Image Restoration

& Reconstruction

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Contents

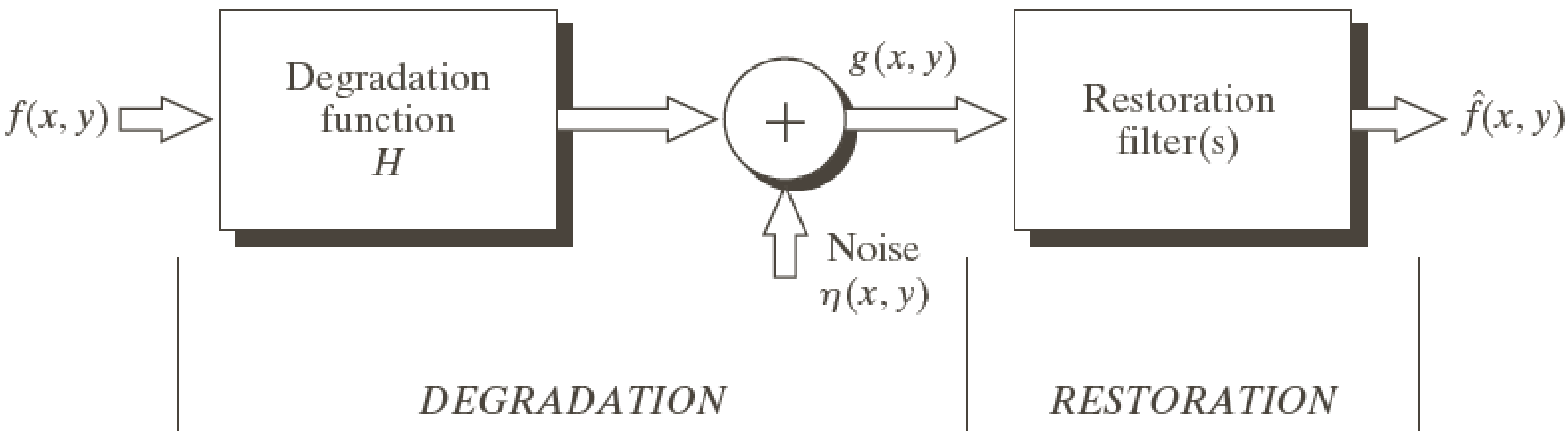
Image Degradation/Restoration Process

Spatial Domain: $$g(x, y) = h(x, y) \star f(x, y) + \eta(x, y)$$

Frequency Domain: $$G(u, v) = H(u, v)F(u, v) + N(u, v)$$

H: linear, position-invariant

N: additive noise



Linear, Position-Invariant Degradations

Properties of the degradation function H

- Linear system

$$H[af_1(x, y) + bf_2(x, y)] = aH[f_1(x, y)] + bH[f_2(x, y)]$$

- Position(space)-invariant system

$$g(x, y) = H[f(x, y)]$$

$$H[f(x - \alpha, y - \beta)] = g(x - \alpha, y - \beta)$$

- c.f. 1-D signal
  - LTI (linear time-invariant system)

Linear, Position-Invariant Degradations

$$f(x,y)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\alpha,\beta)\boxed{\delta(x-\alpha,y-\beta)}d\alpha d\beta$$

impulse

$$g(x,y)=H[f(x,y)]=H\left[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\alpha,\beta)\delta(x-\alpha,y-\beta)d\alpha d\beta\right]$$

linear

$$g(x,y)=H[f(x,y)]=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\alpha,\beta)\boxed{H\left[\delta(x-\alpha,y-\beta)\right]}d\alpha d\beta$$

$$h(x,y)=H\left[\delta(x,y)\right]$$

Impulse response (point spread function)

position-invariant

$$H\left[\delta(x-\alpha,y-\beta)\right]=h(x-\alpha,y-\beta)$$

Linear, Position-Invariant Degradations

$$g(x,y)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\alpha,\beta)h(x-\alpha,y-\beta)d\alpha d\beta = h(x,y) * f(x,y)$$

$$\eta(x,y)\neq 0$$

$$g(x,y)=h(x,y)*f(x,y)+\eta(x,y)$$

$$G(u,v) = H(u,v)F(u,v) + N(u,v)$$

Image Degradation/Restoration Model

- Image restoration:
  for linear, position-invariant degradations,
  find H(u,v) and apply inverse process,
  also called image deconvolution

- Non-linear, position-dependent system
  – May be general and more accurate
  – Difficult to solve computationally

Estimating the Degradation Function

Three principal ways:

- Estimation by Image observation
- Estimation by experimentation
- Estimation by modeling

Estimation by Image Observation

Take a window in the image

- Simple structure
- Strong signal content

Estimate the original image in the window

known

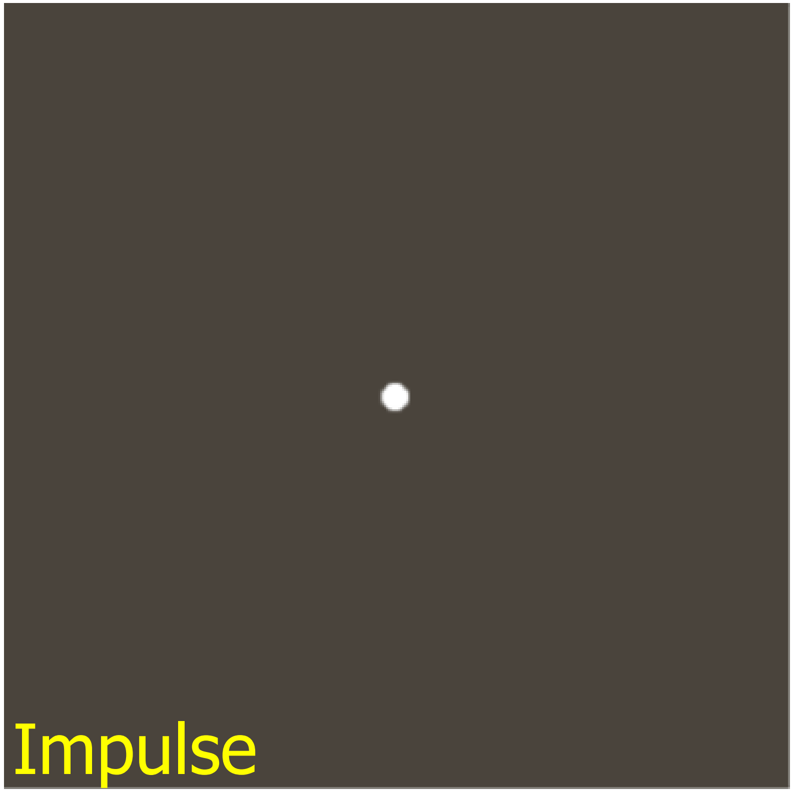
$$H_s(u,v) = \frac{G_s(u,v)}{\hat{F}_s(u,v)}$$

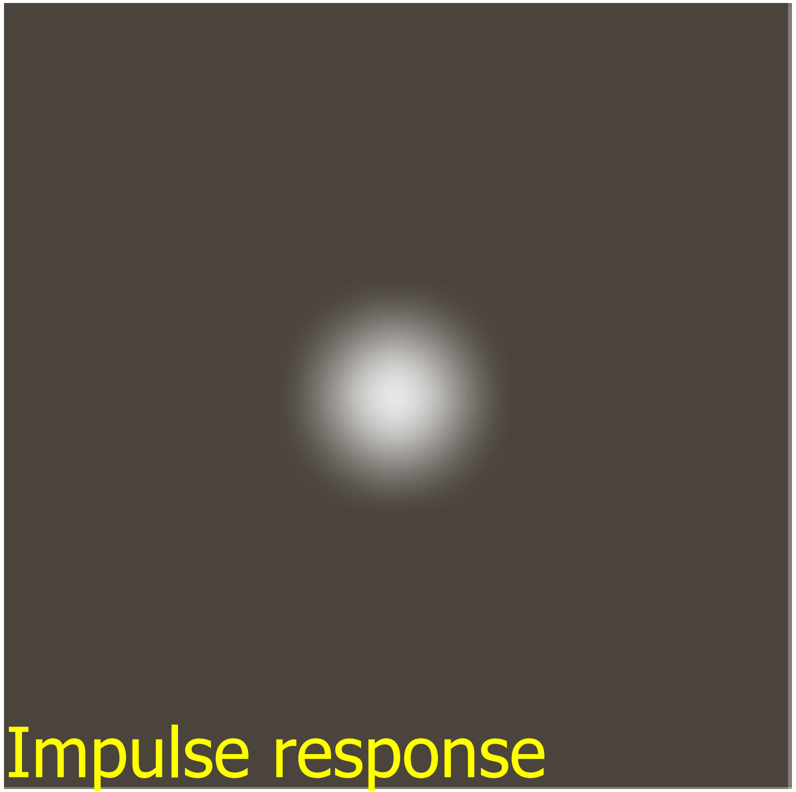
estimate

Estimation by Experimentation

If the image acquisition system is ready, obtain the impulse response, i.e. point spread function (PSF)

$$H(u, v) = \frac{G(u, v)}{A}$$





Estimation by Modeling

Atmospheric turbulence: $H(u,v) = e^{-k(u^2+v^2)^{5/6}}$

$k \approx 0$

$k = 0.0025$

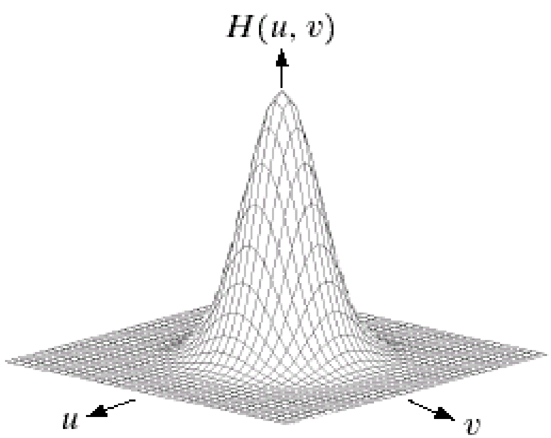
$k = 0.001$

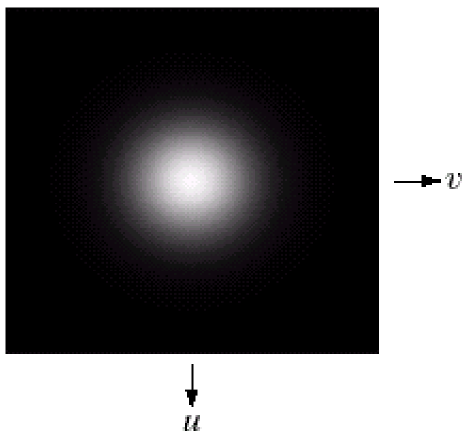
$k = 0.00025$

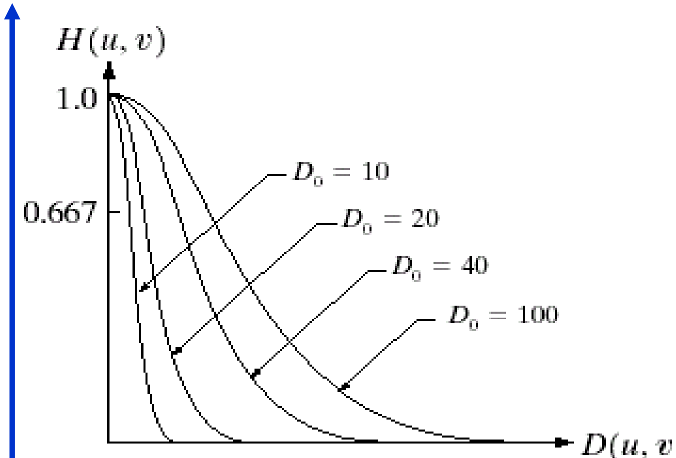
Revisit: Gaussian Lowpass Filters

$$H(u,v)=e^{-D^2(u,v)/2D_0^2}$$

$$D(u,v)=[(u-\boxed{M/2})^2+(v-\boxed{N/2})^2]^{1/2}$$







零频（直流）的坐标在哪儿？两者指的位置不一样！

Atmospheric turbulence: $$H(u,v)=e^{-k(u^2+v^2)^{5/6}}$$

Planar motion during image acquisition

Relative motion between camera & scene

Estimation by Modeling

Planar motion during image acquisition:

$$g(x,y)=\int_0^T f(x-x_0(t),y-y_0(t))dt$$

Planar motion

Fourier transform

$$G(u,v)=F(u,v)\int_0^T e^{-j2\pi[ux_0(t)+vy_0(t)]}dt$$

$$H(u,v)=\int_0^T e^{-j2\pi[ux_0(t)+vy_0(t)]}\,dt$$

Planar motion during image acquisition

$$x_0(t) = at/T$$
$$y_0(t) = 0$$

$$H(u,v) = \int_0^T e^{-j2\pi u x_0(t)}\,dt$$
$$= \int_0^T e^{-j2\pi uat/T}\,dt$$
$$= \frac{T}{\pi ua}\sin(\pi ua)e^{-j\pi ua}$$

