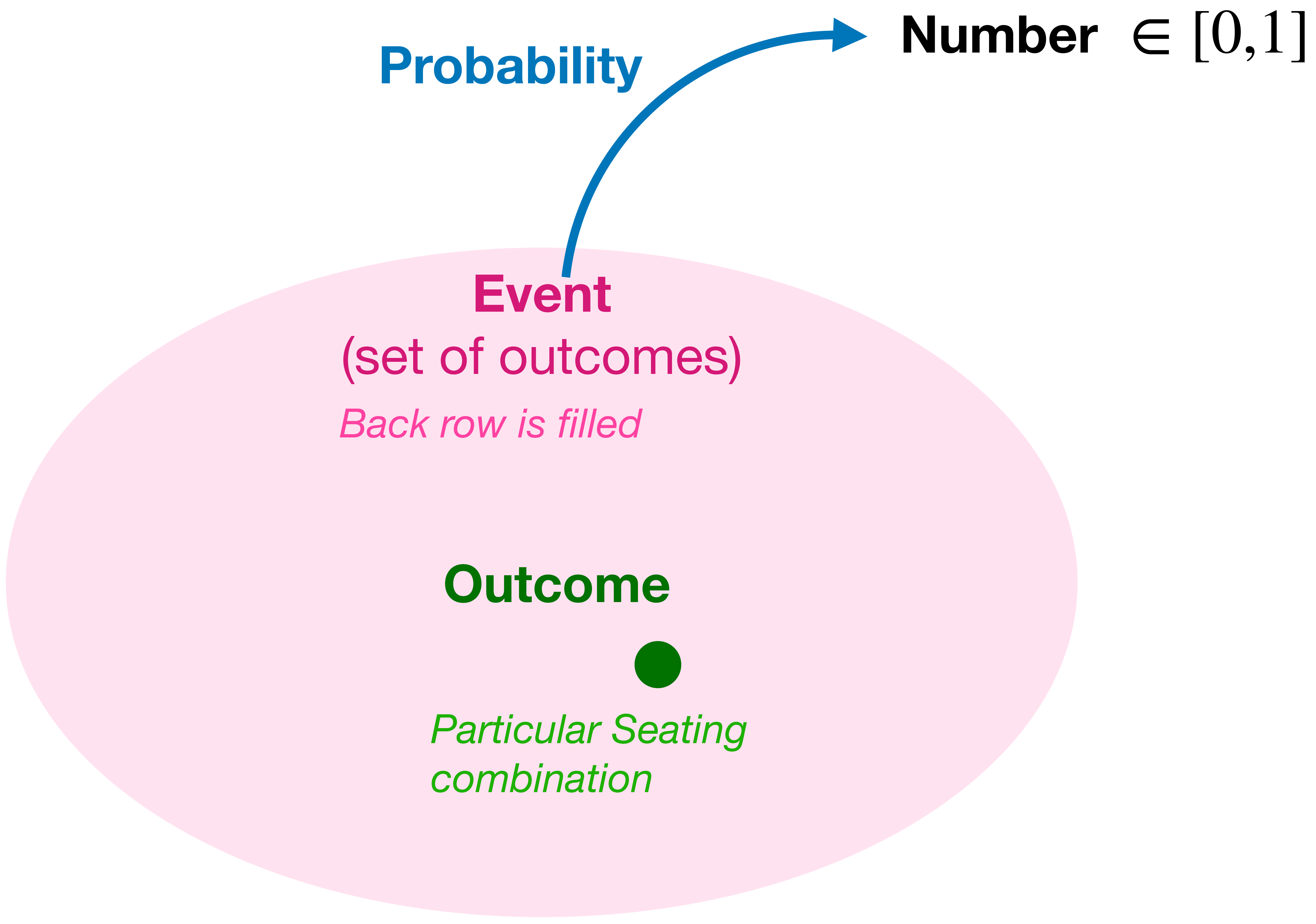
Expectation

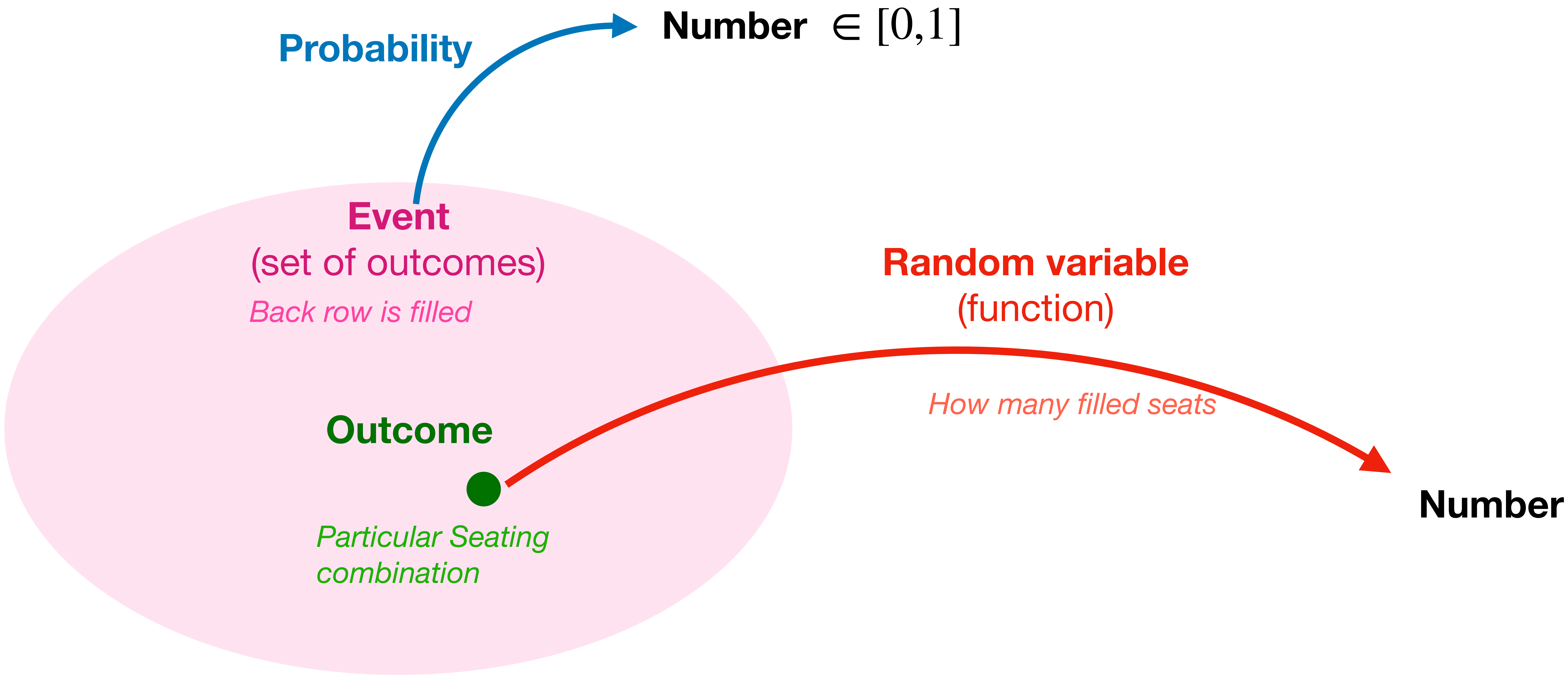
How big do you think it will be?

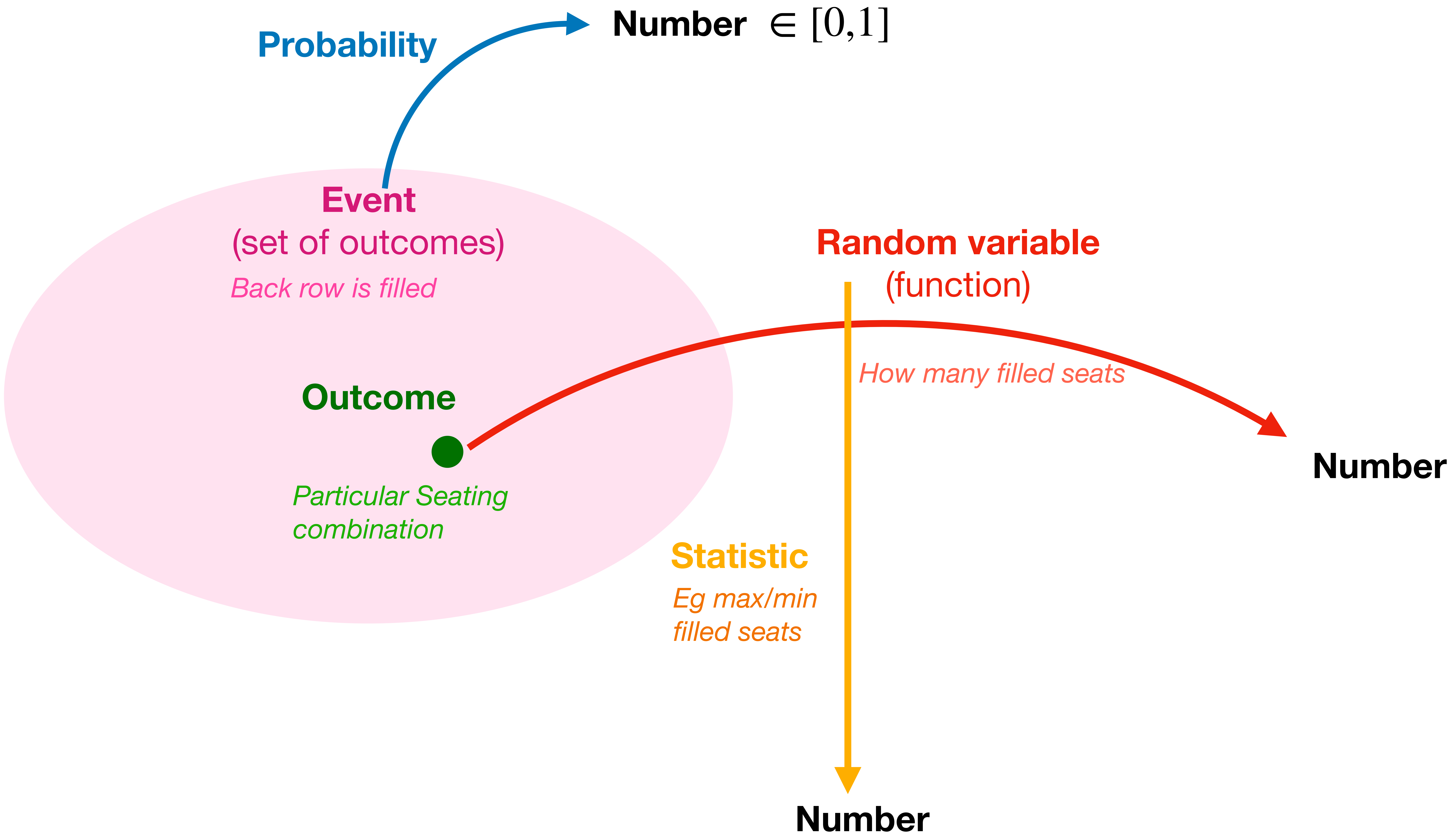
Variance

How uncertain do you think it will be?

⋮


Probability
Number $\in [0,1]$
Event
(set of outcomes)
Back row is filled
Outcome
Particular Seating
combination


Probability
Number $\in [0,1]$
Event
(set of outcomes)
Back row is filled
Random variable
(function)
How many filled seats
Outcome
Particular Seating combination
Number

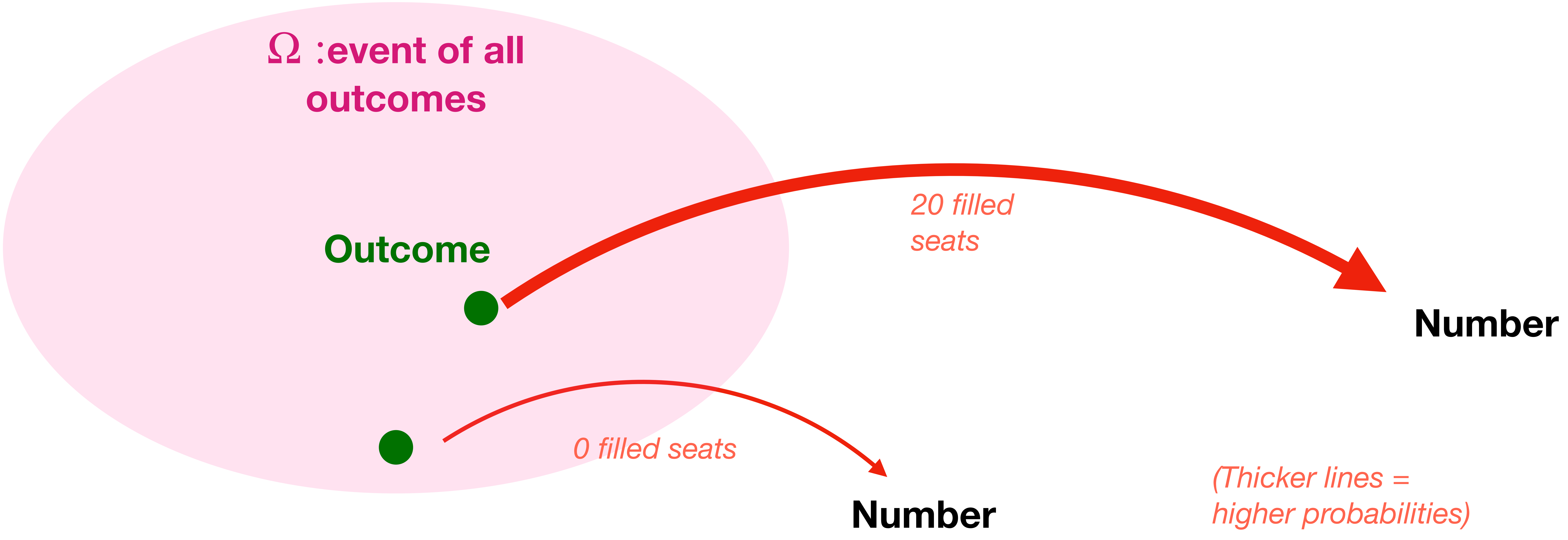


Probability
Number $\in [0,1]$
Event
(set of outcomes)
Back row is filled
Random variable
(function)
How many filled seats
Outcome
Number
Particular Seating
combination
Statistic
Eg max/min
filled seats
Number

Expected value of a random variable

How big will it be?

Average RV value over all outcomes,
weighted by their probabilities

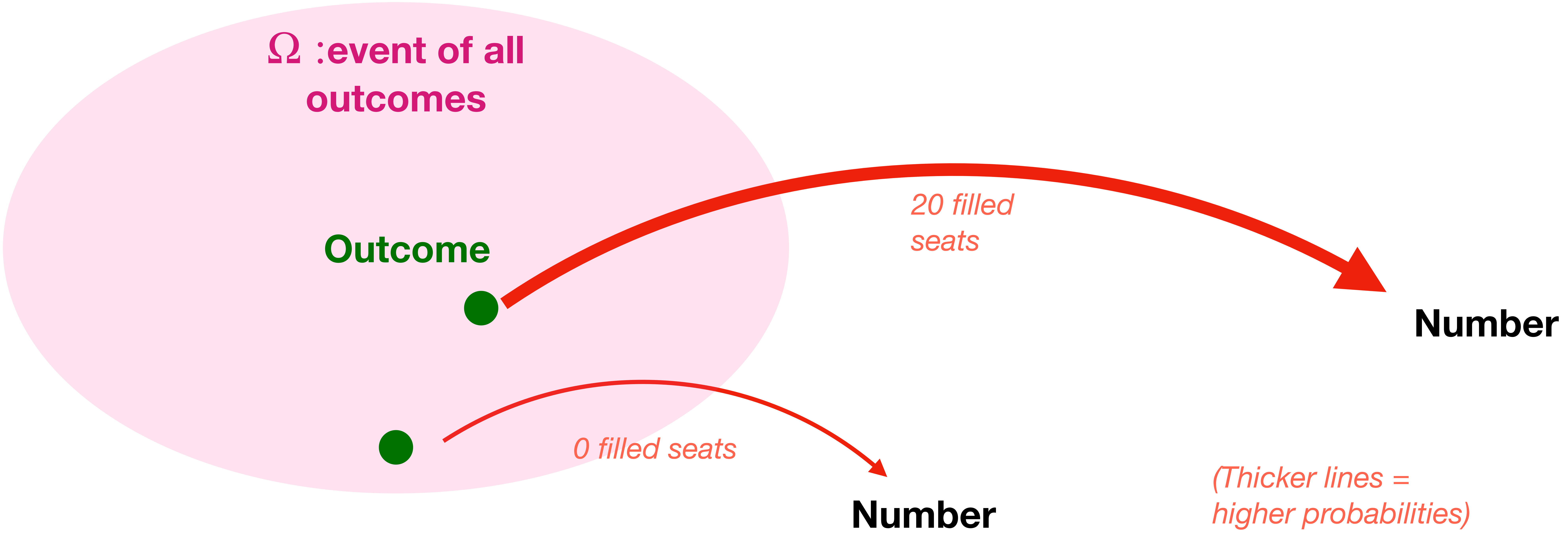


Expected value of a random variable

How big will it be?

