CITS2211: ASSIGNMENT ONE 2024

This assignment has 12 questions with a total value of 60 marks. Follow the instructions on LMS for submission.

1. State whether each of the following propositions is a **tautology** or a **contradiction** or **contingent** (i.e. neither). For each of your answers, give a brief justification.

   (a) $P \vee (Q \vee \neg P)$

   (b) $(P \wedge \neg P) \vee \neg Q$

   (c) $Q \rightarrow (P \wedge \neg Q)$

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2. Use the equivalences listed in lectures to prove that the following equivalence:

$$P \vee \neg(P \vee \neg Q) \equiv P \vee Q$$

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3. We define a natural number to be *fluffy* if it has a factor other than one or itself. Express the statement

   for every number n there exists a fluffy number between n and $n+5$

   in predicate logic. You should use no other predicates other than the $\leq$ and $=$ predicates commonly used in mathematics.

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4. Express the following colloquial English statements using predicate logic, where the domain of discourse is all people. Use the constants $a =$ "Anna" and $b =$ "Ben".

   (a) Anna has no neighbours.

   (b) Ben has two neighbours.

   (c) If somebody is a neighbour of Ben, Ben is also a neighbour of that person.

   (d) Except for Anna, everyone is the neighbour of someone.

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5. Which of the following proposed inference rules are sound for propositional logic? If they are sound then give a brief justification. If they are unsound, then give a counter-example.

$$a)\ \frac{P}{P} \qquad b)\ \frac{P}{P \leftrightarrow Q} \qquad c)\ \frac{P \leftrightarrow Q}{P} \qquad d)\ \frac{P \quad Q}{P \vee Q} \qquad e)\ \frac{P \to Q \quad \neg\neg P}{Q}$$

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6. Fill in the blanks in the following proof that

$$\forall x.(\neg Q(x) \wedge P(x)) \wedge \exists x.(Q(x) \vee (P(x) \wedge R(x))) \to \exists x.R(x)$$

1. $\forall x.(\neg Q(x) \wedge P(x))$	premise
2. $\exists x.(Q(x) \vee (P(x) \wedge R(x)))$	premise
3.	2, exist elimination
4. $\neg Q(a) \wedge P(a)$	
5. $\neg Q(a)$	4, conjunction elimination
6. $\neg Q(a) \to (P(a) \wedge R(a))$	3, implication law
7. $P(a) \wedge R(a)$	
8.	7, conjunction elimination
9. $\exists x.R(x)$	

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7. A number is *squidgy* if it can be represented as a fraction $\frac{p}{q}$ where both p and q are integers. Using a proof by contradiction, show that when you multiple the difference between a squidgy and a non-squidgy number by 2, the result is a non-squidgy number.

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8. Prove that x is an odd integer if and only if $5x - 1$ is an even integer

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9. In a badminton tournament of n players, each player plays exactly one match against every other player. There are no draws. Prove via induction that the players can be arranged in an order $p_1, p_2, ..., p_n$ such that p_i defeats p_{i+1} for all $i \in 1, 2, .., n-1$.

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10. Consider the following formal language (no semantics provided):

    - Alphabet: a, b
    - Syntax: the symbols a and baa are formulae. If ψ and ϕ are formulae then so are $\psi a \phi$ and $aba\psi$.

    Let $A(\psi)$ be the number of a symbols in formula ψ, and let $B(\psi)$ be the number of b symbols.

    Use structural induction to prove that:

$$\forall \phi . A(\phi) \geq 2B(\phi)$$

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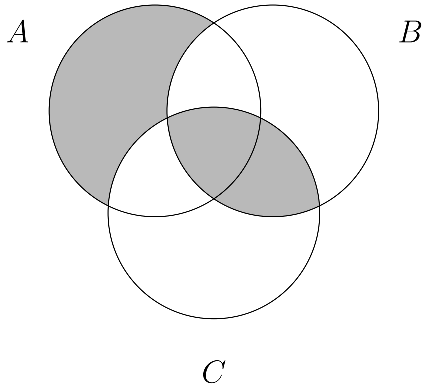
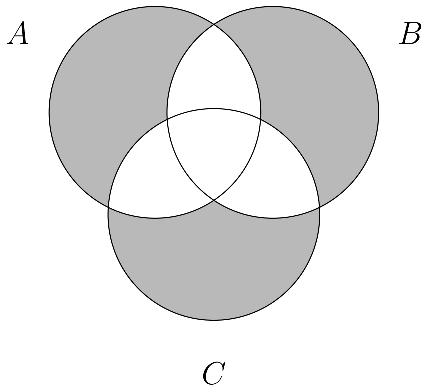
11. For each of the Venn diagrams below, write down a simple set theory expression which equals the shaded area of the diagram.

(i)



(ii)



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12. Consider two relations R, S on a set X. Let the relation T be defined as:

$$T = \{(x, z) \mid \exists y.(x, y) \in R \wedge (y, z) \in S\}$$

Either prove or disprove the following statements:

(a) If T is reflexive then so are R and S.

(b) If R and S are reflexive then so is T.

(c) If R and S are symmetric then so is T.

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