UNIVERSITY OF LAGOS
DISTANCE LEARNING INSTITUTE

BUSINESS FINANCE I

FIN 310

COURSE TEXT

Module 1 Study Session 1 Mr. Charles Onyeiwu
Module 1 Study Sessions 2 to 5 Mr. O. Olaoluniyi
Module 2 Study Sessions 1 to 4 Mr. O. Olaoluniyi
Module 2 Study Sessions 5 & 6 Mr. R. W. Oladiran
Module 3 Study Sessions 1 & 2 Mr. A. A. Shiro
Module 3 Study Sessions 4 & 5 Mr. Segun Odunowo

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Study Session 1 Scope of Financial Management and Goals of the Firm

Introduction

This first study session in our Business Finance I course is basically set to introduce you to what you will be learning in Financial Management. It is to briefly show the extent of coverage of the subject matter of this first course and the one that comes after it (Business Finance II). The study session will also serve the purpose, as the topic suggest, of showing you the main goals and objectives that propels financial managers in their day-to-day endeavors. The session then is introductory and hence concerned with preparing your mind in what to expect in this module, in the course generally and the other study sessions that follows. The issue of the goals and objectives that drive managerial actions in Finance will keep recurring as the study sessions unfolds. That is why you need to take particular note of the main goals and objectives that have evolved over time that has been found to be pursued by financial managers. All that you need as pre-requisites to this course has been covered in the various lower level courses in Principles of Accounting, Business Mathematics and Economics. You are definitely set then to grasp what will be taught in the course with relative ease. Bon voyage as you steer your way through the stuff in this, and all the other modules and study sessions that follow it!

Learning Outcomes for Study Session 1

When you have studied this session, you should be able to:

1.1 Itemize and briefly describe the three main decision themes that pre-occupy financial management.

1.2 Explain what the Profit Maximization goal of the firm entail and various criticisms leveled against it.

1.3 Explain why the goal of maximization of Shareholders’ wealth have been found to be the main functional objective pursued by financial management, and describe how the goal works to drive financial managerial actions.

1.4 Itemize and describe other goals that financial management often pursue and show why these are subsidiary to the goal of maximization of Shareholders’ wealth.

1.1 Scope of Financial Management

Management is concerned with decision making and smooth running of an enterprise. Financial management involves acquisition of assets needed for the operation of a firm, and financing such asset acquisitions through dealings in the financial markets.

It is also concerned with the issue of what to do with the profits being made in the firm – plough it back into the business or return it to the shareholders.

The three types of decisions just mentioned are the basic preoccupations of financial management. Put in another way these decisions are:

(i) Investment or capital budgeting decisions

(ii) Financing Decisions; and

(iii) Dividend policy decisions.

Investment decisions entail ensuring that various tangible assets needed by the firm are available at the right time. These assets include: office accommodations, plants, machinery, factories, farmlands etc. Plans or budgets are normally made to ensure that these items are acquired when needed.

Procuring funds for the acquisition of needed assets often poses variety of problems for business organizations. Financing decisions are made to find solutions to this. It often involve sale of pieces of papers called financial assets, or securities. These financial instruments are claims against the firm's tangible assets and are bought by investors to supply needed fund for the enterprise. Examples of financial assets are share certificates, bonds, bank loans and leasehold papers.

A dividend policy decision is a hybrid of the other two types of financial management activities since it involves the two. Profits are either retained for reinvestment purpose to aid growth of the firm or they are declared as dividends to shareholders, or both.

This is a meeting point between investment and financing decisions and affects the value of the firm.

These essentially are the basic features of the concern of Financial Management or Corporate finance. How some of these decisions are made would be theoretically spelled out in this text. It should be noted however that finance being a branch of economics will normally involve application of statistics, and mathematics to the study of its theory. Econometrics is also applied at higher levels of study.

- Why is the Dividend Policy decision described as a 'hybrid' decision?
    - It is because it has implications for investment/ploughing-back of profit into the business; and also has implications for financing since cash has to be procured to pay shareholders when dividends are declared.

1.2Goals of the Firm

There is little agreement in the literature about what the objectives of firms are or what they ought to be. Firms are also known to pursue diverse objectives at the same time, but a few central ones stand out as major. These includes: Profit maximization, wealth maximization, and ensuring generation of a good public image as a corporate citizen of the community. We will discuss these and other relevant objectives of the business organization and their implications for financial management.

1.2.1 Profit Maximization

The micro-economic model of perfectly competitive market postulates that the private firms should and does behave primarily with the profit maximization motive. This is based on the assumptions that there are many buyers and sellers who are price-takers in the market; and entry into or exit from the market can be made freely by the players in the market who have perfect information about the homogenous good being sold in the market.

To maximize profit, the firm must maximize output for a given set of scarce inputs, or expressed equivalently, minimize the cost of producing a given output. This is tantamount to saying that the firm must be efficient in its use of resources. So from the point of view of economic theory, profit maximization is simply a criterion for economic efficiency.

This goal of profit maximization emanated from the argument that the shareholders are the owners of the firm which therefore should be operated for their benefit by trustee managers. Furthermore, economists have argued that under the restrictive assumptions mentioned above profit maximization by the individual entrepreneur who is assumed to receive all the profits leads to the maximization of overall economic welfare. This thinking is basic to the free enterprise system which postulates that by doing the best for yourself, you are at the same time doing your best to society. Let us now discuss the theory underlying this invisible-hand doctrine.

In an attempt to maximize profit, owners of firms aim at the largest excess of revenues over costs. The costs results from competitive bidding for scarce resources in factor markets. The prices offered for a factor will depend on the revenue the bidding firms expect to receive from using the input. The revenue in turn will be determined by the price of the goods and or services to be produced using the factor. Prices of particular goods are reflections of consumer desires that are backed by money, that is, effective demand. The pattern of consumers' expenditure would then establish a set of prices for final goods that reflect the values placed on commodities. By extension prices in the final goods market represent the value the economy places on the various goods and services that are provided. The bids offered for factors depend on the prices that exist in the final goods market. The bidding process thereby ensures that factors are employed by those whom society thinks will use them in the most worthwhile way.

It is assumed that the price generated for any input by the factor market applies to all the units traded. This price just equates factor demand and supply and will be equal to the expected revenue to be obtained from using the last or marginal unit traded. Additional input at this price would be unprofitable, because the revenue that could have been obtained from using the input would not cover its cost. All the firms that do obtain factors will use them on products that are deemed, at least, as valuable as the commodity which is produced by the marginal unit of the factor. All such enterprises are therefore manufacturing products that are considered more valuable than the goods that would be produced by those organizations that were denied factors by the bidding process. Hence the invisible hand guarantees that factor prices will reflect the value of the output sacrificed by allowing any given firm to use the inputs it requires. This implies that the costs faced by entrepreneurs are opportunity costs reflecting the revenue obtained from factors in their best alternate use.

