Intensity Transformations

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Contents

- What is image enhancement?
- Point processing
- Histogram processing

A Note About Grey Levels

- Generally, in the range [0, 255]
  - where 0 is black and 255 is white
  - stems from display technologies
- For high precision processing,
  use double in the range [0.0, 1.0]
- in Matlab
  - im2uint8()
  - im2unit16()
  - im2double()

What Is Image Enhancement?

- Image enhancement is the process of making images more useful
- The reasons for doing this include:
  - Highlighting interesting detail in images
  - Removing noise from images
  - Making images more visually appealing

Image Enhancement Examples

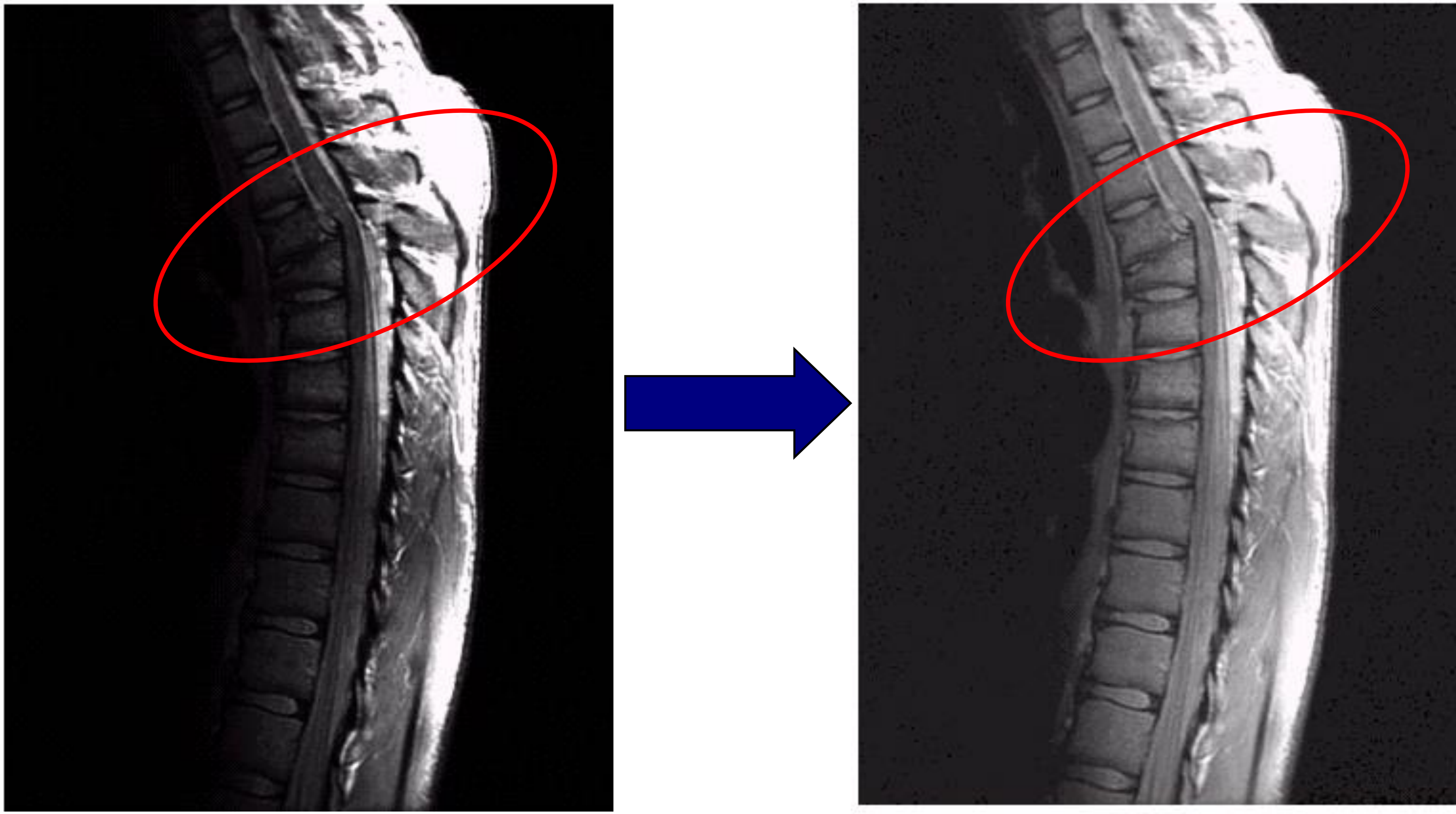
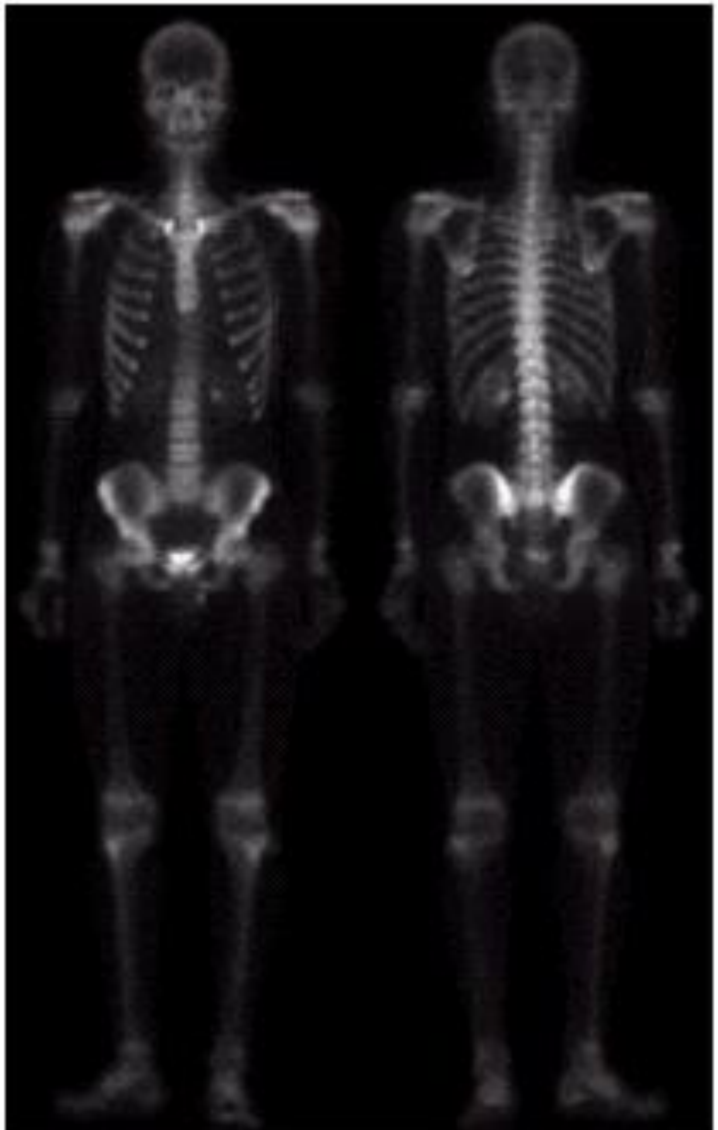


Image Enhancement Examples (cont…)



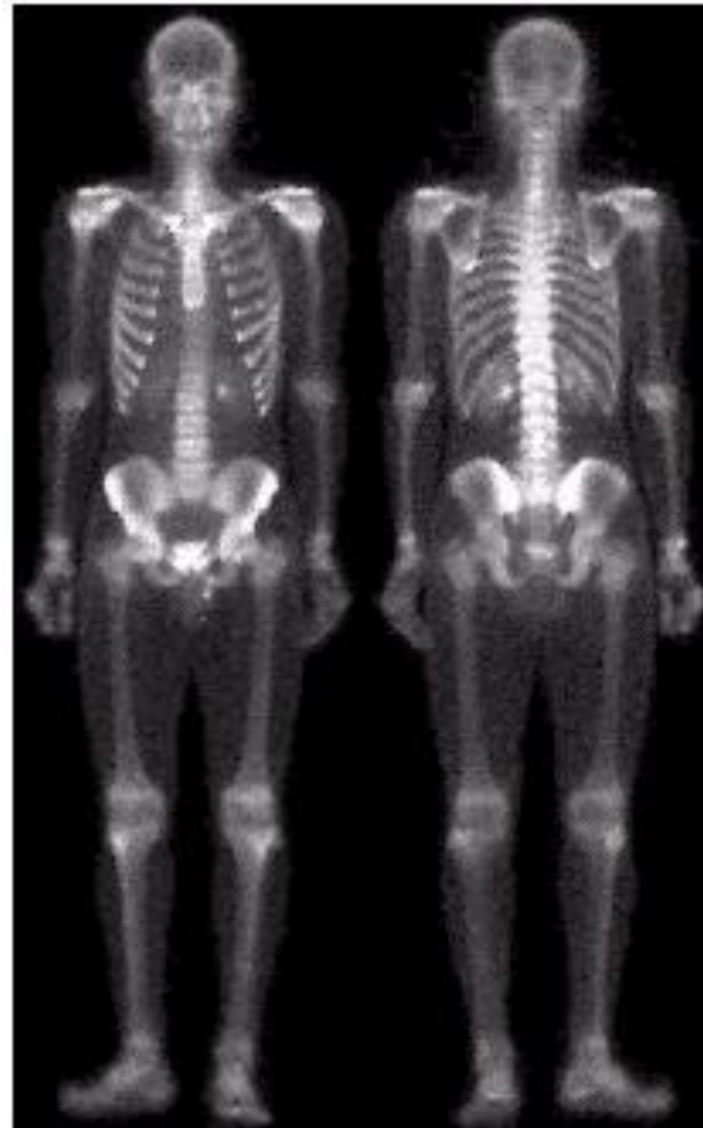


Image Enhancement Examples (cont…)

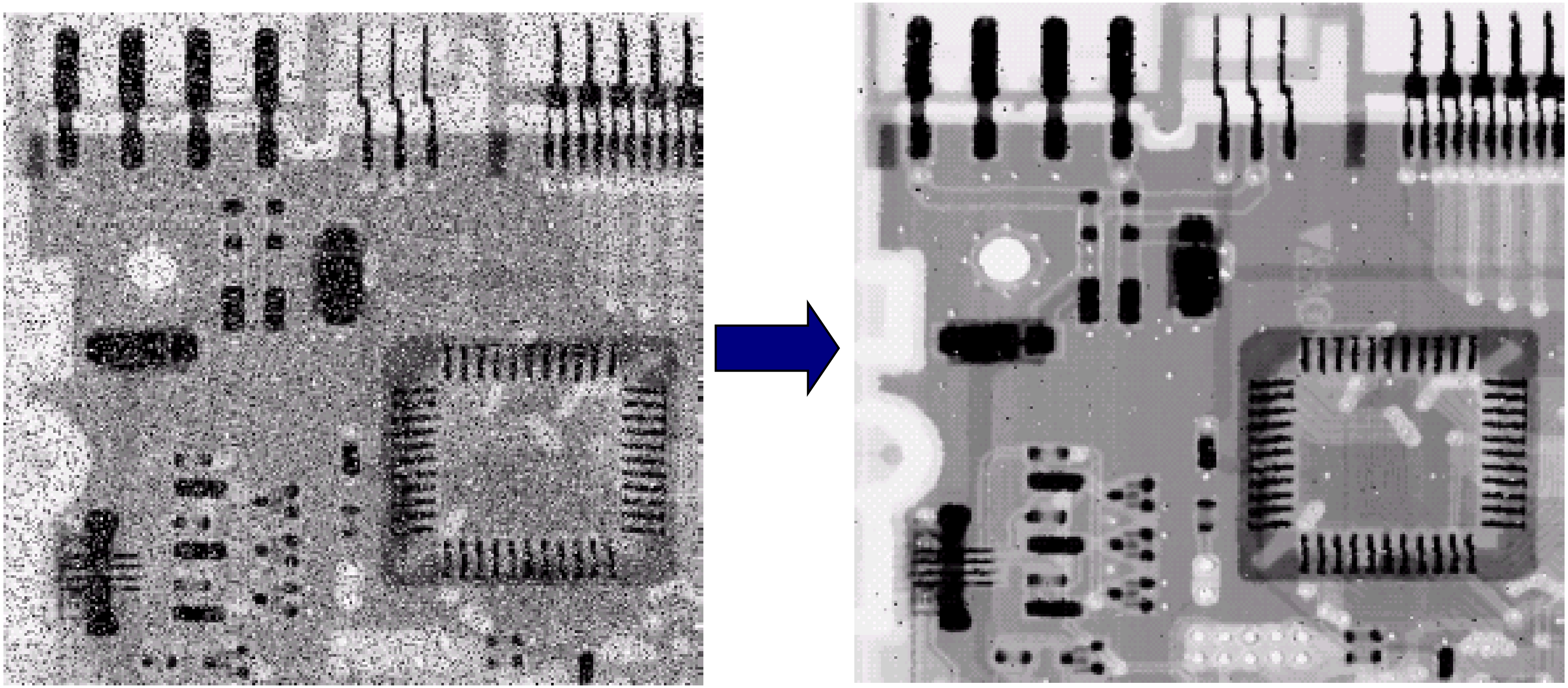


Image Enhancement Examples (cont…)

Spatial & Frequency Domains

- Image enhancement techniques
  - Spatial domain techniques

    Direct manipulation of image pixels
  - Frequency domain techniques

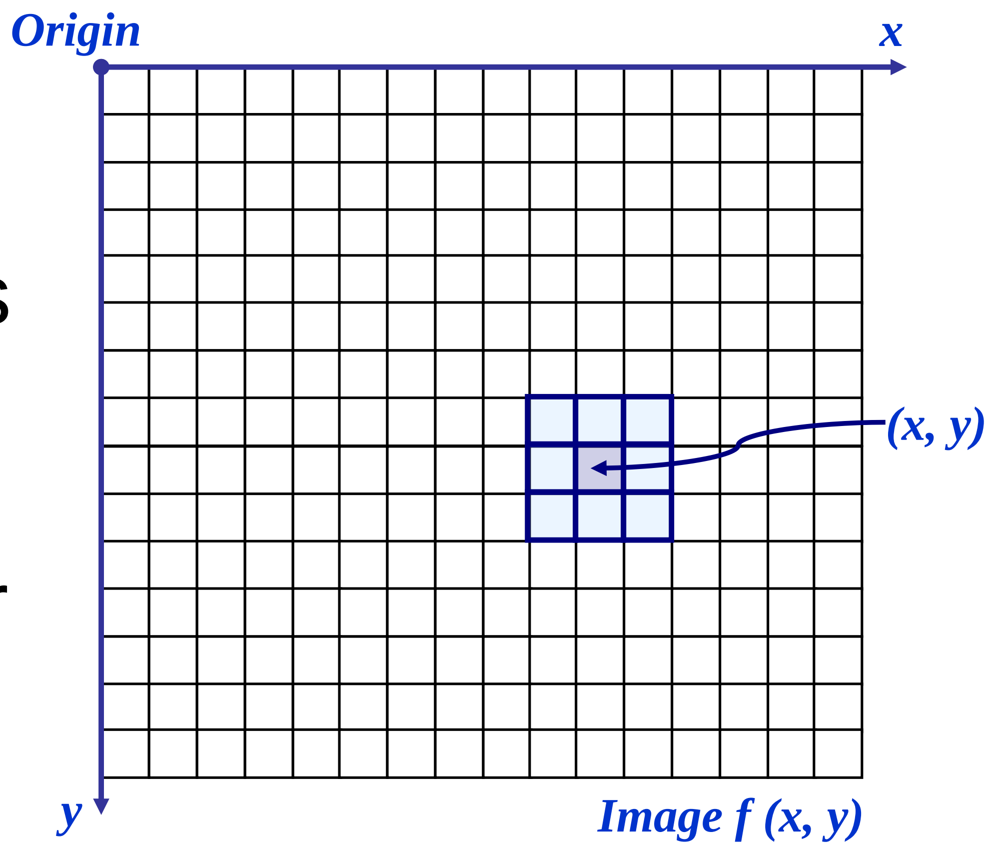
    Manipulation of Fourier transform or wavelet transform of an image

Basic Spatial Domain Image Enhancement

Most spatial domain enhancement operations can be reduced to the form

$g(x, y) = T[f(x, y)]$

where $f(x, y)$ is the input image, $g(x, y)$ is the processed image and T is some operator defined over some neighbourhood of (x, y)



Contents

- What is image enhancement?
- Point processing
- Histogram processing

Point Processing

- What is point processing?
- Negative images
- Thresholding
- Logarithmic transformation
- Power law transforms
- Grey level slicing
- Bit plane slicing

Point Processing

The simplest spatial domain operations occur when the neighbourhood is simply the pixel itself

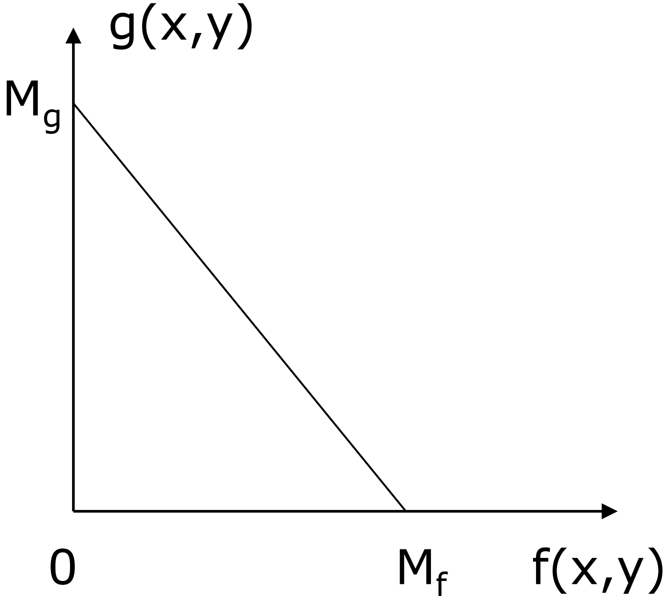
In this case T is referred to as a *grey level transformation function* or a *point processing operation*

Point processing operations take the form

$$s = T(r)$$

where s refers to the processed image pixel value and r refers to the original image pixel value

Example: Negative Images

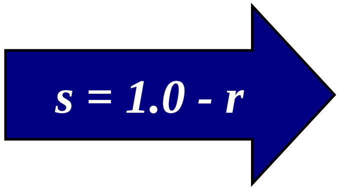


Example: Negative Images

Negative images are useful for enhancing white or grey detail embedded in dark regions of an image

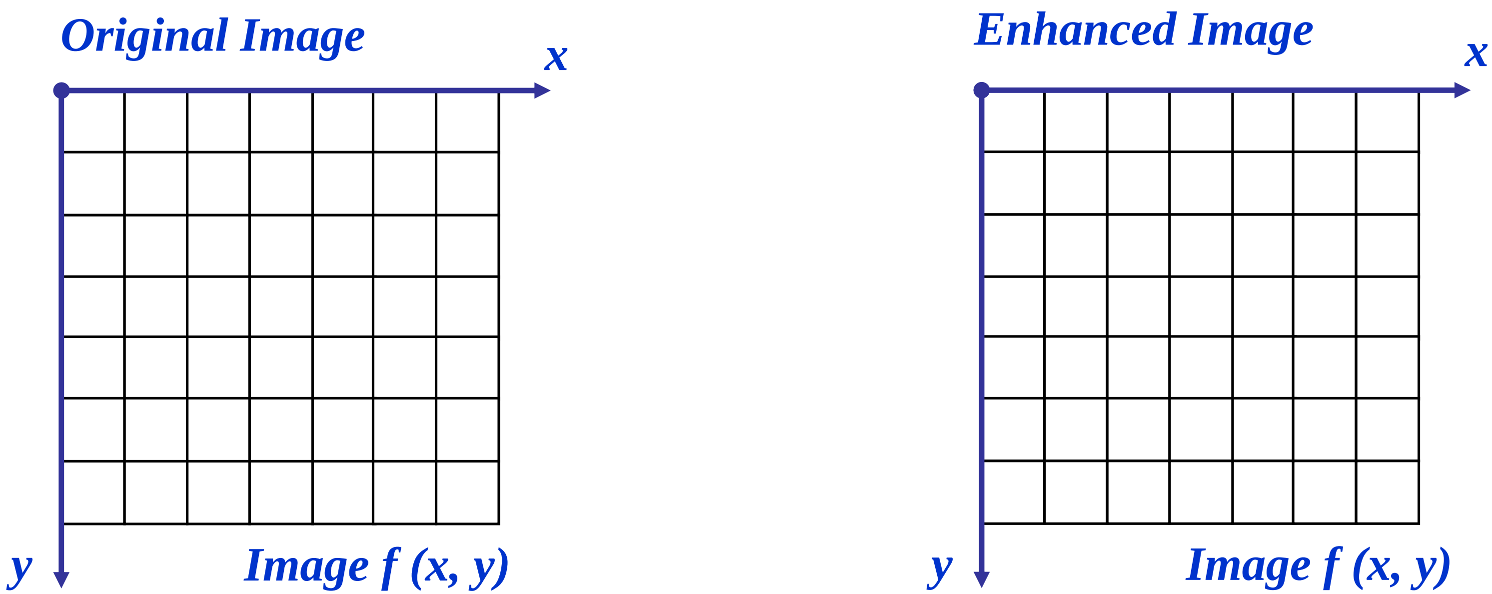
– Note how much clearer the tissue is in the negative image of the mammogram below

Original Image



Negative Image

Example: Negative Images (cont…)

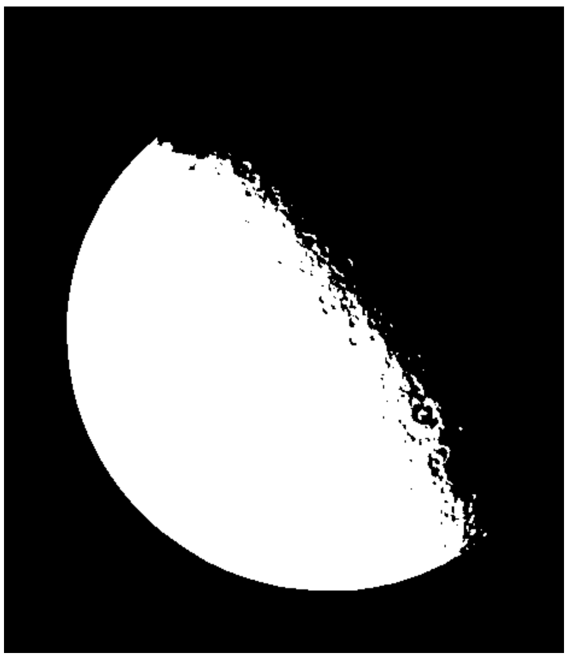


$$s = intensity_{max} - r$$

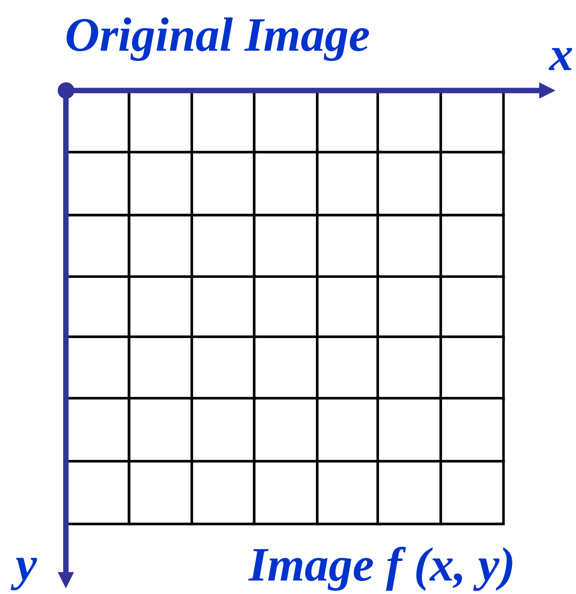
Example: Thresholding

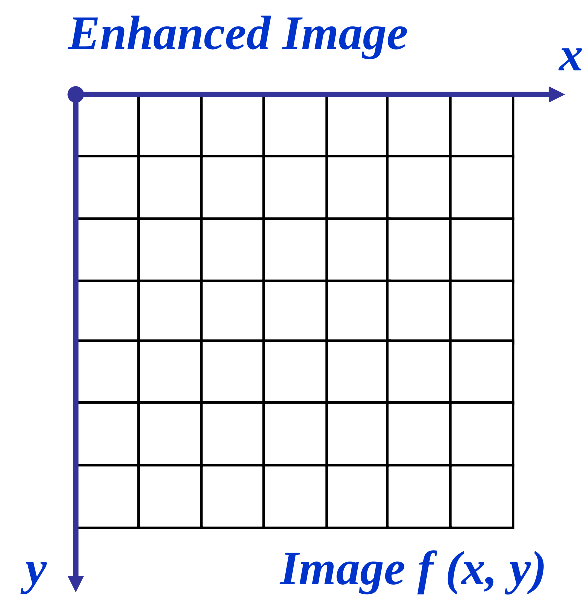
Thresholding transformations are particularly useful for segmentation in which we want to isolate an object of interest from a background

$$s = \begin{cases} 1.0 & r > \text{threshold} \\ 0.0 & r <= \text{threshold} \end{cases}$$