Average RV value over all outcomes, weighted by their probabilities

$$\mathbb{E}[X] = \sum_{\omega \in \Omega} X(\omega)\mathbb{P}(\omega) \quad ?$$



Lévy distribution
How big will it be?

$$\mathbb{E}[X] = \infty$$

Google if interested

Sample of a random variable
Taken from data

Experimental trial

Run the experiment once:
$(\Omega, \mathscr{F}, \mathbb{P})$

ω_i = *outcome on trial* i

$X_i =$ *Value of RV* X *on trial* i

EG $X_i =$ *Filled seats on week* i

Sample of a random variable
Taken from data

Experimental trial

Run the experiment once:
$(\Omega, \mathcal{F}, \mathbb{P})$

ω_i = *outcome on trial* i

$X_i =$ *Value of RV* X *on trial* i

EG $X_i =$ *Filled seats on week* i

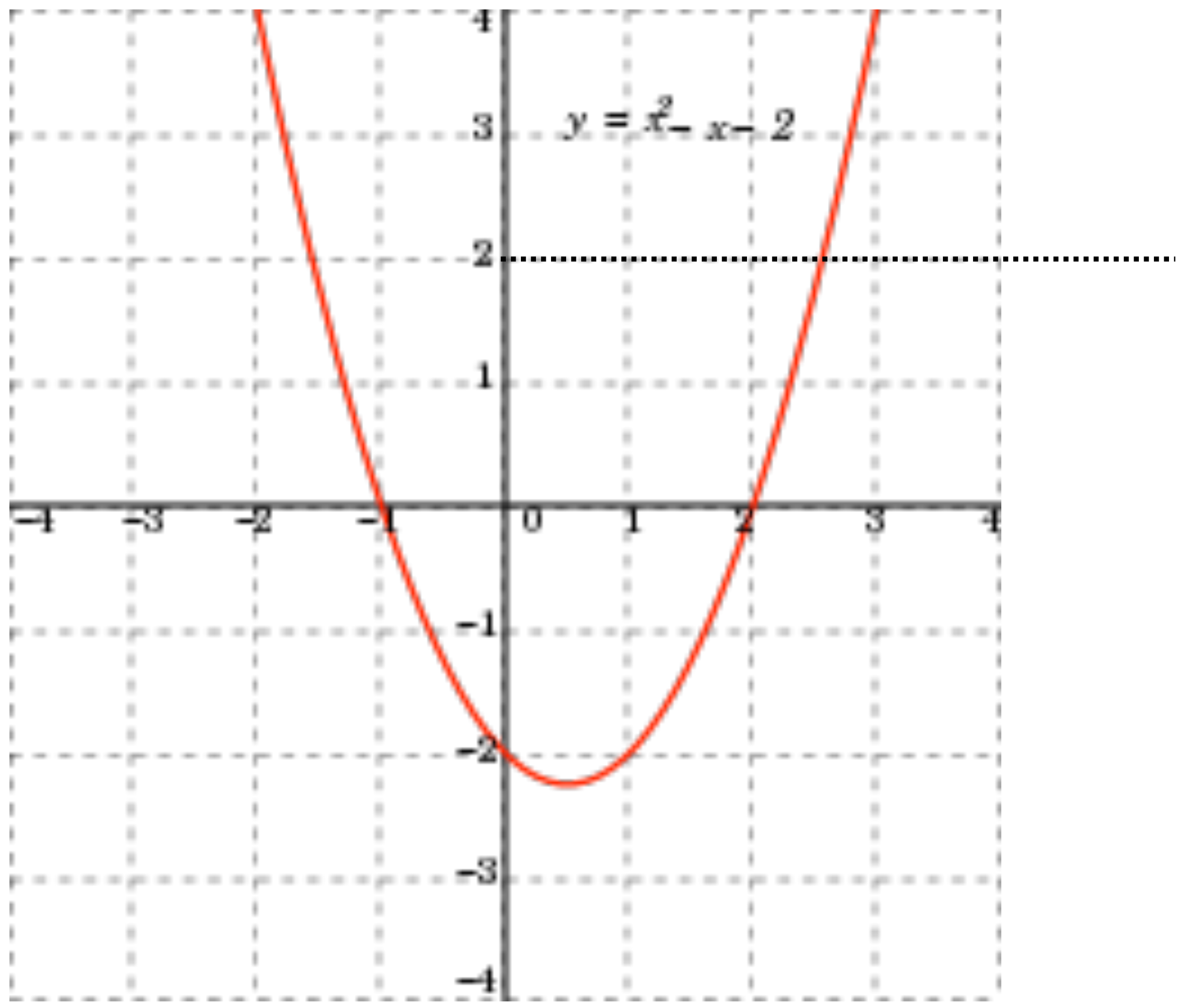
Spot the difference

$X(\omega)$*:* RV is a function

X_i: is a value for the function on trial i

Just like….

Function



Value at ….

Sample statistic
Estimate from data

Sample expected value

$$\bar{X}_N = \frac{X_1 + X_2 + \ldots X_N}{N}$$

e.g. X_i = *Filled seats on week* i

Higher probability outcomes
naturally get weighted more

True statistic

Estimate statistic from infinite data

$$\bar{X}_\infty = \lim_{N \to \infty} \frac{X_1 + X_2 + \ldots X_N}{N}$$

$$\bar{X}_\infty = \mathbb{E}[X]?$$

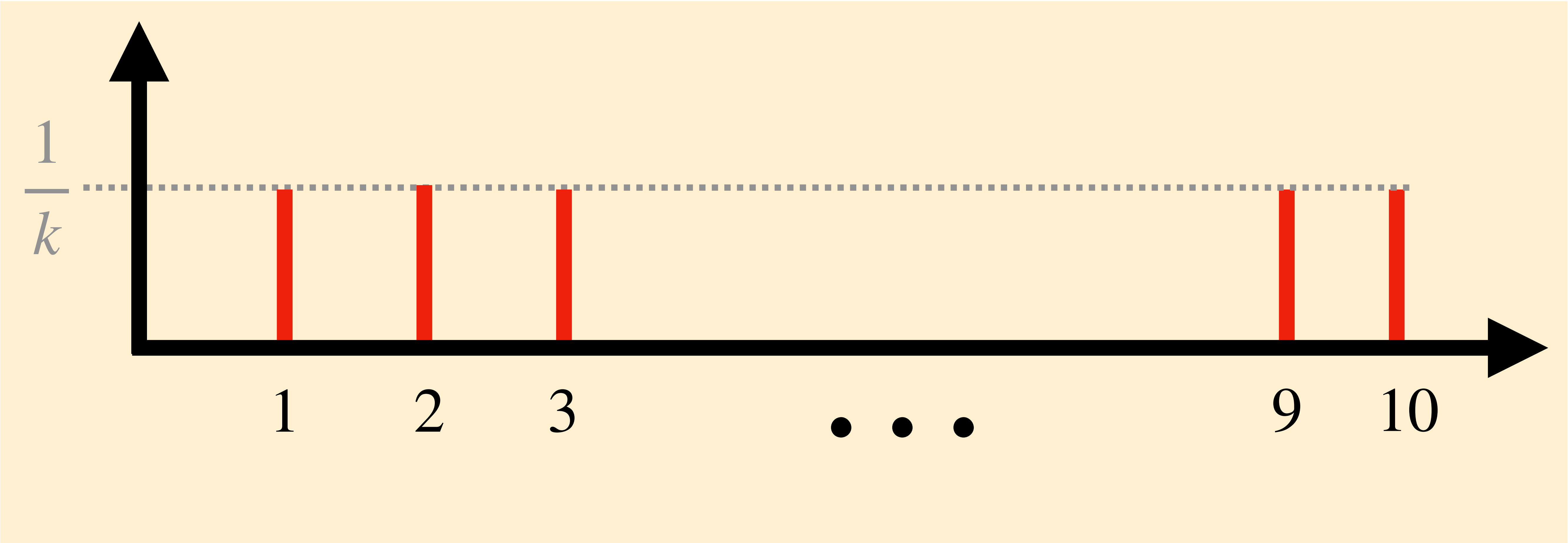
Never have access to infinite trials. How to calculate?

From PMF / PDF…

Recap: Uniform random variable

$$X \sim U\left(\{i\}_{i=1}^{10}\right)$$

Probability mass function of X



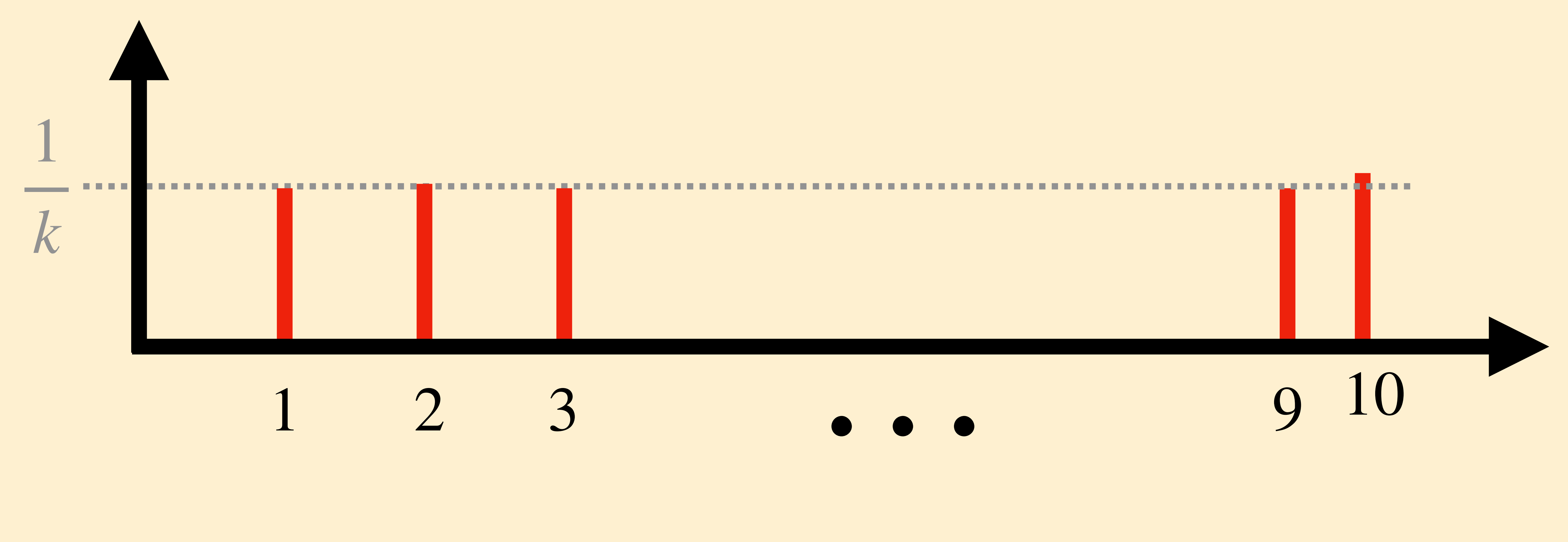
Expectation of a random variable

…calculating from PMF

Goal: evaluate following limit using PMF only

$$\frac{1}{N}(X_1 + X_2 + X_3 + X_4 + X_5 + X_6 + X_7 + \ldots + X_N)$$

Probability mass function of X



Expectation of a random variable

…calculating from PMF

Step 1: rearrange summation according to event

$$\frac{1}{N}(X_1 + X_2 + X_3 + X_4 + X_5 + X_6 + X_7 + \ldots + X_N) =$$

$$\frac{1}{N}(\underbrace{X_4 + X_{19} + X_7}_{\substack{\textbf{Event:}\\ X_i = 1}}\ldots + \underbrace{X_9 + X_6 + X_2}_{\substack{\textbf{Event:}\\ X_i = 2}}\ldots + \underbrace{X_{124} + X_{28} + X_{47}}_{\substack{\textbf{Event:}\\ X_i = 10}})$$

$\cdots$

Expectation of a random variable

…calculating from PMF

Step 2: substitute random variables with their outcomes

$$\frac{1}{N}(X_4 + X_{19} + X_7 \ldots + X_9 + X_6 + X_2 \ldots + X_{124} + X_{28} + X_{47})$$

Event: $X_i = 1$

Event: $X_i = 2$

$\cdots$

Event: $X_i = 10$

$$= 1 \times \frac{|\{X_i = 1\}|}{N} + 2 \times \frac{|\{X_i = 2\}|}{N} \cdots 10 \times \frac{|\{X_i = 10\}|}{N}$$

(Cardinality: number of elements in the set $\{X_i = 1\}$)

Expectation of a random variable

…calculating from PMF

$$\frac{1}{N}(\underbrace{X_4 + X_{19} + X_7}_{\substack{\textbf{Event:}\\ X_i = 1}} \ldots + \underbrace{X_9 + X_6 + X_2}_{\substack{\textbf{Event:}\\ X_i = 2}} \ldots + \underbrace{X_{124} + X_{28} + X_{47}}_{\substack{\textbf{Event:}\\ X_i = 10}})$$

$\cdots$

$$= 1 \times \frac{|\{X_i = 1\}|}{N} + 2 \times \frac{|\{X_i = 2\}|}{N} \cdots 10 \times \frac{|\{X_i = 10\}|}{N}$$

How many instances of each event do we expect?

Expectation of a random variable

…calculating from PMF

$$\lim_{N\to\infty} \frac{|\{X = i\}|}{N} = \mathbb{P}[X = i]$$

(Law of large numbers)

“If you do enough trials, the proportion of times an event happen gets infinitely close to the probability of the event"

Expectation of a random variable

…calculating from PMF

$$\frac{1}{N}\sum_{i=1}^{N} X_i = \ldots$$

$$\boxed{1 \times \frac{|\{X_i = 1\}|}{N}} + \boxed{2 \times \frac{|\{X_i = 2\}|}{N}} \cdots \boxed{10 \times \frac{|\{X_i = 10\}|}{N}}$$

$1 \times \mathbb{P}[X = 1]$ $\quad$ $2 \times \mathbb{P}[X = 2]$ $\quad$ $10 \times \mathbb{P}[X = 10]$

Expectation of a random variable

…calculating from PMF

$$\frac{1}{N}\sum_{i=1}^{N} X_i = \ldots \qquad \ldots = \sum_{i=1}^{10} \frac{i}{N} = 5.5$$

$$1 \times \frac{|\{X_i = 1\}|}{N} + 2 \times \frac{|\{X_i = 2\}|}{N} \cdots 10 \times \frac{|\{X_i = 10\}|}{N}$$

$$1 \times \mathbb{P}[X = 1] \qquad 2 \times \mathbb{P}[X = 2] \qquad 10 \times \mathbb{P}[X = 10]$$

Summary

$$\lim_{N \to \infty} \frac{1}{N}(X_4 + X_{19} + X_7 \ldots + X_9 + X_6 + X_2 \ldots + X_{124} + X_{28} + X_{47})$$

Event: $X_i = 1$

Event: $X_i = 2$

$\cdots$

Event: $X_i = 10$

$$= 1 \times \frac{|\{X_i = 1\}|}{N} + 2 \times \frac{|\{X_i = 2\}|}{N} \cdots 10 \times \frac{|\{X_i = 10\}|}{N}$$

$$\boxed{= \sum_{i=1}^{10} x \times \mathbb{P}[X = x]}$$

Generalising the formula

(discrete random variables)

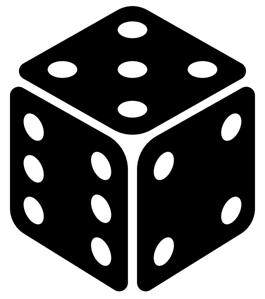
$$\mathbb{E}[X] = \sum_{i=1}^{10} x \times \mathbb{P}[X = x]$$

$$\downarrow$$

$$\boxed{\mathbb{E}[X] = \sum_{x \in \mathrm{supp}(X)} x f_X(x)}$$

Probability mass function

Practice



Formula for expected value?

$$\text{support}(X) = \{1,2,\ldots,6\}$$

$$\mathbb{E}[X] = \sum_{i=1}^{6} i \times \mathbb{P}[i] = \sum_{i=1}^{6} \frac{i}{6}$$

Expectation of function of random variable

What's the mean of $h(X)$ over many trials?

$$\bar{h}_N(X) = \frac{1}{N}\sum_{i=1}^{N} h(X_i)$$

$$\mathbb{E}[h(X)] = \lim_{N\to\infty} \bar{h}_N(X)$$

Expectation of function of random variable

What's the mean of $h(X)$ over many trials?

$$\boxed{\mathbb{E}[h(X)] = \sum_{x \in \text{supp}(X)} h(x) f_X(x)}$$

Show for yourself $\uparrow$ *(follow logic of slides for $\mathbb{E}[X]$)*

$$\bar{h}_N(X) = \frac{1}{N} \sum_{i=1}^{N} h(X_i)$$

$$\mathbb{E}[h(X)] = \lim_{N \to \infty} \bar{h}_N(X)$$

Practice

Formula for $\mathbb{E}[X^2]$?

$$\boxed{\mathbb{E}[h(X)] = \sum_{x \in \text{supp}(X)} h(x) f_X(x)}$$

Practice

Formula for $\mathbb{E}[X^2]$?

support$(X) = \{1, 2, \ldots, 6\}$

$$\mathbb{E}[X^2] = \sum_{i=1}^{6} i^2 \times \mathbb{P}[X = i]$$

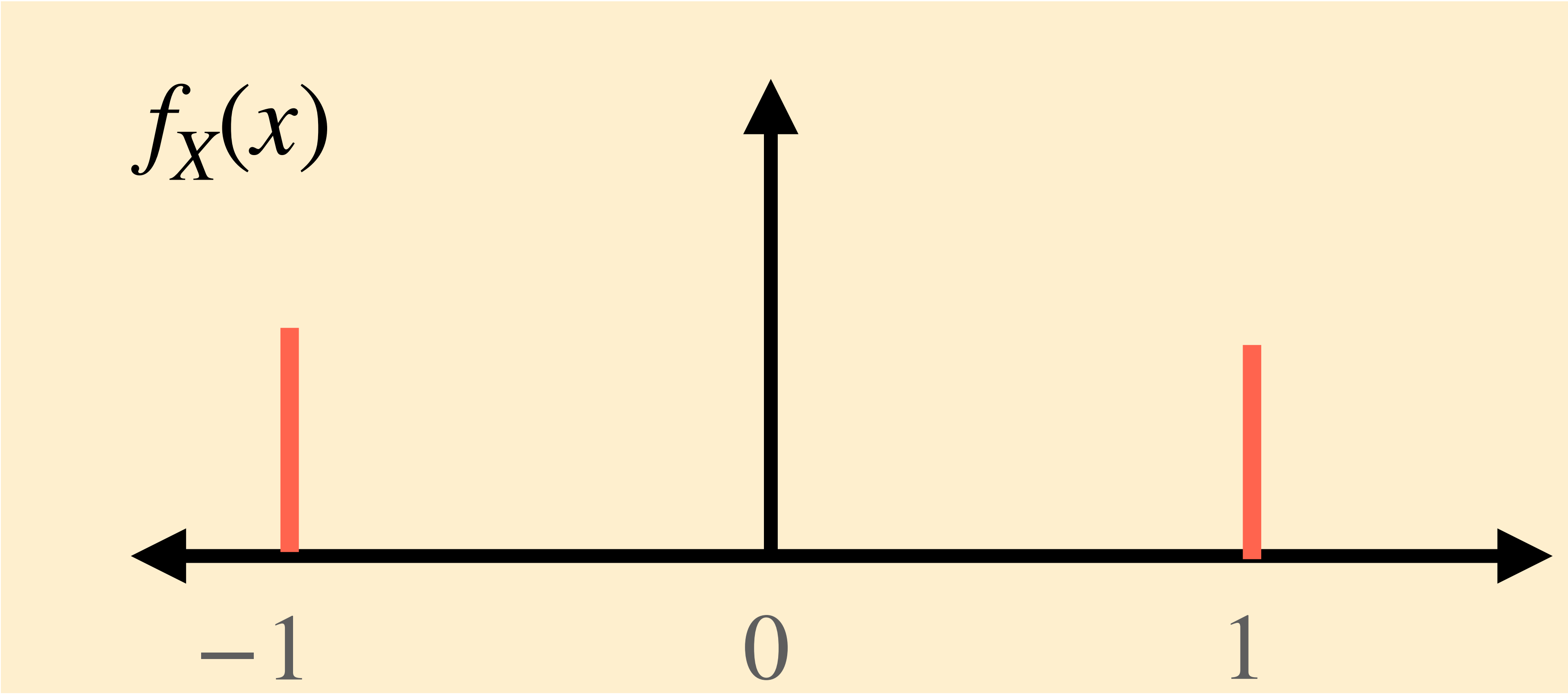
$$= \sum_{i=1}^{6} \frac{i^2}{6} = 15\frac{1}{6}$$

$$\mathbb{E}[X] = 3.5$$

$$\mathbb{E}[X]^2 = 12\frac{1}{4}$$

Getting intuition on expectations

Let's consider $X \sim U(\{-1,1\})$



$\mathbb{E}[X]$?