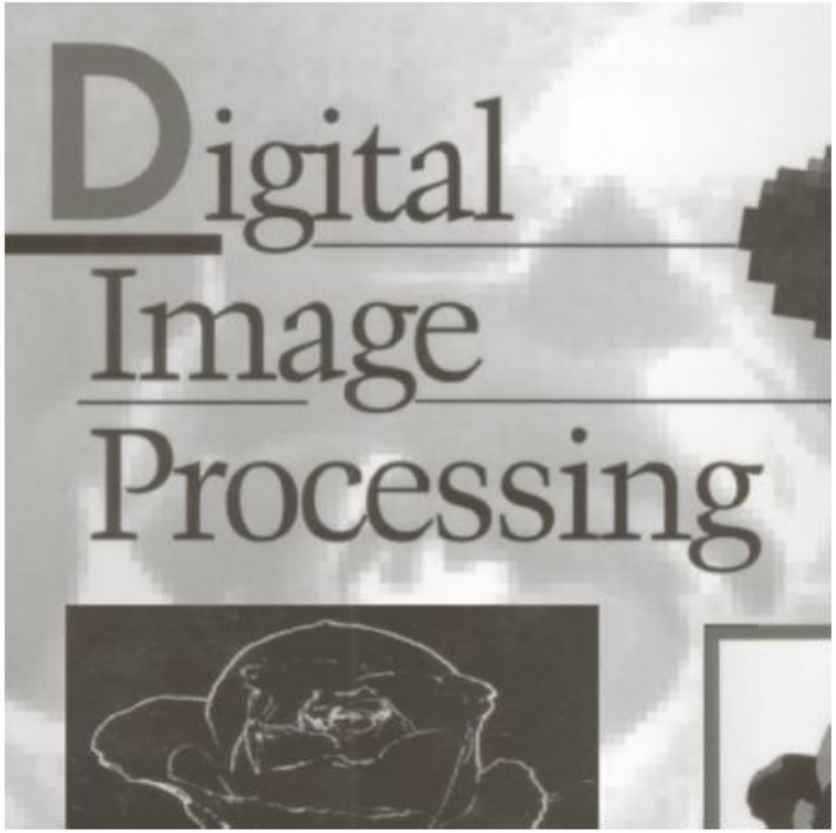
$$x_0(t) = at/T$$
$$y_0 = bt/T$$

$$H(u,v) = \frac{T}{\pi(ua+vb)}\sin[\pi(ua+vb)]e^{-j\pi(ua+vb)}$$

Planar motion during image acquisition

$$x_0(t) = at/T$$
$$y_0 = bt/T$$

$$a = b = 0.1 \text{ and } T = 1$$



Inverse Filtering

Using the estimated degradation function *H*(u,v)

$$G(u,v) = H(u,v)F(u,v) + N(u,v)$$

Unknown noise

$$\hat{F}(u,v) = \frac{G(u,v)}{H(u,v)} = F(u,v) + \frac{N(u,v)}{H(u,v)}$$

Estimate of original image

Ill-posed: 0 or small values

Solution: limit the frequency around the origin

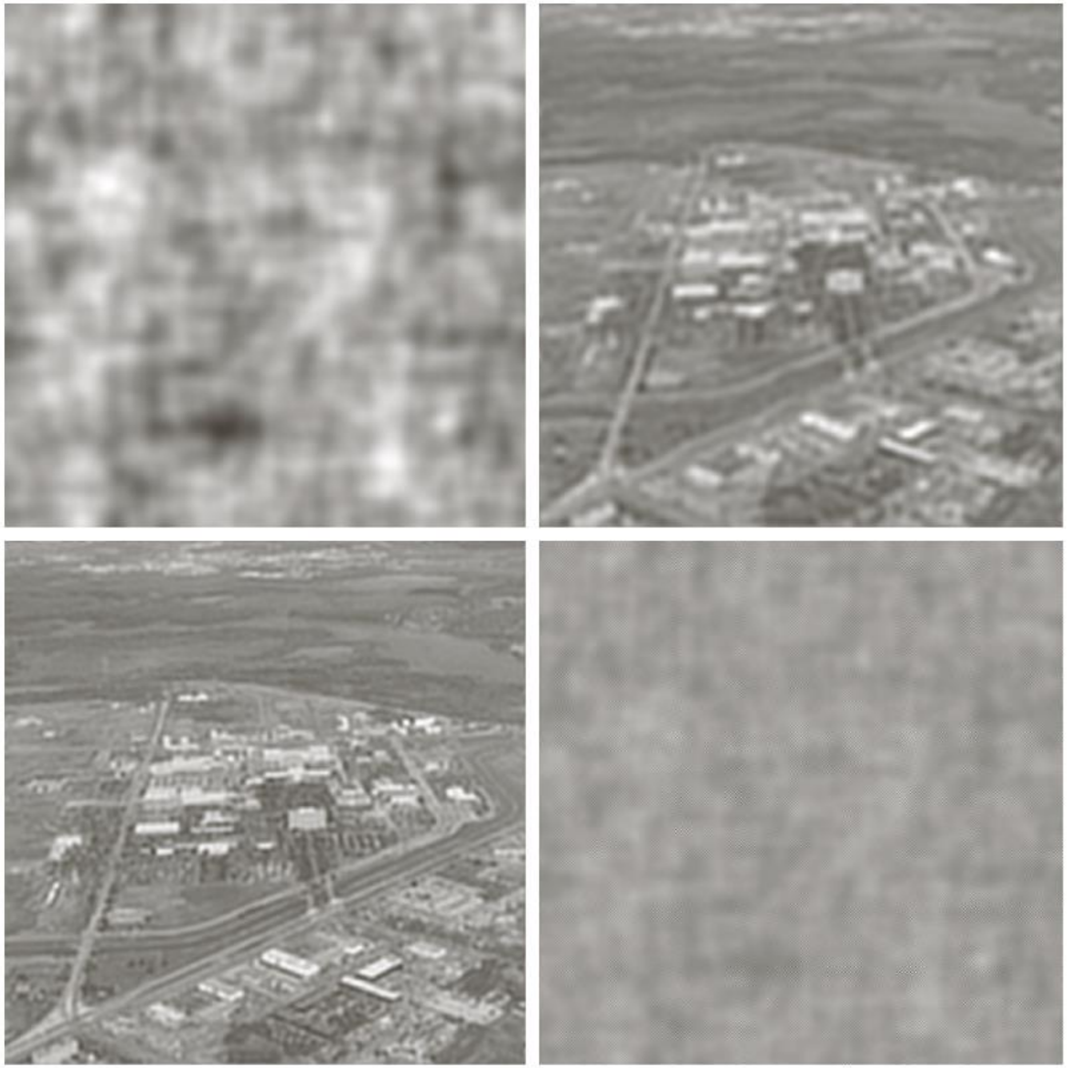
Inverse Filtering Example

For Atmospheric turbulence with $k = 0.0025$

Full inverse filter

Cut outside radius = 40

Butterworth LPF (n=10)



Cut outside radius = 70

Cut outside radius = 85

Wiener Filter

Minimum Mean (Least) Square Error Filter

$$e^2 = E\{(f - \hat{f})^2\}$$

$$\hat{F}(u,v) = \left[\frac{H^*(u,v)S_f(u,v)}{S_f(u,v)|H(u,v)|^2 + S_\eta(u,v)}\right]G(u,v)$$

$$= \left[\frac{H^*(u,v)}{|H(u,v)|^2 + S_\eta(u,v)/S_f(u,v)}\right]G(u,v)$$

$$= \left[\frac{1}{H(u,v)}\frac{|H(u,v)|^2}{|H(u,v)|^2 + S_\eta(u,v)/S_f(u,v)}\right]G(u,v)$$

Wiener Filter

$$\hat{F}(u,v) = \left[\frac{1}{H(u,v)} \frac{|H(u,v)|^2}{|H(u,v)|^2 + S_\eta(u,v)/S_f(u,v)} \right] G(u,v)$$

$H(u,v) =$ degradation function

$H^*(u,v) =$ complex conjugate of $H(u,v)$

$|H(u,v)|^2 = H^*(u,v)H(u,v)$

$S_\eta(u,v) = |N(u,v)|^2 =$ power spectrum of the noise

$S_f(u,v) = |F(u,v)|^2 =$ power spectrum of the undegraded image

Common Metrics

- Mean Square Error

$$\mathrm{MSE} = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} [f(x, y) - \hat{f}(x, y)]^2$$

- Signal-to-Noise Ratio

Frequency domain

$$\mathrm{SNR} = \frac{\sum_{u=0}^{M-1} \sum_{v=0}^{N-1} |F(u, v)|^2}{\sum_{u=0}^{M-1} \sum_{v=0}^{N-1} |N(u, v)|^2}$$

$$\mathrm{SNR} = \frac{\sum_{x=0}^{M-1} \sum_{y=0}^{N-1} \hat{f}(x, y)^2}{\sum_{x=0}^{M-1} \sum_{y=0}^{N-1} [f(x, y) - \hat{f}(x, y)]^2}$$

Spatial domain

Wiener Filter

- White noise:

  $S_\eta(u,v) = |N(u,v)|^2$ is a constant
- Usually $S_f(u,v) = |F(u,v)|^2$ is unknown
- Simplified Wiener Filter

$$\hat{F}(u,v) = \left[\frac{1}{H(u,v)} \frac{|H(u,v)|^2}{|H(u,v)|^2 + K}\right] G(u,v)$$

K is chosen interactively to yield the best results

Comparison of Inverse and Wiener Filtering

Full inverse filter

Cut outside radius = 70

Wiener filter

Comparison of Inverse and Wiener Filtering

Noise variance

1 order less

4 order less

motion blur + noise | Inverse filtering | Wiener filtering

Constrained Least Squares Filter

- Image degradation in vector-matrix form

$$\mathbf{g} = \mathbf{H}\mathbf{f} + \boldsymbol{\eta}$$

$MN \times 1$ $\quad$ $MN \times MN$

- Image restoration model

min

$$C = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1}\left[\nabla^2 f(x, y)\right]^{\boxed{2}}$$

Smoothness Metric

s.t.

$$\|\mathbf{g} - \mathbf{H}\hat{\mathbf{f}}\|^2 = \|\boldsymbol{\eta}\|^2 \qquad \text{where } \|\mathbf{w}\|^2 \triangleq \mathbf{w}^T\mathbf{w}$$

Constrained Least Squares Filter

- Frequency domain solution

$$\hat{F}(u,v) = \left[\frac{H^*(u,v)}{|H(u,v)|^2 + \gamma |P(u,v)|^2} \right] G(u,v)$$

DFT

γ is a parameter tuned for $\|\mathbf{g} - \mathbf{H}\hat{\mathbf{f}}\|^2 = \|\boldsymbol{\eta}\|^2$

$$p(x,y) = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

Laplacian operator

Inverse filtering   Constrained least squares

Noise variance

motion blur + noise   Wiener filtering

How to compute γ

- Residual vector $\mathbf{r} = \mathbf{g} - \mathbf{H}\hat{\mathbf{f}}$

$\phi(\gamma) = \mathbf{r}^T\mathbf{r} = \|\mathbf{r}\|^2$ monotonically increases with γ

- Adjust γ so that $\|\mathbf{r}\|^2 = \|\boldsymbol{\eta}\|^2 \pm a$

where a is an accuracy factor

1. Specify an initial value of γ.
2. Compute $\|\mathbf{r}\|^2$.
3. Stop if Eq. (5.9-8) is satisfied; otherwise return to step 2 after increasing γ if $\|\mathbf{r}\|^2 < \|\boldsymbol{\eta}\|^2 - a$ or decreasing γ if $\|\mathbf{r}\|^2 > \|\boldsymbol{\eta}\|^2 + a$. Use the new value of γ in Eq. (5.9-4) to recompute the optimum estimate $\hat{F}(u, v)$.