Thus firms, by seeking to maximize revenue over and above these opportunity costs, would be maximizing the economic welfare of society.

In addition, the profit-maximization system has a built-in control device which ensures that in the long run the allocation of resources reflects society's preferences as expressed in the market place. Profits that is greater than those regarded as normal would attract new entrepreneurs into the industry and this competition ensures that excess profits are removed. Thus we have a purely automatic system which, provided everyone is looking after himself, ensures the optimal economic welfare of society.

But we have various criticisms leveled against the profit maximization objectives since this El Dorado is not easily achieved. It is suggested that being based on private ownership, profit maximization could lead to serious in-equality in distribution of income and wealth among different groups. We find however that the whole society is better off if its production of goods and services is as efficient as possible.

Other criticisms are leveled against the profit maximization concept as a goal of the firm. The definition of profit, that is, what constitutes profit, need further clarification. The economist's definition of profit would likely differ from that of the accountant. Besides, the concept does not take into consideration the influence of uncertainty which is capable of altering profit estimation. The objective also does not take into account the time value of money.

It is further argued that market prices may not in fact reflect true values and that pure price competition exists only in few industries, making for existence of oligopolies and monopolies in many market situations.

There are also barriers to entry because of costs, technology and capital requirements. Government regulations, price controls and subsidies also produce other distortions. Furthermore, the market system cannot easily cope with the benefits or costs of an action which do not fall on those taking the action but on others. Industrial pollution is an example of this as managers and employees are said not to always behave legally and ethically.

We agree that these problems are real enough. But the remedy for such difficulties does not lie in abandoning the market economy and profit maximization as a criterion. Instead the market deficiencies can and are being corrected by government intervention. This is why we have legislation on pollution, and natural monopolies among others.

- Mention two (2) criticisms leveled against the Profit Maximization objective of the firm.
  - Firstly, it could lead to serious in-equality in distribution of income and wealth among different groups; secondly, the concept does not take into consideration the influence of uncertainty which is capable of altering profit estimation.

1.3Maximization of Shareholders' Wealth

Not withstanding the above identified deficiencies we assume in this text that firms seek to maximize profit which will be shown to be equivalent to seeking to maximize shareholders' welfare. Maximization of shareholder's wealth is stressed due to some practical problems that arise in the use of the pure profit maximization criterion. Profit maximization does not help us to decide between options such as ₦500.00 with certainty or ₦1,500 if things go right but nothing otherwise. Normally profit maximization is assumed to apply to the long run as projects will contribute profits through time and not just in one year. Thus we must devise some means of comparing profits received at different times. Furthermore we need to make our objective operational by being more precise about what we mean by profits. The accountant's profit figure will need some adjustments here. We will need to settle for net cash flows received from a project throughout its life.

So for operational use, profit maximization does not take account of risk and the time value of money. Besides this, it is ambiguous. For these reasons wealth maximization has replaced profit maximization as the operational criterion for financial management decisions. By measuring benefits in terms of cash flows we would avoid much of the ambiguity of profits. By discounting those cash flows over time using concept of interest compounding we can take account of both risk and time value of money.

The value of an asset is best viewed in terms of the benefits it can produce. The benefits of an investment or financing decision can be measured in terms of the stream of future expected cash flows generated by the decision. This must take account not only of the magnitude (expected value) of the flows, but also their degree of uncertainty. All other things being equal, less uncertain flows are valued more highly than more uncertain flows. So the wealth maximization criterion involves a comparison of value to cost. An action that has discounted value reflecting both time and risk that exceeds its cost can be said to create value. Such actions increase the value or wealth of the firm and should be undertaken. Conversely, actions with value less than cost reduce the value of the firm and should be rejected. In the case of alternatives that are mutually exclusive, when just one is to be chosen, the alternative with the greatest excess of inflow over outflows should be selected. In short, the objective of financial management is to maximize the value of the firm and in turn the value of shareholders' investment since they provide the operational funds of the firm. It is important to note that maximization of shareholders' wealth is as explained here simply an extension of profit maximization to a scenario that is uncertain and multi period in nature. Where the time period is short and the degree of uncertainty is not great, wealth-maximization and profit-maximization almost amount to essentially the same thing.

You may start wondering at this point on how exactly wealth maximization is determined. On the final analysis, wealth maximization is tantamount to maximization of the price of a firm's shares in the capital market (Stock Exchange). The price of the shares is determined by the firm's present earnings and expected future earnings which in turn is determined by its overall cash outflows and cash inflows from investment projects. We can conclude from this that wealth maximization is equivalent to stock price maximization. Hence what managers do to maximize shareholders' wealth is to take decisions that positively enhance the firm's stock price. The following factors will affect a firm's stock price.

a. Estimated earnings per share;
b. Timing of the earnings stream;
c. Risks attached to projected earnings;
d. The various combinations of securities used in financing the firm, that is, how debt and equity securities are combined (the firm's capital structure) Cost of capital;
e. Dividend policy

It has been found that shareholders benefit more when share prices appreciate in the market. This way there is a lock-in of capital gain when they sell their shares to other investors.

So every major investment and financing decision made in a business firm would normally be analyzed in terms of its effects on the factors enumerated above and hence on the price of the firm's shares.

There are limitations to the shareholders' wealth maximization objective. For instance, using share price as a yardstick might be misleading. This is because other factors not directly related to the company's performance do affect share prices e.g. war, event in other countries, natural disasters etc. All these are not related to the ability of the financial manager to perform.

- What main feature makes maximization of shareholders' wealth functional as appropriate goal of financial management?

  - It measures benefits that accrue to the firm in terms of cash flows and hence avoid much of the ambiguity of 'profit'.

1.4Other Relevant Objectives

The disenchantment with the profit maximization criterion discussed earlier has brought the idea that firms do pursue other goals. A firm for example may be thought to "satisfice" rather than maximize; that is seek "satisfactory" rather than maximum profit. The definition of "satisficing" however almost always includes such concepts as survival, preservation of market shares, or even some growth of market share, since the institution would invariably operate subject to competition from alternate providers of its goods and services, it is forced to seek relatively efficient solutions. This ensures that whatever goals are pursued would bring out results that do not differ substantially from the profit maximization objective.

Furthermore, the question of management of many firms being separate from owners (shareholders) has made many writers in this field to suggest pursuit of goals that management favours. These would include:

(i) Improving employees' welfare;
(ii) Creating a good community relationship or a good public image
(iii) Improving welfare of the management team by increasing emoluments and other perquisites and devoting money to even recreational facilities like golf or tennis.

