CITS2211 Assignment 1

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Question 1

a) $\mathbf{P \lor (Q \lor \neg P)}$

P	Q	$\neg P$	$Q \lor \neg P$	$P \lor (Q \lor \neg P)$
T	T	F	T	T
T	F	F	F	T
F	T	T	T	T
F	F	T	T	T

As the truth table demonstrates, the proposition is a tautology since it's always true regardless of the truth values of Q or P

b) $\mathbf{(P \land \neg P) \lor \neg Q}$

P ∧ ¬P = F (by contradiction)
¬Q (By absorption)

the proposition depends on the truth value of Q hence it's a contingent

c) $\mathbf{Q \rightarrow (P \land \neg Q)}$

P	Q	$\neg Q$	$P \land \neg Q$	$Q \rightarrow (P \land \neg Q)$
T	T	F	F	F
T	F	T	T	T
F	T	F	F	F
F	F	T	F	T

As the truth table demonstrates, the truth value of the proposition depends on the truth values of P and Q. Hence, the proposition is a contingent.

Question 2

$\mathbf{P \lor \neg(P \lor \neg Q) \equiv P \lor \neg Q}$

1. P ∨ ¬ (P ∨ ¬ Q)	Premise
2. P ∨ (¬ P ∧ Q)	1, Demorgan's laws
3. (P ∨ ¬P) ∧ (P ∨ Q)	2, distributivity
4. T ∧ (P ∨ Q)	3, excluded middle
5. P ∨ Q	4, absorption

5. $P \lor Q$
Q.E.D.

Question 3

$\forall x\ \exists n\ (x \leq n \leq x{+}5 \land (\exists a\ \exists b\ (a \neq n) \land (a \neq 1) \land (b \neq n) \land (b \neq 1) \land (a \times b = n))$

Question 4

let N(x, y) be ”x is a neighbour of y”

a) Anna has no neighbours

$\neg(\exists x.N(x, a))$

b) Ben has two neighbours

$\exists x\, \exists y\, (N(x, b) \land N(y, b) \land x \neq y \land \forall z\, (N(z, b) \rightarrow (z = x \lor z = y)))$

c) If somebody is a neighbour of Ben, Ben is also a neighbour of that person

$\forall x\, (N(x, b) \rightarrow N(b, x))$

d) Except for Anna, everyone is the neighbour of someone

$\forall x\, (x \neq a \rightarrow \exists y(N(x, y)))$

Question 5

a) $\dfrac{\boldsymbol{P}}{\boldsymbol{P}}$

$P \equiv P$ by rule of identity
hence, the inference rule is sound

b) $\dfrac{\boldsymbol{P}}{\boldsymbol{P \leftrightarrow Q}}$

if Q if false, the axiom is true but the conclusion is false
Hence, the inference rule is unsound

c) $\dfrac{\boldsymbol{P \leftrightarrow Q}}{\boldsymbol{P}}$

if P and Q are false, the axiom is true but the conclusion is false
Hence, the inference rule is unsound

d) $\dfrac{\boldsymbol{P\ Q}}{\boldsymbol{P \lor Q}}$

From the premise P we can infer $P \lor Q$ by disjunction introduction rule
Hence, the inference rule is sound

e) $\dfrac{\boldsymbol{P \to Q \quad \neg\neg P}}{\boldsymbol{Q}}$

1. ¬¬P	Premise
2. P → Q	Premise
3. P	1, double negation
4. Q	3, 2, Demorgan's laws

Hence the rule is sound

Question 6

∀x.(¬ Q(x) ∨ P(x)) ∨ ∃ x(Q(x) ∨ (P(x) ∧ R(x))) → ∃x.R(x)

1. $\forall x(\neg Q(x) \wedge P(x))$	premise
2. $\exists x(Q(x) \vee (P(x) \wedge R(x)))$	premise
3. $Q(a) \vee (P(a) \wedge R(a))$	2, Exist elimination
4. $\neg Q(a) \wedge P(a)$	1, Forall elimination
5. $\neg Q(a)$	4, conjunction elimination
6. $\neg Q(a) \to (P(a) \wedge R(a))$	3, implication law
7. $P(a) \wedge R(a)$	5, 6, Modus ponens
8. R(a)	7, conjunction elimination
9. $\exists x.R(x)$	8, Exists introduction

Question 7

Assuming the difference beetween a squidgy and a non-squidgy number multiplied by 2 produces a squidgy number

we have a non-squidgy number k and a squidgy number l,
l can be represented as $\frac{p}{q}$ where p and q are integers

$$(\frac{p}{q} - k) * 2 = \frac{p\prime}{q\prime}$$

where $p\prime$ and $q\prime$ are integers (by the assumption that the result is a squidgy number)

$$k = \frac{2pq\prime - p\prime q}{2q\prime q}$$

By arithmetic

we know that the product and the difference of integers is an integer.

2p$q\prime$ - $p\prime$q and 2$q\prime$q are integers

Hence, k can be represented in the form of $\frac{a}{b}$ where a and b are integers,
which contradicts with our premise that k is a non-squidgy number.
Our assumption that the result is a squidgy number must be false.
Hence, the result of multiplying the difference betweeen a squidgy and a non-squidgy number by 2 is a non-squidgy number
Q.E.D.

Question 8

this is a bidirection proof so firstly, we prove that if x is odd then 5x-1 is even,
and then secondly, prove that if 5x-1 is even then x is odd.

first direction: if x is odd then 5x-1 is even

let x be an arbitary odd integer then x is of the form 2k-1 where k is an integer
x = 2k-1
5x = 10k - 5 (multiplying both sides by 5)
5x - 1 = 10k - 6 (substracting 1 from both sides)
=2(5k - 3) (factoring out 2)
=2n (where n = 5k-3)
which is an odd number.

therefore, for any odd integer x, 5x -1 is an even integer

second direction: if 5x-1 is an even integer then x is an odd integer

The contrapositive of the statement is -
if x is not an odd integer then 5x-1 is not an even integer
an integer not being odd implies that the integer is even
and similarly an integer not begin even implies that the integer is odd
Hence, we need to prove that if x is an even integer then 5x-1 is an odd integer

let x be an arbitary even integer then x is of the form 2n where n is an integer
x = 2n
5x = 10n (multiplying both sides by 5)

5x - 1 = 10n - 1 (substracting 1 from both sides)
= 2 (5n) - 1 (factoring out 2)
= 2k - 1 (where k is 5n)

Hence, 5x - 1 is of the form 2k - 1 which is an odd number
therefore, for any even integer x, 5x - 1 is an odd integer **Conclusion** therefore, we have proved that if if x is odd, then 5x-1 is even
and if 5x-1 is even then x is odd.
Hence, it follows that x is odd if and only if 5x-1 is even

Question 9

Skill issues

Question 10