How to compute $\|\mathbf{r}\|^2$ & $\|\boldsymbol{\eta}\|^2$

- Compute $\|\mathbf{r}\|^2$

$$R(u,v) = G(u,v) - H(u,v)\hat{F}(u,v)$$

$$\|\mathbf{r}\|^2 = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} r^2(x,y)$$

- Compute $\|\boldsymbol{\eta}\|^2$

$$\|\boldsymbol{\eta}\|^2 = MN[\boxed{\sigma_\eta^2} + \boxed{m_\eta^2}]$$

$$\sigma_\eta^2 = \frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}[\eta(x,y) - m_\eta]^2$$

Noise estimation

$$m_\eta = \frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}\eta(x,y)$$

Constrained least squares filtering

Using correct noise parameters

Using wrong noise parameters

Geometric Mean Filter

$$\hat{F}(u,v) = \left[\frac{H^*(u,v)}{|H(u,v)|^2}\right]^{\alpha}\left[\frac{H^*(u,v)}{|H(u,v)|^2 + \beta\left[\frac{S_\eta(u,v)}{S_f(u,v)}\right]}\right]^{1-\alpha} G(u,v)$$

with α and β being positive, real constants

$\alpha = 1 \longrightarrow$ Inverse Filter

$\alpha = 0 \longrightarrow$ Parametric Wiener Filter

$\alpha = 0, \beta = 1 \longrightarrow$ Standard Wiener Filter

$\alpha = 0.5, \beta = 1 \longrightarrow$ Geometric Mean

Spectrum Equalization Filter

Image Reconstruction from Projections

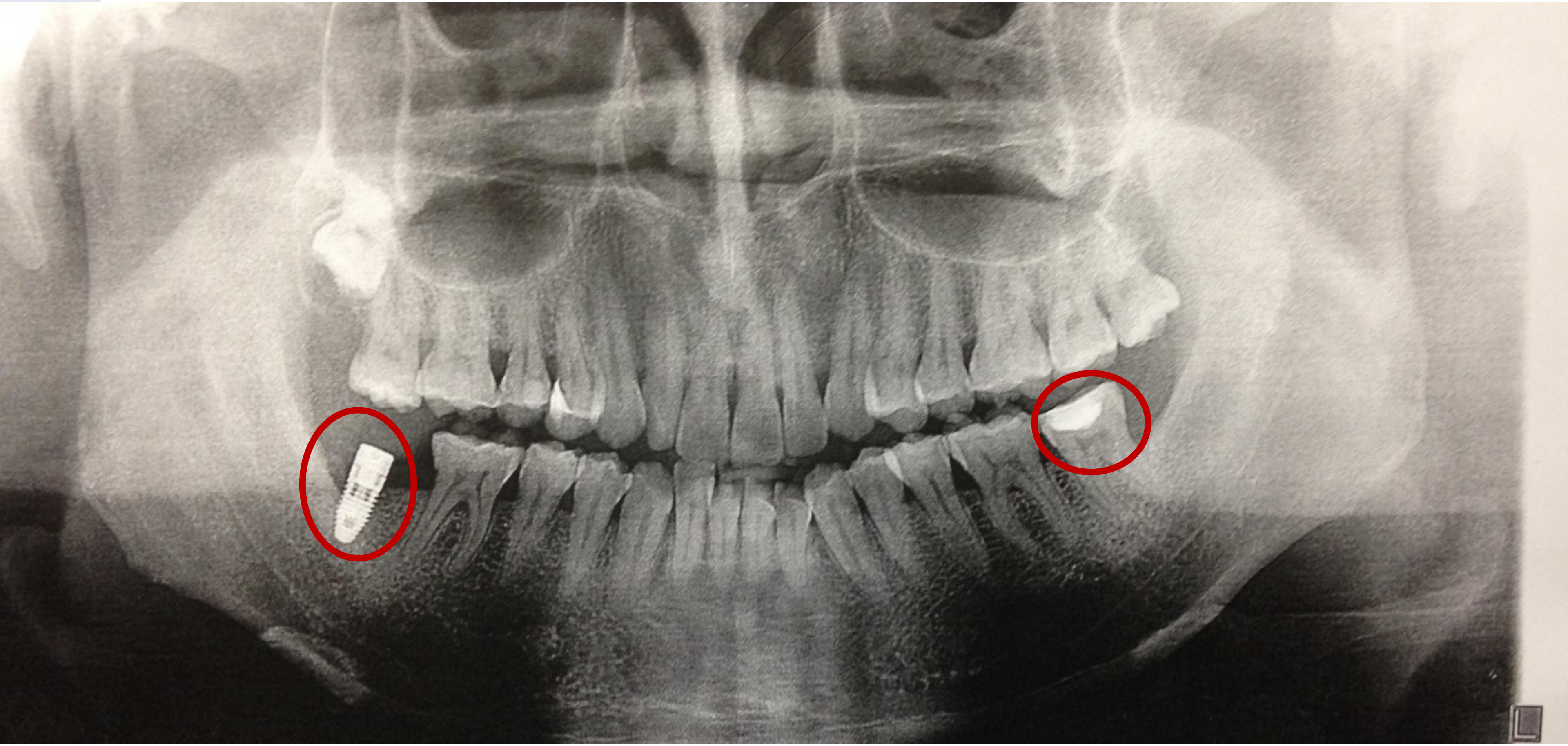
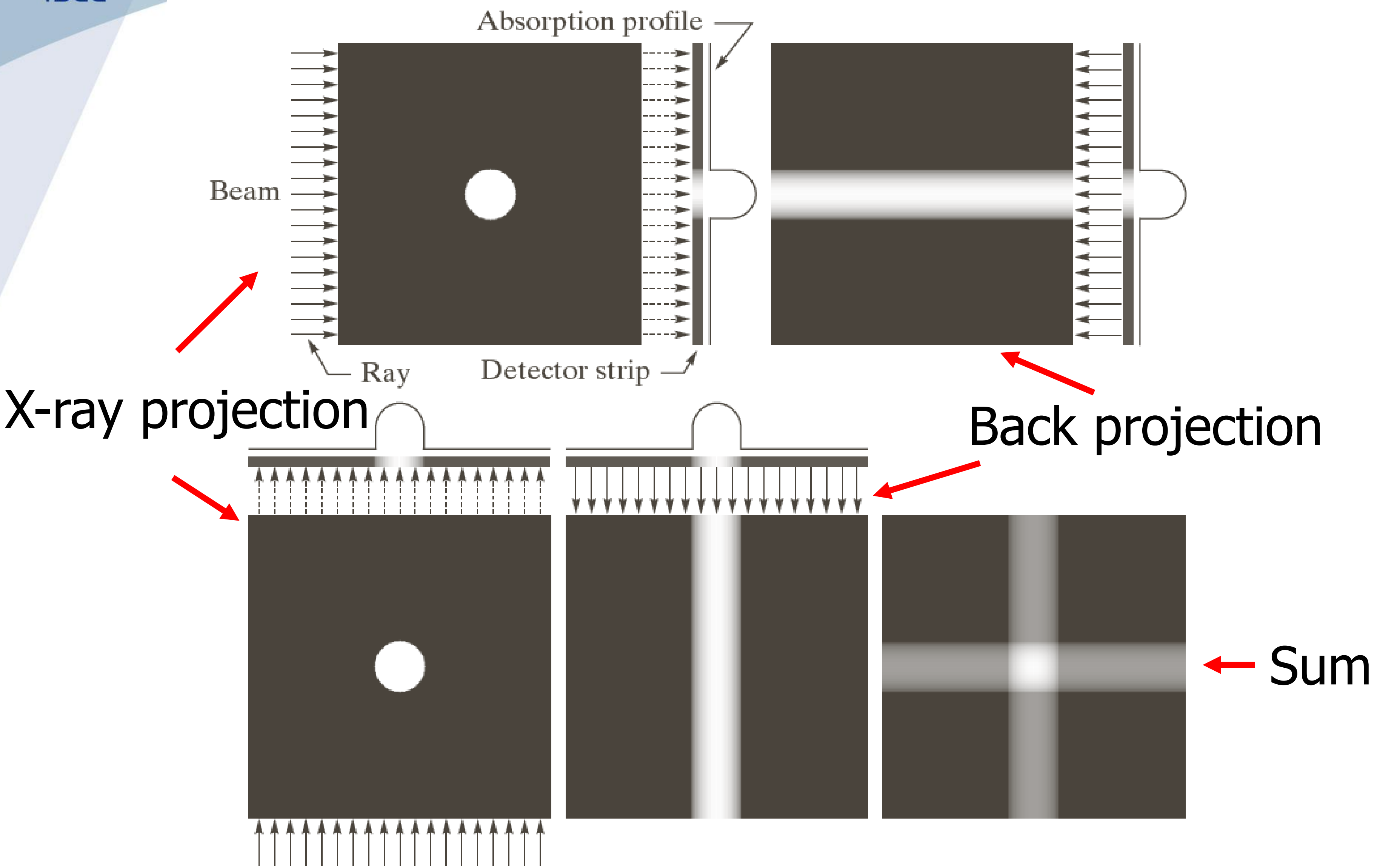


Image Reconstruction from Projections

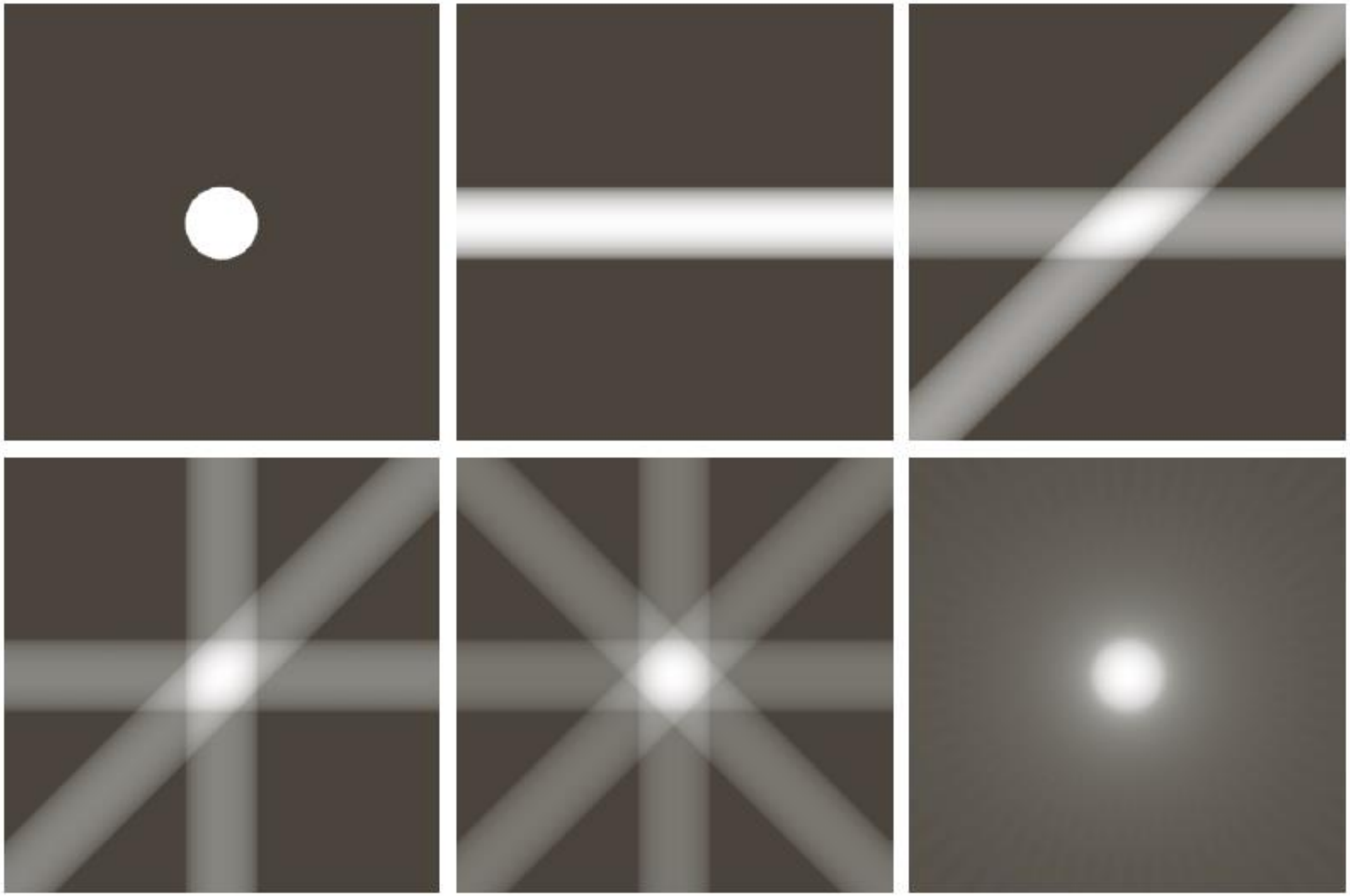
- Basic Idea
- Principles of Computed Tomography (CT)
- Projections and the Radon Transform
- The Fourier-Slice Theorem
- Reconstruction Using Parallel-Beam Filtered Backprojections
- ~~Reconstruction Using Fan-Beam Filtered Backprojections~~