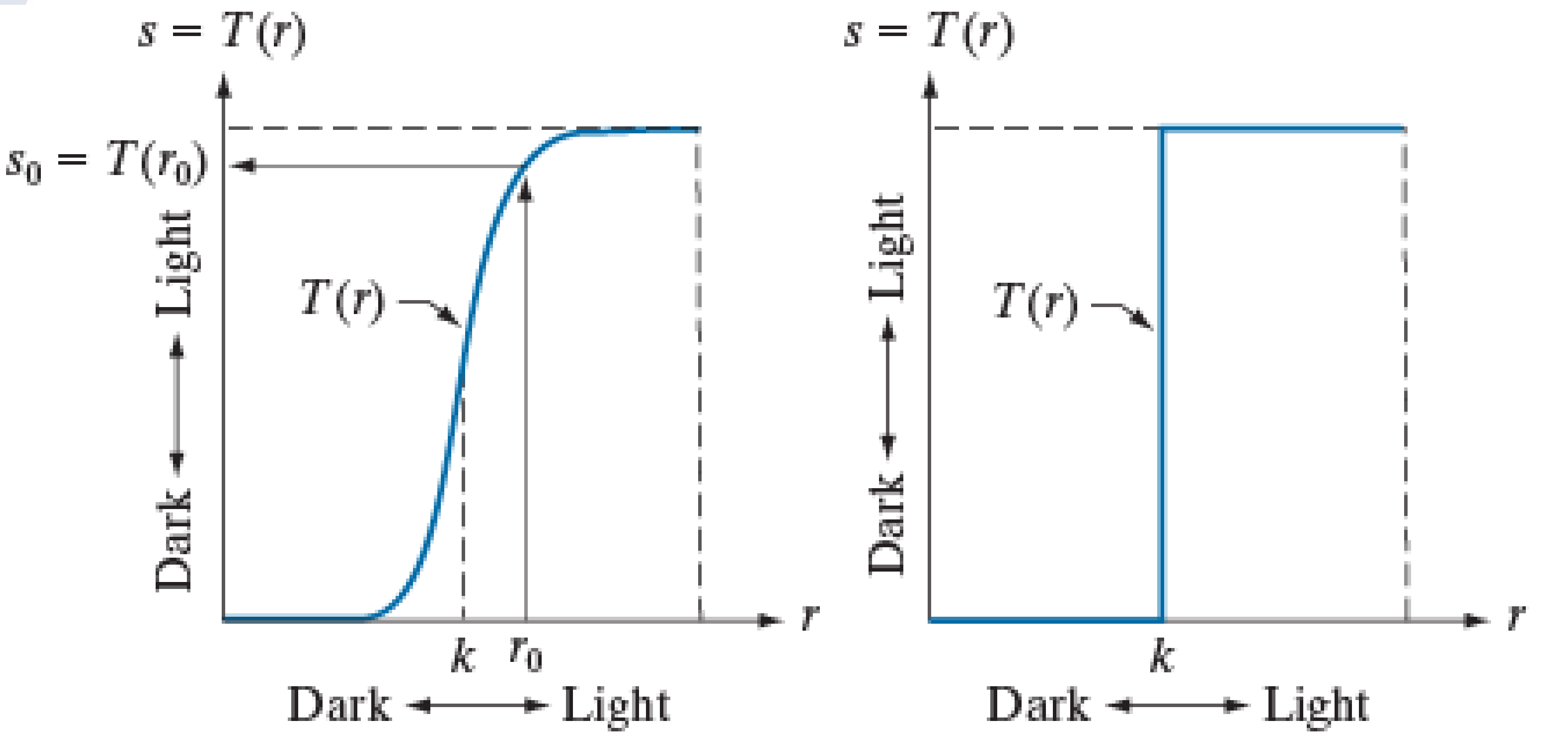
Example: Thresholding (cont…)





$$s = \begin{cases} 1.0 & r > threshold \\ 0.0 & r <= threshold \end{cases}$$

Thresholding (cont…)

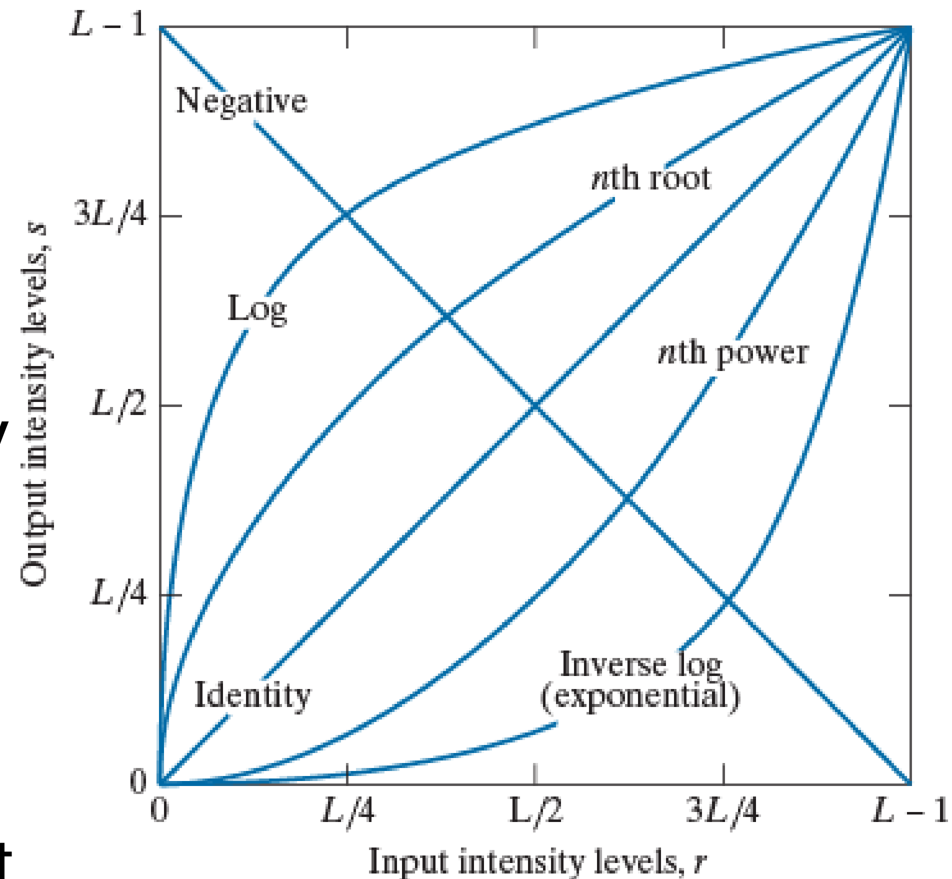


Basic Grey Level Transformations

There are many different kinds of grey level transformations

Three of the most common are shown here

- Linear
  - Negative / Identity
- Logarithmic
  - Log / Inverse log
- Power law
  - n^{th} power / n^{th} root



Logarithmic Transformations

The general form of the log transformation is

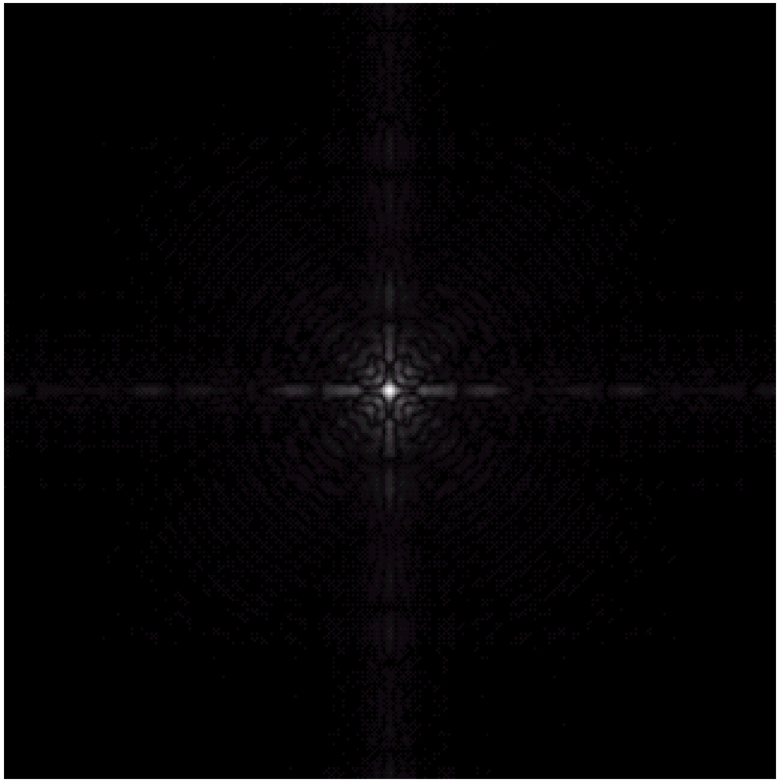
$$s = c \times log(1 + r)$$

The log transformation maps a narrow range of low input grey level values into a wider range of output values

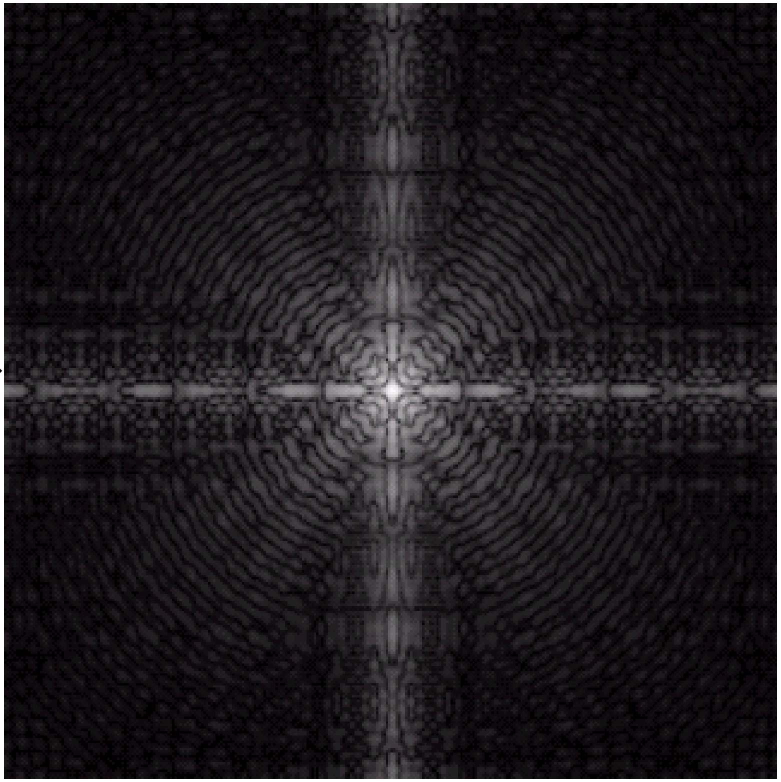
The inverse log transformation performs the opposite transformation

Logarithmic Transformations (cont…)

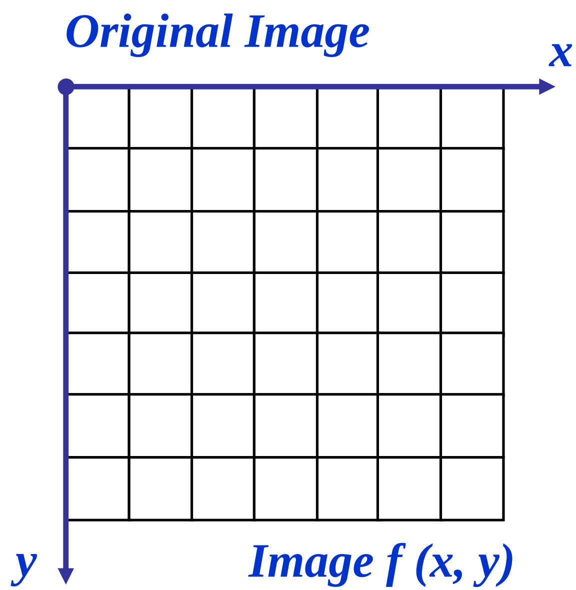
In the following example, the Fourier transform of an image is put through a log transform to reveal more detail

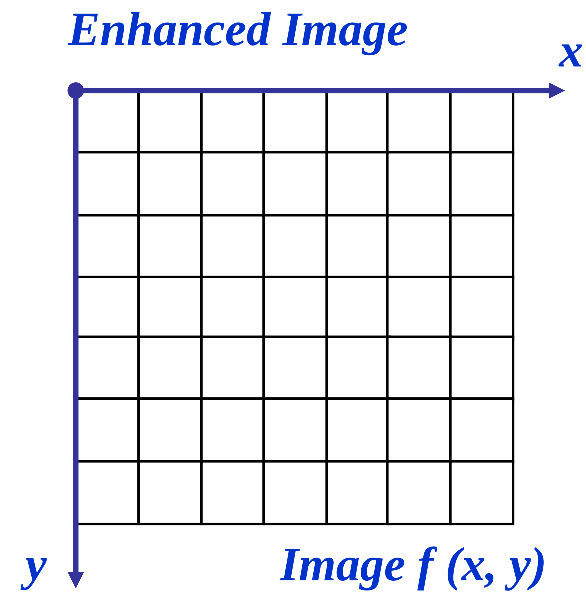


$s = log(1 + r)$



Logarithmic Transformations (cont…)





We usually set c to 1

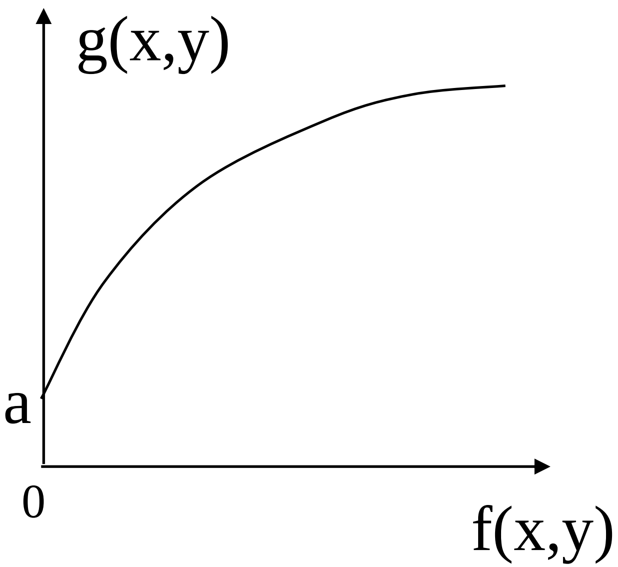
$$s = c \times \log(1 + r) \quad \rightarrow \quad s = \log(1 + r)$$

Grey levels must be in the range [0.0, 1.0]

Logarithmic Transformations (cont…)

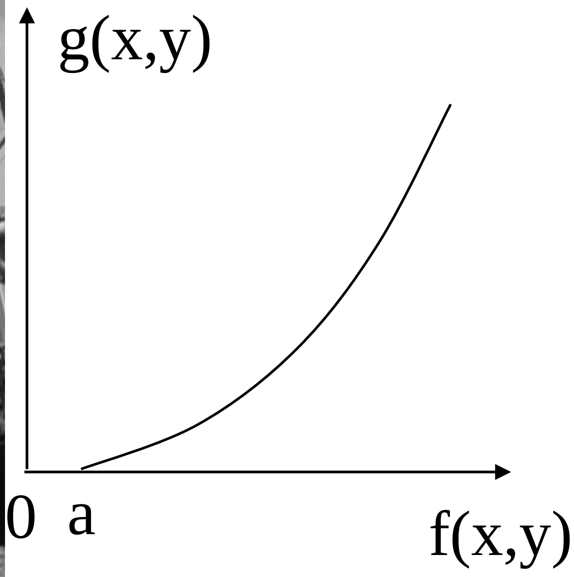
Original

Log Transformed



$$g(x,y) = a + \frac{\ln[f(x,y)+1]}{b}$$

Exponential Transformations



Original

Exp Transformed

$$g(x,y) = b^{c[f(x,y)-a]} - 1$$

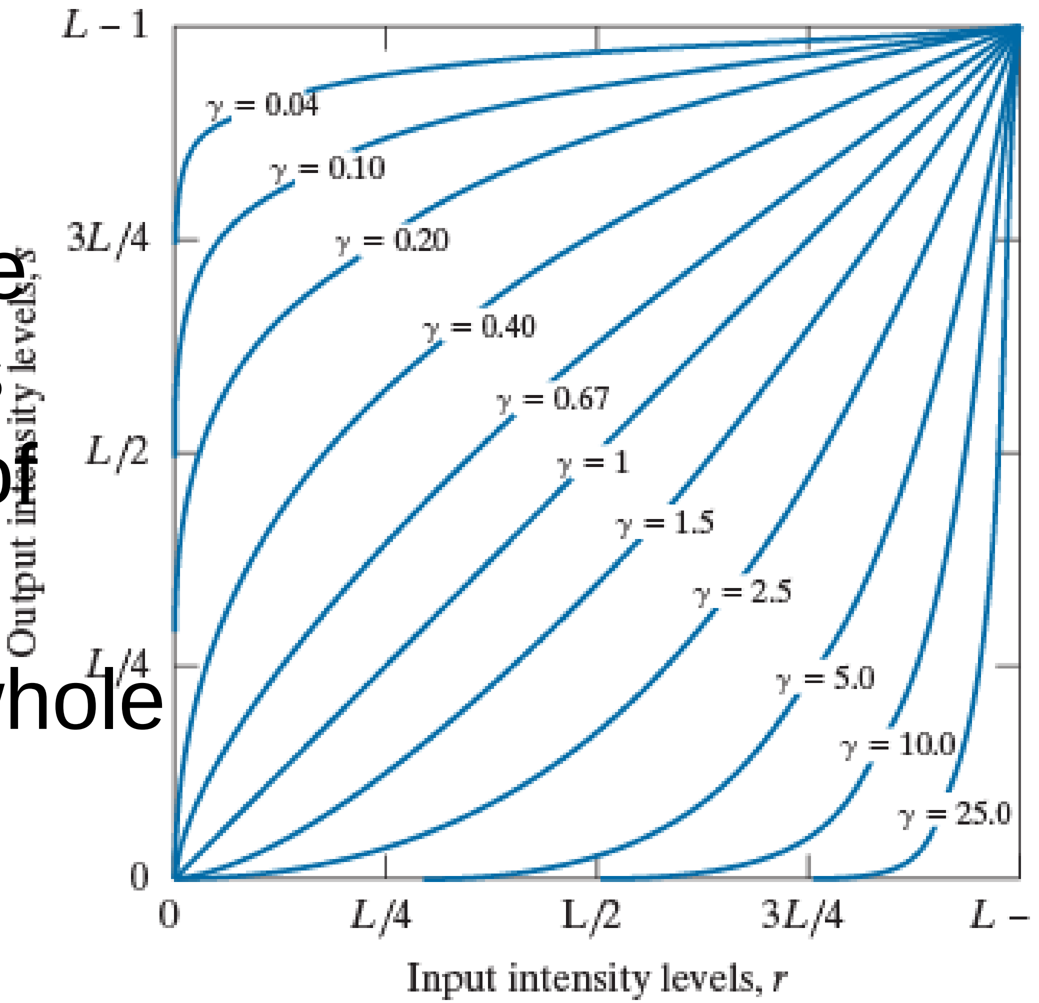
Power Law Transformations

Power law transformations have the following form

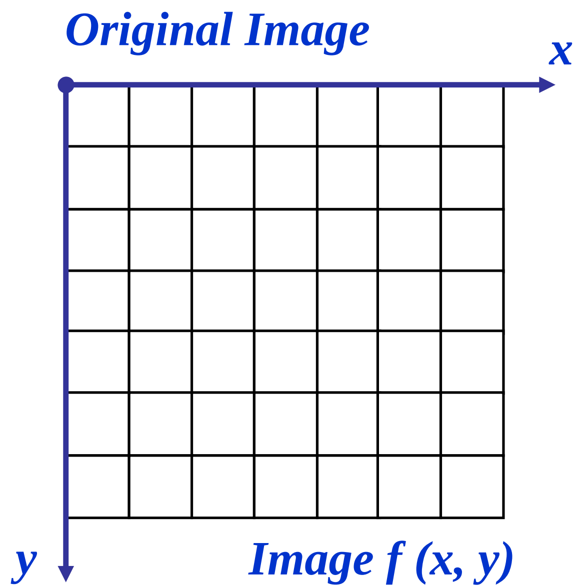
$$s = c \times r^{\gamma}$$

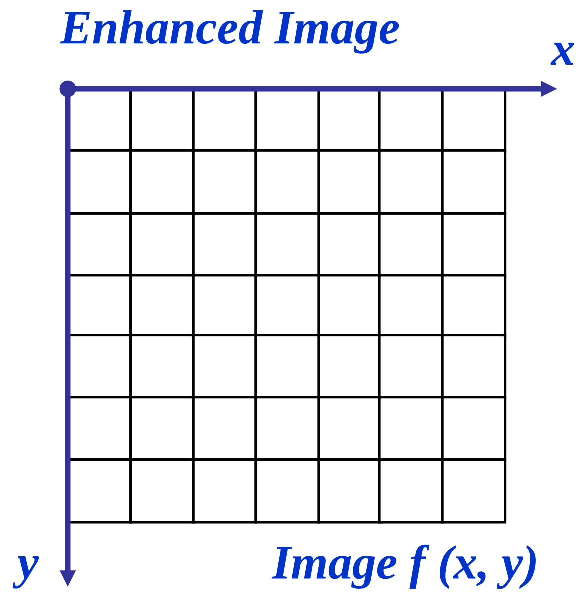
Map a narrow range of dark input values into a wider range of output when $\gamma < 1$

Varying γ gives a whole family of curves



Power Law Transformations (cont…)





$$s = r^{\gamma}$$

We usually set c to 1

Grey levels must be in the range [0.0, 1.0]

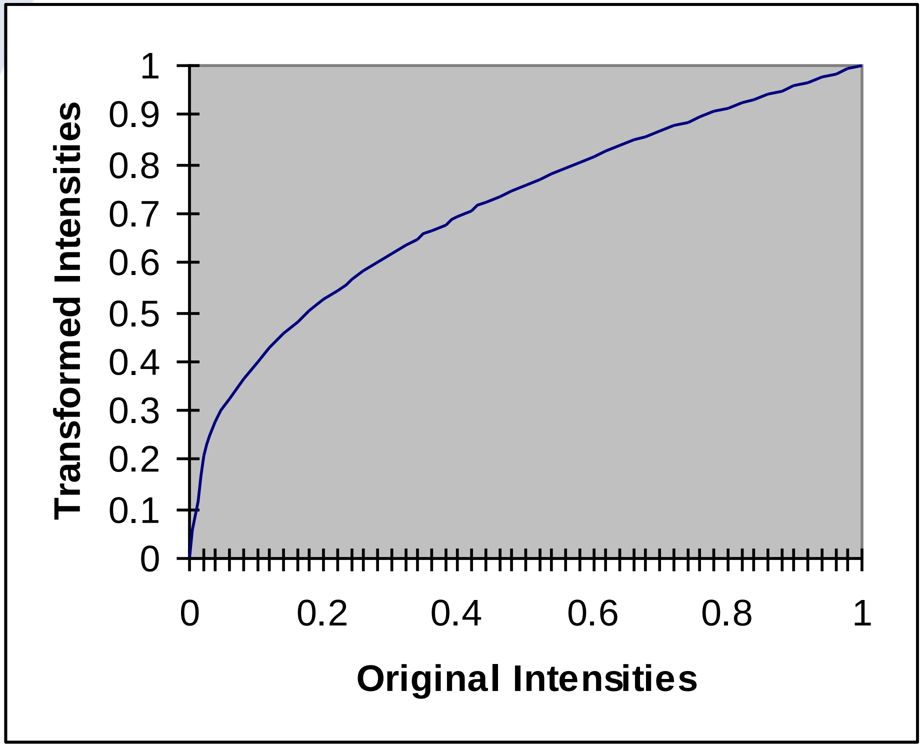
Power Law Example

Power Law Example (cont…)