We may not be able to rule out the possibility of management pursuing other objectives apart from profit maximization conceding the reason of management's independence from shareholders. But the share-holding of various stockholders of publicly quoted companies are diversified. So if one company is operating at a loss, a stockholder will not be losing much as other companies would bring required returns. Management on the other hand are not diversified and so setbacks affect them more seriously. So they tend to play it safe. Besides, managements tend to be careful being mindful of takeover bids (especially in advanced countries) that could displace them. Furthermore, many firms do tie management's compensation to the company's performance, and there are indications that this motivates managers to operate in a manner consistent with stock price maximization.

1.4.1Social Programs

Business organizations with normal or supernormal profits can and do engage in community welfare programs. These social services increase costs however. In highly competitive industries, there would be a minimization of such efforts. It is conceivable also that those who hold companies' shares would prefer those of companies that increases value of shareholders' wealth to those of "social do-gooders" who reduce shareholders' wealth.

So normally, social cost-increasing actions may have to be put on mandatory rather than on voluntary basis, at least initially. This is to ensure that the burden of such actions falls uniformly across business organizations. For this reason, and the deficiencies of the invisible-hand philosophy that we saw earlier; government legislation and regulations are introduced for

(i) Minimum wage rules
(ii) Industrial training fund laws
(iii) Regulation on standard qualities of industrial output
(iv) Workmen safety regulations
(v) Environmental pollution regulations etc.

These and other regulations to ensure social responsibility of business organizations become constraints. Firms would normally strive to maximize shareholders' wealth subject to these constraints.

- Why are we sure that firms would not normally pursue goals that are substantially different from maximization of shareholders wealth?
  - Other goals could lead to losses in the firm that could lead to management losing their position, so they learn to play safe.

Summary of Study Session 1

The subject matter of this study session has been the enumeration of the basic pre-occupations of financial officers of a business organization, and why they carry out these activities. The economic concept of profit maximization as a motivating factor of business enterprise which prompted the *laissez faire* doctrine was explained. The operational defects of the profit maximization motive however, led to the search for more relevant concepts to practically tackle the goals financial management should pursue. This is shareholders' wealth maximization. This objective ensure that the ambiguity in application of the profit maximization criterion is avoided. Furthermore, difficulties posed by projects that entail cash flows in multiple periods and risks inherent in business decisions are handled with relative ease. You will see how these work out in the application of the concept of time value of money and how to handle uncertainties in subsequent chapters.

Self-Assessment Questions (SAQs) Study Session 1

Having completed this study session, you can assess how well you have achieved its Learning Outcomes by answering the following questions. You can check your answers at the Appendix to the course text.

SAQ 1.1 (tests Learning Outcome 1.1)

Explain the components of major decisions of financial management in a business firm

SAQ 1.2 (tests Learning Outcome 1.2)

List the major goals of financial management in a business organization

SAQ 1.3 (tests Learning Outcome 1.3)

How can the key goal of financial management be achieved in a business firm?

SAQ 1.4 (tests learning Outcome 1.4)

List three (3) subsidiary goals that firms could pursue. State why these would not be in conflict nor displace the key goal of financial management?

REFERENCES

1. Bromwich, Michael, The Economics of Capital Budgeting Penguin U.K. 1967.

2. Solomon, Ezra and John J. Pringle, An introduction to Financial Management 2nd Edition. Santa Monica California. Good Year Publishing Company. 1980.

Should you require more explanation on this study session, please do not hesitate to contact your e-tutor via the LMS.

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iag@dli.unilag.edu.ng
08033366677

Study Session 2: Compounding Methods

Introduction

The focus of this study session is basically compounding methods i.e. problems that involve calculating the future values of lump sum payments and series of payments (annuities).

The issue of time value of money, a concept that is basic to the field of financial management is first discussed. Thereafter the nitty-gritty of deriving future values of lump sum payments and related issues are discussed. Compounding problems of series of payments (annuities) were then tackled.

What you need, to understand and grasp the materials in this study session is simple calculation skills. This you definitely would have acquired and mastered at the lower levels of your program having studied some basic courses in Business Mathematics and Economics.

Learning Outcomes for Study Session 2

When you have studied this session, you should be able to:

2.1 Itemize and explain the facts of life that confer time value on money

2.2 Calculate the compound (future) value of a lump sum payment, using the basic compounding formula.

2.3 Calculate number of periods needed to attain a particular level of growth in initial investment when other relevant variables are known, for a lump sum payment.

2.4 Calculate the interest rate when other relevant variables are known for a lump sum payment.

2.5 Calculate the effective interest rate when nominal interest rate and frequency of compounding in a year are known.

2.6 Compute the compound (future) values of ordinary annuities (annuities in arrears).

2.7 Compute the compound (future) values of annuities due (annuities in advance) when relevant information is supplied.

2.1 The Concept of time value of money

A concept that is basic to Business Finance is the time value of money. Money is a scarce resource which its owners can use to acquire other resources. The timing of cash flows, profits or benefits generated by an investment matters since ₦500 that is available today is worth more to an individual than the same amount that is promised to be paid three months or one year hence.

Some basic facts of life would make a rational decision maker prefer to have a certain amount of money now rather than wait to have the same amount sometime later. Inflation is a basic reality of our economic life that makes money lose some of its value over time. Uncertainty of projected returns on investments is another feature making a certain sum of money now, ordinarily incomparable to equivalent sum sometime in the future. Furthermore like owners of other scarce resources owners of money can permit others (borrowers) to rent the use of their money for a period of time, at an agreed interest rate. All these features of our economic situation confer time value on money.

You will in the course of this text learn various techniques of dealing with inflation, uncertainties of time and other issues relating to risk in investment analyses. It is sufficient for now to note that the medium of interest rates can be and are used to deal with the basic features that gives time value to money. Apart from interest charges on passage of time, appropriate additional interest rates can be used to account for inflation and uncertainty of cash flows of an investment. This is why financial analysts use interest calculations to make amounts of money to be paid or received at different times comparable. We will see how this is done in this and several subsequent study sessions. By the use of appropriate interest rates we can compute the present equivalent of future sums of money and so make them comparable. Similarly we can place all naira to be paid (or received) in the future on a comparable basis so that they can be added or subtracted. Thus the rate of interest provides the tool needed for adjusting the value of all cash flows, whenever they are expected to occur, to a single common point of time.

Most financial decisions such as purchase or procurement of funds, lease versus purchase, capital budgeting, dividend policy, and bond refunding among others affects the firm’s cash flows in different time periods. For instance, if the firm borrows fund from a bank or from any other source it receives cash now and commits an obligation to pay interest and repay principal in future periods. Sound decision making requires that the cash flows which a firm is expected to give up over a period should be logically comparable. Cash flows become logically comparable when they are appropriately adjusted for their differences in timing and risk. For this reason the recognition of the time value of money is extremely vital in financial decision making.

