Morphological Image Processing

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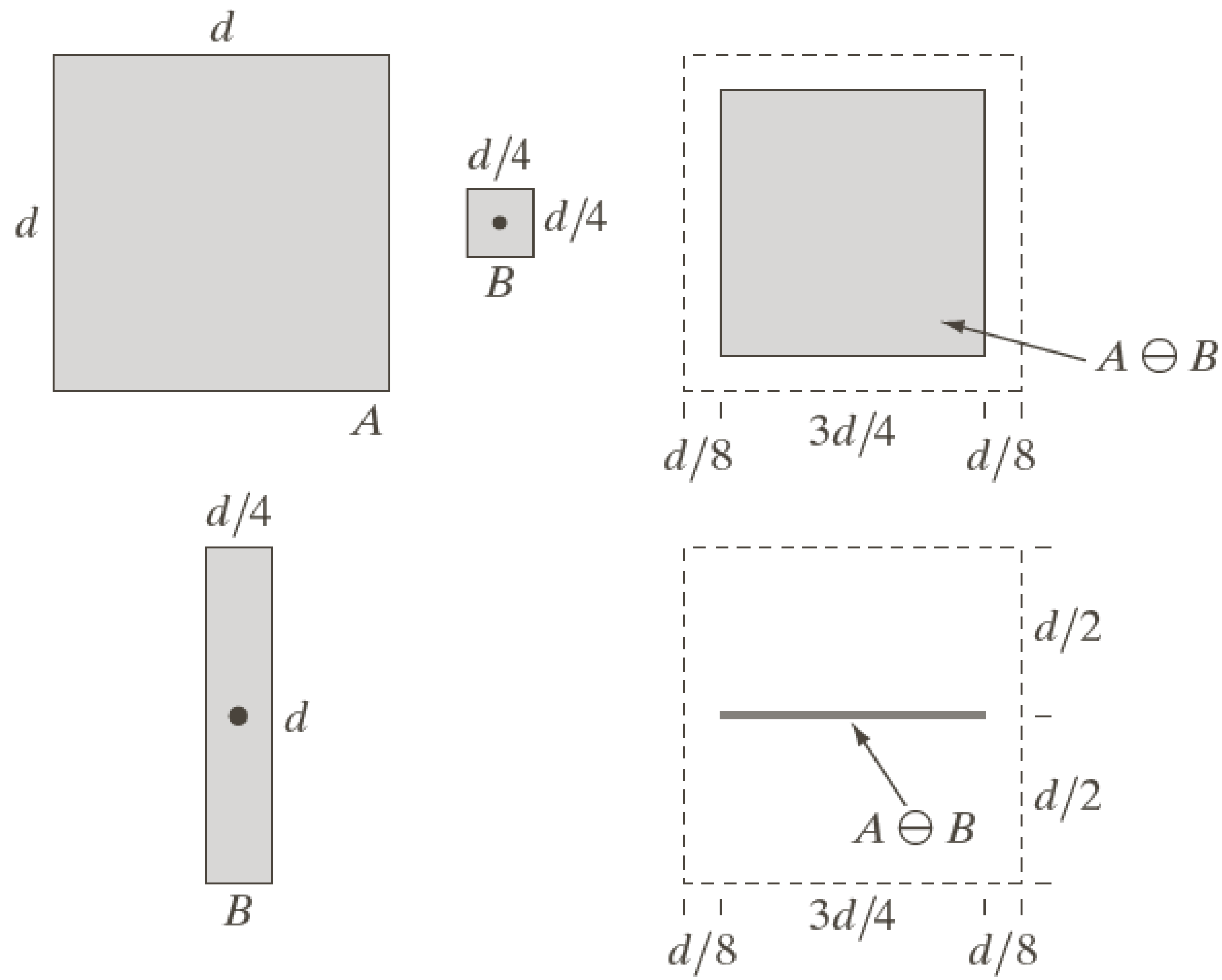
Contents

Erosion

$$A \ominus B = \{z \mid (B)_z \subseteq A\} = \{z \mid (B)_z \cap A^c = \varnothing\}$$



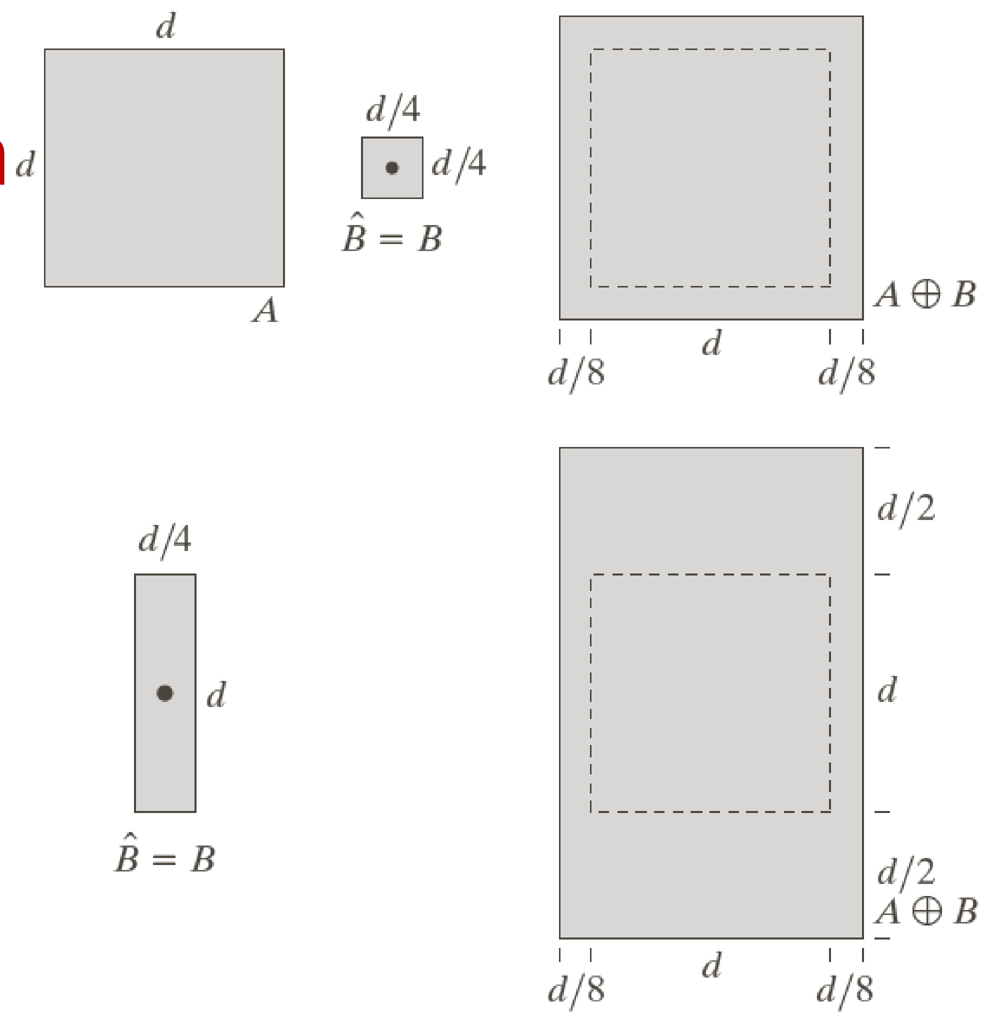
Dilation

$$A \oplus B = \left\{ z \,|\, (\hat{B})_z \cap A \neq \varnothing \right\}$$

- Why reflection?

Make Duality between Erosion & Dilation

- The reflection and shifting of B is analogous to spatial convolution



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Duality between Erosion & Dilation

$$(A \ominus B)^c = A^c \oplus \hat{B}$$

$$(A \oplus B)^c = A^c \ominus \hat{B}$$

$$A \oplus B = \bigcup_{b \in B} A_b$$

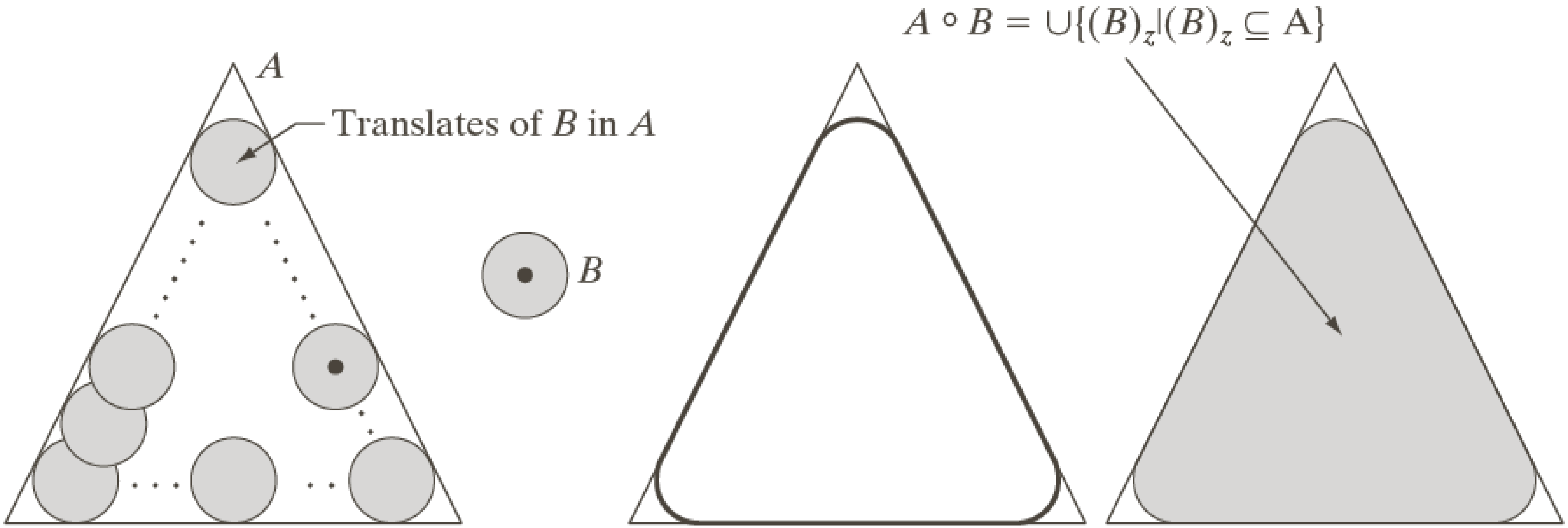
$$A \Theta B = \bigcap_{b \in \hat{B}} A_b$$

Opening

$$A \circ B = (A \ominus B) \oplus B$$

$$A \circ B = \bigcup \{(B)_z | (B)_z \subseteq A\}$$

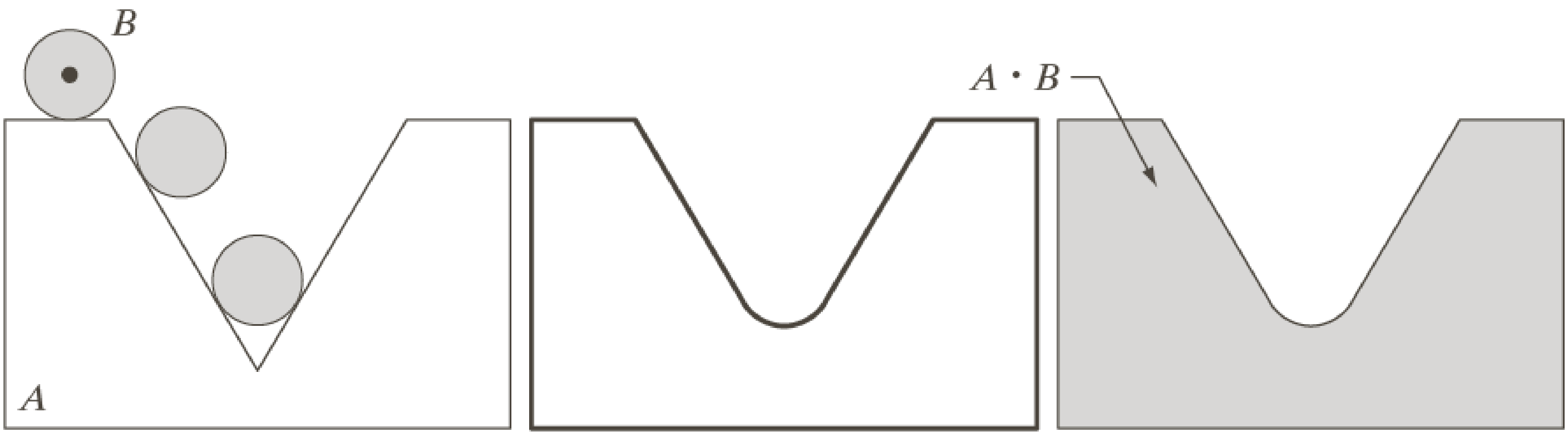
Structuring element B rolling along the inner boundary of A



Closing

$$A \bullet B = (A \oplus B) \ominus B$$

Structuring element B rolling along the outer boundary of A



Opening and Closing Properties

- Duality

$$(A \bullet B)^c = (A^c \circ \hat{B})$$
$$(A \circ B)^c = (A^c \bullet \hat{B})$$

- Opening

(a) $A \circ B$ is a subset (subimage) of A.
(b) If C is a subset of D, then $C \circ B$ is a subset of $D \circ B$.
(c) $(A \circ B) \circ B = A \circ B$.

- Closing

(a) A is a subset (subimage) of $A \bullet B$.
(b) If C is a subset of D, then $C \bullet B$ is a subset of $D \bullet B$.
(c) $(A \bullet B) \bullet B = A \bullet B$.

Opening, Closing & Set Operations

- Set Union

$$\boxed{\left(\bigcup_{i=1}^{n} A_i\right) \circ B \supseteq \bigcup_{i=1}^{n} (A_i \circ B)}$$

$$\left(\bigcup_{i=1}^{n} A_i\right) \bullet B \supseteq \bigcup_{i=1}^{n} (A_i \bullet B)$$

$$\xrightarrow{\text{Proof}} \quad \bigcup_{i=1}^{n} A_i \supseteq A_i$$

$$\Downarrow$$

$$\left(\bigcup_{i=1}^{n} A_i\right) \circ B \supseteq A_i \circ B$$

$$\Downarrow$$

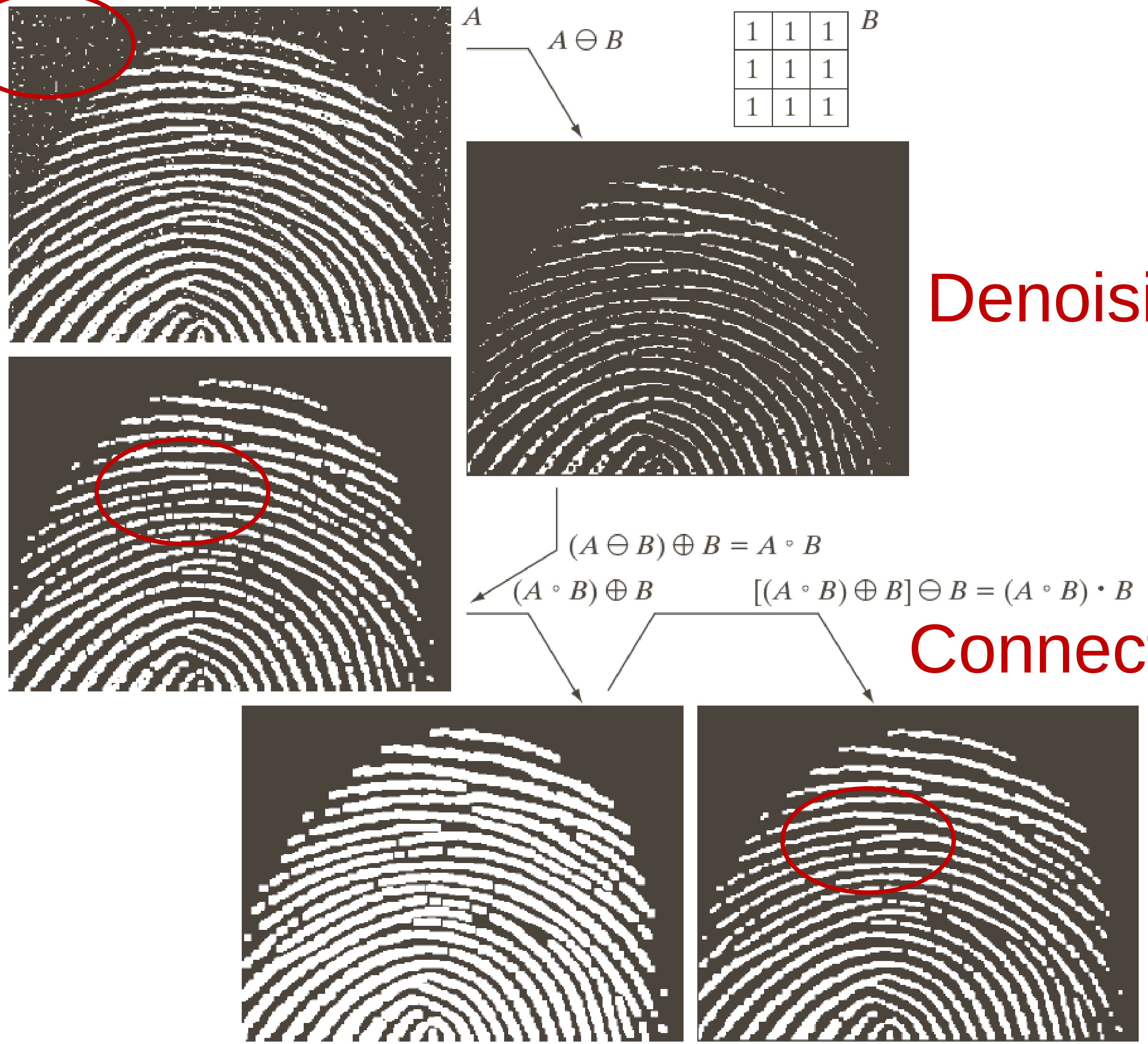
$$\left(\bigcup_{i=1}^{n} A_i\right) \circ B \supseteq \bigcup_{i=1}^{n} (A_i \circ B)$$