$\gamma = 0.6$

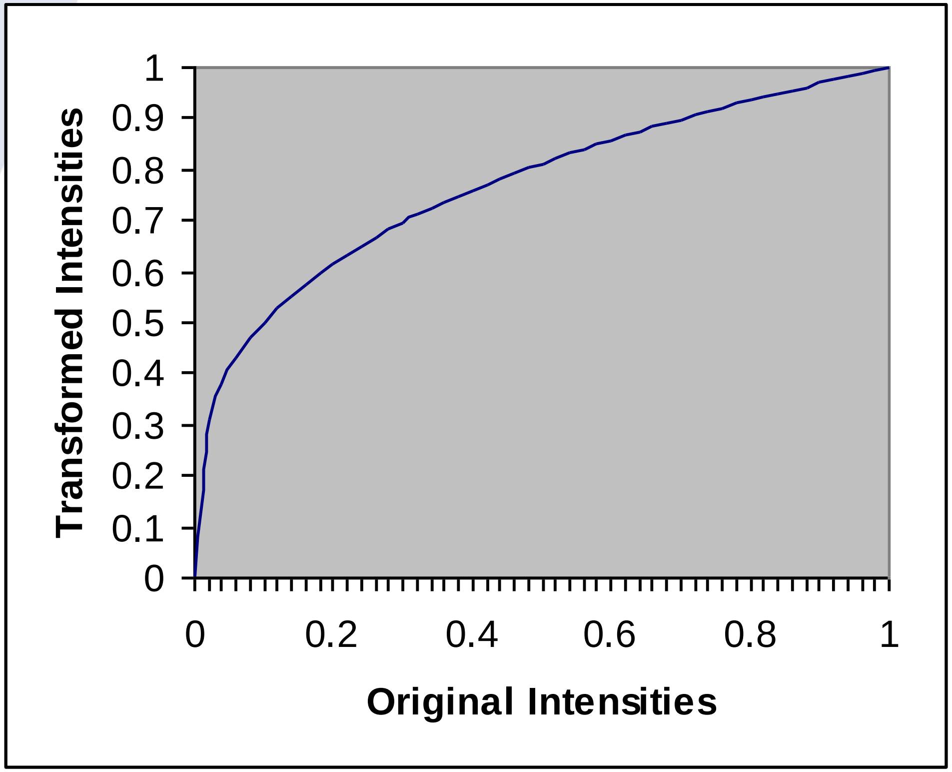
Power Law Example (cont…)

$$\gamma = 0.4$$



Power Law Example (cont…)

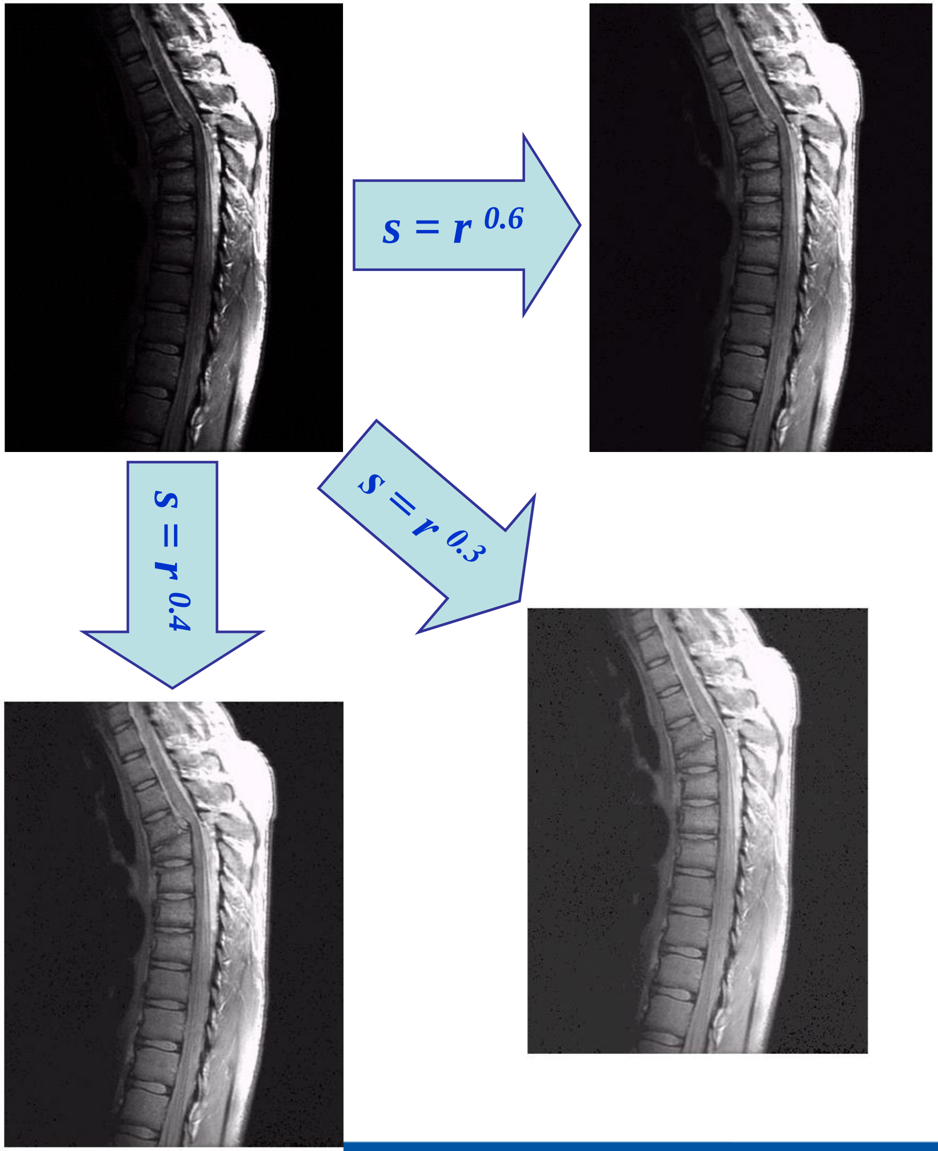
$\gamma = 0.3$



Power Law Example (cont…)

The images to the right show a magnetic resonance (MR) image of a fractured human spine

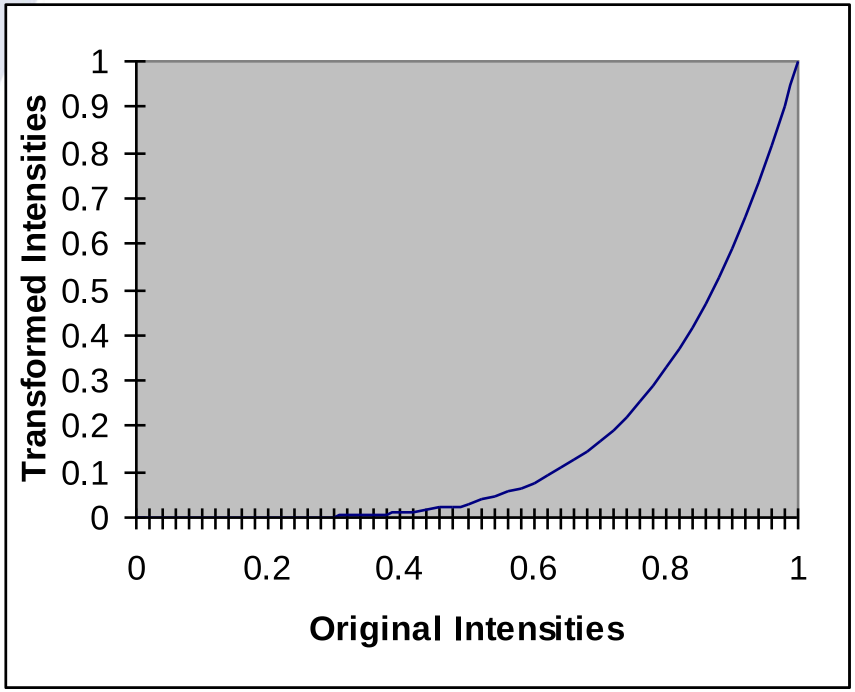
Different curves highlight different detail



Power Law Example

Power Law Example (cont…)

$$\gamma = 5.0$$

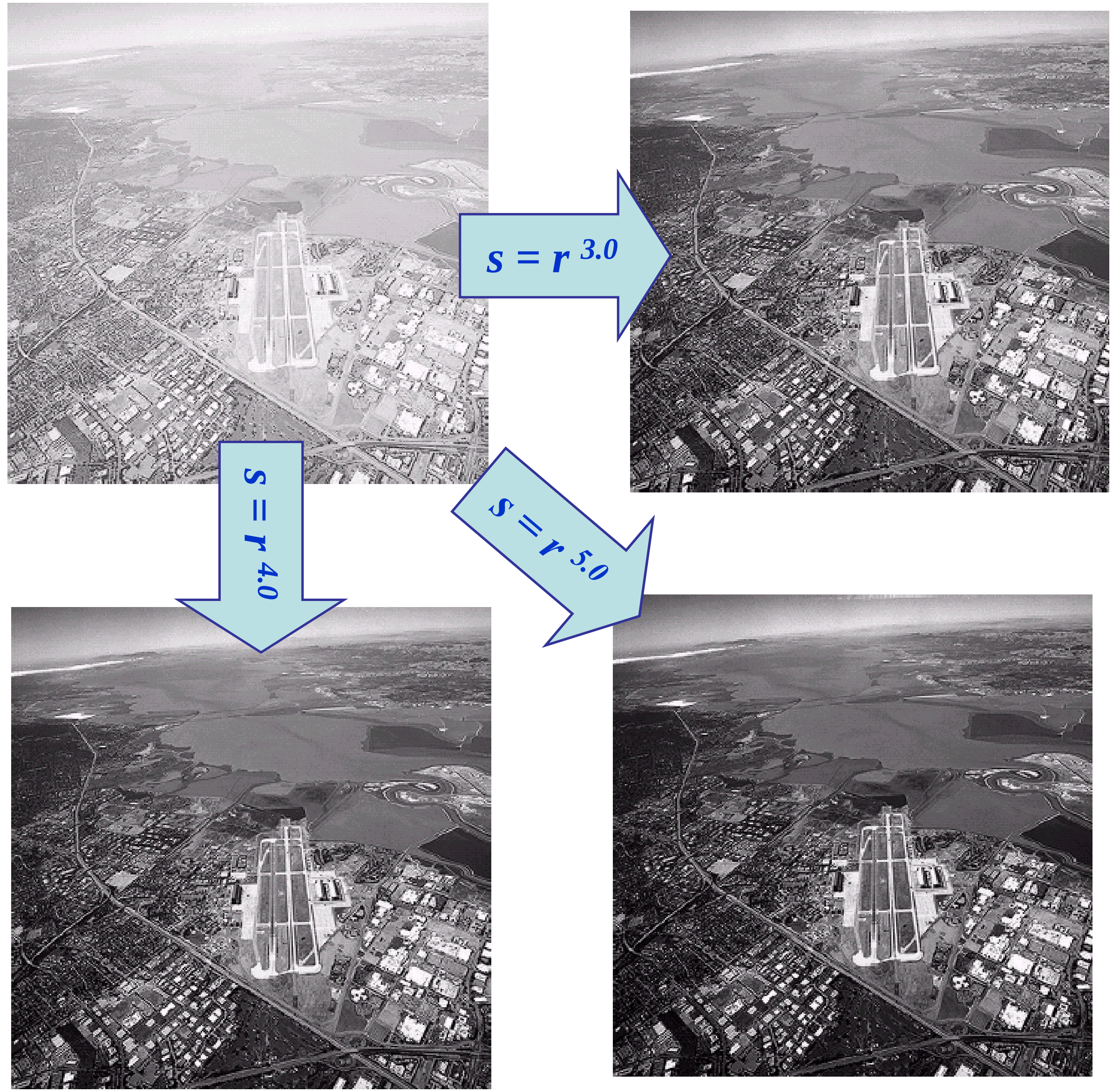


Power Law Example (cont…)

An aerial photo of a runway is shown

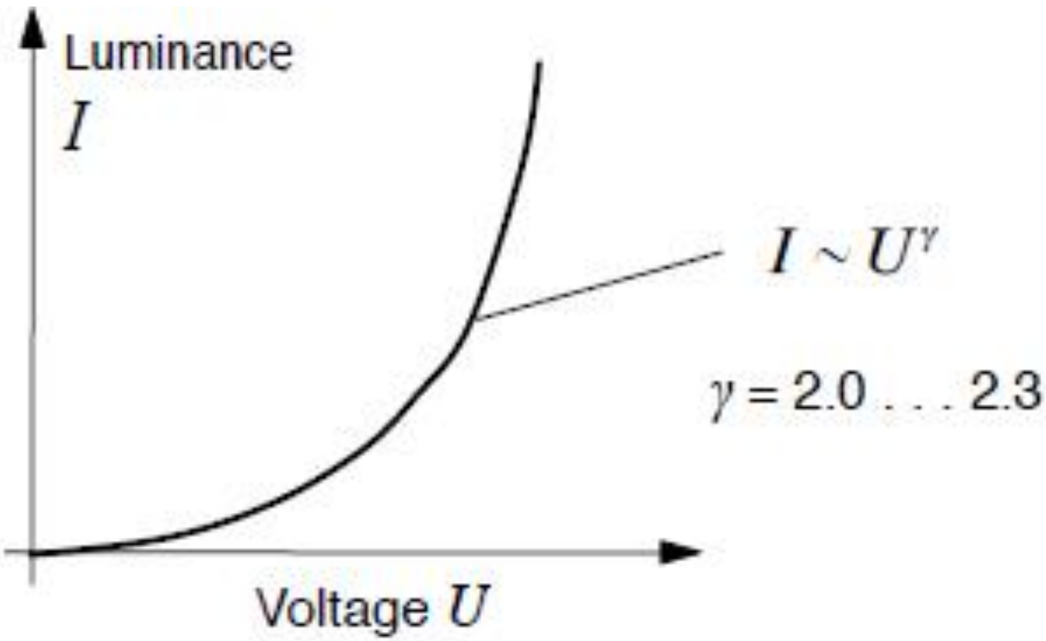
This time power law transforms are used to darken the image

Different curves highlight different detail



Gamma Correction

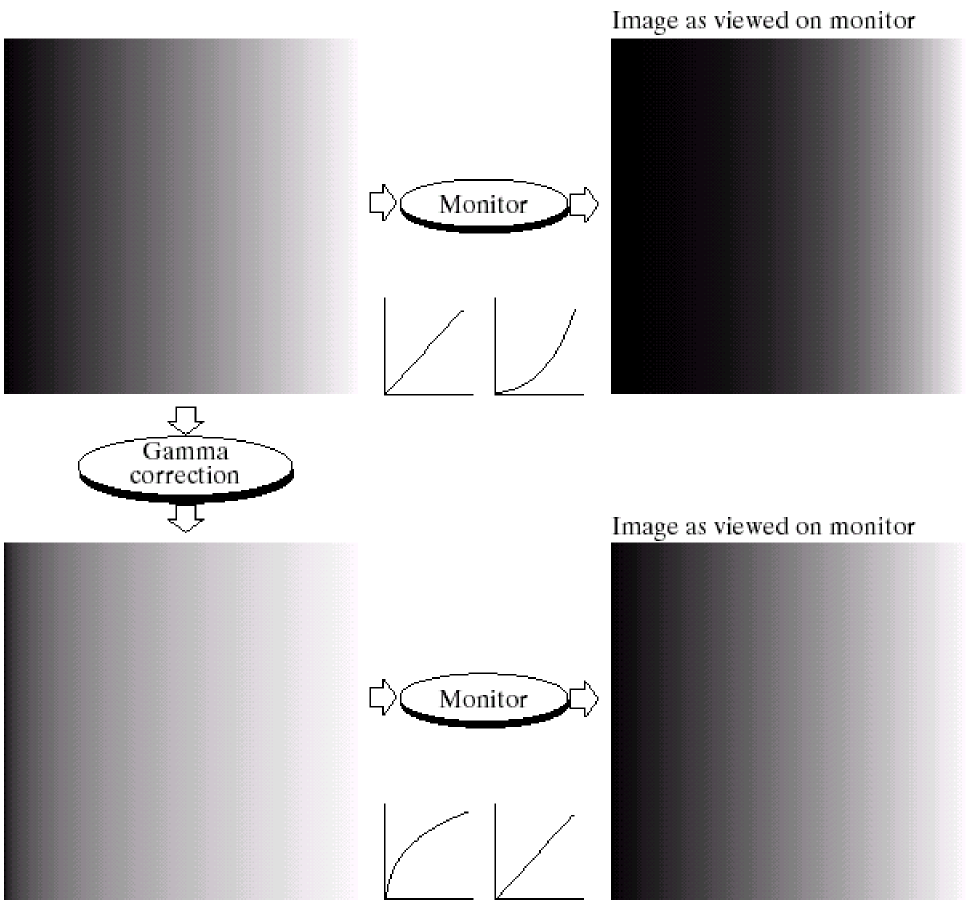
Many of you might be familiar with gamma correction of CRT monitors



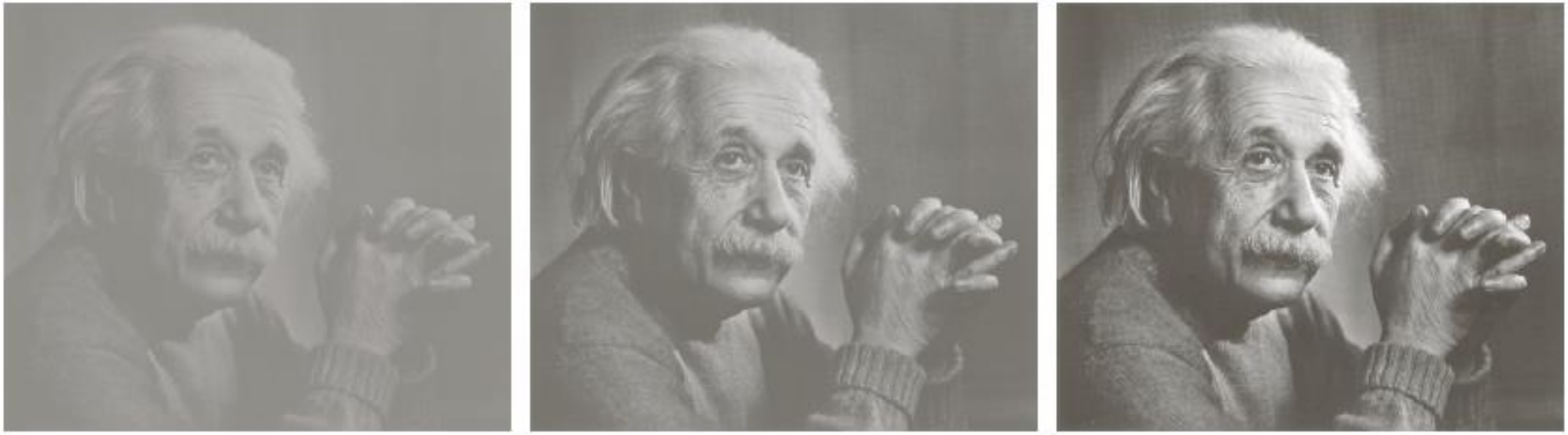
Gamma Correction

Problem is that display devices do NOT respond linearly to different intensities

Can be corrected using an inverse gamma transform



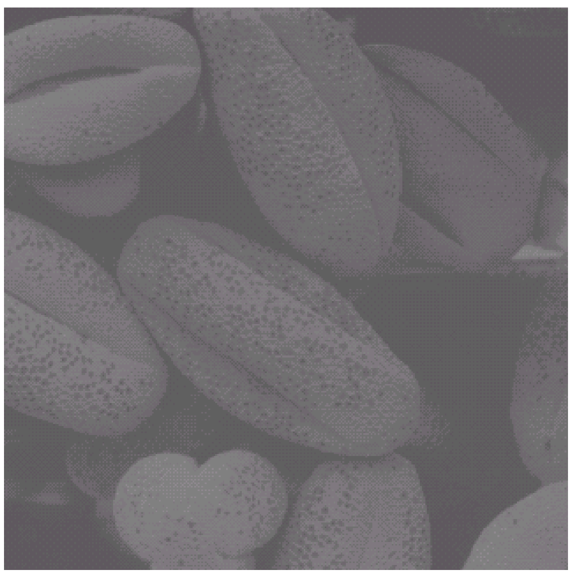
More Contrast Issues

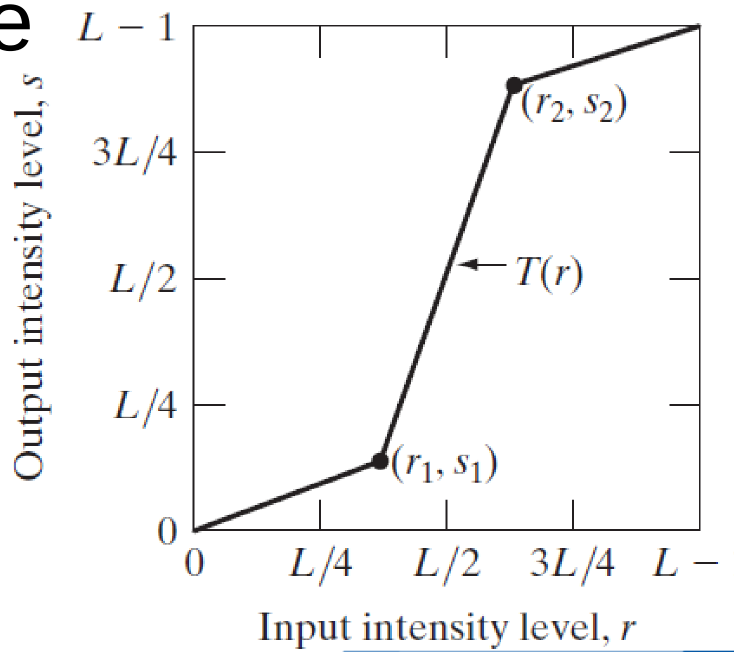


Piecewise Linear Transformation

Rather than using a well defined mathematical function we can use arbitrary user-defined transforms

The images below show a contrast stretching linear transform to add contrast to a poor quality image

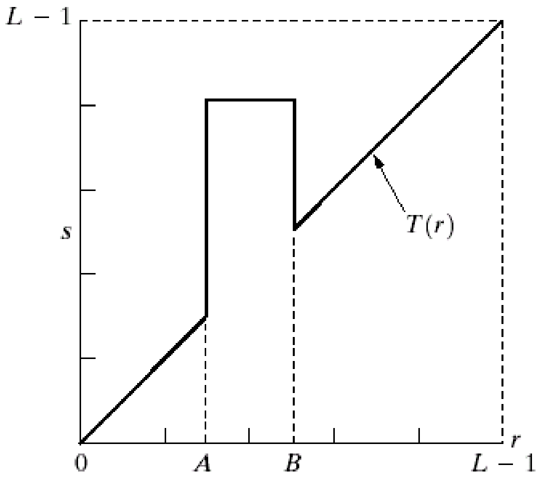


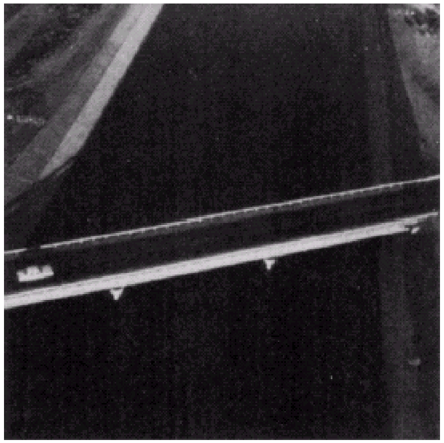


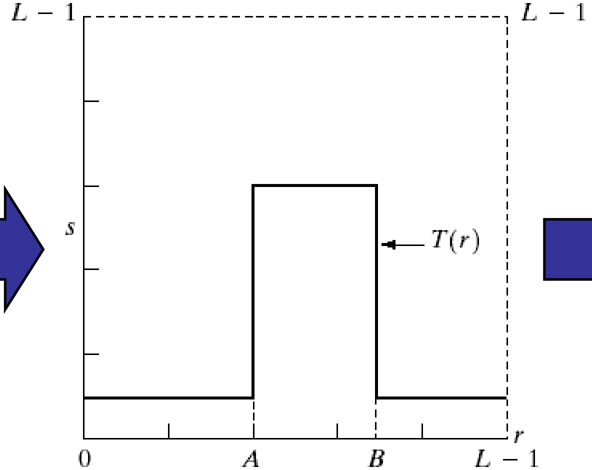
Grey Level Slicing

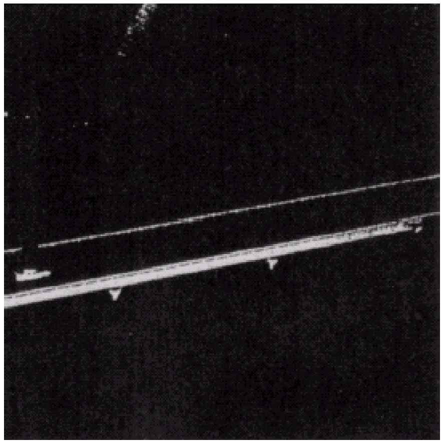
Highlights a specific range of grey levels

