Image Segmentation 2

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Contents

Thresholding

- Edge detection:

First find edge segments, then link them

- Thresholding:

Partition the image directly into regions based on intensity values or properties

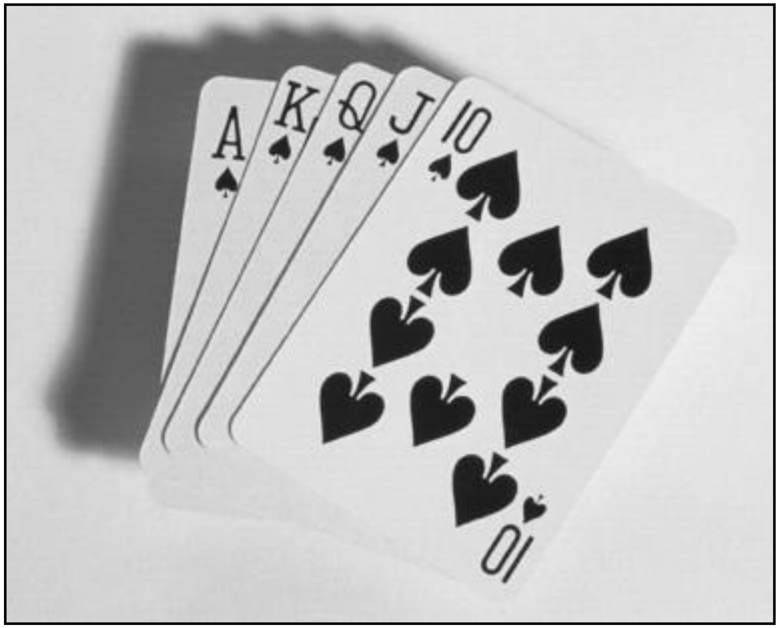
- Global thresholding

$$g(x,y)=\begin{cases}1 & if \;\; f(x,y)>T \\ 0 & if \;\; f(x,y)\le T\end{cases}$$

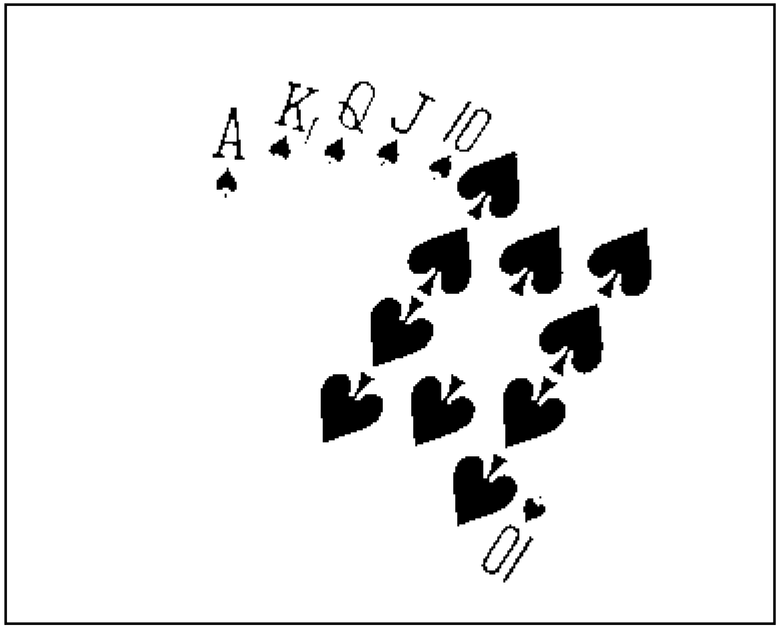
→ Local / regional / variable thresholding

Thresholding Example

- Imagine a poker playing robot that needs to visually interpret the cards in its hand



