CITS2211 Assignment 2

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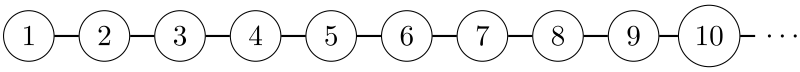
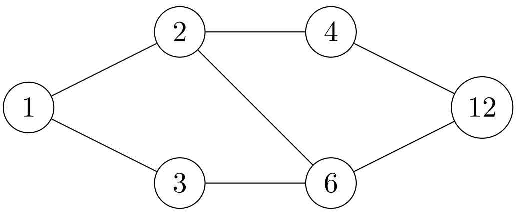
Question 1

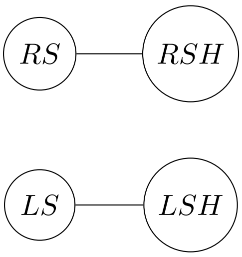
a)



b)



c)



Question 2

a)

the relation is reflexive if $\forall x \in \mathbb{R}((x, x) \in R)$
subtrating any real number by itself results in zero which is an integer, Hence R is reflexive.

The relation is symmetric if for all $(x, y) \in R$, $(y, x) \in R$ if $x \neq y$ we need to show that if x - y is an integer, y - x is an integer as well

$$\begin{aligned} &\text{let } x - y = k \text{ where } x \in \mathbb{Z} \\ &y - x = -k \text{ (arithmetic)} \end{aligned}$$

k is an integer so -k is an integer as
hence, y - x is an integer
Therefore, we have proved that is $(x, y) \in R$ then $(y, x) \in R$ and relation R is symmetric
The relation R is transitive if for all $x, y, z \in \mathbb{R}$ if xRy and yRz then xRz holds
so we need to prove that if $x - y \in \mathbb{Z}$ and $y - z \in \mathbb{Z}$ then $x - z \in \mathbb{Z}$

$$\begin{aligned}
\text{let } x - y &= k_1 \text{ where } k_1 \text{ is an integer} \\
\text{let } y - z &= k_2 \text{ where } k_2 \text{ is an integer} \\
x - z &= (x - y) + (y - z) \text{ (adding and substracting y)} \\
&= k_1 + k_2
\end{aligned}$$

both k_1 and k_2 are integers so $k_1 + k_2$ is integer as well, so x - z is an integer and $(x, z) \in R$ and the relation R is transitive

Q.E.D.

b)

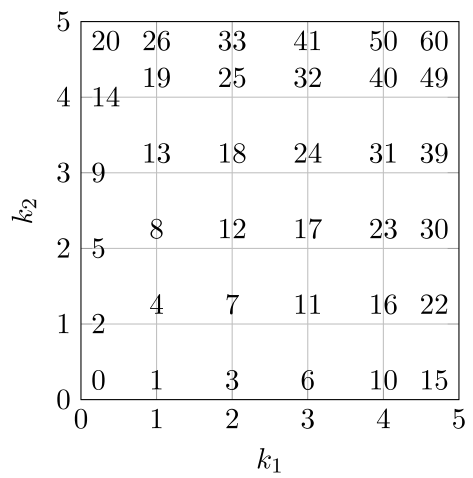
i.
the equivalence class of any real number x is given by
$[x] = \{y \in \mathbb{R} \mid y - x \in \mathbb{Z}\}$
let y - x be an integer k then y = x + k
Therefore, the equivalence class of any real number x consists of all numbers produced by adding an integer to x since the numbers integers is infinite, each equivalence class is infinite

ii.
as proved in part **i** every real number has an equivalence classes therefore, the total number of equivalence classes are infinite.

Question 4

a)

Values of $\pi(k_1, k_2)$



b)

The function π is bijection between $\mathbb{N}^2$ and $\mathbb{N}$. We need to prove that there exists a bijection between $\mathbb{N}^n$ and $\mathbb{N}$
We will prove this using induction
let $P(k)$ be $|\mathbb{N}^k| = |\mathbb{N}|$

<u>**Base case:**</u> The base case is n = 1
$\mathbb{N}^1 = \mathbb{N}$
$|\mathbb{N}| = |\mathbb{N}|$ is trivially true

<u>**Inductive Case:**</u>
We need to prove that $P(k) \rightarrow P(k+1)$ for some arbitary $k \geq 1$

Inductive hypothesis:
We can assume that $P(k)$ holds for an arbitary $k \geq 1$

Inductive step:
now we need to show that $P(k+1)$ holds given $P(k)$ is true
we know that there is a bijection f from $\mathbb{N}^n$ to $\mathbb{N}$ from the inductive hypothesis
So, $\mathbb{N}^n$ can be mapped to a single natural number $\mathbb{N}^{n+1}$ can be written as $\mathbb{N}^n \times \mathbb{N}$
$\mathbb{N}^n \times \mathbb{N}$ can further be mapped using the function π
since we know that both π and f function are bijections, they can be combined to a single bijective function from $\mathbb{N}^{n+1}$ to $\mathbb{N}$
Therefore, $|\mathbb{N}^{n+1}| = |\mathbb{N}|$
Q.E.D.