$\mathbb{E}[X^2]$?

Getting intuition on expectations

Expectations don’t play nicely with nonlinearities!

$$\mathbb{E}[X] = 0$$

$$\mathbb{E}[X^2] = 1$$

Expectation satisfies linearity

$$\mathbb{E}[aX + bY] = a\mathbb{E}[X] + b\mathbb{E}[Y] \qquad \forall a, b \in \mathbb{R}$$

(For any real numbers a and b)

Examples

$$\mathbb{E}[\sum_i a_i X_i] = \sum_i a_i \mathbb{E}[X_i] \qquad \forall a_i \in \mathbb{R}$$

$$\mathbb{E}[XY] \neq \mathbb{E}[X]\mathbb{E}[Y]$$ if X and Y are both random variables

Summarising random variables

How big is a random variable (on average)

Expected value: $\mathbb{E}[X]$ or $\mu(X)$

How variable is a random variable (on average)

Variance: $\mathrm{Var}[X]$ or $\sigma^2(X)$

Conceptualising variance

How far does a random variable usually fall from its expected value?

↕

How much uncertainty in the random variable?

↕

$$\boxed{\mathbb{E}[(X - \mathbb{E}[X])^2]}$$

Conceptualising variance

How far does a random variable usually fall from its expected value?

↕

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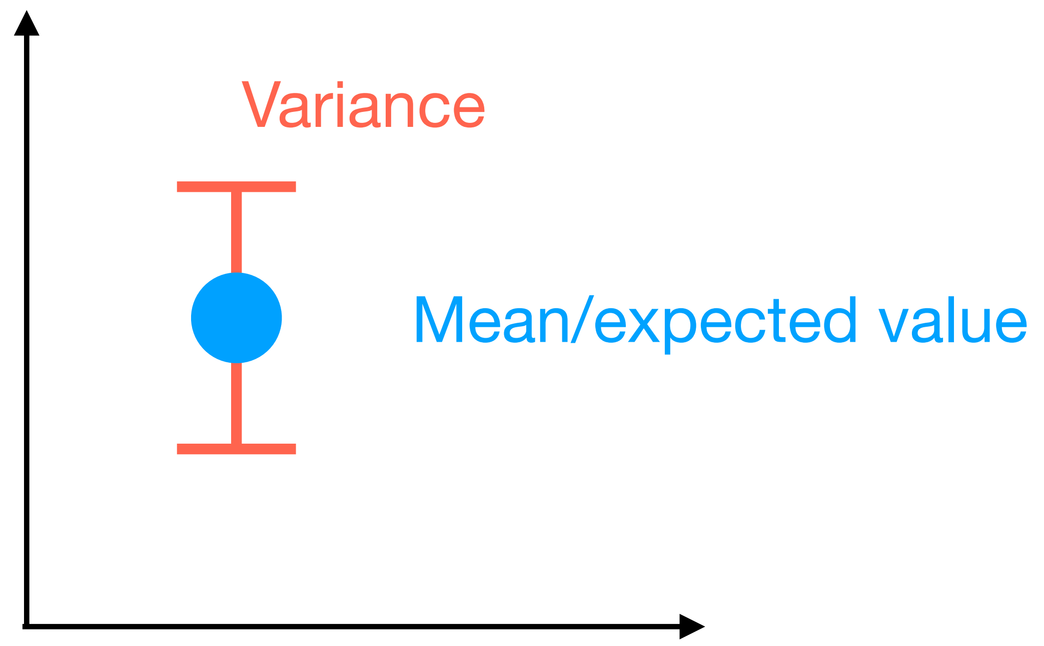
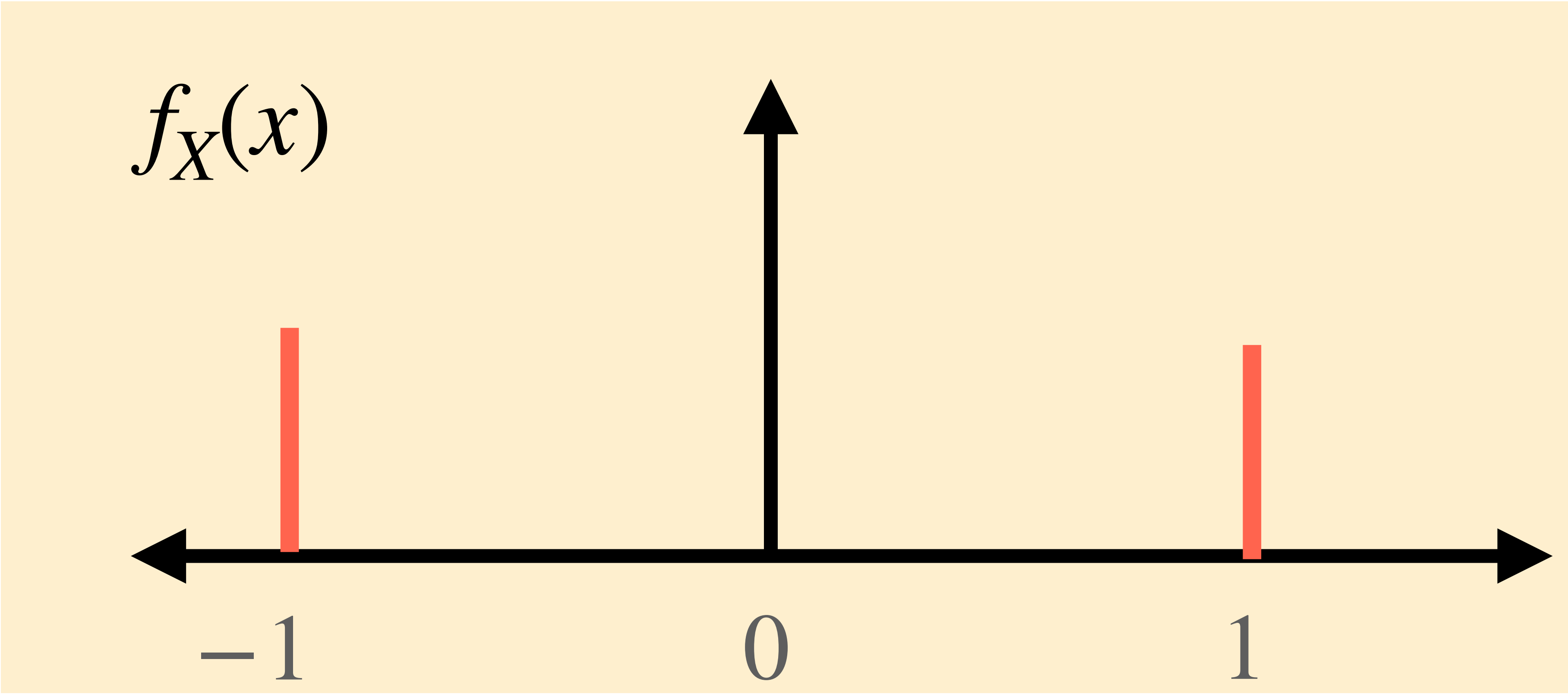
↕

$$\boxed{\mathbb{E}[(X - \mathbb{E}[X])^2]}$$



Variance of an example random variable

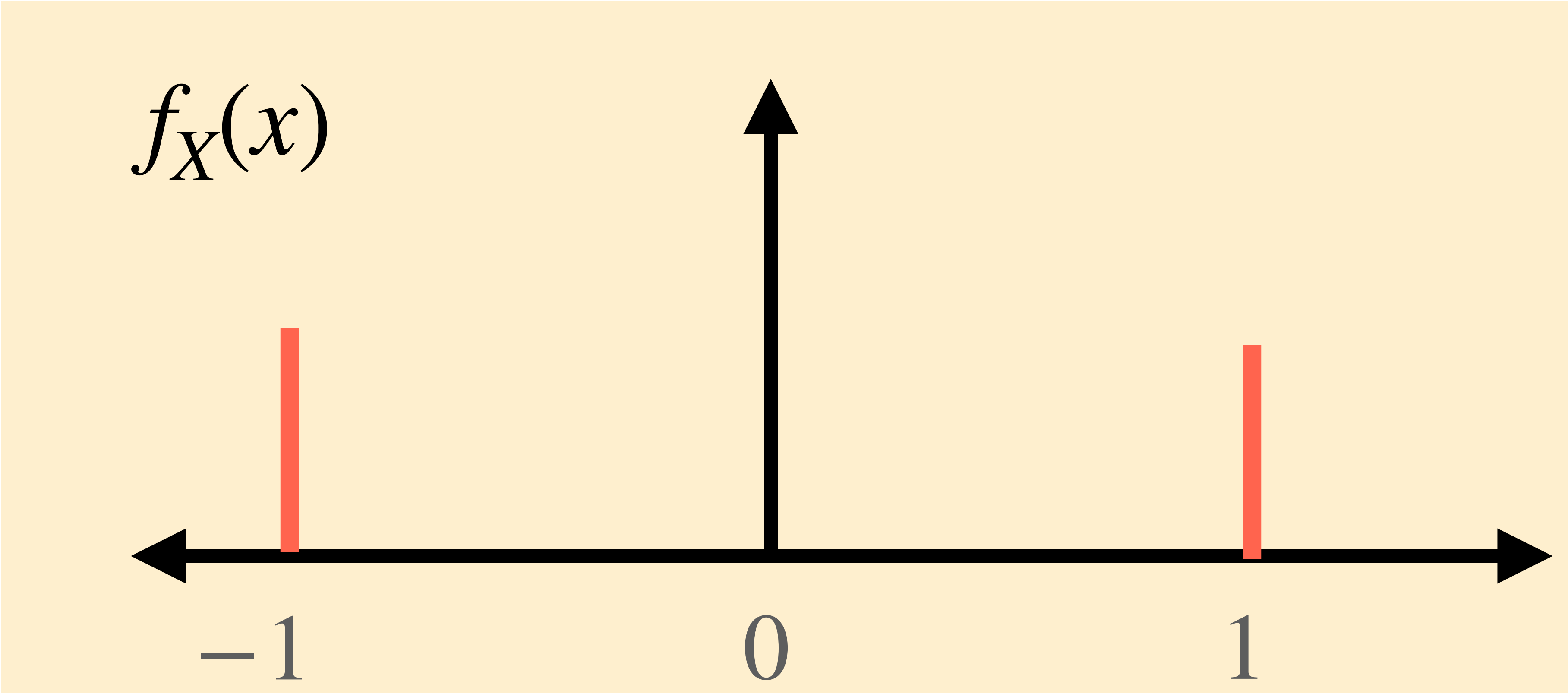
Let's consider $X \sim U(\{-1, 1\})$ again



$$\mathbb{E}[(X - \mathbb{E}[X])^2]?$$

Variance of an example random variable

Let's consider $X \sim U(\{-1, 1\})$ again



$$\mathbb{E}[(X - \mathbb{E}[X])^2] = \mathbb{E}[X^2] = 1$$

Algebra on the expected value

$$\mathbb{E}[(X - \mathbb{E}[X])^2]$$

$$= \mathbb{E}\left[X^2 - 2X\mathbb{E}[X] + \mathbb{E}[X]^2\right]$$

$$= \mathbb{E}[X^2] - 2\mathbb{E}\left[X\mathbb{E}[X]\right] + \mathbb{E}\left[\mathbb{E}[X]^2\right]$$

(Linearity of expectation)

Algebra on the expected value

How to simplify $\mathbb{E}\left[\mathbb{E}[X]\right]$ and similar?

Well $\mathbb{E}[X]$ is a real number, so can
apply linearity!

$$= \mathbb{E}[X^2] - 2\mathbb{E}\left[X\mathbb{E}[X]\right] + \mathbb{E}\left[\mathbb{E}[X]^2\right]$$

Algebra on the expected value

How to simplify $\mathbb{E}\left[\mathbb{E}[X]\right]$ and similar?

Well $\mathbb{E}[X]$ is a real number, so can
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$$= \mathbb{E}[X^2] - 2\mathbb{E}\left[X\mathbb{E}[X]\right] + \mathbb{E}\left[\mathbb{E}[X]^2\right]$$

$$= \mathbb{E}[X^2] - 2\mathbb{E}[X]^2 + \mathbb{E}[X]^2\mathbb{E}[1]$$

Two equivalent expressions for the variance

$$\boxed{\mathbb{E}[(X - \mathbb{E}[X])^2 = \mathbb{E}[X^2] - \mathbb{E}[X]^2}$$

$$= \mathbb{E}[X^2] - 2\mathbb{E}\left[X\mathbb{E}[X]\right] + \mathbb{E}\left[\mathbb{E}[X]^2\right]$$

$$= \mathbb{E}[X^2] - 2\mathbb{E}[X]^2 + \mathbb{E}[X]^2\mathbb{E}[1]$$

Algebra on the variance

Homework

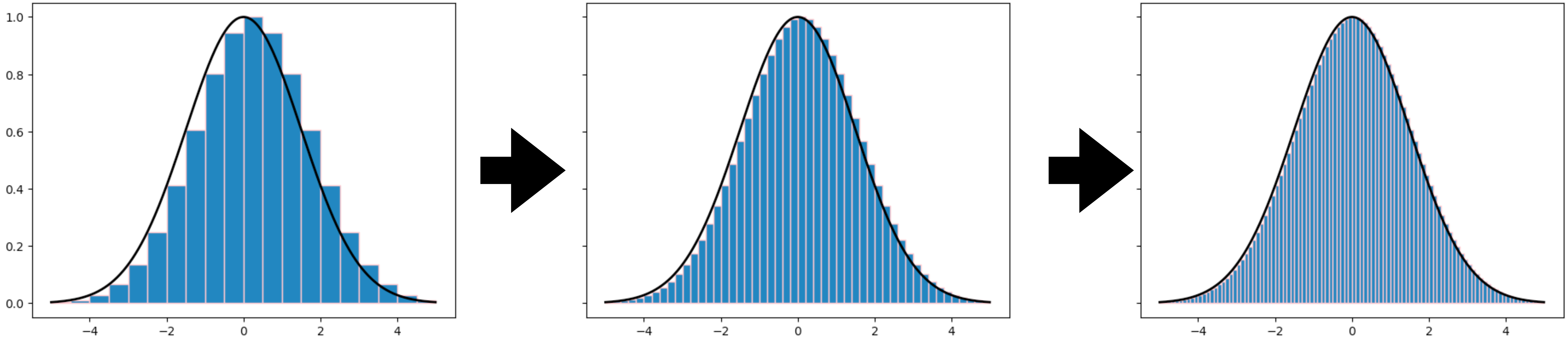
Use linearity of expectation to convince yourself that:

$$\mathrm{Var}(cX) = c^2\mathrm{Var}(X)$$

$$c \in \mathbb{R}$$

Expectation for continuous random variables

Approximate continuous random variable with an improving series of discrete random variables

