TDT4171 Artificial Intelligence Methods
Assigment 1

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1.1a. The probability of more than 2 bananas are eaten is the same as the sum of the probabilities of 3, 4 and 5 bananas eaten.

$P(Bananas > 2) = P(Bananas = 3) + P(Bananas = 4) + P(Bananas = 5) = 0.28 + 0.10 + 0.17 = \underline{\underline{0.55}}$

1.1b. The probability of at most 4 bananas eaten is the same as the sum of the probabilities of 0, 1, 2, 3 and 4 bananas eaten, which is also the same as not 5 bananas eaten.

$P(Bananas <= 4) = 1 - P(Bananas = 5) = 1 - 0.17 = \underline{\underline{0.83}}$

1.1c. The probability of 4 or more bananas are eaten is the same as the sum of the probabilities of 4 and 5 bananas eaten.

$P(Bananas >= 4) = P(Bananas = 4) + P(Bananas = 5) = 0.17 + 0.10 = \underline{\underline{0.27}}$

1.2a. We start by denoting the batches on no rotten apples, 1 rotten apple and 2 rotten apples as respectively batches A, B and C, and picking 2 fine apples as 2F. We can now find $P(2F|A) = 1$, $P(2F|B) = \frac{19}{20} \cdot \frac{18}{19} = 0.9$ and $P(2F|C) = \frac{18}{20} \cdot \frac{17}{19} = 0.805$.
Now, we also need to know the probability to choose 2 fine apples, given that we do not know which batch we have selected from. $P(2F) = P(2F|A) \cdot P(A) + P(2F|B) \cdot P(B) + P(2F|C) \cdot P(C) = 0.6 + 0.9 \cdot 0.3 + 0.805 \cdot 0.1 = 0.9505$.

This means that the probability of the batch that is picked has no rotten apples is the same as the selected batch is A given that we pick 2 fine apples.
$P(A|2F) = \frac{P(A) \cdot P(2F|A)}{P(2F)} = \underline{\underline{0.6312}}$

1.2b. We use values from the previous task, but this time in regards to B.
$P(B|2F) = \frac{P(B) \cdot P(2F|B)}{P(2F)} = \underline{\underline{0.2841}}$

1.3b. We use values from the task a, but this time in regards to C.
$P(C|2F) = \frac{P(C) \cdot P(2F|C)}{P(2F)} = \underline{\underline{0.0847}}$

1.3a. We want to find the probability that a man over 50 with a negative test result (T) has the common cold (C).
We have that $P(C) = 0.07$, $P(T|C) = 0.1$ and $P(\overline{T}|\overline{C}) = 0.05$.
This means that $P(T) = P(T|C) \cdot P(C) + P(T|\overline{C}) \cdot P(\overline{C}) = 0.1 \cdot 0.07 + 0.95 \cdot 0.93 = 0.8905$.
We now have everything we need to apply Bayes' formula.

$P(C|T) = \frac{P(T|C) \cdot P(C)}{P(T)} = \frac{0.1 \cdot 0.07}{0.8905} \approx \underline{0.0078}$

This means that a man over the age of 50, who gets a negative result of the test has a 0.78% chance of having the common cold.

1.4. There is in total 9 non-defective sets and 3 defective. If one is to pick 2 defective sets and 3 non-defective, this can be done in $\binom{9}{3} \cdot \binom{3}{2} = 360$ different ways.
Now, if one were to choose 3 defective sets and 2 non-defective, this can be done in $\binom{3}{3} \cdot \binom{9}{2} = 36$ ways.
In total, this gives us 396 possible combinations which contains 2 or more defective sets.

2. See figure 1. One of the conditional independence properties are Household Income which is conditionally independent from Number of Children, given Working Parents.
Religion av household income could have been modelled differently, such that religion depends on household income, but I've decided to go with household income being indirectly dependent on religion through number of children and working parents, as I believe this to be stronger dependencies than the former.

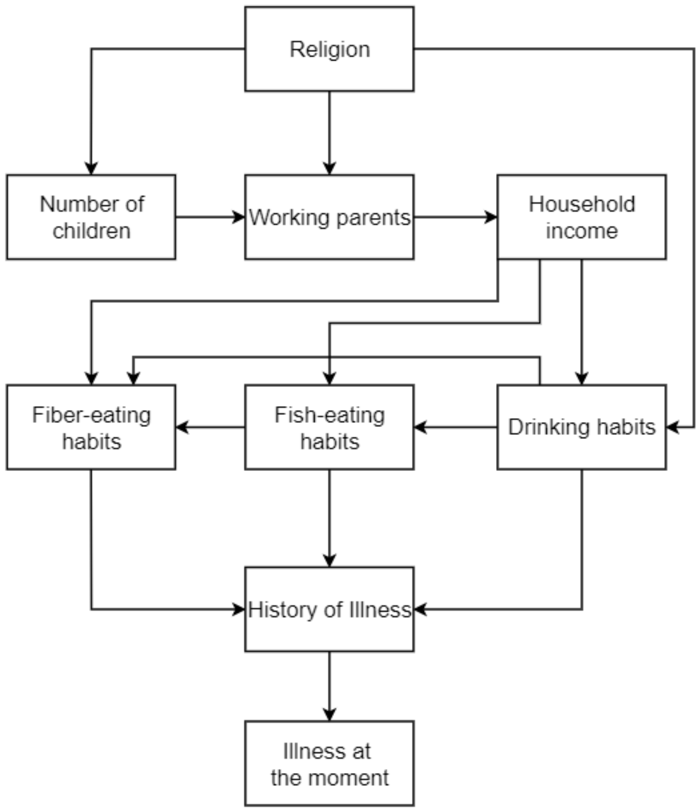

Figure 1: Bayesian network for task 2

3. See figures 2, 3, 4 and 5.
In figure 2 we can see the general probabilities given that MyChoice is A and no decision as to which door is opened is made.