Basic Idea


Absorption profile
Beam
Ray
Detector strip
X-ray projection
Back projection
Sum

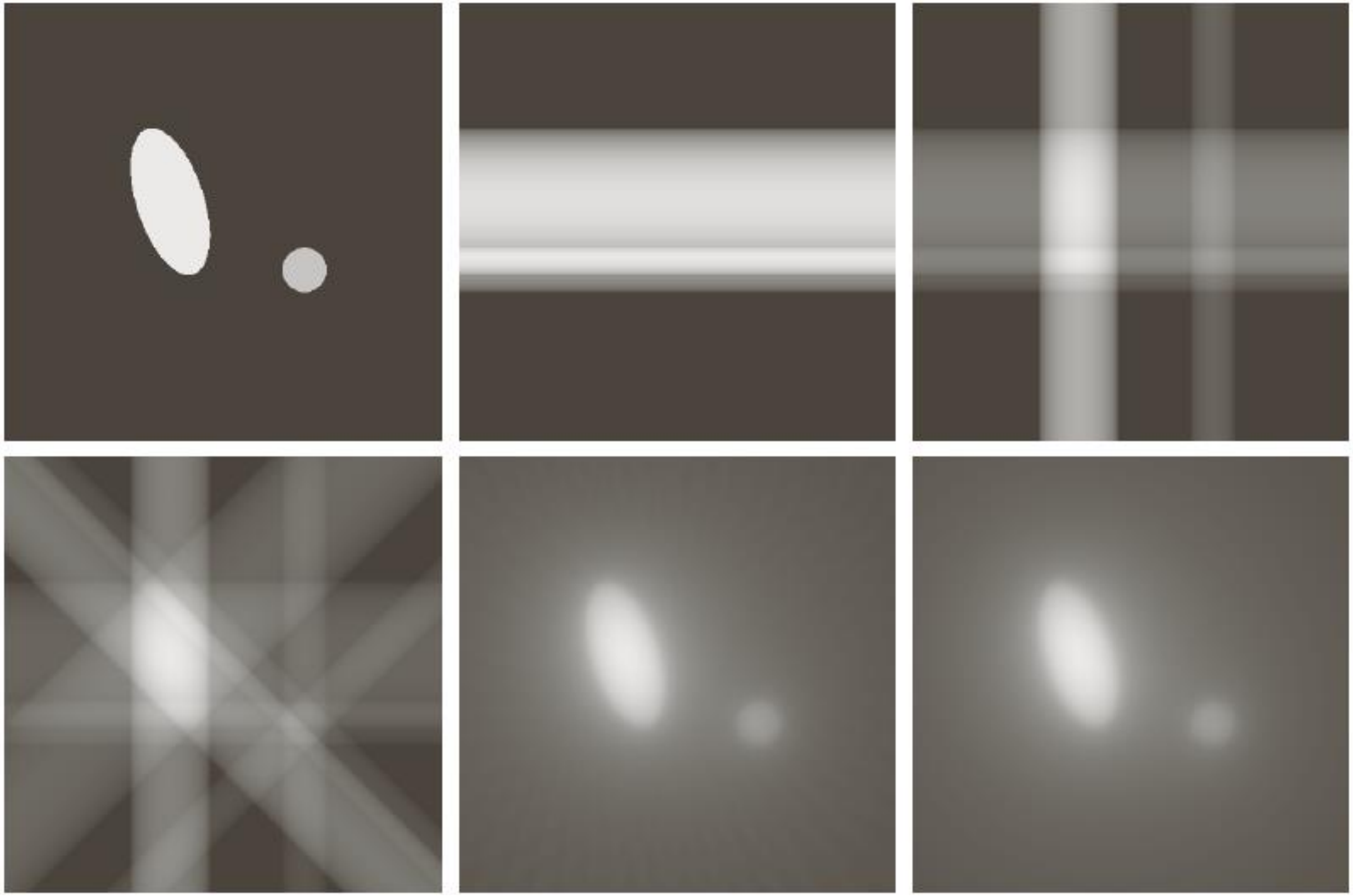
Taking More Views

Reconstruction using 1,2,3,4,32 backprojections



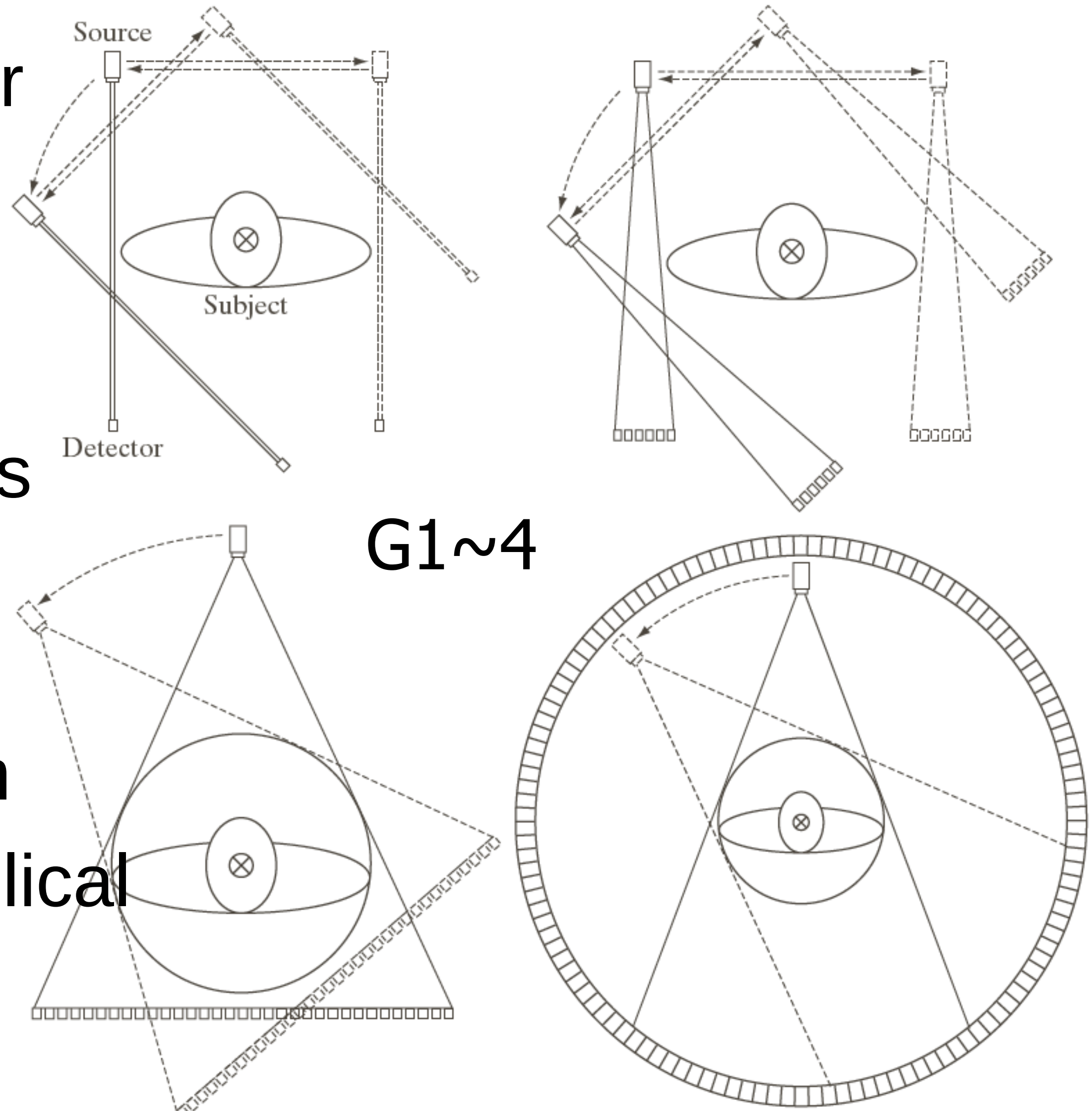
Imaging Two Objects

Reconstruction using 1,2,4,32,64 backprojections



Principles of Computed Tomography (CT)

- Generations of CT scanners
  - G1: single detector
  - G2: fan beam, multiple detectors
  - G3: wide beam, a bank of detectors
  - G4: a circular ring of detectors
  - G5: electron beam
  - G6: continuous helical
  - G7: multislice CT

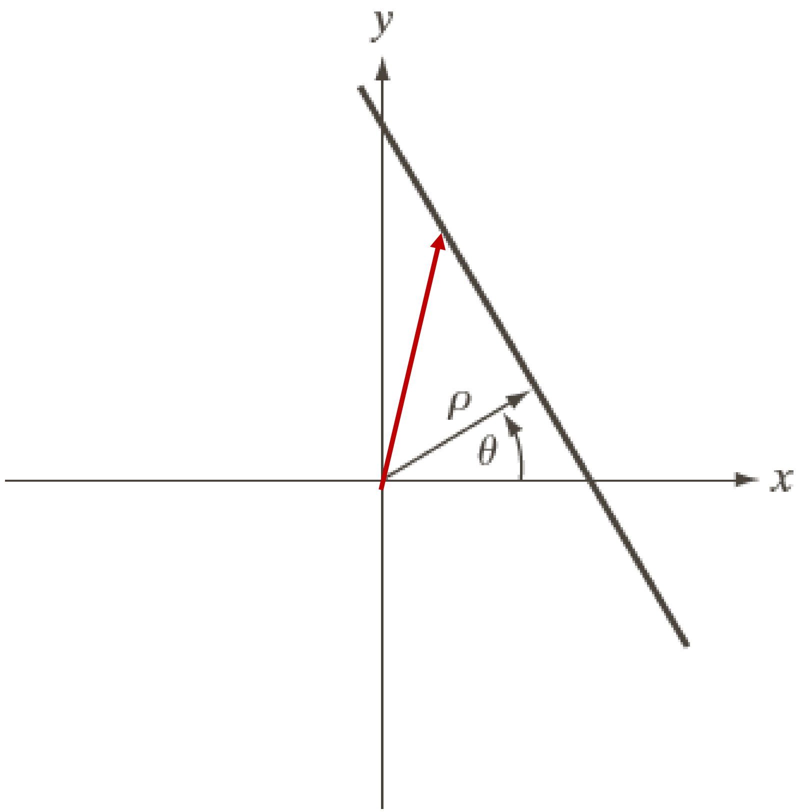


Projections and the Radon Transform

- Normal representation of a line

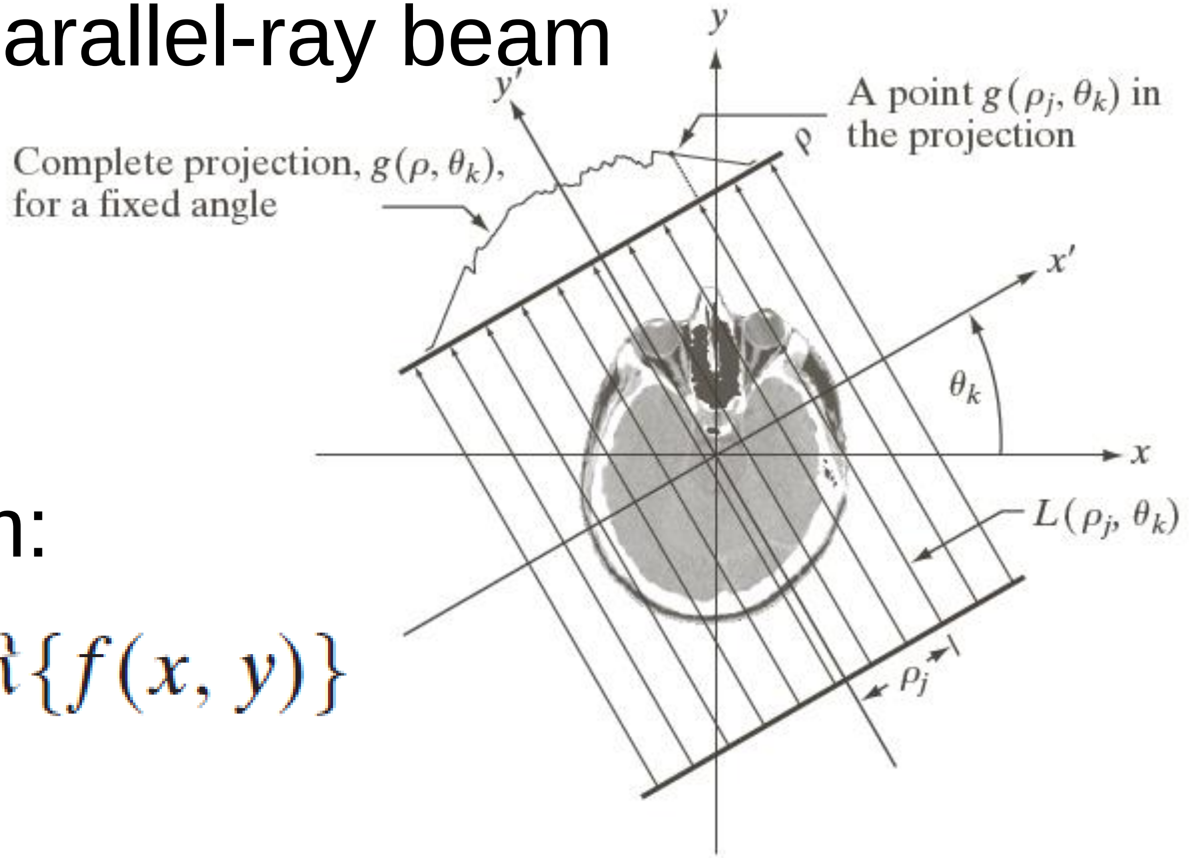
$$x\cos\theta + y\sin\theta = \rho$$

$$\begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix} = \rho$$



Projections and the Radon Transform

- Geometry of a parallel-ray beam



- Radon transform:

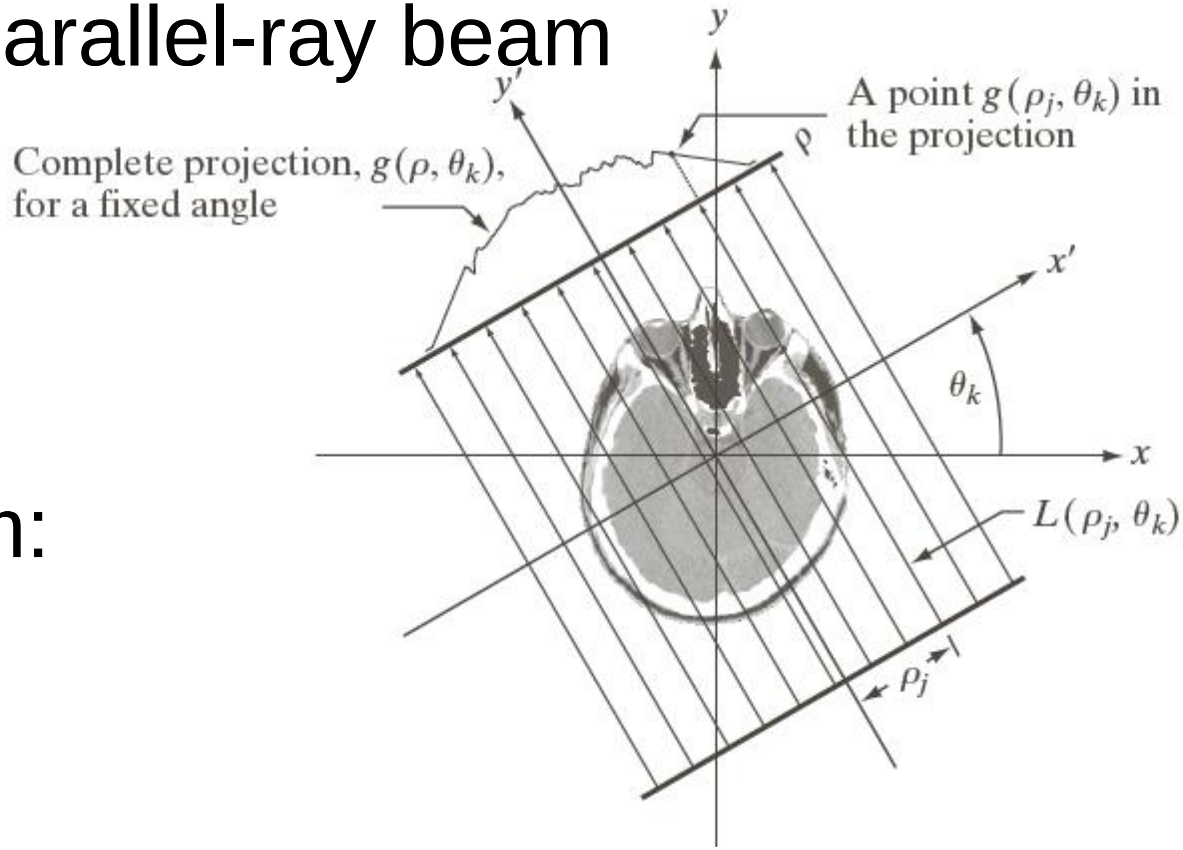
  line integral $\Re\{f(x, y)\}$

$$g(\rho_j, \theta_k) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x, y)\delta(x\cos\theta_k + y\sin\theta_k - \rho_j)\,dx\,dy$$

$$g(\rho, \theta) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x, y)\delta(x\cos\theta + y\sin\theta - \rho)\,dx\,dy$$

Projections and the Radon Transform

- Geometry of a parallel-ray beam



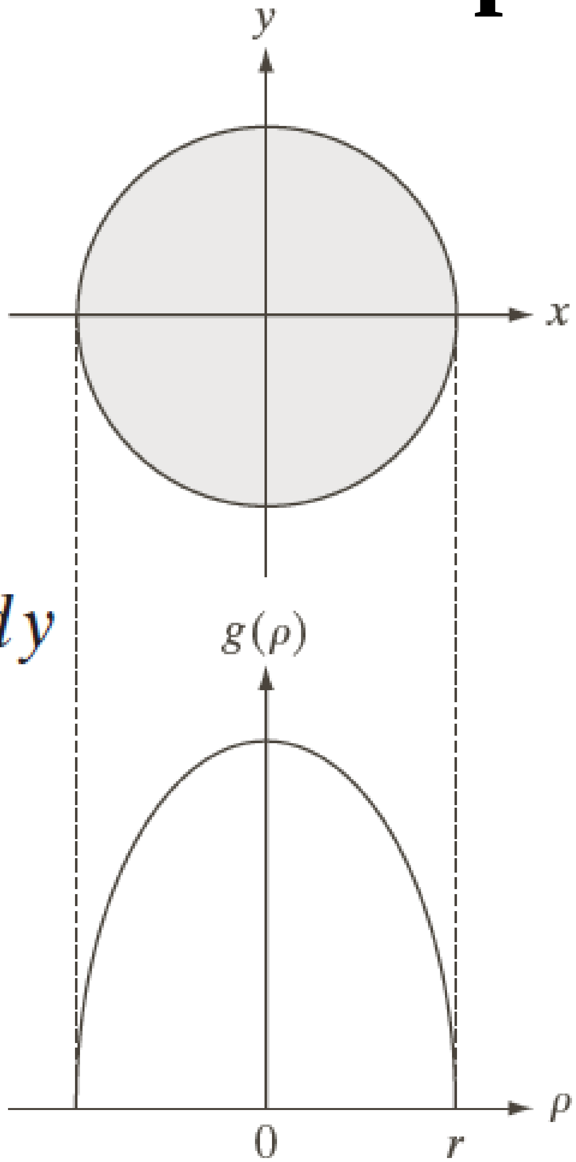
- Radon transform:
  discrete form

$$g(\rho, \theta) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x, y)\delta(x\cos\theta + y\sin\theta - \rho)$$

Radon Transform Example

- Projection of a circular object

$$f(x,y) = \begin{cases} A & x^2 + y^2 \le r^2 \\ 0 & \text{otherwise} \end{cases}$$

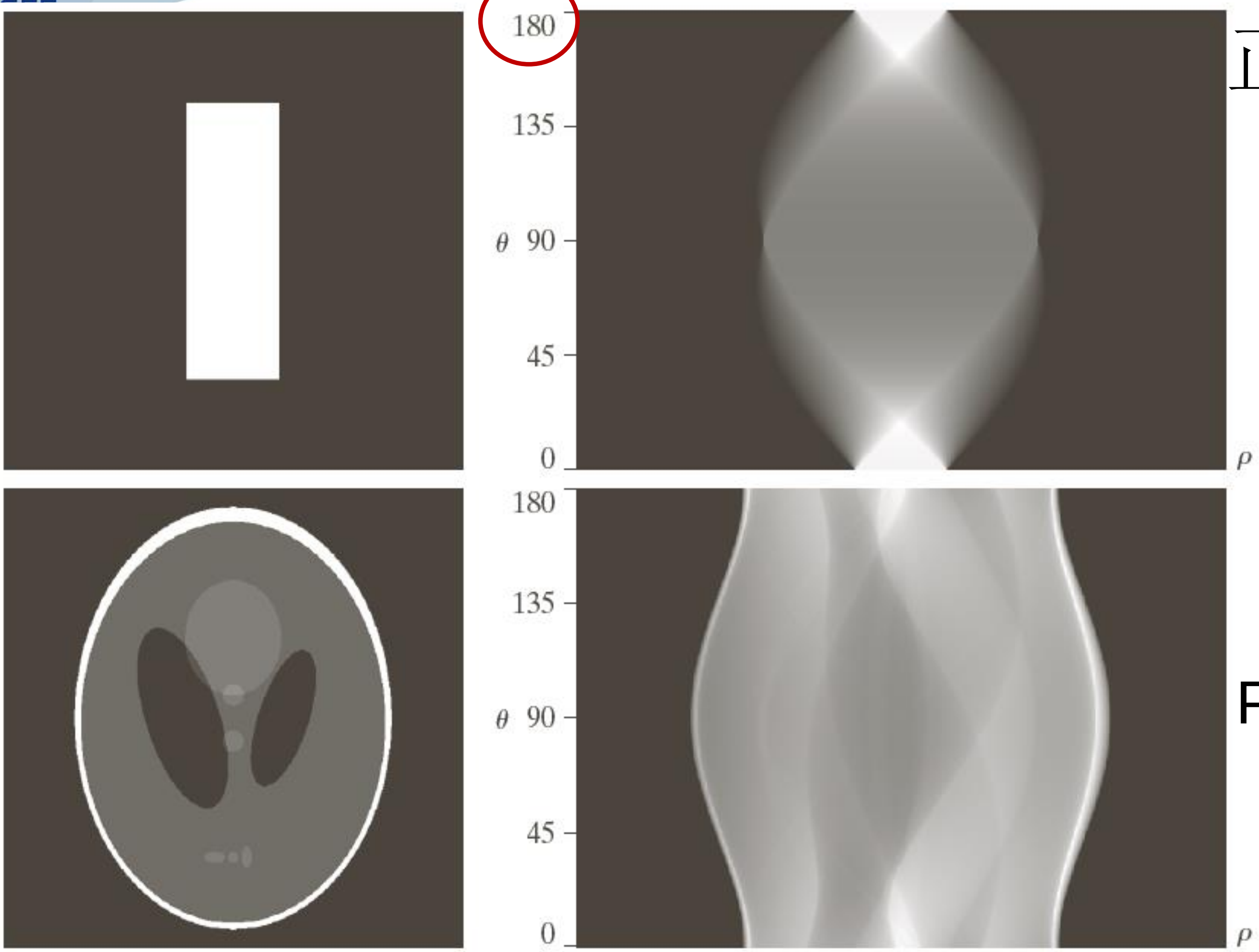


$$g(\rho,\theta) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\delta(x-\rho)\,dx\,dy$$

$$= \int_{-\infty}^{\infty} f(\rho,y)\,dy$$

$$= \begin{cases} 2A\sqrt{r^2-\rho^2} & |\rho| \le r \\ 0 & \text{otherwise} \end{cases}$$

Radon Transform: Image → Sinogram



正弦图

Phantom
体模

Backprojection: Sinogram→Laminogram

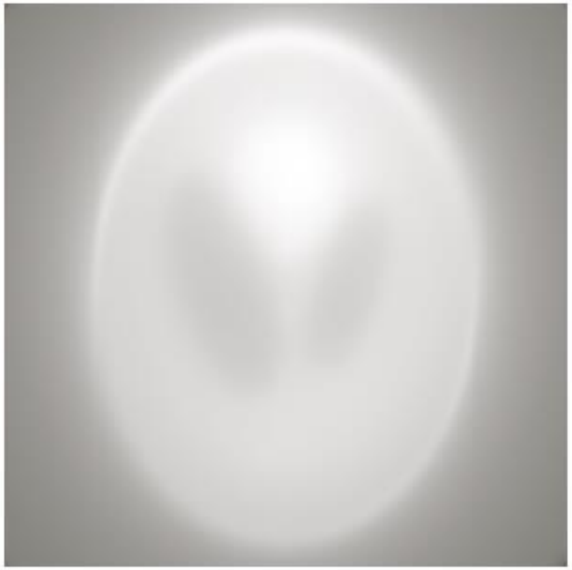
层图

$$f_{\theta_k}(x, y) = g(\rho, \theta_k) = g(x\cos\theta_k + y\sin\theta_k, \theta_k)$$

$$f_\theta(x, y) = g(x\cos\theta + y\sin\theta, \theta)$$

$$f(x, y) = \int_0^\pi f_\theta(x, y)\, d\theta$$

$$f(x, y) = \sum_{\theta=0}^{\pi} f_\theta(x, y)$$



The Fourier-Slice Theorem

- 1-D Fourier transform of a projection

$$G(\omega, \theta) = \int_{-\infty}^{\infty} g(\rho, \theta) e^{-j2\pi\omega\rho}\, d\rho$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x, y)\delta(x\cos\theta + y\sin - \rho)e^{-j2\pi\omega\rho}\, dx\, dy\, d\rho$$