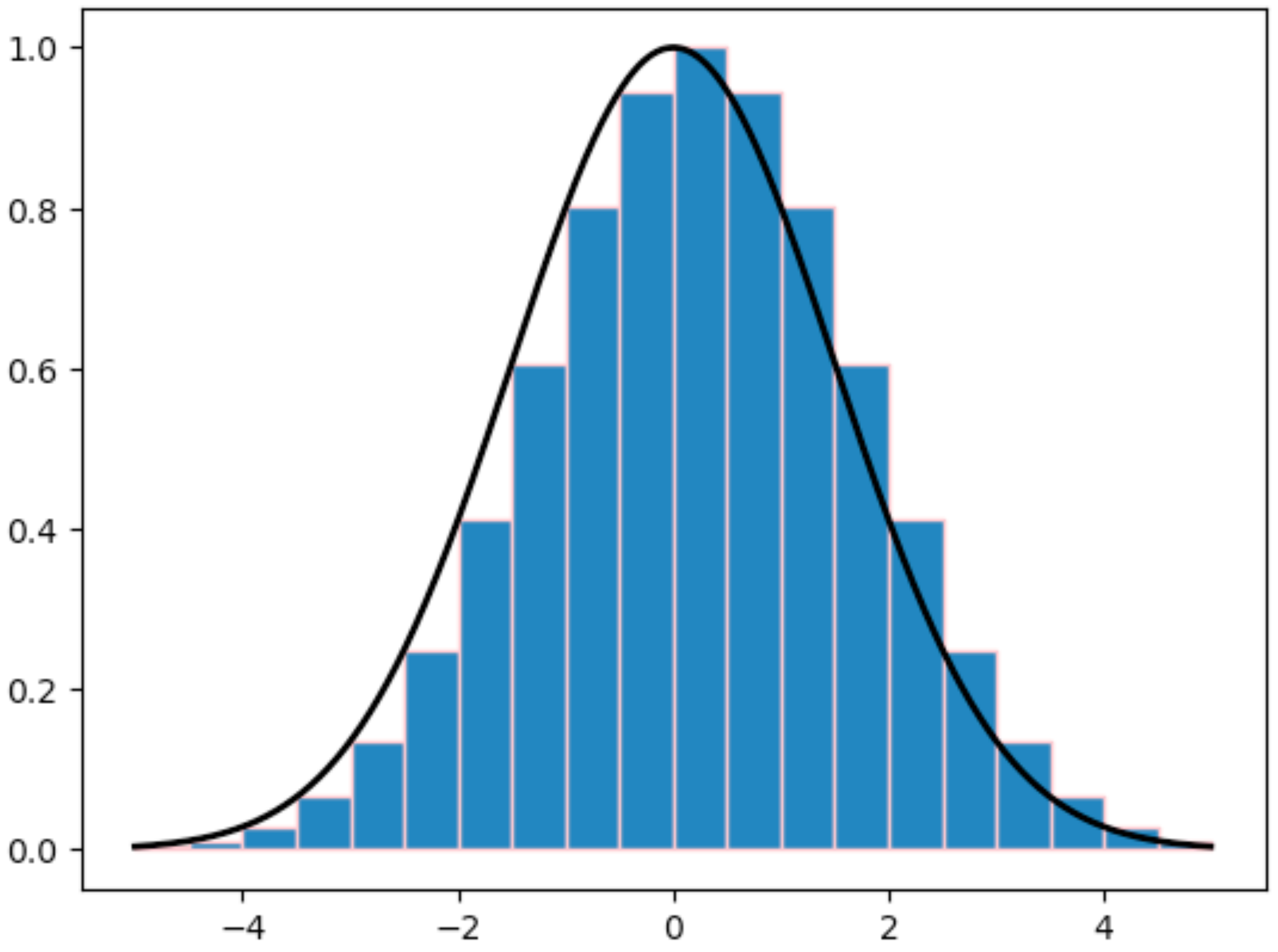


Discrete RV only takes values in the centre of each rectangle

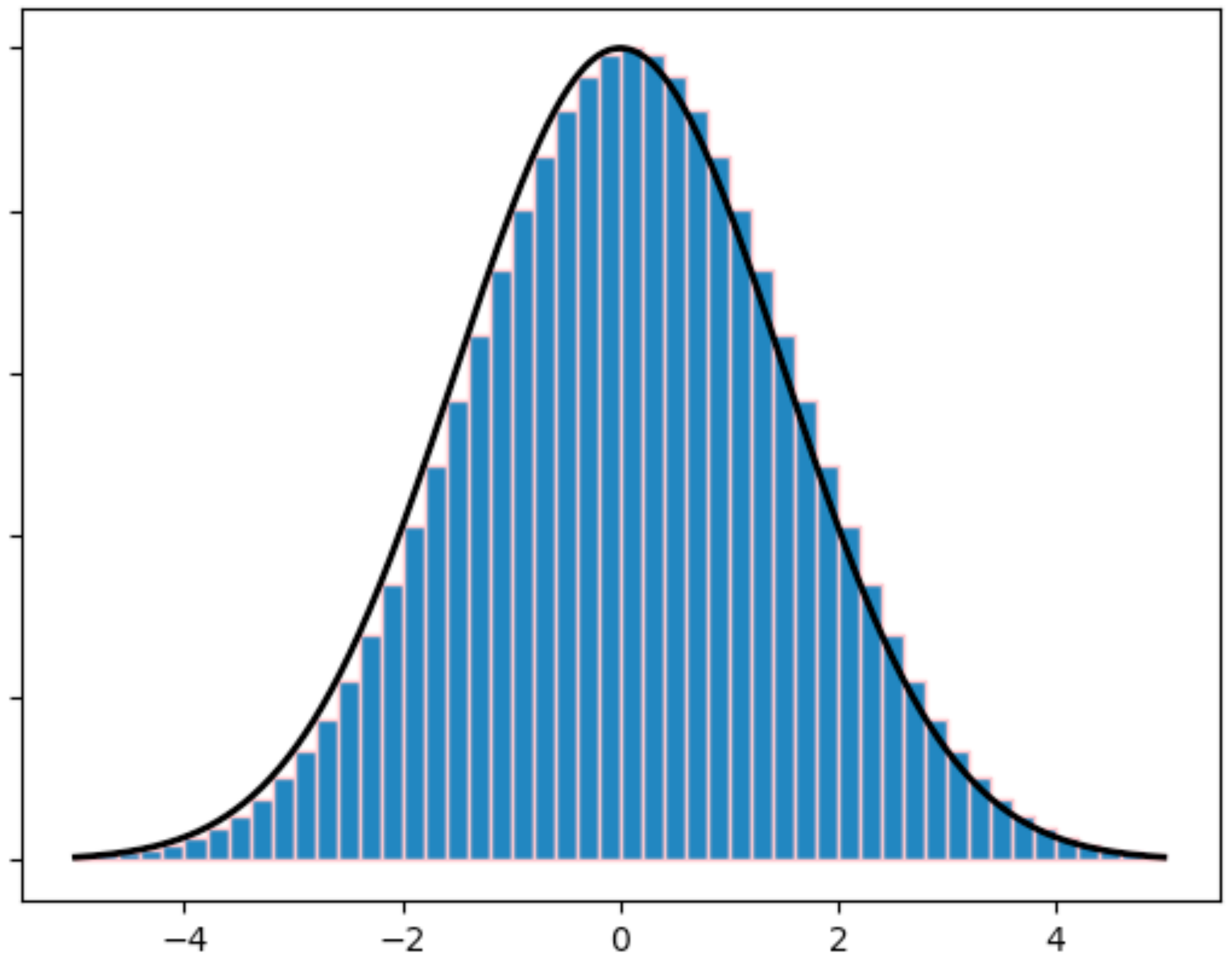
Expectation for continuous random variables

$\{\hat{X}_j\}_{j=1}^{\infty}$ = sequence of discrete random variables approximating X

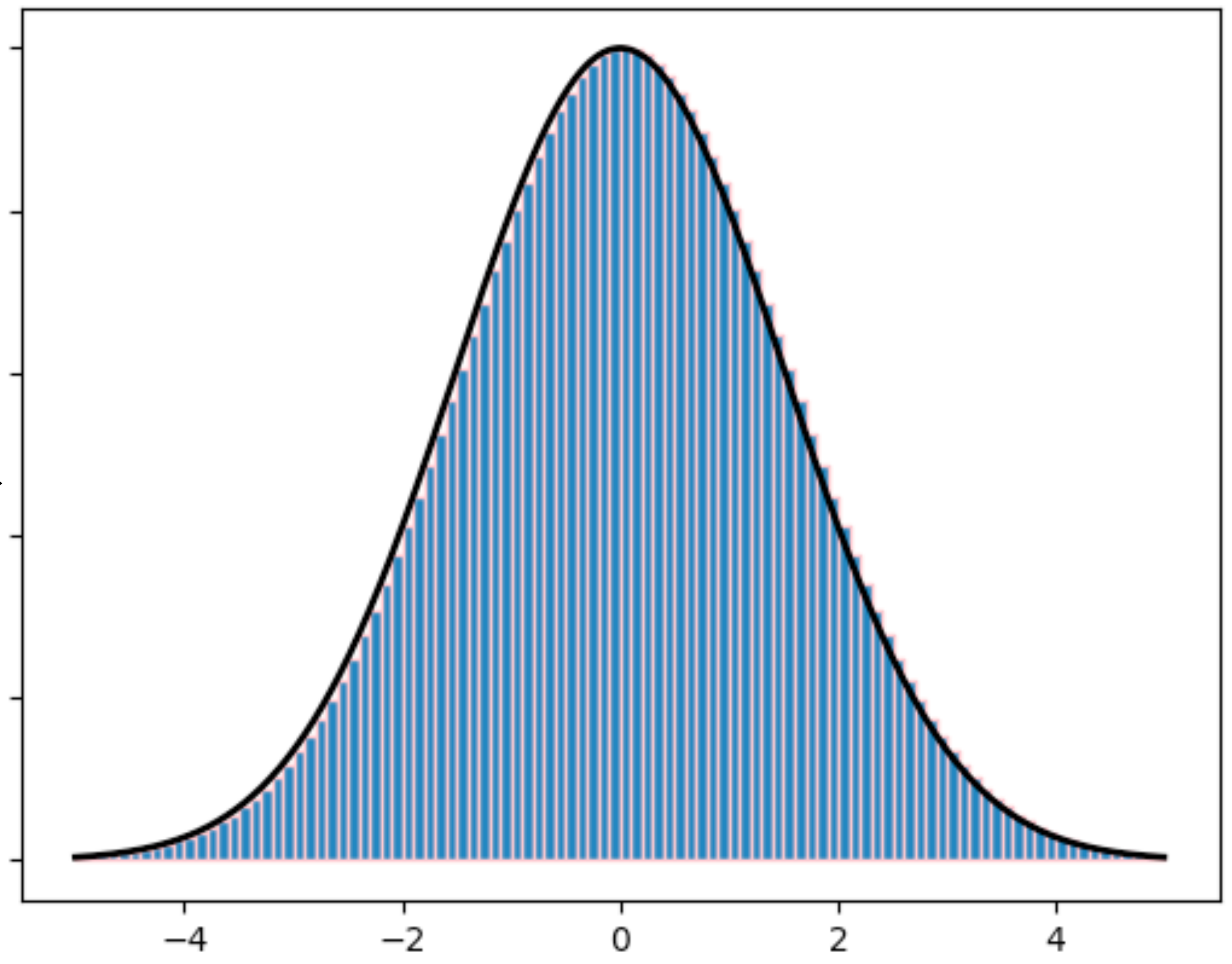
$\hat{X}_1$



$\hat{X}_{10}$



$\hat{X}_{100}$



Discrete RV only takes values in the centre of each rectangle

$\hat{X}_1 = 0$ if $X \in [-1,1]$

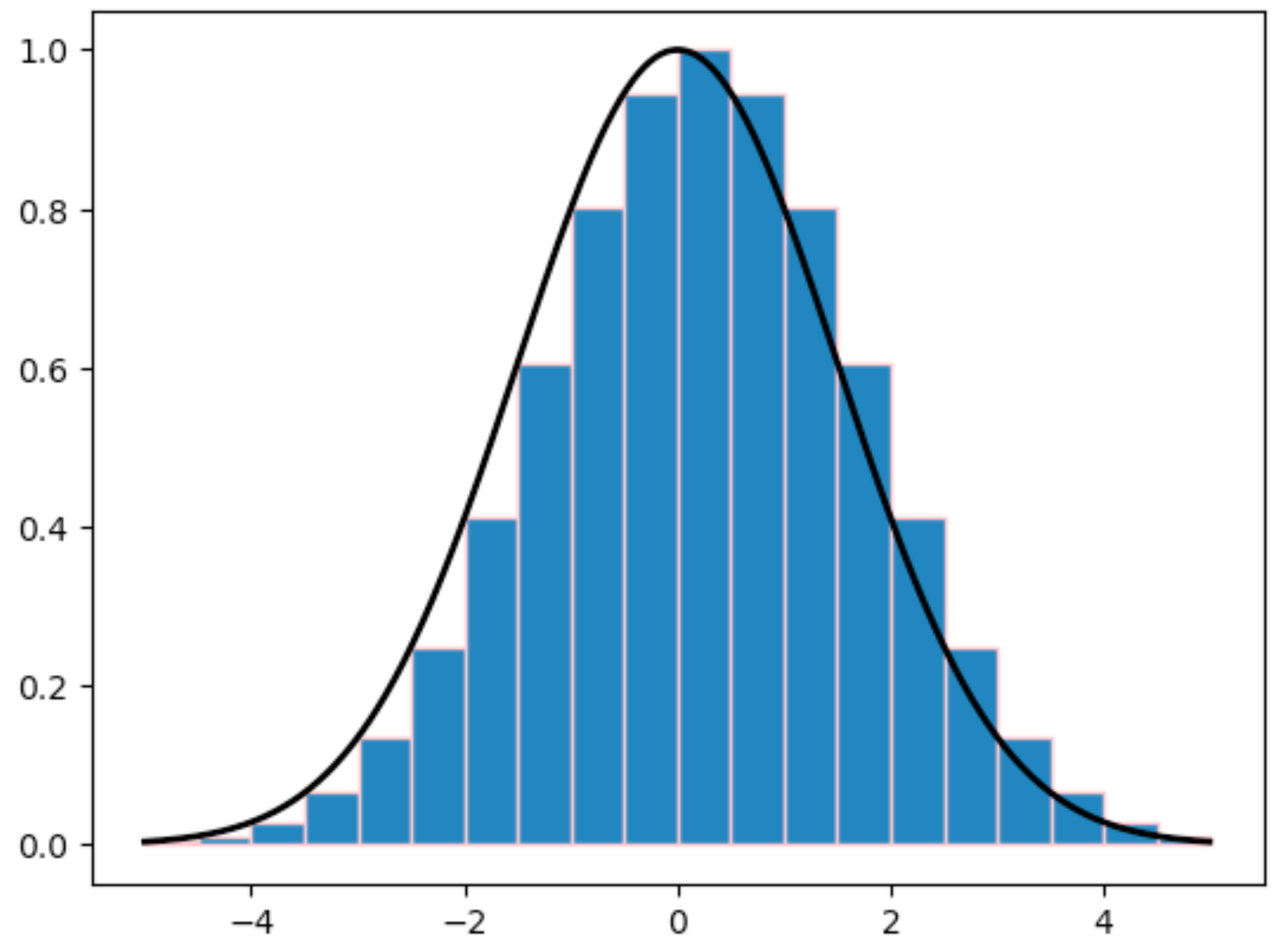
$\hat{X}_{10} = 0$ if $X \in [-0.1,0.1]$

Expectation for continuous random variables

$\{\hat{X}_j\}_{j=1}^{\infty}$ = sequence of discrete random variables approximating X

$$\mathbb{P}[\hat{X}_j = x] \approx \int_{L_i^j}^{U_i^j} f_X(x)\ dx$$

$[L_i^j, U_i^j]$: i^{th} rectangle
j^{th} approximation



Expectation for continuous random variables

$\{\hat{X}_j\}_{j=1}^{\infty}$ = sequence of discrete random variables approximating X

$$\mathbb{E}[\hat{X}_j] = \sum_{\text{supp}(\hat{X}_j)} x\mathbb{P}[\hat{X}_j = x] \quad \Rightarrow \quad \boxed{\mathbb{E}[X] = \int_{\text{supp(X)}} x f_X(x)\, dx}$$

Probability density function

Expectation for continuous random variables

$$\boxed{\mathbb{E}[X] = \int_{\text{supp}(X)} x f_X(x) \; dx}$$

Probability density function

Hard, optional homework

Prove above identity by formalising intuition of previous three slides

Easier homework

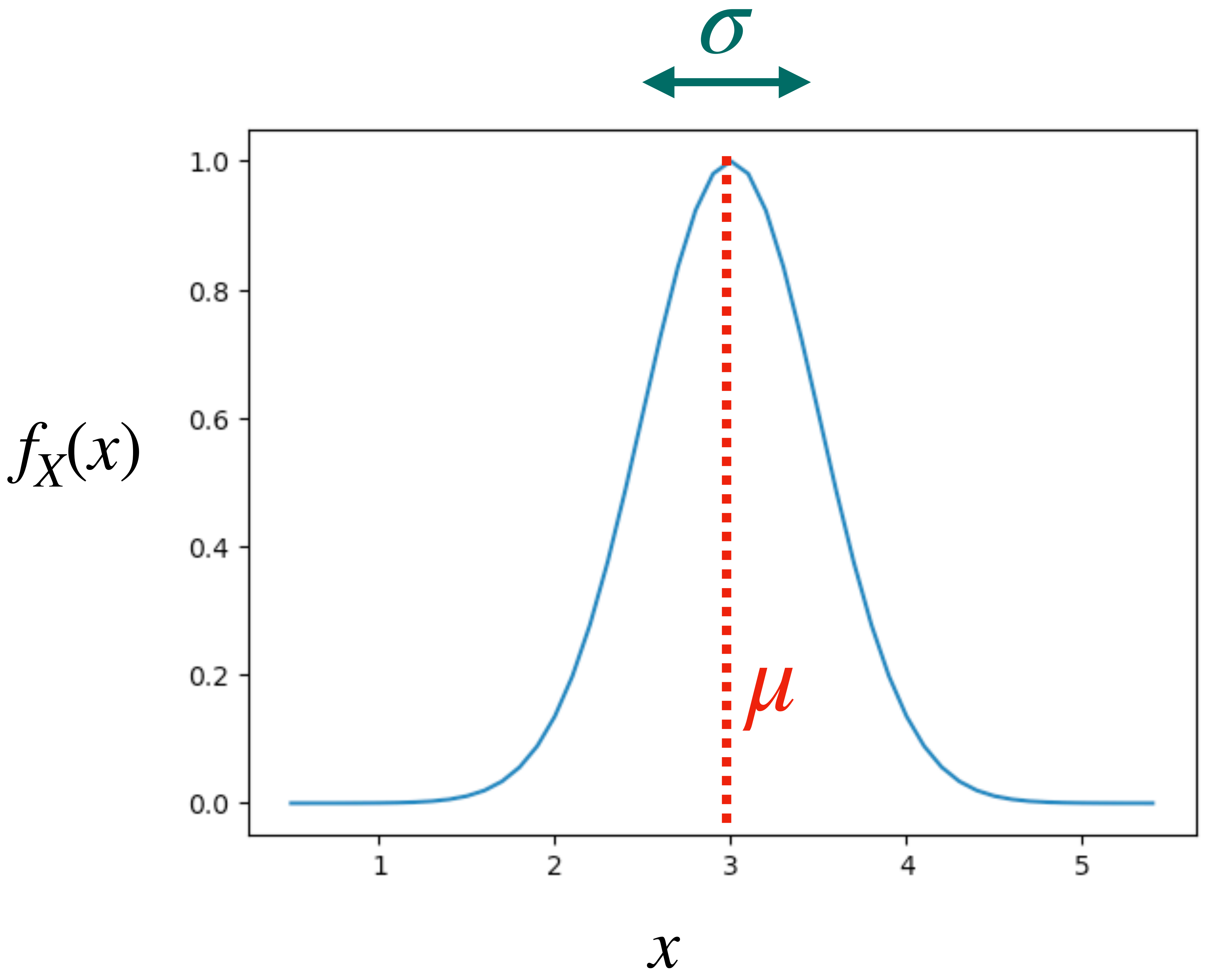
What's the highest value a PDF can theoretically take?