- Set Intersection

$$\left(\bigcap_{i=1}^{n} A_i\right) \circ B \subseteq \bigcap_{i=1}^{n} (A_i \circ B)$$

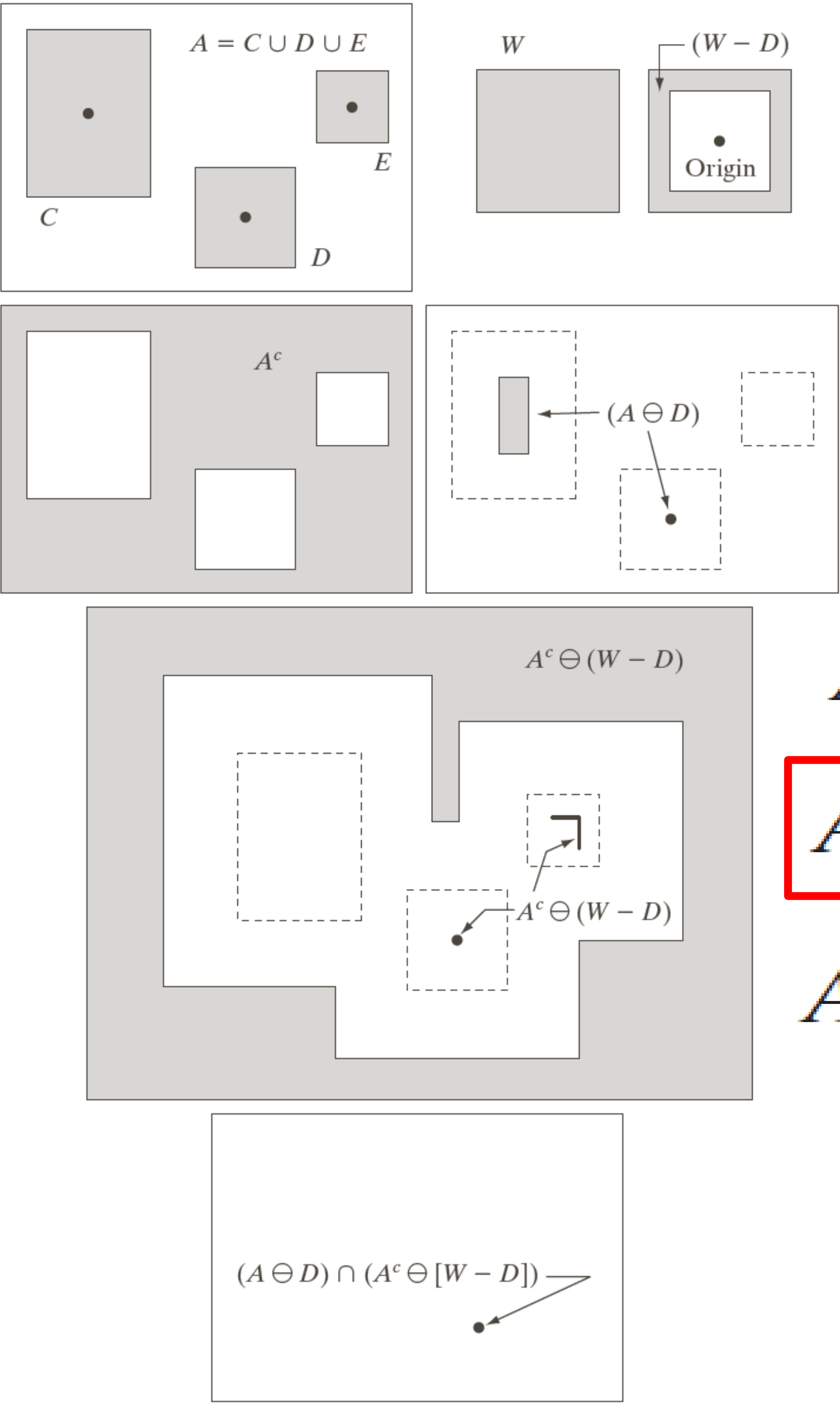
$$\left(\bigcap_{i=1}^{n} A_i\right) \bullet B \subseteq \bigcap_{i=1}^{n} (A_i \bullet B)$$

Morphological Processing Example



Denoising

Connecting



Hit-or-Miss Transformation

Find the location of a specific shape

$$A \circledast B = (A \ominus D) \cap \left[A^c \ominus (W - D)\right]$$

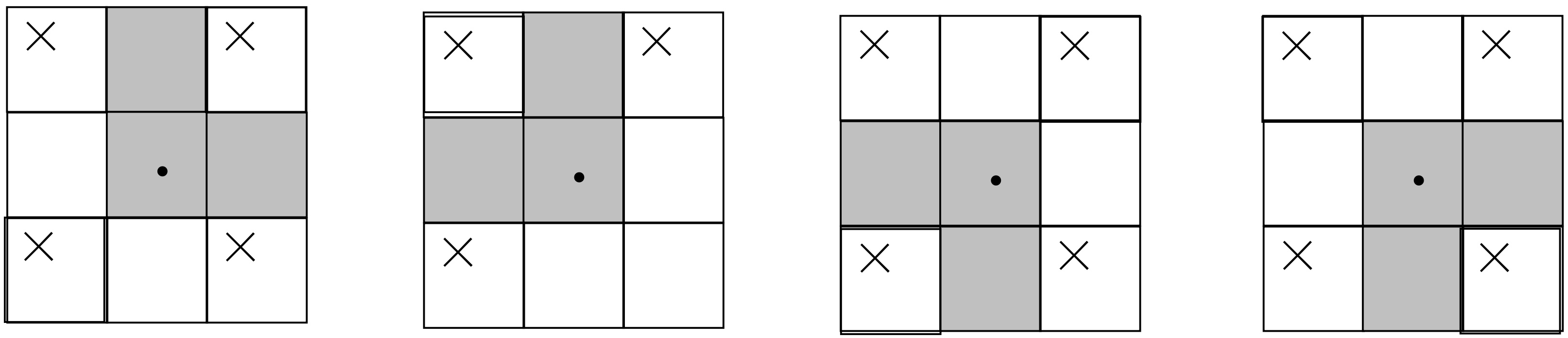
$$\boxed{A \circledast B = (A \ominus B_1) \cap (A^c \ominus B_2)}$$

$$\Downarrow$$

$$A \circledast B = (A \ominus B_1) - (A \oplus \hat{B}_2)$$

Hit-or-Miss Transformation

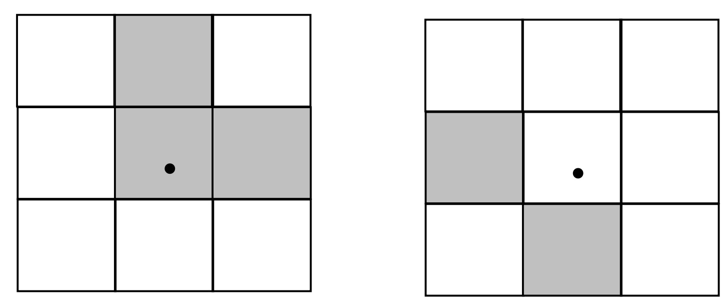
× : don't care pixels



(a) (b) (c) (d)

Structuring elements for corner detection

$$A \circledast B = (A \ominus B_1) \cap (A^c \ominus B_2)$$



(a) B_1 (b) B_2

Properties

运算 \ 性质	膨胀	腐蚀	开	闭
位移不变性	$(A)_x \oplus B = (A \oplus B)_x$	$(A)_x \Theta B = (A \Theta B)_x$	$A \circ (B)_x = A \circ B$	$A \bullet (B)_x = A \bullet B$
互换性	$A \oplus B = B \oplus A$			
组合性	$(A \oplus B) \oplus C = A \oplus (B \oplus C)$	$(A \Theta B) \Theta C = A \Theta (B \oplus C)$		
增长性	$A \subseteq B \Rightarrow A \oplus C \subseteq B \oplus C$	$A \subseteq B \Rightarrow A \Theta C \subseteq B \Theta C$	$A \subseteq B \Rightarrow A \circ C \subseteq B \circ C$	$A \subseteq B \Rightarrow A \bullet C \subseteq B \bullet C$
同前性			$(A \circ B) \circ B = A \circ B$	$(A \bullet B) \bullet B = A \bullet B$
外延性	$A \subseteq A \oplus B$	$A \Theta B \subseteq A$	$A \circ B \subseteq A$	$A \subseteq A \bullet B$

上表中膨胀和腐蚀的外延性只当结构元原点在内部时成立

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Basic Morphological Algorithms

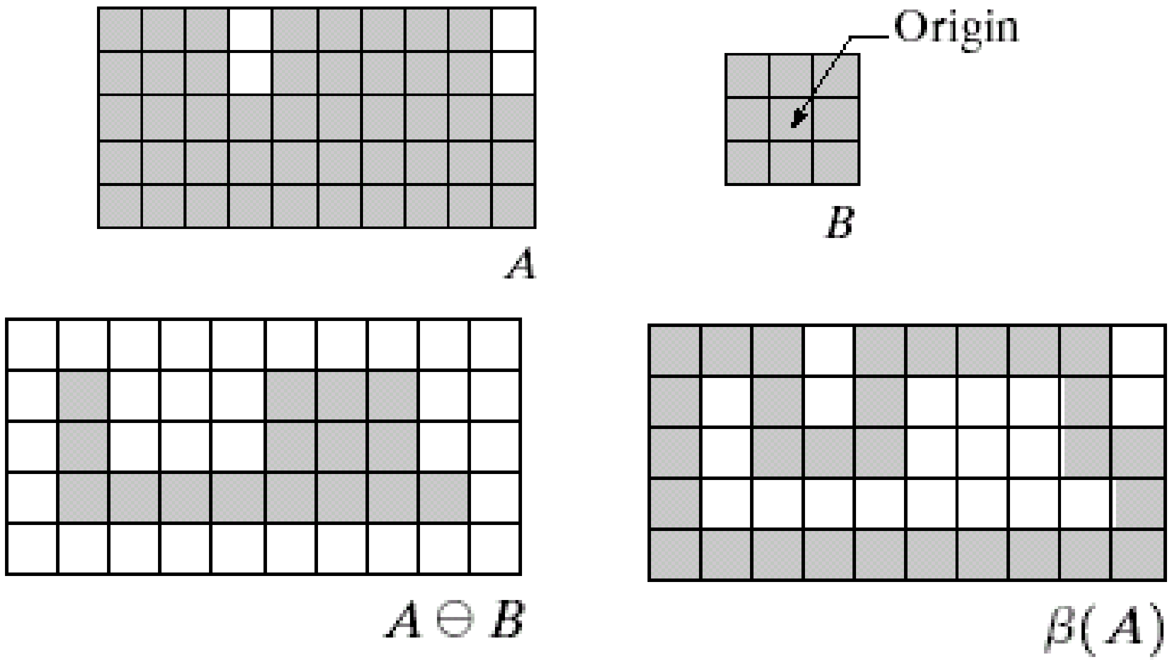
- Boundary Extraction
- Region Filling
- Extraction of Connected Components
- Convex Hull
- Thinning
- Thickening
- Skeletons
- Pruning（修剪）
- Morphological Reconstruction

Boundary Extraction

Extracting the boundary (or outline) of an object is often extremely useful

The boundary can be given simply as

$$\beta(A) = A - (A \ominus B)$$

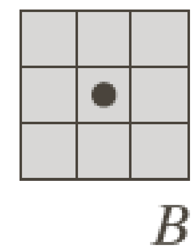


Boundary Extraction Example

A simple image and the result of performing boundary extraction using a square 3*3 structuring element

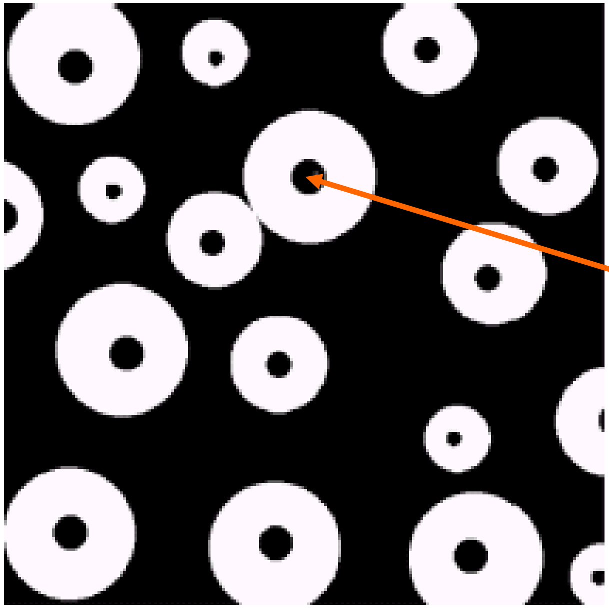
Original Image

Extracted Boundary



Region Filling

Given a pixel inside a boundary, *region filling* attempts to fill that boundary with object pixels (1s)



Given a point inside here, can we fill the whole circle?

Region Filling (cont…)

The key equation for region filling is

$$X_k = (X_{k-1} \oplus B) \cap A^c \qquad k = 1,2,3.....$$

Where X_0 is simply the starting point inside the boundary, B is a simple structuring element and A^c is the complement of A

This equation is applied repeatedly until X_k is equal to X_{k-1}

Finally the result is unioned with the original boundary

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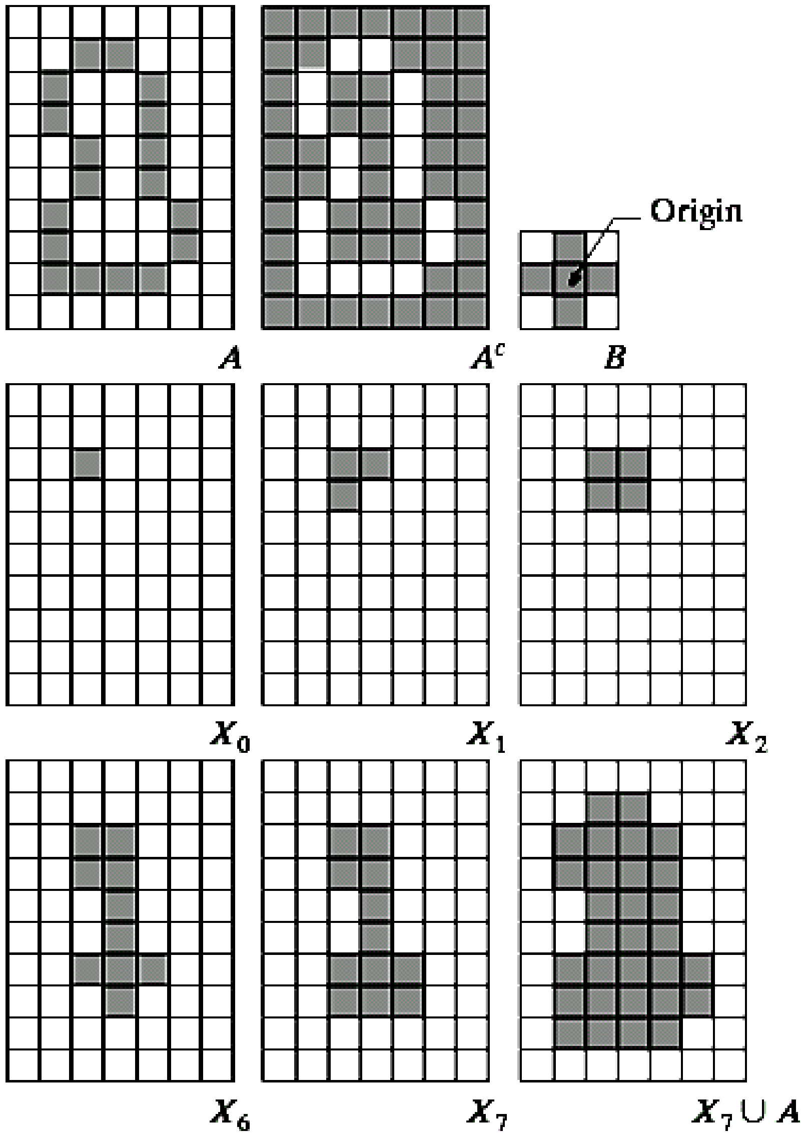
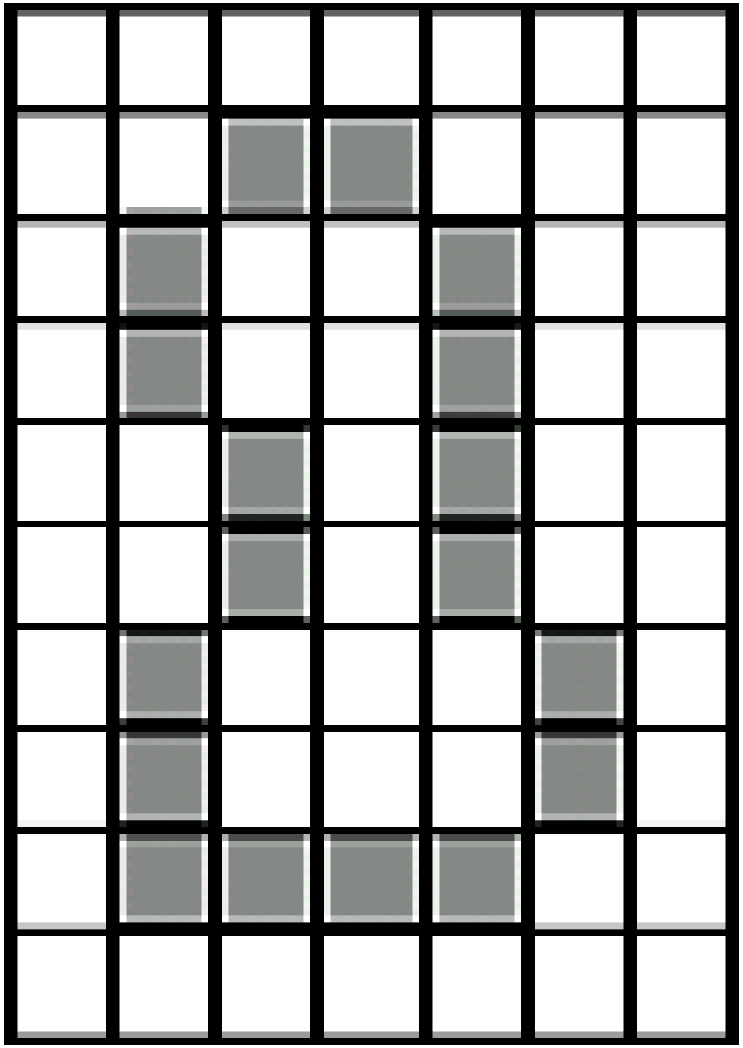
Region Filling Step By Step