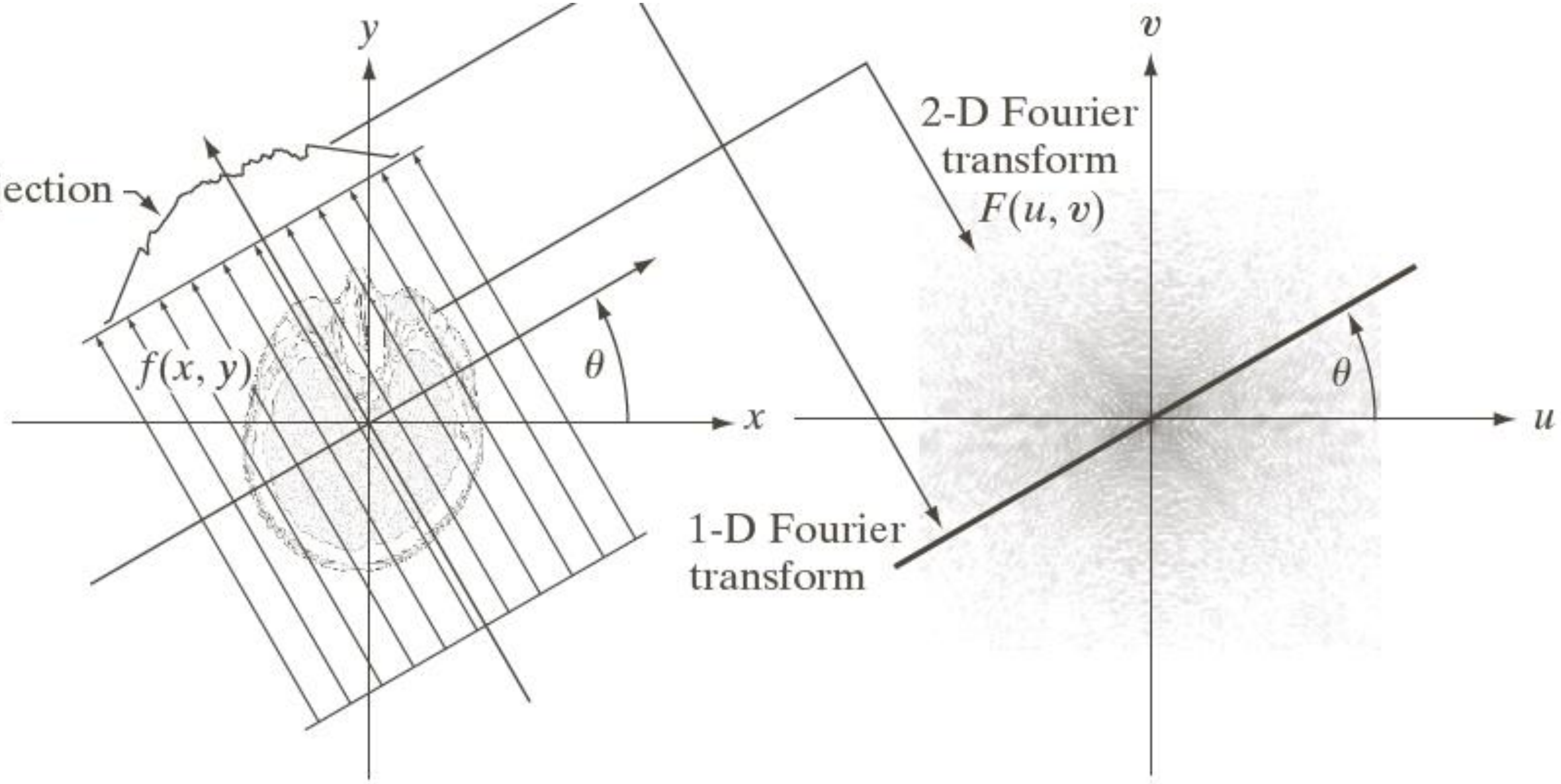
$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x, y)\left[\int_{-\infty}^{\infty} \underbrace{\delta(x\cos\theta + y\sin\theta - \rho)e^{-j2\pi\omega\rho}\, d\rho}_{\text{采样}}\right] dx\, dy$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x, y) e^{-j2\pi\omega(x\cos\theta + y\sin\theta)}\, dx\, dy$$

$$= [F(u, v)]_{u=\omega\cos\theta;\, v=\omega\sin\theta} = F(\omega\cos\theta, \omega\sin\theta)$$

The Fourier-Slice Theorem

$$G(\omega,\theta) = [F(u,v)]_{u=\omega\cos\theta;\, v=\omega\sin\theta} = F(\omega\cos\theta, \omega\sin\theta)$$



Reconstruction Using Parallel-Beam Filtered Backprojections

$$f(x, y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F(u, v) e^{j2\pi(ux+vy)}\, du\, dv$$

$u = \omega \cos\theta \quad v = \omega \sin\theta$

$$= \int_{0}^{2\pi}\int_{0}^{\infty} F(\omega\cos\theta, \omega\sin\theta) e^{j2\pi\omega(x\cos\theta + y\sin\theta)} \omega\, d\omega\, d\theta$$

Fourier-slice theorem

$$= \int_{0}^{2\pi}\int_{0}^{\infty} G(\omega, \theta) e^{j2\pi\omega(x\cos\theta + y\sin\theta)} \omega\, d\omega\, d\theta$$

$G(\omega, \theta + 180°) = G(-\omega, \theta)$

$$= \int_{0}^{\pi}\int_{-\infty}^{\infty} |\omega| G(\omega, \theta) e^{j2\pi\omega(x\cos\theta + y\sin\theta)}\, d\omega\, d\theta$$

$$F(-\omega,\theta)=F(\omega,\theta+\pi)=F(\omega,\theta-\pi)$$

$$\Rightarrow \int_{\pi}^{2\pi}\int_{0}^{+\infty} F(\omega,\theta)\exp(j2\pi\omega t)\omega\, d\omega d\theta$$

$$=\int_{\pi}^{2\pi}\int_{0}^{-\infty} F(-\omega',\theta)\exp(-j2\pi\omega' t)(-\omega')\, d(-\omega')\, d\theta$$

$$=\int_{\pi}^{2\pi}\int_{0}^{-\infty} F(\omega',\theta-\pi)\exp(-j2\pi\omega' t)\omega'\, d\omega'\, d\theta$$

$$=\int_{0}^{\pi}\int_{0}^{-\infty} F(\omega',\theta')\exp(-j2\pi\omega'(x\cos(\theta'+\pi)+y\sin(\theta'+\pi)))\omega'\, d\omega'\, d(\theta'+\pi)$$

$$=\int_{0}^{\pi}\int_{0}^{-\infty} F(\omega',\theta')\exp(j2\pi\omega'(x\cos\theta'+y\sin\theta'))\omega'\, d\omega'\, d\theta'$$

$$=\int_{0}^{\pi}\int_{0}^{-\infty} F(\omega,\theta)\exp(j2\pi\omega(x\cos\theta+y\sin\theta))\omega d\omega d\theta$$

$$=\int_{0}^{\pi}\int_{-\infty}^{0} F(\omega,\theta)\exp(j2\pi\omega t)(-\omega) d\omega d\theta$$

$$=\int_{0}^{\pi}\int_{-\infty}^{0} F(\omega,\theta)\exp(j2\pi\omega t)\left|\omega\right| d\omega d\theta$$

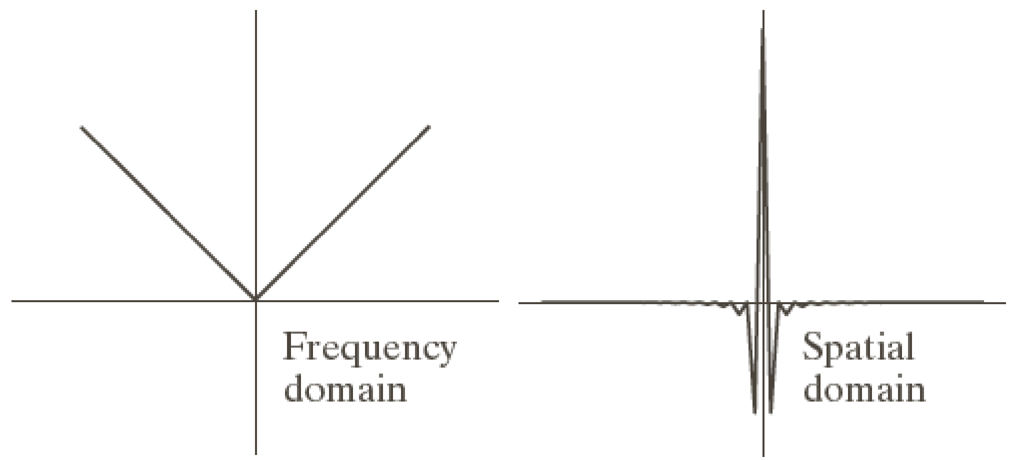
Reconstruction Using Parallel-Beam Filtered Backprojections

$$f(x, y) = \int_0^{\pi} \int_{-\infty}^{\infty} |\omega| G(\omega, \theta) e^{j2\pi\omega(x\cos\theta + y\sin\theta)}\, d\omega\, d\theta$$

$$= \int_0^{\pi} \underbrace{\left[\int_{-\infty}^{\infty} |\omega| G(\omega, \theta) e^{j2\pi\omega\rho}\, d\omega \right]}_{\text{Inverse 1-D Fourier Transform}}{}_{\rho = x\cos\theta + y\sin\rho}\, d\theta$$

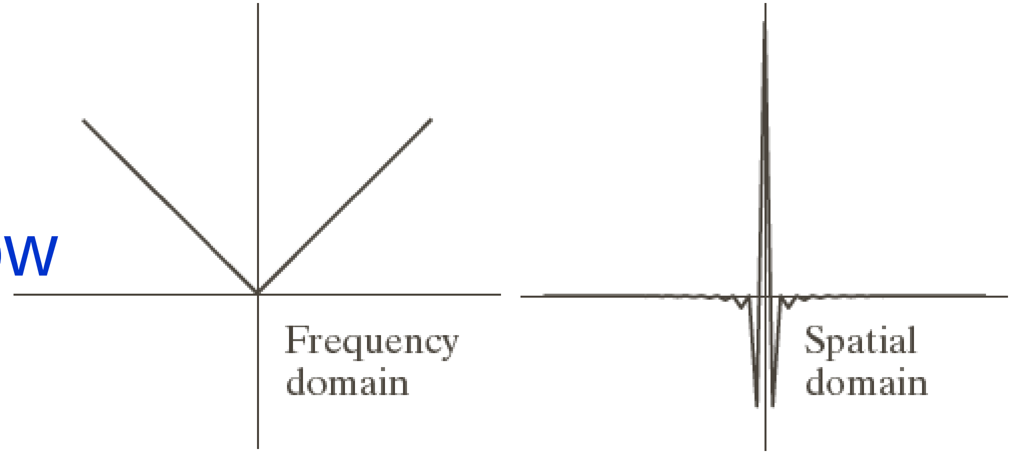
Ramp Filter

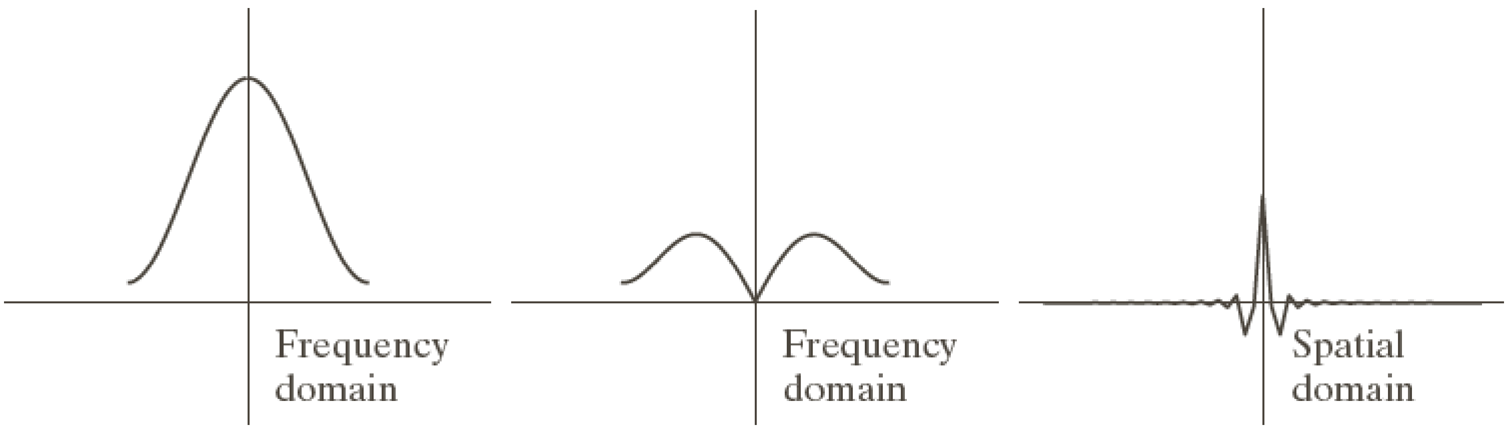
After band limited



Windowed Ramp Filter

Box window





c=0.54: Hamming window

c=0.5: Hann window

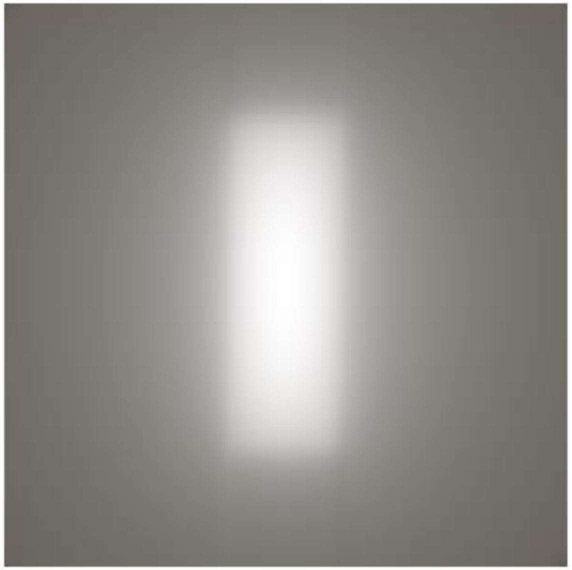
$$h(\omega) = \begin{cases} c + (c-1)\cos\dfrac{2\pi\omega}{M-1} & 0 \le \omega \le (M-1) \\ 0 & \text{otherwise} \end{cases}$$