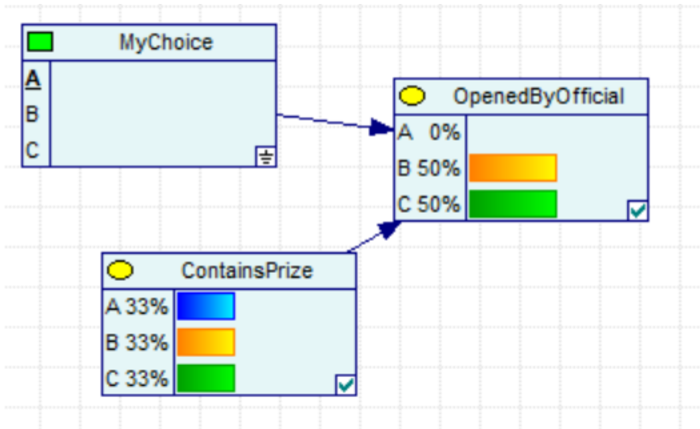

Figure 2: General probabilities when we have no knowledge other than our choice

In figure 3 we can see the probability of the host opening either door B or C given that we have chosen the door with the prize. At this point, one could be unsure whether the host was forced to open one of the doors or not.

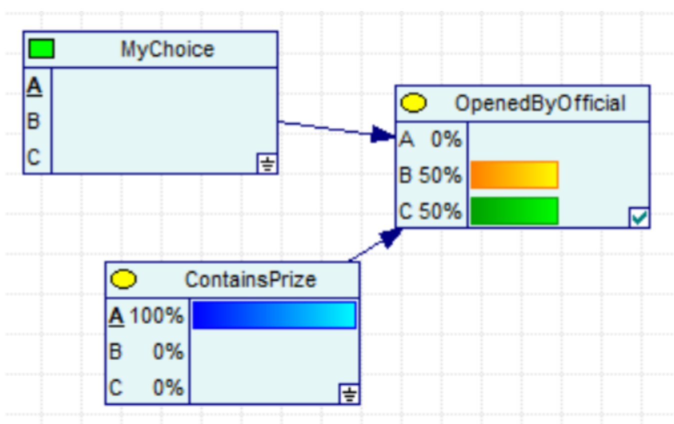

Figure 3: Probabilities of host opening door B or C given that MyChoice is A and ContainsPrize is A

If we change ContainsPrize to B as we can see in figure 4, we can see that the host is now forced to open door C, and if we were to know that the host was forced, we would also know with 100% certainty to switch doors.

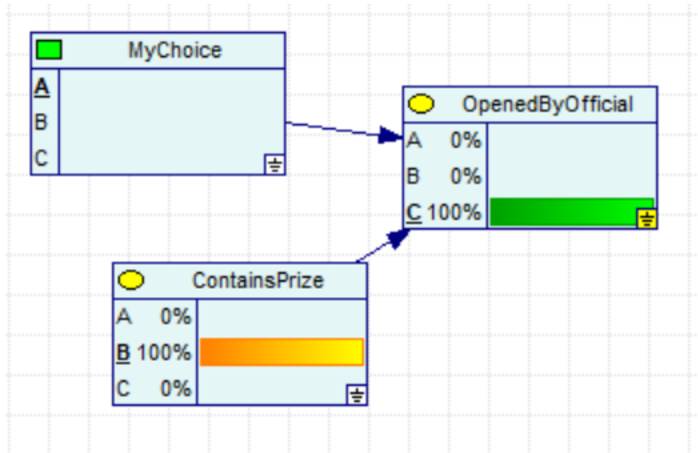

Figure 4: The host is now forced to open door C, as MyChoice and ContainsPrize are A and B

The last figure we will look at is how the probabilities look when the host chooses a door, and we don't know which door the prize is behind. This is shown in figure 5. Here we see that there is a 67% chance of the prize being behind door C and a 33% chance of the prize being behind A. This can be explained from the previous figures, as there are 2 ways that you can choose the wrong door on your first guess, and only 1 way to choose the right one. In the case of a wrong guess, the host is forced to open a door, and you will know to switch when this happens. In other words; one should always switch doors when given the chance.

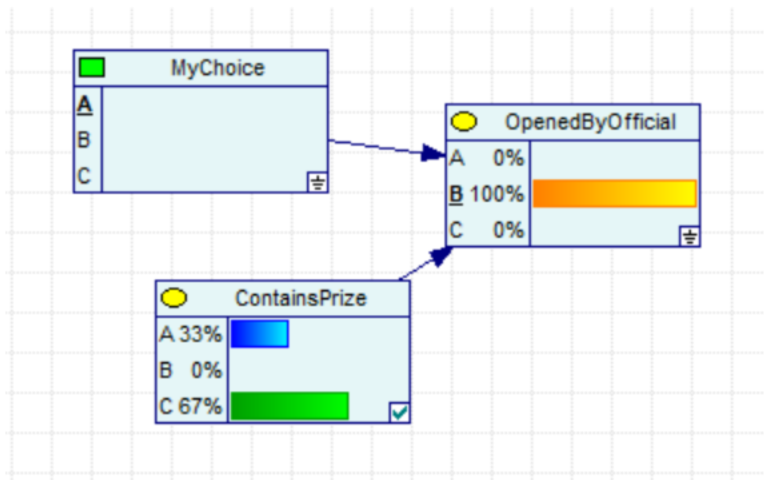

Figure 5: Probability of door containing the prize given MyChoice A and OpenedByOfficial B