Gaussian random variable are special

$$X \sim \mathcal{N}(\mu, \sigma^2)$$

$\Updownarrow$

"X is a Gaussian with mean μ and variance σ^2"



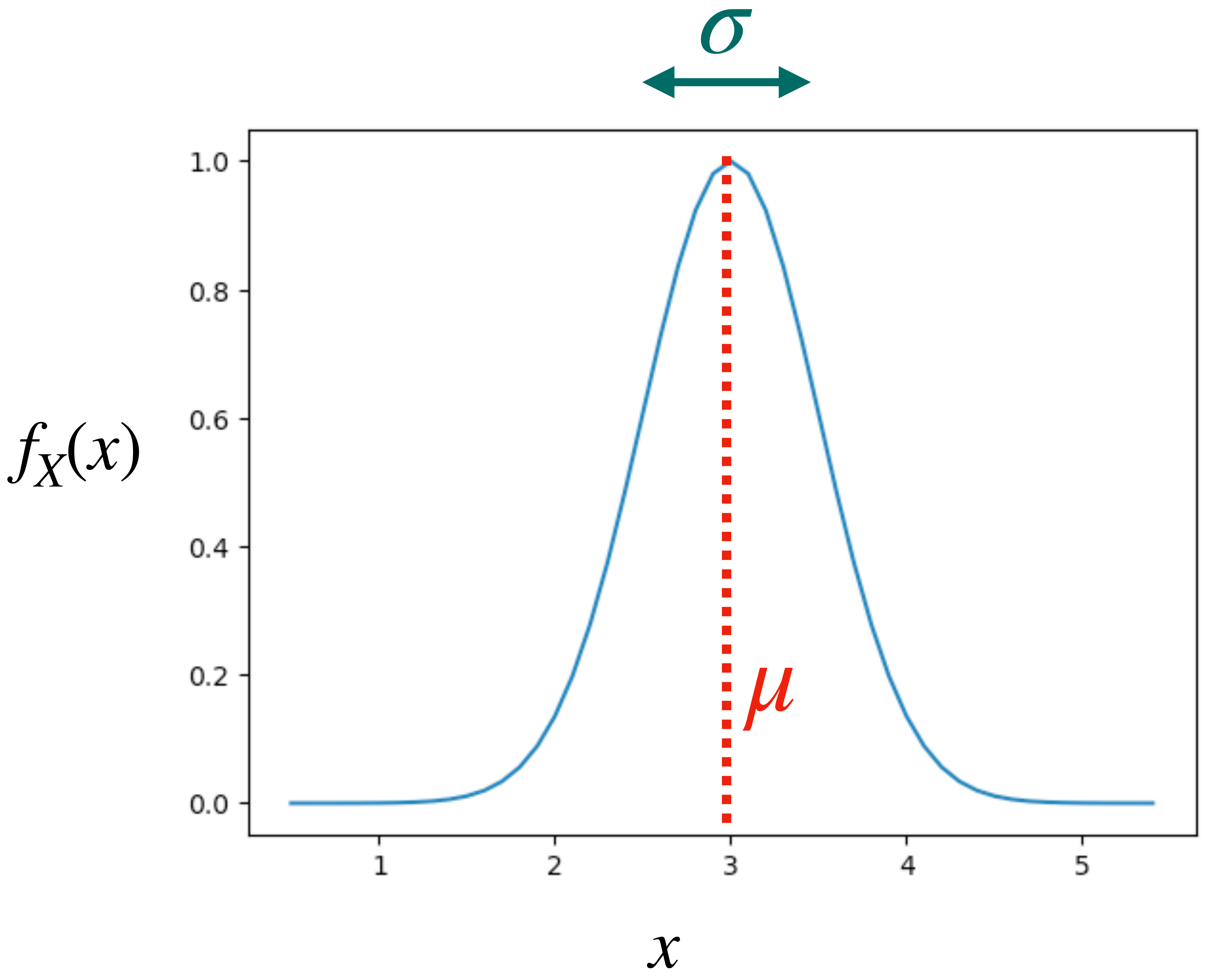
Gaussian random variable are special

$$X \sim \mathcal{N}(\mu, \sigma^2)$$

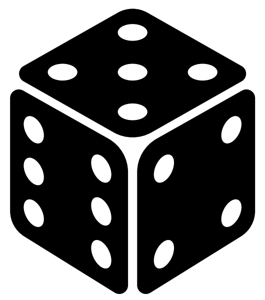
$$\Updownarrow$$

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$

(Don't have to remember)



Gaussians are special

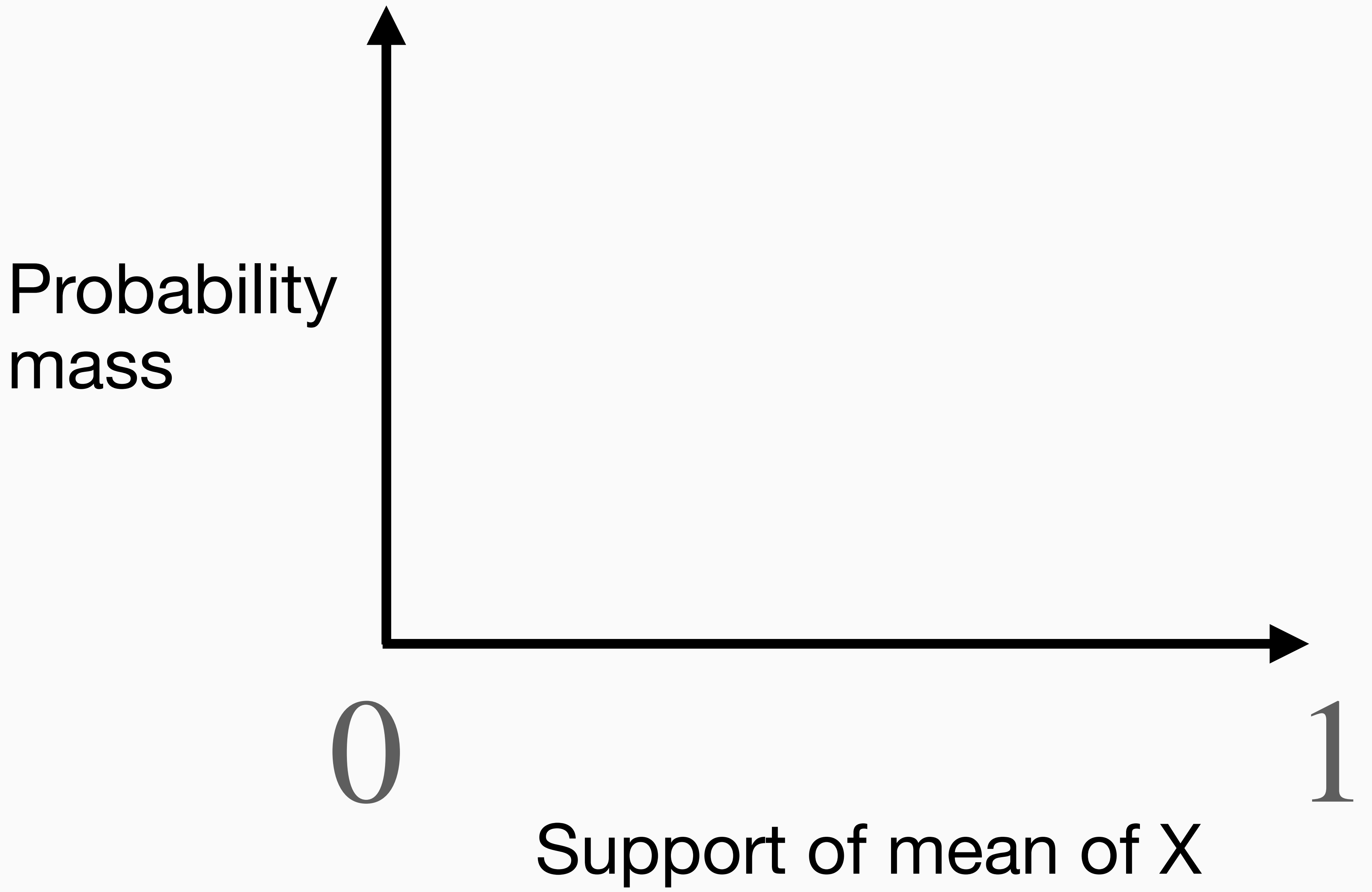


Single trial

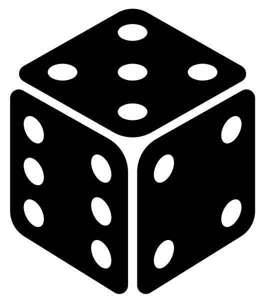
Bernoulli RV: X is 3

What is the PMF for the mean of X over e.g. 100 trials?

$$\bar{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i$$



Gaussians are special



Single trial

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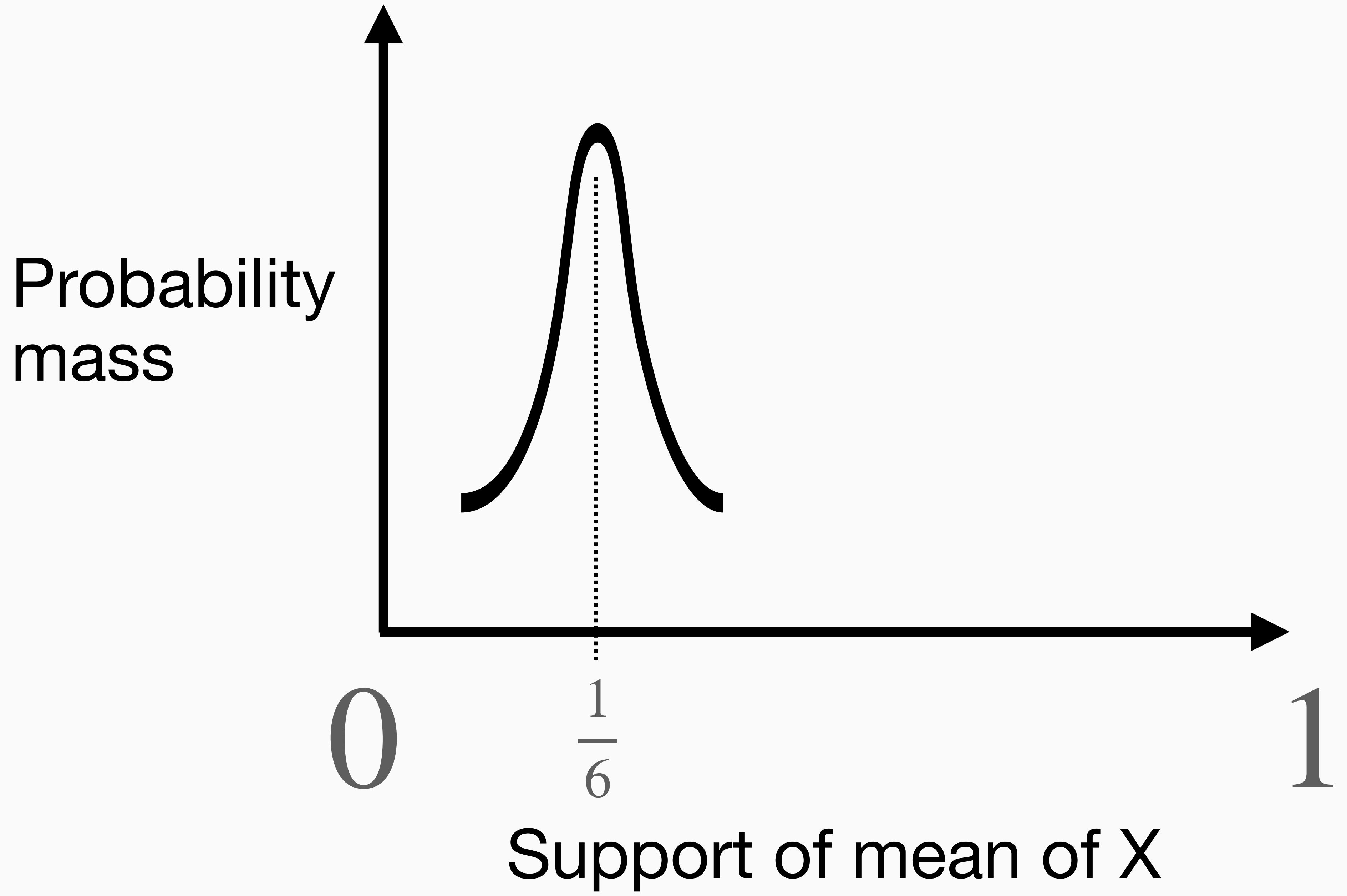


Gaussians are special

More generally…

Random variables depend on
single experiments

What about their expectation over
groups of *independent* experiments?

Gaussians are special

Random variables depend on
single experiments

Arbitrary random variable X:

$$\mathbb{E}[X] = \mu \qquad \mathrm{Var}[X] = \sigma^2$$

What about their expectation over
groups of *independent* experiments?

$\{X_1 \ldots X_n\}$ are i.i.d.

"Independent, identically distributed"
(remember abbreviation)

$$\bar{X}_n = \frac{1}{n} \sum_{i=1}^{n} X_i$$

?

Gaussians are special: Central Limit Theorem

$$\bar{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i$$

Average outcome over groups of n experiments

Central Limit Theorem

$\bar{X}_n \to^d \mathcal{N}\left(\mu, \frac{\sigma^2}{n}\right)$ regardless of X_i distribution

Converges in distribution

Gaussians are special: Central Limit Theorem

Caveat

How large should n be for the Central Limit Approximation to be reasonable?

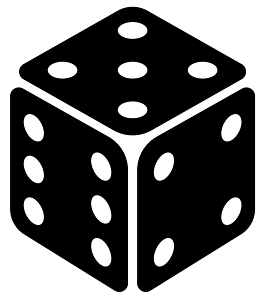
Pragmatic: n=30
Theoretical: arbitrarily large! (depends on how 'non-Gaussian' X is)

Central Limit Theorem

$$\bar{X}_n \xrightarrow{d} \mathcal{N}\left(\mu, \frac{\sigma^2}{n}\right)$$ regardless of X_i distribution

Converges in distribution

Gaussians are special: Central Limit Theorem



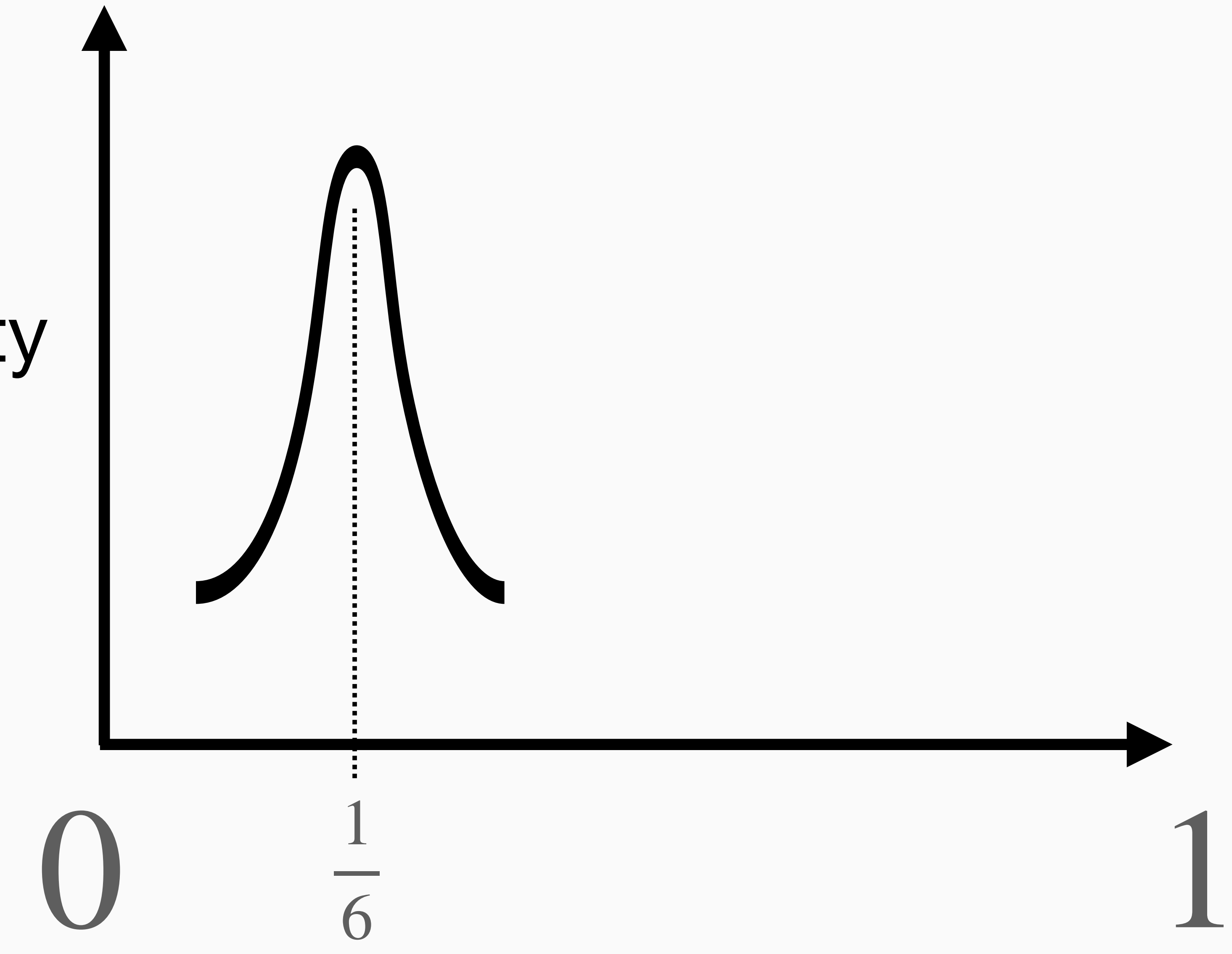
Single trial

Bernoulli RV: X is 3

What is the PDF for the mean of X over e.g. 100 trials?

$$\bar{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i$$

Probability density



Support of mean of X

Gaussians are special: Central Limit Theorem

Is height approximately normally (gaussian) distributed?

