Intro To Artificial Intelligence - Exercise 2

Eran Ston and Oded Vaalany

June 8, 2024

1 CNF

P	Q	$\neg Q$	$P \lor Q$	$P \land Q$	$\neg Q \lor P$	$\neg Q \land P$	$Q \Rightarrow P$	$P \iff Q$
true	true	false	true	true	true	false	true	true
true	false	true	true	false	true	true	true	false
false	true	false	true	false	false	false	false	false
false	false	true	false	false	true	false	true	true

so we can conclude that:

1. $Q \Rightarrow P \equiv \neg Q \lor P$
2. $Q \iff P \equiv P \Rightarrow Q \land P \Leftarrow Q$
3. $P \iff Q \equiv Q \Rightarrow P \land P \Rightarrow Q \equiv (\neg Q \lor P) \land (\neg P \lor Q)$
4. $\neg\neg P \equiv P$

convert each of the following formulas to CNF:

1.1 1. $(P \Rightarrow \neg Q) \iff R$

$$Q \Rightarrow P \equiv \neg Q \lor P \tag{1}$$

1.2 2. $\neg(P \land \neg Q) \Rightarrow \neg R \lor \neg Q$

$$\begin{aligned}
(P \Rightarrow \neg Q) \iff R &\overbrace{\equiv}^{\text{rule (2)}} (P \Rightarrow \neg Q) \Rightarrow R \land (P \Rightarrow \neg Q) \Leftarrow R \\
&\overbrace{\equiv}^{\text{rule (1)}} (\neg(P \Rightarrow \neg Q) \lor R) \land ((P \Rightarrow \neg Q) \lor \neg R) \\
&\overbrace{\equiv}^{\text{rule (1)}} (\neg(\neg P \lor \neg Q) \lor R) \land ((\neg P \lor \neg Q) \lor \neg R) \\
&\overbrace{\equiv}^{\text{De Morgan}} ((P \land Q) \lor R) \land (\neg P \lor \neg Q \lor \neg R) \\
&\overbrace{\equiv}^{\text{Distribution}} (P \lor R) \land (Q \lor R) \land (\neg P \lor \neg Q \lor \neg R)
\end{aligned} \tag{2}$$

1.3 3. $\neg(P \land \neg Q) \Rightarrow (\neg R \lor \neg Q)$

$$\begin{aligned}
\neg(P \land \neg Q) \Rightarrow \neg R \lor \neg Q &\overbrace{\equiv}^{\text{rule (1)}} (\neg(\neg(P \land \neg Q)) \lor (\neg R \lor \neg Q)) \\
&\overbrace{\equiv}^{\text{rule (4)}} ((P \land \neg Q) \lor \neg R \lor \neg Q) \\
&\overbrace{\equiv}^{\text{Distribution}} (P \lor \neg R \lor \neg Q) \land (\neg Q \lor \neg R)
\end{aligned} \tag{3}$$

1.4 4. $\neg(P \iff \neg Q) \Rightarrow R$

$$\begin{aligned}
\neg(P \iff \neg Q) \Rightarrow R &\overbrace{\equiv}^{\text{rule (3)}} \neg((\neg P \vee \neg Q) \wedge (Q \vee P)) \Rightarrow R \\
&\overbrace{\equiv}^{\text{rule (1)}} \neg(\neg((\neg P \vee \neg Q) \wedge (Q \vee P))) \vee R \\
&\overbrace{\equiv}^{\text{rule (4)}} ((\neg P \vee \neg Q) \wedge (Q \vee P)) \vee R \\
&\overbrace{\equiv}^{\text{Distribution}} ((\neg P \vee \neg Q) \vee R) \wedge ((Q \vee P) \vee R) \\
&\overbrace{\equiv} (\neg P \vee \neg Q \vee R) \wedge (Q \vee P \vee R)
\end{aligned} \tag{4}$$