- How much will ₦2,000 deposited today in an account at a simple interest rate of 8 per cent amount to in five years?

- ₦2,800

2.2 Compound (future) value

We will start our discussion on the application of the time value of money concept, with a look at compound interest. The **rate of interest** is the rate at which an individual or firm will be compensated for exchanging money now for money later in the future. The rate of interest is usually expressed as a percentage of the amount borrowed per unit of time, say one year. Examples are 12 percent per year, 2 per cent every quarter and so on.

2.2.1 Compound Interest: To **compound** interest is to pay the lender at the end of the period, the interest earned during the period. In other words, it is to add the interest in with the principal so that the principal for next interest period is larger. The period between interest calculations during which the principal earns interest is called the **compounding period**. This can also be referred to as the **conversion period**. This last term is derived from the fact that the interest earned is converted (added) to principal at the end of the interest-earning period.

Illustration: Let us now discuss an example. Mr. James Nwike deposited ₦10,000 in a savings account that pays compound interest at the rate of 6 per cent per year. How much will he have at the end of one year?

Solution: Let us assume that interest is compounded annually, that is, interest is added to the principal balance at the end of the year. Then at the end of the first year Mr. Nwike will have

Initial Amount (Principal)	₦10,000.00
Interest for 1 year at 6 percent	600.00
Amount at end of year	₦ 10,600.00

If we assume that Mr. Nwike leave the money in the savings account untouched for the second year, what will he have at the end of the second year?

Balance at beginning of second year	₦10,600.00
Interest for second year at 6 percent	636.00
Amount at end of second year	₦11,236.00

During the second year the principal of ₦10,600 will earn ₦636 interest: ₦600 on the initial deposit of ₦10,000 and ₦36 on the ₦600 earned in the previous year.

The effect of compounding interest is more substantial than you may realize. For example, compounded at 12 percent, money doubles itself in about 6 years. In other words, if you put ₦ 1013.26 in a savings account that pays 12 percent compounded annually, you will have ₦2000 in 6 years. You should note in contrast that Simple Interest calculation deals only with addition of the interest at the tail end of the agreed period. For example at Simple Interest (computed as principal multiplied by rate multiplied by time i.e. S.I = p x r x t) ₦500 deposited for 6 years at

12 per cent will earn total interest of ₦500 x 0.12 x 6 = ₦360. The total amount at the end of six years, when principal is reckoned with simple interest, will be ₦860.

Let us look at this problem in a systematic way for general application purpose. We will define a few terms:

P_0 = Principal or initial amount at time 0.

r = Interest rate

I = Total amount of interest earned (in Naira)

P_n = Principal value at the end of n periods. When n equals 1,

P_n can be calculated as follows:

$$P_1 = P_0 + I$$

$$= P_0 + P_0 r$$

$$= P_0 (1 + r) \qquad (2 – 1)$$

In equation 2-1 the ending amount (P_1) is equal to the beginning amount (P_0) multiplied by the factor (1 + r). In our example where P_0 = ₦10,000; r = 6 percent (or 0.06 when expressed in decimal notation for computational purpose) and n = 1, P_n is determined as follows:

P_1 = ₦10,000 (1.0 + 0.06)

= ₦10,000 (1.06) = ₦10,600

Let us extend this to multiple periods. If Mr. Nwike leaves the ₦10,000 in the savings account for five years, to what will it have grown at the end of that time? We can simply calculate how it grows from year to year for a couple of years before we generalize.

Note that P_2, the balance at the end of the second year is found as follows:

$$P_2 = P_1 (1 + r)$$

$$= P_0 (1 + r) (1 + r)$$

$$= P_0 (1+r)^2$$

Similarly, P_3, the balance at the end of 3 years is found as:

$$P_3 = P_2 (1 + r)$$

$$= P_0 (1 + r)^3$$

Equation 2-1 can be used to construct a table (Table 2-1) which follows the growth of the money deposited in the savings account till we arrive at what it will amount to at the end of five years.

TABLE 2-1 COMPOUND (FUTURE) VALUE CALCULATIONS

Period	Initial Amount	*x* (1 + r)	= Ending Amount (P_n)
1	₦10,000.00	1.06	₦10,600.00
2	₦10,600.00	1.06	₦11,236.00
3	₦11,236.00	1.06	₦11,910.16
4	₦11,910.16	1.06	₦12,624.77
5	₦12,624.77	1.06	₦13,382.26

We can conclude this section by noting that in general, P_n, the compound (future) value amount at the end of year n, is found as

$$P_n \quad = \quad P_0 (1 + r)^n \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(2-2)}$$

Equation 2-2 is the basic equation of compound interest. It should be noted that equation 2-1 is a special case of equation 2-2 where n = 1.

Getting a quick knack of the derivation of equation 2-2 is not difficult. You need this in order to understand much of the materials covered in this course. However the concept can be applied quite readily by rote. This is because tables have been constructed for values of (1 + r) for wide ranges of r and n. Table 1 in the "Interest factor Tables" (Appendix) is an example where compound value interest factors for a wide selection of combinations of r and n can easily be read off the table and used by multiplying the factor with the principal amount involved. For example the interest factor for our example of ₦10,000 deposited for five years at 6 percent interest rate is read from the table as 1.3382. You determine this by looking under the 6 percent column at row 5(for 5 years or periods). Using this factor we find the compound value of ₦10,000 at the end of five years as follows:

$P_5 \quad = \quad$ ₦10,000 (1.3382)

$\quad\quad = \quad$ ₦13,382.26

You will realize that this is exactly what we arrived at by use of table 2-1.

Illustration: We will now discuss a few sample problems involving compound or future values.

1. How much will ₦25,000 deposited today at 8 per cent compounded annually amount to in 10 years?

Solution: ₦1.00 deposited today at 8 per cent will grow to ₦2.1589 in 10 years using the table figure; therefore ₦25,000 will grow to ₦25000 $(1.08)^{10}$ = ₦25,000 x 2.1589 = ₦53,973.12

2. What will be the amount accumulated by ₦5,400 in 37 years at 9 per cent compounded annually.

Solution: $P_n = P_0 (1 + r)^n$

P_0 = ₦5,400 r = 0.09 n = 37 years

P_n = ₦5400 $(1.09)^{37}$

= ₦5400 (24.25384)

= ₦130,970.71

What is simply done is the use of calculator to derive the compound value of ₦1 at 9 percent interest rate for 37 years. This gives a factor of 24.25384 which we use in multiplying the initial principal of ₦5400 to arrive at the answer above.