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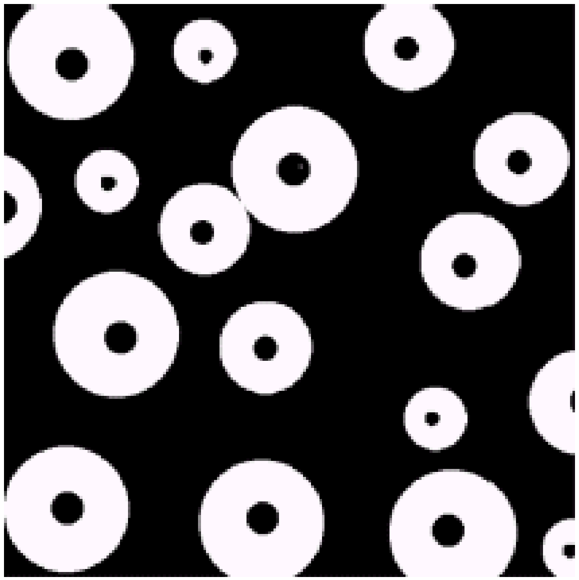
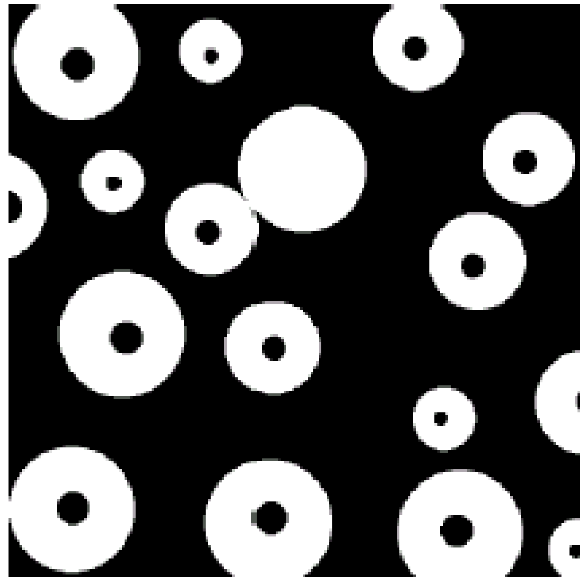
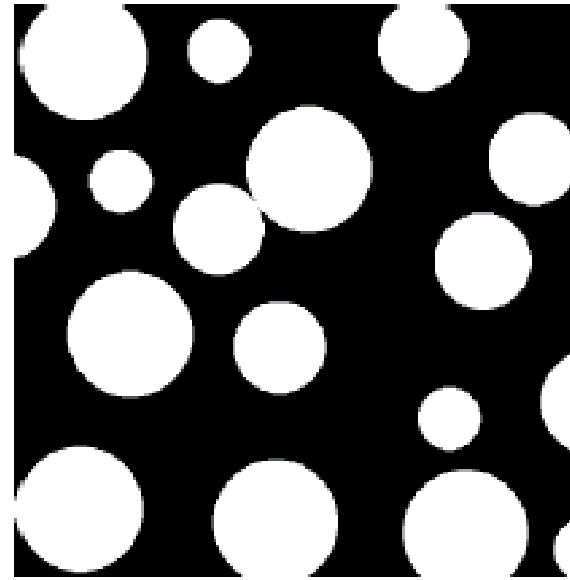
Region Filling Example



Original Image



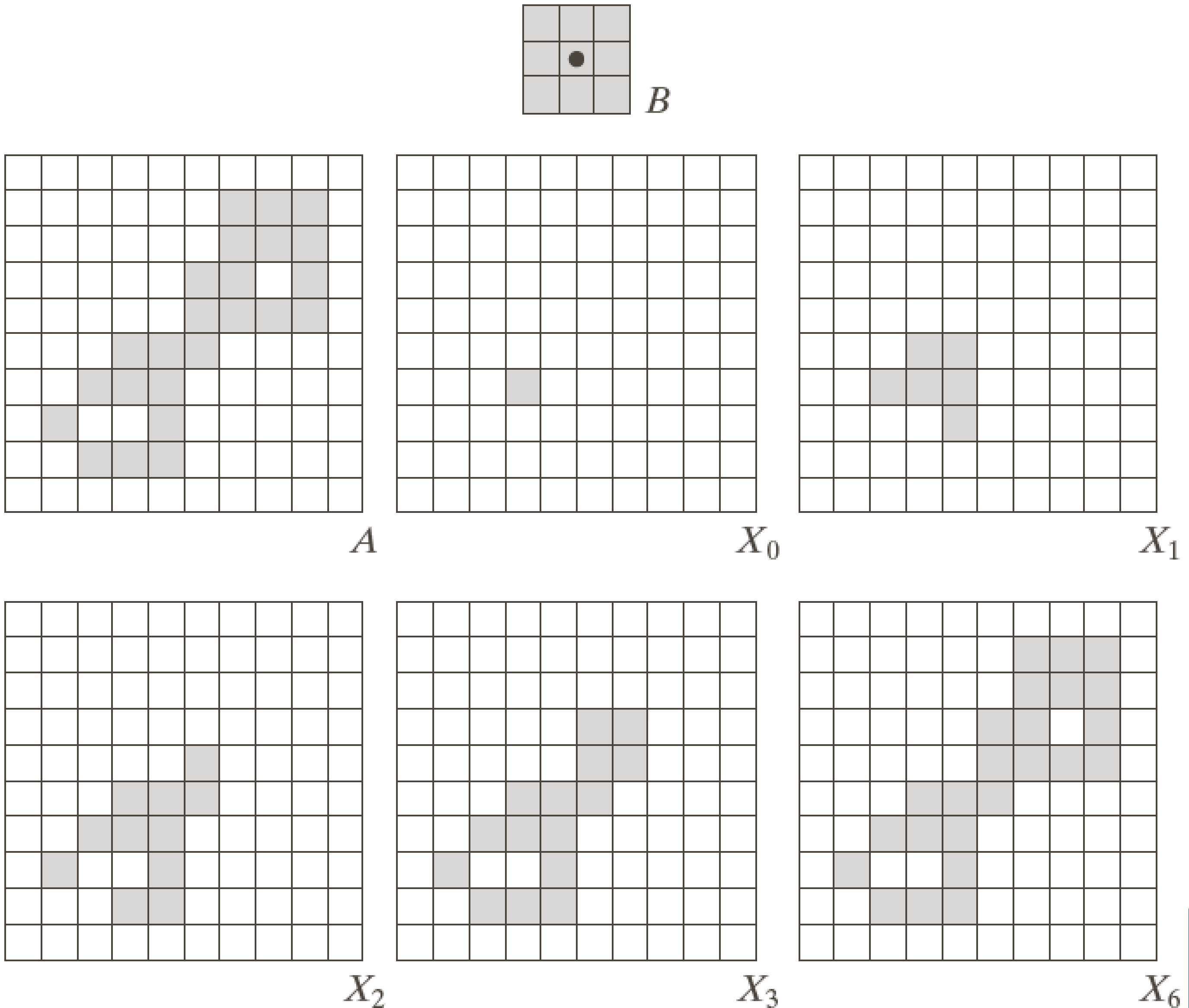
One Region Filled



All Regions Filled

Extraction of Connected Components

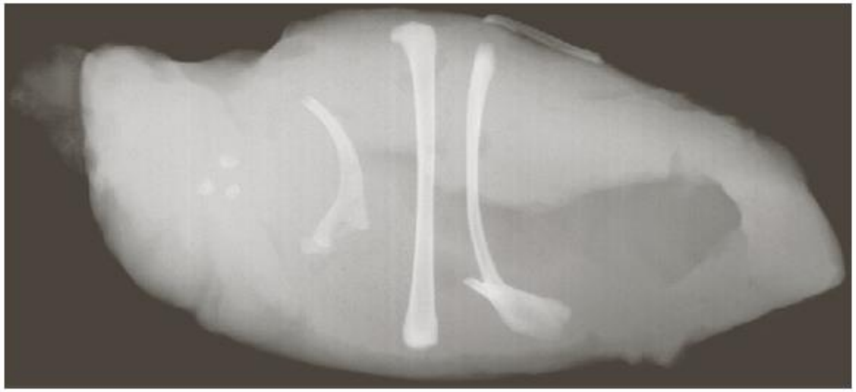
$$X_k = (X_{k-1} \oplus B) \cap A \qquad k = 1, 2, 3, \ldots$$



Extraction of Connected Components

$$X_k = (X_{k-1} \oplus B) \cap A \qquad k = 1, 2, 3, \ldots$$

X-ray Image of Chicken breast



Thresholded

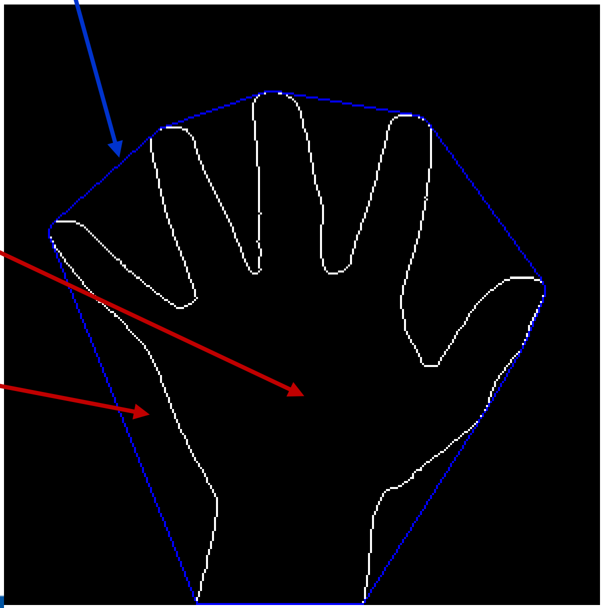
Eroded with a 5x5 SE of 1s

Connected component	No. of pixels in connected comp
01	11
02	9
03	9
04	39
05	133
06	1
07	1
08	743
09	7
10	11
11	11
12	9
13	9
14	674
15	85

Convex Hull

- Convex (凸): set *A* is said to be *convex* if the straight line segment joining any two points in *A* lies entirely within *A*.
- Convex Hull (凸包): *convex hull H* of an arbitrary set *S* is the smallest convex set containing *S*
- Convex Deficiency (凸缺):

set difference *H - S* is called the *convex deficiency* of *S*



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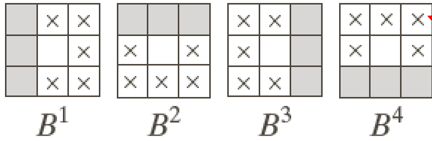
Convex Hull



Don't care

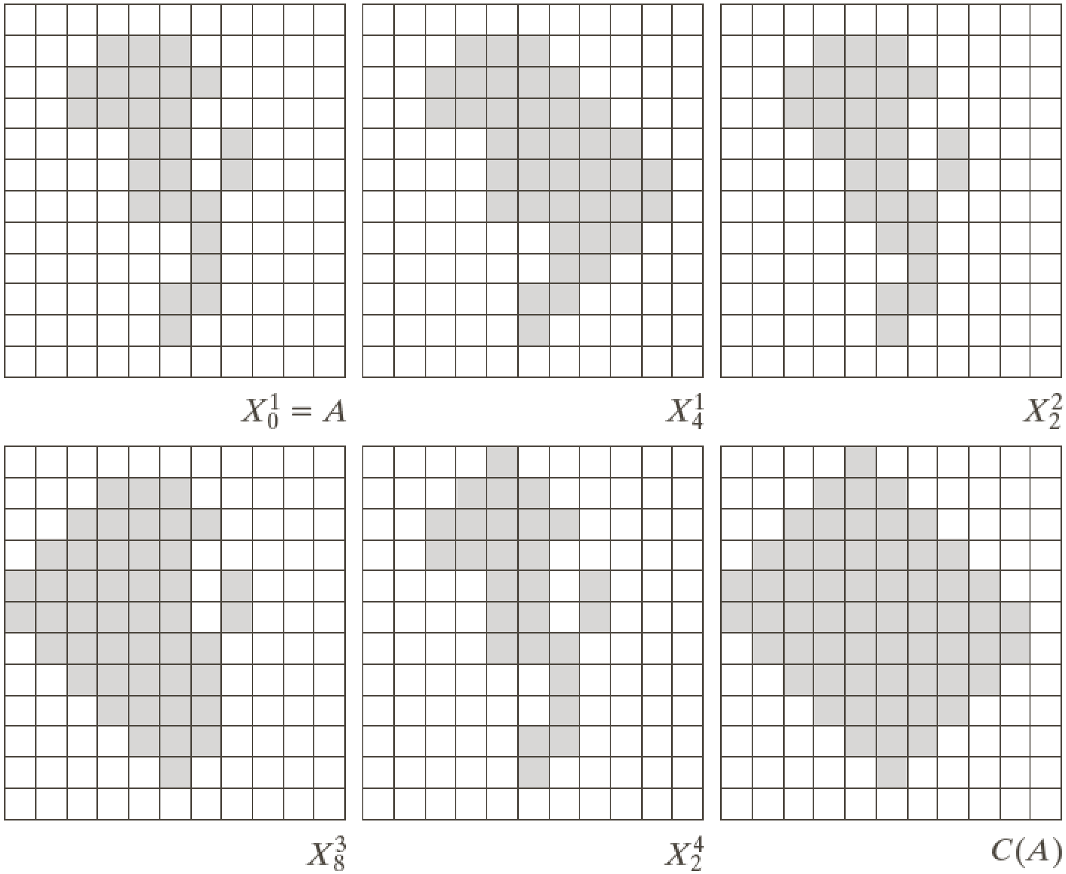
$$X_k^i = (X_{k-1} \circledast B^i) \cup A$$

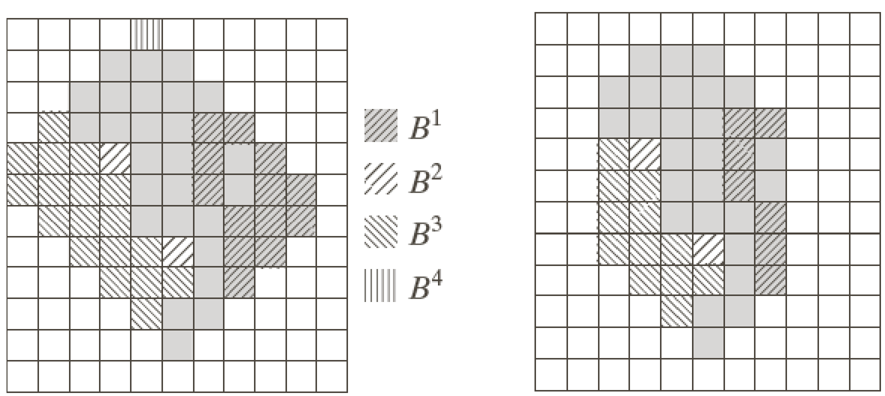
$$i = 1, 2, 3, 4$$

$$k = 1, 2, 3, \ldots$$

$$X_0^i = A \qquad D^i = X_k^i$$

$$C(A) = \bigcup_{i=1}^{4} D^i$$





Limiting growth of the convex hull

Thinning

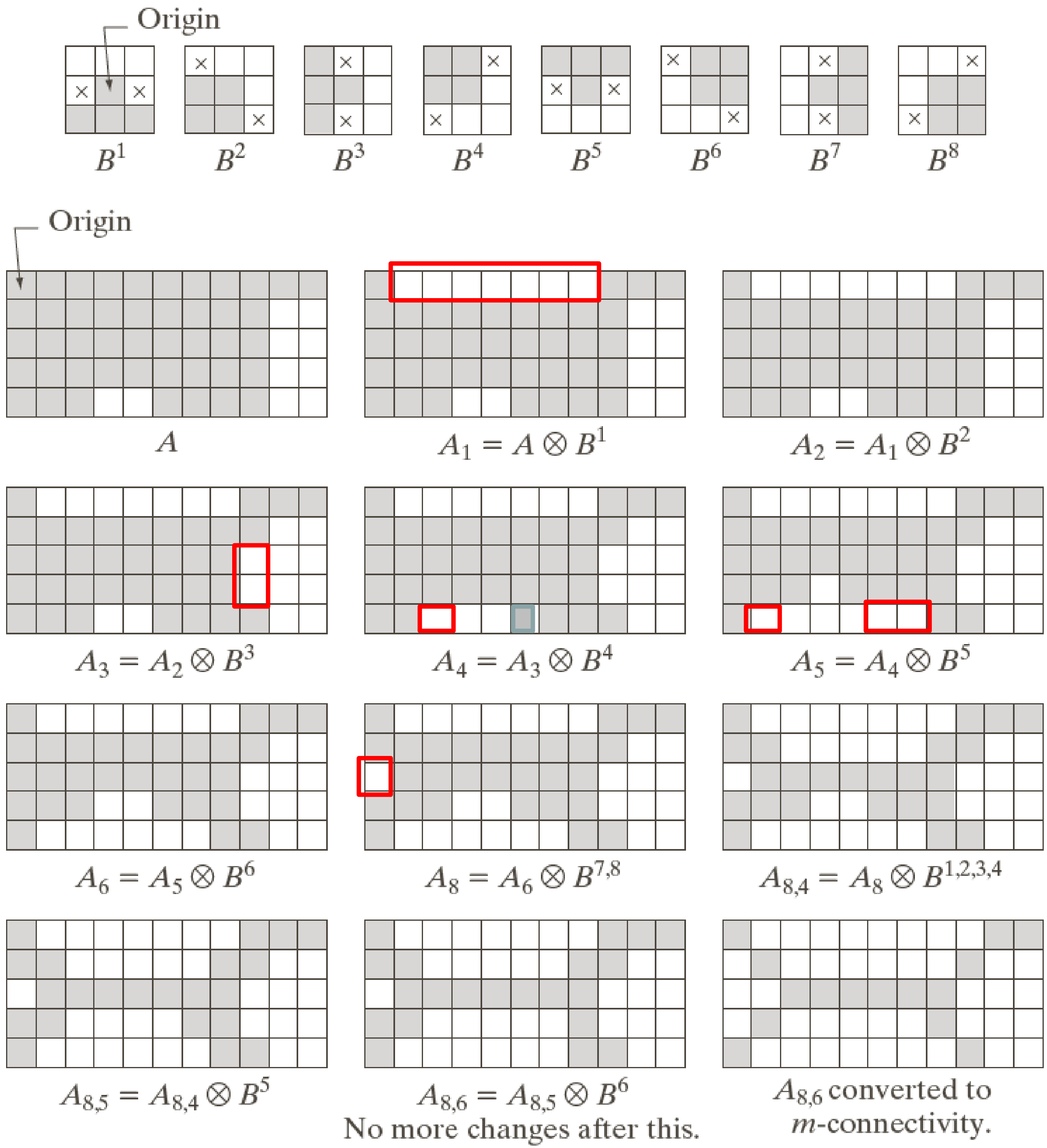
$$A \otimes B = A - (A \circledast B) = A \cap (A \circledast B)^c$$

$$\{B\} = \{B^1, B^2, B^3, \ldots, B^n\}$$

$$B^i \text{ is a rotated version of } B^{i-1}$$

$$A \otimes \{B\} = ((\ldots((A \otimes B^1) \otimes B^2) \ldots) \otimes B^n)$$

Thinning Example



Thickening

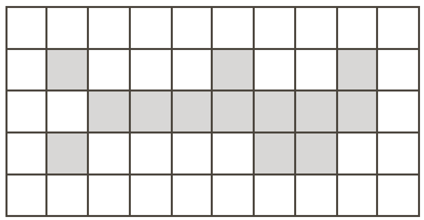
$$A \odot B = A \cup (A \circledast B)$$