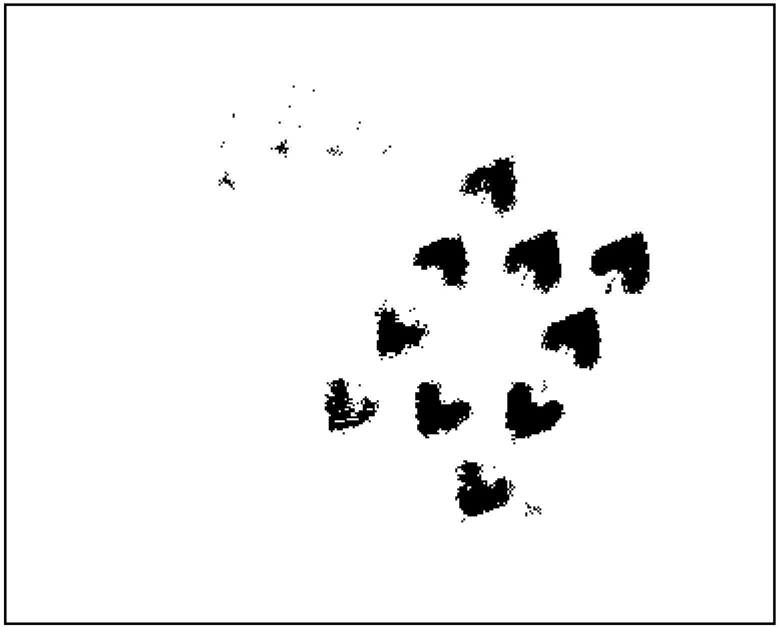
Original Image



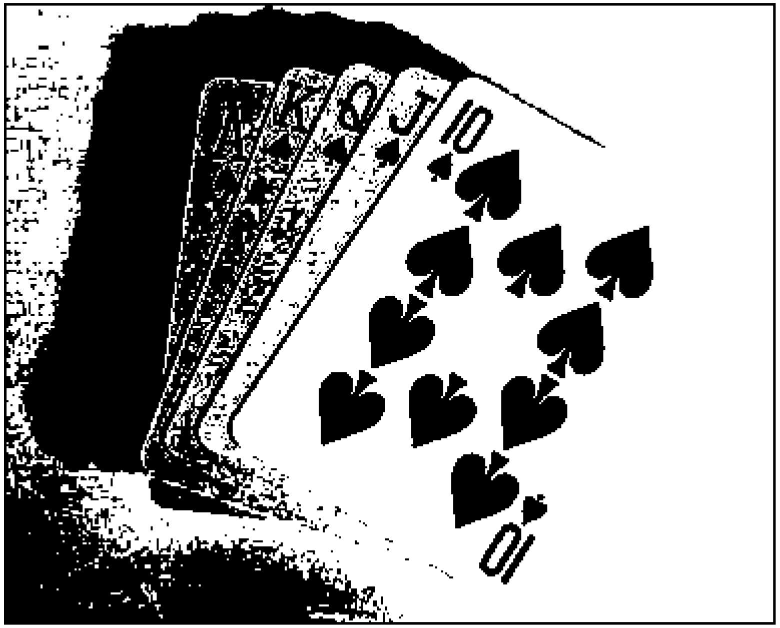
Thresholded Image

But Be Careful

- If you get the threshold wrong, the results can be disastrous



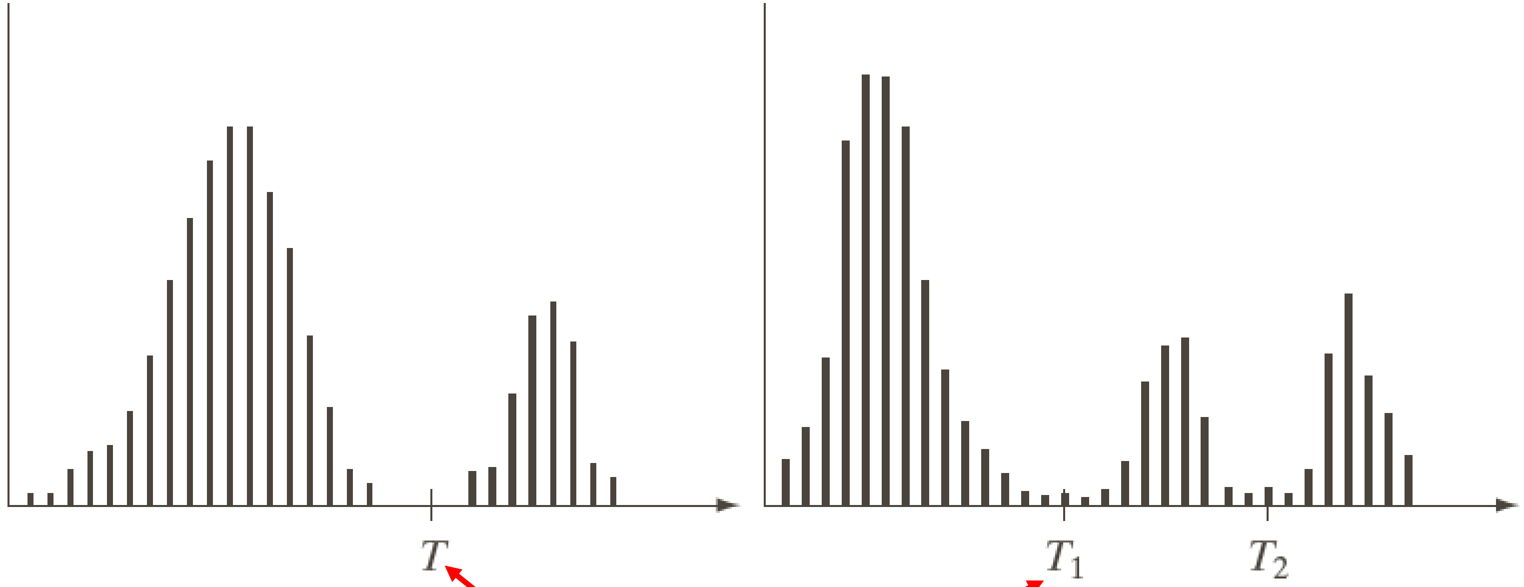
Threshold Too Low



Threshold Too High

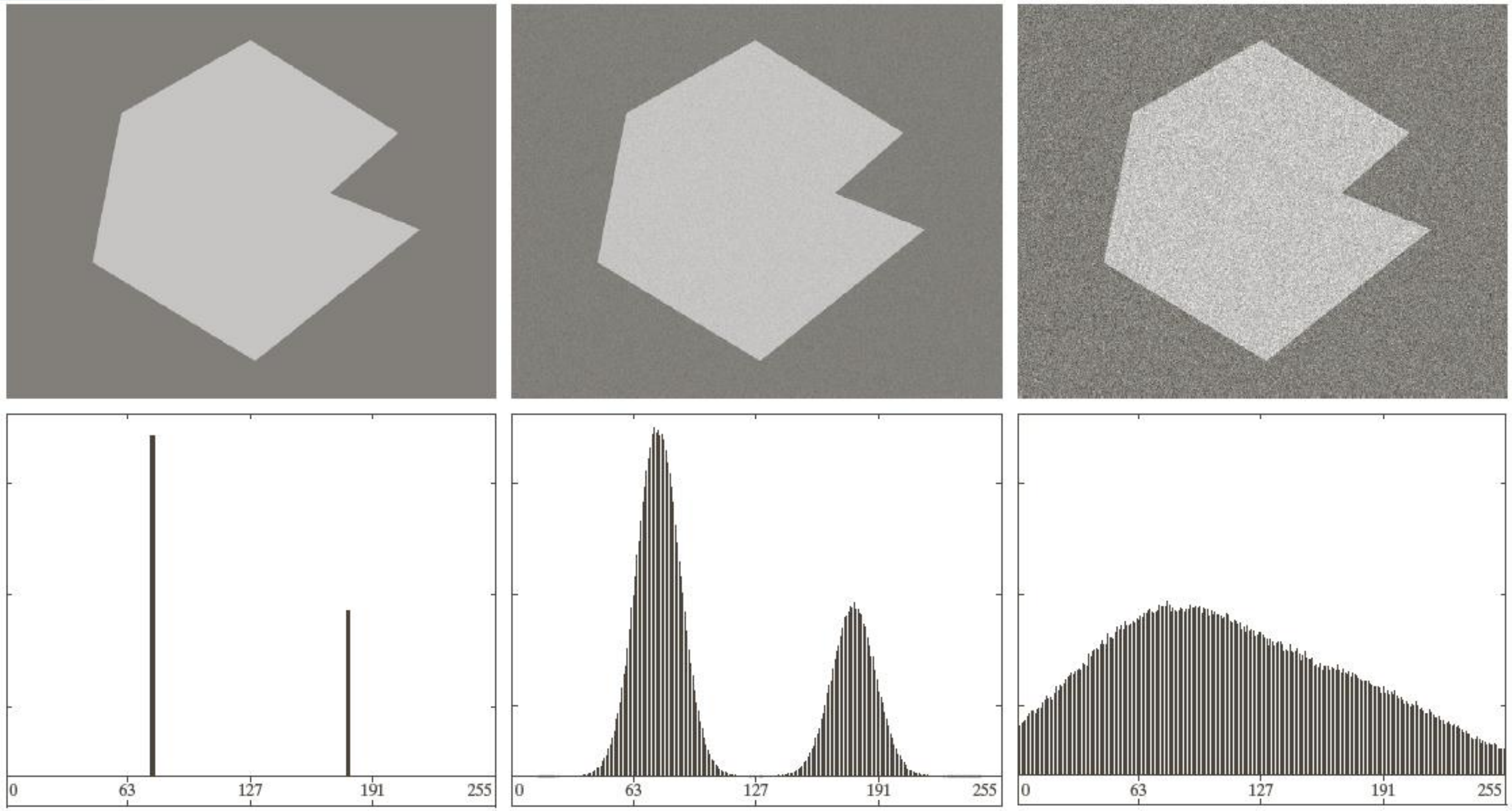
Basic Global Thresholding

Based on the histogram of an image



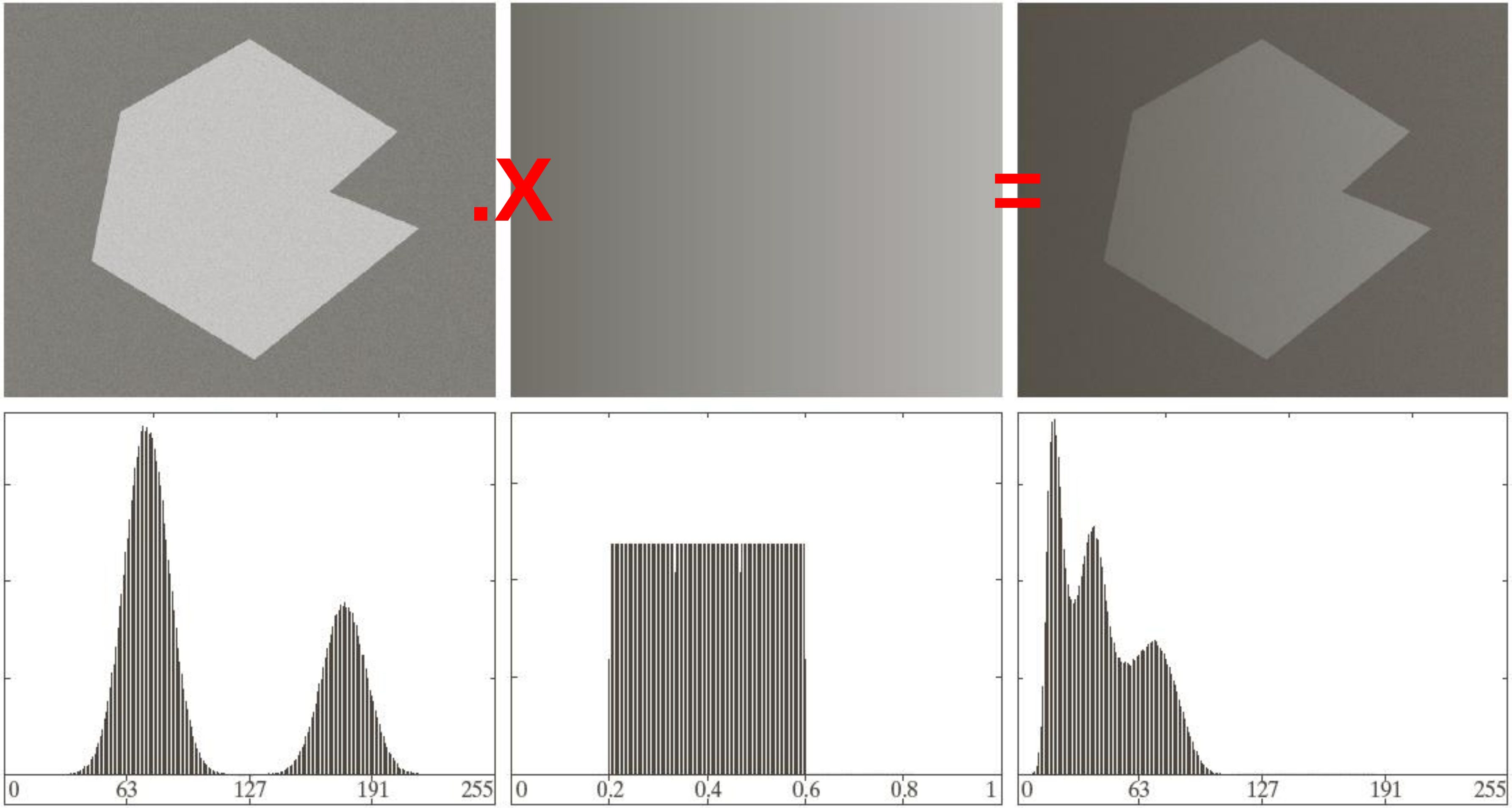
Thresholds

The Role of Noise in Thresholding



Noiseless + N(0, 10) + N(0, 50)

The Role of Illumination & Reflectance



Noisy image Intensity ramp Product

Basic Global Thresholding Algorithm

- The basic global threshold, T, is calculated as follows:
  1. Select an initial estimate for T (typically the average grey level in the image)
  2. Segment the image using T to produce two groups of pixels: G_1 consisting of pixels with grey levels >T and G_2 consisting pixels with grey levels ≤ T
  3. Compute the average grey levels of pixels in G_1 to give μ_1 and G_2 to give μ_2

Basic Global Thresholding Algorithm

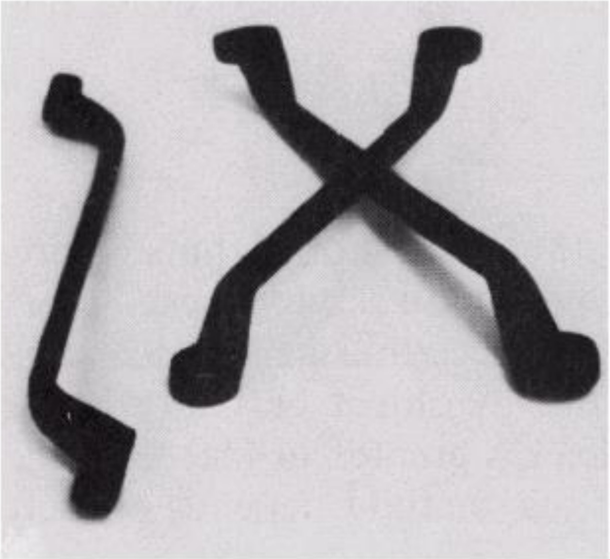
4. Compute a new threshold value:

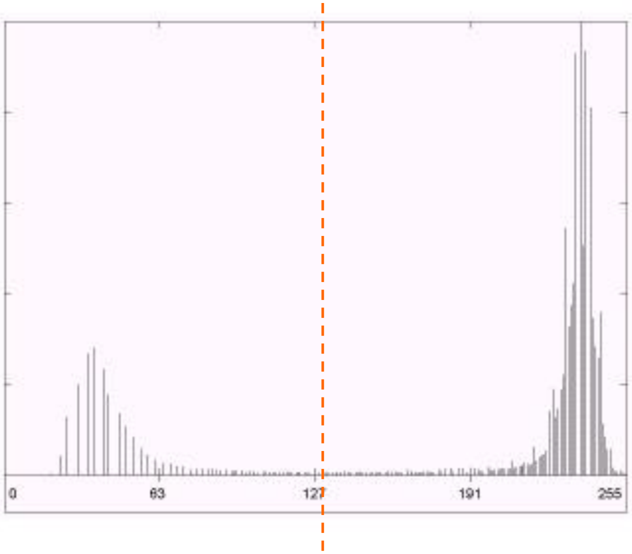
$$T = \frac{\mu_1 + \mu_2}{2}$$

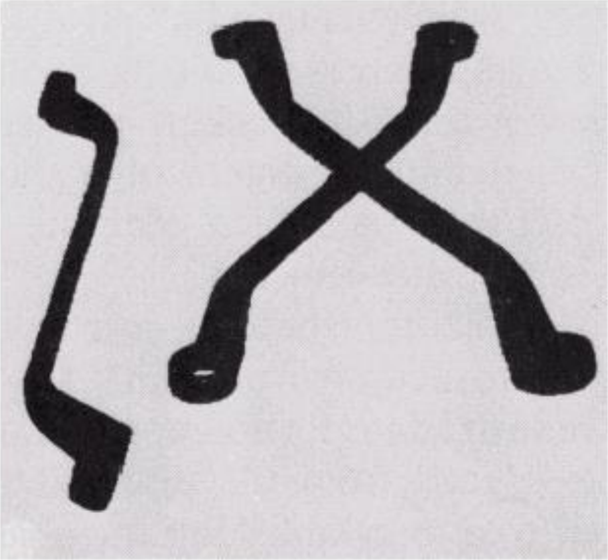
5. Repeat steps 2 – 4 until the difference in T in successive iterations is less than a predefined limit

- This algorithm works very well for finding thresholds when the histogram is suitable

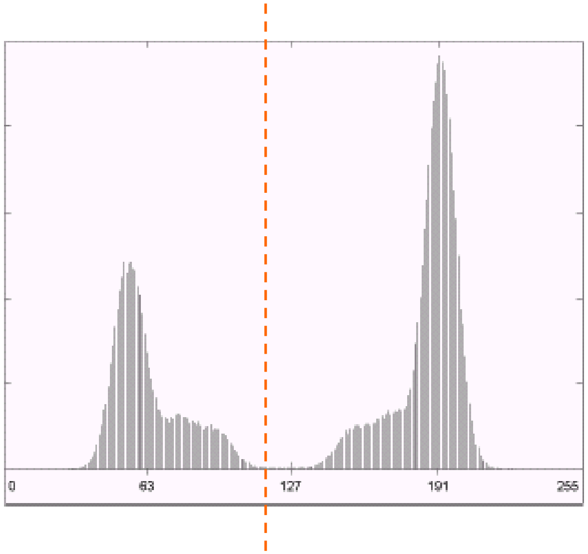
Thresholding Example 1







Thresholding Example 2



Optimum Global Thresholding Using Otsu's Method

- Idea: Max $\eta = \dfrac{\sigma_B^2}{\sigma_G^2}$
- Global variance $\sigma_G^2 = \sum_{i=0}^{L-1} (i - m_G)^2 p_i$
- Between-class variance

$$\sigma_B^2 = P_1(m_1 - m_G)^2 + P_2(m_2 - m_G)^2 = P_1 P_2 (m_1 - m_2)^2$$

$$= \frac{(m_G P_1 - m)^2}{P_1(1 - P_1)}$$

- Threshold = *k* $p_i = n_i / MN$

$$m_G = \sum_{i=0}^{L-1} i p_i \qquad P_1(k) = \sum_{i=0}^{k} p_i \qquad m(k) = \sum_{i=0}^{k} i p_i$$

Optimum Global Thresholding Using Otsu's Method

- 日本学者大津（Nobuyuki Otsu）
- Idea: Max

$$\eta(k) = \frac{\sigma_B^2(k)}{\sigma_G^2}$$

$$\sigma_B^2(k) = \frac{\left[m_G P_1(k) - m(k)\right]^2}{P_1(k)\left[1 - P_1(k)\right]}$$

- Threshold = *k**

$$\sigma_B^2(k^*) = \max_{0 \le k \le L-1} \sigma_B^2(k)$$

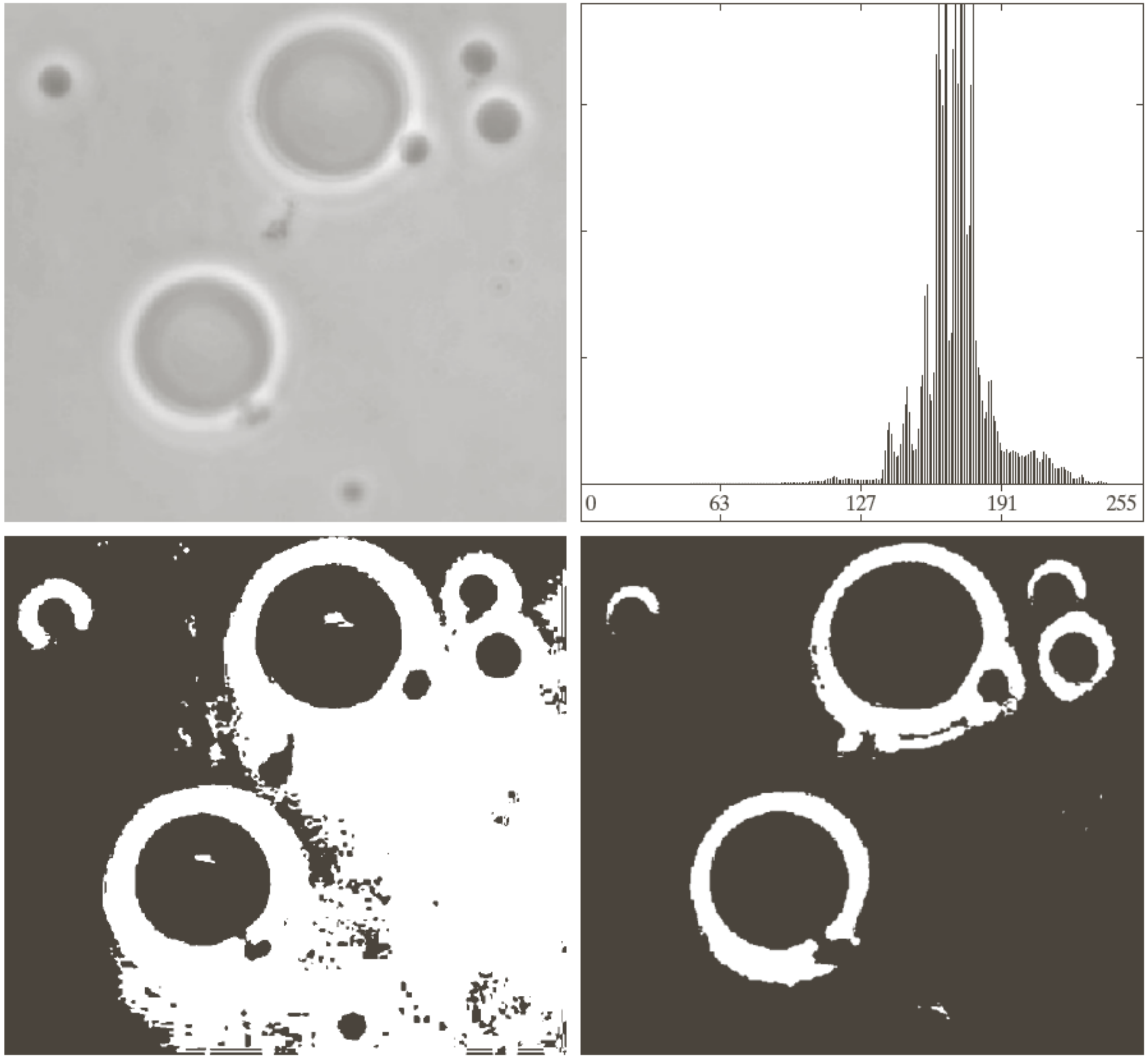
- Segmentation

$$g(x, y) = \begin{cases} 1 & \text{if } f(x, y) > k^* \\ 0 & \text{if } f(x, y) \le k^* \end{cases}$$