Is the average height of each MSc course normally distributed?

Conditional probability

Partial knowledge of an outcome

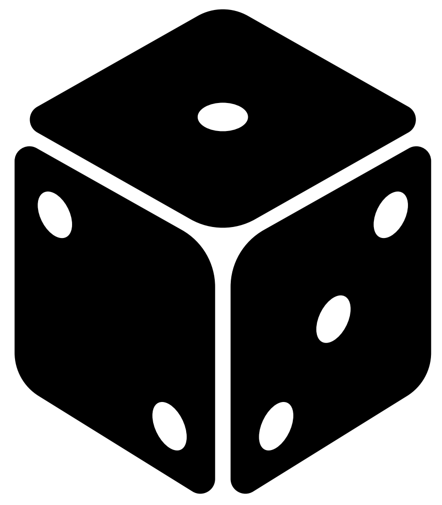
Partial knowledge of an outcome

Other events are still uncertain, but their probability has changed

Y : first seat in back row filled?

Z : all seats unfilled?

Partial knowledge of an outcome



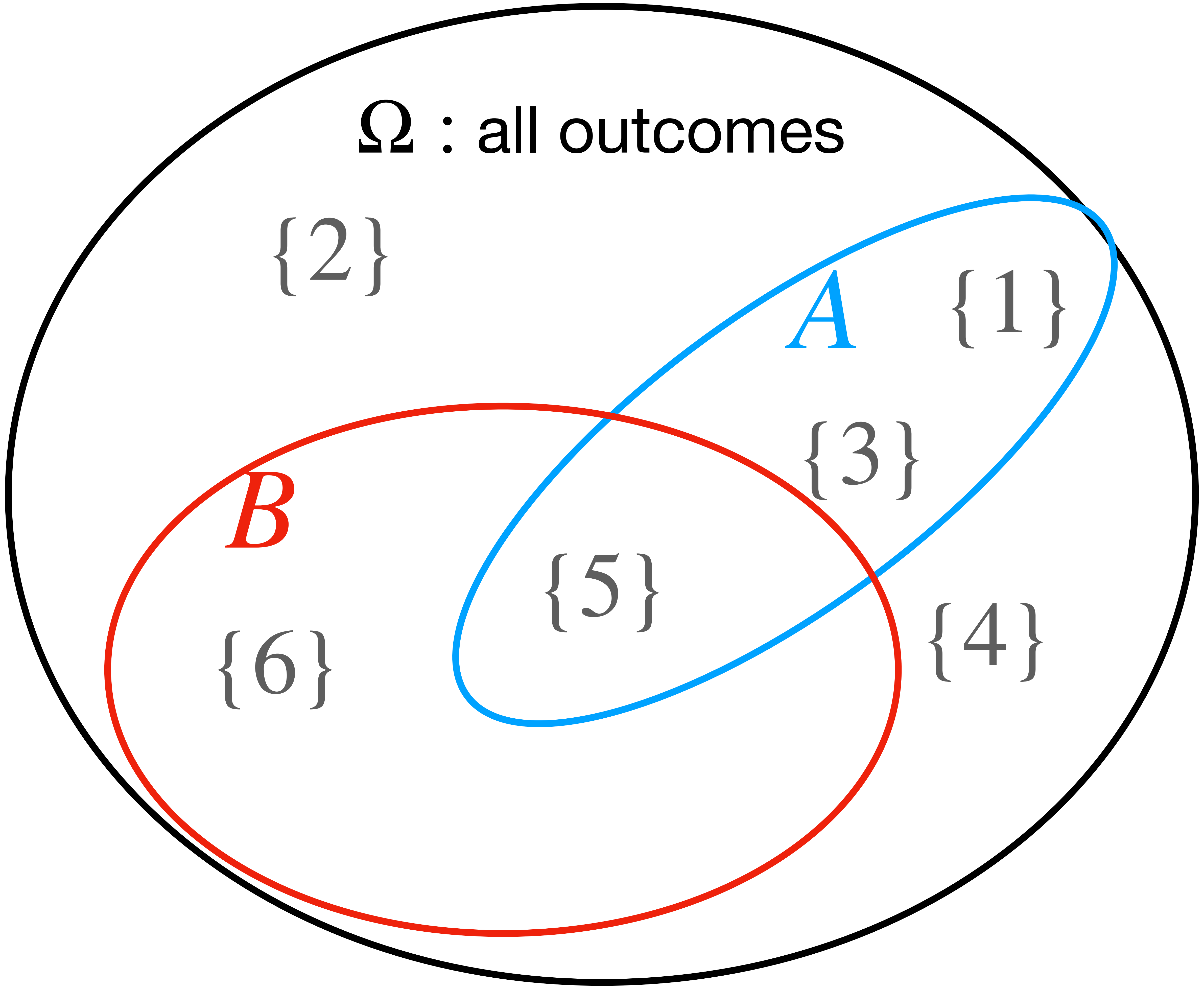
$$B = \{ \text{ roll} \geq 5 \ \}$$

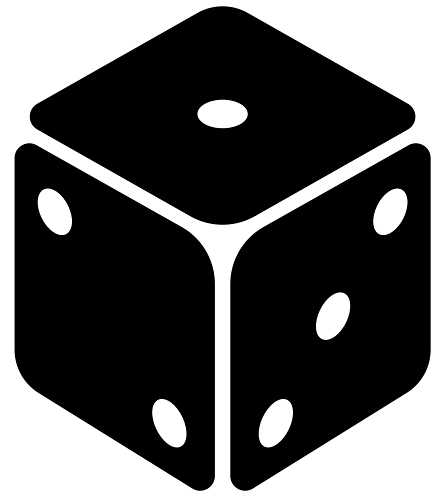
$$A = \{ \text{ roll is odd} \}$$

$$\mathbb{P}[A \mid B]?$$

"Probability of A given B"

Partial knowledge of an outcome





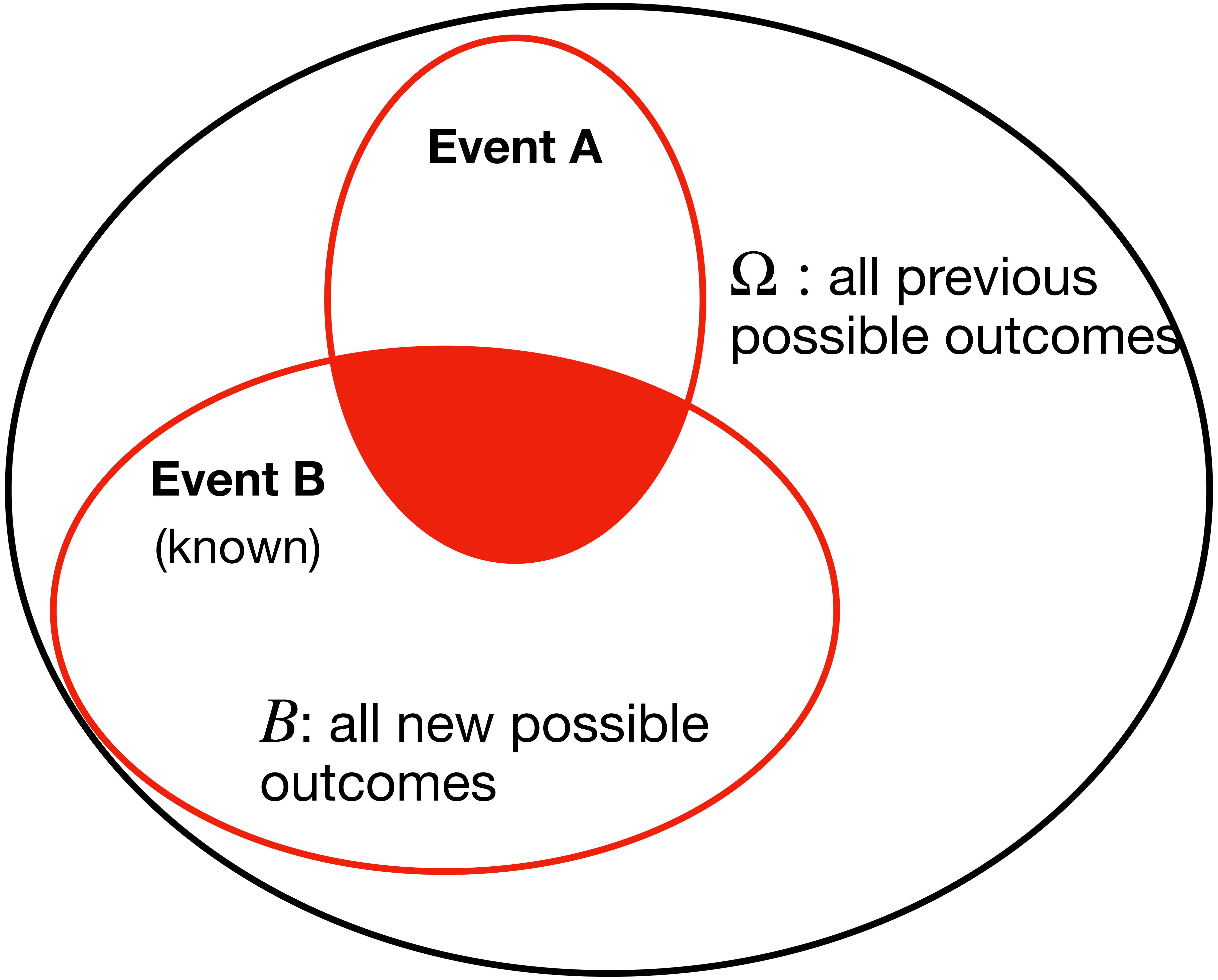
$B = \{ \text{ roll} \geq 5 \text{ } \}$

$A = \{ \text{ roll is odd} \}$

$\mathbb{P}[A \mid B]$?

"Probability of A given B"

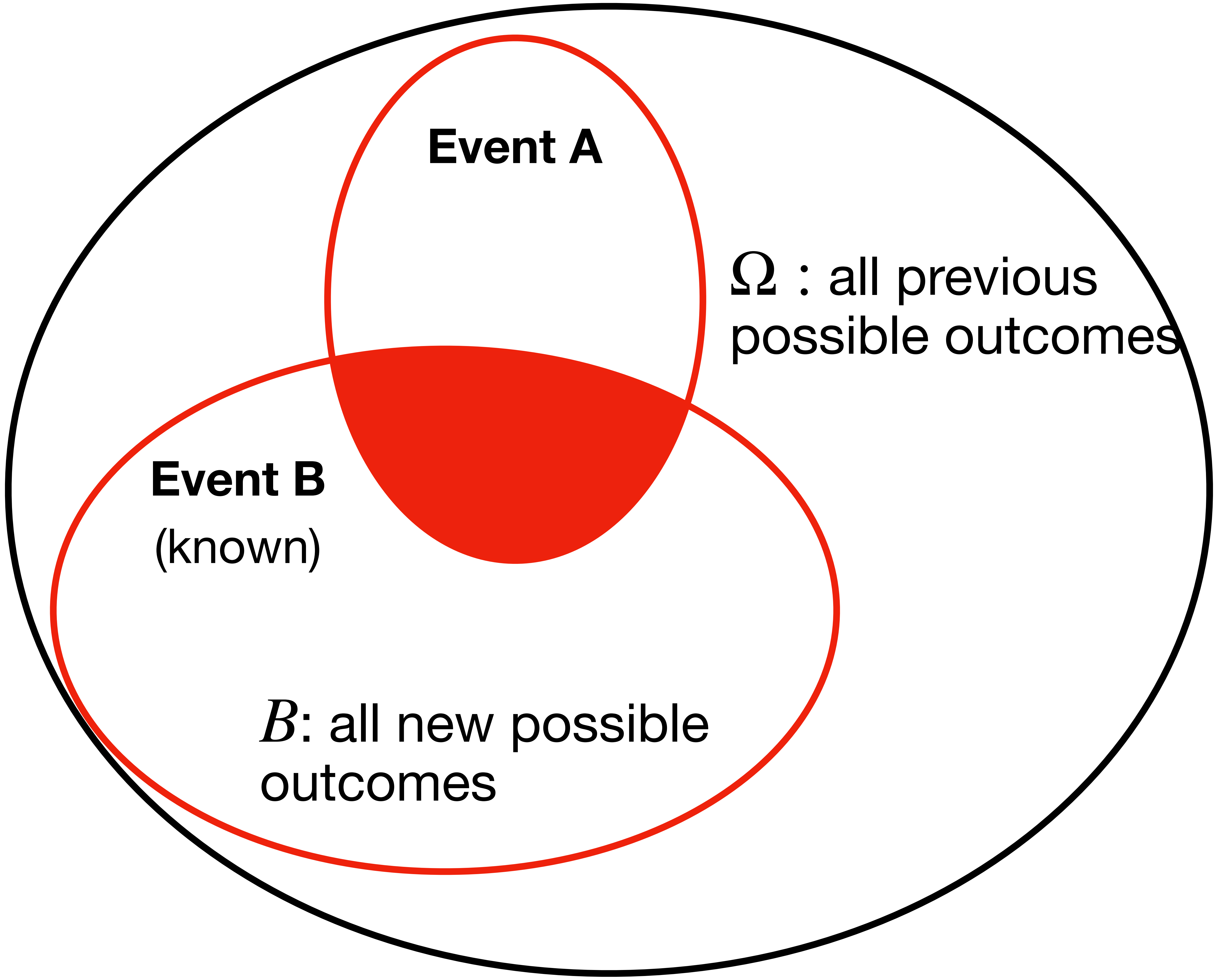
New Probability Space when Event B occurred



New set of outcomes in which event A happens: $A \cap B$

New set of all possible outcomes: B

New Probability Space when Event B occurred

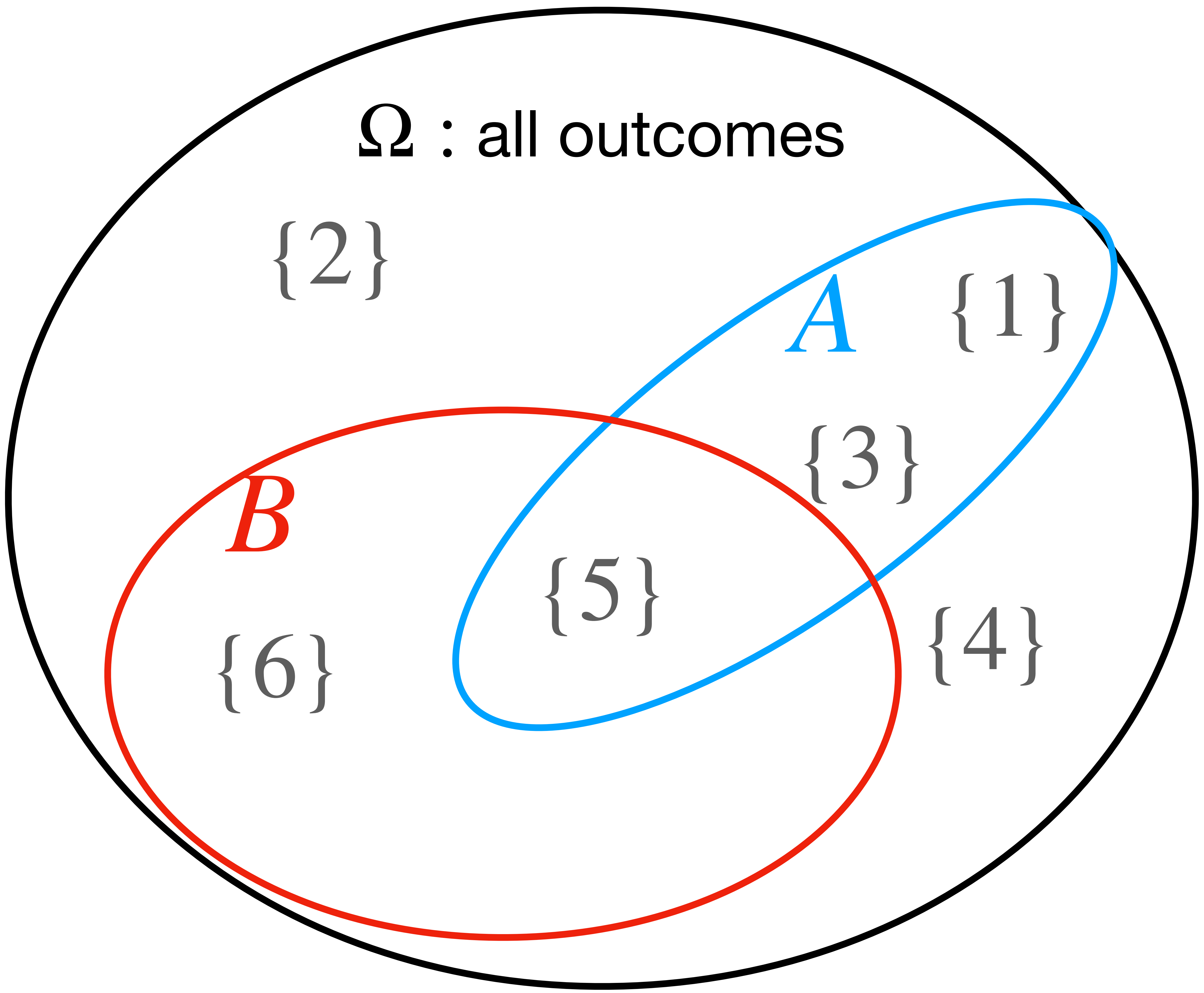


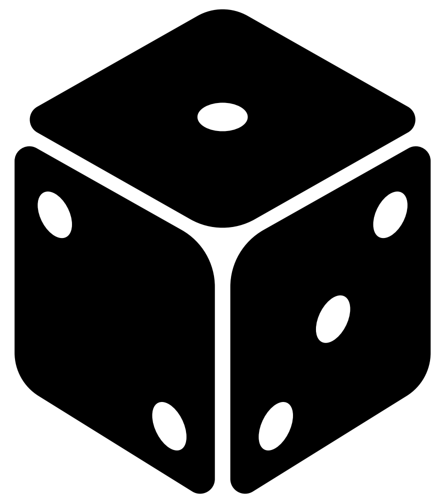
New set of outcomes in which event A happens: $A \cap B$