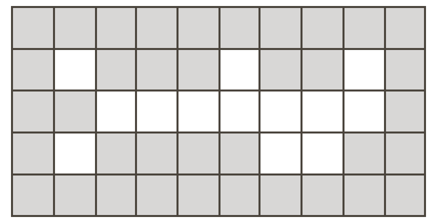
$$A \odot \{B\} = ((\dots((A \odot B^1) \odot B^2) \dots) \odot B^n)$$

Duality:

Thickening the foreground = Thinning the background

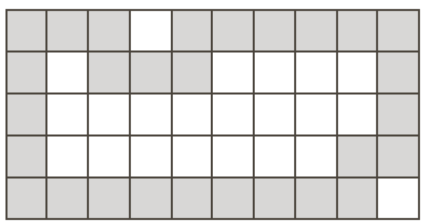
A

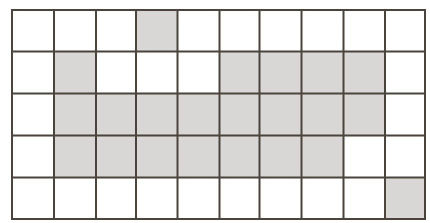




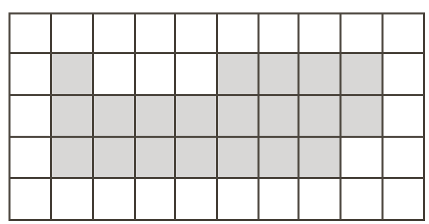
A^c

Thinning of A^c





Thickening of A



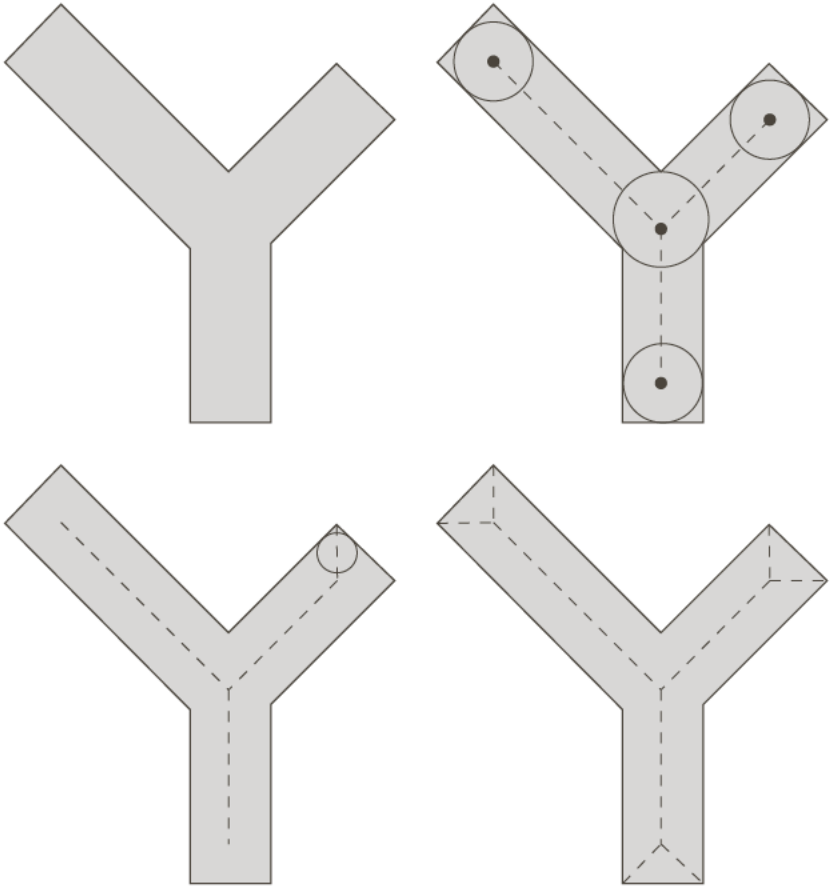
Remove disconnected points

Skeletons（骨架、中轴）

(a) If z is a point of $S(A)$ and $(D)_z$ is the largest disk centered at z and contained in A, one cannot find a larger disk (not necessarily centered at z) containing $(D)_z$ and included in A. The disk $(D)_z$ is called a *maximum disk*.

(b) The disk $(D)_z$ touches the boundary of A at two or more different places.

- 区域边界内切圆的圆心的集合
- 火烧草地：边界上同时点火，假设火蔓延的速度处处相同，火线相遇的地方构成中轴



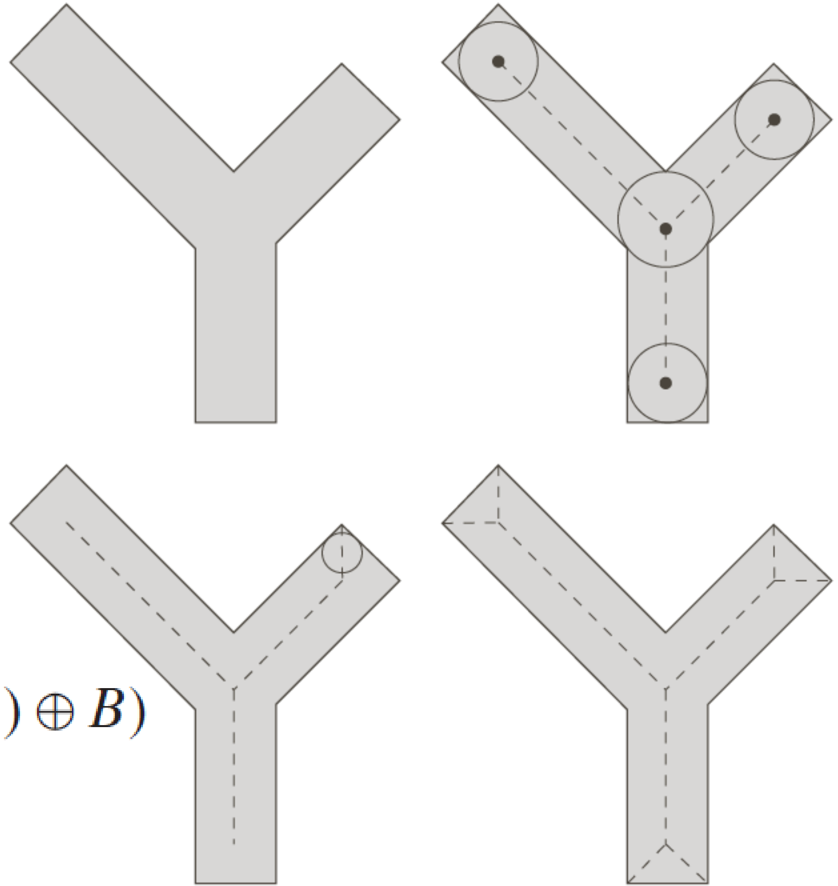
Skeletons（骨架、中轴）

$$S(A) = \bigcup_{k=0}^{K} S_k(A)$$

$$S_k(A) = (A \ominus kB) - (A \ominus kB) \circ B$$

$$(A \ominus kB) = ((\ldots((A \ominus B) \ominus B) \ominus \ldots) \ominus B)$$

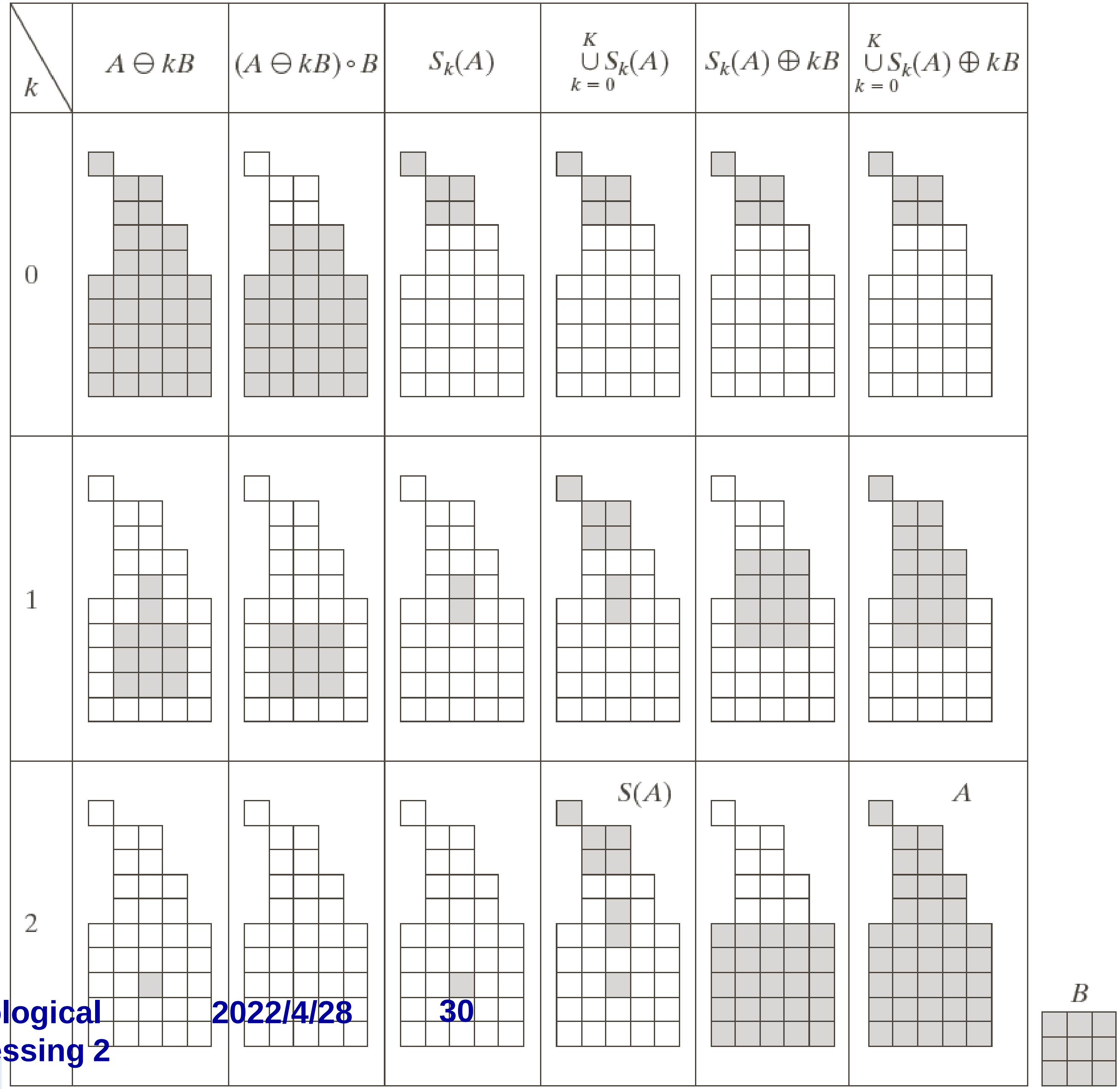
$$K = \max\{k \,|\, (A \ominus kB) \neq \varnothing\}$$



$$A = \bigcup_{k=0}^{K} (S_k(A) \oplus kB)$$

$$(S_k(A) \oplus kB) = ((\ldots((S_k(A) \oplus B) \oplus B) \oplus \ldots) \oplus B)$$

Skeletons Example



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Pruning

- Problem: “spurs” (parasitic components) after thinning and skeletonizing

$$X_1 = A \otimes \{B\}$$

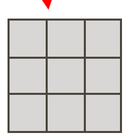
3 times

Thinning

$$X_2 = \bigcup_{k=1}^{8} (X_1 \circledast B^k)$$

Hit-or-Miss

$$X_3 = (X_2 \oplus H) \cap A$$



3 times

$$X_4 = X_1 \cup X_3$$

Morphological Reconstruction

- Geodesic dilation（测地膨胀）
  - *Marker* image *F*: contains the starting points
  - *Mask* image *G*: constrains the transformation
  - *Structuring Element: B*
  - Geodesic dilation of size 1

$$D_G^{(1)}(F) = (F \oplus B) \cap G \qquad\qquad F \subseteq G$$

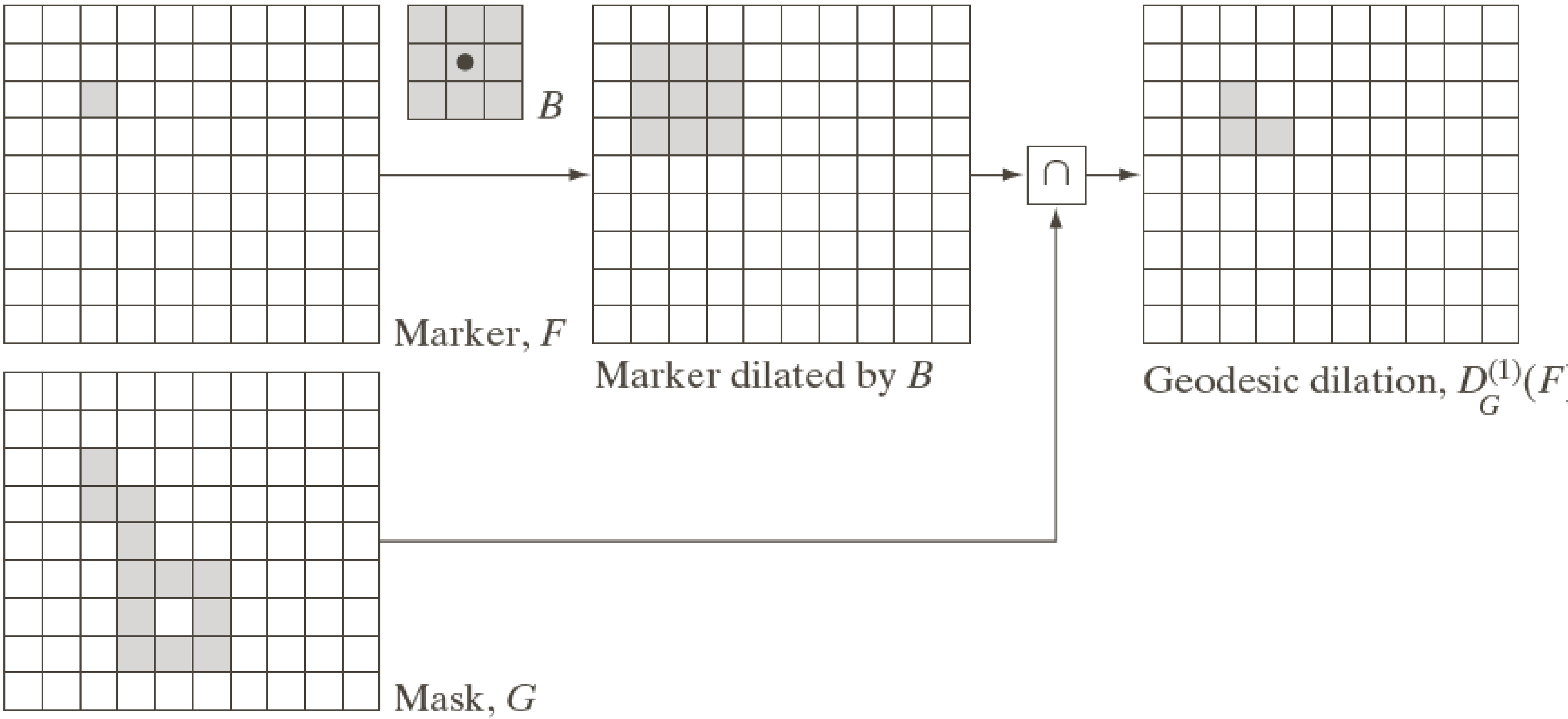
  - Geodesic dilation of size *n*

$$D_G^{(0)}(F) = F$$

$$D_G^{(n)}(F) = D_G^{(1)}\left[D_G^{(n-1)}(F)\right]$$

Morphological Reconstruction

- Geodesic dilation（测地膨胀）



Converges after a finite number of iterations

Morphological Reconstruction

- Geodesic erosion（测地腐蚀）
  - *Marker* image *F*: contains the starting points
  - *Mask* image *G*: constrains the transformation
  - *Structuring Element: B*
  - Geodesic erosion of size 1