1.5 5. $\neg(P \Rightarrow (\neg R \vee \neg Q)) \Rightarrow \neg R$

$$\begin{aligned}
\neg(P \Rightarrow (\neg R \vee \neg Q)) \Rightarrow \neg R &\overbrace{\equiv}^{\text{rule (1)}} \neg(\neg P \vee \neg R \vee \neg Q) \Rightarrow \neg R \\
&\overbrace{\equiv}^{\text{rule (1)}} \neg(\neg(\neg P \vee \neg R \vee \neg Q)) \vee \neg R \\
&\overbrace{\equiv}^{\text{rule (4)}} \neg P \vee \neg R \vee \neg Q \vee \neg R \\
&\overbrace{\equiv}^{\text{rule (4)}} \neg P \vee \neg R \vee \neg Q \vee \neg R \\
&\overbrace{\equiv}^{\text{rule (4)}} \neg P \vee \neg R \vee \neg Q \vee \neg R
\end{aligned} \tag{5}$$

2 Resolution

2.1 Given G = Nitay carries an umbrella

Let's define the following predicates:

- P = Nitay carries an umbrella
- R = If it rains
- W = Nitay dosen't get wet

So we can write the given statement as:

1. $R \Rightarrow P \equiv \neg R \vee P$
2. $P \Rightarrow W \equiv \neg P \vee W$
3. $\neg R \Rightarrow W \equiv R \vee W$
4. W
5. G = P

Now we can start the algorithm, where KB=$(\neg R \vee P) \wedge (\neg P \vee W) \wedge (R \vee W) \wedge W$ and $\alpha = P$

1. we will define C = $KB \wedge \neg P \equiv \{(\overline{R} + P), (\overline{P} + W), (R + W), W, \overline{P}\}$
2. we will define new = $\emptyset$
3. forall C_i, C_j in C: (we will write only paris that have a resolution)
   (a) $(\overline{R} + P)(\overline{P} + W)$ Resolution: $\overline{R} + W$
   (b) $(\overline{R} + P)(R + W)$ Resolution: $\overline{P} + W$
   (c) $(\overline{R} + P)(\overline{P})$ Resolution: $\overline{R}$

4. new $= \{(\overline{P}+W), (\overline{R}+W), (\overline{R})\}$

5. new $\not\subset$ C

6. C = C $\cup$ new $\equiv \{(\overline{R}+P), (R+W), W, (\overline{P}+W), (\overline{R}+W), (\overline{R}), \overline{P}\}$

7. forall C_i, C_j in C:
   (a) $(\overline{R}+P)(R+W)$ Resolution: $P+W$
   (b) $(\overline{R}+P)(\overline{P}+W)$ Resolution: $\overline{R}+W$
   (c) $(\overline{R}+P)(\overline{P})$ Resolution: $\overline{R}$
   (d) $(R+W)(\overline{R}+W)$ Resolution: W
   (e) $(R+W)(\overline{R})$ Resolution: W

8. new $= \{(P+W), (\overline{R}+W), \overline{R}, W\}$

9. new $\not\subset$ C

10. C = C $\cup$ new $\equiv \{(P+W), (\overline{R}+P), (R+W), W, (\overline{P}+W), (\overline{R}+W), (\overline{R}), \overline{P}\}$

11. for all C_i, C_j in C:
    (a) $(P+W)(\overline{P}+W)$ Resolution: W
    (b) $(P+W)\overline{P}$ Resolution: W
    (c) $(\overline{R}+P)(R+W)$ Resolution: $P+W$
    (d) $(\overline{R}+P)(\overline{P}+W)$ Resolution: $\overline{R}+W$
    (e) $(\overline{R}+P)(\overline{P})$ Resolution: $\overline{R}$
    (f) $(R+W)(\overline{R}+W)$ Resolution: W
    (g) $(R+W)(\overline{R})$ Resolution: W

12. new $= \{W, (P+W), (\overline{R}+W), \overline{R}\}$

13. since new $\subset$ C we will return false

We dosn't recived a contradiction, so we can conclude that Nitay carries an umbrella.

