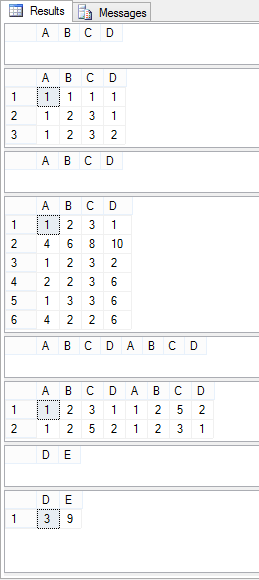
Yang Du

CS 145

**Problem 1**



**Problem 2**

n = 2 do not need to satisfy any FD

R(A1, A2) : can have any combination of A1, A2 such as (A, 0), (B, 0), (A, 1), (B, 1)

n=3 A1, A2 -> A3

R(A1, A2, A3): if A1, A2 is the same, A3 has to be the same, thus, have exactly same tuples, otherwise, if A1, A2 is not the same, any combination is allowed, such as:

(A,E,0)

(A,E,0)

(A,F,0)

(B,E,0)

(B,F,0)

(C,G,0)

(C,H,0)

(D,G,0)

(D,H,0）

(A,G,1)

(B,G,1)

(A,H,1)

(B,H,1)

(C,E,1)

(C,F,1)

(D,E,1)

(D,F,1)

To make sure that this violates all other functional dependencies, all combination of A1, A2, A3 has to exists except the one violate the A1, A2 -> A3.

R(A1, A2, A3, A4): if A1, A2 is the same, A3, and A4, has to be the same, if A1 is not the same, but A2, A3 is the same, A4 has to be the same, otherwise, it can have any combination

(a,A,E,0)

(a,A,E,0)

(a,A,F,0)

(a,B,E,0)

(a,B,F,0)

(a,C,G,0)

(a,C,H,0)

(a,D,G,0)

(a,D,H,0）

(a,A,G,1)

(a,B,G,1)

(a,A,H,1)

(a,B,H,1)

(a,C,E,1)

(a,C,F,1)

(a,D,E,1)

(a,D,F,1)

(b,A,E,0)

...

If n is large, then when An-2, An-1 is the same, An has to be the same.

If An-3, An-2 is the same, then An-1, An has to be the same.

Thus, in the table, for any i <= n-2, if Ai, Ai+1 is the same, then, Ai+2, Ai+3, Ai+4, ..., An has to be the same. But before Ai, the table allows any combination in A1 to Ai-1.

**Problem 3**

A -> B

I1:

(1, 2, 3)

I2:

(1, 2, 3)

(1, 5, 3)

I2 can be larger than I1 because of the above example where if for any tuples with A being the same, B has to equals, thus, I2 violates A -> B.

However, I1 cannot be larger than I2 because for any tuples with A being the same in I1, B is the same in I1. Thus, if any tuples can exists in I1, then it must holds A -> B. and remove any tuples from I1 doesn't affect the A -> B dependency.

A ->> B

I1:

(1, 2, 3)

(1, 3, 5)

(1, 2, 5)

(1, 3, 3)

I2 smaller:

(1, 2, 3)

(1, 3, 5)

(1, 2, 5)

I2 larger:

(1, 2, 3)

(1, 3, 5)

(1, 2, 5)

(1, 3, 3)

(1, 7, 8)

In I1, the A->>B holds because for t = (1, 2, 3), u = (1, 3, 5), exists v = (1, 2, 5), for for t = (1, 3, 5), u = (1,2, 3), exists v = (1, 3, 3), ... Thus for any combination of t, and u in I1 where t. A = u.A, there exists v that t.A = v.A and t.B = v.B and v.C = u.C

However, in I2 small, for t =(1, 3, 5), u = (1, 2, 3), doesn't exists v = (1, 3, 3), and in I2 larger, for t = (1, 7, 8), u =(1, 2, 3), doesn't exists v = (1, 7, 3), and for t = (1, 2, 3), u = (1, 7, 8), doesn't exists v = (1, 2, 8). Thus, I2 smaller, and I2 larger doesn't holds A ->> B

Thus, I2 can be smaller or larger than I1

**Problem 4**

A ->> B

then

for all t1[A] = t2[A], there exists t3, such that t1[A] = t3[A], t1[B] = t3[B], and t2[R-B] - t3[R-B]

Thus, to prove AD ->>BC where D includes C

for all t1[AD] = t2[AD], we know that t1[A] = t2[A] and t1[D] = t2[D].

From the above A ->> B, we know that there exists t3, such that t1[A] = t3[A], t1[B] = t3[B], and t2[R-B] = t3[R-B].

Because t2[R-B] - t3[R-B] and D is included in R-B, so t2[D] = t3[D]

Because t1[D] = t3[D], then t1[D] = t2[D] = t3[D]

Because C is included in D, then t1[C] = t2[C] = t3[C]

Because t2[R-B] = t3[R-B] and t2[C] = t3[C], then t2[R-BC] = t3[R-BC]

Thus, we have t3, where t1[A] = t3[A] = t2[A] and t1[D] = t2[D] = t3[D], t1[B] = t3[B] and t1[C] = t3[C], and t2[R-BC] = t3[R-BC]

Thus, we have t3, where t1[AD] = t3[AD] = t2[AD], and t1[BC] = t3[BC], and t2[R-BC] = t3[R-BC]

Thus, AD ->> BC