Table can be used. You will notice however that the table provided do not show values for 37 years (or periods). We can get around this either by using calculator as above or picking values from the table and multiplying them (provided the n values of factors picked under 9 percent sum up to 37). Try this for values like n = 17 and n = 20 or n = 15 and n = 22, etc. You will arrive at the same answer as above.

- An investor in the shares of Tina PLC expects a dividend of ₦1.20 per share. The investor's holding of the company's shares is 10,000. If the company's dividends are expected to grow at the rate of 22% per annum for the next 15 years, how much is the projected value of the dividend at the end of the period?

  - ₦236,907.44

2.2.2 Solving for the number of periods

Sometimes the financial manager needs to know how long it will take for a certain amount invested today to grow to a certain value, given that it earns a known rate of interest.

For example a pension fund administrator may have ₦250 million to invest today to meet future pension needs. The administrator might want to know how long it would take for that money to grow to ₦1 billion needed to meet the firm's obligations. The analytical solution is derived from our equation 2.2. Solving the expression for n gives

$$n = \frac{\ln\left[\frac{p_n}{p_0}\right]}{\ln(1+r)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \; 2.3$$

Here, we use the natural logarithm but any logarithm measure, such as base 10 logarithms would equally work. The only restriction is that the logarithms used in the numerator and denominator must use the same base.

Illustration:

If you earn an annual rate of 17 percent compounded annually, how many years will it take for money to triple?

Solution: Since you wish to triple your money we can assume that you start out with ₦1, and after n years you will have ₦3. Substituting these numbers and the 17 percent interest rate in equation 2.3, we compute the value of n:

$$n = \frac{\ln\left[\frac{₦3}{₦1}\right]}{1n(1+0.17)} = \frac{1.09861}{0.1570} = 6.997 \quad = \text{approx 7 years}$$

- How many years will it take for an initial investment to double at 1% compounded monthly? (hint: note that each period is a month)
    - 69.66 months or approx. 5years 10 months

2.2.3 Solving of for an unknown Interest Rate

There are situations where financial managers and investors must deal with a time value of money problem in which the present value and future values are known, as well as the time between them, but not the interest rate that connects those values. For example many bonds require a payment now and return a specified larger payment at some future dates, but the implicit interest rate the bond earns is not stated, so it must be computed separately.

We can solve this and similar problems in several ways. We can use financial calculator. A second technique is to solve the problem directly. To do this we need to find the interest rate from our compound value equation (equation 2.2). Since the time between the present and future value is n, what is required essentially is to isolate the term containing r and taking the nth root of both sides of the equation.

The expression for r is then:

$$r = \left[\frac{p_n}{P_o}\right]^{1/n} - 1.0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad 2.4$$

Illustration:

Mr. James Adu a fast talking salesman, comes into your office and offers you a deal. James offers to pay you ₦5,960 eight years from now. He says the price you must pay today is only ₦1000. At what rate of interest is this offer made?

We solve this by applying equation 2.4

$$r = \left[\frac{\overline{N}5960}{1000}\right]^{1/8} - 1.0 \qquad = 0.25$$

$$r \qquad = \qquad 25\%$$

- Joseph's parents have estimated that his university education will cost ₦120,000 when he enrolls 7 years from today. At what annual compounding rate of interest would Joseph's parents invest ₦20,000 to meet the expenses of his education?

o 29.2% or approximately 29%

2.2.4 Pictorial (Graphical) Representation of the Compounding Process.

Our discussion on solving for number of periods now leads us to consider a pictorial view of the compounding process. Figure 2 – 1 below shows how the interest factors increase or grow the longer the compounding period, and the higher the interest rate. We can draw curves for any interest rate, including fractional rates. What we have plotted are the curves for 0 percent, 7 per cent and 14 percent. The information in Table I in the "Interest Factor Tables" (See appendix) can be used to plot the relationship between compound or future value, interest rates and time for any interest rate. A cursory look at figure 2-1 shows that the higher the rate of interest, the faster the rate of growth.

2.3 Changing The Compounding Period; Nominal And Effective Interest Rates.

We have assumed in our examples so far that interest is added or converted into principal once a year. In practice, we often find that compounding takes place more than once a year. Interest payments on loans and bonds are usually made semiannually. Many commercial banks add interest to deposits quarterly or monthly. What is the effect of such increase in the frequency of compounding? This we will address presently.

If ₦1,000 is invested today at 16 per cent compounded annually, its future value one year later is ₦1,160. If the rate of interest is 16 per cent compounded semiannually, then 8 per cent interest is added to the principal every six months. At the end of the first six months, ₦1,000 will have grown to ₦1080. At the end of the second six months interest at 8 per cent is computed on a principal of ₦1,080 so that the accumulation will be ₦1,080 x 1.08 = ₦1,166.40 by the end of the year. You will notice from this that 16 per cent compounded semiannually is equivalent to 16.64 per cent compounded annually.

Let us assume that interest is added to the deposit on a quarterly basis. This means that an additional 4 per cent of the principal will be added to, or converted into principal every three months. By the end of the year ₦1,000 will grow to: ₦1000 x $(1.04)^4$ = ₦1,169.86 We see here that 16 per cent compounded quarterly is equivalent to 16.6986% compounded annually. To carry it a little bit further, if 16 per cent is compounded monthly, then ₦1.00 will grow to ₦1 x $(1+0.16/12)^{12}$ = ₦1.17227 at the end of one year. Thus 16 per cent compounded monthly is equivalent to 17.227 per cent compounded annually.

We will now generalize this. You will have noticed that given a nominal rate such as the 16% in the example above, the more often interest is compounded or converted into principal, the higher the effective rate of interest paid. If a nominal rate, r is compounded m times per year, then the effective rate (Eff. Rate) is:

$$\text{Eff. rate} = \left(1+\frac{r}{m}\right)^m - 1$$

So when a nominal rate of 10 per cent is compounded semiannually the effective rate is $\left(1+\frac{0.10}{2}\right)^2-1$ or 10.25 per cent. If nominal rate is 10 per cent and compounding is quarterly, the effective rate will be $\left(1+\frac{0.10}{4}\right)^4-1$ or 10.38 per cent.

2.3.1 Continuous Compounding:

There are some situations both in practice and theoretically when compounding is done weekly, daily or continuously, that is, instantaneously. Continuous compounding is the limit to which we can carry our frequency of compounding period. We will now discuss how this works.

It was derived in equation 2-2 that the compound value at the end of any period, n is:

$$P_n = P_o(1+r)^n \quad \ldots. \quad \ldots\ldots \quad \ldots\ldots. \quad (2.2)$$

$$P_n = P_o(1+\tfrac{r}{m})^{mn} \quad \ldots. \quad \ldots\ldots \quad \ldots\ldots. \quad (2.2a)$$

When interest on deposit is paid more than once in a year.

