Exponents 1,

<u>What is an exponent?</u> Let's start by talking about other ***<u>mathematical operators</u>*** first.

The first thing we learn to do with numbers is learn to *<u>count</u>*, one number at a time -- 1 then 2 then 3 then 4... With plain counting, given any number, you can find which number is *next*. The number after 99 is 100. ***<u>Counting upwards</u> is where mathematics starts.***

Next comes *<u>addition</u>*. If I want to add 1 to a number, I count upwards once. If I want to add two, I count upwards twice. If I want to start with 5 and add 10, I could count upwards 10 times (*5*, 6, 7, 8, 9, 10, 11, 12, 13, 14, *15*) -- but I could also use *addition* and realise that 10+5 = *15*. If I want to count up by 100, it is ***much faster*** to trust addition than to count upwards by one 100 times.
<u>Addition</u> is a shortcut that we use to <u>count upwards</u> multiple times in one step.

Next comes *<u>multiplication</u>*. Let's say I have a lonely 2. If I add another one, I have 2+2 = 4. Another, 2+2+2 = 6. Another, 2+2+2+2 = 8. Put another way, if I add a stack of *two 2's* together, their total is 4. If I add a stack of *three 2's*, I get 6. If there are *four 2's* added together, they equal 8. If I add 2 to itself a total of *four times*, I get 8. 2 x 4 = 8. 2 x 3 = 6. 2 x 2 = 4. 2 x 1 = 2.
<u>Multiplication</u> is a shortcut that we use to <u>add a number to itself</u> multiple times at once.

Next come the *<u>exponents</u>*. Let's say I have a lonely 3. If I multiply it by another three, I have 3x3=9. Another, so there are *three 3's*, makes for 3x3x3 = 27. Another makes *four 3's*, which is 3x3x3x3=81. What if there was an easier, faster way to write it when multiplying stacks of the same number like this? What if we wrote the base number, the one that keeps being multiplied by itself, and then just wrote how many of there were, much smaller, floating off above its right side? So that $3^4 = 81$. $3^3 = 27$. $3^2 = 9$. $3^1 = 3$. We call the small floating number the power, and describe this as raising a number to the (something)th power.
<u>Exponents</u> are a shortcut we use to <u>multiply a number by itself</u> multiple times at once.

Here's a list of examples.

$1^0 = 1$	$2^0 = 1$	$3^0 = 1$	$4^0 = 1$	$5^0 = 1$
$1^1 = 1$	$2^1 = 2$	$3^1 = 3$	$4^1 = 4$	$5^1 = 5$
$1^2 = 1x1 = 1$	$2^2 = 2x2 = 4$	$3^2 = 3x3 = 9$	$4^2 = 4x4 = 16$	$5^2 = 5x5 = 25$
$1^3 = 1x1x1 = 1$	$2^3 = 2x2x2 = 8$	$3^3 = 3x3x3 = 27$	$4^3 = 4x4x4 = 64$	$5^3 = 5x5x5 = 125$
$1^4 = 1x1x1x1 = 1$	$2^4 = 2x2x2x2 = 16$	$3^4 = 3x3x3x3 = 81$	$4^4 = 4x4x4x4 = 256$	$5^4 = 5x5x5x5 = 625$

Often, people refer to raising a number to the second power as ***squaring*** that number, or say that that number has been ***squared***. This is because a common way to think about multiplication involves drawing a rectangle on a piece of paper. If you draw a rectangle that is two boxes tall and 3 boxes wide, then count the number of boxes in the rectangle, you will get six boxes total -- which can also be written as 2x3=6.

When you raise a number to the second power, also known as multiplying it by itself once, both sides of the box drawing will be the same length. A 2x2 or 3x3 or 4x4 box forms a perfect ***Square***. For similar reasons, raising things to the third power is often called ***cubing*** them. Two cubed is eight, or $2^3 = 2x2x2 = 8$.

Exponents 1,

<u>What is a root?</u>

Every one of these operations has an opposite. We can count forwards (1, 2, 3, 4...), but we can also count backwards (4, 3, 2, 1...). If you count forwards by one, then backwards by one, you end up where you started -- so, these two operations ***undo*** eachother. Knowing how to undo something can be very useful.

We can add 3+2=5, but what if we have that answer and want to get back to three? 5-2=3, so ***subtracting*** two is the opposite of ***adding*** two, and ***<u>Addition and Subtraction are opposites.</u>***

Using similar logic, 2x3=6, and 6 ÷ 3=2, so ***<u>Multiplication and Division are opposites.</u>***

<u>So what is the opposite of an exponent?</u>

We can write that $3^2 = 9$, and that $\sqrt{9} = 3$. Also, $4^2 = 4x4 = 16$ and $\sqrt{16} = 4$. A **<u>square root</u>** is the answer to the question "What number, if we multiply two copies of it together, equals this number?" A square root is the second root, and it is the opposite of raising a number to the second power. A square root is the opposite of squaring a number. If you know the area of a square, the square root of that area will be the length of one of the sides.