Optimum Global Thresholding Using Otsu's Method

Example of Otsu's Method

Original image



Histogram

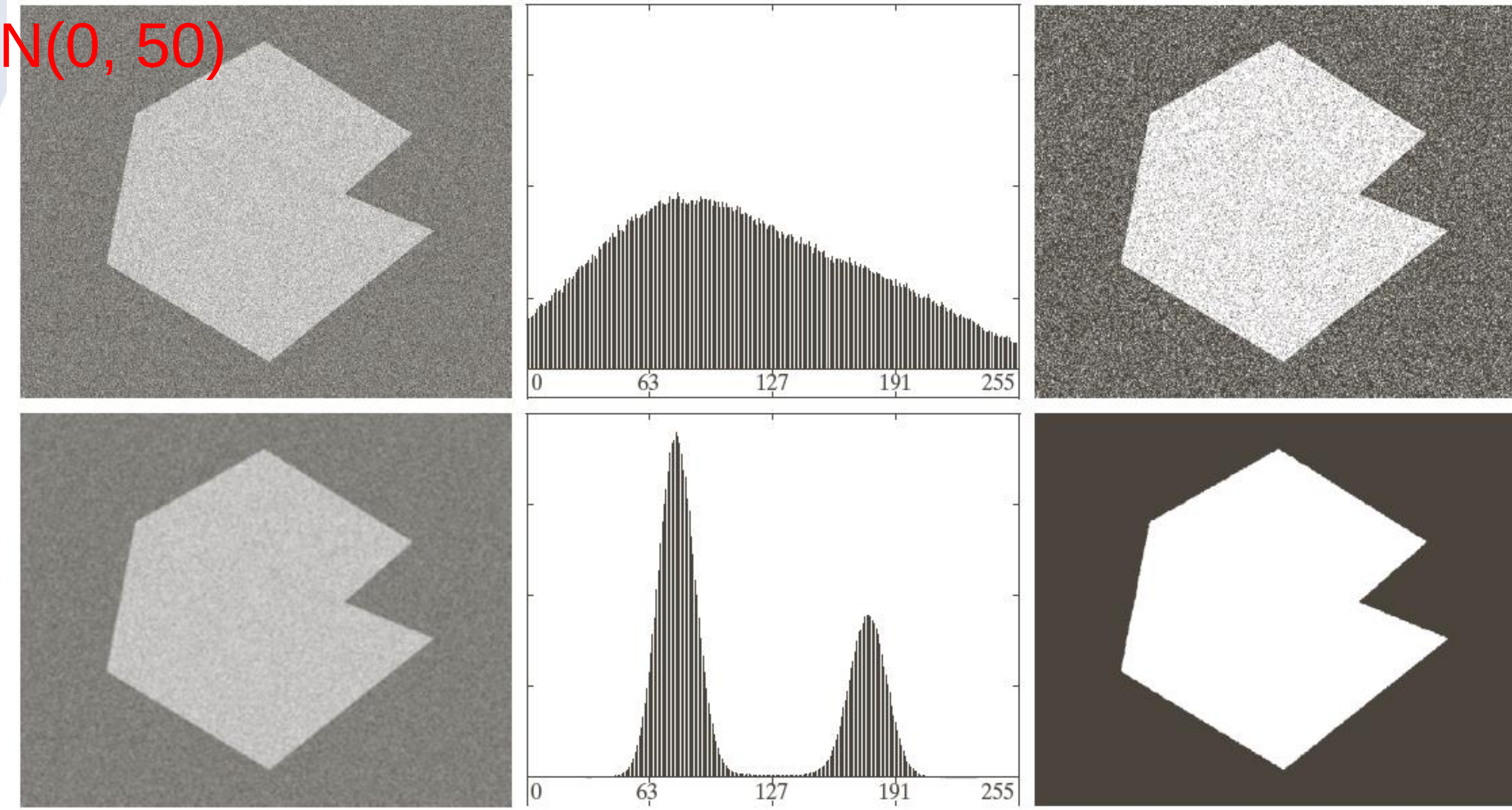
Basic: T=169

Otsu: T=181

Using Image Smoothing to Improve Global Thresholding

Noisy image | Histogram | Otsu's result

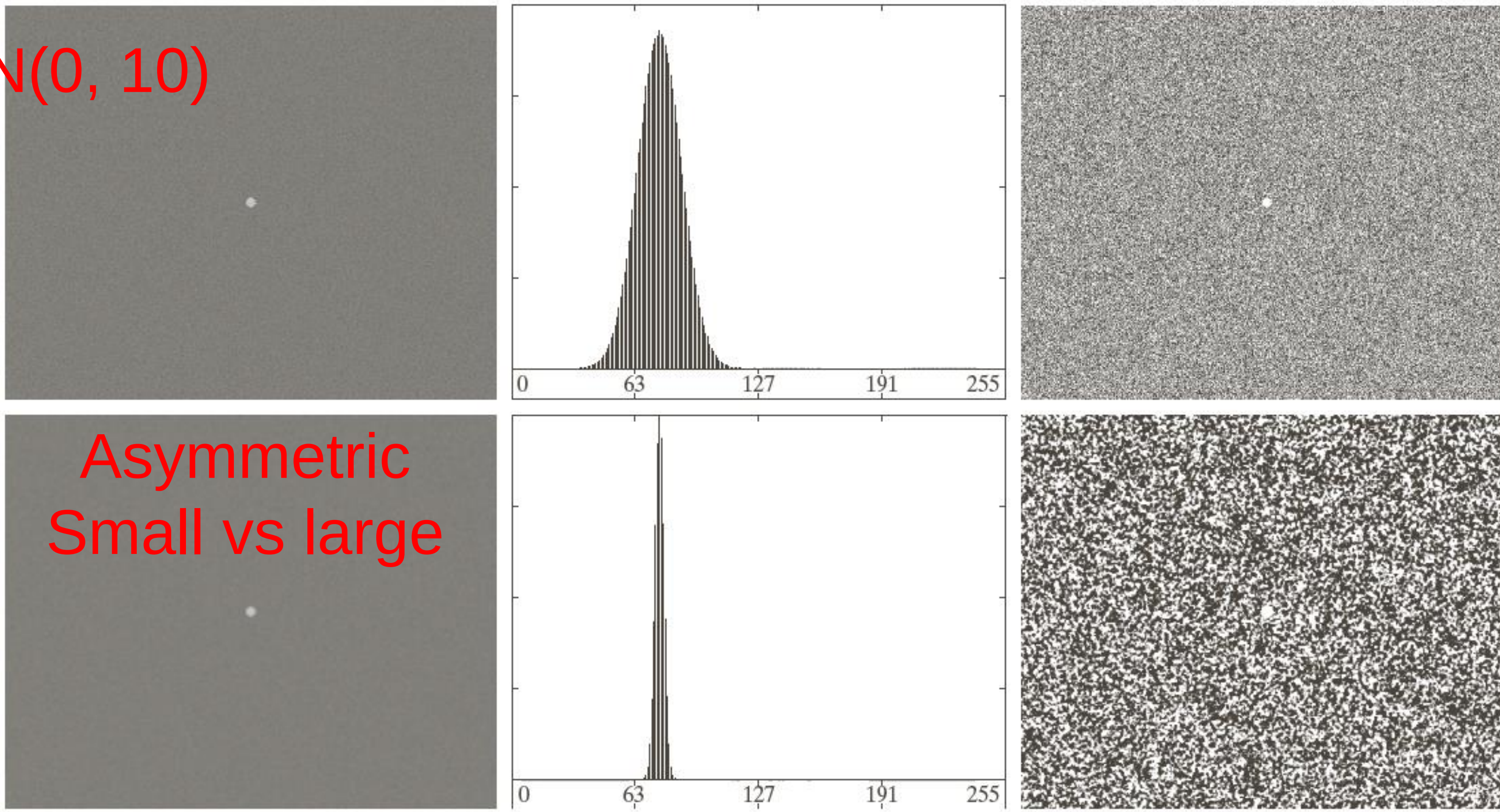
+ N(0, 50)