$$E_G^{(1)}(F) = (F \ominus B) \cup G$$

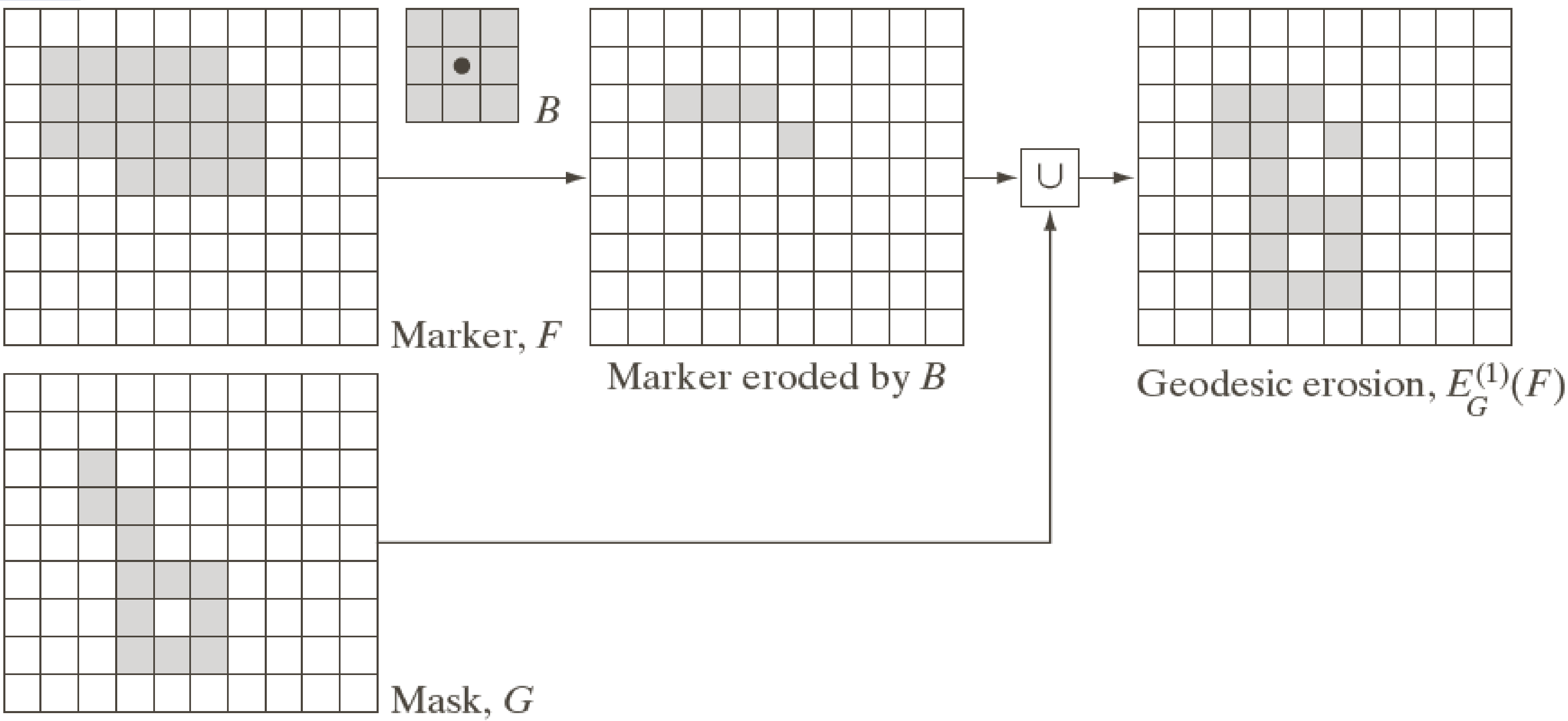
  - Geodesic erosion of size *n*

$$E_G^{(0)}(F) = F$$

$$E_G^{(n)}(F) = E_G^{(1)}\left[E_G^{(n-1)}(F)\right]$$

Morphological Reconstruction

- Geodesic erosion（测地腐蚀）

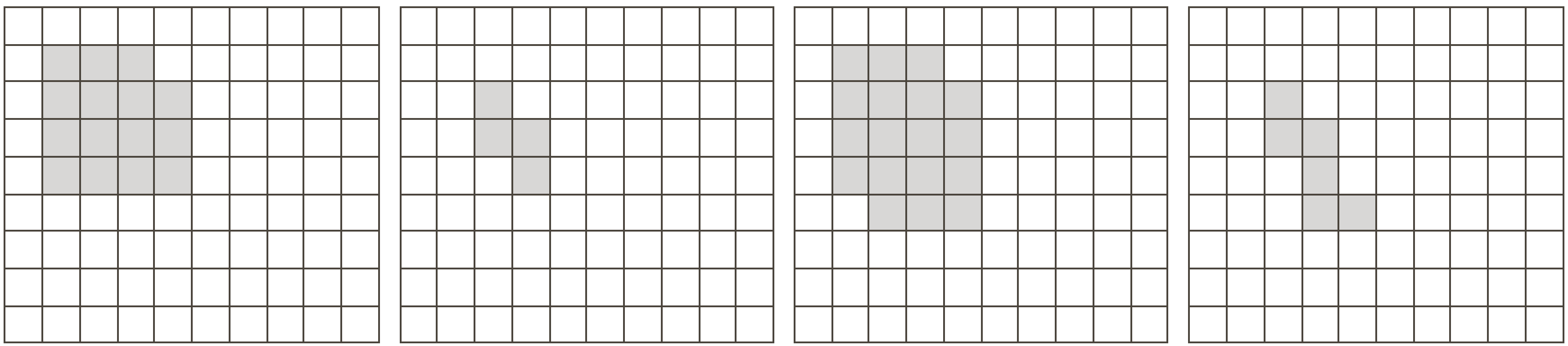


Converges after a finite number of iterations

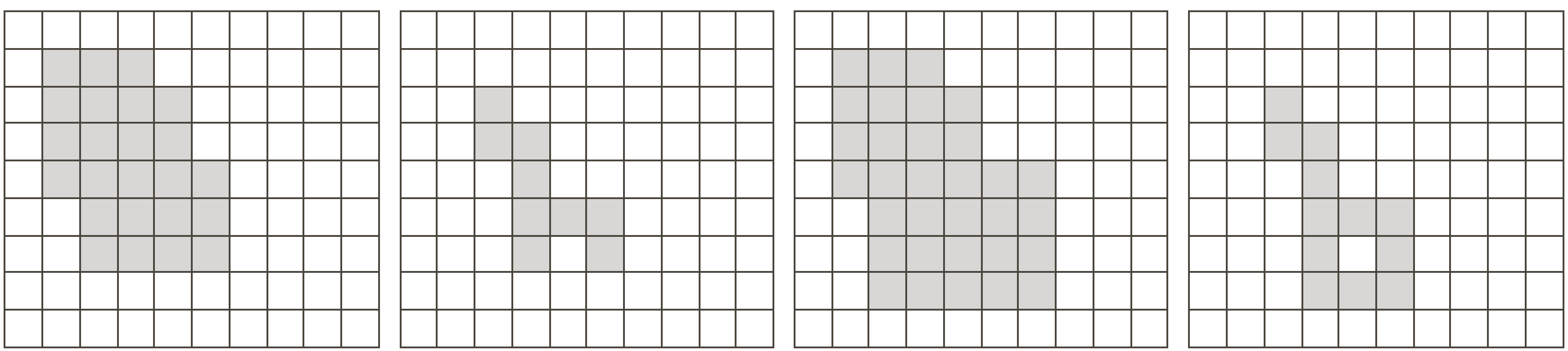
Morphological Reconstruction

- By dilation

$$R_G^D(F) = D_G^{(k)}(F) \text{ with } k \text{ such that } D_G^{(k)}(F) = D_G^{(k+1)}(F)$$



$D_G^{(1)}(F)$ dilated by B | $D_G^{(2)}(F)$ | $D_G^{(2)}(F)$ dilated by B | $D_G^{(3)}(F)$



$D_G^{(3)}(F)$ dilated by B | $D_G^{(4)}(F)$ | $D_G^{(4)}(F)$ dilated by B | $D_G^{(5)}(F) = R_G^D(F)$

Morphological Reconstruction

- By dilation

$$R_G^D(F) = D_G^{(k)}(F) \quad \text{with } k \text{ such that } D_G^{(k)}(F) = D_G^{(k+1)}(F)$$

- By erosion

$$R_G^E(F) = E_G^{(k)}(F) \quad \text{with } k \text{ such that } E_G^{(k)}(F) = E_G^{(k+1)}(F)$$

Sample Application

- Opening by reconstruction

Restores exactly the shapes of the objects that remain after erosion, whereas conventional opening may not

$$O_R^{(n)}(F) = R_F^D\left[(F \ominus nB)\right]$$

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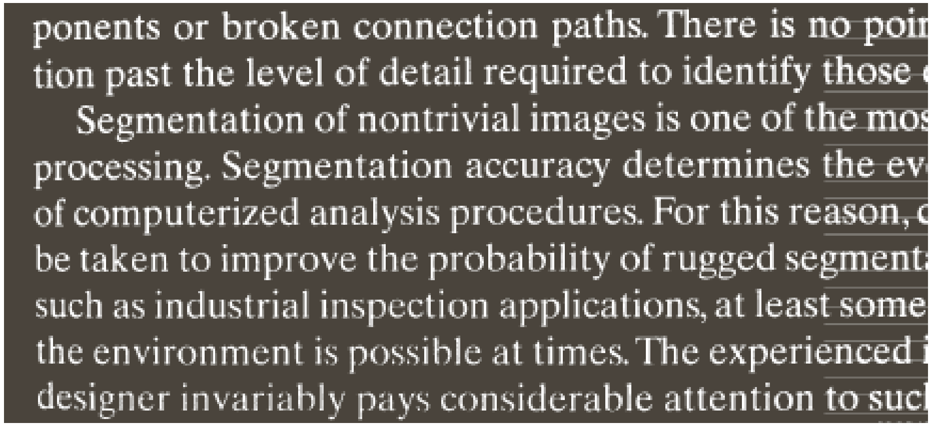
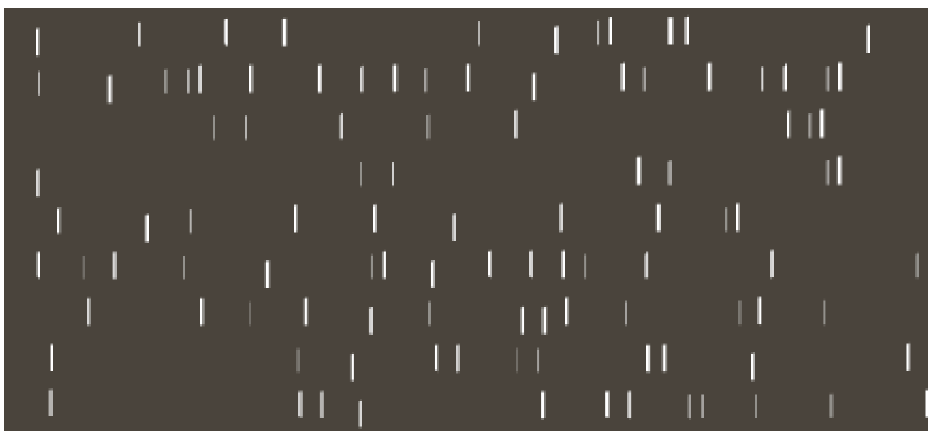
Opening by reconstruction example

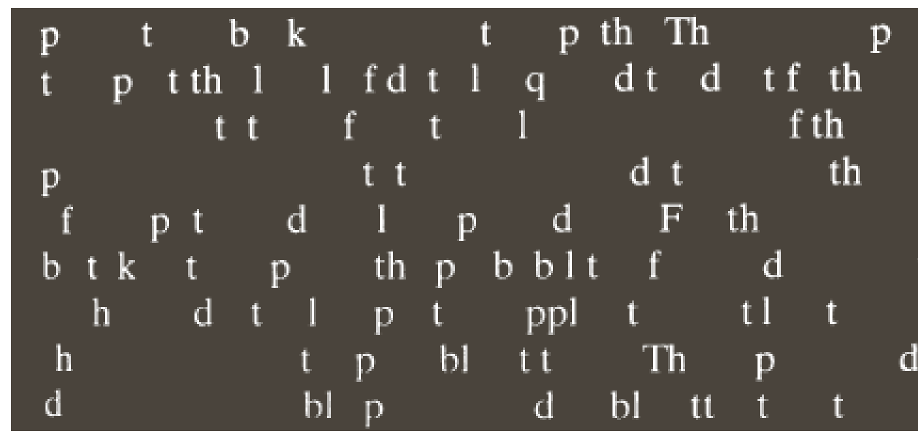
Text image



Erosion by a SE of 51x1 pixels



Opening with the same SE



Opening by reconstruction

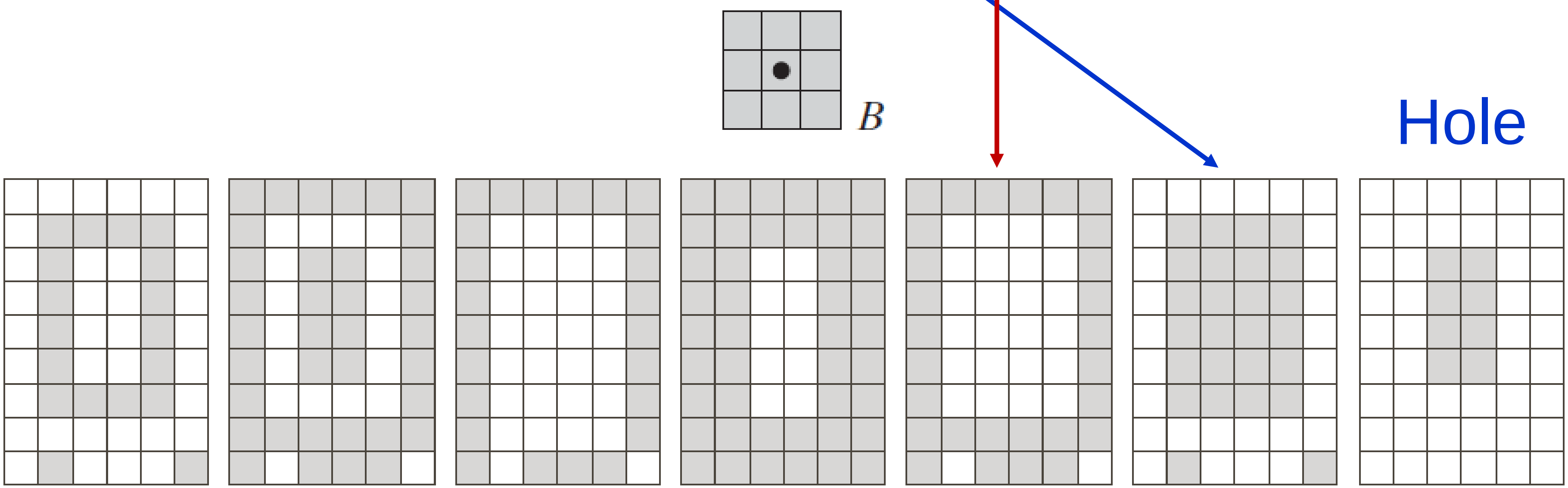
Automated Hole Filling example

$$F(x, y) = \begin{cases} 1 - I(x, y) & \text{if } (x, y) \text{ is on the border of } I \\ 0 & \text{otherwise} \end{cases}$$

Then

$$H = \left[R_{I^c}^{D}(F) \right]^c$$

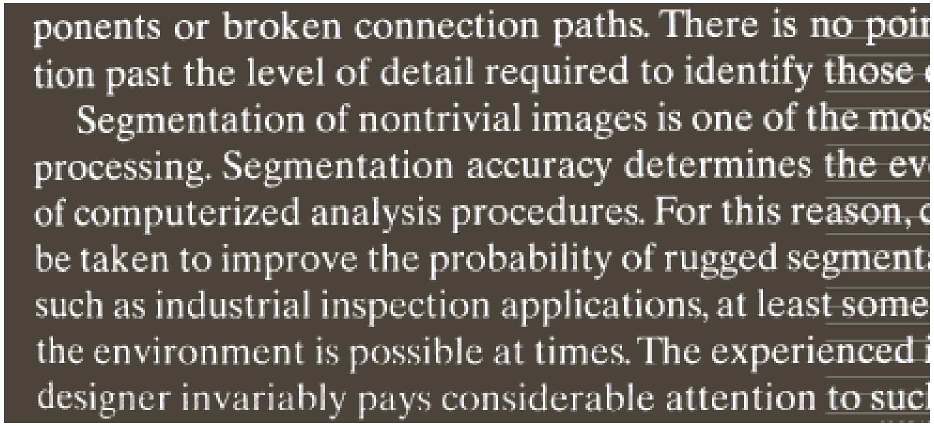
is a binary image equal to I with all holes filled.