Likewise, $2^3 = 8$ and $\sqrt[3]{8} = 2$. A **<u>cube root</u>** is the answer to the question "What number, if we multiply three copies of it together, equals this number?" It is the third root, and it is the opposite of raising a number to the third power. The cube root is the opposite of cubing a number. If you know the volume of a cube, the cube root of that volume will be the length of one of the sides.

In general, the opposite of raising a number to the nth power is taking the nth root. Outside of college level math, you will deal almost exclusively with square roots.

<u>Exponents and roots are opposites.</u>

ANSWER KEY to Questions On Next Page

SECTION 1: 1C, 2A, 3C, 4E, 5B, 6E, 7D, 8E, 9B, 10A
SECTION 2: 1A, 2A, 3B, 4C, 5C.
SECTION 3:

1: 16 square inches
2: 4 square inches
3: 2.25 square inches
4: 5 inches
5: 10 inches

6: 24 square inches. The area of a square is equal to its side squared -- so 8^2 = 8 x 8 = 64. If we cut away 64 out of 88, then 88 - 64 = 24.
7a: 108 square inches total. $9 x 12 = 108$!

7b: 3 square inches per brownie. The total area is 108 inches, if we divide that into 36 pieces, $108 \div 36 = 3$

7c: 99 square inches left. We know there are three square inches per brownie, and someone took three of those. 3 x 3 = 9. The total, full pan of brownies was 108 inches before any were taken -- so, 108 - 9 = 99 square inches of brownies left over.

Exponents 1,

SECTION 1: Please circle the biggest number in each group.

1. A: $3 x 10^{3}$ B: $4 x 10^{3}$ C: 10^{5} D: $9 x 10^{2}$ E: $5 x 10^{1}$
2. A: $5 x 10^{5}$ B: $1 x 10^{4}$ C: $1 x 10^{2}$ D: $1 x 10^{3}$ E: $3 x 10^{-2}$
3. A: $1 x 10^{-1}$ B: $1 x 10^{-2}$ C: $5 x 10^{-1}$ D: $5 x 10^{-3}$ E: $2 x 10^{-2}$ (tricky)
4. A: $6 x 10^{2}$ B: $2 x 10^{2}$ C: $5 x 10^{2}$ D: $3 x 10^{2}$ E: $70 x 10^{1}$ (tricky)
5. A: 5^{2} B: 10^{2} C: 2^{2} D: 8^{2} E: 2^{3}
6. A: 2^{2} B: 2^{3} C: 3^{2} D: 3^{3} E: 4^{2}
7. A: $\sqrt{9}$ B: $\sqrt{36}$ C: $\sqrt{16}$ D: $\sqrt{49}$ E: $\sqrt{25}$
8. A: 2^{2} B: 4 C: $3 x 10^{-1}$ D: $\sqrt{16}$ E: $A,\ B,\ and\ D$
9. A: $1 \div 10$ B: $2 x 10^{-1}$ C: 10^{-1} D: 10^{-2} E: $1 \div 10^{2}$ (tricky)
10. A: 5^{3} B: 11^{2} C: $11^{2} - 22$ D: $10^{2} + 24$ E: 10^{2} (tricky)

SECTION 2: Which answer is equal to the number given at the left?

1. 5^{3} A: 125 B: 15 C: 53
2. $2 x 10^{6}$ A: 2 million B: 1 million C: 2 hundred thousand
3. $\sqrt{16}$ A: 6 B: 4 C: 8
4. $\$2 x 10^{-2}$ A: $0.25 B: $-20.00 C: $0.02
5. $\sqrt{8^{2}}$ A: 82 B: 16 C: 8 (tricky)

SECTION 3: Geometry, Not Multiple Choice.

A square is a shape with four equal sides. Its area is equal to the length of its side, squared. A rectangle is similar, but it has a long side and a short side. Its area is equal to the long side times the short side. In formulas this would look like: $A = (side)^{2}$ *and* $A = (Long)\ x\ (Short)$.

1. What is the area of a square with sides four inches long?
2. What is the area of a square with sides 2 inches long?
3. What is the area of a square with sides 1.5 inches long?
4. How long is the side of a square with an area of 25 square inches?
5. How long is the side of a square with an area of 100 square inches?

6 (tricky): An 8 x 11 inch piece of paper has an area of 8 x 11 = 88 square inches. If you cut an 8x8 inch square out of that paper (subtracting its area from the whole sheet), what is the area of the thin strip that is left?

7 (tricky): Someone bakes a 9x12 inch pan of brownies.
A: What is the total area of the brownies in the pan, in square inches?
B: If they cut the pan into 36 brownies, how many square inches is each brownie?
C: If someone takes three brownies, how many square inches of brownie is left?