If we have the following assumptions

(a) P_0 = ₦1.00 (b) n = 1 year and (c) m = frequency of compounding.

We will change equation (2-2) to take assumptions (a) to (c) here into consideration. So we have

$$P_1 = ₦\,1\,(1+\frac{r}{m})^{mn} \quad \ldots\ldots. \quad \ldots\ldots.. \quad \ldots\ldots\ldots \quad (2.3)$$

Let us further assume that r = 1 per cent.

As m, increases, we compound more often over shorter time intervals. This causes the right hand side (RHS) of equation (2.3) to increase. At the limit when m approaches infinity (and compounding is instantaneous or continuous), the RHS of equation (2.3) approaches the value 2.71828… This value is mathematically denoted as e; where e is the base of the natural logarithm.

$$e = \lim_{m \to \infty} \left(1 + \frac{1}{m}\right)^m = 2.71828.. \qquad (2.4)$$

To extend the coverage of equation 2.4 for n periods (say 2 or more years) and for interest rates above 1 per cent, we simply raise e to power r x n (that is e^{rn}) for whatever value of interest rates and compounding periods are involved, respectively. Remember that what we derived in equations (2.3) and (2.4) is for ₦1.00 investment. You will need to multiply by the total amount invested, that is

$P_n = P_0 e^{rn}$ when principal or initial value is greater than ₦1.00.

So if interest is compounded continuously at nominal rate r per cent per year, then effective annual rate is $e^r - 1$. For example, 16 per cent per year compounded continuously is equivalent to 17.351 per cent compounded annually.

You should however not confuse the compounding period with the payment period. Tables or calculators are normally used in practice to derive an equivalent effective rate to apply at the end of a month or quarter.

In practice, to solve problems that require computation of interest quoted at a nominal rate of r per cent per year compounded m times per year for n years, merely use the tables for rate r/m and m x n periods. For example 12 per cent compounded quarterly for 5 years is equivalent to the rate found in the compound value tables for r = 12/4 = 3 per cent for m x n periods = 4 x 5 = 20 periods. Besides, calculators can easily be used for these problems once you understand the basic principles and the formulae involved. The idea of the compounding period not necessarily being a year is all pervading to our subject matter of "Mathematics of Finance". In other words you should note very well that the issue of more than one compounding period in a year does not apply only to compound (future) value problems. It is also applicable to present value, annuity, sinking funds, amortization and other related topics.

2.3.2 Sample problems on Nominal and Effective Interest Rates

1. What effective annual interest rate correspond to (a) 9 per cent compounded monthly (b) 17 per cent compounded quarterly.

Solution:

(a) The expression discussed in the text is used for this problem.

r = 9% n = 1 m = 12

$(1 + .09/12)^{12} – 1$ gives us the effective annual interest rate. This is $(1.0075)^{12} – 1 = 0.938$ or 9.38 per cent

(b) The effective annual interest rate when 17 per cent is compounded quarterly is calculated as follows:

r = 0.17 m = 4 n = 1. Effective rate is $(1 + 0.17/4)^4 – 1 = (1.0425)^4 –1 = 0.18115$ or 18.115 per cent.

2. If an annual effective interest rate of 12 per cent is desired, what nominal rate should be asked if compounding is to be semiannually?

Solution:

This problem is a reverse of number 1 above. The solution could be got from first principles to derive r which is unknown in the following equation:

$$\left(1+\frac{r}{2}\right)^2 -1 = 0.12$$

By expansion of the term in bracket we have

$$1+r+\left(\frac{r}{2}\right)^2 -1 = 0.12$$

$r^2 + 4r - 0.48 = 0$

By solving the last equation being a quadratic equation (remember the high school almighty formula in algebra!), we have r = -4.1166 or 0.1166. Since interest rates cannot be negative we know the correct answer is 0.1166 or 11.66 per cent. So to get an effective yield of 12 per cent per annum, 11.66 percent compounded semiannually should be demanded.

3. Compute the compound amount when ₦5,000 is invested at 10 per cent compounded continuously for 4 years.

Solution:

What we need do is to simply apply the formula we discussed on continuous compounding using the relevant natural logarithm table or a calculator. So we have:

P_o = ₦5000 n = 4 years r = 10% or 0.10 P_n = ?

$P_n = P_o \; e^{rn}$

P_4 = ₦5,000 $e^{0.10x4}$

= ₦5,000 (1.49182) = ₦7,459.12

- What effective rate of interest correspond to 13% p. a. compounded quarterly?
- (ii) Compute the compound amount when ₦7200 is invested at 7.5% compounded continuously for 6 years
- (iii) What is the effective rate of interest for the transaction in (ii)?
    - (i) 13.65% (ii) ₦11,291.85 (iii) 7.89%

2.4 Compound (Future) Value of Ordinary Annuity (Annuity in Arrears)

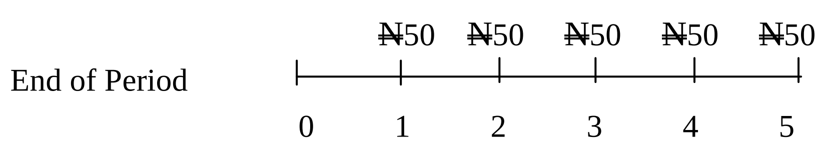
An annuity can be defined as a series of equal payments made at the beginning or end of equal periods of time. Some examples of annuities include mortgage payments, installment contracts, monthly rental payments; semi-annual corporate bond coupon (or interest) payments, and annual or monthly payments to a retired employee under a pension plan. We will now discuss a few important concepts on annuity.

An annuity whose payments occur at the end of each period is called an ordinary annuity or annuity in arrears or simple annuity. Corporate bond coupon payments are usually made in arrears, that is, the first payment does not occur until the bond has been outstanding for six months.

An annuity whose payments occur at the beginning of each period is called an annuity due or an annuity in advance. A series of rental payments can then be termed an annuity due. When the first payment of an annuity is at some time later than the end of the first period it is called deferred annuity. Annuities can be paid for ever. These are called perpetuities.

We will now get into the discussion on how to tackle problems of compound (future) value of an ordinary annuity. Our means of presentation of the subject matter will be two-fold to aid your understanding. We will discuss the use of time lines and later show the derivation of the compound value interest factor of an annuity.

A time line marks the end of each period, numbers the period, and it shows the payments to be received or paid and shows the time at which the annuity is valued. An example is shown in the diagram below.



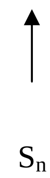


Figure 2-2: Time line for a 5 period ordinary annuity.

What we have in figure 2-2 represents an ordinary annuity (annuity in arrears) for 5 periods of ₦50 to be valued at the end of period 5. The end of period 0 is now.