- Similar to thresholding
- Other levels can be suppressed or maintained
- Useful for highlighting features in an image





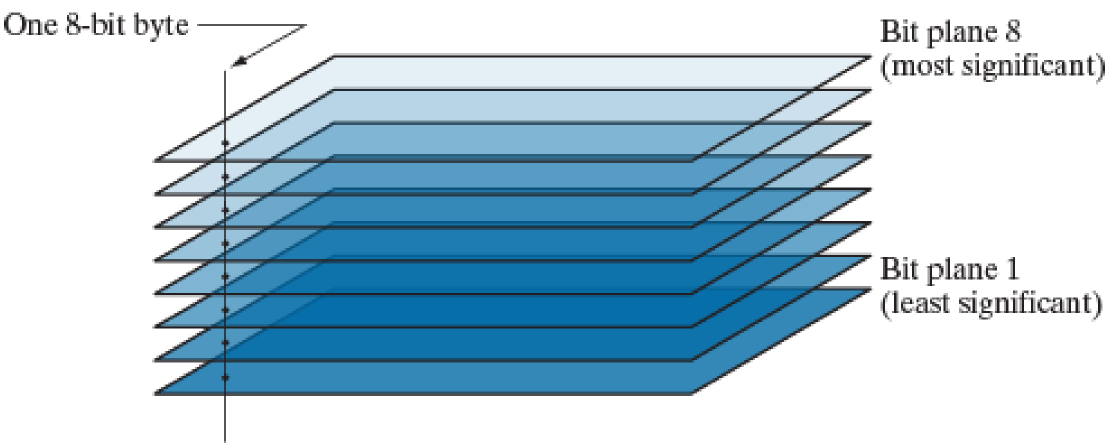


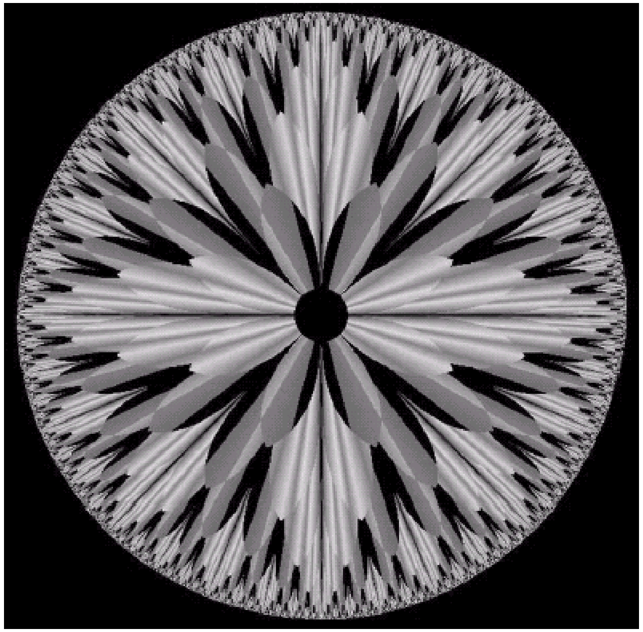


Bit Plane Slicing

Often by isolating particular bits of the pixel values in an image we can highlight interesting aspects of that image

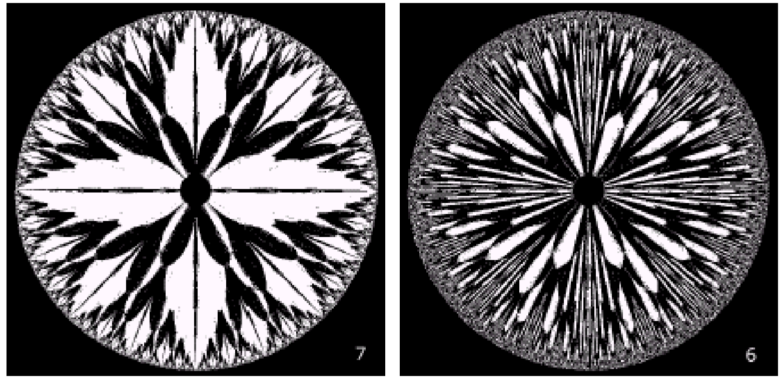
- Higher-order bits usually contain most of the significant visual information
- Lower-order bits contain subtle details





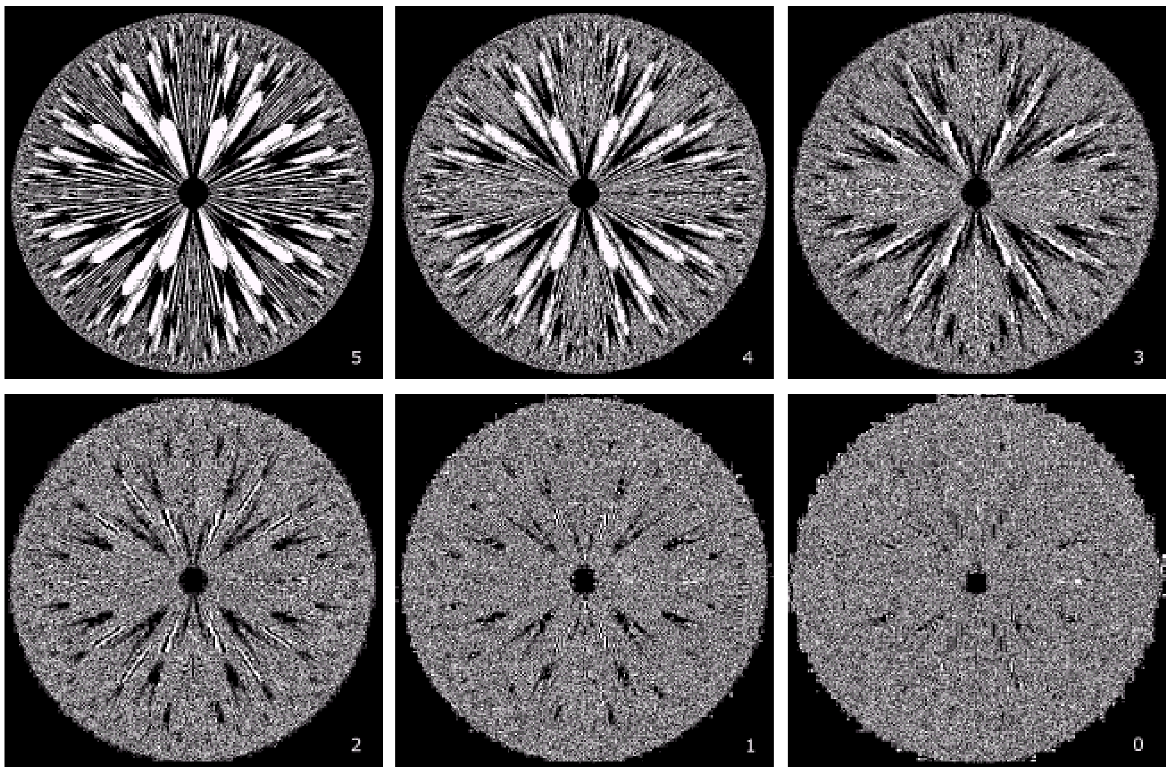
Bit Plane Slicing (cont…)

[10000000]



[01000000]

[00100000]



[00001000]

[00000100]

[00000001]

Bit Plane Slicing (cont…)

a b c
d e f
g h i

FIGURE 3.14 (a) An 8-bit gray-scale image of size 500 × 1192 pixels. (b) through (i) Bit planes 0 through 7, with bit plane 0 corresponding to the least significant bit. Each bit plane is a binary image.

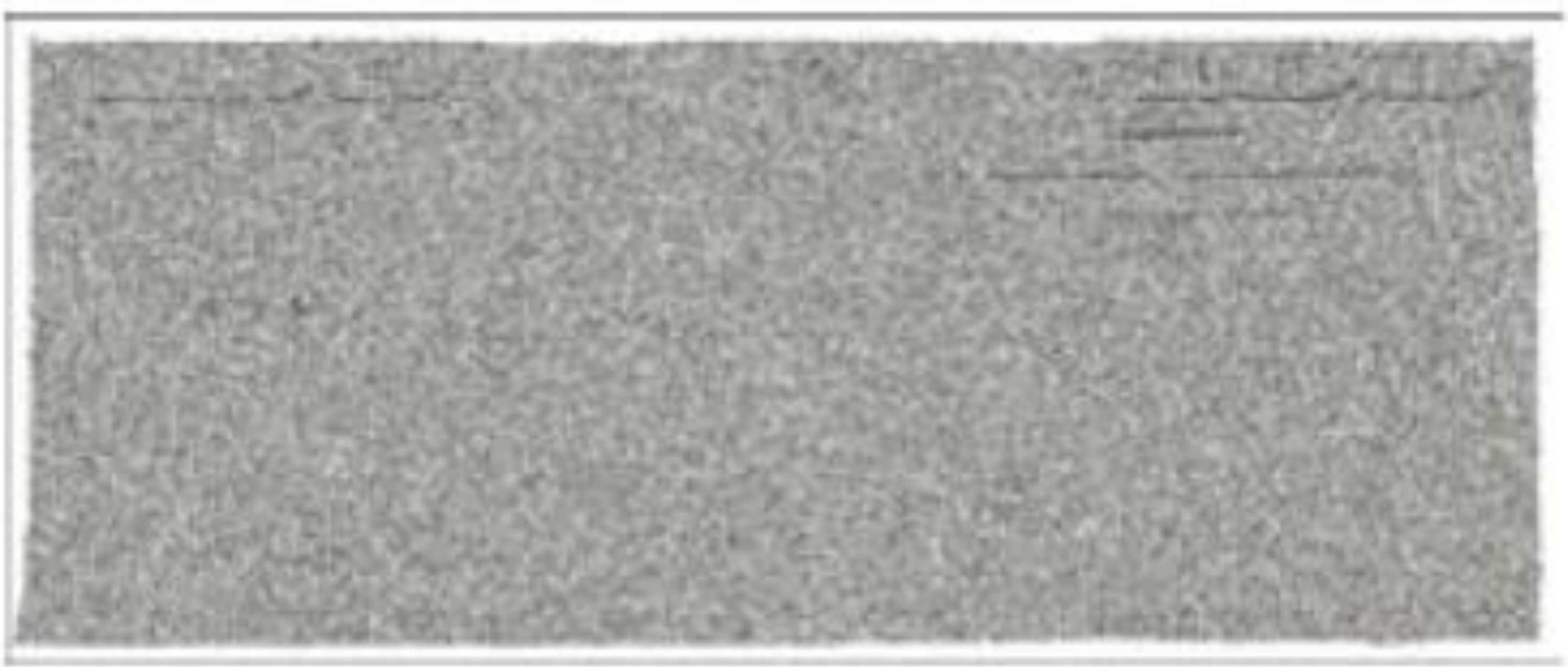
Bit Plane Slicing (cont…)

Bit Plane Slicing (cont…)

Bit plane 0

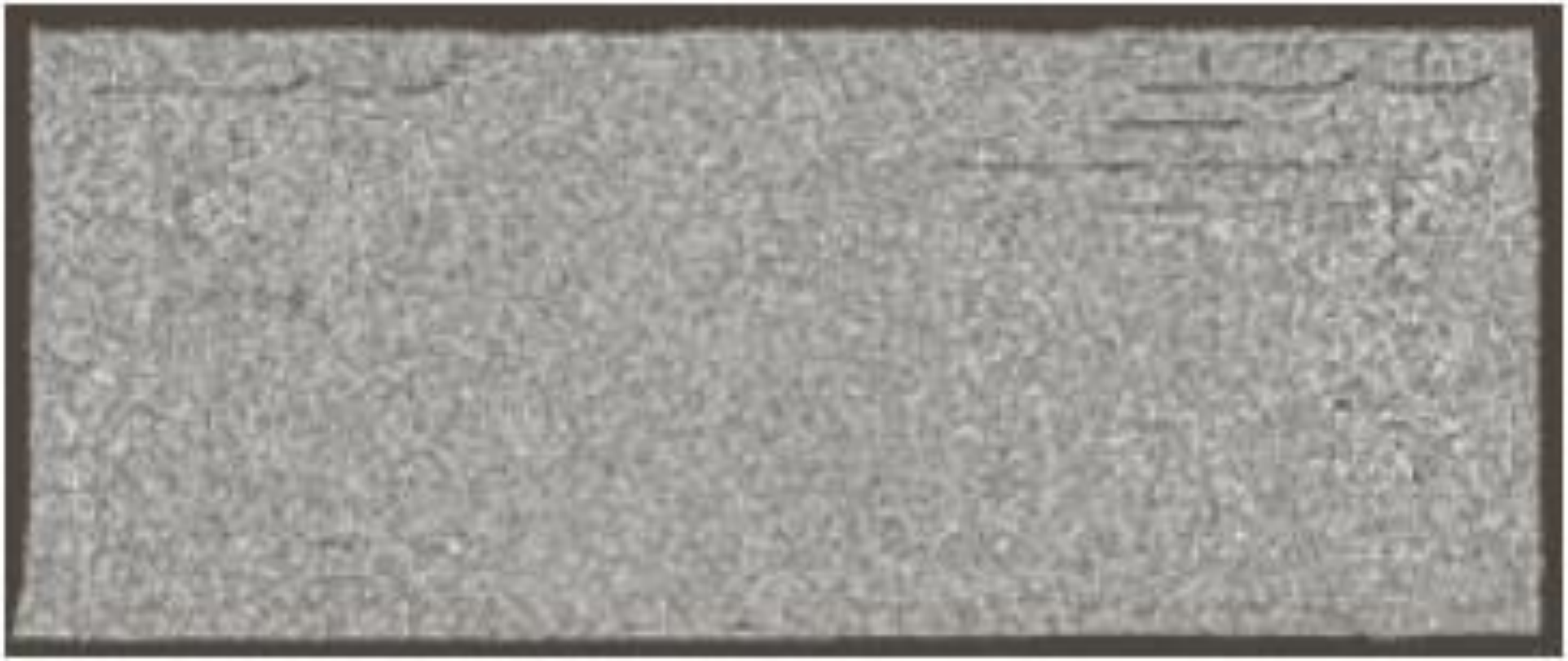
Bit Plane Slicing (cont…)

Bit plane 1



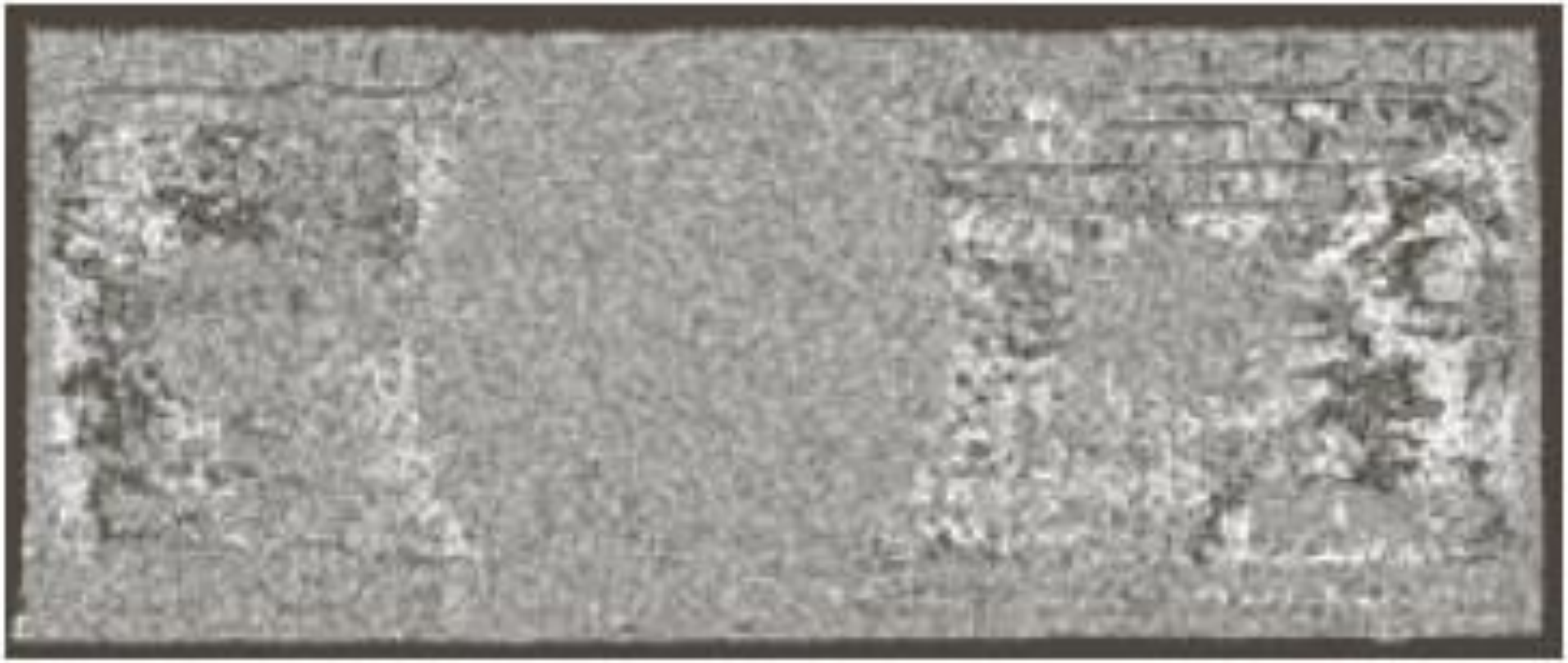
Bit Plane Slicing (cont…)

Bit plane 2



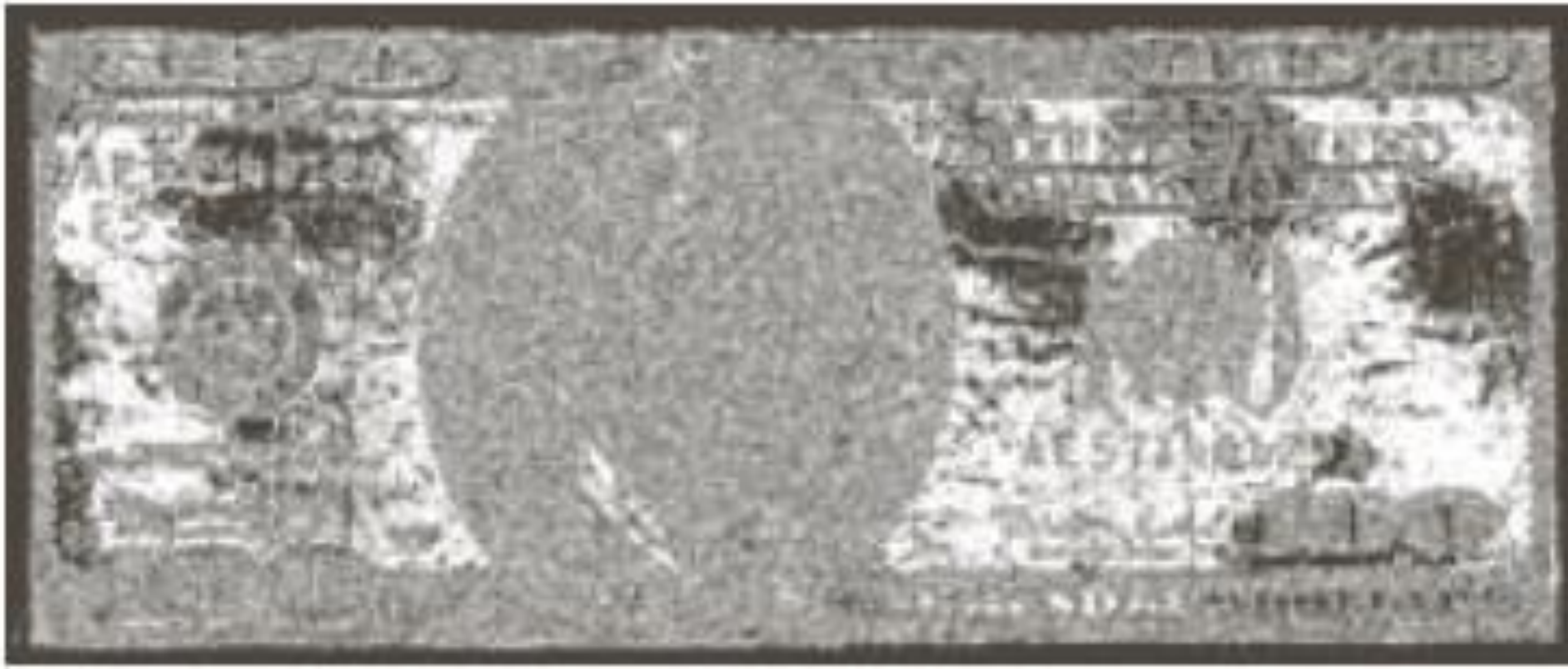
Bit Plane Slicing (cont…)

Bit plane 3



Bit Plane Slicing (cont…)

Bit plane 4



Bit Plane Slicing (cont…)

Bit plane 5

Bit Plane Slicing (cont…)

Bit plane 5

Bit Plane Slicing (cont…)

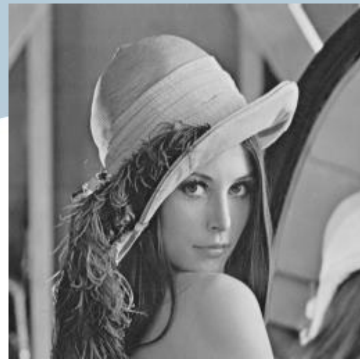
Bit plane 7

Bit Plane Slicing (cont…)

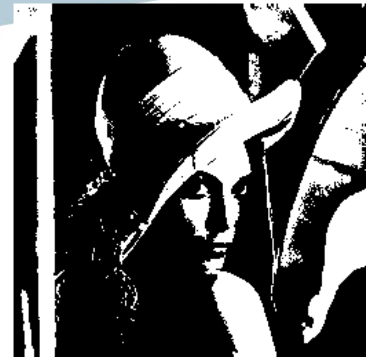
Reconstructed image using only bit planes 7 and 6

