

## Exponents 2

Let's calculate  $2^4$ , then  $2^3$ , then  $2^2$ , and so on.

$2^4 = 2 \times 2 \times 2 \times 2$ , and $2 \times 2 = 4$ , so $= 4 \times 2 \times 2$ , and $4 \times 2 = 8$ , so $= 8 \times 2$ , and $8 \times 2 = 16$ , so $= 16$	$2^3 = 2 \times 2 \times 2$ , $2 \times 2 = 4$ , so $= 4 \times 2$ , and $4 \times 2 = 8$ , so $= 8$	$2^2 = 2 \times 2$ , $2 \times 2 = 4$ , so $= 4$
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The difference between each of these numbers is a single step, a single  $\times 2$ . To raise the exponent by 1, we must multiply by two -- so to **lower** the exponent by 1, we must divide by two.

$2^4 = 16$ $\uparrow$	$2^3 = 2^4 \div 2$ $2^3 = 16 \div 2$ $2^3 = 8$	$2^2 = 2^3 \div 2$ $2^2 = 8 \div 2$ $2^2 = 4$
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This method is giving us the same answers as we were getting before. Let's keep going.

$2^1 = 2^2 \div 2$ $2^1 = 4 \div 2$ $2^1 = 2$	$2^0 = 2^1 \div 2$ $2^0 = 2 \div 2$ $2^0 = 1$	$2^{-1} = 2^0 \div 2$ $2^{-1} = 1 \div 2$ $2^{-1} = \frac{1}{2}$	$2^{-2} = 2^{-1} \div 2$ $2^{-2} = \frac{1}{2} \div 2$ $2^{-2} = \frac{1}{2 \times 2}$ $2^{-2} = \frac{1}{2^2}$	$2^{-3} = 2^{-2} \div 2$ $2^{-3} = \frac{1}{2 \times 2} \div 2$ $2^{-3} = \frac{1}{2 \times 2 \times 2}$ $2^{-3} = \frac{1}{2^3}$
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The same pattern works for any number except zero. This is why a number to the first power ( $x^1$ ) is always equal to that number. This is why a number to the zeroth power ( $x^0$ ) is always equal to one. Any number to a *negative power* is equal to one divided by the positive version of that power. To say that in a mathy way, we could write that, for any number "a" (except zero), and any number b:

$a^1 = a$	$a^0 = 1$	$a^b = a^{b-1} \times a$	$a^b \div a = a^{b-1}$	$a^{-b} = \frac{1}{a^b}$
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You can raise negative numbers to powers too.

$(-2)^2 = (-2) \times (-2)$ $(-2)^2 = +4$  $+(2^2) = +4$	$(-2)^3 = (-2) \times (-2) \times (-2)$ $(-2)^3 = +4 \times (-2)$ $(-2)^3 = -8$  $-(2^3) = -8$	$(-2)^4 = (-2) \times (-2) \times (-2) \times (-2)$ $(-2)^4 = +4 \times (-2) \times (-2)$ $(-2)^4 = -8 \times (-2)$ $(-2)^4 = +16$ $+(2^4) = +16$
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Any time the exponent is **even**, your result will be positive because every negative sign will be able to cancel itself out against another negative sign. Any time the exponent is **odd**, your result will be negative, because there will be one negative sign left over.