I $\quad I^c \quad$ F $\quad F \oplus B \quad$ $F \oplus B \cap I^c \quad$ $H \quad$ $H \cap I^c$

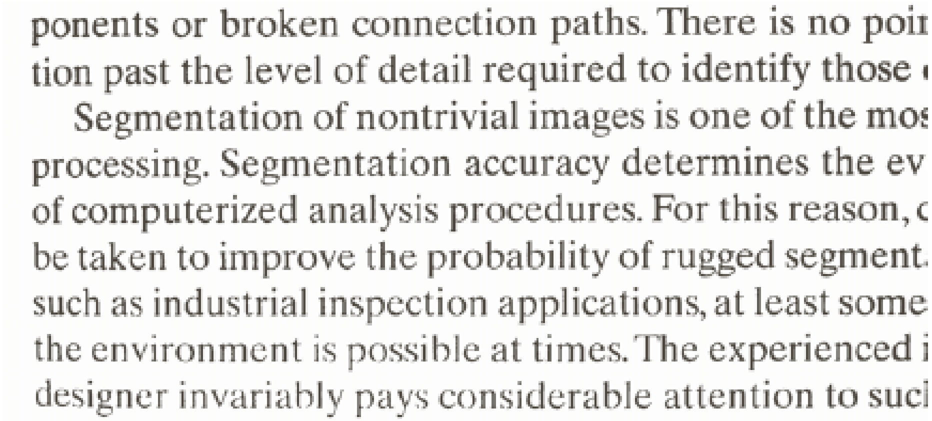
Automated Hole Filling example

Text image



ponents or broken connection paths. There is no poi
tion past the level of detail required to identify those
Segmentation of nontrivial images is one of the mos
processing. Segmentation accuracy determines the ev
of computerized analysis procedures. For this reason, c
be taken to improve the probability of rugged segment
such as industrial inspection applications, at least some
the environment is possible at times. The experienced i
designer invariably pays considerable attention to suc

Complement image



ponents or broken connection paths. There is no poi
tion past the level of detail required to identify those
Segmentation of nontrivial images is one of the mos
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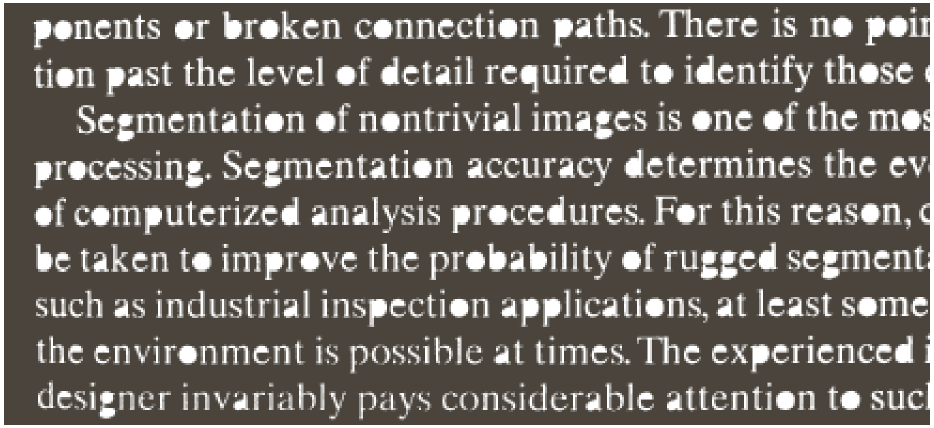
of computerized analysis procedures. For this reason, c
be taken to improve the probability of rugged segment
such as industrial inspection applications, at least some
the environment is possible at times. The experienced i
designer invariably pays considerable attention to suc

Marker image



ponents or broken connection paths. There is no poi
tion past the level of detail required to identify those
Segmentation of nontrivial images is one of the mos
processing. Segmentation accuracy determines the ev
of computerized analysis procedures. For this reason, c
be taken to improve the probability of rugged segment
such as industrial inspection applications, at least some
the environment is possible at times. The experienced i
designer invariably pays considerable attention to suc

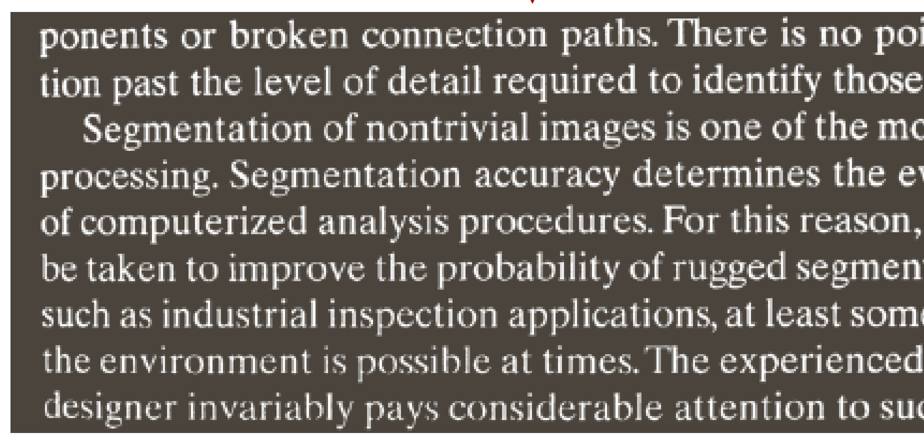
Holes Filled

Border Clearing example

$$F(x, y) = \begin{cases} I(x, y) & \text{if } (x, y) \text{ is on the border of } I \\ 0 & \text{otherwise} \end{cases}$$

$$X = I - R_I^D(F)$$

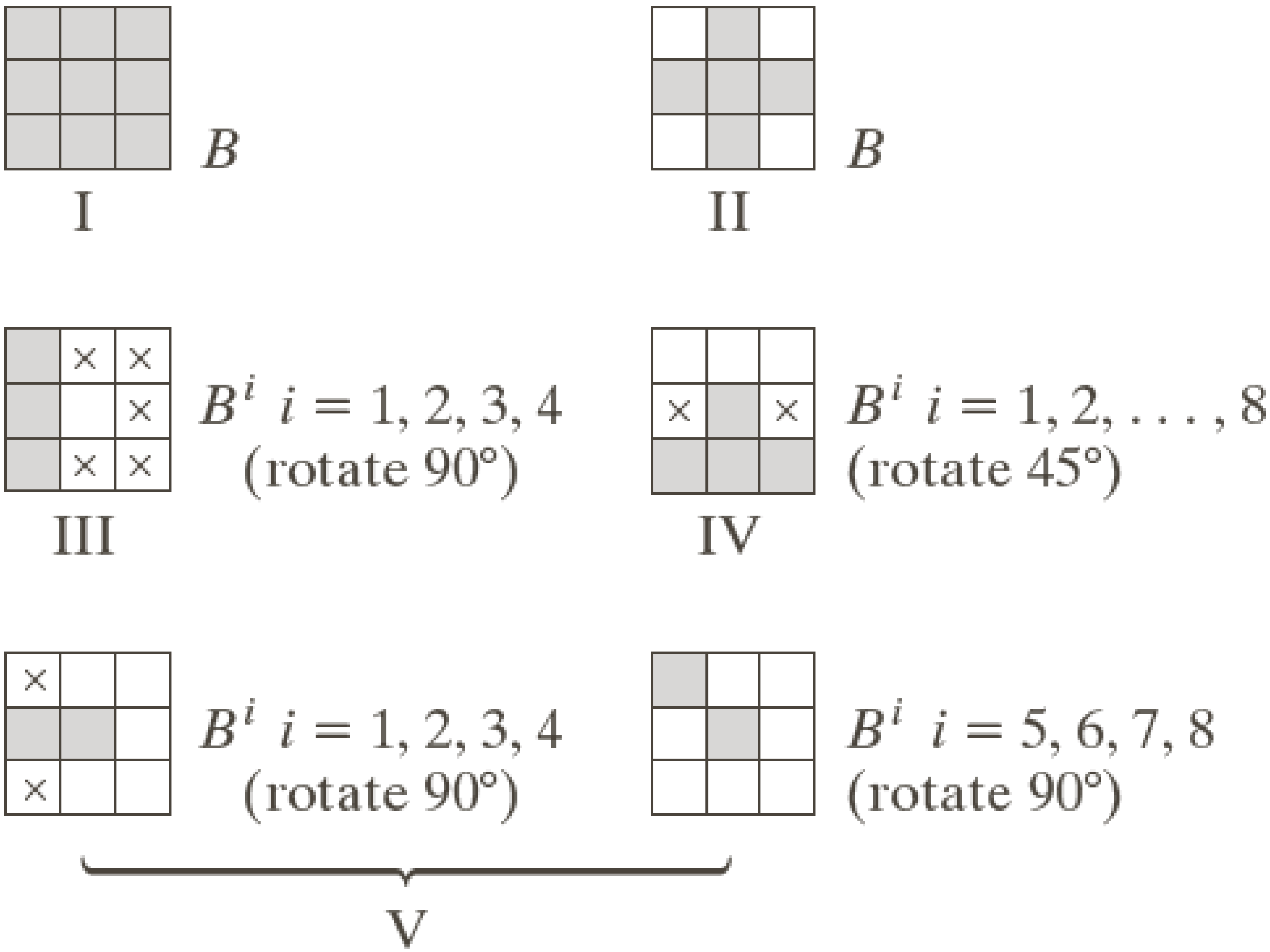
Marker image



ponents or broken connection paths. There is no poi
tion past the level of detail required to identify those
Segmentation of nontrivial images is one of the mo
processing. Segmentation accuracy determines the ev
of computerized analysis procedures. For this reason,
be taken to improve the probability of rugged segment
such as industrial inspection applications, at least some
the environment is possible at times. The experienced
designer invariably pays considerable attention to suc

Border Cleared

Summary of Morphological Operations on Binary Images



Basic types of structuring elements

Summary (cont…)

Operation	Equation	Comments (The Roman numerals refer to the structuring elements in Fig. 9.33.)
Translation	$(B)_z = \{w \mid w = b + z, \text{ for } b \in B\}$	Translates the origin of B to point z.
Reflection	$\hat{B} = \{w \mid w = -b, \quad \text{for } b \in B\}$	Reflects all elements of B about the origin of this set.
Complement	$A^c = \{w \mid w \notin A\}$	Set of points not in A.
Difference	$A - B = \{w \mid w \in A, w \notin B\} = A \cap B^c$	Set of points that belong to A but not to B.
Dilation	$A \oplus B = \left\{z \mid (\hat{B}_z) \cap A \neq \varnothing\right\}$	"Expands" the boundary of A. (I)
Erosion	$A \ominus B = \left\{z \mid (B)_z \subseteq A\right\}$	"Contracts" the boundary of A. (I)
Opening	$A \circ B = (A \ominus B) \oplus B$	Smoothes contours, breaks narrow isthmuses, and eliminates small islands and sharp peaks. (I)

(*Continued*)

Summary (cont…)

Operation	Equation	Comments (The Roman numerals refer to the structuring elements in Fig. 9.33.)
Closing	$A \bullet B = (A \oplus B) \ominus B$	Smoothes contours, fuses narrow breaks and long thin gulfs, and eliminates small holes. (I)
Hit-or-miss transform	$A \circledast B = (A \ominus B_1) \cap (A^c \ominus B_2) = (A \ominus B_1) - (A \oplus \hat{B}_2)$	The set of points (coordinates) at which, simultaneously, B_1 found a match ("hit") in A and B_2 found a match in A^c
Boundary extraction	$\beta(A) = A - (A \ominus B)$	Set of points on the boundary of set A. (I)
Hole filling	$X_k = (X_{k-1} \oplus B) \cap A^c$; $k = 1, 2, 3, \ldots$	Fills holes in A; $X_0 =$ array of 0s with a 1 in each hole. (II)
Connected components	$X_k = (X_{k-1} \oplus B) \cap A$; $k = 1, 2, 3, \ldots$	Finds connected components in A; $X_0 =$ array of 0s with a 1 in each connected component. (I)
Convex hull	$X_k^i = (X_{k-1}^i \circledast B^i) \cup A$; $i = 1, 2, 3, 4$; $k = 1, 2, 3, \ldots$; $X_0^i = A$; and $D^i = X_{\text{conv}}^i$	Finds the convex hull $C(A)$ of set A, where "conv" indicates convergence in the sense that $X_k^i = X_{k-1}^i$. (III)

Summary (cont…)

Thinning	$A \otimes B = A - (A \circledast B)$ $= A \cap (A \circledast B)^c$ $A \otimes \{B\} =$ $((\ldots((A \otimes B^1) \otimes B^2)\ldots) \otimes B^n)$ $\{B\} = \{B^1, B^2, B^3, \ldots, B^n\}$	Thins set A. The first two equations give the basic definition of thinning. The last equations denote thinning by a sequence of structuring elements. This method is normally used in practice. (IV)
Thickening	$A \odot B = A \cup (A \circledast B)$ $A \odot \{B\} =$ $((\ldots(A \odot B^1) \odot B^2 \ldots) \odot B^n)$	Thickens set A. (See preceding comments on sequences of structuring elements.) Uses IV with 0s and 1s reversed.
Skeletons	$S(A) = \bigcup_{k=0}^{K} S_k(A)$ $S_k(A) = \bigcup_{k=0}^{K} \{(A \ominus kB) - [(A \ominus kB) \circ B]\}$ Reconstruction of A: $A = \bigcup_{k=0}^{K} (S_k(A) \oplus kB)$	Finds the skeleton $S(A)$ of set A. The last equation indicates that A can be reconstructed from its skeleton subsets $S_k(A)$. In all three equations, K is the value of the iterative step after which the set A erodes to the empty set. The notation $(A \ominus kB)$ denotes the kth iteration of successive erosions of A by B. (I)

(*Continued*)

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Summary (cont…)

Pruning	$X_1 = A \otimes \{B\}$ $X_2 = \bigcup_{k=1}^{8} (X_1 \circledast B^k)$ $X_3 = (X_2 \oplus H) \cap A$ $X_4 = X_1 \cup X_3$	X_4 is the result of pruning set A. The number of times that the first equation is applied to obtain X_1 must be specified. Structuring elements V are used for the first two equations. In the third equation H denotes structuring element I.
Geodesic dilation of size 1	$D_G^{(1)}(F) = (F \oplus B) \cap G$	F and G are called the *marker* and *mask* images, respectively.
Geodesic dilation of size n	$D_G^{(n)}(F) = D_G^{(1)}\left[D_G^{(n-1)}(F)\right]$; $D_G^{(0)}(F) = F$	
Geodesic erosion of size 1	$E_G^{(1)}(F) = (F \ominus B) \cup G$	
Geodesic erosion of size n	$E_G^{(n)}(F) = E_G^{(1)}\left[E_G^{(n-1)}(F)\right]$; $E_G^{(0)}(F) = F$	

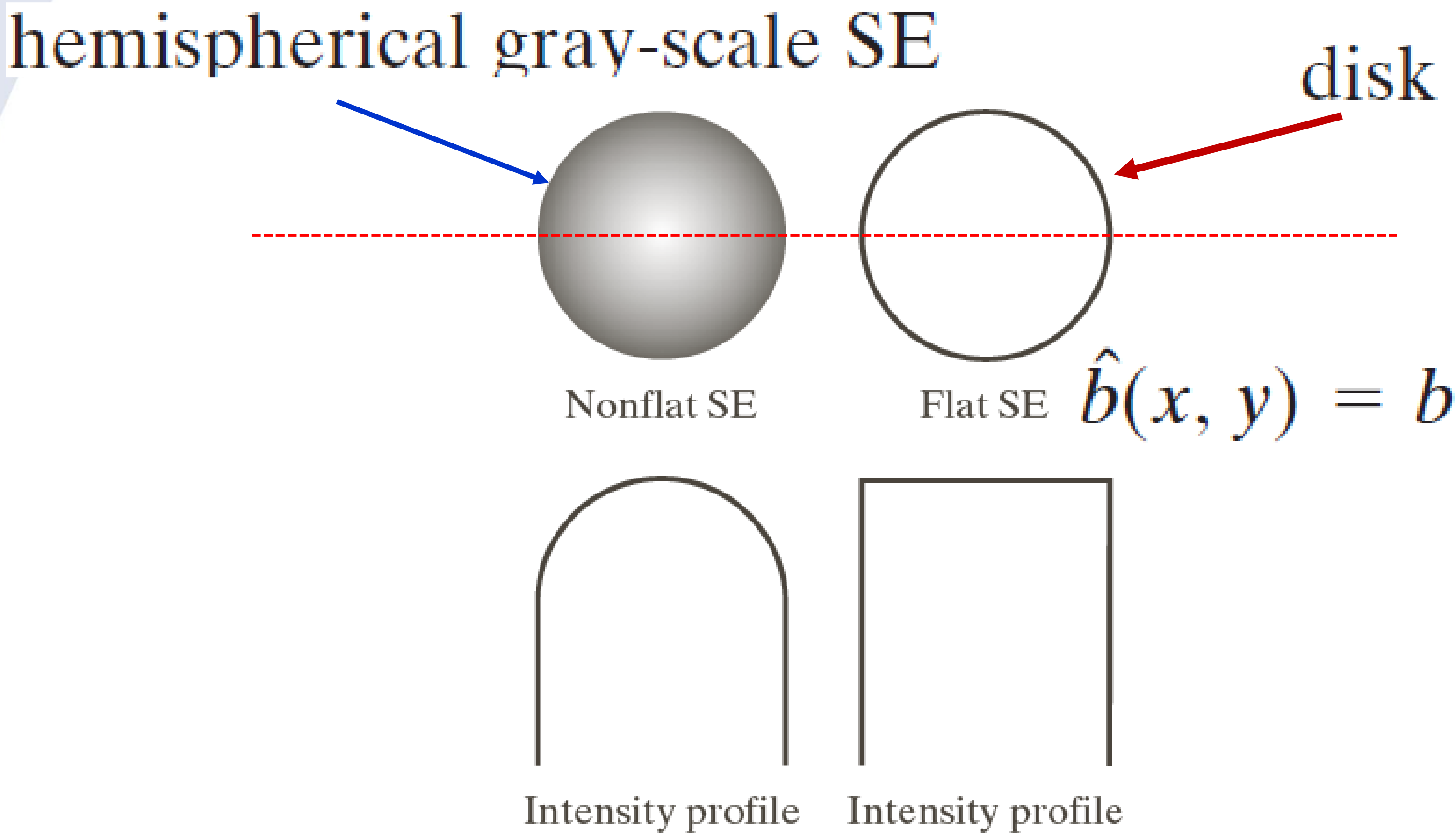
Summary (cont…)

Morphological reconstruction by dilation	$R_G^D(F) = D_G^{(k)}(F)$	k is such that $D_G^{(k)}(F) = D_G^{(k+1)}(F)$
Morphological reconstruction by erosion	$R_G^E(F) = E_G^{(k)}(F)$	k is such that $E_G^{(k)}(F) = E_G^{(k+1)}(F)$
Opening by reconstruction	$O_R^{(n)}(F) = R_F^D[(F \ominus nB)]$	$(F \ominus nB)$ indicates n erosions of F by B.
Closing by reconstruction	$C_R^{(n)}(F) = R_F^E[(F \oplus nB)]$	$(F \oplus nB)$ indicates n dilations of F by B.
Hole filling	$H = \left[R_{I^c}^D(F)\right]^c$	H is equal to the input image I, but with all holes filled. See Eq. (9.5-28) for the definition of the marker image F.
Border clearing	$X = I - R_I^D(F)$	X is equal to the input image I, but with all objects that touch (are connected to) the boundary removed. See Eq. (9.5-30) for the definition of the marker image F.