New set of all possible outcomes: B

$$\boxed{\mathbb{P}[A \mid B] = \frac{\mathbb{P}(A \cap B)}{\mathbb{P}(B)}}$$

Partial knowledge of an outcome





$$\boxed{\mathbb{P}[A \,|\, B] = \frac{\mathbb{P}(A \cap B)}{\mathbb{P}(B)}}$$

Law of conditional probability

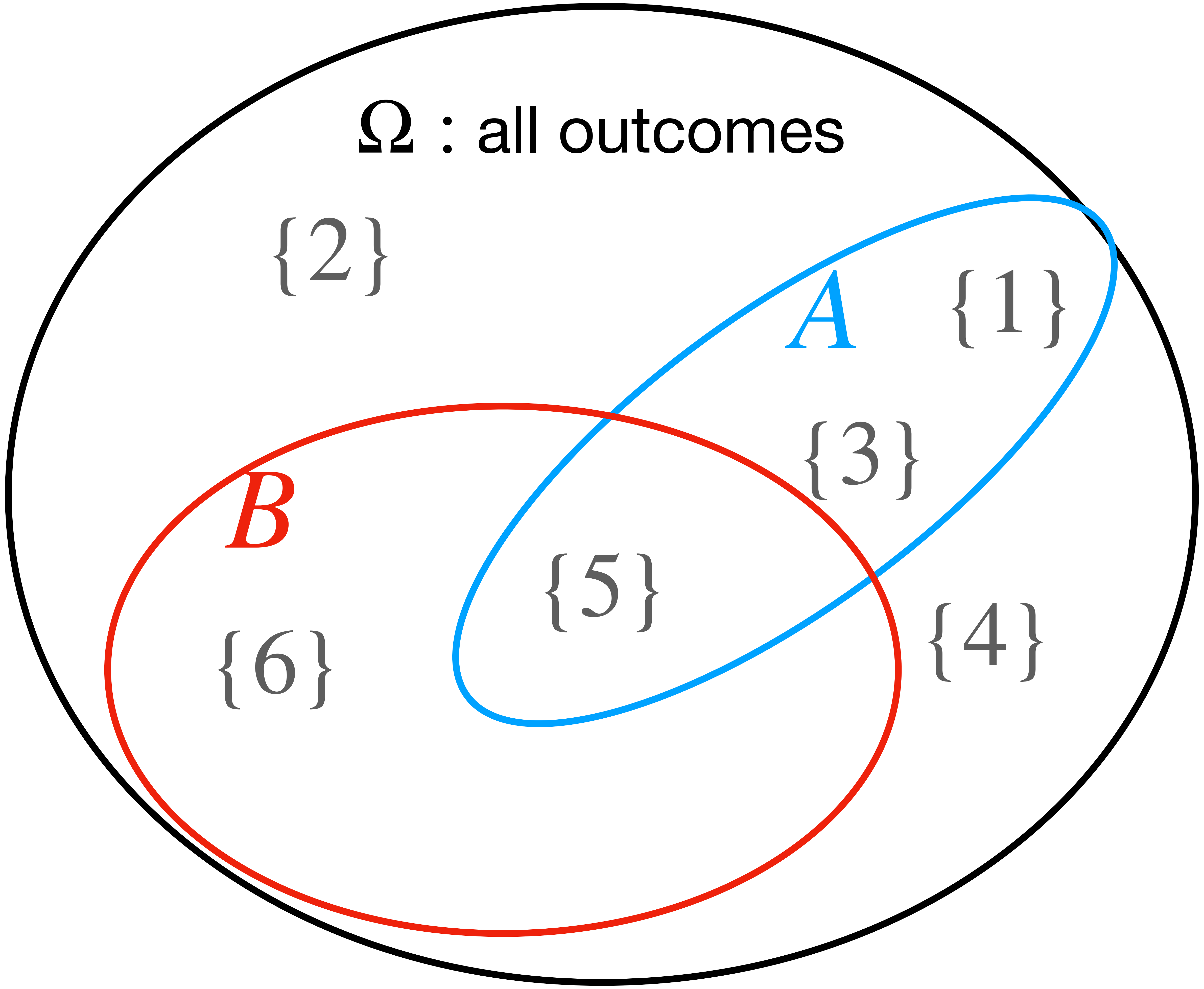
$$\boxed{\mathbb{P}[A \mid B] = \frac{\mathbb{P}(A \cap B)}{\mathbb{P}(B)}}$$

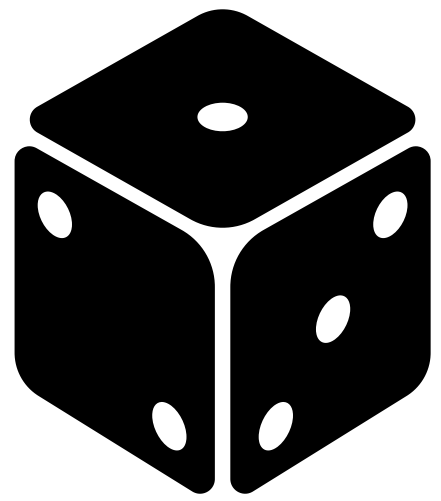
Independent events

$$\mathbb{P}[A \mid B] = \mathbb{P}[A]$$

$$\Rightarrow \mathbb{P}[A \cap B] = \mathbb{P}[A]\mathbb{P}[B]$$

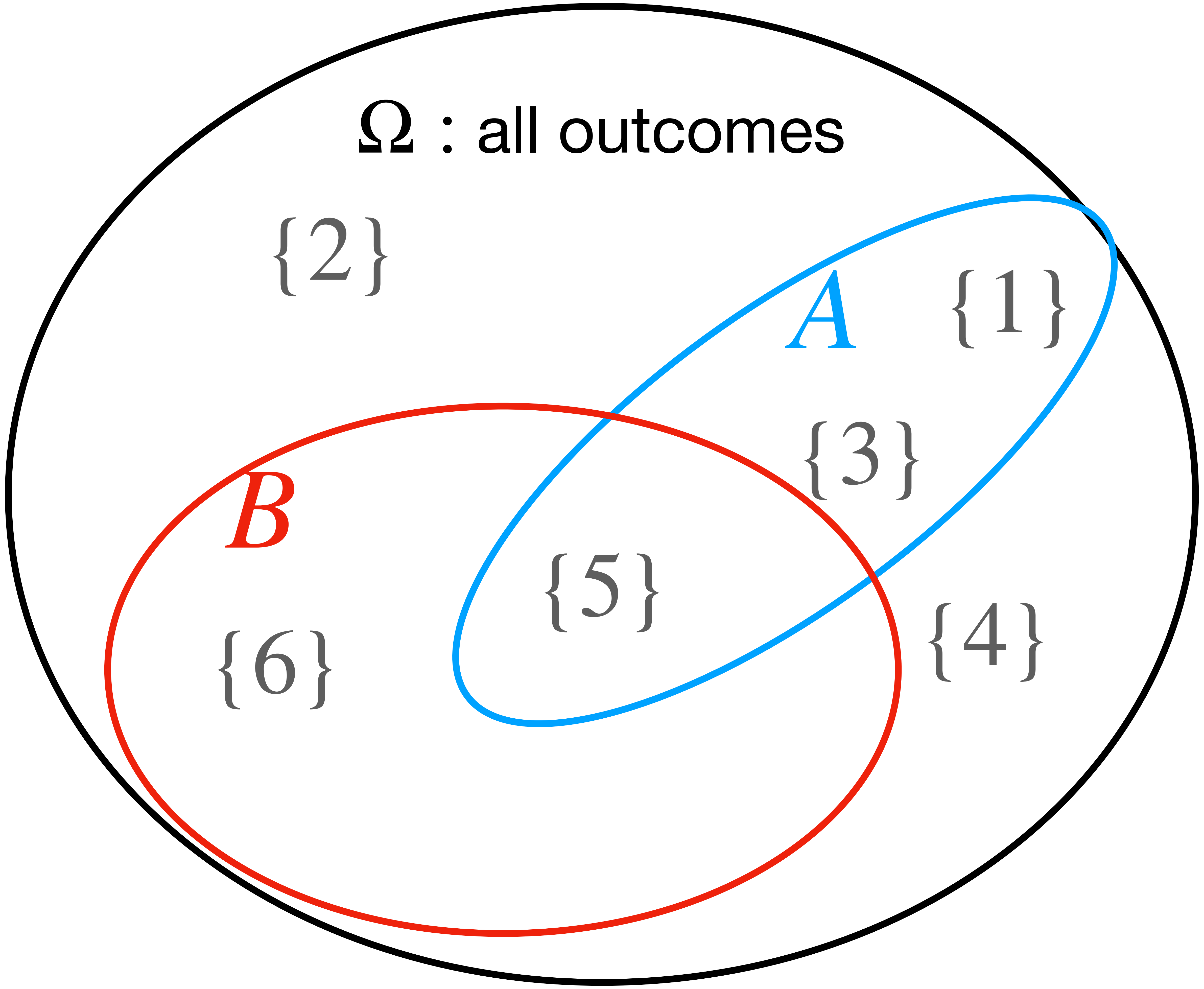
Independent events?

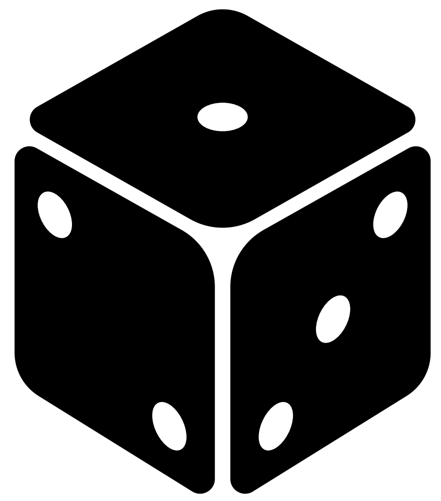




$$\boxed{\mathbb{P}[A \mid B] = \frac{\mathbb{P}(A \cap B)}{\mathbb{P}(B)}}$$

Independent events?

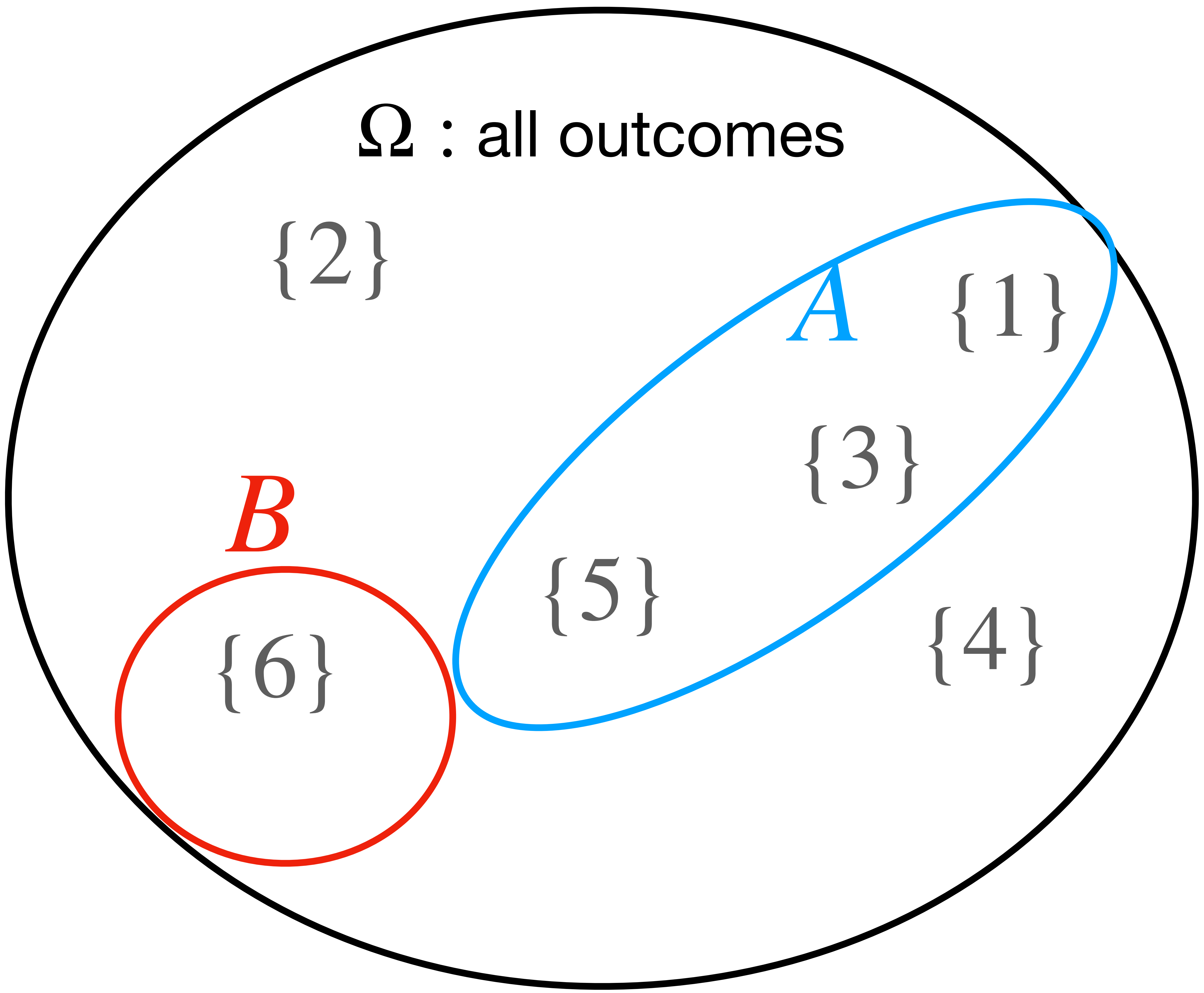




Can exclusive events (no intersection) **ever be independent?**

$$\mathbb{P}[A \cap B] = \mathbb{P}[A]\mathbb{P}[B]$$

Independent events?



