Assignment Title

Your Name

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1 CNF

P	Q	$\neg Q$	$P \vee Q$	$P \wedge Q$	$\neg Q \vee P$	$\neg Q \wedge P$	$Q \Rightarrow P$	$P \iff Q$
true	true	false	true	true	true	false	true	true
true	false	true	true	false	true	true	true	false
false	true	false	true	false	false	false	false	false
false	false	true	false	false	true	false	true	true

so we can conclude that:

1. $Q \Rightarrow P \equiv \neg Q \vee P$
2. $Q \iff P \equiv P \Rightarrow Q \wedge P \Leftarrow Q$
3. $P \iff Q \equiv Q \Rightarrow P \wedge P \Rightarrow Q \equiv (\neg Q \vee P) \wedge (\neg P \vee Q)$
4. $\neg\neg P \equiv P$

convert each of the following formulas to CNF:

1.1 1. $(P \Rightarrow \neg Q) \iff R$

$$Q \Rightarrow P \equiv \neg Q \vee P \tag{1}$$

1.2 2. $\neg(P \wedge \neg Q) \Rightarrow \neg R \vee \neg Q$

$$\begin{aligned}
(P \Rightarrow \neg Q) \iff R &\overbrace{\equiv}^{\text{rule (2)}} (P \Rightarrow \neg Q) \Rightarrow R \wedge (P \Rightarrow \neg Q) \Leftarrow R \\
&\overbrace{\equiv}^{\text{rule (1)}} (\neg(P \Rightarrow \neg Q) \vee R) \wedge ((P \Rightarrow \neg Q) \vee \neg R) \\
&\overbrace{\equiv}^{\text{rule (1)}} (\neg(\neg P \vee \neg Q) \vee R) \wedge ((\neg P \vee \neg Q) \vee \neg R) \\
&\overbrace{\equiv}^{\text{De Morgan}} ((P \wedge Q) \vee R) \wedge (\neg P \vee \neg Q \vee \neg R) \\
&\overbrace{\equiv}^{\text{Distribution}} (P \vee R) \wedge (Q \vee R) \wedge (\neg P \vee \neg Q \vee \neg R)
\end{aligned} \tag{2}$$

1.3 3. $\neg(P \wedge \neg Q) \Rightarrow (\neg R \vee \neg Q)$

$$\begin{aligned}
\neg(P \wedge \neg Q) \Rightarrow \neg R \vee \neg Q &\overbrace{\equiv}^{\text{rule (1)}} (\neg(\neg(P \wedge \neg Q)) \vee (\neg R \vee \neg Q)) \\
&\overbrace{\equiv}^{\text{rule (4)}} ((P \wedge \neg Q) \vee \neg R \vee \neg Q) \\
&\overbrace{\equiv}^{\text{Distribution}} (P \vee \neg R \vee \neg Q) \wedge (\neg Q \vee \neg R)
\end{aligned} \tag{3}$$

1.4 4. $\neg(P \iff \neg Q) \Rightarrow R$

$$
\begin{aligned}
\neg(P \iff \neg Q) \Rightarrow R &\overbrace{\equiv}^{\text{rule (3)}} \neg((\neg P \vee \neg Q) \wedge (Q \vee P)) \Rightarrow R \\
&\overbrace{\equiv}^{\text{rule (1)}} \neg(\neg((\neg P \vee \neg Q) \wedge (Q \vee P))) \vee R \\
&\overbrace{\equiv}^{\text{rule (4)}} ((\neg P \vee \neg Q) \wedge (Q \vee P)) \vee R \\
&\overbrace{\equiv}^{\text{Distribution}} ((\neg P \vee \neg Q) \vee R) \wedge ((Q \vee P) \vee R) \\
&\overbrace{\equiv} (\neg P \vee \neg Q \vee R) \wedge (Q \vee P \vee R)
\end{aligned} \tag{4}
$$

1.5 5. $\neg(P \Rightarrow (\neg R \vee \neg Q)) \Rightarrow \neg R$

$$
\begin{aligned}
\neg(P \Rightarrow (\neg R \vee \neg Q)) \Rightarrow \neg R &\overbrace{\equiv}^{\text{rule (1)}} \neg(\neg P \vee \neg R \vee \neg Q) \Rightarrow \neg R \\
&\overbrace{\equiv}^{\text{rule (1)}} \neg(\neg(\neg P \vee \neg R \vee \neg Q)) \vee \neg R \\
&\overbrace{\equiv}^{\text{rule (4)}} \neg P \vee \neg R \vee \neg Q \vee \neg R \\
&\overbrace{\equiv}^{\text{rule (4)}} \neg P \vee \neg R \vee \neg Q \vee \neg R \\
&\overbrace{\equiv}^{\text{rule (4)}} \neg P \vee \neg R \vee \neg Q \vee \neg R
\end{aligned} \tag{5}
$$

2 Resolution

2.1 Given G = Nitay carries an umbrella

Let's define the following predicates:

- P = Nitay carries an umbrella
- R = If it rains
- W = Nitay dosen't get wet

So we can write the given statement as:

1. $R \Rightarrow P \equiv \neg R \vee P$
2. $P \Rightarrow W \equiv \neg P \vee W$
3. $\neg R \Rightarrow W \equiv R \vee W$
4. W
5. G = P

Now we can start the algorithm, where KB=$(\neg R \vee P) \wedge (\neg P \vee W) \wedge (R \vee W) \wedge W$ and $\alpha = P$
(for the sake of simplicity we will use the following notation: $Q \vee P = Q + P$ and $\neg Q = Q'$ and $Q \wedge P = QP$)