Suppose you are promised ₦100,000 a year for four years by your employers under their staff training program to partly fund your Bachelor’s program. This is a four-year ordinary annuity. Let us further assume that you have no need for the money until some years hence and you deposit each annual payment in a commercial bank savings account paying interest at the rate of 10 per cent per year. How much would you have at the end of the fourth year? A time line is used to give answer to the problem as shown in figure 2-2 below.

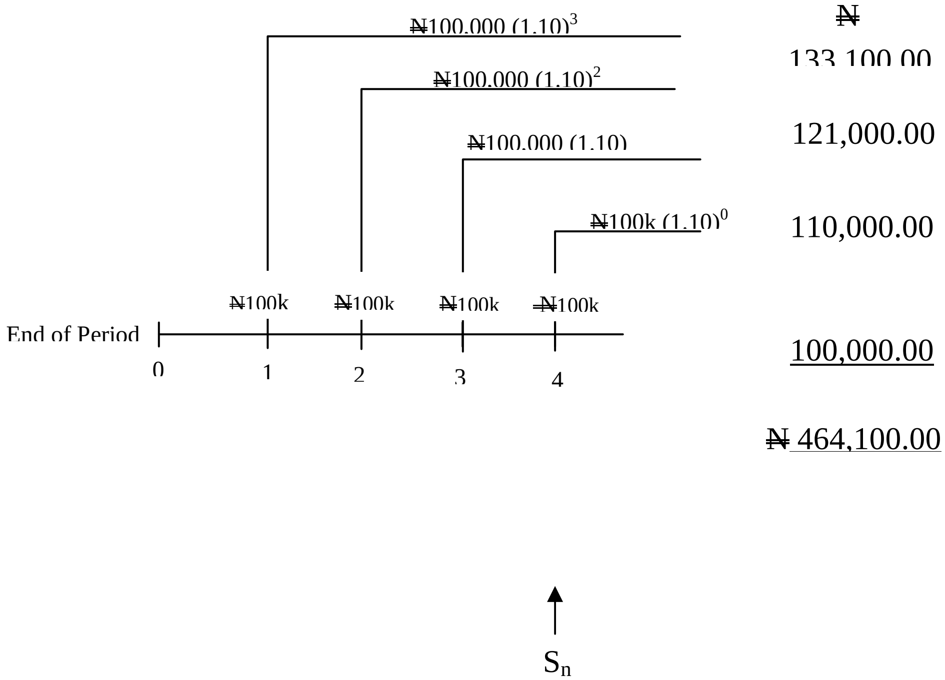


Figure 2-3 Time line for ₦100,000 4-year ordinary annuity at 10 per cent per year (compound or future value)

The first ₦100,000 received at the end of the first period earns interest for three years, so it is worth ₦100,000 $(1.10)^3$ = ₦133,100 at the end of the fourth year. The ₦100,000 received at the end of the second year grows to ₦100,000 $(1.10)^2$ = ₦121,000.00. It earns interest for two years. The ₦100,000 received at the end of the third year grows to ₦110,000 since it earns only 1 year interest. The ₦100,000 received at the end of the fourth year is of course worth ₦100,000 at the end of year 4. The entire annuity is worth ₦464,100.00, when we add up what each installment grow to at the end of the fourth year.

Let us now translate this to a mathematical expression so you can see how to derive the compound value interest factor of an annuity. We will define S_n as the compound sum; R, as the periodic receipt and n as the length of an annuity. The compound value of an annuity is derived as follows:

$S_n = R\,(1+r)^{n-1} + R\,(1+r)^{n-2} + \ldots + R\,(1+r) + R\,(i+r)^0.$

We can rewrite the right hand side (RHS) of the equation starting with last addendum to the first, and designate it as equation (1) for now.

$S_n = R\ (1+r)^0 + R\ (1+r) + \ldots + R\ (1+r)^{n-1}$ …………………….. (1)

Multiply both sides by (1+r)

$S_n\ (1+r) = R\ (1+r)^1 + R\ (1+r)^2 + \ldots + R\ (1+r)^n$ …………….. (2)

Subtract equation (1) from (2)

$S_n\ (1+r) - S_n = R\ (1+r)^n - R\ (1+1)^0$

Solve for S_n

$S_n + S_{nr} - S_n = R[(1+r)^n - 1\ ]$

$$S_n = \frac{R\left[(1+r)^n - 1\right]}{r} \qquad \ldots\ldots\ldots \qquad \ldots\ldots\ldots\ldots \qquad (2.5)$$

Equation 2.5 gives us the formula for the compound value interest factor of an annuity. There are tables constructed for this for various combinations of the values of n and i. We can tackle a lot of problems with the use of the tables and time line.

Applying equation (2-5) to the 4-year ordinary annuity problem we discussed earlier gives us the same answer that we derived, howbeit using a shorter method.

R = ₦100,000 r = 10% or 0.10

n = 4 S_n = ?

$$S_n = \text{N}50{,}000\left[\frac{(1.10)^4 - 1}{0.10}\right]$$

= ₦100,000 x 4.641

= ₦464,100

2.4.1 Sample Problems on Compound (Future) Value of Ordinary Annuity

1. What is the future value of ₦1 invested per year compounded at 12 per cent for 33 years if the first payment is made one year from now (an ordinary annuity for 33 years)

Solution:

We can use a time line to represent the problem to start with as in Figure 2.4 below:

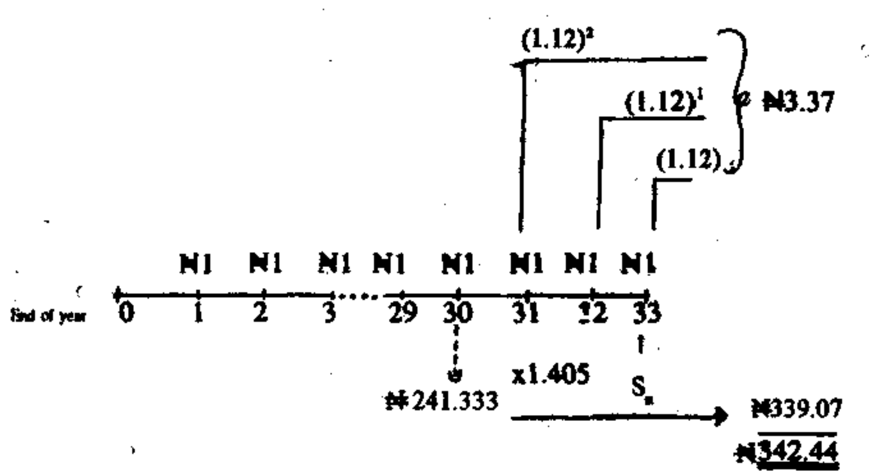


Figure 2- 4: Time Line for ₦1; 33 years ordinary annuity compounded at 12% per year.