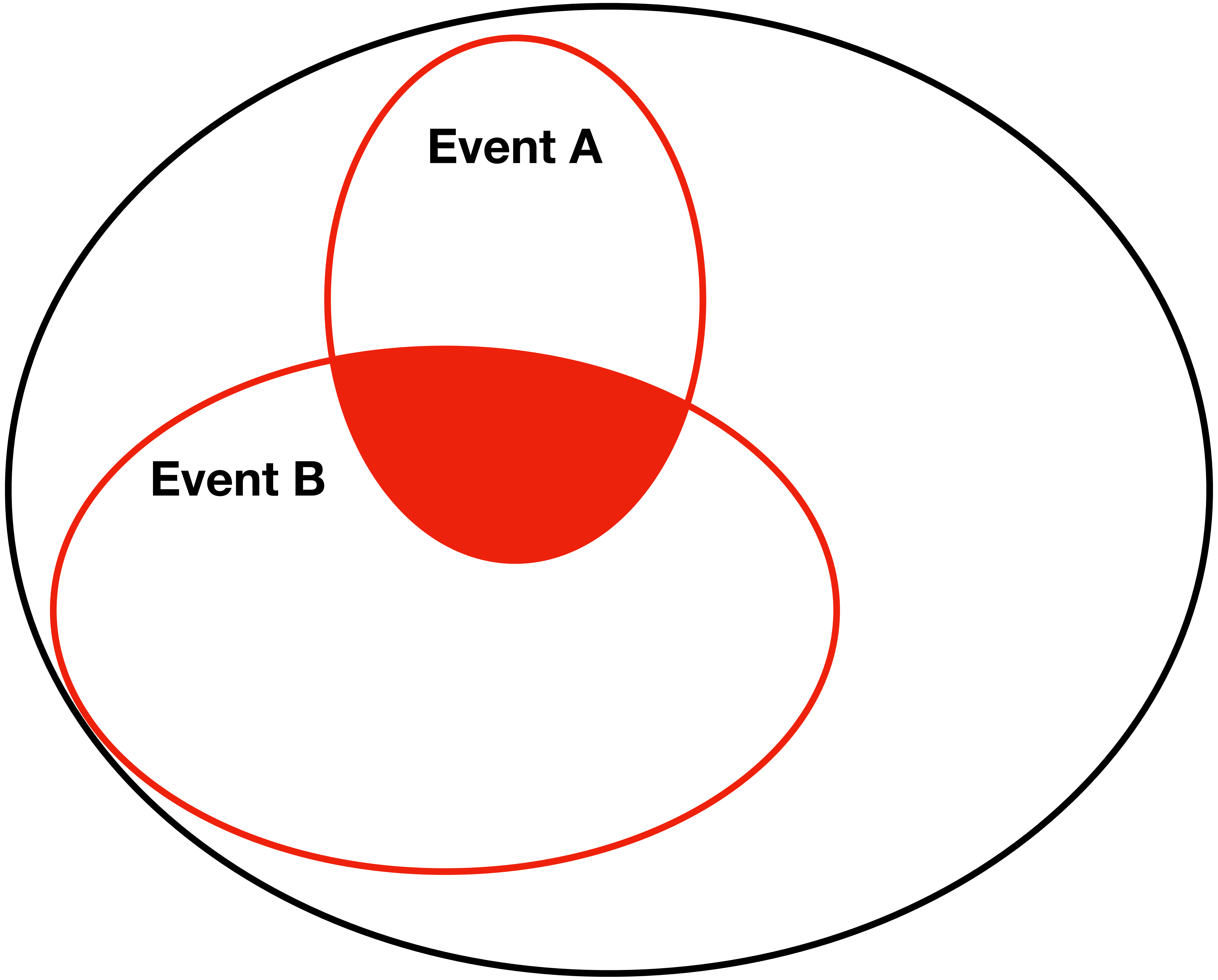
Exclusive events:

$$\boxed{\mathbb{P}[A \mid B] = \frac{\overset{0}{\mathbb{P}(A \cap B)}}{\mathbb{P}(B)}}$$

$$\mathbb{P}[A \cap B] = \mathbb{P}[A]\mathbb{P}[B]$$

…only if one of their initial probabilities are zero

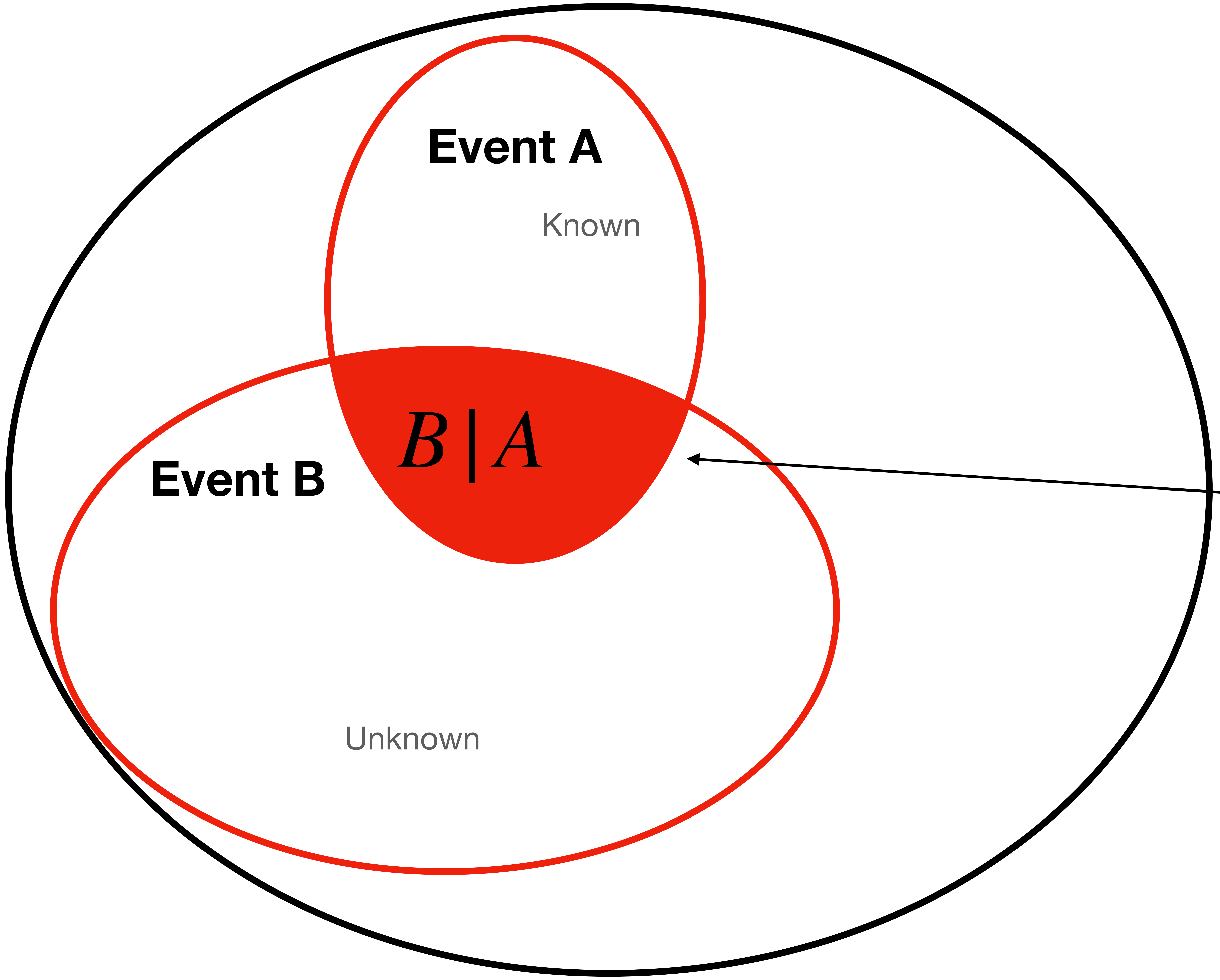
Bayes' Theorem



$$\boxed{\mathbb{P}[A \,|\, B] = \frac{\mathbb{P}(B \,|\, A)\mathbb{P}[A]}{\mathbb{P}(B)}}$$

Bayes' Theorem

$B|A:$



$$\boxed{\mathbb{P}[A|B] = \frac{\mathbb{P}(B|A)\mathbb{P}[A]}{\mathbb{P}(B)}}$$

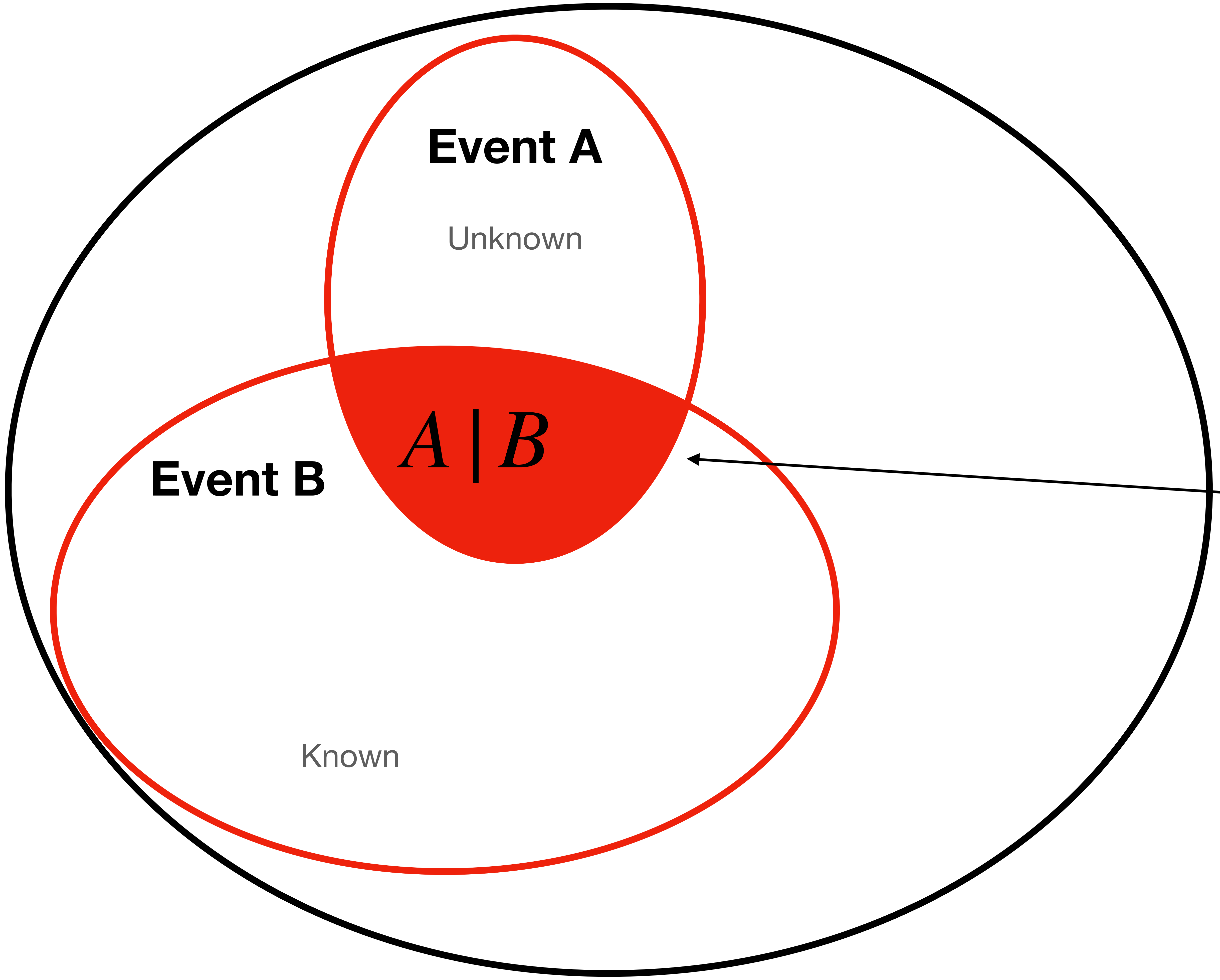
$$\mathbb{P}[B|A] = \frac{\mathbb{P}[A \cap B]}{\mathbb{P}[A]}$$

$$\Downarrow$$

$$\mathbb{P}[A \cap B] = \mathbb{P}[B|A]\mathbb{P}[A]$$

Bayes' Theorem

$A \mid B:$



$$\boxed{\mathbb{P}[A \mid B] = \frac{\mathbb{P}(B \mid A)\mathbb{P}[A]}{\mathbb{P}(B)}}$$

$$\mathbb{P}[A \mid B] = \frac{\mathbb{P}[A \cap B]}{\mathbb{P}[B]}$$

$$\Downarrow$$

$$\mathbb{P}[A \cap B] = \mathbb{P}[A \mid B]\mathbb{P}[B]$$

Bayes' Theorem

$A \,|\, B :$

$\mathbb{P}[A \cap B] = \mathbb{P}[A \,|\, B]\mathbb{P}[B]$

$B \,|\, A :$

$\mathbb{P}[A \cap B] = \mathbb{P}[B \,|\, A]\mathbb{P}[A]$

$$\Rightarrow \quad \mathbb{P}[A \,|\, B]\mathbb{P}[B] = \mathbb{P}[B \,|\, A]\mathbb{P}[A]$$

Bayes' Theorem

$$\boxed{\mathbb{P}[A \,|\, B] = \frac{\mathbb{P}(B \,|\, A)\mathbb{P}[A]}{\mathbb{P}(B)}}$$

$A \,|\, B :$

$$\mathbb{P}[A \cap B] = \mathbb{P}[A \,|\, B]\mathbb{P}[B]$$

$$\mathbb{P}[A \,|\, B]\mathbb{P}[B]$$

$$=$$

$$\mathbb{P}[B \,|\, A]\mathbb{P}[A]$$

$B \,|\, A :$

$$\mathbb{P}[A \cap B] = \mathbb{P}[B \,|\, A]\mathbb{P}[A]$$

Bayes' Theorem is useful

$$\boxed{\mathbb{P}[A \,|\, B] = \frac{\mathbb{P}(B \,|\, A)\mathbb{P}[A]}{\mathbb{P}(B)}}$$

Probability of COVID | sore throat?

Probability of COVID = 1%

Probability of sore throat = 5%

Percentage of Covid patients with sore throat = 30%

Bayes' Theorem is useful

$$\boxed{\mathbb{P}[A \mid B] = \frac{\mathbb{P}(B \mid A)\mathbb{P}[A]}{\mathbb{P}(B)}}$$

Probability of COVID | sore throat?

$$= \frac{0.3 \times 0.01}{0.05}$$

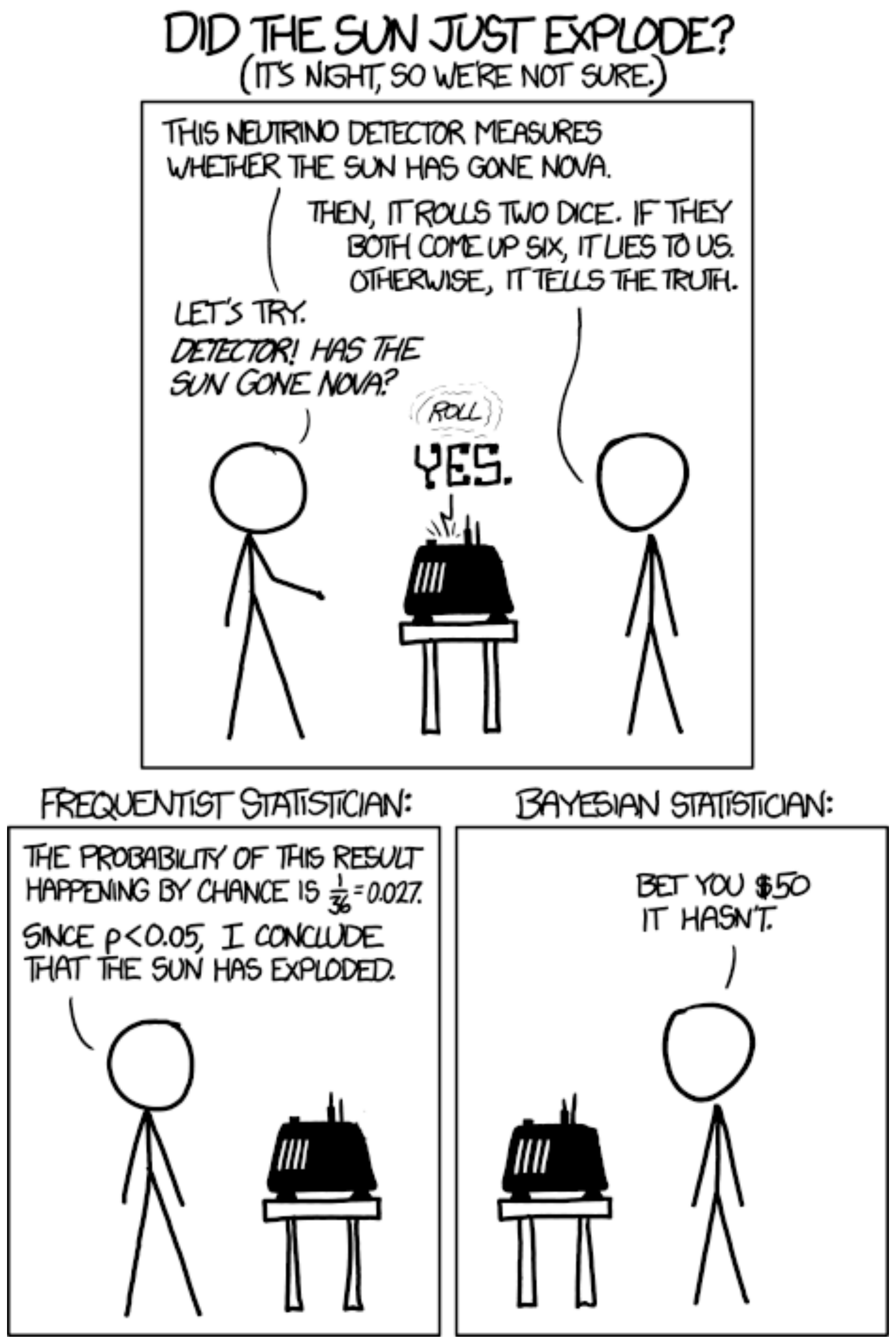
$$= 6\,\%$$

Probability of COVID = 1%

Probability of sore throat = 5%

Percentage of Covid patients with sore throat = 30%

Bayes' Theorem is useful



$$\boxed{\mathbb{P}[A \mid B] = \frac{\mathbb{P}(B \mid A)}{\mathbb{P}(B)}}$$

Reading mathematics

For any $A \in \mathbb{R}^{m \times n}$, where $m, n \in \mathbb{N} : \ker(A) = \{x : Ax = 0\}$

Definition

The **span** of a set of vectors

Consider a vector space V, with associated field K (usually $K = \mathbb{R}$). Pick two elements $e_1, e_2 \in V$. Consider the set

$$S = \{v \in V : v = \alpha_1 e_1 + \alpha_2 e_2, \quad \text{where} \quad \alpha_1, \alpha_2 \in K\}$$

1. S is known as the **span** of the vectors e_1 and e_2.

Reading mathematics

Let K be a field:

$$K[x] = \{a_0 + a_1x + a_2x^2 + a_3x^3 + \ldots a_nx^n \ \ | a_i \in K\}$$

$$K[x]_{\leq N} = \{a_0 + a_1x + a_2x^2 + a_3x^3 + \ldots a_nx^n \ \ | \quad 0 \leq n \leq N, \ \ a_i \in K\}$$