CITS2211 Assignment 2

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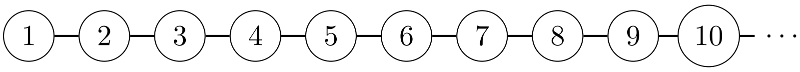
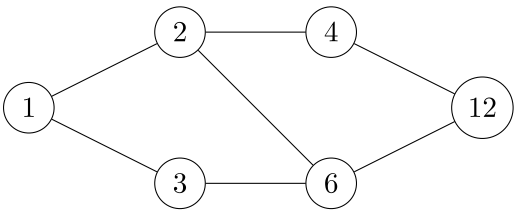
Question 1

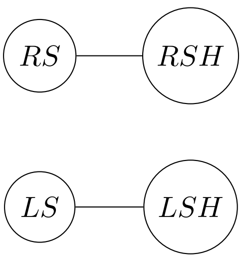
a)



b)



c)



Question 2

a)

the relation is reflexive if $\forall x \in \mathbb{R}((x, x) \in R)$
subtrating any real number by itself results in zero which is an integer, Hence R is reflexive.

The relation is symmetric if for all $(x, y) \in R$, $(y, x) \in R$ if $x \neq y$ we need to show that if x - y is an integer, y - x is an integer as well

$$\begin{aligned} \text{let } x - y &= k \text{ where } x \in \mathbb{Z} \\ y - x &= -k \text{ (arithmetic)} \end{aligned}$$

k is an integer so -k is an integer as
hence, y - x is an integer
Therefore, we have proved that is $(x, y) \in R$ then $(y, x) \in R$ and relation R is symmetric
The relation R is transitive if for all $x, y, z \in \mathbb{R}$ if xRy and yRz then xRz holds
so we need to prove that if $x - y \in \mathbb{Z}$ and $y - z \in \mathbb{Z}$ then $x - z \in \mathbb{Z}$

$$\begin{aligned}
&\text{let } x - y = k_1 \text{ where } k_1 \text{ is an integer} \\
&\text{let } y - z = k_2 \text{ where } k_2 \text{ is an integer} \\
x - z &= (x - y) + (y - z) \text{ (adding and substracting y)} \\
&= k_1 + k_2
\end{aligned}$$

both k_1 and k_2 are integers so $k_1 + k_2$ is integer as well, so x - z is an integer and $(x, z) \in R$ and the relation R is transitive

Q.E.D.

b)

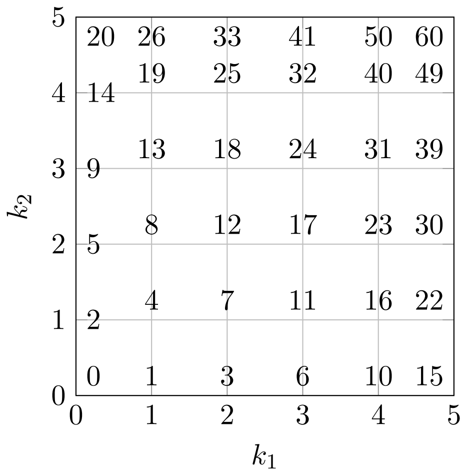
i.
the equivalence class of any real number x is given by
$[x] = \{y \in \mathbb{R} \mid y - x \in \mathbb{Z}\}$
let y - x be an integer k then y = x + k
Therefore, the equivalence class of any real number x consists of all numbers produced by adding an integer to x since the numbers integers is infinite, each equivalence class is infinite

ii.
as proved in part **i** every real number has an equivalence classes therefore, the total number of equivalence classes are infinite.