Reconstructed image using only bit planes 7, 6 and 5

Reconstructed image using only bit planes 7, 6, 5 and 4



(a) Original



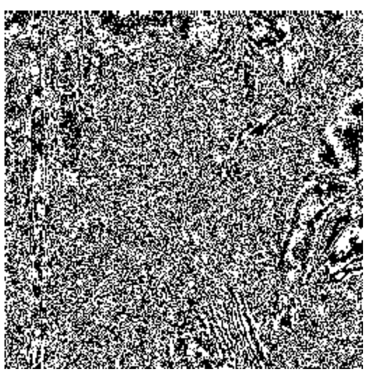
(b) Bit plane 7

(c) Bit plane 6

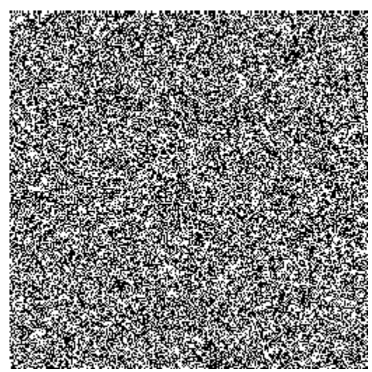
(d) Bit plane 5

(e) Bit plane 4

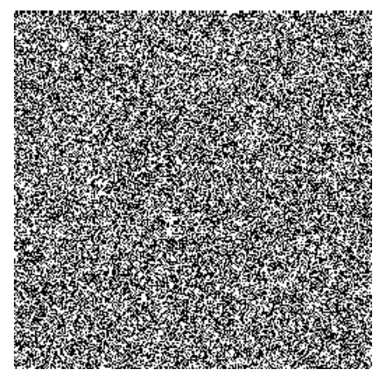
(f) Bit plane 3



(g) Bit plane 2



(h) Bit plane 1



(i) Bit plane 0

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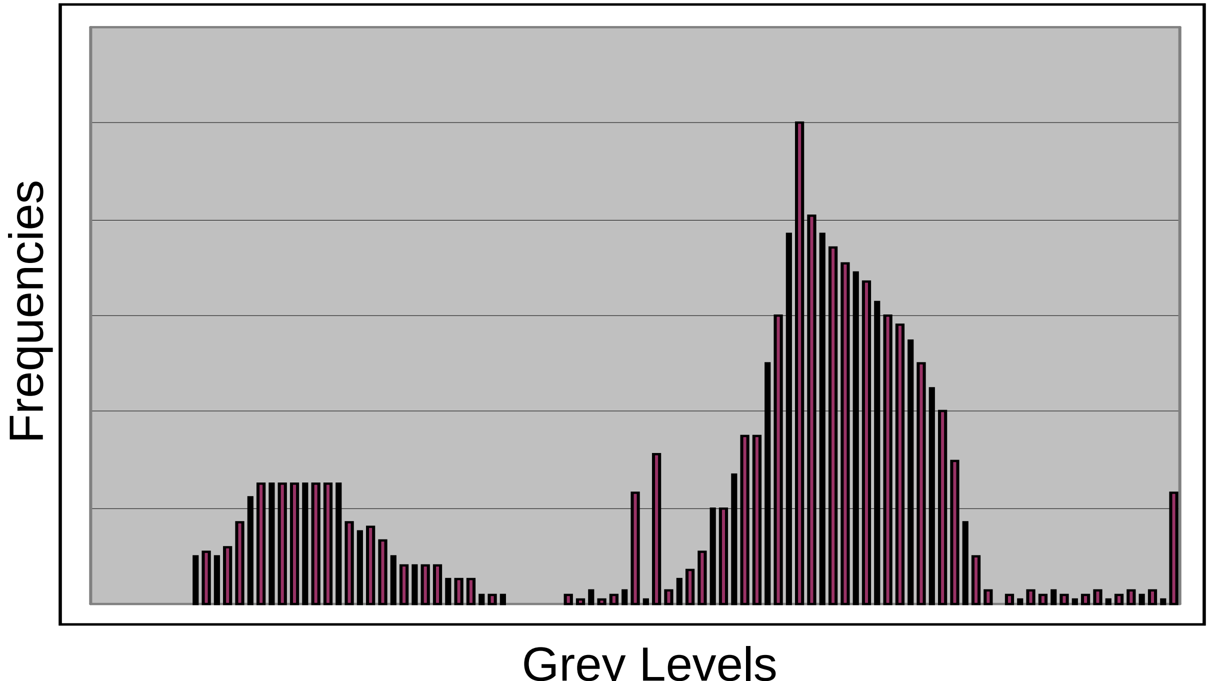
- Point processing
- Histogram processing

Image Histograms

The histogram of an image shows us the distribution of grey levels in the image

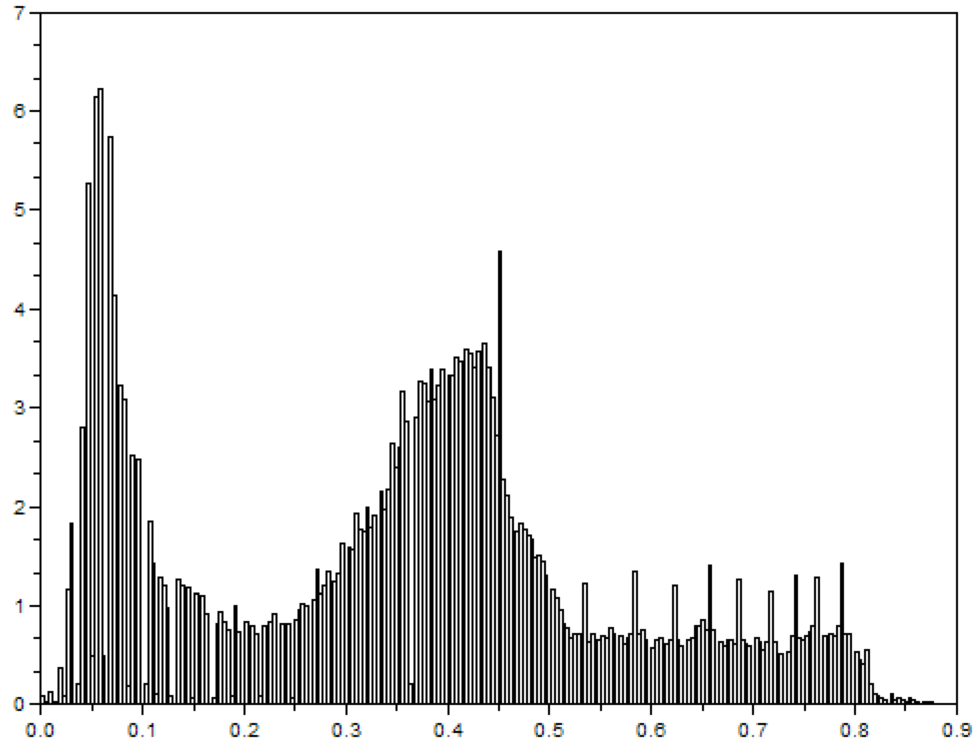
Massively useful in image processing, especially in segmentation



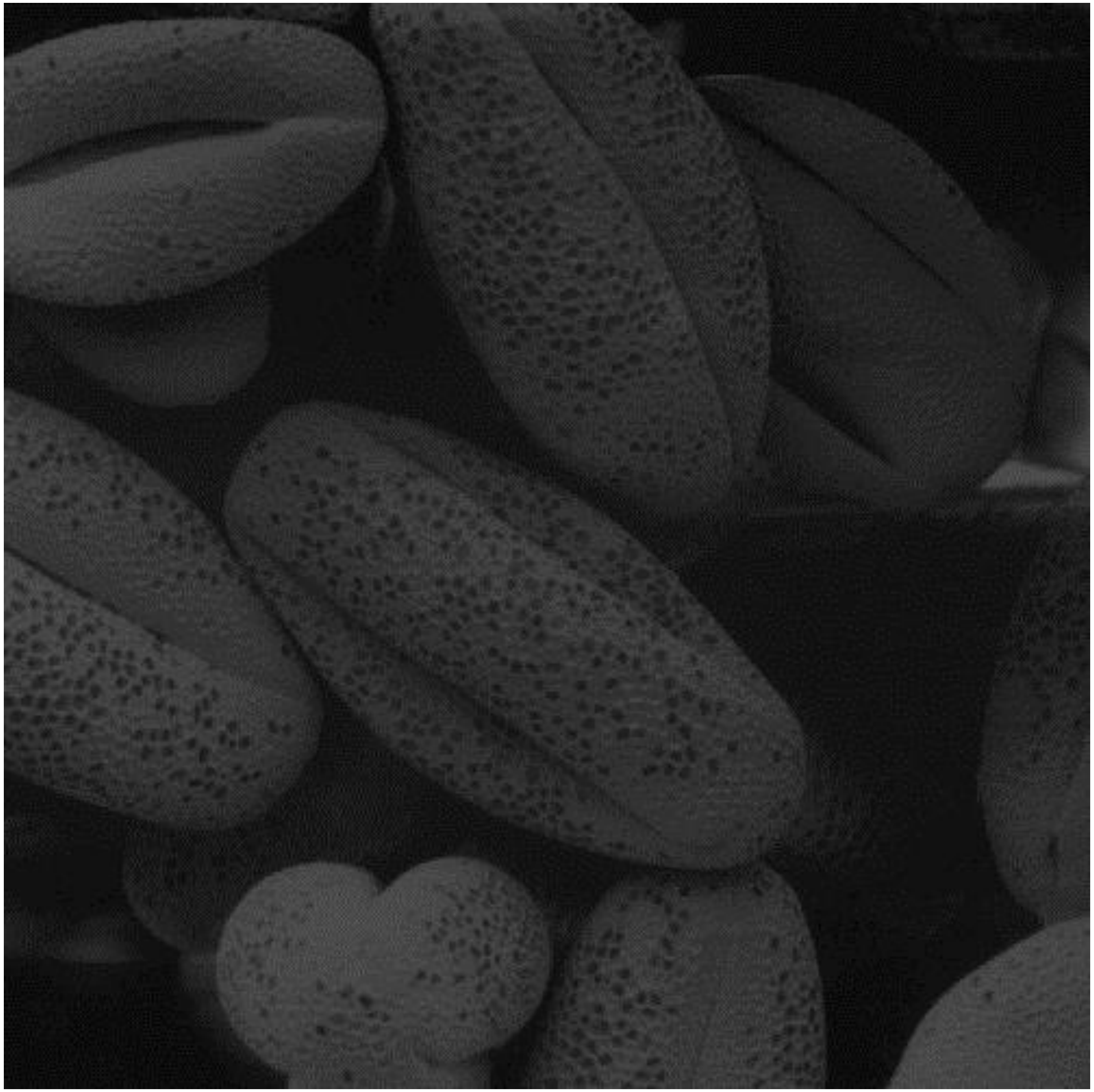
Histogram Examples

Lena

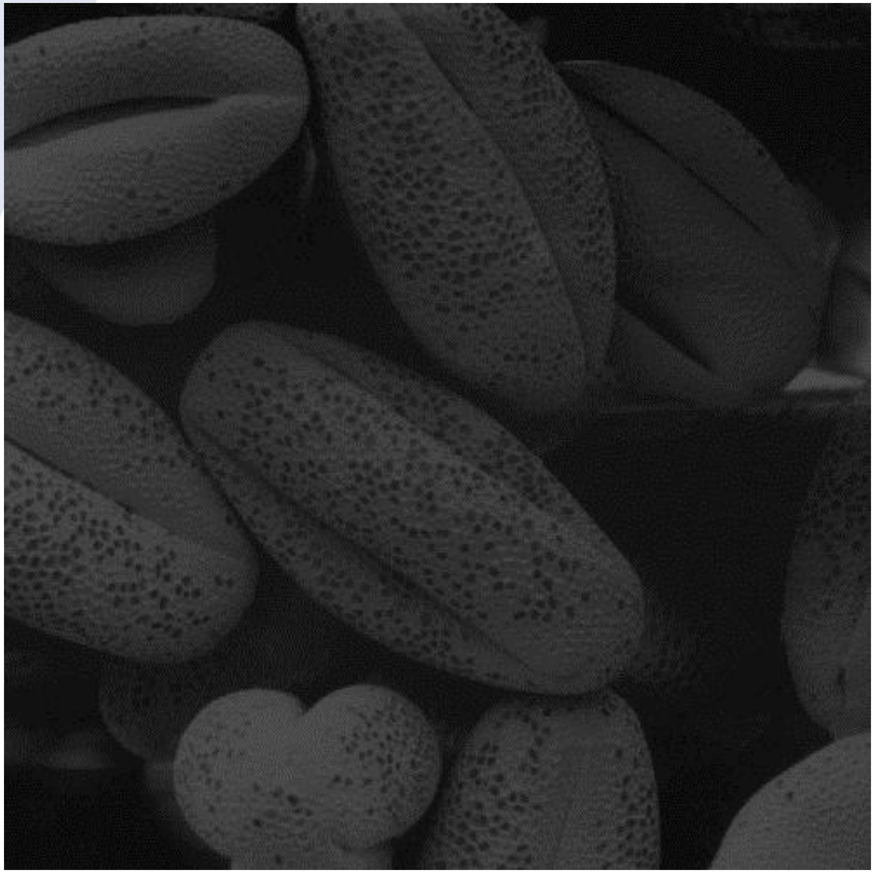
Histogram Examples (cont…)

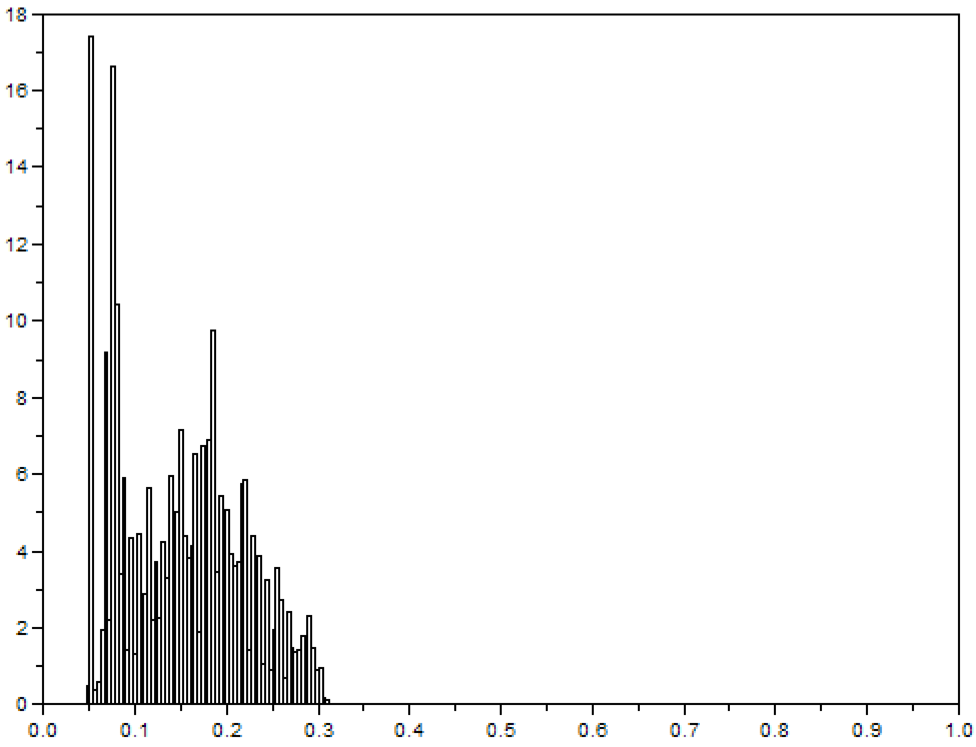


Histogram Examples (cont…)



Histogram Examples (cont…)





Histogram Examples (cont…)

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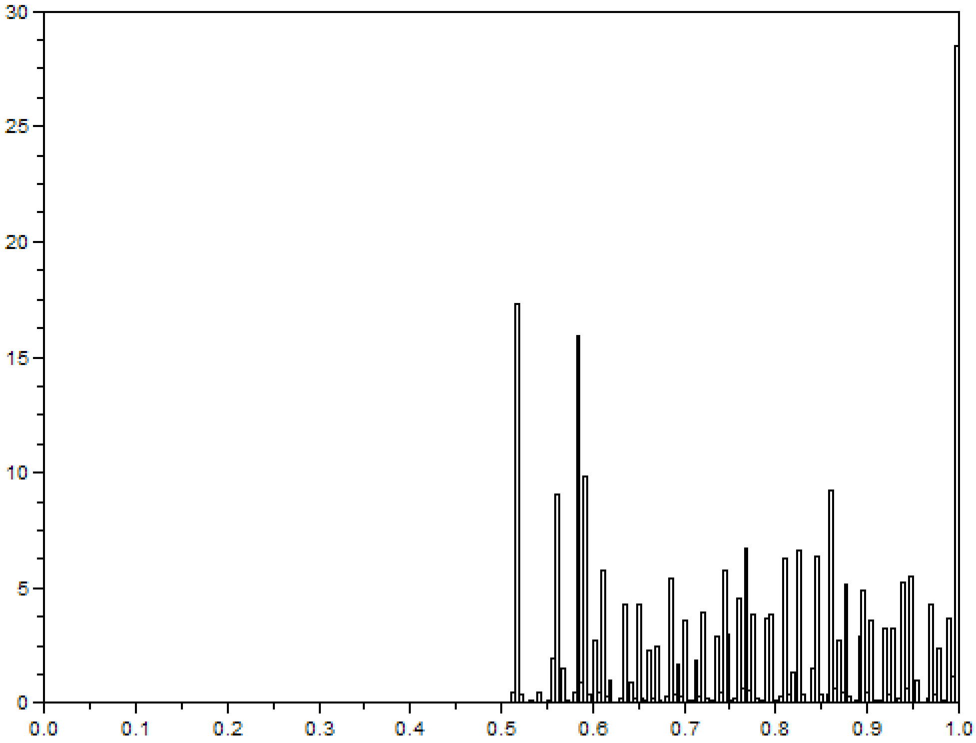
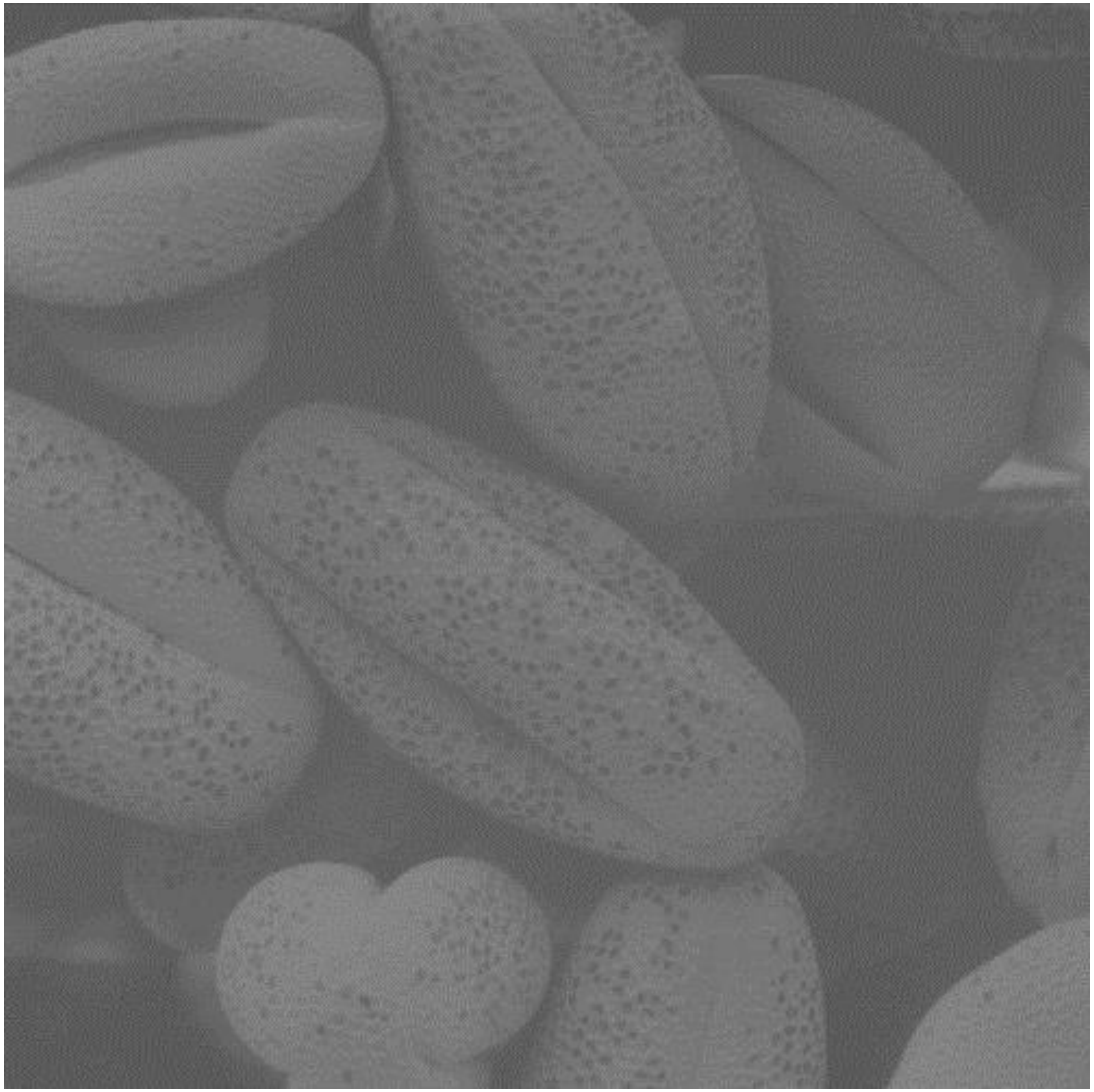
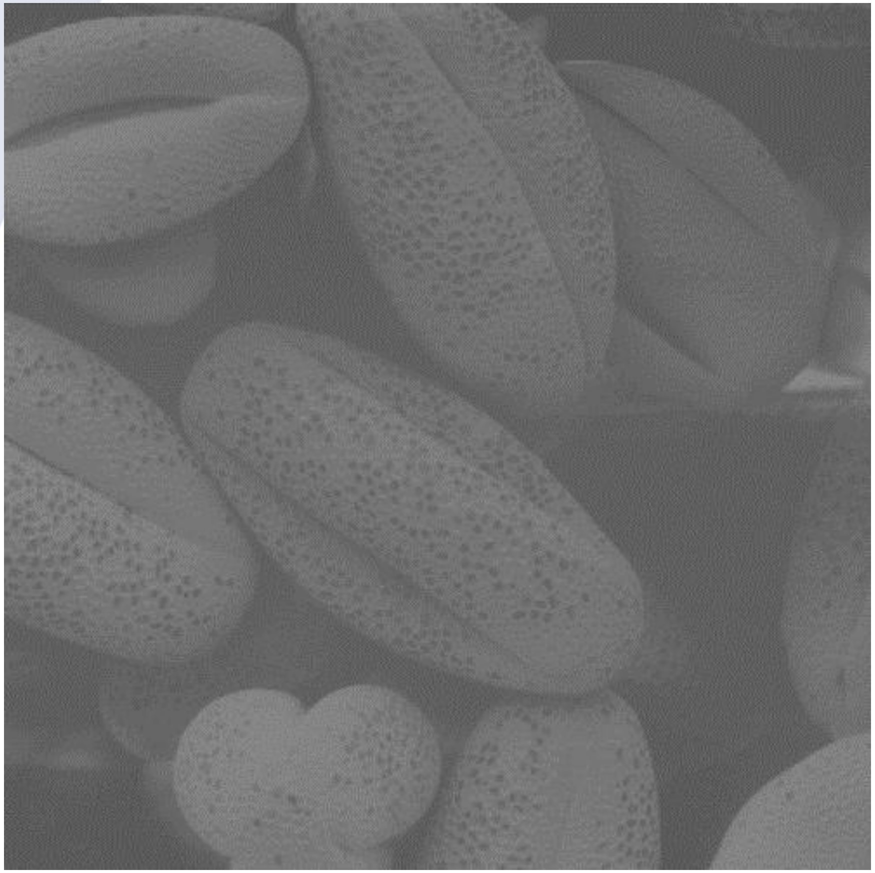
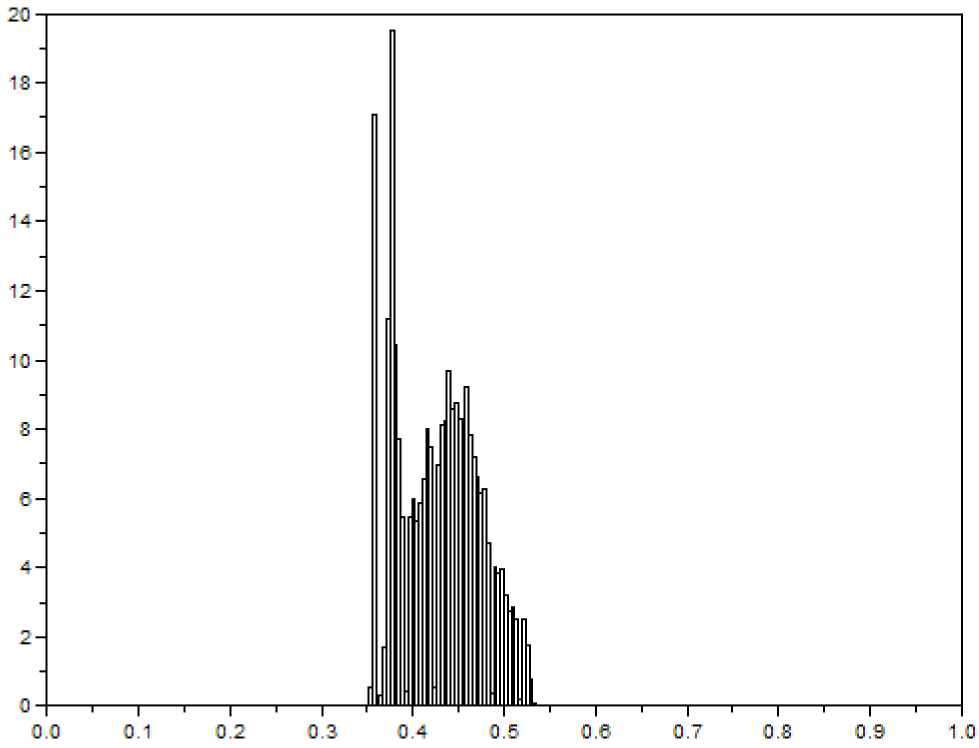


Histogram Examples (cont…)