L09
Imag

Gray-Scale Morphology

- f(x, y): gray-scale image
- b(x, y): structuring element



$$\hat{b}(x, y) = b(-x - y)$$

Erosion and Dilation by a Nonflat SE

$$[f \ominus b_N](x, y) = \min_{(s,t)\in b_N}\{f(x+s, y+t) - b_N(s,t)\}$$

$$[f \oplus b_N](x, y) = \max_{(s,t)\in b_N}\{f(x-s, y-t) + b_N(s,t)\}$$

Duality:

$$(f \ominus b)^c(x, y) = (f^c \oplus \hat{b})(x, y)$$

where $f^c = -f(x, y)$ and $\hat{b} = b(-x, -y)$

$$(f \oplus b)^c = (f^c \ominus \hat{b})$$

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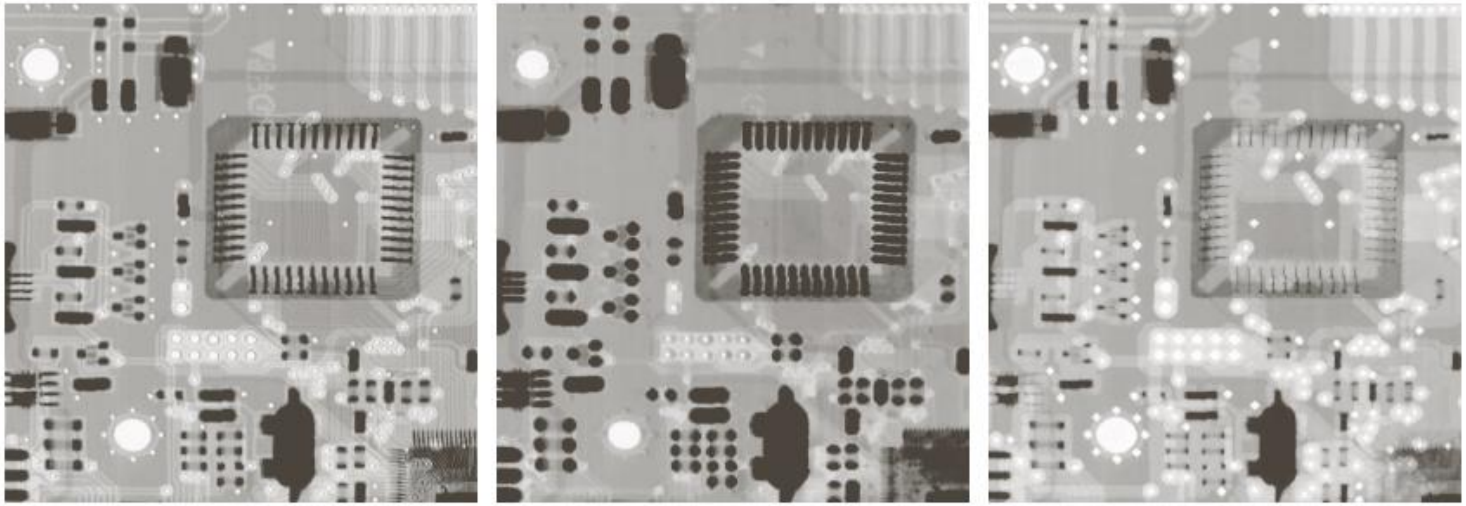
Erosion and Dilation by a Flat SE

$$[f \ominus b](x, y) = \min_{(s,t)\in b} \{f(x+s, y+t)\}$$

$$[f \oplus b](x, y) = \max_{(s,t)\in b} \{f(x-s, y-t)\}$$



Original image After Erosion After Dilation

SE: a flat disk with a radius of two pixels

Erosion and Dilation by a Flat SE

$$[f \ominus b](x, y) = \min_{(s,t)\in b}\{f(x+s, y+t)\}$$

$$[f \oplus b](x, y) = \max_{(s,t)\in b}\{f(x-s, y-t)\}$$

Original image

After Erosion

After Dilation

Opening and Closing

$$f \circ b = (f \ominus b) \oplus b$$

$$f \bullet b = (f \oplus b) \ominus b$$

Duality:

$$(f \bullet b)^c = f^c \circ \hat{b}$$

$$f^c = -f(x, y) \quad \Longrightarrow \quad -(f \bullet b) = (-f \circ \hat{b})$$

$$(f \circ b)^c = f^c \bullet \hat{b}$$

1D Opening and Closing Example

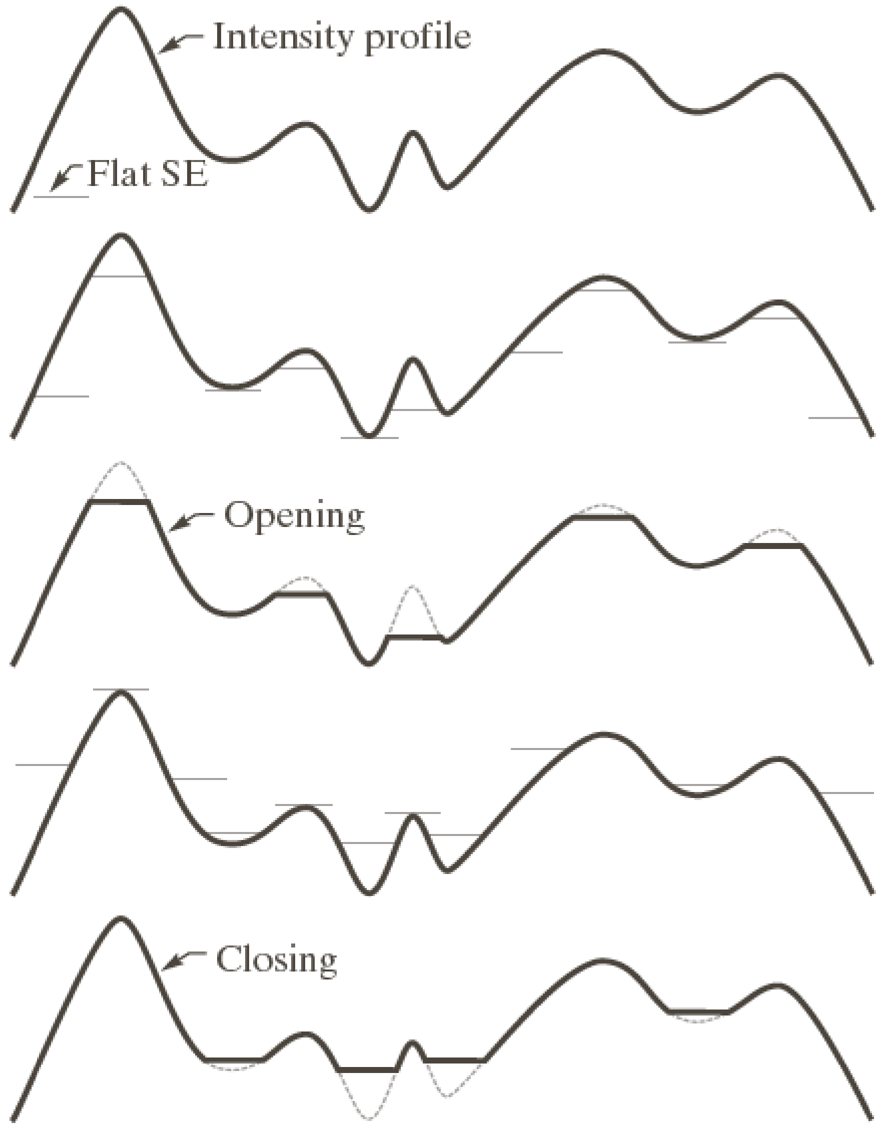
Original signal

Flat SE pushed up underneath the signal

Opening

Flat SE pushed down along the top of the signal

Closing



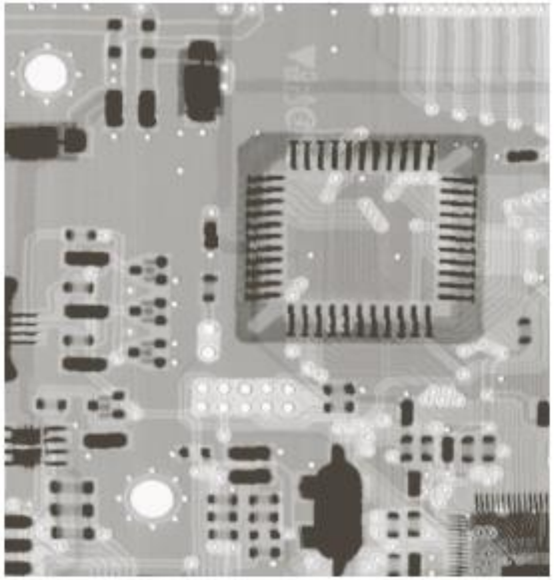
Opening and Closing Properties

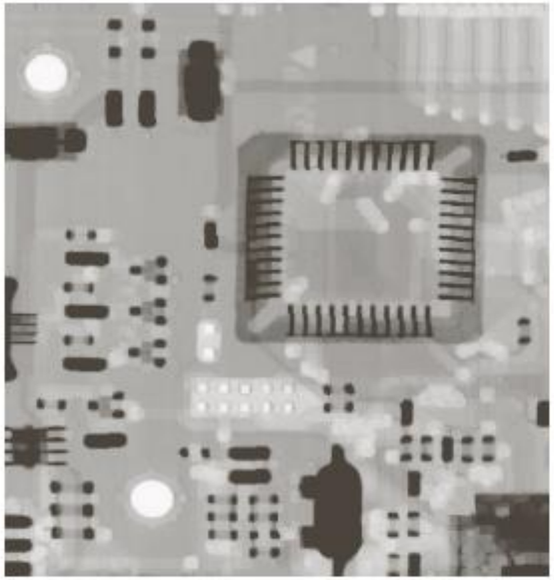
(a) $f \circ b \dashv f$
(b) If $f_1 \dashv f_2$, then $(f_1 \circ b) \dashv (f_2 \circ b)$
(c) $(f \circ b) \circ b = f \circ b$

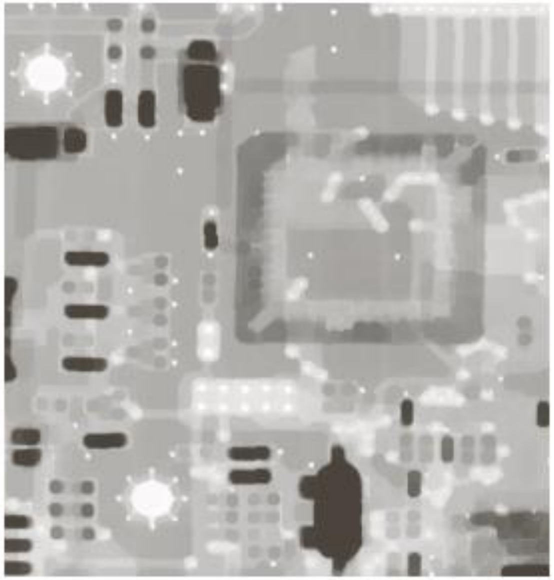
The notation $e \dashv r$ is used to indicate that the domain of e is a subset of the domain of r, and also that $e(x, y) \leq r(x, y)$ for any (x, y) in the domain of e.

(a) $f \dashv f \bullet b$
(b) If $f_1 \dashv f_2$, then $(f_1 \bullet b) \dashv (f_2 \bullet b)$
(c) $(f \bullet b) \bullet b = f \bullet b$

1D Opening and Closing Example







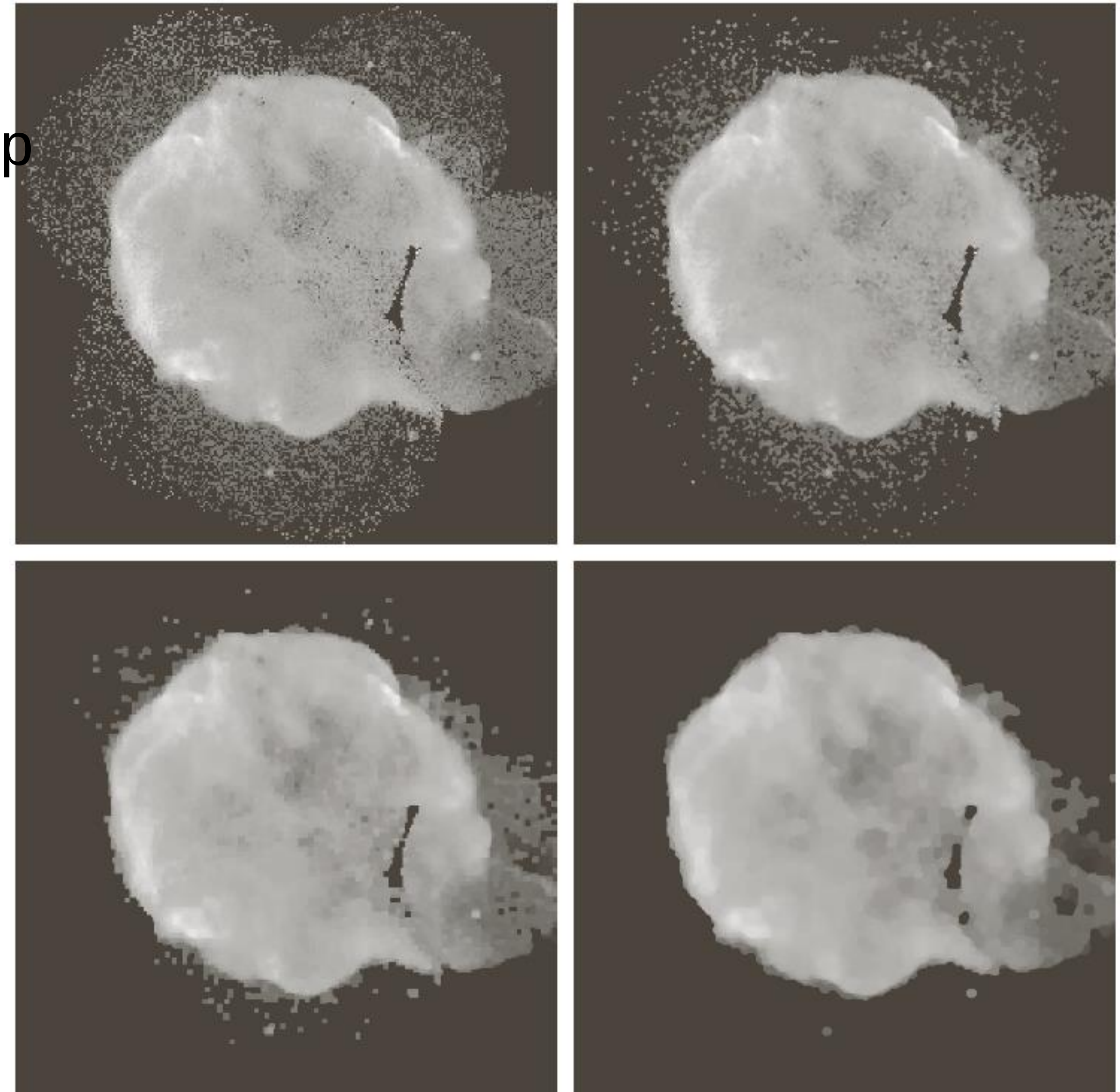
Original image
448 x 425

Opening using a disk SE of radius 3

Closing using a disk SE of radius 5

Morphological Smoothing: opening + closing

Cygnus Loop supernova
天鹅座环超新星
X-ray band



Disk SE of radius 1

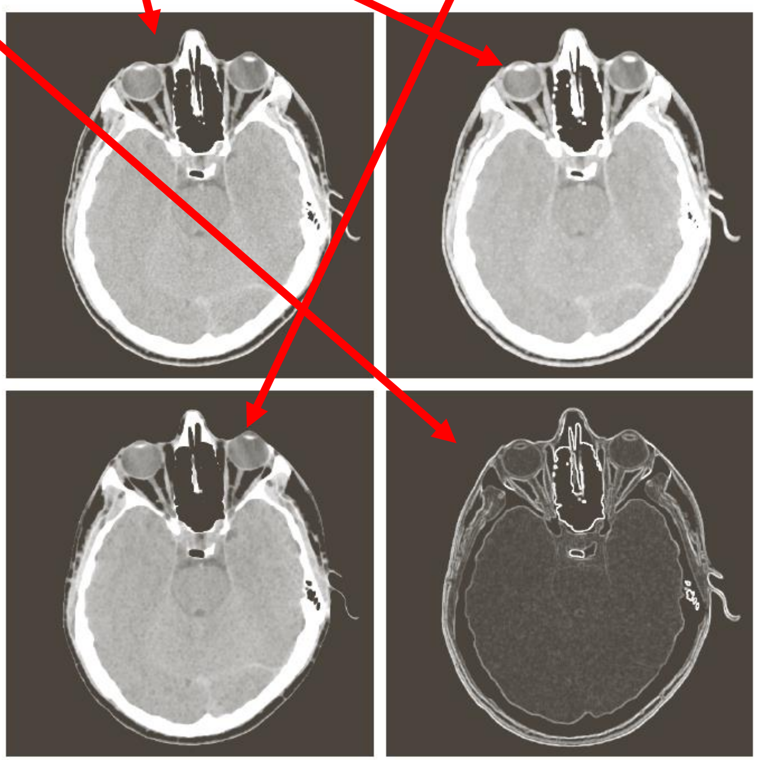
Disk SE of radius 3

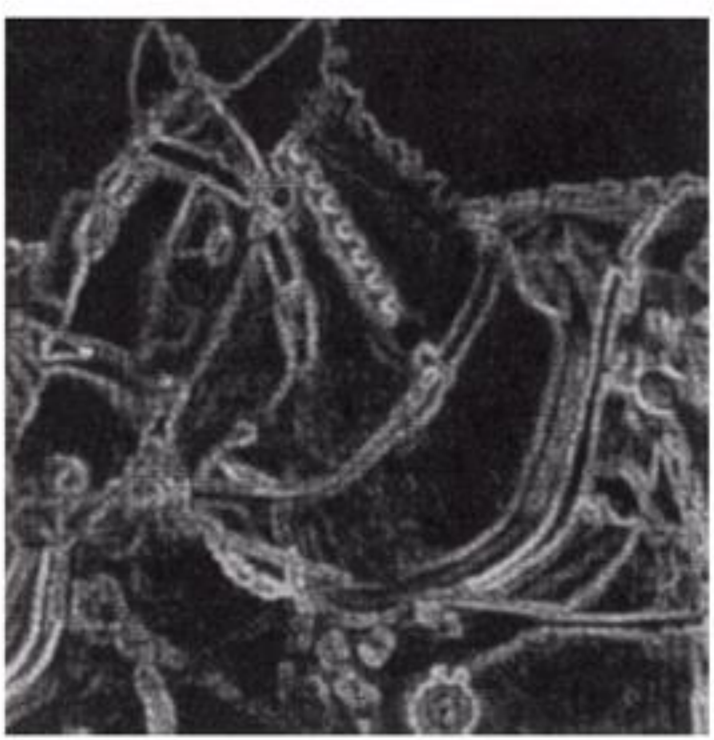
Disk SE of radius 5

Morphological gradient

- Dilation thickens regions and erosion shrinks them, so the difference emphasizes the boundaries between regions