Question 4

a)

Values of $\pi(k_1, k_2)$



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b)

The function π is bijection between $\mathbb{N}^2$ and $\mathbb{N}$. We need to prove that there exists a bijection between $\mathbb{N}^n$ and $\mathbb{N}$
We will prove this using induction
let $P(k)$ be $|\mathbb{N}^k| = |\mathbb{N}|$

<u>**Base case:**</u> The base case is n = 1
$\mathbb{N}^1 = \mathbb{N}$
$|\mathbb{N}| = |\mathbb{N}|$ is trivially true

<u>**Inductive Case:**</u>
We need to prove that $P(k) \to P(k+1)$ for some arbitary $k \geq 1$

Inductive hypothesis:
We can assume that $P(k)$ holds for an arbitary $k \geq 1$

Inductive step:
now we need to show that $P(k+1)$ holds given $P(k)$ is true
we know that there is a bijection f from $\mathbb{N}^n$ to $\mathbb{N}$ from the inductive hypothesis
So, $\mathbb{N}^n$ can be mapped to a single natural number $\mathbb{N}^{n+1}$ can be written as $\mathbb{N}^n \times \mathbb{N}$
$\mathbb{N}^n \times \mathbb{N}$ can further be mapped using the function π
since we know that both π and f function are bijections, they can be combined to a single bijective function from $\mathbb{N}^{n+1}$ to $\mathbb{N}$
Therefore, $|\mathbb{N}^{n+1}| = |\mathbb{N}|$
Q.E.D.

Question 5

We need to prove that a bijection from B to A^B doesn't exist
assume a bijection f from B to A^B exists
for every $x \in B$, $f(x) \in A^B$. f(x) is a function from B to A.
Now, we can define new function f' such that $f'(x) \neq (f(x))(x)$
since, $|A| \geq 2$ we can define this function by choosing a different value from set A for every value of $x \in B$ (using the diagonal argument)
the function f is a bijection so there must be some value of $x \in B$ such that $f(x) = f'$

This contradicts with the definition of f' function since this would imply that $f(x)(y) = f'(y)$ for all $y \in B$
our assumption that f is a bijection must be false Therefore, a bijection from set B to A^B doesn't exist and $|A^B| \neq |B|$.
Q.E.D.

Question 6

a)

States: $Q = \{q_1, q_2, q_3\}$
Start state: $q_0 = q_1$

Alphabet: $\Sigma = \{0, 1\}$
Accepting states: $F = \{q_1, q_3\}$
State transition: $\delta : Q \times \Sigma -> \mathcal{P}(Q)$

b)

FSM recognises the symbols 1 and 0

- q_1 is the starting state and it stays there if it reads 0 and moves to q_2 if it reads 1
- in state q_2 it doesn't move if it reads 0 and moves to q_3 if it reads 1
- in state q_3 it only accepts the symbol 0 and stays in q_3

states q_1 and q_3 are the accepting states
Thus, the FSM accepts strings that contain exactly two 1s and any number of 0s including none anywhere

c)

0*10*10*

Question 7

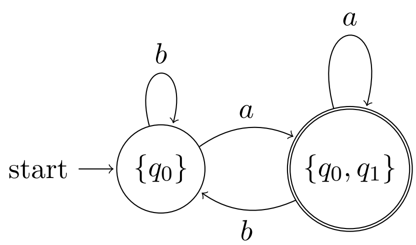
States:
set of States of the DFSM is the powerset of the states of the original NFSM
$Q = \{\phi, \{q_0\}, \{q_1\}, \{q_0, q_1\}\}$

Initial state:
The initial state of the DFSM is a singleton set containing the initial state of the NFSM
$\{q_0\}$ is the initial state

Accepting states:
accepting states of the DFSM are the states containing any accepting states of NFSM
$\{q_1\}$ and $\{q_0, q_1\}$ are the accepting states

The Transition Function:
The transition function, δ , returns the union of all states that are reachable via the original transition function δ' , by consuming the input from any of the NFSM states in the current DFSM state, i.e., $\delta(s, x) = \bigcup\{\delta'(q, x) \mid q \in s\}$

The Result could be simplified by removing the state $\{q_1\}$ since there is no way to reach the state and it's part of the combined state $\{q_0, q_1\}$



Question 8

a)

b*ab*aab*aaab*

b)

- ((11)*)1*)* can be simplified to 1* since (11)* means even number of 1s and the 1* provides additional 1s which means an empty string of any string made of 1s is accepted
- (11 + 1)* can be simplified to 1* because 11* would be even number of 1s and 1* would make any number of 1s including none.
- (0 + ϵ)* can be simplified to 0* since ϵ* can be covered by no zeroes included in 0*
- the expression can be further simplified by combining 1* + 1* to 1*

final expression is 1* + 0*