2.2

First we will define the following predicates:

- Z: animals that make noise at night
- N: Its night
- D: Have dog
- C: Have cat
- M: Have mice
- P: Struggling to fall asleep
- A: Amit

So we can write the given statement as:

1. All hounds (dogs) bark at night $\equiv D \Rightarrow Z$
2. If you have a cat, you don't have mice $\equiv C \Rightarrow \neg M$
3. If you have problems falling a sleep you don't keep animals that make sound at night $\equiv P \Rightarrow \neg Z$
4. Amit has either a dog or a cat $A(C \vee D)$
5. G = If Amit have problems falling a sleep, then Amit has no mice $\equiv A \wedge P \Rightarrow \neg M \wedge A \equiv \neg A + \neg P + \neg M$

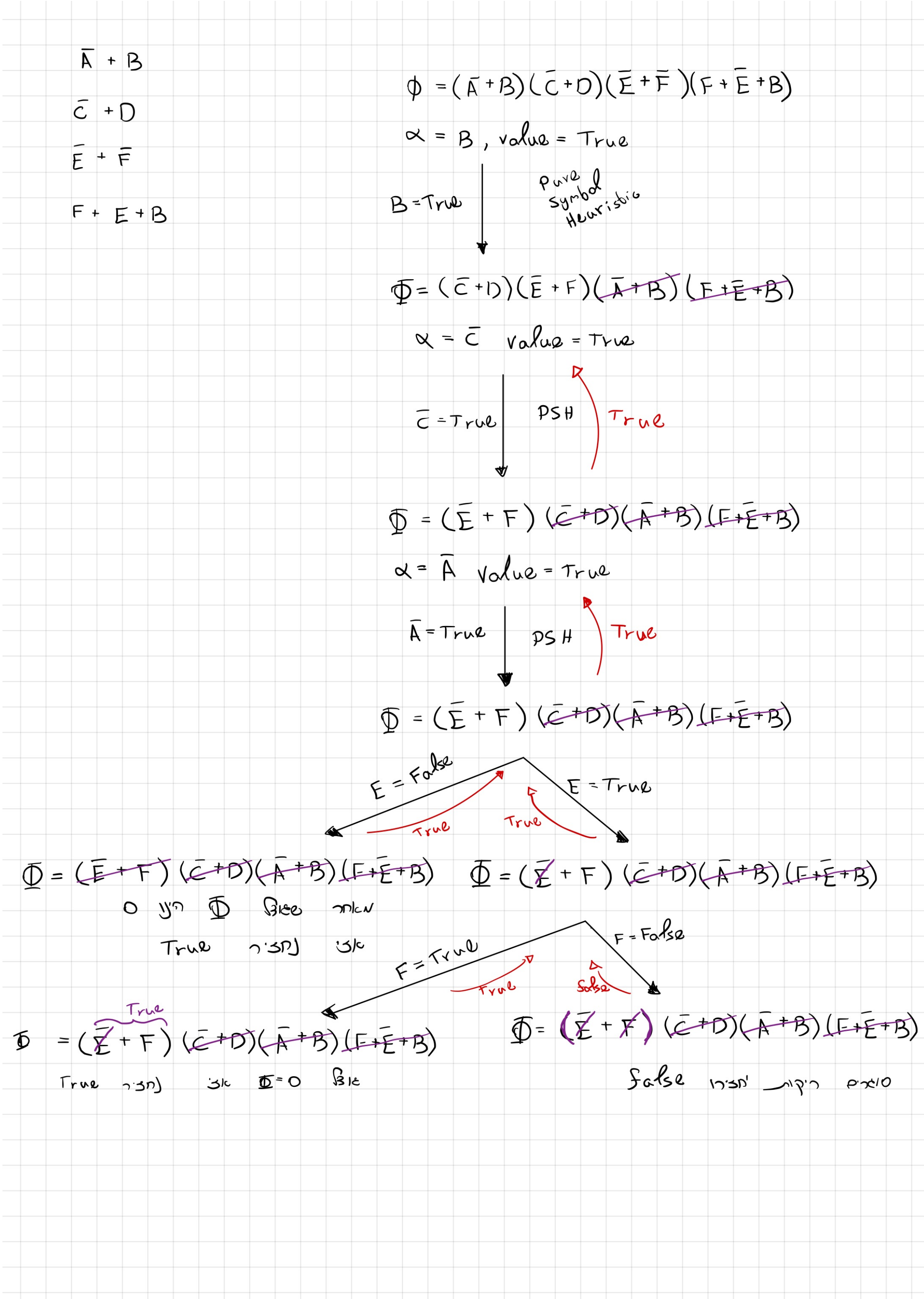
Now we can start the algorithm, where KB=$\{(\overline{D}+Z),(\overline{C}+\overline{M}),(\overline{P}+\overline{Z}),A,(C+D)\}$ and $\alpha = \neg M \vee \neg A \vee \neg P$

