TDT4171 Artificial Intelligence Methods
Assigment 2

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Part A

The Umbrella domain described in Artificial Intelligence: A Modern Approach has a hidden variable in whether it rains or not. The variable used to denote this in the book is R_t. In other words:

$$\mathbf{X}_t = \{R_t\}$$

This also means that there is one variable that is observable which is whether or not the director brings an umbrella. This is denoted as U_t in the book.

$$\mathbf{E}_t = \{U_t\}$$

The probabilities in the dynamic model has a 0.7 chance of the same weather persisting from one time slice to the next. By using one day as one time slice, we can use this to present the dynamic model as

$$\mathbf{T} = \mathbf{P}(\mathbf{X}_t|\mathbf{X}_{t-1}) = \begin{bmatrix} 0.7 & 0.3 \\ 0.3 & 0.7 \end{bmatrix}$$

The observation model in the Umbrella domain will then be the probability of whether or not an umbrella is brought, given whether it rains or not. In matrix form, this gives

$$\mathbf{O} = \mathbf{P}(\mathbf{E}_t|\mathbf{X}_t) = \begin{bmatrix} 0.9 & 0.1 \\ 0.8 & 0.2 \end{bmatrix}$$

When describing the umbrella domain we have to make a few assumptions. The first assumption is a sensor Markov assumption, that is an assumption that whether or not the director brings an umbrella is only dependent on the weather of the current time slice. This might not be very reasonable, as one could be inclined to believe that the director would adapt to consecutive days of the same weather and therefore more likely to bring an umbrella on the 5th day of rain for example.
Another assumption that is made is that this domain is a first order Markov

process. With the limited information about the state that we have, this is not that unreasonable to assume, but in a more realistic scenario this could lead to a lower accuracy.
Another important assumption is that the probability of consecutive rain days stays constant, and does not change with time. This is pretty unrealistic if one is to consider the seasons of a year, but in this model it will not make that much of a difference if one is to assume that the probability is the mean throughout a year.
All in all, in such a limited and simple model these assumptions might not make that much of a difference from a more complex dynamic model.

Part B

With the given observations as described in the task and an implementation that is verified to give a probability of rain on day 2 is 0.883, we can calculate the probability of rain on day 5 with the forward-method. The sequence of normalized forward messages then becomes:

$$\mathbf{f}_{1:0} = \begin{bmatrix} 0.500 \\ 0.500 \end{bmatrix}$$

$$\mathbf{f}_{1:1} \approx \begin{bmatrix} 0.818 \\ 0.182 \end{bmatrix}$$

$$\mathbf{f}_{1:2} \approx \begin{bmatrix} 0.883 \\ 0.117 \end{bmatrix}$$

$$\mathbf{f}_{1:3} \approx \begin{bmatrix} 0.191 \\ 0.809 \end{bmatrix}$$

$$\mathbf{f}_{1:4} \approx \begin{bmatrix} 0.731 \\ 0.269 \end{bmatrix}$$

$$\mathbf{f}_{1:5} \approx \begin{bmatrix} 0.867 \\ 0.133 \end{bmatrix}$$

In other words, we have a probability of rain on the 5th day of 0.867.

Part C

With the same evidence as in part B and a forward-backward method that is confirmed to give the desired result, we are able to use this to calculate the probability of rain at day 1. The backward messages during the execution is:

$$\mathbf{b}_{5:5} = \begin{bmatrix} 1.000 \\ 1.000 \end{bmatrix}$$

$$\mathbf{b}_{4:5} = \begin{bmatrix} 0.690 \\ 0.410 \end{bmatrix}$$

$$\mathbf{b}_{3:5} \approx \begin{bmatrix} 0.459 \\ 0.244 \end{bmatrix}$$

$$\mathbf{b}_{2:5} \approx \begin{bmatrix} 0.091 \\ 0.150 \end{bmatrix}$$

$$\mathbf{b}_{1:5} \approx \begin{bmatrix} 0.066 \\ 0.046 \end{bmatrix}$$

And finally, the smoothed probability estimate for day 1 is:

$$\mathbf{P}(\mathbf{X}_1 | \mathbf{e}_{1:5}) \approx \begin{bmatrix} 0.867 \\ 0.133 \end{bmatrix}$$