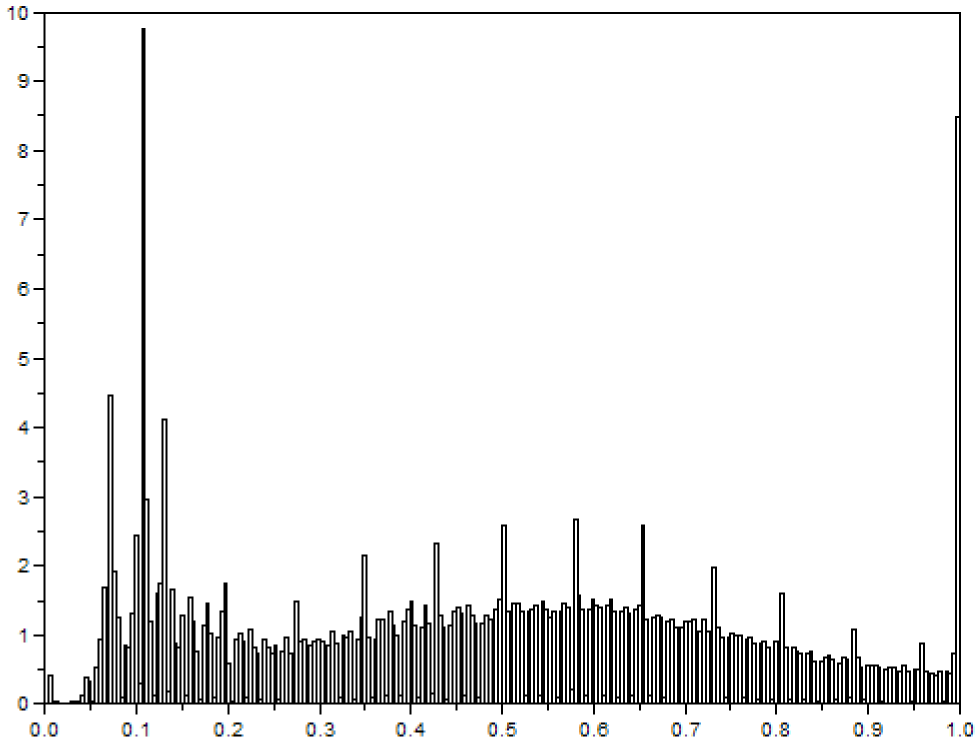
Histogram Examples (cont…)





Histogram Examples (cont…)

Histogram Examples (cont…)

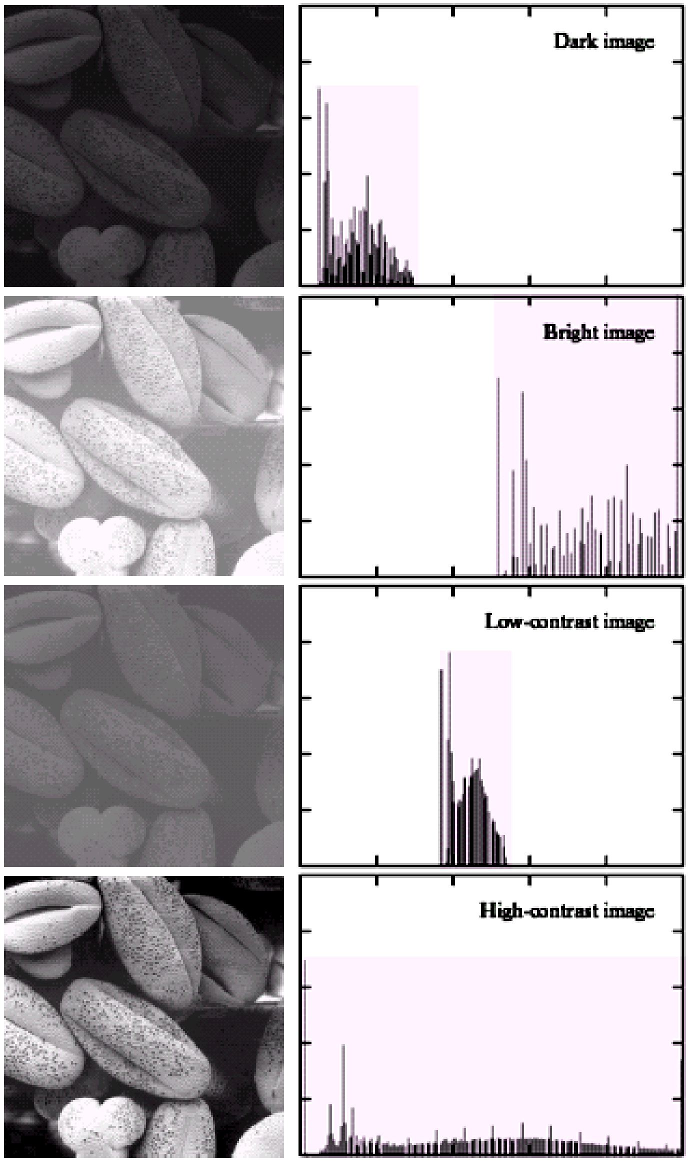


Histogram Examples (cont…)

A selection of images and their histograms

Notice the relationships between the images and their histograms

Note that the high contrast image has the most evenly spaced histogram



Contrast Stretching

We can fix images that have poor contrast by applying a pretty simple contrast specification

The interesting part is how do we decide on this transformation function?

Histogram Equalisation

Spreading out the frequencies in an image (or equalising the image) is a simple way to improve dark or washed out images

- When pixel intensity is continuous

$$p_s(s)ds = p_r(r)dr \longrightarrow \frac{ds}{dr} = \frac{p_r(r)}{p_s(s)}$$

$$p_s(s) = \frac{1}{L-1} \longrightarrow s = T(r) = (L-1)\int_0^r p_r(w)dw$$

Histogram Equalisation Example

$$s = T(r) = (L-1)\int_0^r p_r(w)\,dw$$

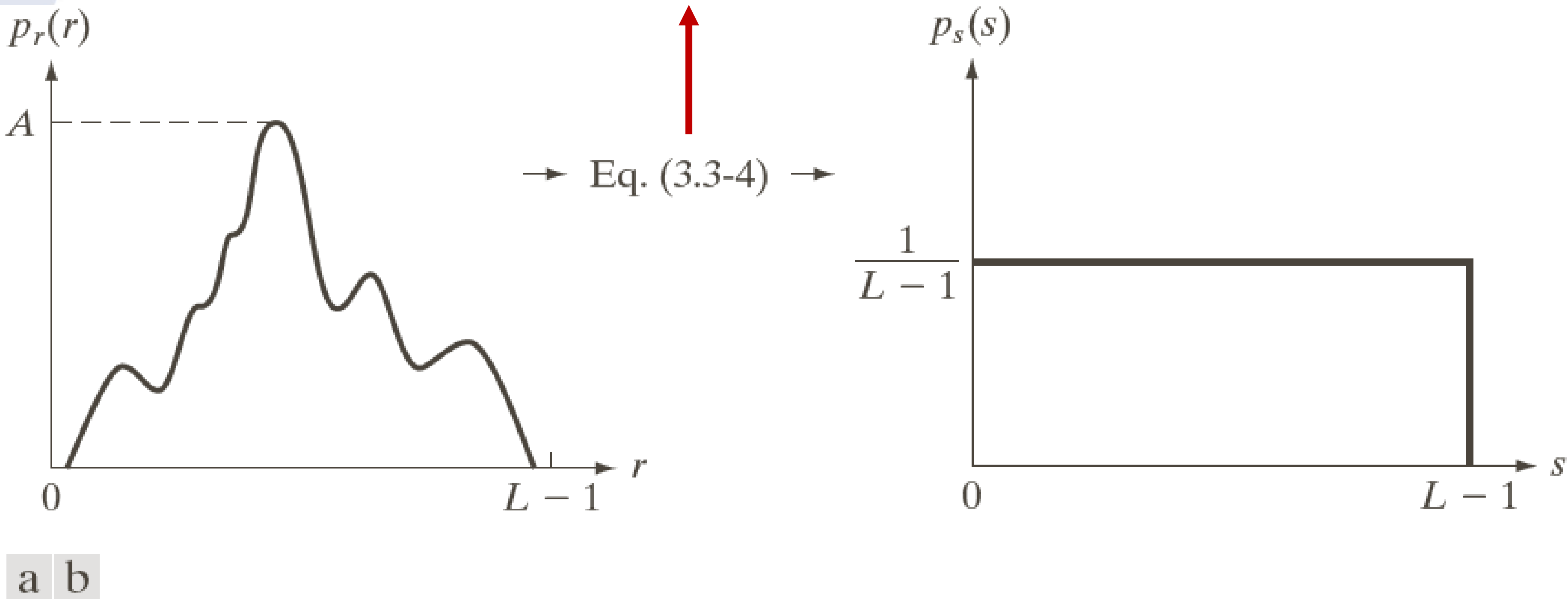


FIGURE 3.18 (a) An arbitrary PDF. (b) Result of applying the transformation in Eq. (3.3-4) to all intensity levels, r. The resulting intensities, s, have a uniform PDF, independently of the form of the PDF of the r's.

Histogram Equalisation Example

$$p_r(r) = \begin{cases} \dfrac{2r}{(L-1)^2} & \text{for } 0 \le r \le L-1 \\ 0 & \text{otherwise} \end{cases}$$

$$s = T(r) = (L-1)\int_0^r p_r(w)\,dw = \frac{2}{L-1}\int_0^r w\,dw = \frac{r^2}{L-1}$$

$$p_s(s) = p_r(r)\left|\frac{dr}{ds}\right| = \frac{2r}{(L-1)^2}\left|\left[\frac{ds}{dr}\right]^{-1}\right|$$

$$= \frac{2r}{(L-1)^2}\left|\left[\frac{d}{dr}\frac{r^2}{L-1}\right]^{-1}\right| = \frac{2r}{(L-1)^2}\left|\frac{(L-1)}{2r}\right| = \frac{1}{L-1}$$

Multiple / One to One Mapping

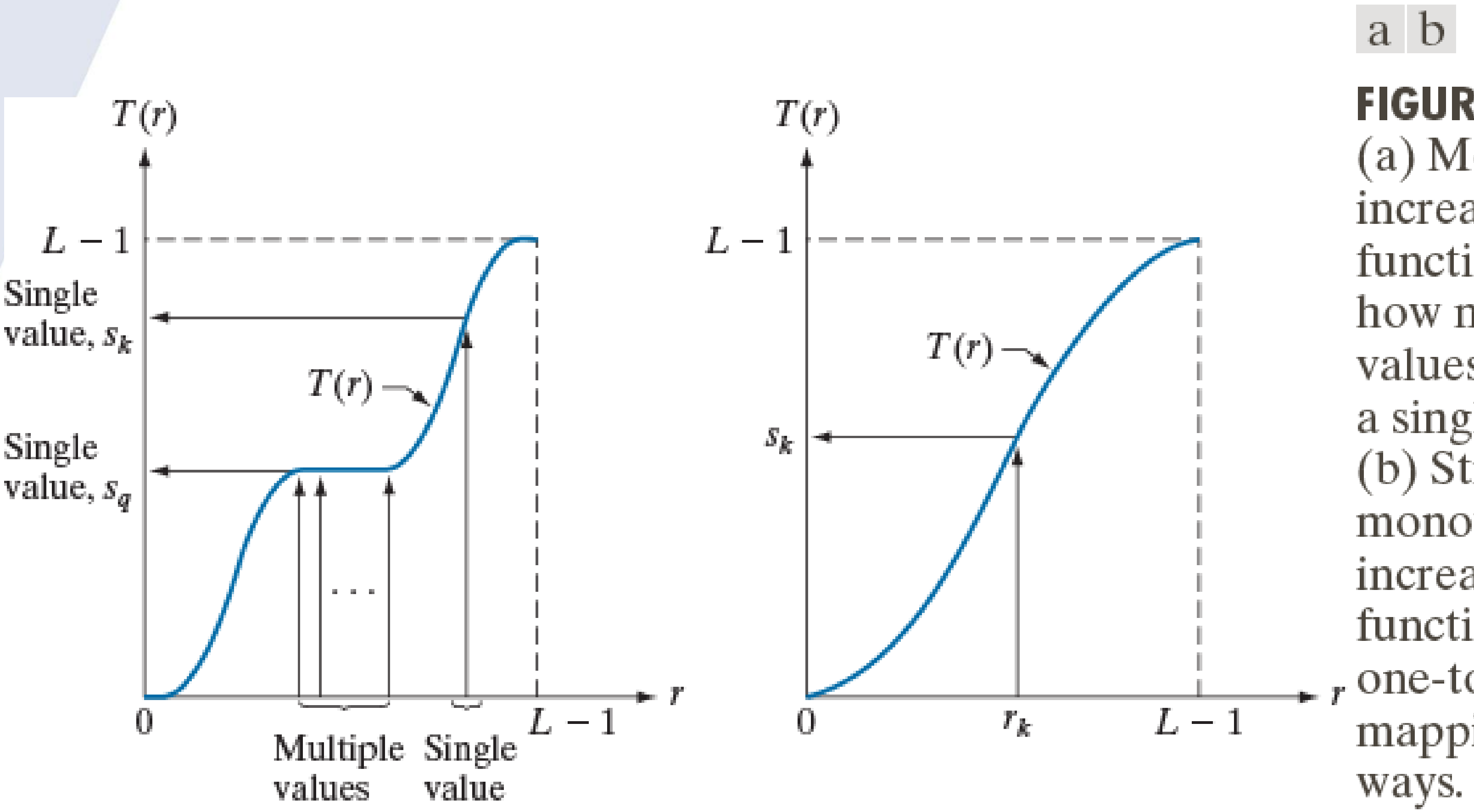


FIGURE 3.17
(a) Monotonically increasing function, showing how multiple values can map to a single value.
(b) Strictly monotonically increasing function. This is a one-to-one mapping, both ways.

Discrete Histogram Equalisation

- When pixel intensity is discrete, the formula for histogram equalisation is given

$$s_k = (L-1)T(r_k) = (L-1)\sum_{j=0}^{k} p_r(r_j)$$

where

$$= (L-1)\sum_{j=0}^{k} \frac{n_j}{n}$$

  - r_k: input intensity
  - s_k: processed intensity
  - k: the intensity range
    $k = 0, 1, 2, \ldots, L\text{-}1$
  - n_j: the frequency of intensity j
  - n: the sum of all frequencies

Discrete Histogram Equalisation

r_k	n_k	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02

TABLE 3.1 Intensity distribution and histogram values for a 3-bit, 64×64 digital image.

Discrete Histogram Equalisation

$$s_0 = T(r_0) = 7\sum_{j=0}^{0} p_r(r_j) = 7p_r(r_0) = 1.33 \rightarrow 1$$

$$s_1 = T(r_1) = 7\sum_{j=0}^{1} p_r(r_j) = 7p_r(r_0) + 7p_r(r_1) = 3.08 \rightarrow 3$$

$$s_2 = 4.55 \rightarrow 5$$

$$s_3 = 5.67 \rightarrow 6$$

$$s_4 = 6.23 \rightarrow 6$$

$$s_5 = 6.65 \rightarrow 7$$

$$s_6 = 6.86 \rightarrow 7$$

$$s_7 = 7.00 \rightarrow 7$$

Discrete Histogram Equalisation

NOT perfectly flat

Problem 3.6

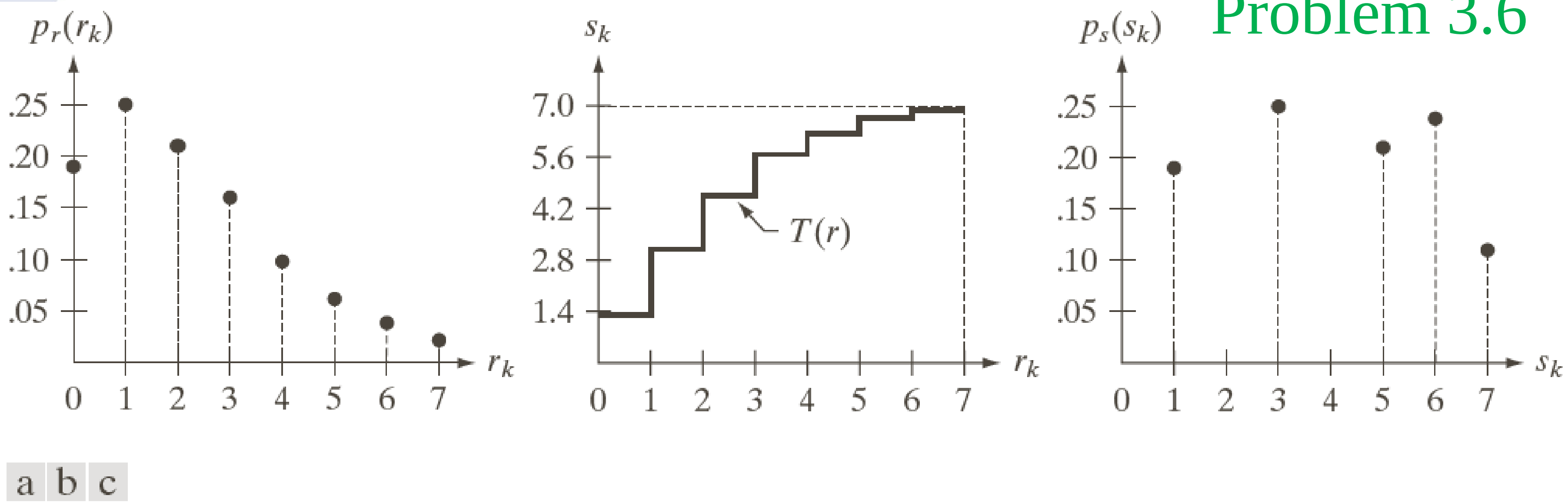
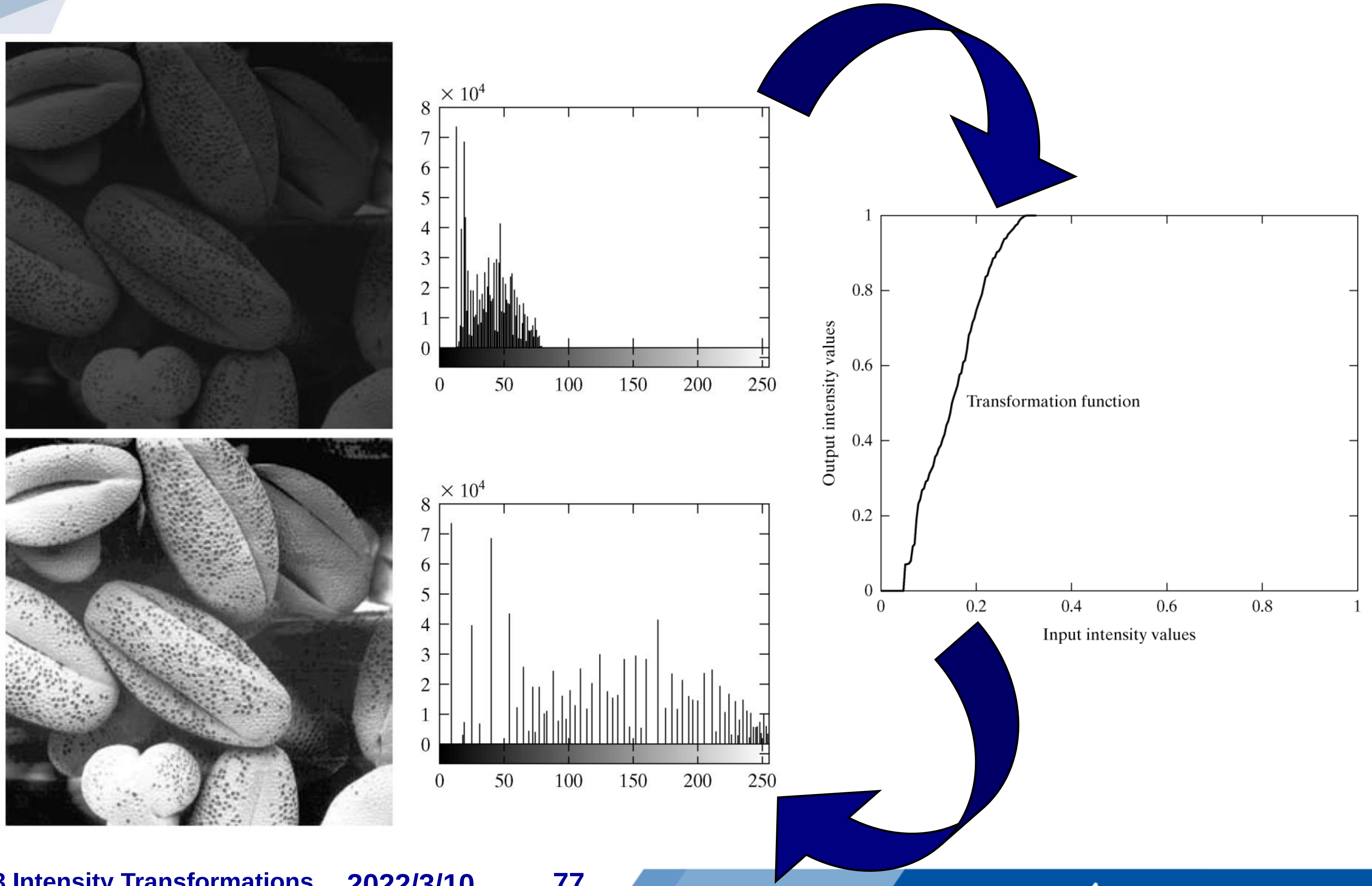
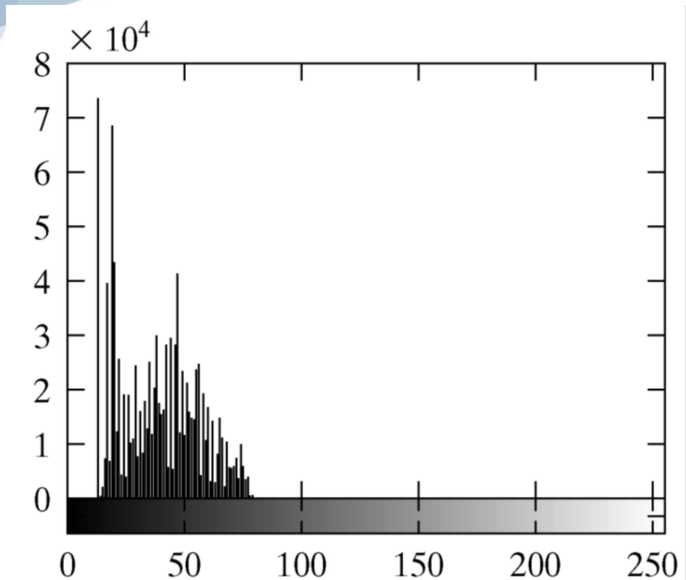


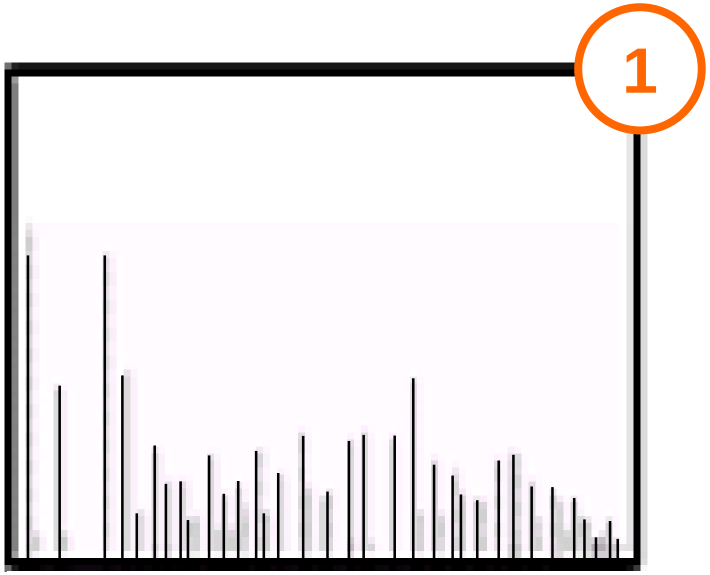
FIGURE 3.19 Illustration of histogram equalization of a 3-bit (8 intensity levels) image. (a) Original histogram. (b) Transformation function. (c) Equalized histogram.