1. we will define C $= KB \wedge \neg\alpha \equiv \{(\overline{D}+Z),(\overline{C}+\overline{M}),(\overline{P}+\overline{Z}),A,(C+D),M,P\}$
2. new $= \emptyset$
3. forall C_i, C_j in C: (we will write only paris that have a resolution)
   (a) $(\overline{D}+Z)(\overline{P}+\overline{Z})$ Resolution: $\overline{D}+\overline{P}$
   (b) $(\overline{D}+Z)(C+D)$ Resolution: $Z+C$
   (c) $(\overline{C}+\overline{M})(C+D)$ Resolution: $\overline{M}+P$
   (d) $(\overline{C}+\overline{M})(M)$ Resolution: $\overline{C}$
   (e) $(\overline{P}+\overline{Z})P$ Resolution: $\overline{Z}$
4. new $= \{\overline{D}+\overline{P}, Z+C, \overline{M}+P, \overline{C}, \overline{Z}\}$
5. new $\not\subset$ C
6. C $=$ C $\cup$ new $\equiv \{(\overline{D}+Z),(\overline{C}+\overline{M}),(\overline{P}+\overline{Z}),A,(C+D),M,P,(\overline{D}+\overline{P}),(Z+C),(\overline{M}+P),\overline{C},\overline{Z}\}$
7. new $= \emptyset$
8. forall C_i, C_j in C: (we will write only paris that have a resolution)
   (a) $(\overline{D}+Z)(\overline{Z})$ Resolution: $\overline{D}$
   (b) $(\overline{C}+\overline{M})(Z+C)$ Resolution $\overline{M}+Z$
   (c) $(\overline{P}+\overline{Z})(Z+C)$ Resolution: $\overline{P}+C$
   (d) $(\overline{P}+\overline{Z})(\overline{M}+P)$ Resolution: $\overline{Z}+\overline{M}$
   (e) $(C+D)(\overline{D}+\overline{P})$ Resolution: $C+\overline{P}$
   (f) $(C+D)\overline{C}$ Resolution: D
   (g) $(M)(\overline{M}+P)$ Resolution: P
   (h) $P(\overline{D}+\overline{P})$ Resolution: $\overline{D}$
   (i) $(\overline{D}+\overline{P})(\overline{M}+P)$ Resolution: $\overline{D}+\overline{M}$
   (j) $(Z+C)\overline{Z}$ Resolution: C
   (k) $(Z+C)\overline{C}$ Resolution: Z
9. new $= \{(\overline{M}+Z),(\overline{P}+C),(\overline{Z}+\overline{M}),(C+\overline{P}),D,P,\overline{D},(\overline{D}+\overline{M}),C,Z\}$
10. new $\not\subset$ C
11. C $=$ C $\cup$ new
12. forall C_i, C_j in C: (we will write only paris that have a resolution)
    (a) $\overline{D}\cdot D$ Resolution: $\emptyset$
    (b) we will return True

We received a contradiction, so we can conclude that if Amit has problems to sleep at night so amit has no mice.

3 DPLL Algorithm

given the following CNF formula: $(\overline{A} + B)(\overline{C} + D)(\overline{E} + \overline{F})(F + \overline{E} + B)$



We can see that the algorithm is working correctly and we can conclude that the formula is satisfiable. One of the optional Models is: $B = Ture \wedge C = False \wedge A = False \wedge E = False \wedge F = True$
- If on the Third call we could define E as a pure symbol, we could have finished the algorithm in 3 steps.