CITS2211 Assignment 1

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Question 1

a) P ∨ (Q ∨ ¬P)

P	Q	$\neg P$	$Q \vee \neg P$	$P \vee (Q \vee \neg P)$
T	T	F	T	T
T	F	F	F	T
F	T	T	T	T
F	F	T	T	T

As the truth table demonstrates, the proposition is a tautology since it's always true regardless of the truth values of Q or P

b) (P ∧ ¬P) ∨ ¬Q

P ∧ ¬P = F (by contradiction)
¬Q (By absorption)

the proposition depends on the truth value of Q hence it's a contingent

c) Q → (P ∧ ¬Q)

P	Q	$\neg Q$	$P \wedge \neg Q$	$Q \rightarrow (P \wedge \neg Q)$
T	T	F	F	F
T	F	T	T	T
F	T	F	F	F
F	F	T	F	T

As the truth table demonstrates, the truth value of the proposition depends on the truth values of P and Q. Hence, the proposition is a contingent.

Question 2

P ∨ ¬(P ∨ ¬Q) ≡ P ∨ ¬Q

1. P ∨ ¬ (P ∨ ¬ Q)	Premise
2. P ∨ (¬ P ∧ Q)	1, Demorgan's laws
3. (P ∨ ¬P) ∧ (P ∨ Q)	2, distributivity
4. T ∧ (P ∨ Q)	3, excluded middle
5. P ∨ Q	4, absorption

5. $P \lor Q$
Q.E.D.

Question 3

$\forall x\ \exists n\ (x \leq n \leq x+5 \land (\exists a\ \exists b\ (a \neq n) \land (a \neq 1) \land (b \neq n) \land (b \neq 1) \land (a \times b = n))$

Question 4

let N(x, y) be "x is a neighbour of y"

a) Anna has no neighbours

$\neg(\exists x.N(x, a))$

b) Ben has two neighbours

$\exists x \exists y\,(N(x, b) \land N(y, b) \land x \neq y \land \forall z\,(N(z, b) \rightarrow (z = x \lor z = y)))$

c) If somebody is a neighbour of Ben, Ben is also a neighbour of that person

$\forall x\,(N(x, b) \rightarrow N(b, x))$

d) Except for Anna, everyone is the neighbour of someone

$\forall x\,(x \neq a \rightarrow \exists y(N(x, y)))$

Question 5

a) $\dfrac{\boldsymbol{P}}{\boldsymbol{P}}$

$P \equiv P$ by rule of identity
hence, the inference rule is sound

b) $\dfrac{\boldsymbol{P}}{\boldsymbol{P \leftrightarrow Q}}$

if Q if false, the axiom is true but the conclusion is false
Hence, the inference rule is unsound

c) $\dfrac{\boldsymbol{P \leftrightarrow Q}}{\boldsymbol{P}}$

if P and Q are false, the axiom is true but the conclusion is false
Hence, the inference rule is unsound

d) $\dfrac{\boldsymbol{P\ Q}}{\boldsymbol{P \lor Q}}$

From the premise P we can infer $P \lor Q$ by disjunction introduction rule
Hence, the inference rule is sound

e) $\dfrac{\boldsymbol{P \to Q} \quad \boldsymbol{\neg\neg P}}{\boldsymbol{Q}}$

1. ¬¬P	Premise
2. P → Q	Premise
3. P	1, double negation
4. Q	3, 2, Demorgan's laws

Hence the rule is sound

Question 6

∀x.(¬ Q(x) ∨ P(x)) ∨ ∃ x(Q(x) ∨ (P(x) ∧ R(x))) → ∃x.R(x)

1. $\forall x(\neg Q(x) \land P(x))$	premise
2. $\exists x(Q(x) \lor (P(x) \land R(x)))$	premise
3. $Q(a) \lor (P(a) \land R(a))$	2, Exist elimination
4. $\neg Q(a) \land P(a)$	1, Forall elimination
5. $\neg Q(a)$	4, conjunction elimination
6. $\neg Q(a) \to (P(a) \land R(a))$	3, implication law
7. $P(a) \land R(a)$	5, 6, Modus ponens
8. R(a)	7, conjunction elimination
9. $\exists x.R(x)$	8, Exists introduction

Question 7

Assuming the difference beetween a squidgy and a non-squidgy number multiplied by 2 produces a squidgy number

we have a non-squidgy number k and a squidgy number l,
l can be represented as $\frac{p}{q}$ where p and q are integers

$$(\frac{p}{q} - k) * 2 = \frac{p\prime}{q\prime}$$

where $p\prime$ and $q\prime$ are integers (by the assumption that the result is a squidgy number)

$$k = \frac{2pq\prime - p\prime q}{2q\prime q}$$

By arithmetic

we know that the product and the difference of integers is an integer.

2p$q\prime$ - $p\prime$q and 2$q\prime$q are integers

k can be represented in the form of $\frac{a}{b}$ where a and b are integers,
which contradicts with our premise that k is a non-squidgy number.
Our assumption that the result is a squidgy number must be false.
Hence, the result of multiplying the difference betweeen a squidgy and a non-squidgy number by 2 is a non-squidgy number
Q.E.D.

Question 8

Case 1: x is an odd integer
Case 2: x is an even integer