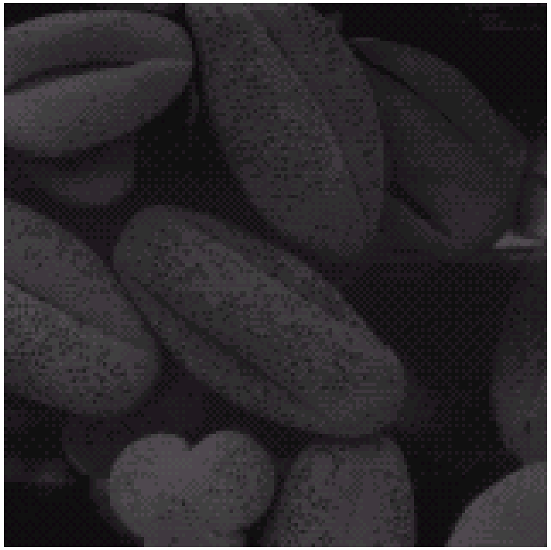
Equalisation Transformation Function



Equalisation Examples

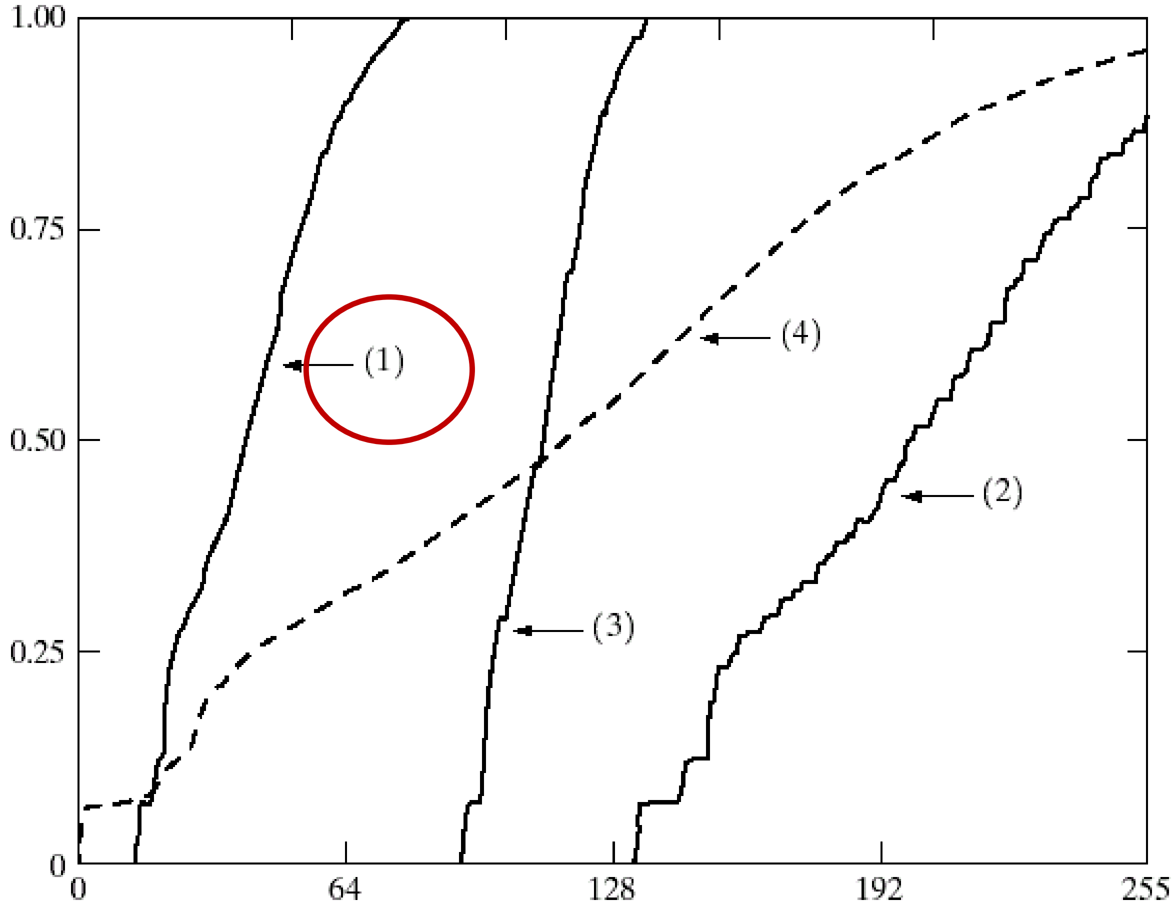




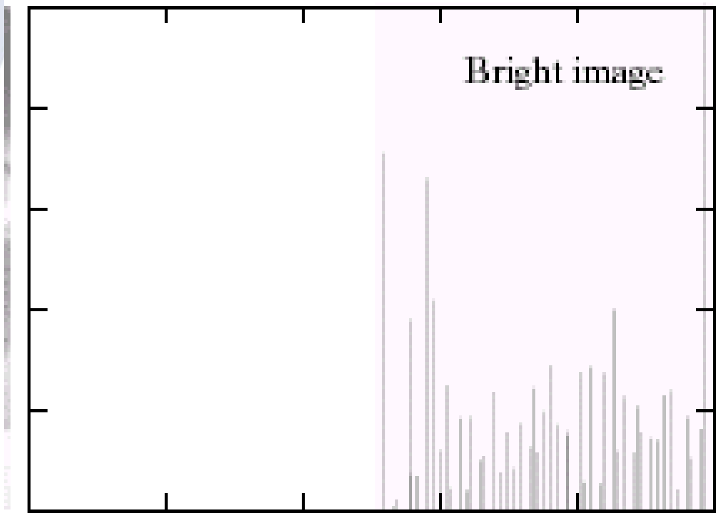


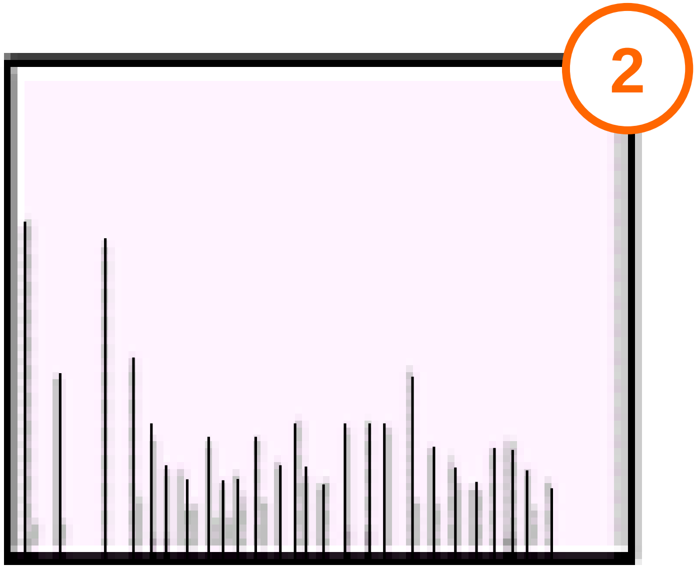
Equalisation Transformation Functions

The functions used to equalise the images in the previous example



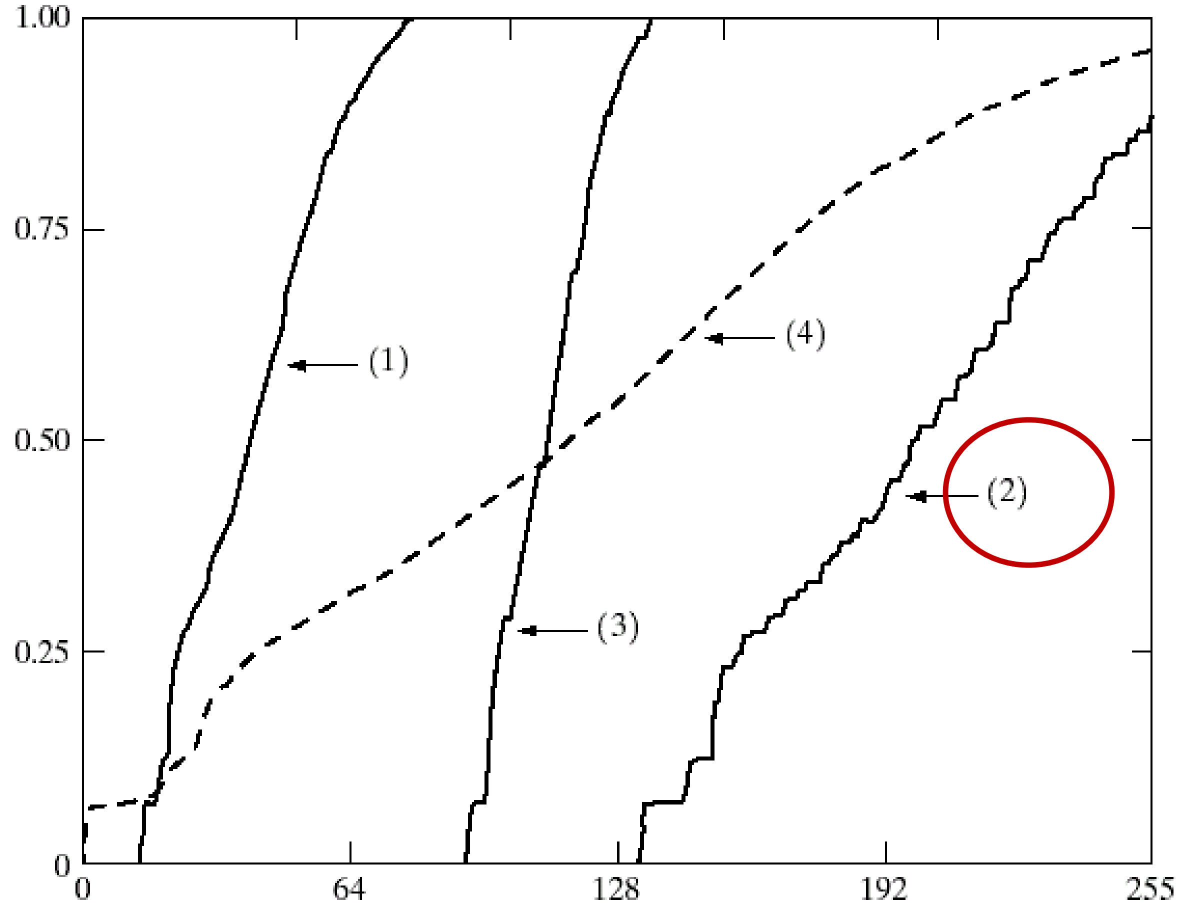
Equalisation Examples



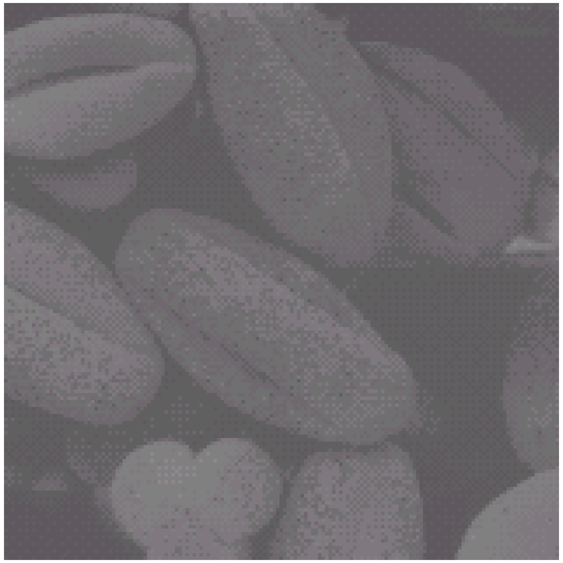


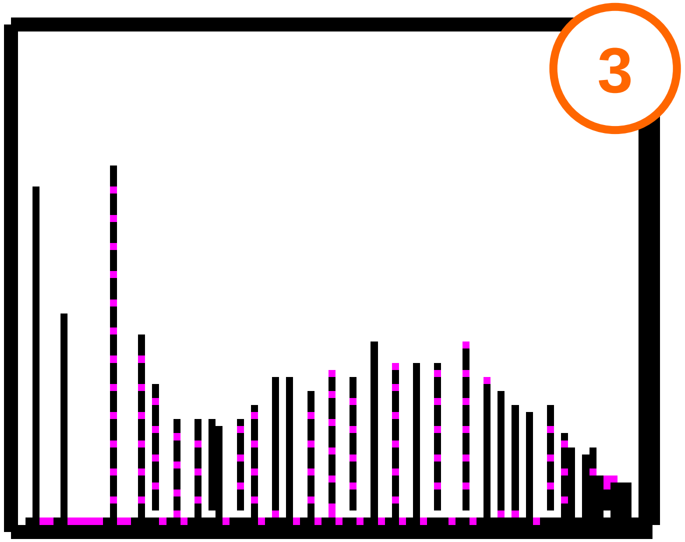
Equalisation Transformation Functions

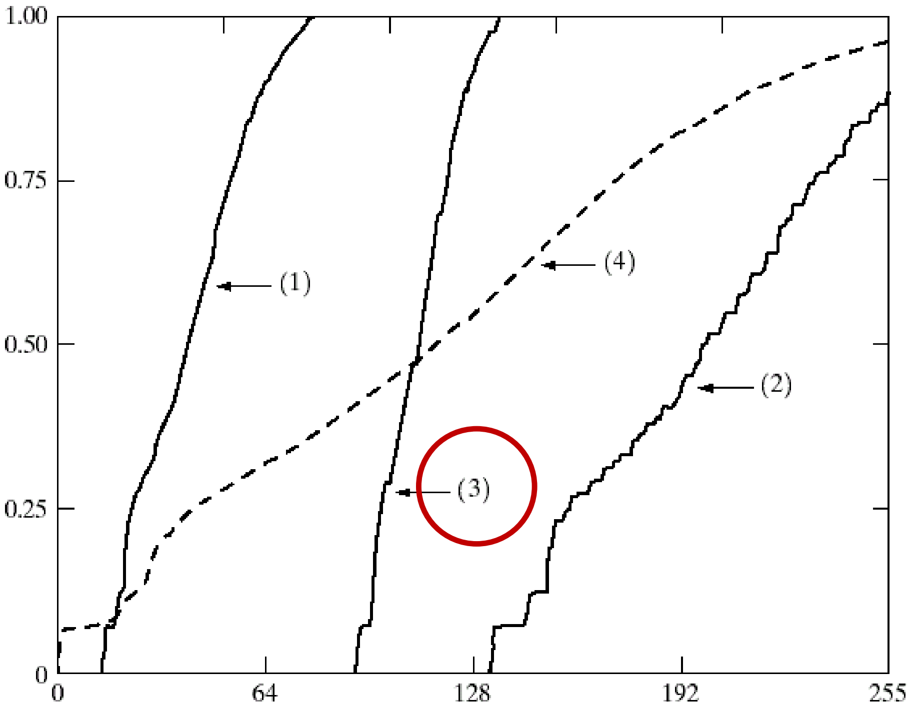
The functions used to equalise the images in the previous example



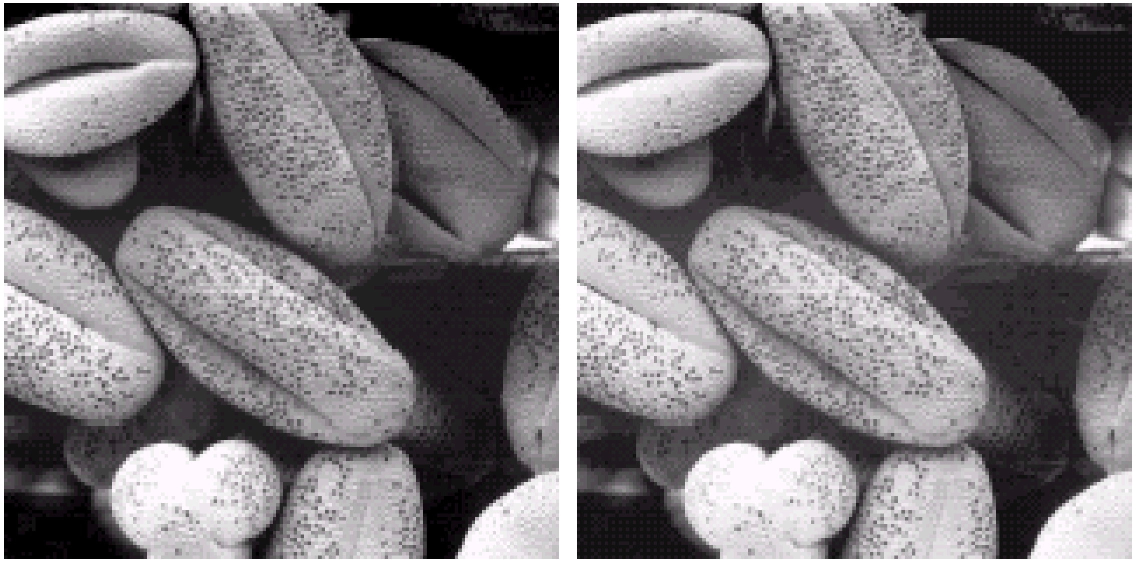
Equalisation Examples (cont…)

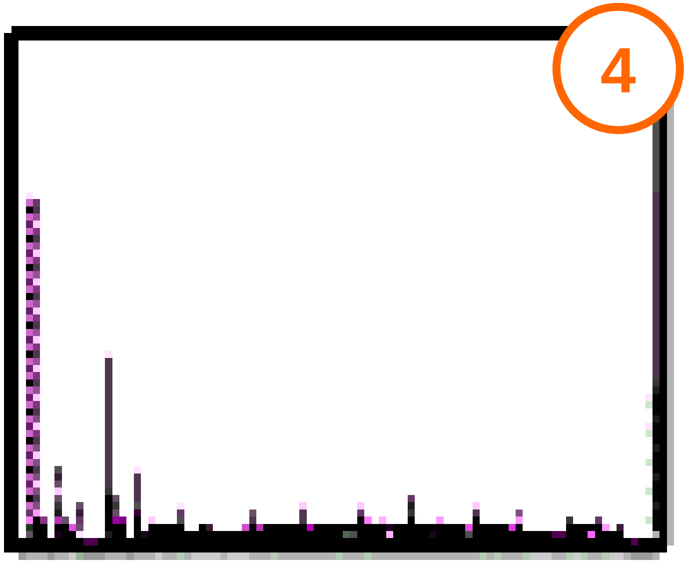


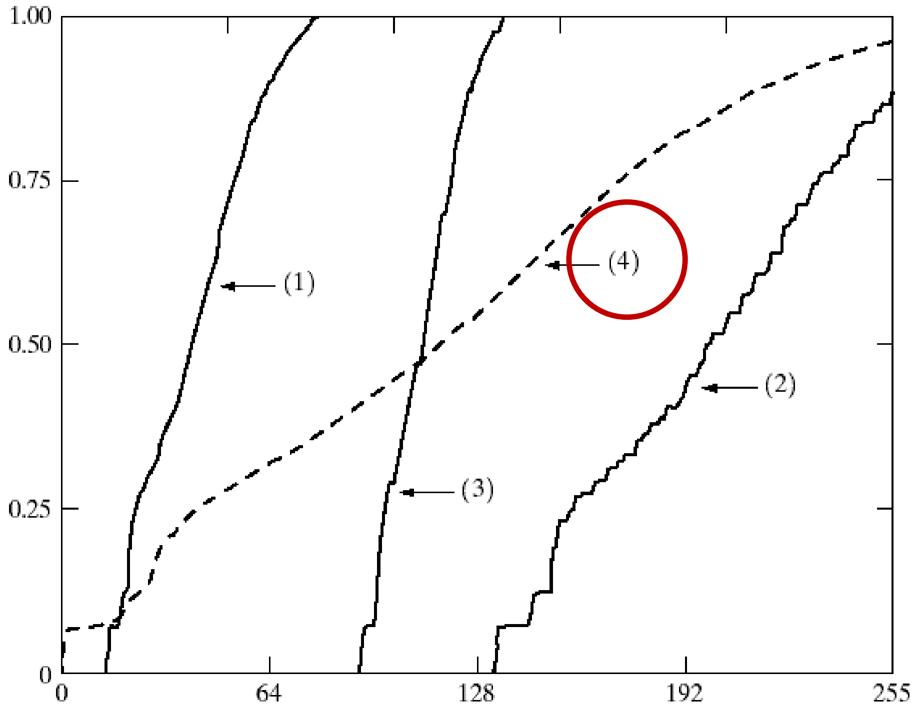




Equalisation Examples (cont…)







Discrete Histogram Equalisation

- A second pass of histogram equalization will produce exactly the same result as the first pass
  - 1st pass

$$s_l = T(l) = (L-1)\sum_{k=0}^{l}\frac{n_{0k}}{n}$$

$$l \rightarrow s \Rightarrow [0,l] \rightarrow [0,s]$$

  - 2nd pass

$$t_l = T(s_l) = (L-1)\sum_{k=0}^{s_l}\frac{n_{1k}}{n} = (L-1)\sum_{k=0}^{l}\frac{n_{0k}}{n} = s_l$$

Histogram Matching (Specification)

- Use histogram equalization as the bridge
- For continuous intensities $p_r(r) \rightarrow p_z(z)$

$$s = T(r) = (L-1)\int_0^r p_r(w)\,dw$$

$$G(z) = (L-1)\int_0^z p_z(t)\,dt = s$$

$$z = G^{-1}[T(r)] = G^{-1}(s)$$

Does the inverse mapping always exist? NO

Histogram Matching (Specification)

- For discrete intensities

$$s_k = T(r_k) = (L-1)\sum_{j=0}^{k} p_r(r_j) = \frac{(L-1)}{MN}\sum_{j=0}^{k} n_j$$

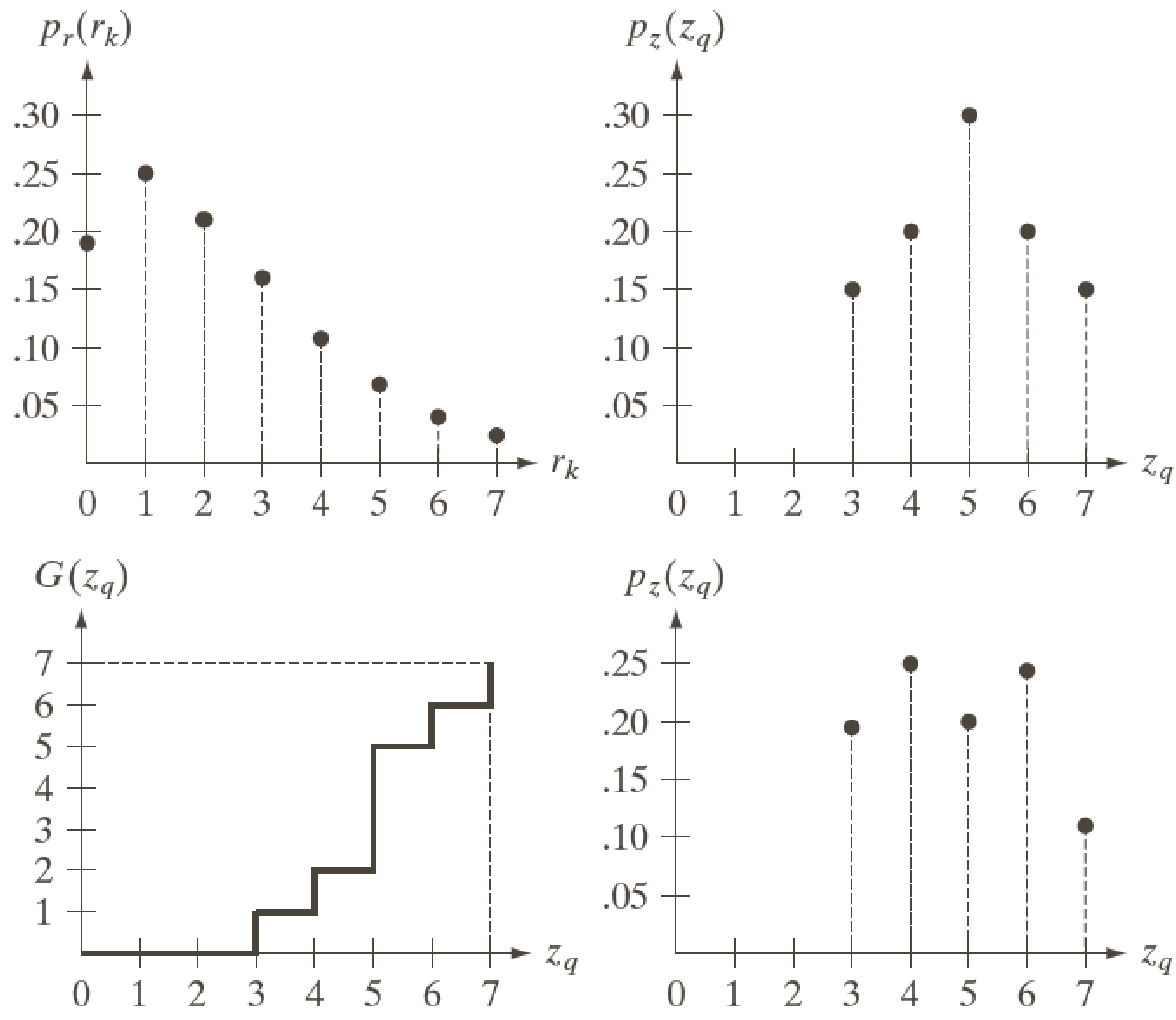
$$G(z_q) = (L-1)\sum_{i=0}^{q} p_z(z_i) = s_k$$

$$z_q = G^{-1}(s_k)$$

Approximation

Histogram Matching (Specification)

- For discrete intensities



a	b
c	d

FIGURE 3.22
(a) Histogram of a 3-bit image. (b) Specified histogram.
(c) Transformation function obtained from the specified histogram.
(d) Result of performing histogram specification. Compare (b) and (d).