Smoothed with 5x5 averaging mask

Failed for Small Regions

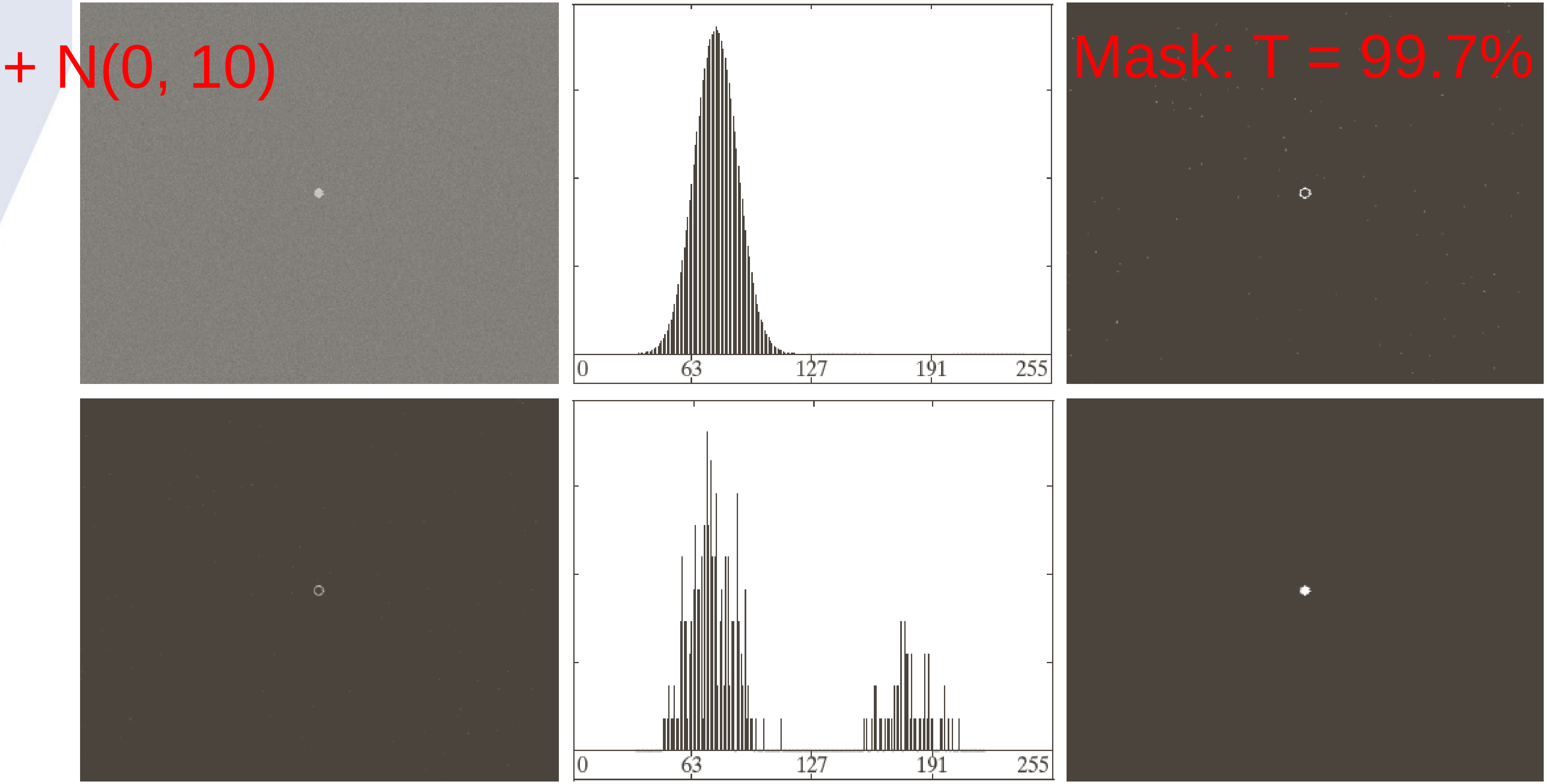
Noisy image | Histogram | Otsu's result



Smoothed with 5x5 averaging mask

Using Edges to Improve Global Thresholding

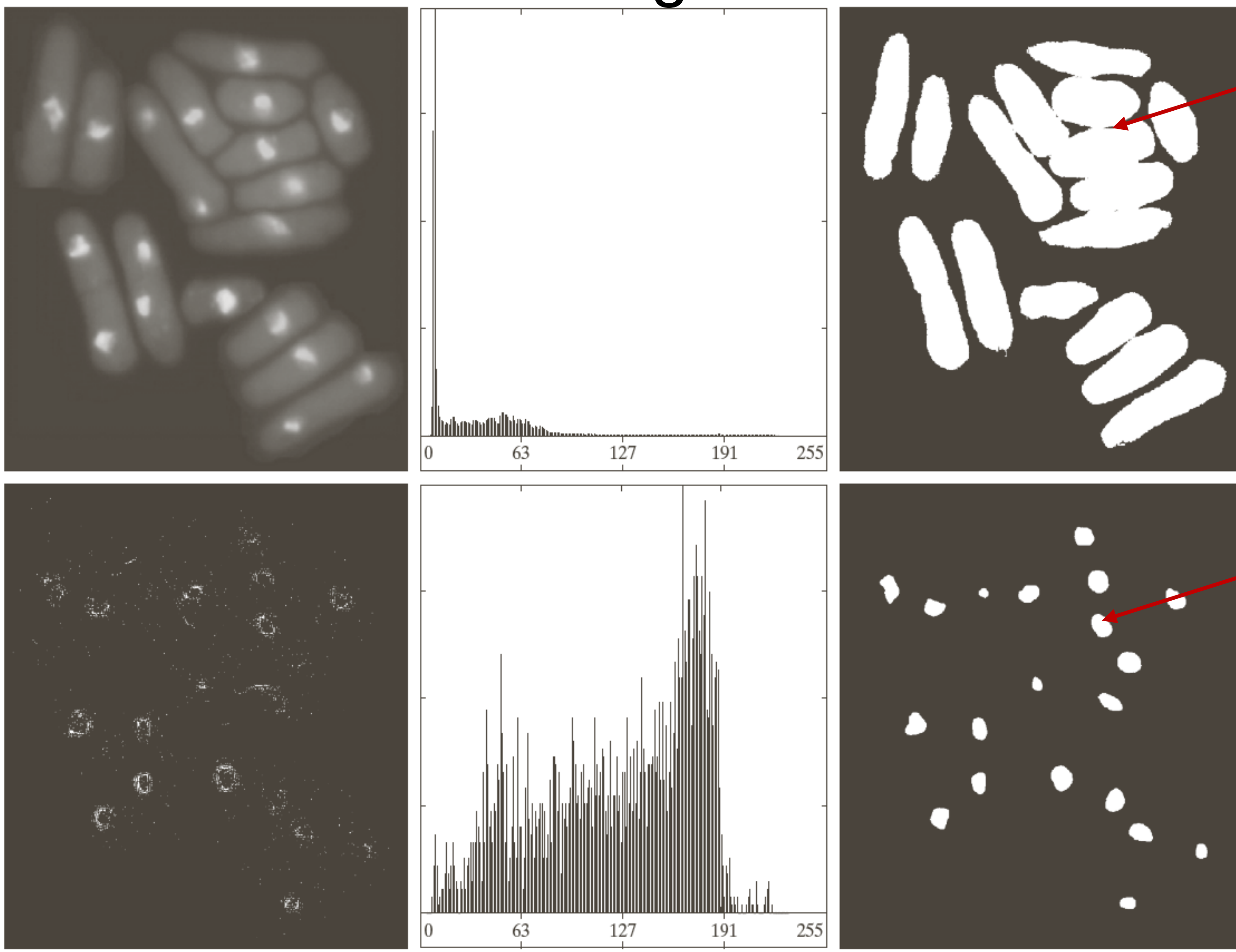
Noisy image Histogram Gradient magnitude



Masked image Histogram Otsu’s result

Another Example

Original image Histogram



NOT separated

Mask

Thresholded (99.5%) absolute Laplacian Otsu's result

Another Example

- Use a lower value to threshold the absolute Laplacian image

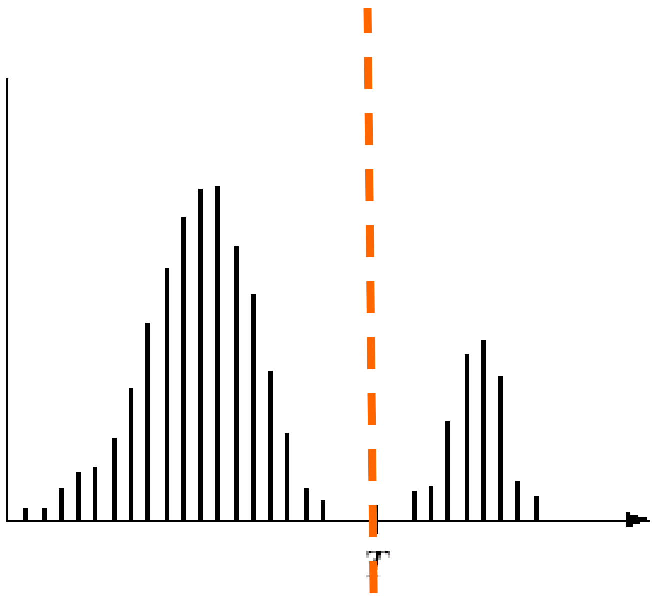
5% max ~ 53.9% percentile

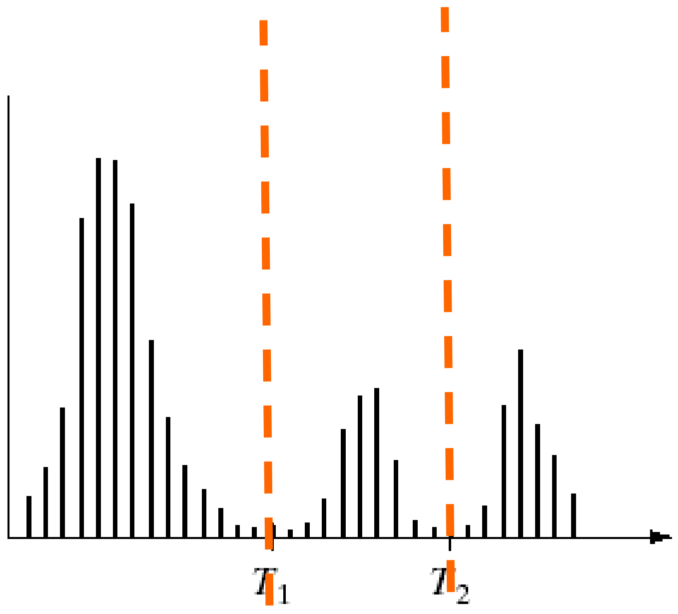
→ enlarge the mask

separated

Problems With Single Value Thresholding

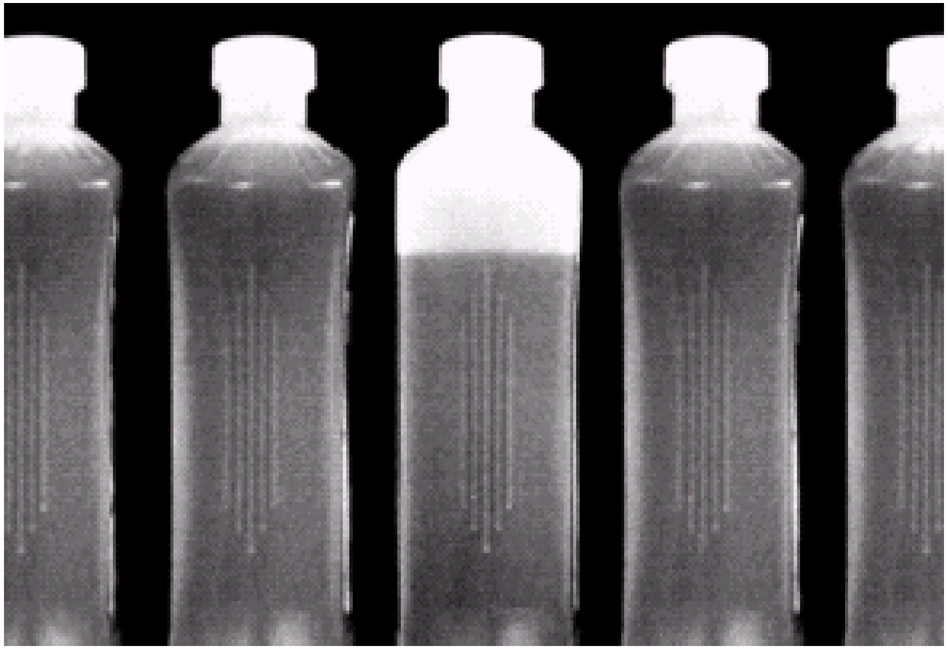
- Single value thresholding only works for bimodal histograms
- Images with other kinds of histograms need more than a single threshold





Problems With Single Value Thresholding

Let's say we want to isolate the contents of the bottles



Think about what the histogram for this image would look like

What would happen if we used a single threshold value?

Multiple Thresholds

- Generalization of Otsu's method to K classes

$$C_1, C_2, \ldots C_K$$

$$P_k = \sum_{i \in C_k} p_i \qquad m_k = \frac{1}{P_k} \sum_{i \in C_k} i p_i$$

$$\sigma_B^2 = \sum_{k=1}^{K} P_k (m_k - m_G)^2$$

$$\sigma_B^2(k_1^*, k_2^*, \ldots, k_{K-1}^*) = \max_{0<k_1<k_2<\ldots k_{n-1}<L-1} \sigma_B^2(k_1, k_2, \ldots k_{K-1})$$

- K > 2: additional descriptors will be used

Multiple Thresholds

- K = 2: hysteresis （迟滞） thresholding

$$\sigma_B^2 = P_1(m_1 - m_G)^2 + P_2(m_2 - m_G)^2 + P_3(m_3 - m_G)^2$$

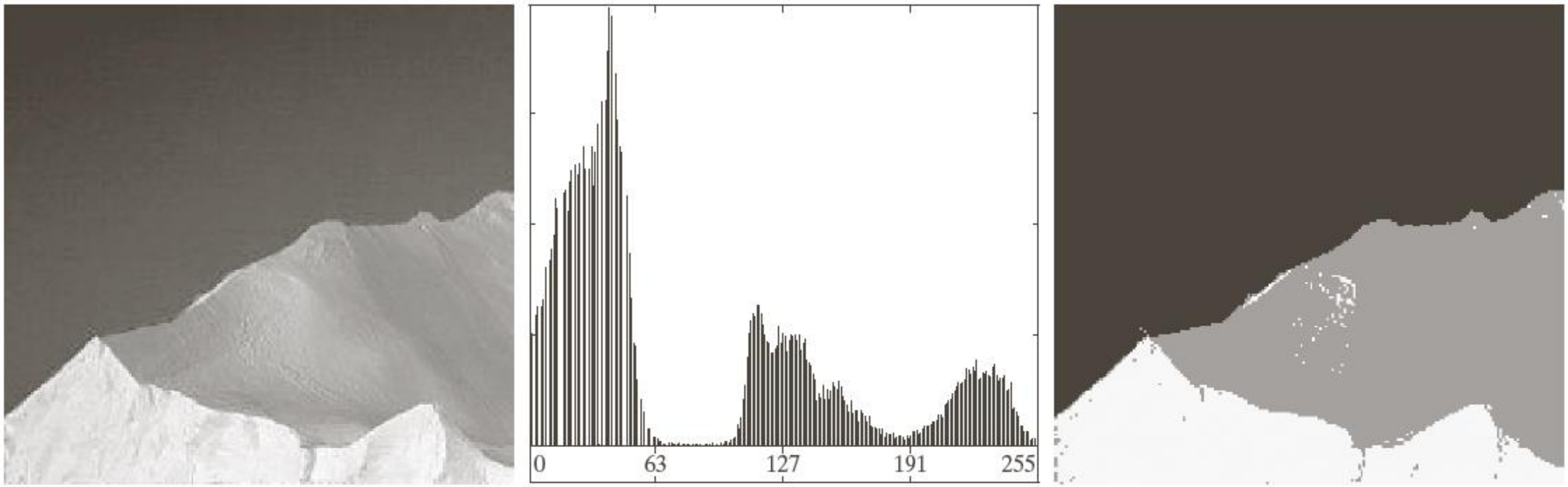
- Compute 2-D array, $\sigma_B^2(k_1, k_2)$

- Optimum thresholds

$$\sigma_B^2(k_1^*, k_2^*) = \max_{0<k_1<k_2<L-1} \sigma_B^2(k_1, k_2)$$

- Segmentation $g(x, y) = \begin{cases} a & \text{if } f(x, y) \le k_1^* \\ b & \text{if } k_1^* < f(x, y) \le k_2^* \\ c & \text{if } f(x, y) > k_2^* \end{cases}$