Reconstruction Using Parallel-Beam Filtered Backprojections

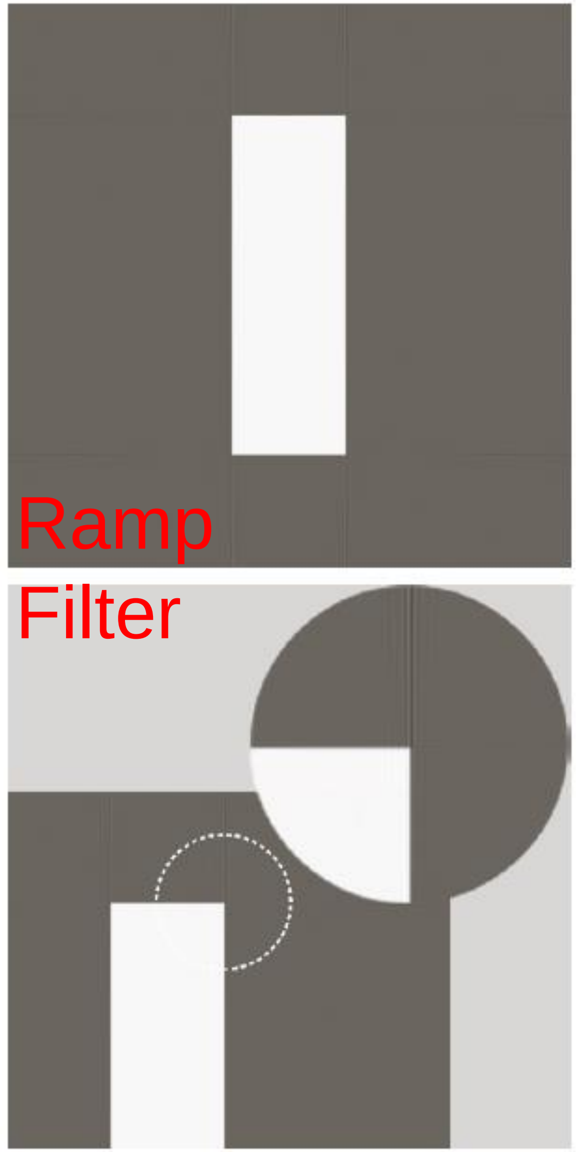
$$f(x, y) = \int_0^{\pi} \left[\int_{-\infty}^{\infty} |\omega| G(\omega, \theta) e^{j2\pi\omega\rho} \, d\omega \right]_{\rho = x\cos\theta + y\sin\rho} d\theta$$

1. Compute the 1-D Fourier transform of each projection.
2. Multiply each Fourier transform by the filter function $|\omega|$ which, as explained above, has been multiplied by a suitable (e.g., Hamming) window.
3. Obtain the inverse 1-D Fourier transform of each resulting filtered transform.
4. Integrate (sum) all the 1-D inverse transforms from step 3.

Comparison



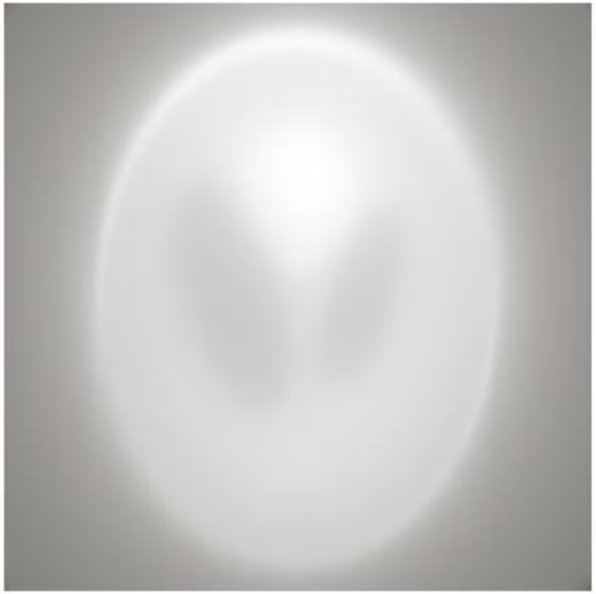
Laminogram



Ramp Filter

Hamming-windowed Ramp Filter

Comparison



Laminogram

Ramp
Filter

Hamming-
windowed
Ramp
Filter

Reconstruction Using Parallel-Beam Filtered Backprojections

$$f(x, y) = \int_0^{\pi}\left[\int_{-\infty}^{\infty} |\omega| G(\omega, \theta) e^{j2\pi\omega\rho}\, d\omega\right]_{\rho = x\cos\theta + y\sin\rho} d\theta$$

$$= \int_0^{\pi} \left[\underline{s(\rho) \bigstar g(\rho, \theta)}\right]_{\rho = x\cos\theta + y\sin\theta} d\theta$$

$$= \int_0^{\pi} \left[\underline{\int_{-\infty}^{\infty} g(\rho, \theta) s(x\cos\theta + y\sin\theta - \rho)\, d\rho}\right] d\theta$$

In spatial domain: convolution with ramp filter

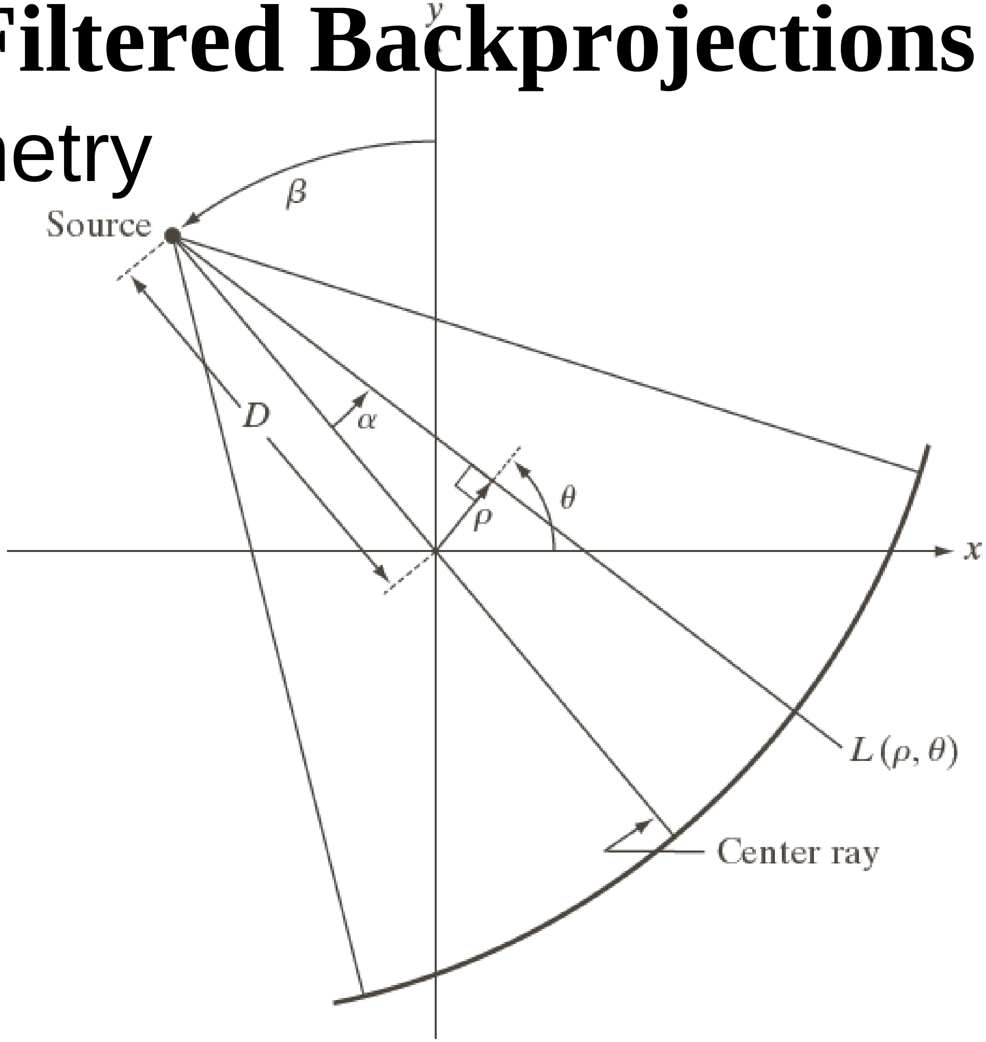
Reconstruction Using Fan-Beam Filtered Backprojections

- Basic fan-beam geometry

$$p(\alpha, \beta) \longleftrightarrow L(\rho, \theta)$$

$$\theta = \beta + \alpha$$

$$\rho = D \sin \alpha$$



$$g(\rho, \theta) = 0 \text{ for } |\rho| > T$$

$$f(x, y) = \frac{1}{2} \int_0^{2\pi} \int_{-T}^{T} g(\rho, \theta) s(x \cos \theta + y \sin \theta - \rho)\, d\rho d\theta$$

Reconstruction Using Fan-Beam Filtered Backprojections

polar coordinates (r, φ) $x = r\cos\varphi$ $y = r\sin\varphi$

$$x\cos\theta + y\sin\theta = r\cos\varphi\cos\theta + r\sin\varphi\sin\theta$$

$$= r\cos(\theta - \varphi)$$

$$f(x, y) = \frac{1}{2}\int_0^{2\pi}\int_{-T}^{T} g(\rho, \theta)s[r\cos(\theta - \alpha) - \rho]\,\underline{d\rho\, d\theta}$$

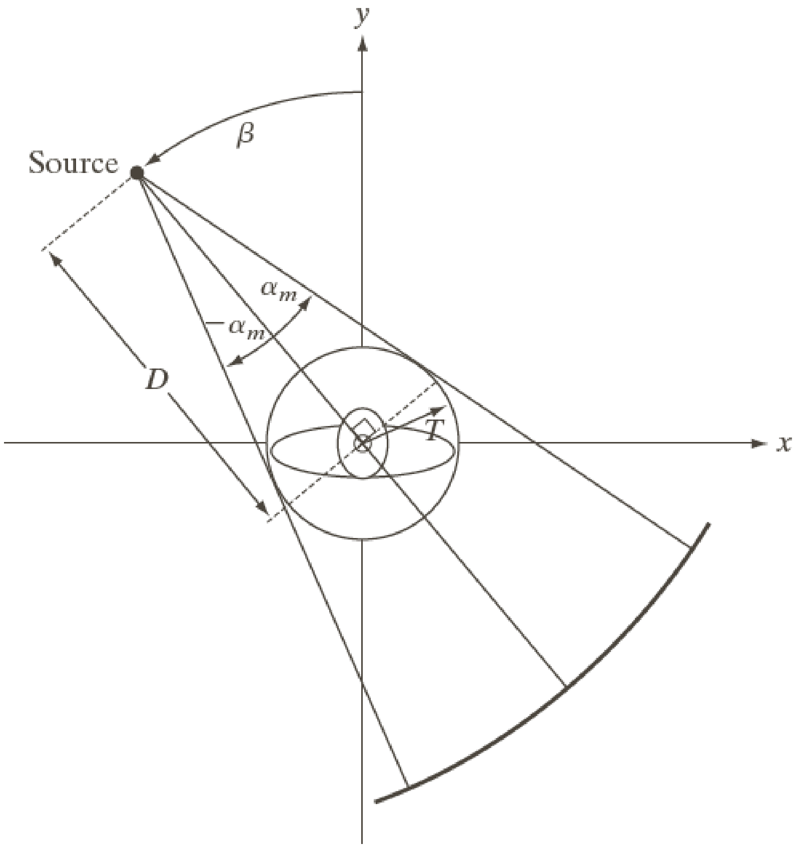
$$f(r, \varphi) = \frac{1}{2}\int_{\underline{-\alpha}}^{\underline{2\pi-\alpha}}\int_{\sin^{-1}(-T/D)}^{\sin^{-1}(T/D)} g(D\sin\alpha, \alpha + \beta)$$

$$s[r\cos(\beta + \alpha - \varphi) - D\sin\alpha]\underline{D\cos\alpha\, d\alpha\, d\beta}$$