Question 5

We need to prove that a bijection from B to A^B doesn't exist
assume a bijection f from B to A^B exists
for every $x \in B$, $f(x) \in A^B$. f(x) is a function from B to A.
Now, we can define new function f' such that $f'(x) \neq (f(x))(x)$
since, $|A| \geq 2$ we can define this function by choosing a different value from set A for every value of $x \in B$ (using the diagonal argument)
the function f is a bijection so there must be some value of $x \in B$ such that $f(x) = f'$

This contradicts with the definition of f' function since this would imply that $f(x)(y) = f'(y)$ for all $y \in B$
our assumption that f is a bijection must be false Therefore, a bijection from set B to A^B doesn't exist and $|A^B| \neq |B|$.
Q.E.D.

Question 6

a)

States: $Q = \{q_1, q_2, q_3\}$
Start state: $q_0 = q_1$

Alphabet: $\Sigma = \{0, 1\}$
Accepting states: $F = \{q_1, q_3\}$
State transition: $\delta : Q \times \Sigma- > \mathcal{P}(Q)$

b)

FSM recognises the symbols 1 and 0

- q_1 is the starting state and it stays there if it reads 0 and moves to q_2 if it reads 1
- in state q_2 it doesn't move if it reads 0 and moves to q_3 if it reads 1
- in state q_3 it only accepts the symbol 0 and stays in q_3

states q_1 and q_3 are the accepting states
Thus, the FSM accepts strings that contain exactly two 1s and any number of 0s including none anywhere

c)

0*10*10*

Question 7

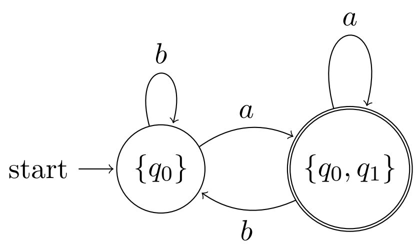
States:
set of States of the DFSM is the powerset of the states of the original NFSM
$Q = \{\phi, \{q_0\}, \{q_1\}, \{q_0, q_1\}\}$

Initial state:
The initial state of the DFSM is a singleton set containing the initial state of the NFSM
$\{q_0\}$ is the initial state

Accepting states:
accepting states of the DFSM are the states containing any accepting states of NFSM
$\{q_1\}$ and $\{q_0, q_1\}$ are the accepting states

The Transition Function:
The transition function, δ , returns the union of all states that are reachable via the original transition function δ' , by consuming the input from any of the NFSM states in the current DFSM state, i.e., $\delta(s, x) = \bigcup\{\delta'(q, x) \mid q \in s\}$

The Result could be simplified by removing the state $\{q_1\}$ since there is no way to reach the state and it's part of the combined state $\{q_0, q_1\}$



Question 8

a)

b*ab*aab*aaab*

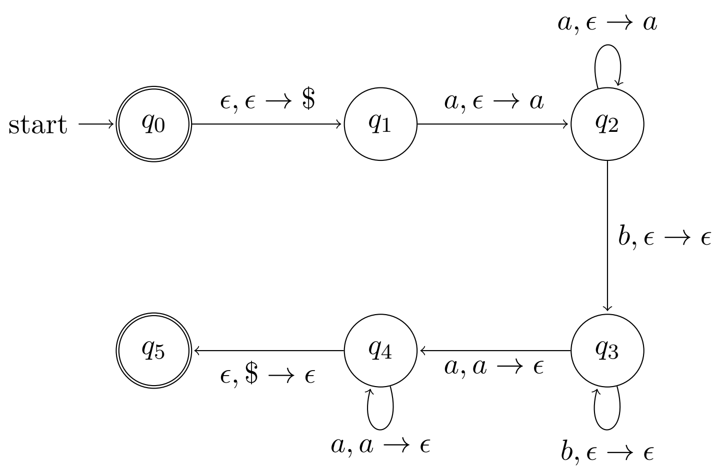
b)

- ((11)*)1*)* can be simplified to 1* since (11)* means even number of 1s and the 1* provides additional 1s which means an empty string of any string made of 1s is accepted
- (11 + 1)* can be simplified to 1* because 11* would be even number of 1s and 1* would make any number of 1s including none.
- (0 + ϵ)* can be simplified to 0* since ϵ* can be covered by no zeroes included in 0*
- the expression can be further simplified by combining 1* + 1* to 1*

final expression is 1* + 0*

Question 10

a) $\{a^n b^m a^n \mid m, n \in \mathbb{N}\}$



The automata requires one a to transition to q2 and additional a's to stay in q2, all of which are pushed on to the stack
It requires at least one b to transition to q3 and any additional b's keeps the state unchanged from q3
Then it requires one a to transition to q4 and any additional a's keeps the state unchanged and all those a's are popped from the stack, once the stack is empty it moves to the accepting state q5 given no input.