$$g = (f \oplus b) - (f \ominus b)$$

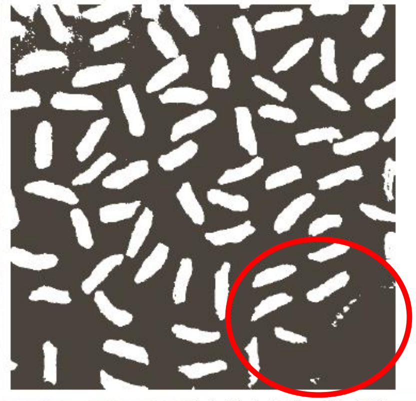




Top-hat and bottom-hat transformations

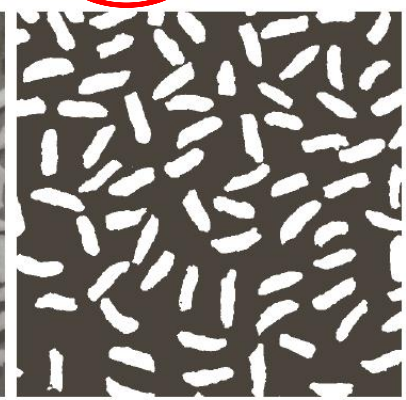
- White Top-hat $T_{\mathrm{hat}}(f) = f - (f \circ b)$
- Black Bottom-hat $B_{\mathrm{hat}}(f) = (f \bullet b) - f$

Original image



Thresholding

Nonuniform illumination



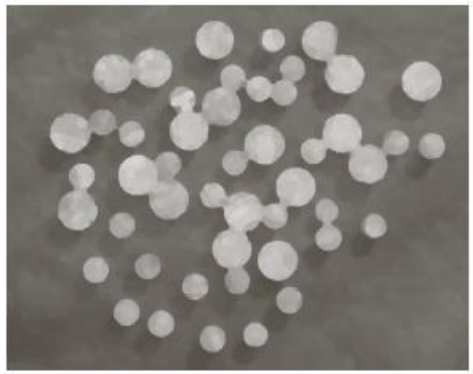
Opening Top-hat Thresholding

Granulometry

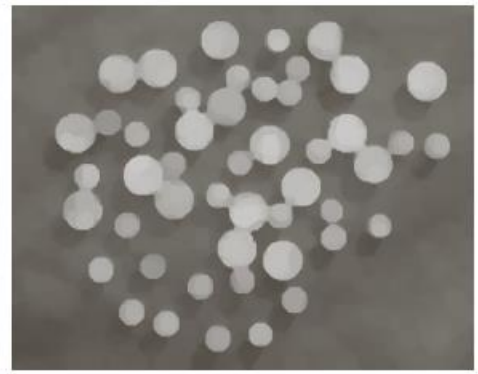
- Determine the size distribution of particles

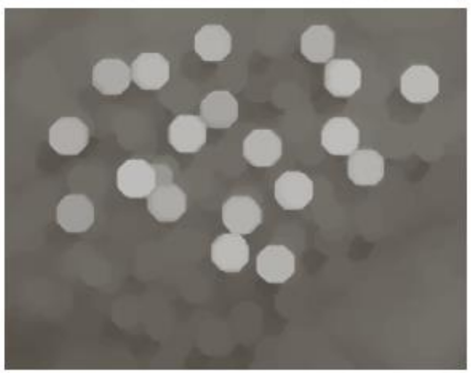
wood dowels

smoothing

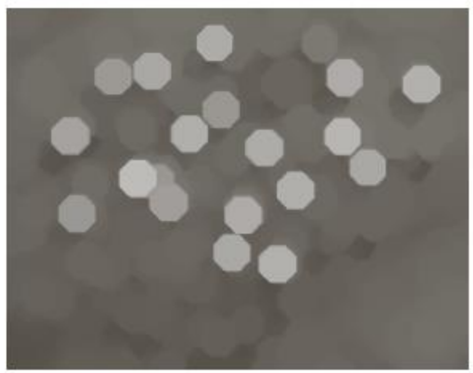


Opening with a disk of radius 10





radius 20

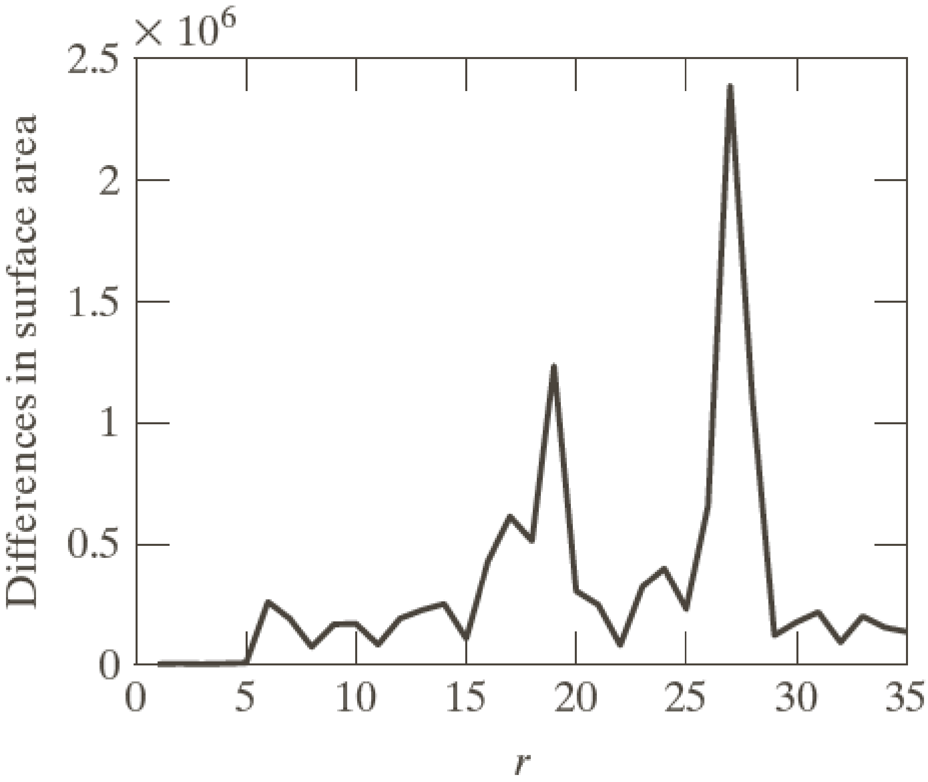


radius 25

radius 30

Granulometry

Differences in surface area as a function of SE disk radius, *r*. The two peaks are indicative of two dominant particle sizes in the image.



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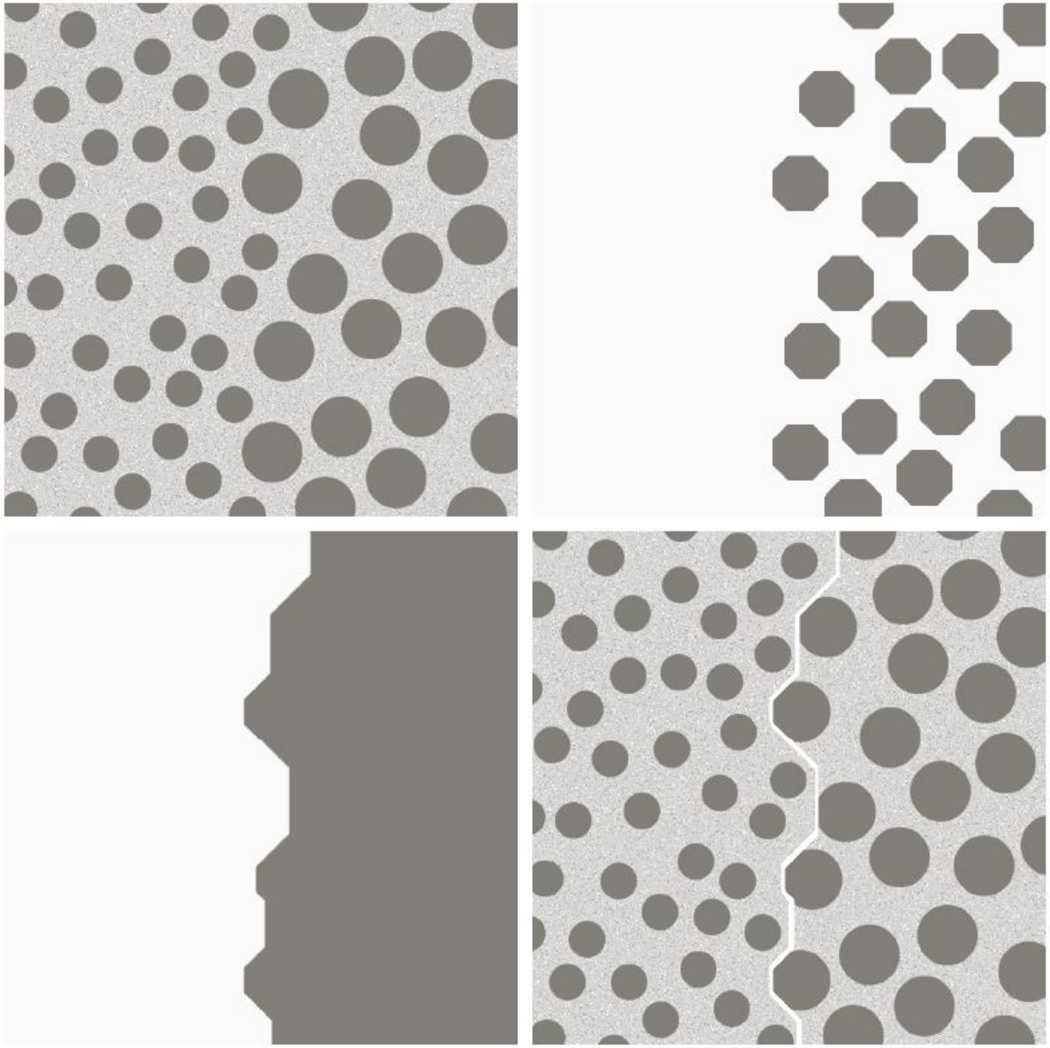
Textual Segmentation

Original

Closing



Opening

Gradient

Gray-Scale Morphological Reconstruction

- Geodesic Dilation of size 1

Marker image Structuring element Mask image

$$D_g^{(1)}(f) = (f \oplus b) \wedge g \qquad f \le g$$

Point-wise minimum

- Geodesic Dilation of size *n*

$$D_g^{(n)}(f) = D_g^{(1)}[D_g^{(n-1)}(f)] \quad \text{with } D_g^{(0)}(f) = f$$

- Morphological reconstruction by dilation

$$R_g^D(f) = D_g^{(k)}(f) \text{ with } k \text{ such that } D_g^{(k)}(f) = D_g^{(k+1)}(f)$$

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Gray-Scale Morphological Reconstruction

- Geodesic Erosion of size 1

Marker image Structuring element Mask image

$$E_g^{(1)}(f) = (f \ominus b) \vee g \qquad f \geq g$$

Point-wise maximum

- Geodesic Erosion of size *n*

$$E_g^{(n)}(f) = E_g^{(1)}\left[E_g^{(n-1)}(f)\right] \quad \text{with } E_g^{(0)}(f) = f$$

- Morphological reconstruction by erosion

$$R_g^E(f) = E_g^{(k)}(f) \text{ with } k \textit{ such } \text{that } E_g^{(k)}(f) = E_g^{(k+1)}(f)$$

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Gray-Scale Morphological Reconstruction

- Opening by reconstruction of size *n*

$$O_R^{(n)}(f) = R_f^D\left[(f \ominus nb)\right]$$

- Top-hat by reconstruction

$$T_R^{(n)} = f - O_R^{(n)}(f)$$

- Closing by reconstruction of size *n*

$$C_R^{(n)}(f) = R_f^E\left[(f \oplus nb)\right]$$

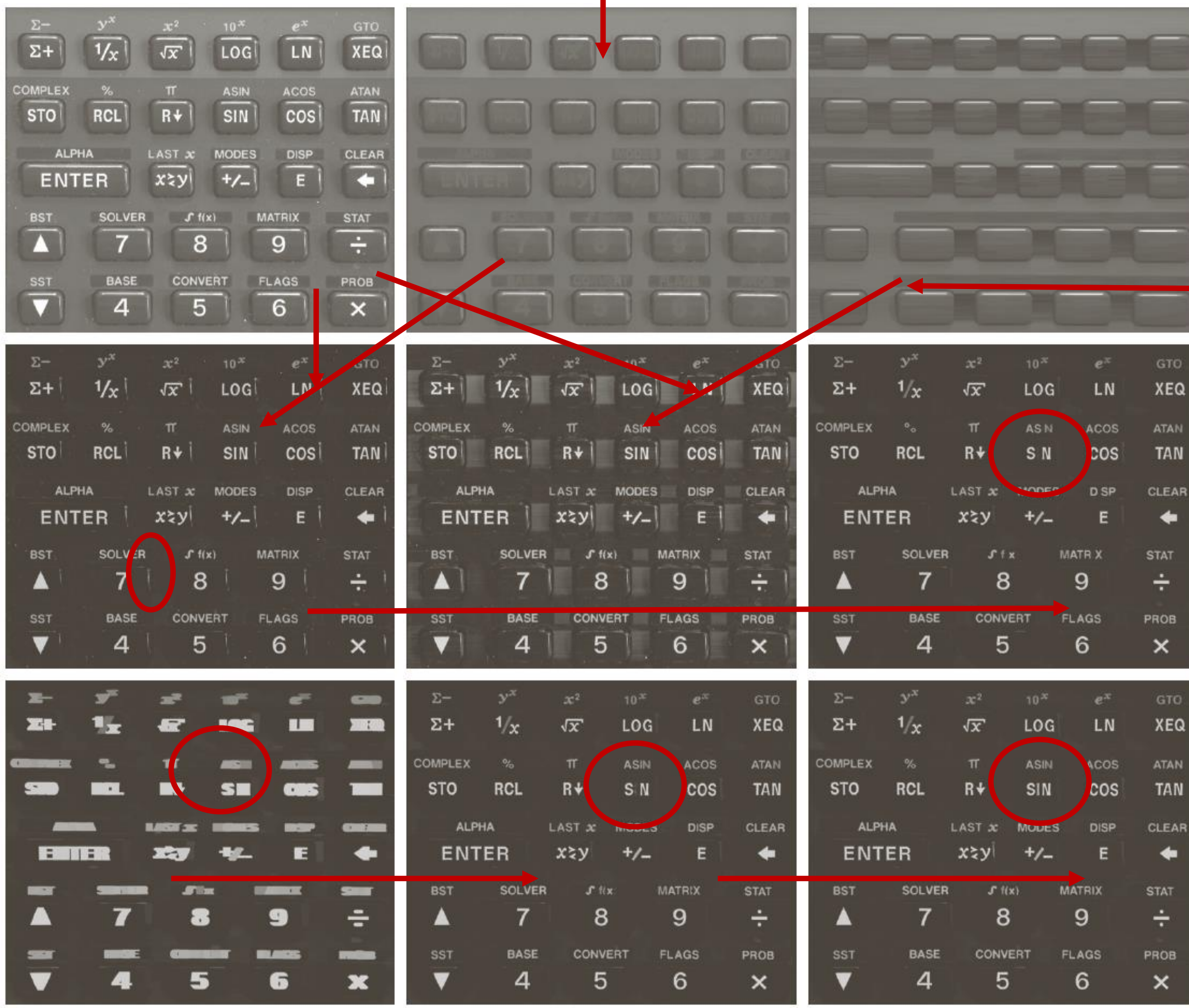
Example: normalize the irregular background

Opening by reconstruction with SE of 1x71

Original

TOP-hat
By recon.
a-b

Dilation
with SE of
1x21



Opening X

TOP-hat
a-c X

Opening
by recon.

Recon.
by dilation

Assignments

- 9.50， 9.51

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