CITS2211 Assignment 1

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Question 1

a) $P \lor (Q \lor \neg P)$

P	Q	$\neg P$	$Q \lor \neg P$	$P \lor (Q \lor \neg P)$
T	T	F	T	T
T	F	F	F	T
F	T	T	T	T
F	F	T	T	T

As the truth table demonstrates, the proposition is a tautology since it's always true regardless of the truth values of Q or P

b) $(P \land \neg P) \lor \neg Q$

$P \land \neg P$ is a contradiction since both P and it's negation can't be true.
$\neg Q$ can either be true of false
hence, the proposition is a contingent

c) $Q \implies (P \land \neg Q)$

P	Q	$\neg Q$	$P \land \neg Q$	$Q \rightarrow (P \land \neg Q)$
T	T	F	F	F
T	F	T	T	T
F	T	F	F	F
F	F	T	F	T

As the truth table demonstrates, the truth value of the proposition depends on the truth values of P and Q. Hence, the proposition is a contingent.

Question 2

$P \lor \neg(P \lor \neg Q) \equiv P \lor \neg Q$

By De Morgan's laws: $P \lor \neg(P \lor \neg Q) \equiv P \lor (\neg P \land Q)$
By distributivity: $P \lor (\neg P \land Q) \equiv (P \lor \neg P) \land (P \lor Q)$
By excluded middle: $(P \lor \neg P) \land (P \lor Q) \equiv T \land (P \lor Q)$
By Absorption: $T \land (P \lor Q) \equiv P \lor Q$

Q.E.D.

Question 3

$\forall x\ \exists n\ (x \leq n \leq x+5 \land (\exists a\ \exists b\ (a \neq n) \land (a \neq 1) \land (b \neq n) \land (b \neq 1) \land (a \times b = n))$

Question 4

let $N(x, y)$ be "x is a neighbour of y"

a) Anna has no neighbours
$\neg(\exists x.N(x, a))$

b) Ben has two neighbours
$\exists x \exists y(N(x, b) \land N(y, b) \land x \neq y \land \forall z(N(z, b) \implies (z = x \lor z = y)))$

c) If somebody is a neighbour of Ben, Ben is also a neighbour of that person

$\forall \mathrm{x}(\mathrm{N}(\mathrm{x}, \mathrm{b}) \implies \mathrm{N}(\mathrm{b}, \mathrm{x}))$

d) Except for Anna, everyone is the neighbour of someone

$\forall \mathrm{x}(\mathrm{x} \neq \mathrm{a} \implies \exists \mathrm{y}(\mathrm{N}(\mathrm{y}, \mathrm{x})))$