Multiple Thresholds Example

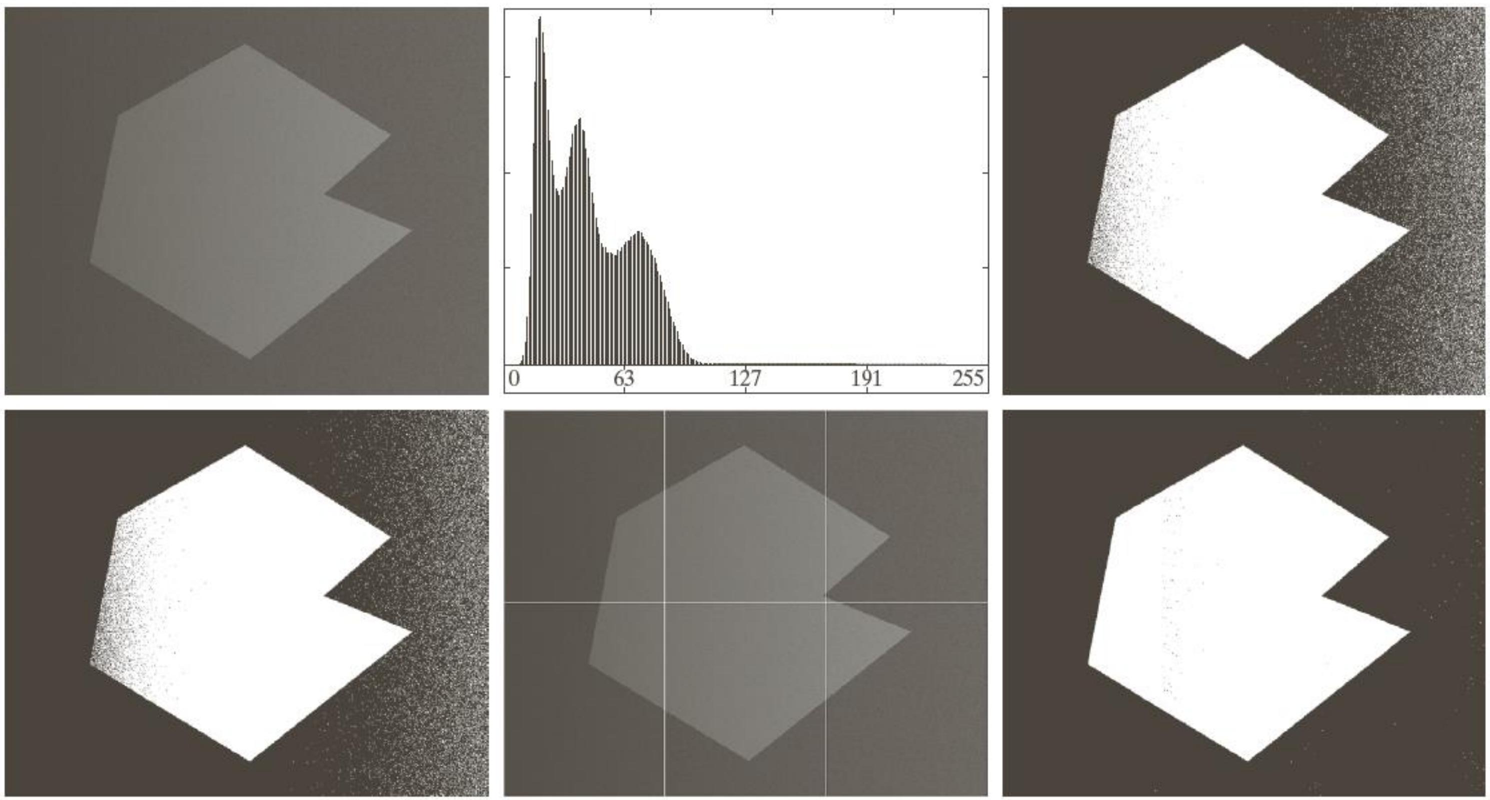


$$k_1^* = 80 \text{ and } k_2^* = 177$$

Variable Thresholding

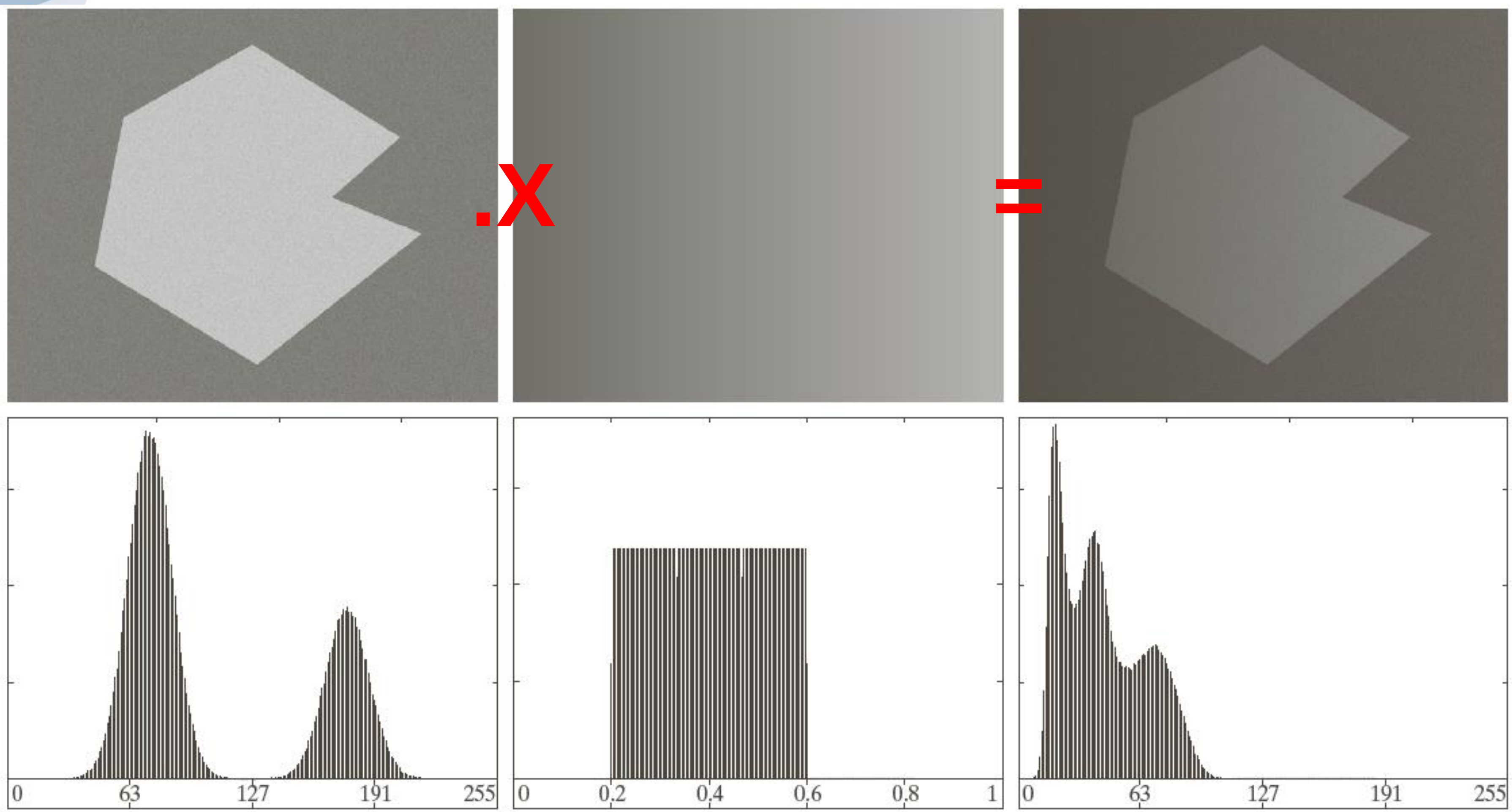
- Image partitioning

Noisy shade image | Histogram | Iterative global threshold



Otsu's result | Partition | Otsu's result

The Role of Illumination & Reflectance

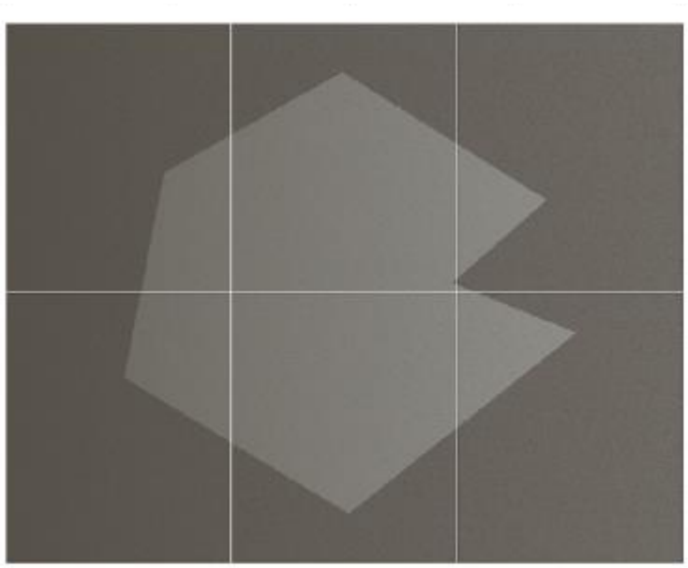


Noisy image Intensity ramp Product

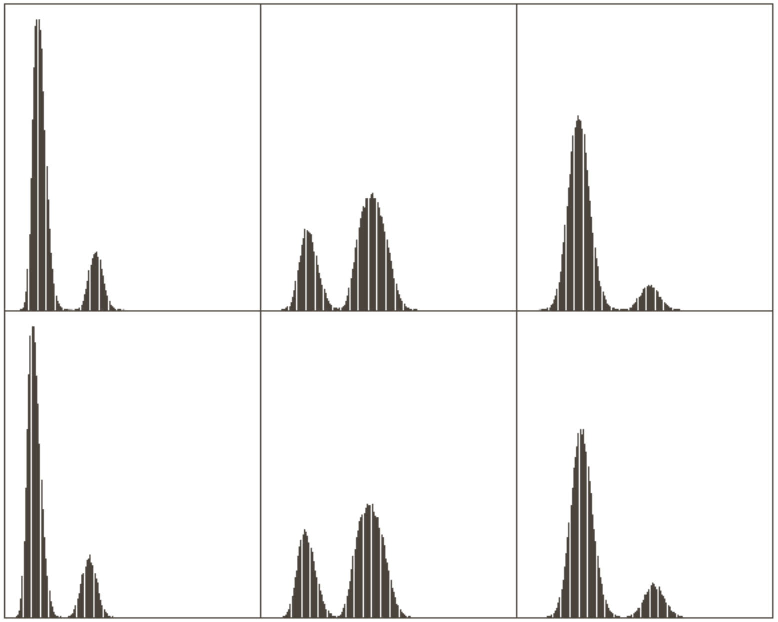
Variable Thresholding

- Image partitioning

Partitioned image



Histogram



Variable Thresholding

- Based on local image properties

$$T_{xy} = a\sigma_{xy} + bm_{xy} \quad \text{or} \quad T_{xy} = a\sigma_{xy} + bm_G$$

- Segmentation

$$g(x, y) = \begin{cases} 1 & \text{if } f(x, y) > T_{xy} \\ 0 & \text{if } f(x, y) \le T_{xy} \end{cases}$$

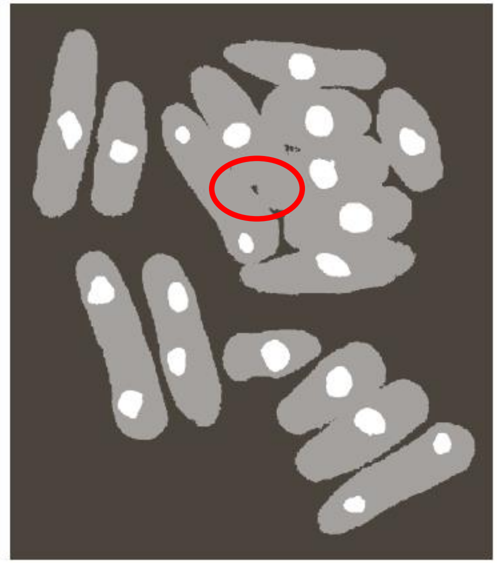
- General form

$$g(x, y) = \begin{cases} 1 & \text{if } Q(\text{local parameters}) \text{ is true} \\ 0 & \text{if } Q(\text{local parameters}) \text{ is false} \end{cases}$$

Variable Thresholding

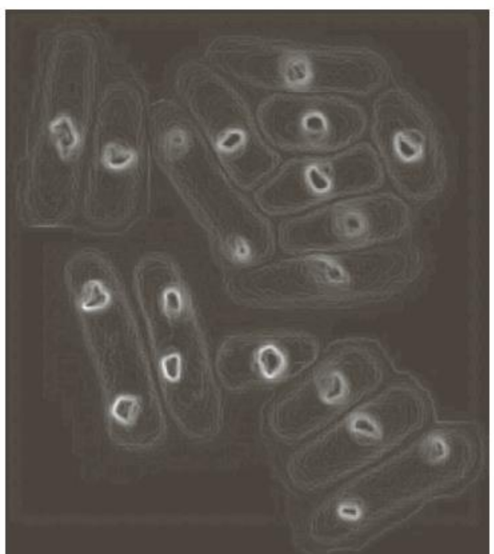
$$Q(\sigma_{xy}, m_{xy}) = \begin{cases} \text{true} & \text{if } f(x, y) > a\sigma_{xy} \text{ AND } f(x, y) > bm_G \\ \text{false} & \text{otherwise} \end{cases}$$

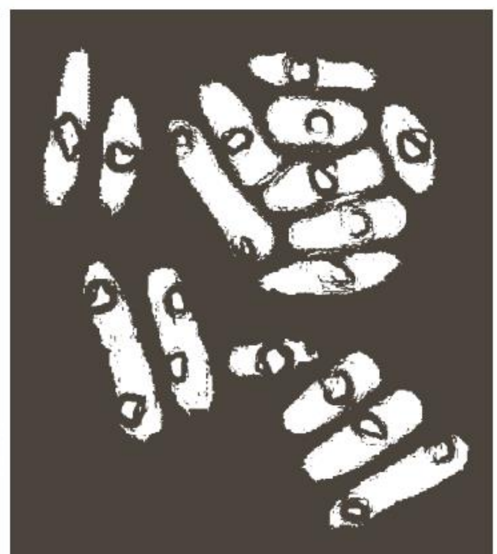
Yeast image



Double thresholding

Local standard deviation Of size 3x3





Local thresholding a=30 b=1.5

Variable Thresholding

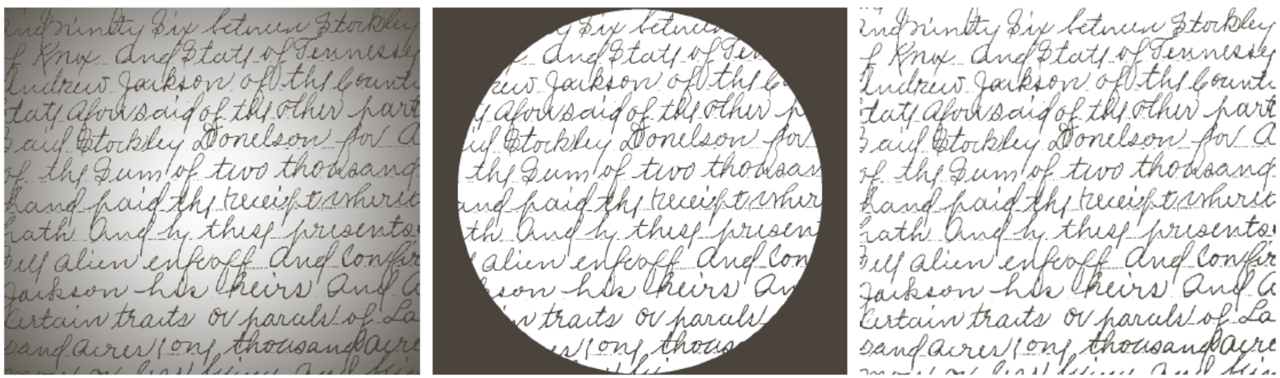
- Using moving averages in Zigzag scanning