1. we will define C = $KB \wedge \neg P$
2. we will define new = $\emptyset$
3. forall C_i, C_j in C:
   (a) $(R' + P)(P' + W) = P'P + P'R' = P'R'$
   (b) $(R' + P)(R + W) \equiv PR + PW + R'W$
   (c) $(R' + P)(W) \equiv WR' + PW$

(d) $(R' + P)(P') \equiv P'R' + PP' \equiv R'P'$
(e) $(P' + W)(R + W) \equiv W + RP'$
(f) $(P' + W)(W) \equiv P'W + W$
(g) $(P' + W)(P') \equiv P' + P'W$
(h) $(R + W)(W) \equiv RW + W$
(i) $(R + W)(P') \equiv P'R + WP'$
(j) WP'

4. new = $\{P', R, (R' + P), (R + W), W, R', P', (P' + W)\}$
5. new $\not\subset$ C
6. C = C $\cup$ new
7. forall C_i, C_j in C:
   (a) $R'R \equiv \emptyset$
   (b) since we got an empty clause will return true

according to the algorithm, we can cann't conclude that the Nitay carries an umbrella.

2.2

First we will define the following predicates:

- Z: There is Noise
- N: Its night
- D: Have dog
- C: Have cat
- M: Have mice
- P: Struggling to fall asleep
- A: Amit

So we can write the given statement as:

1. All hounds (dogs) bark at night $\equiv (N \wedge D) \Rightarrow Z$
2. If you have a cat, you don't have mice $\equiv C \Rightarrow \neg M$
3. If you have problems falling a sleep you don't keep animals that make sound at night $\equiv P \Rightarrow \neg(N \wedge Z)$
4. Amit has either a dog or a cat $\equiv A \Rightarrow (C \vee D)$
5. G = If Amit have problems falling a sleep, then Amit has no mice $\equiv \neg M \vee \neg A \vee \neg P$

We would like to check if it is possible to conclude that if Amit has problems falling asleep, then Amit has no mice. Let's start the algorithm, where KB=$\{\overline{N} + \overline{D} + Z, \overline{C} + M, \overline{P} + N + Z, \overline{A} + C + D\}$ and $\alpha = \overline{M} + \overline{A} + \overline{P}$

1. we will define C = $KB \wedge \neg\alpha$
2. we will define new = $\emptyset$
3. forall C_i, C_j in C:
   (a) $(\overline{N} + \overline{D} + Z)(\overline{C} + M)$ true when: $\overline{N} \cdot \overline{C} + \overline{D} \cdot \overline{C} + Z\overline{C} + \overline{N}M + \overline{D}M + ZM$
   (b) $(\overline{N} + \overline{D} + Z)(\overline{P} + N + Z)$ true when: $\overline{N} \cdot \overline{P} + \overline{N}N + \overline{N}Z + \overline{D} \cdot \overline{P} + \overline{D}N + \overline{D}Z + Z\overline{P} + ZN + ZZ$
   (c) $(\overline{N} + \overline{D} + Z)(\overline{A} + C + D)$ true when: $\overline{N} \cdot \overline{A} + \overline{N}C + \overline{N}D + \overline{D} \cdot \overline{A} + \overline{D}C + \overline{D}D + Z\overline{A} + ZC + ZD$
   (d) $(\overline{N} + \overline{D} + Z)(M)$ true when: $\overline{N} \cdot M + \overline{D} \cdot M + Z \cdot M$

(e) $(\overline{N}+\overline{D}+Z)(A)$ true when: $\overline{N}\cdot A+\overline{D}\cdot A+Z\cdot A$

(f) $(\overline{N}+\overline{D}+Z)(P)$ true when: $\overline{N}\cdot P+\overline{D}\cdot P+Z\cdot P$

(g) $(\overline{C}+M)(\overline{P}+N+Z)$ true when: $\overline{C}\cdot\overline{P}+\overline{C}N+\overline{C}Z+M\overline{P}+MN+MZ$

(h) $(\overline{C}+M)(\overline{A}+C+D)$ true when: $\overline{C}\cdot\overline{A}+\overline{C}C+\overline{C}D+M\overline{A}+MC+MD$

(i) $(\overline{C}+M)(M)$ true when: $\overline{C}\cdot M+MM$

(j) $(\overline{C}+M)(A)$ true when: $\overline{C}\cdot A+MA$

(k) $(\overline{C}+M)(P)$ true when: $\overline{C}\cdot P+MP$

(l) $(\overline{P}+N+Z)(\overline{A}+C+D)$ true when: $\overline{P}\cdot\overline{A}+\overline{P}C+\overline{P}D+N+\overline{A}+NC+ND+Z\overline{A}+ZC+ZD$

(m) $(\overline{P}+N+Z)(M)$ true when: $\overline{P}M+NM+ZM$

(n) $(\overline{P}+N+Z)(A)$ true when: $\overline{P}A+NA+ZA$

(o) $(\overline{P}+N+Z)(P)$ true when: $\overline{P}P+NP+ZP$

(p) $(\overline{A}+C+D)(M)$ true when: $\overline{A}M+CM+DM$

(q) $(\overline{A}+C+D)(A)$ true when: $\overline{A}A+CA+DA$

(r) $(\overline{A}+C+D)(P)$ true when: $\overline{A}P+CP+DP$

4. new $=\{(\overline{N}+\overline{D}+Z),(\overline{C}+M),(\overline{P}+N+Z),(\overline{A}+C+D),M,A,P\}$

5. we got that new $\subset$ C so we return false there is no condradiction between KB to "if Amit has problems falling asleep, then Amit has no mice".