Reconstruction Using Fan-Beam Filtered Backprojections

$$p(\alpha, \beta) = g(D \sin \alpha, \alpha + \beta)$$

$$f(r, \varphi) = \frac{1}{2}\int_0^{2\pi}\int_{-\alpha_m}^{\alpha_m} p(\alpha, \beta) s[r\cos(\beta + \alpha - \varphi) - D\sin\alpha] D\cos\alpha \, d\alpha \, d\beta$$



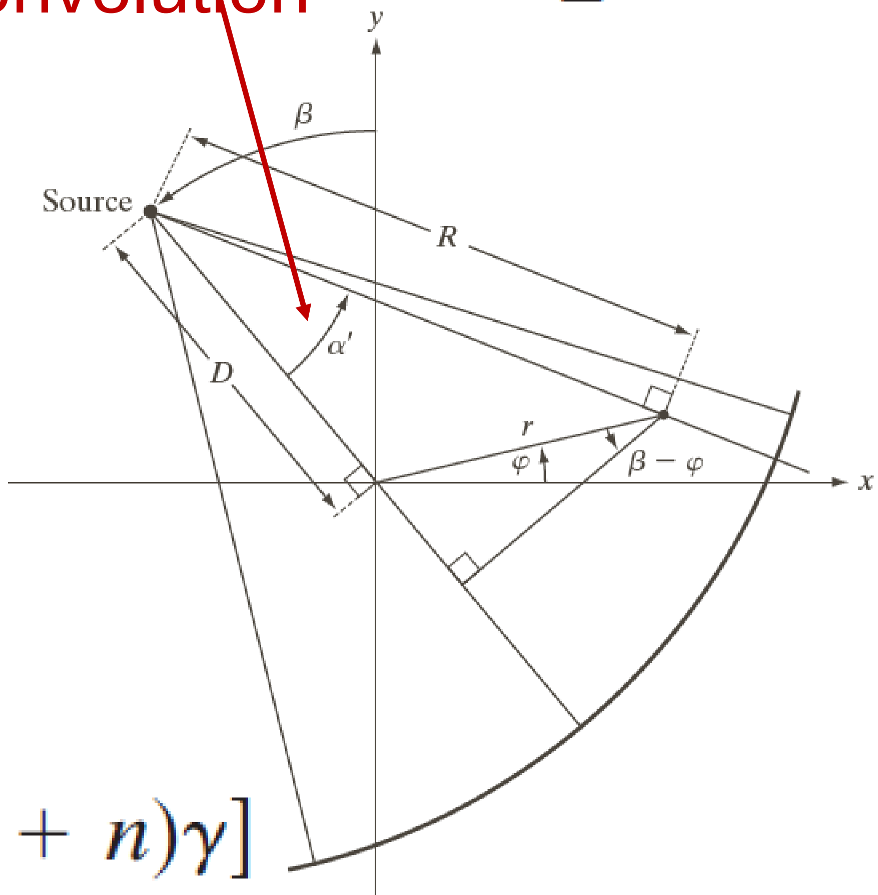
Reconstruction Using Fan-Beam Filtered Backprojections

$$f(r,\varphi) = \int_0^{2\pi} \frac{1}{R^2}\left[\int_{-\alpha_m}^{\alpha_m} q(\alpha,\beta)h(\alpha' - \alpha)\,d\alpha\right] d\beta$$

Convolution

$$h(\alpha) = \frac{1}{2}\left(\frac{\alpha}{\sin\alpha}\right)^2 s(\alpha)$$

$$q(\alpha,\beta) = p(\alpha,\beta)\,D\cos\alpha$$



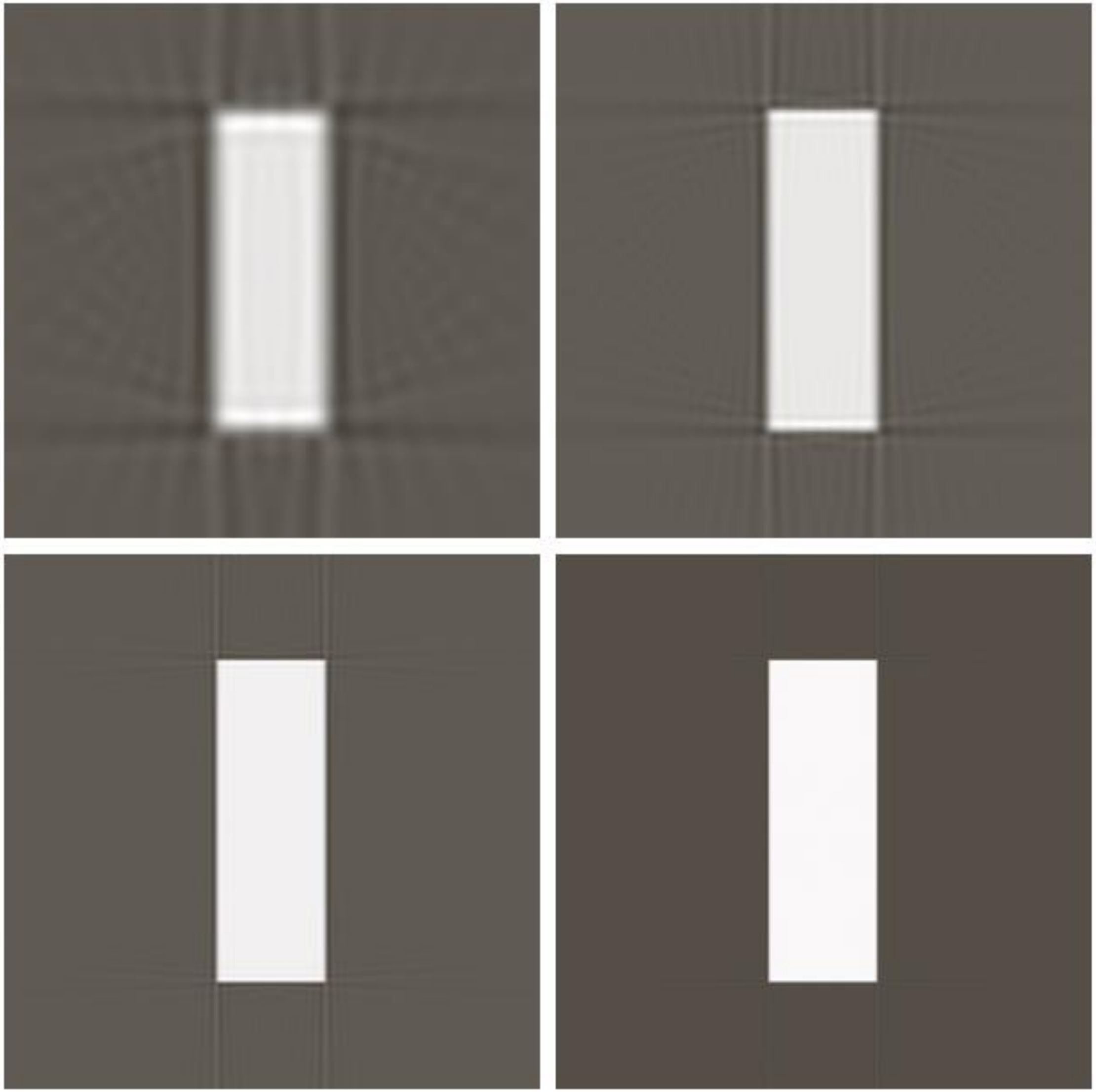
Let $\Delta\beta = \Delta\alpha = \gamma$

$$p(n\gamma, m\gamma) = g[D\sin n\gamma, (m+n)\gamma]$$

Example

$\Delta\alpha = \Delta\beta$
$= 1^\circ$

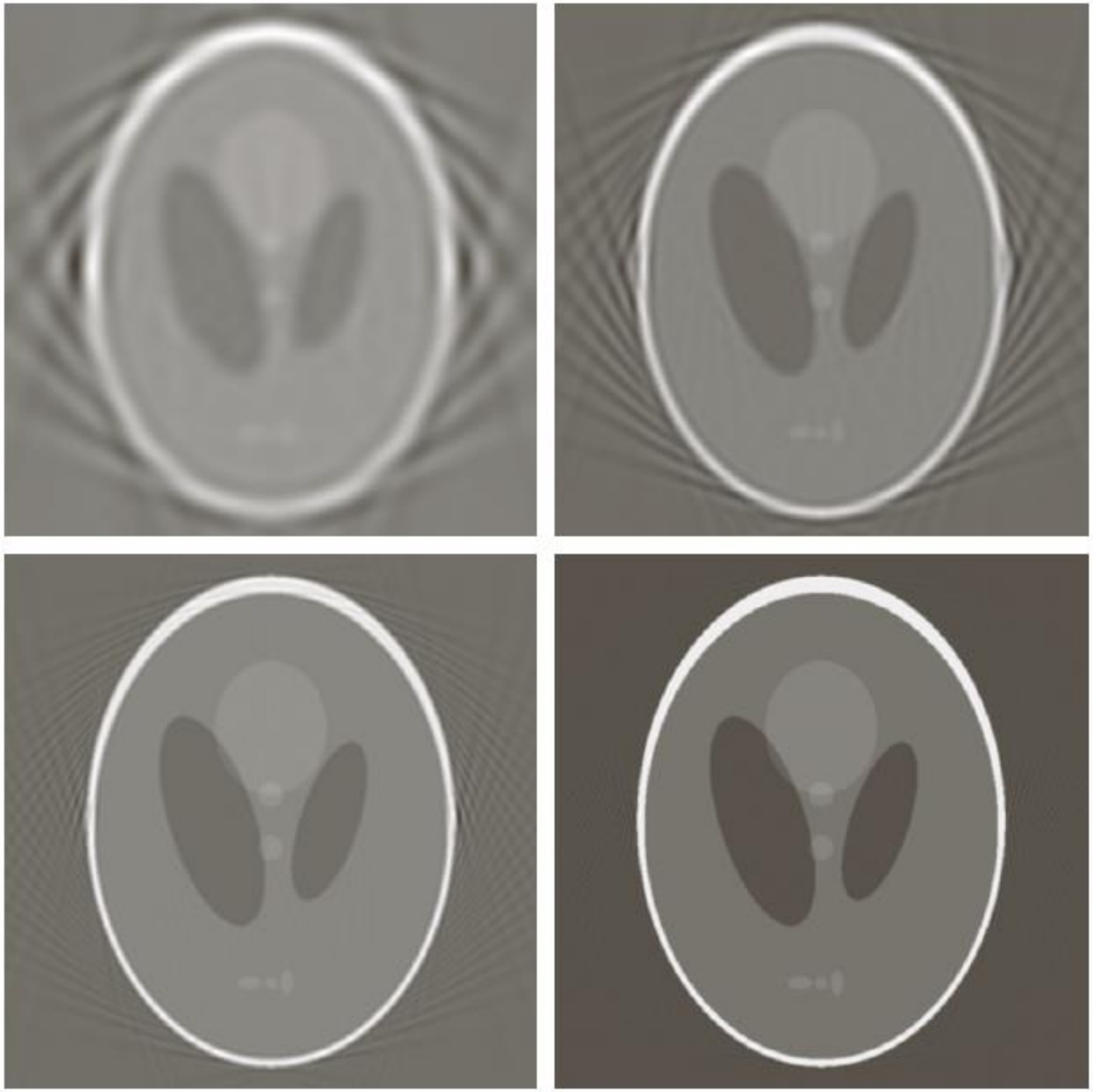
$\Delta\alpha = \Delta\beta$
$= 0.5^\circ$



$\Delta\alpha = \Delta\beta$
$= 0.25^\circ$

$\Delta\alpha = \Delta\beta$
$= 0.125^\circ$

Example



$\Delta\alpha = \Delta\beta$
$= 1°$

$\Delta\alpha = \Delta\beta$
$= 0.5°$

$\Delta\alpha = \Delta\beta$
$= 0.25°$

$\Delta\alpha = \Delta\beta$
$= 0.125°$

Assignments

- 5.25, 5.26, 5.31, 5.35, 5.42