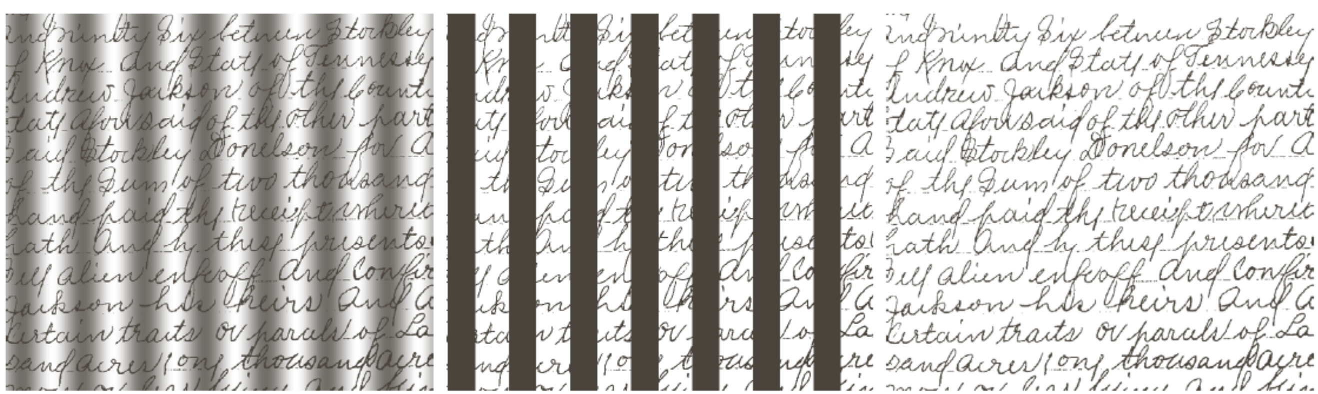
$$m(k+1) = \frac{1}{n}\sum_{i=k+2-n}^{k+1} z_i = m(k) + \frac{1}{n}(z_{k+1} - z_{k-n})$$

- Threshold: $T_{xy} = bm_{xy}$

Otsu's result

Moving averages
n=20, b=0.5





Multivariable Thresholding

- RGB Colors

$$\mathbf{z} = (z_1, z_2, z_3)^T$$

- Distance measure

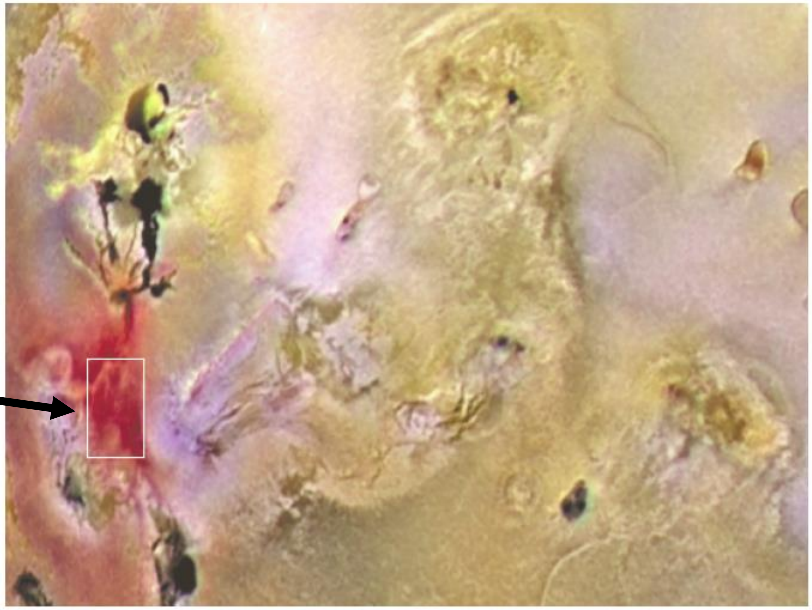
$$D(\mathbf{z}, \mathbf{a}) = \|\mathbf{z} - \mathbf{a}\|$$

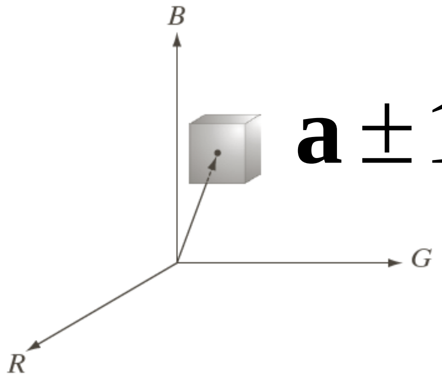
- Segmentation

$$g = \begin{cases} 1 & \text{if } D(\mathbf{z}, \mathbf{a}) < T \\ 0 & \text{otherwise} \end{cases}$$

Segmentation in RGB Vector Space

Original image
with colors of interest





$\mathbf{a} \pm 1.25\boldsymbol{\sigma}$

Contents

- Image Segmentation Fundamentals
- Point, Line, and Edge Detection

Part 1

- Thresholding
- Region-Based Segmentation
- Segmentation Using Morphological Watersheds
- The Use of Motion in Segmentation

Part 2

Region Growing

$$Q = \begin{cases} \text{TRUE} & \text{if the absolute difference of the intensities} \\ & \text{between the seed and the pixel at } (x, y) \text{ is} \le T \\ \text{FALSE} & \text{otherwise} \end{cases}$$

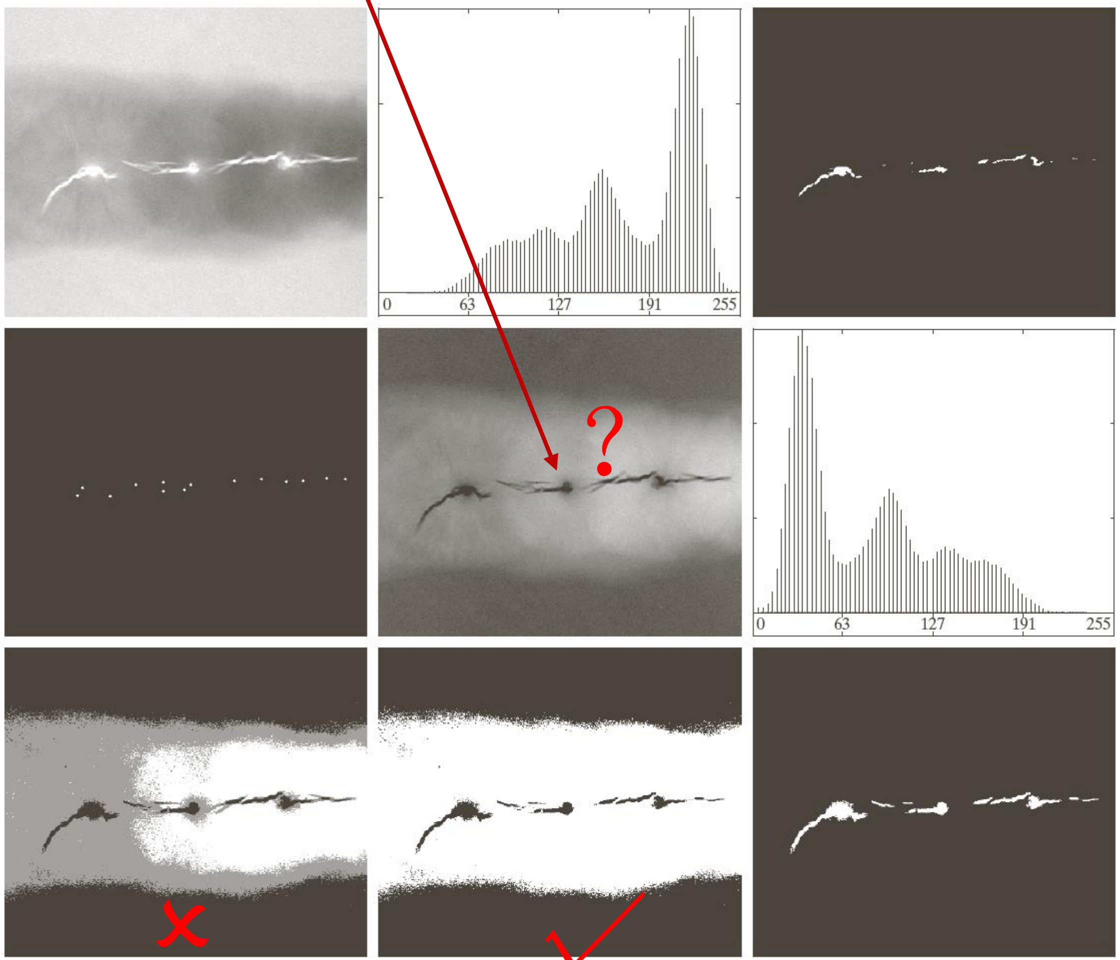
X-ray image　Histogram　Initial seed

T = 254

Final seed

$T_1 = 68$

$T_2 = 126$



Dual thresholds　Single threshold　Region growing

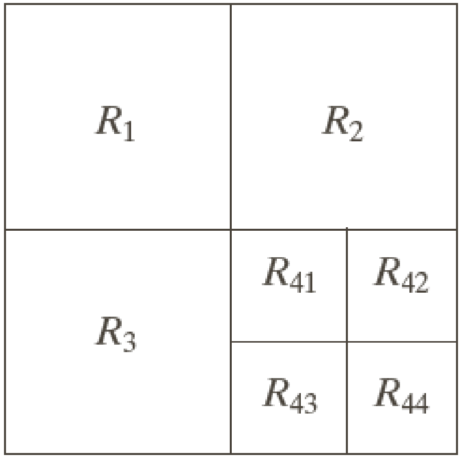
$T_1 = 68$

Region Growing

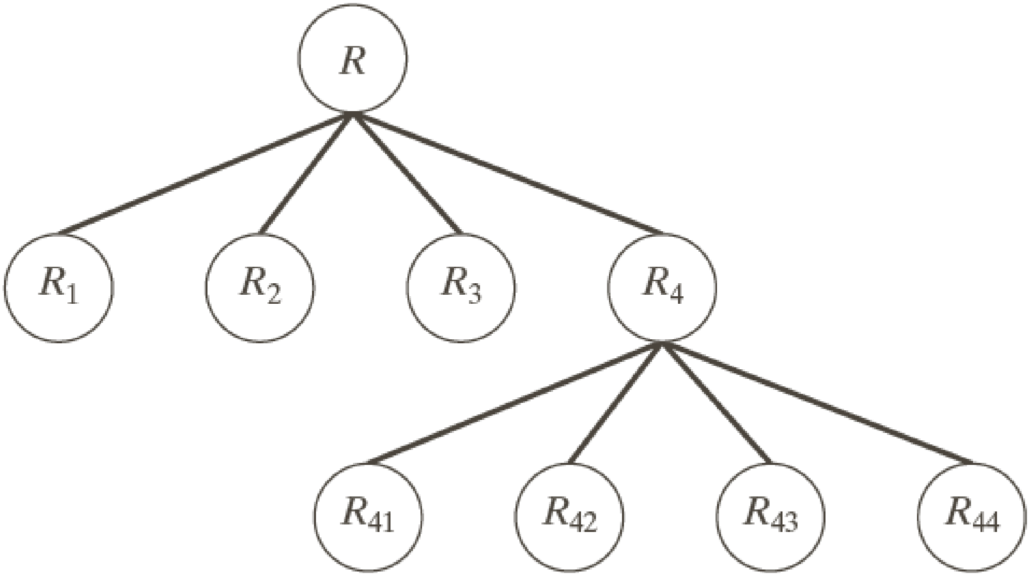
1. Find all connected components in $S(x, y)$ and erode each connected component to one pixel; label all such pixels found as 1. All other pixels in S are labeled 0.
2. Form an image f_Q such that, at a pair of coordinates (x, y), let $f_Q(x, y) = 1$ if the input image satisfies the given predicate, Q, at those coordinates; otherwise, let $f_Q(x, y) = 0$.
3. Let g be an image formed by appending to each seed point in S all the 1-valued points in f_Q that are 8-connected to that seed point.
4. Label each connected component in g with a different region label (e.g., 1, 2, 3, ...). This is the segmented image obtained by region growing.

Region Splitting and Merging

1. Split into four disjoint quadrants any region R_i for which $Q(R_i) = \text{FALSE}$.
2. When no further splitting is possible, merge any adjacent regions R_j and R_k for which $Q(R_j \cup R_k) = \text{TRUE}$.
3. Stop when no further merging is possible.