Histogram Matching (Specification)

the smallest value of z_q so that the value $G(z_q)$ is the closest to s_k

$$s_0 = 1.33 \rightarrow 1$$
$$s_1 = 3.08 \rightarrow 3$$
$$s_2 = 4.55 \rightarrow 5$$
$$s_3 = 5.67 \rightarrow 6$$
$$s_4 = 6.23 \rightarrow 6$$
$$s_5 = 6.65 \rightarrow 7$$
$$s_6 = 6.86 \rightarrow 7$$
$$s_7 = 7.00 \rightarrow 7$$

s_k	$\rightarrow$	z_q
1	$\rightarrow$	3
3	$\rightarrow$	4
5	$\rightarrow$	5
6	$\rightarrow$	6
7	$\rightarrow$	7

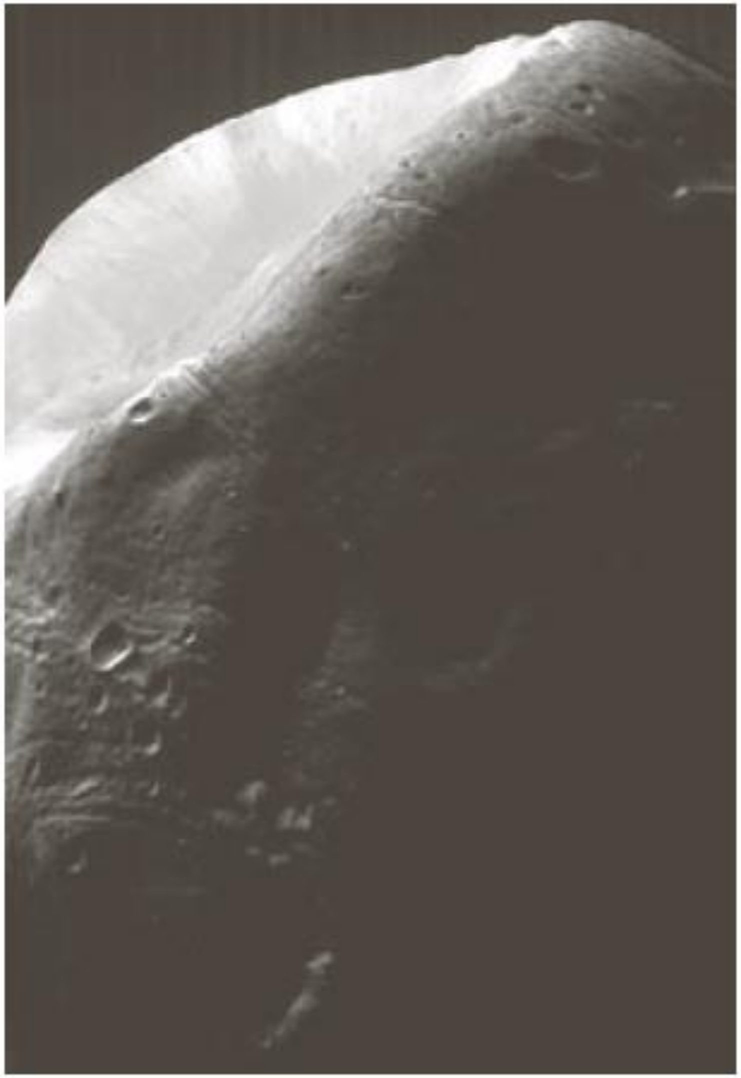
$$G(z_0) = 0.00 \rightarrow 0$$
$$G(z_1) = 0.00 \rightarrow 0$$
$$G(z_2) = 0.00 \rightarrow 0$$
$$G(z_3) = 1.05 \rightarrow 1$$
$$G(z_4) = 2.45 \rightarrow 2$$
$$G(z_5) = 4.55 \rightarrow 5$$
$$G(z_6) = 5.95 \rightarrow 6$$
$$G(z_7) = 7.00 \rightarrow 7$$

Histogram Matching (Specification)

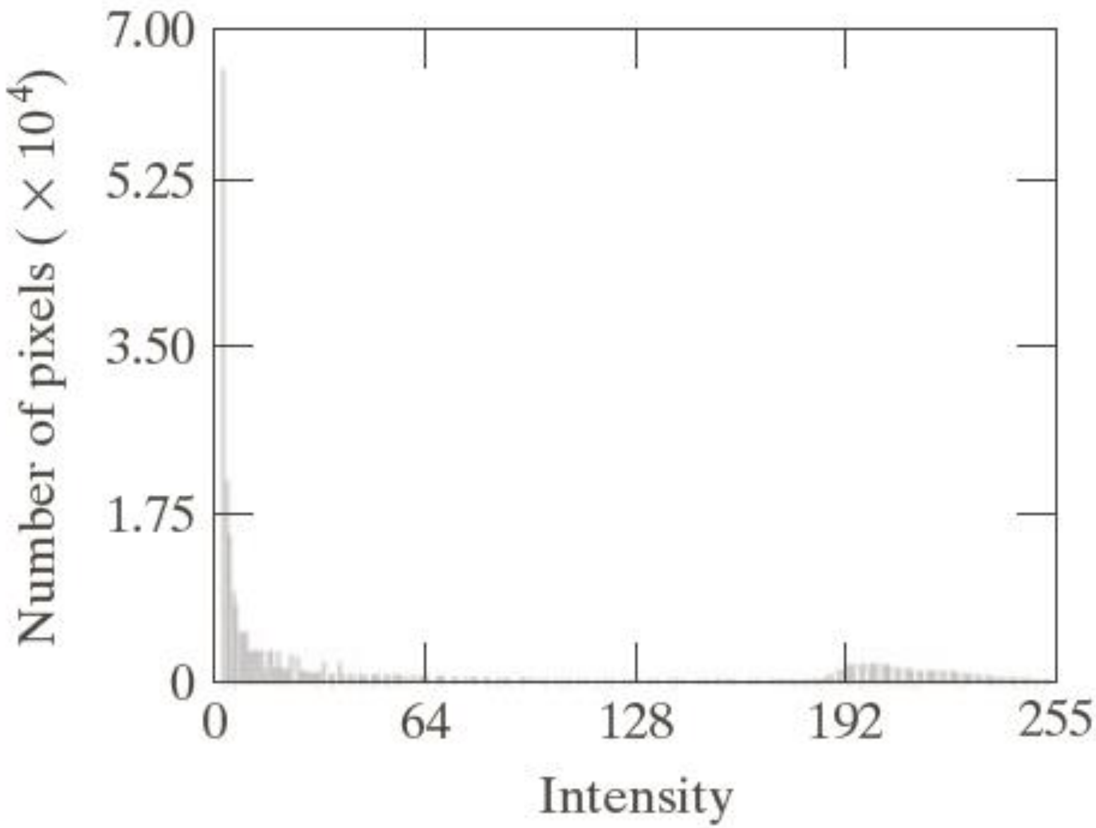
- For discrete intensities

z_q	Specified $p_z(z_q)$	Actual $p_z(z_k)$
$z_0 = 0$	0.00	0.00
$z_1 = 1$	0.00	0.00
$z_2 = 2$	0.00	0.00
$z_3 = 3$	0.15	0.19
$z_4 = 4$	0.20	0.25
$z_5 = 5$	0.30	0.21
$z_6 = 6$	0.20	0.24
$z_7 = 7$	0.15	0.11

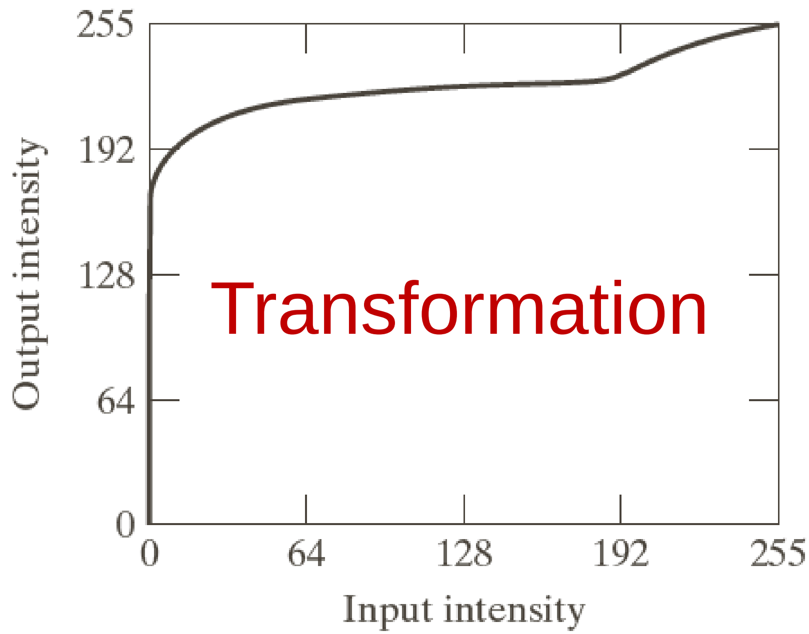
Histogram Equalization vs Matching

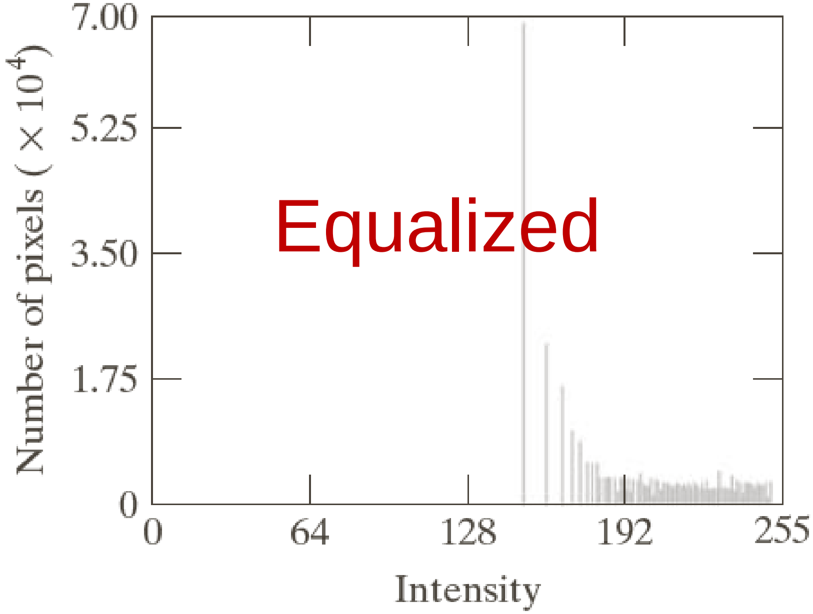


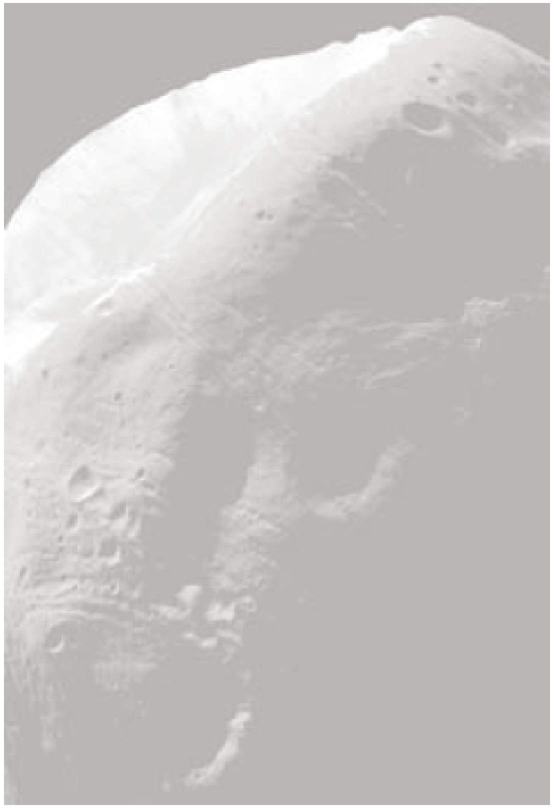
Mars Global Surveyor



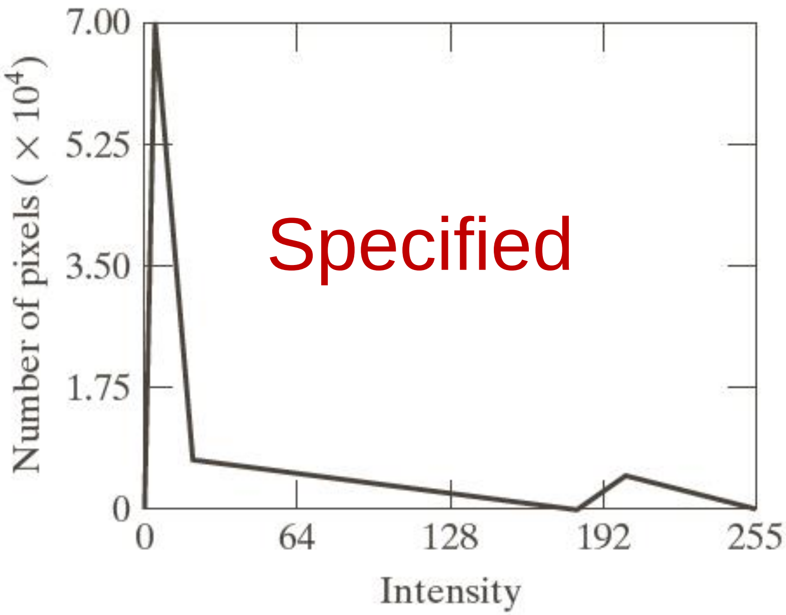
Result of Histogram Equalization



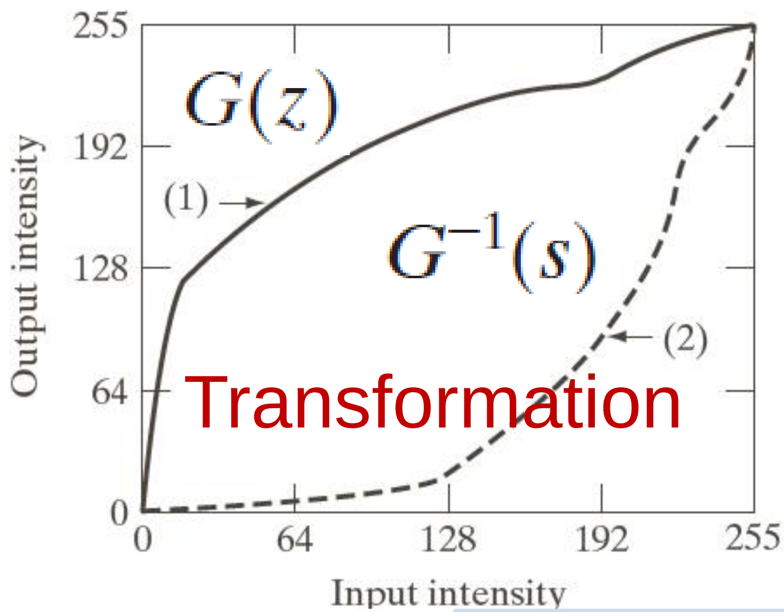


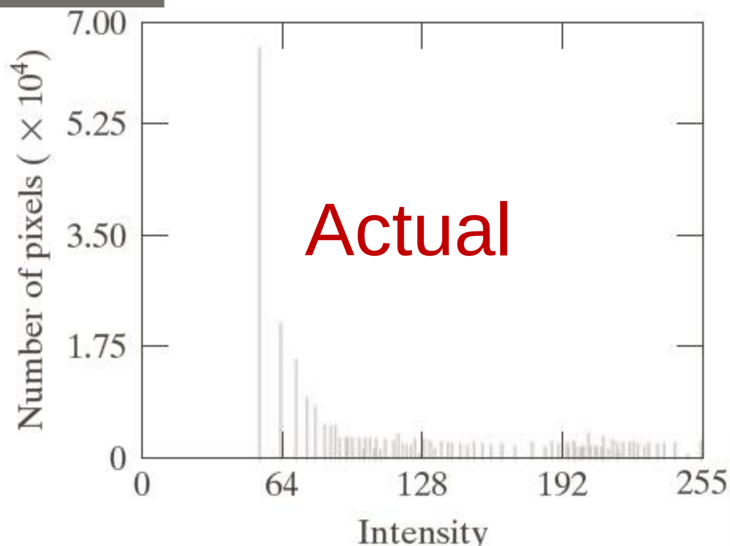


Result of Histogram Matching



case-by-case
trial-and-error





Local Histogram Processing

- Global: entire image
- Local: based on the histogram of a neighborhood

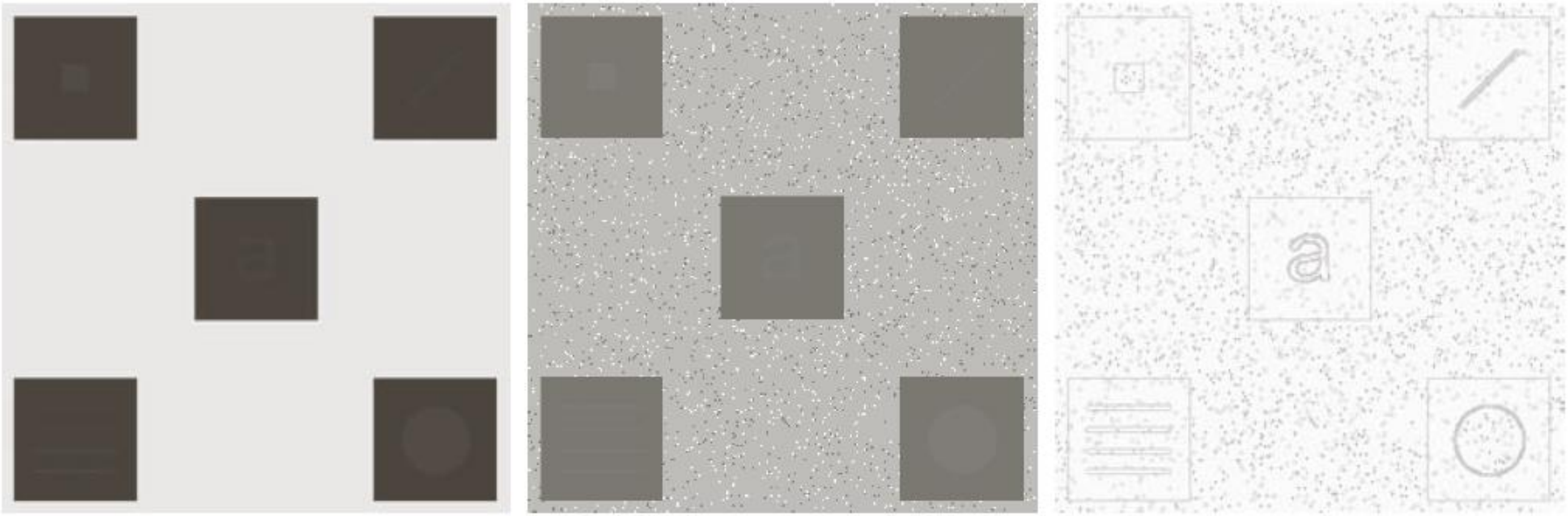


FIGURE 3.26 (a) Original image. (b) Result of global histogram equalization. (c) Result of local histogram equalization applied to (a), using a neighborhood of size 3×3.

Using Histogram Statistics for Image Enhancement

- mean (average intensity) $$m = \sum_{i=0}^{L-1} r_i p(r_i)$$

$$m = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x, y)$$

- *n*-th moment $$\mu_n(r) = \sum_{i=0}^{L-1} (r_i - m)^n p(r_i)$$
- 2-nd moment (variance)

$$\mu_2(r) = \sum_{i=0}^{L-1} (r_i - m)^2 p(r_i)$$

$$\sigma^2 = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} \left[f(x, y) - m \right]^2$$

Using Histogram Statistics for Image Enhancement

- Local mean (average intensity)

$$m_{S_{xy}} = \sum_{i=0}^{L-1} r_i p_{S_{xy}}(r_i)$$

- Local variance

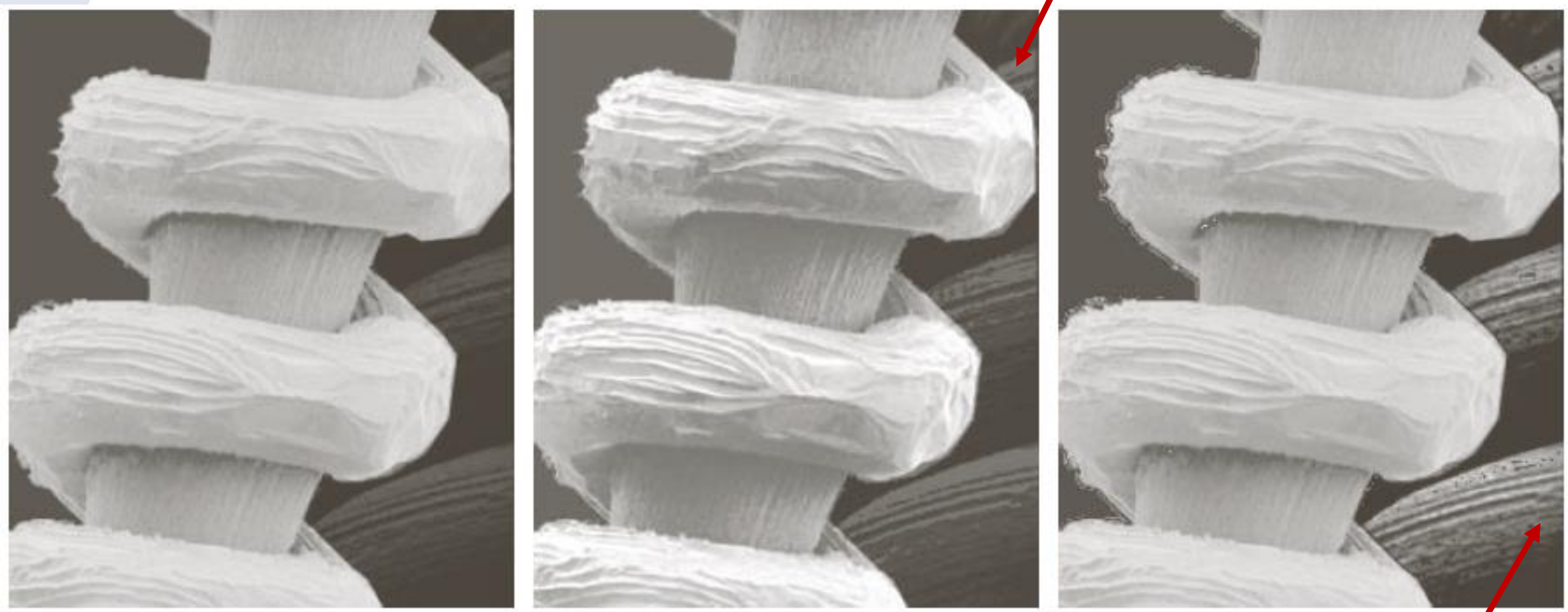
$$\sigma^2_{S_{xy}} = \sum_{i=0}^{L-1} (r_i - m_{S_{xy}})^2 p_{S_{xy}}(r_i)$$

- neighborhood

$$S_{xy}$$

Using Histogram Statistics for Image Enhancement

- Example: enhance the dark filament



Original　　Globally Equalized　　Locally Enhanced

$$g(x, y) = \begin{cases} \underset{4.0}{\underline{E}} \cdot f(x, y) & \text{if } m_{S_{xy}} \le \underset{0.4}{\underline{k_0}} m_G \text{ AND } \underset{0.02}{\underline{k_1}} \sigma_G \le \sigma_{S_{xy}} \le \underset{0.4}{\underline{k_2}} \sigma_G \\ f(x, y) & \text{otherwise} \end{cases}$$

Assignments

课后作业题目请对照参考第4版英文原版

- 3.1, 3.5, 3.6, 3.9