Partitioned image



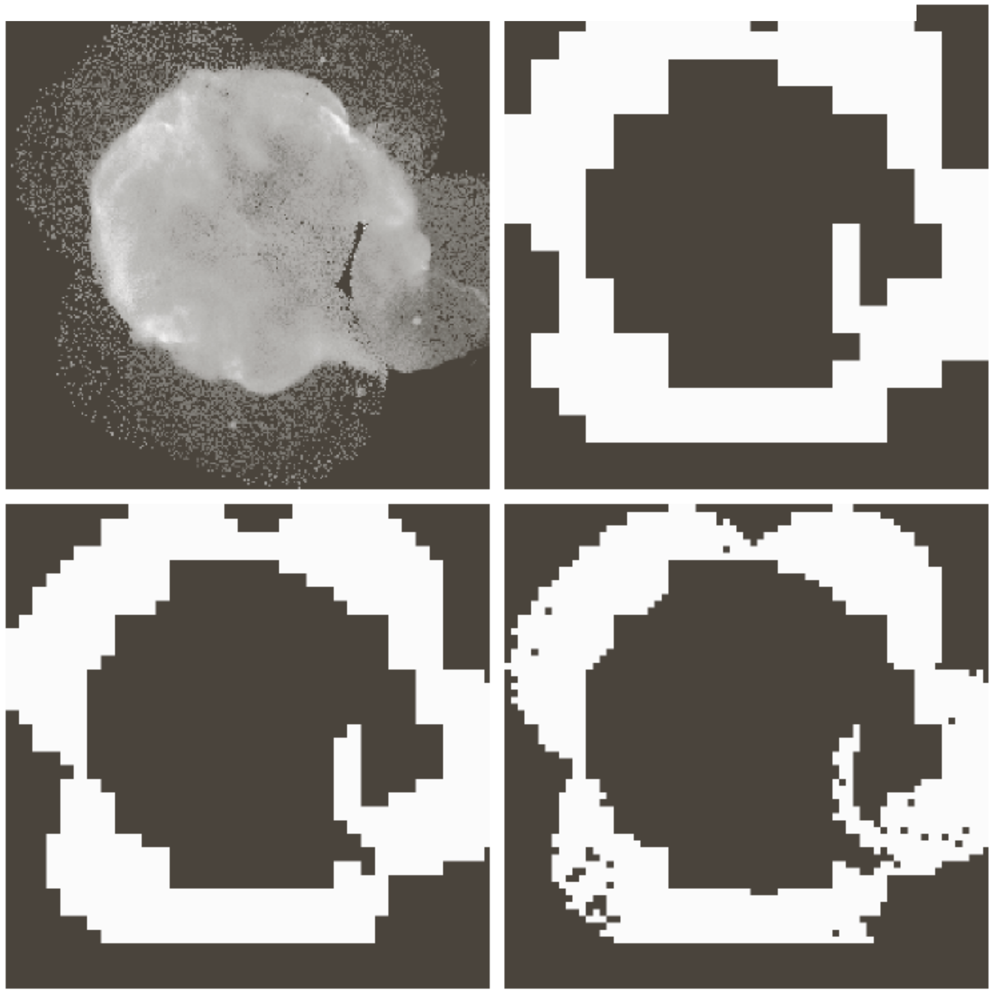
Quadtree

Region Splitting and Merging

$$Q = \begin{cases} \text{TRUE} & \text{if } \sigma > a \quad \text{AND} \quad 0 < m < b \\ \text{FALSE} & \text{otherwise} \end{cases}$$

Cygnus Loop supernova

Smallest: 32x32



Smallest: 16x16

Smallest: 8x8

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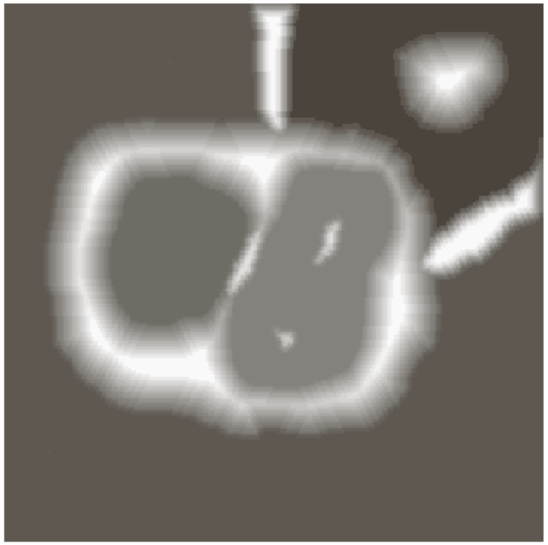
Part 2

Watersheds

- Watershed lines: points at which water would be equally likely to fall to more than one minimum

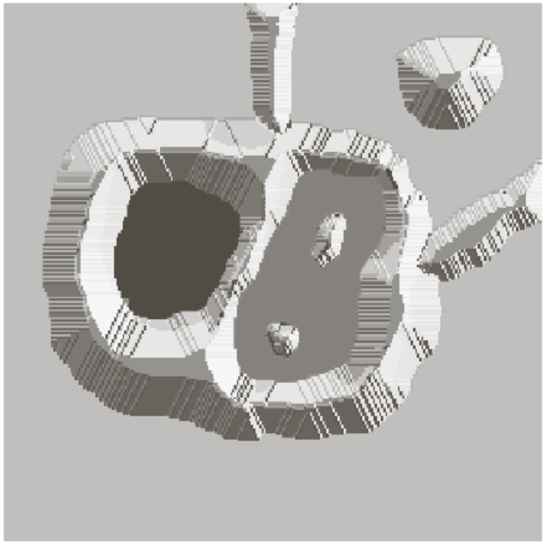
Watersheds

Original image



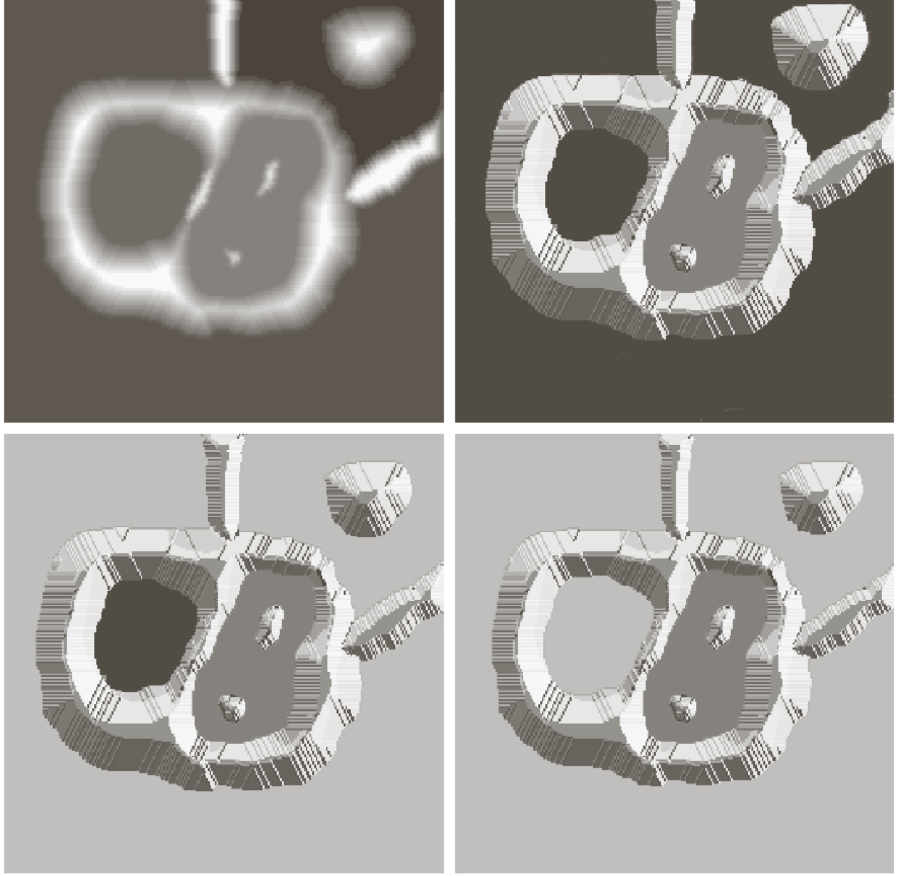
Topographic view

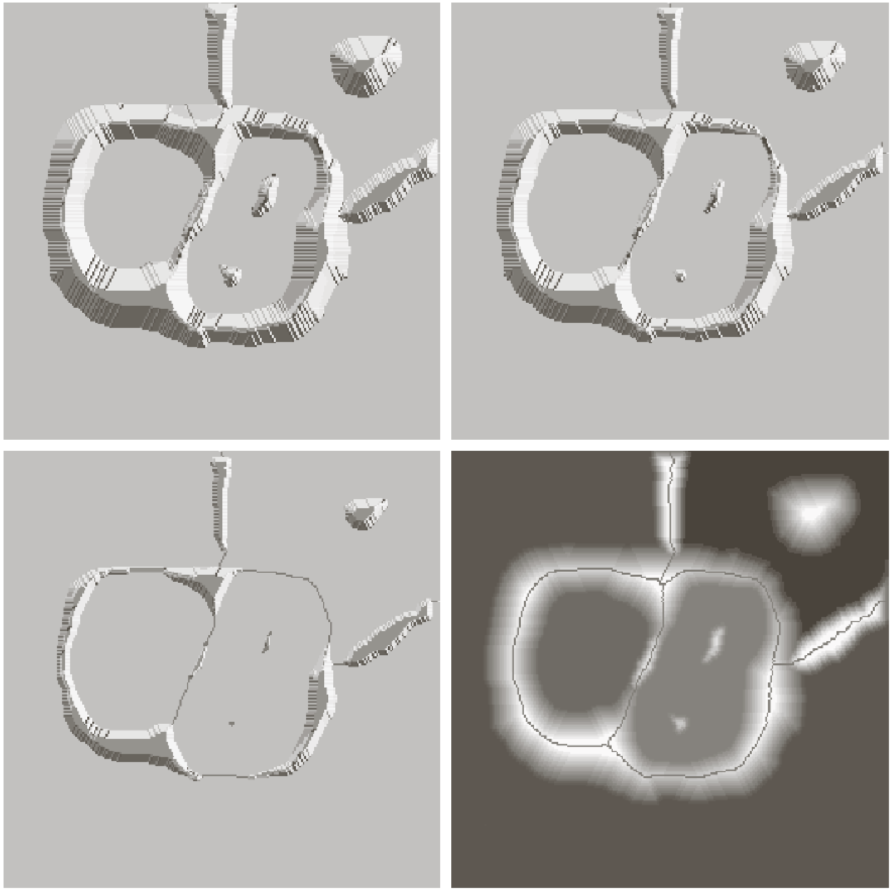
Flooding Stage 1



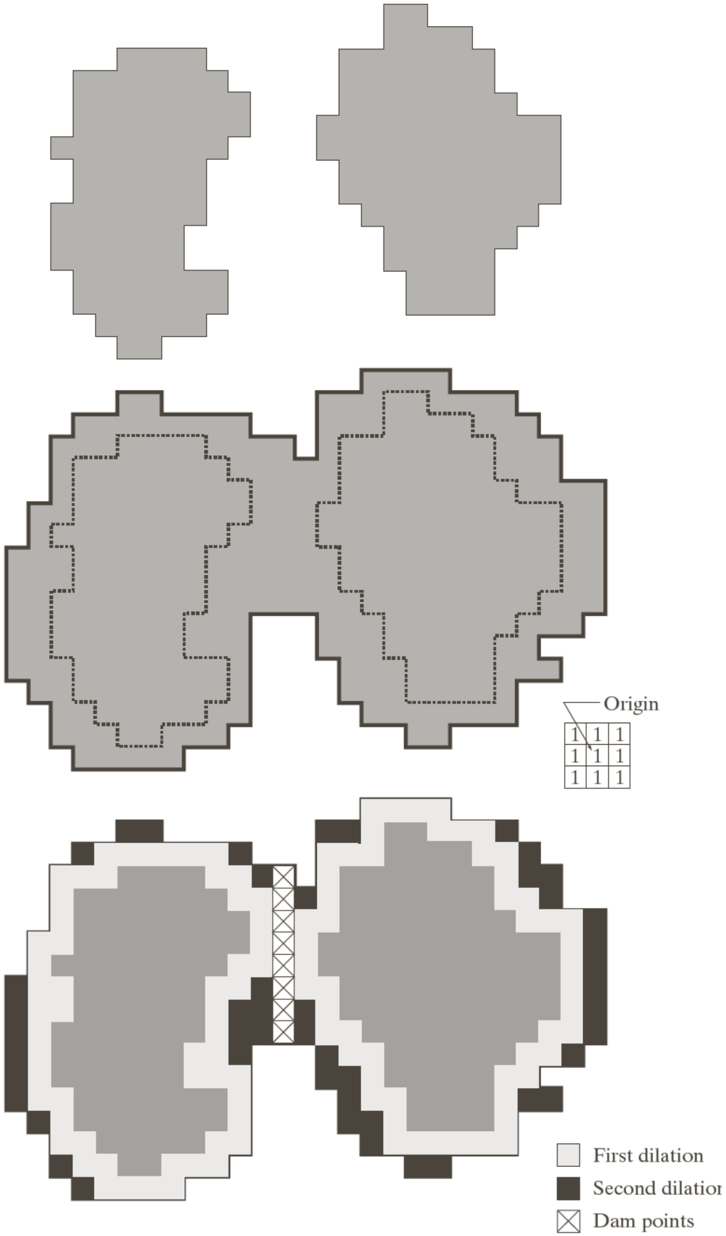
Flooding Stage 2

Watersheds





Dam Construction using Morphological Dilation



Flooding stage n-1

Flooding stage n

Result of dilation
and dam construction

Watershed Segmentation Algorithm

- Flooding from n = min + 1 to n = max + 1

$$T[n] = \{(s,t) \mid g(s,t) < n\}$$

- Flooded Catchment basin

$$C_n(M_i) = C(M_i) \cap T[n]$$

- Union

$$C[n] = \bigcup_{i=1}^{R} C_n(M_i)$$

- Initialization

$$C[\min + 1] = T[\min + 1]$$

Watershed Segmentation Algorithm

Let Q denote the set of connected components in $T[n]$
for each connected component $q \in Q[n]$

1. $q \cap C[n-1]$ is empty

new minimum   flooded catchment basin

2. $q \cap C[n-1]$ contains one connected component of $C[n-1]$

q is incorporated into $C[n-1]$ to form $C[n]$

3. $q \cap C[n-1]$ contains more than one connected component of $C[n-1]$

dams must be built within q to prevent overflow

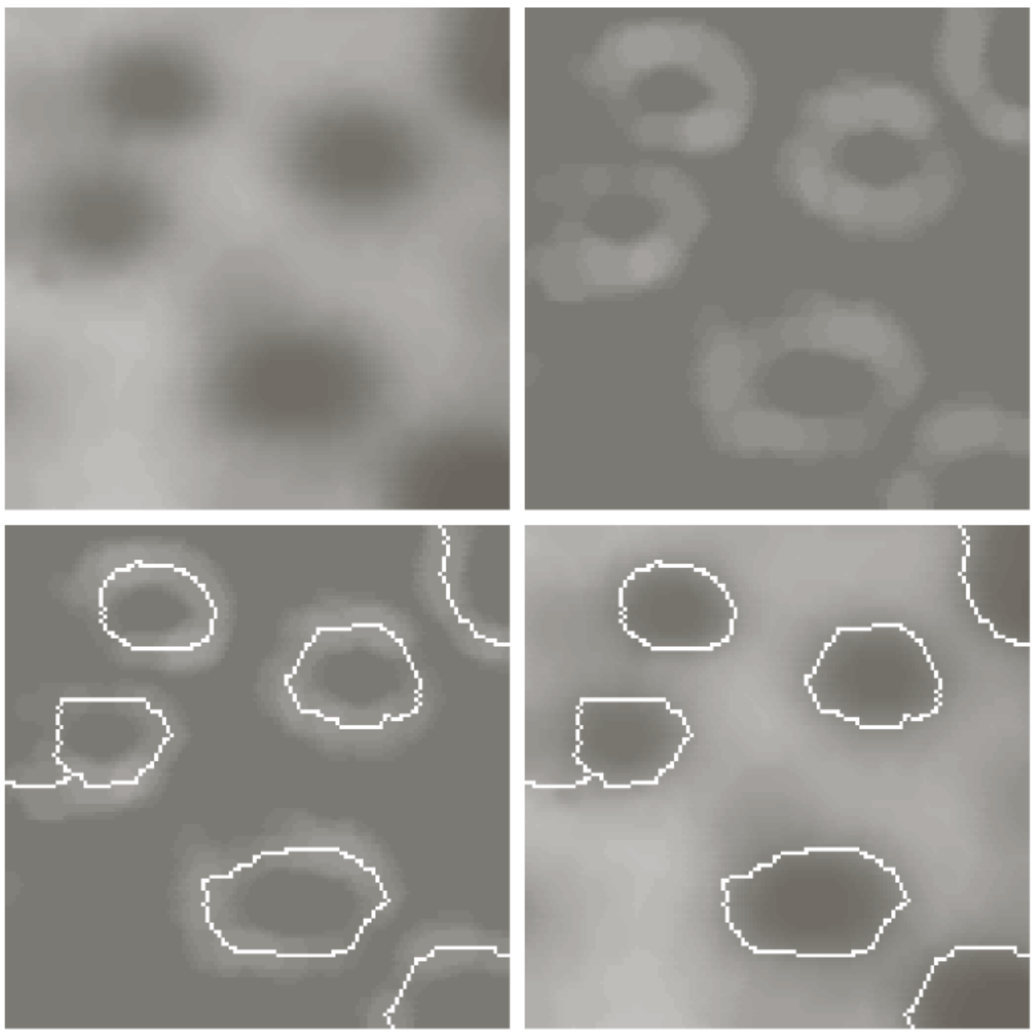
Watershed Segmentation Algorithm

- Flooding from n = min + 1 to n = max + 1

$$T[n] = \{(s,t) \mid g(s,t) < n\}$$

Original Image

Gradient Image

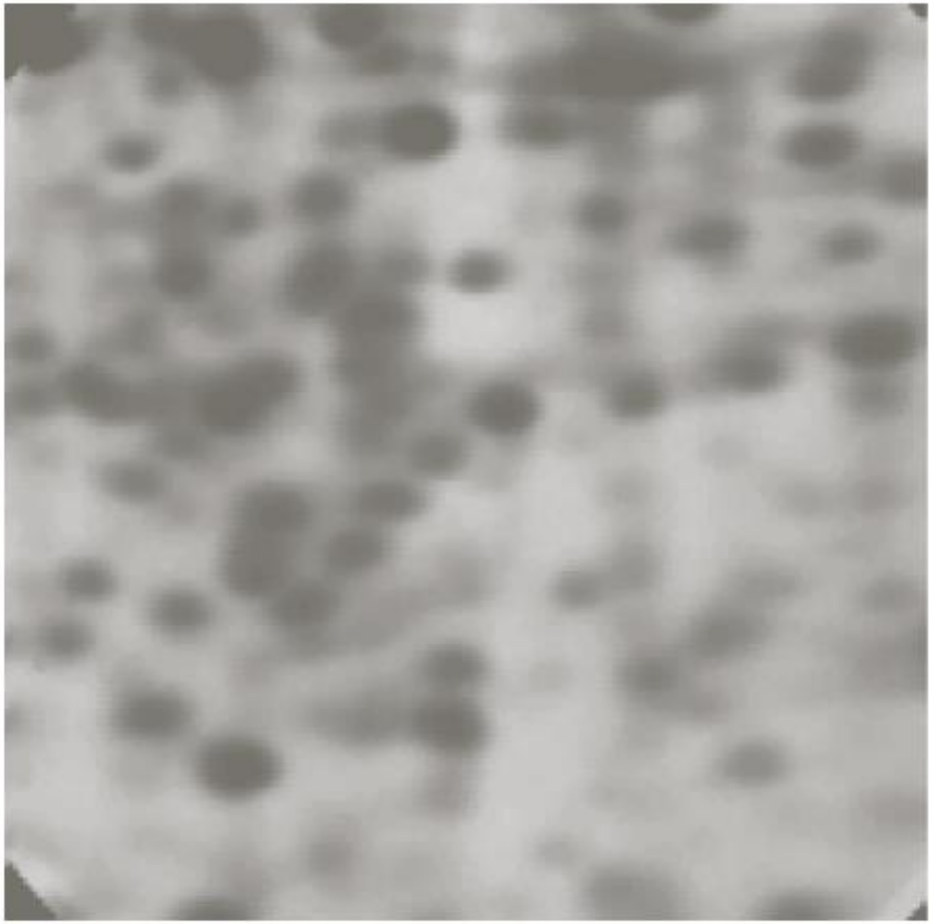


Watershed lines

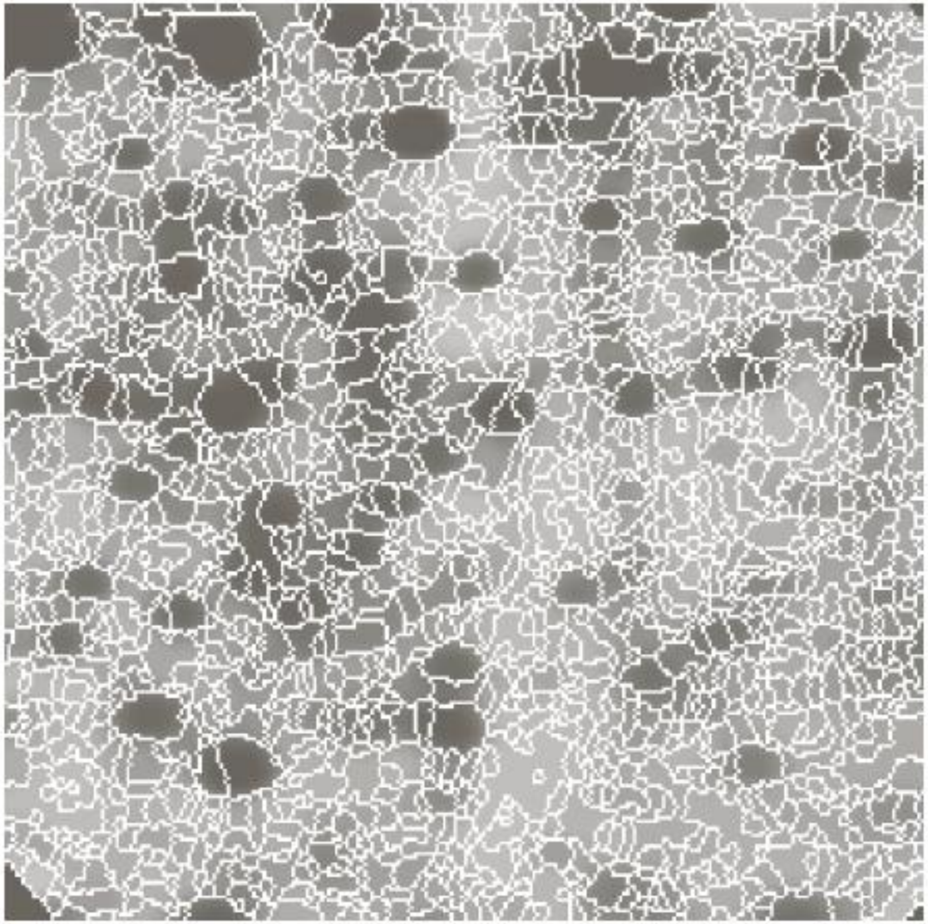
Superimposed on the original image

The Use of Markers

- Oversegmentation

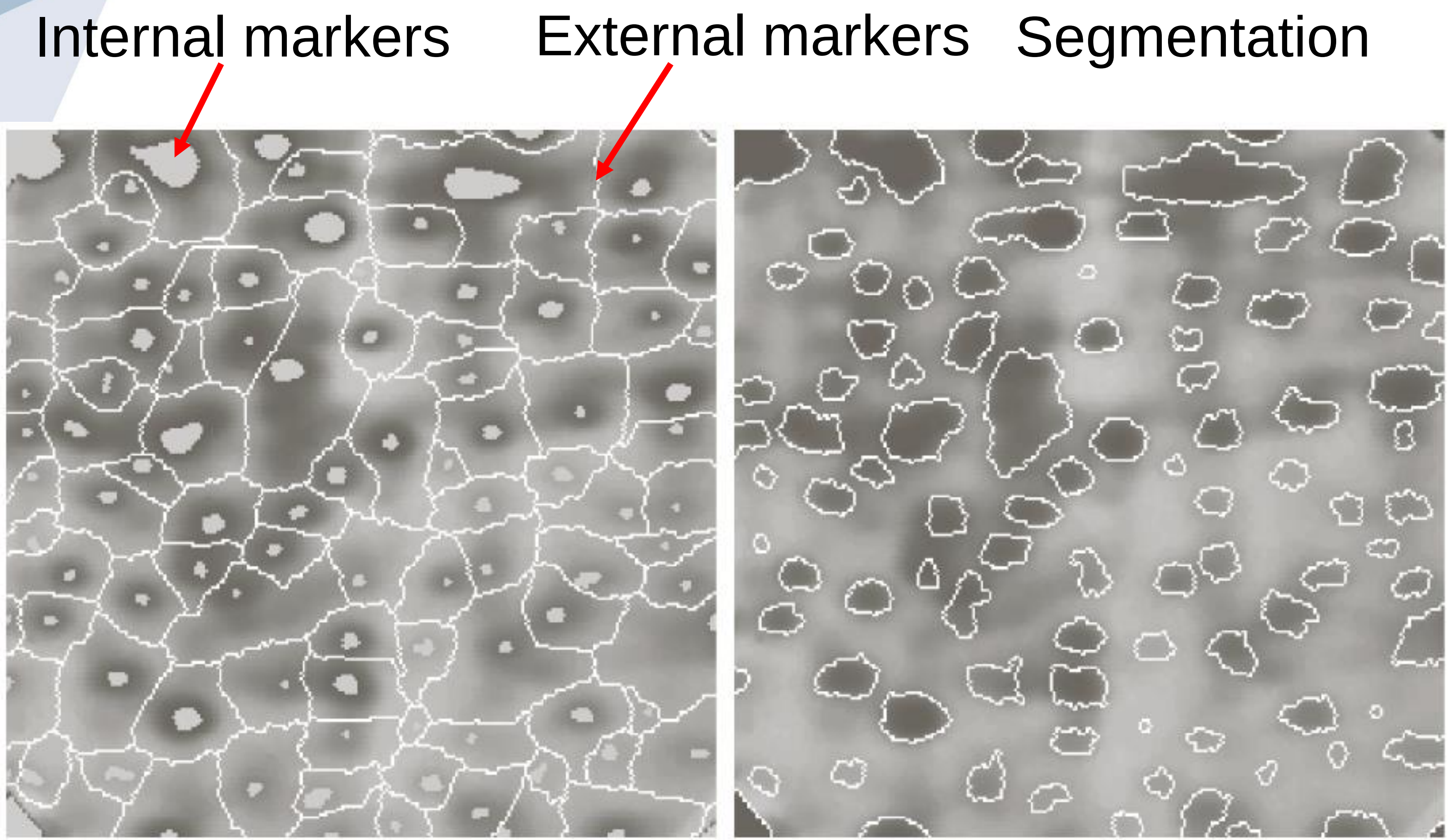


Original Image



Watershed lines of the gradient image

The Use of Markers



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- Image Segmentation Fundamentals
- Point, Line, and Edge Detection

Part 1

- Thresholding
- Region-Based Segmentation
- Segmentation Using Morphological Watersheds
- The Use of Motion in Segmentation
  - Spatial Techniques
  - ~~Frequency Domain Techniques~~

Part 2

Spatial Techniques in Motion Segmentation

- Basic approach: difference image

$$d_{ij}(x, y) = \begin{cases} 1 & \text{if } \left| f(x, y, t_i) - f(x, y, t_j) \right| > T \\ 0 & \text{otherwise} \end{cases}$$

- Accumulative difference image (ADI)
  - Absolute ADI

$$A_k(x, y) = \begin{cases} A_{k-1}(x, y) + 1 & \text{if } \left| R(x, y) - f(x, y, k) \right| > T \\ A_{k-1}(x, y) & \text{otherwise} \end{cases}$$

  - Positive ADI

$$P_k(x, y) = \begin{cases} P_{k-1}(x, y) + 1 & \text{if } \left[R(x, y) - f(x, y, k) \right] > T \\ P_{k-1}(x, y) & \text{otherwise} \end{cases}$$

  - Negative ADI

$$N_k(x, y) = \begin{cases} N_{k-1}(x, y) + 1 & \text{if } \left[R(x, y) - f(x, y, k) \right] < -T \\ N_{k-1}(x, y) & \text{otherwise} \end{cases}$$

ADI Example

Moving rectangular object

Absolute ADI　　Positive ADI　　Negative ADI

Establishing a Reference Image for ADI

Moving object removed

Frame 1 Frame 2 Reference image

Assignments

- 10.31, 10.34, 10.49