Our tables do not show future values of annuities for 33 years. We consequently need to do some interpolations. At the end of year 30 the annuity will grow to ₦241.333 x 1.405 =

₦339.0729. In addition as of the end of year 30, the payments from the ends of years 31, 32, 33 are an ordinary annuity of 3 years with future value of ₦3.374. So the entire annuity has future value of ₦339.0729 + ₦3.374 = ₦342.45.

A quick alternative is to use your calculator by putting in the relevant values of i and n (12 per cent and 33 years respectively) into the compound value interest factor of an annuity formula where r = 0. 12; n = 33

$$S_n = \frac{R\left[(1+r)^n - 1\right]}{r}$$

$$S_n = \frac{\left[(1.12)^{33} - 1\right]}{0.12}$$

S_{33} = ₦342.43

The little difference between this and what we get by interpolation from the table is because of rounding in the latter situation.

2. How much money will you have to deposit every 6 months into an account paying 14 per cent compounded semi-annually in order to accumulate ₦1.5 million in 5 years?

Solution:

You can either use a table or your calculator to arrive at the compound value interest factor of an ordinary annuity that is involved. This is then divided into the compound sum to arrive at the amount of equal payment involved.

$$S_n = \frac{\left[(1+\frac{r}{m})^{mn} - 1\right]}{r/m}$$

$$R = \frac{S_n}{\frac{\left[(1+\frac{r}{m})^{mn} - 1\right]}{r/m}}$$

Here S_n = ₦1,500,000; r= 0.14; n=5; m=2; $m \times n = 2x5 = 10$; $\frac{r}{m} = \frac{0.14}{2} = 0.07$

$$R = \frac{₦1{,}500{,}000}{\left[\frac{(1.07)^{10} - 1}{0.07}\right]}$$

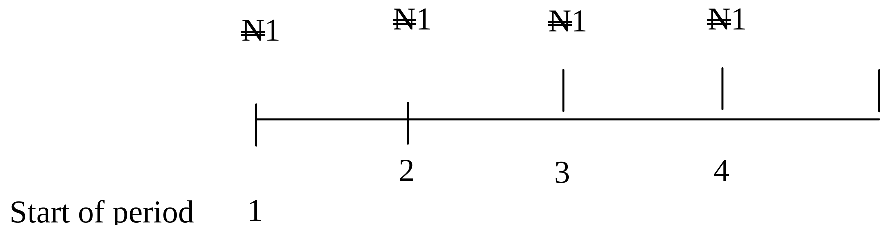
$$= \frac{₦1{,}500{,}000}{13.8164} = ₦108{,}566.25$$

- ABC Ltd has a ₦2.5 million bond issue outstanding. Assuming it can earn 12.5% p.a. on its investment, how much would it need to invest each year in order to accumulate enough money to retire the bond at the end of 10 years?
  - ₦139,054.45

2.5 Compound (Future) Value of an Annuity Due

Annuities due or annuities in advance relates to situations where the payments are made at the beginning of each period. In a fixed number of periods, an annuity due has one more interest earning period than an ordinary annuity but no installment payment is made at the end of the last of those periods. Take a look at the diagrams below (figure 2.5) showing the time line for the future value of an annuity in advance for 4 periods with the time axis relabeled to show start of period and the time line for the future value of an ordinary annuity (in arrears) for 4 periods as solid lines represents the interest earning periods.

Annuity in Advance



(b) Annuity in Arrears

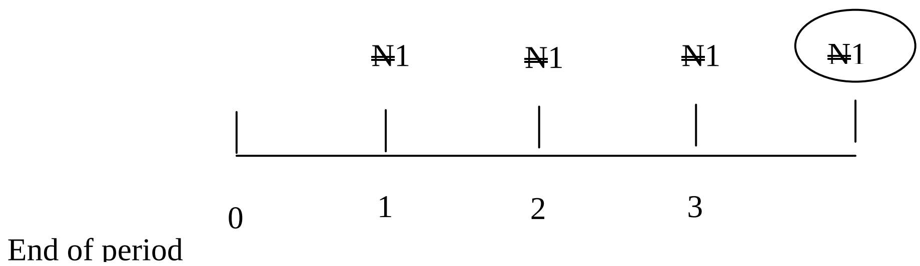


Figure 2- 5 Comparison of Annuity in Advance with ordinary Annuity

We see from these diagrams that a ₦1.00 annuity in advance for n periods has future value equal to the future value of a ₦1.00 annuity in arrears for n+1periods minus ₦1.00 that must be subtracted to calculate the future value of an annuity in advance. So to convert a table of future (compound) value of ordinary annuity to values of an annuity in advance, take one more period and subtract 1.00.

This also applies to the formula for the compound value interest factor of an annuity in advance. We can derive the formula for the compound sum of an annuity in advance from first principles using familiar symbols.

$$S_{ndue} = R(1+r)^{n} + R(1+r)^{n-1} + \ldots. + R(1+r)$$

We can rewrite the RHS of the equation starting with the last addendum to the first, and designate it as equation (1) for now.

$$S_{ndue} = R(1+r) + R(1+r)^2 + \ldots + R(1+r)^n \quad .. \quad .. \quad \ldots \quad (1)$$

Multiply each side by (1+r)

$$S_{ndue}(1+r) = R(1+r)^2 + R(1+r)^3 + \ldots + R(1+r)^{n+1} \quad \ldots \quad \ldots \quad (2)$$

Subtract equation (1) from (2)

$$S_{ndue}(1+r) - S_{ndue} = R(1+r)^{n+1} - R(1+r)$$

Solve for S_{ndue}

$$S_{ndue} + S_{ndue}\, r - S_{ndue} = R[(1+r)^{n+1} - (1+r)]$$

$$S_{ndue} = \frac{R[(1+r)^{n+1} - (1+r)]}{r} \quad \ldots\ldots \quad \ldots\ldots.. \quad \ldots\ldots \quad (2.7)$$

2.5.1 Sample Problems Involving Annuities Due (Compound or Future Value)

In the illustration given previously on your employer promising ₦100,000 a year for four years under a staff training programme. We will now assume that each of the instatement is paid at the beginning of each year for four years, we will further assume that the money is deposited in a savings account that pays 10 per cent per annum. How much will be in the account at the end of the year?

Solution: there are a number of approaches to this problem. We can use a time line and then compound each installment to the end of year 4 at the interest rate given and thereafter sum up to arrive at the compound sum of the annuity due. Try out this method. Secondly, you can use table 3 in the interest factor tables (see appendix) under 10% column and 5-period (n+ 1) row. We then subtract 1.00 from the factor written there, and use the result to multiply the regular payment into the annuity. Try this also on your own, and compare your answer for first and second approach to the third approach discussed below.

A second approach is to use the compound sum formula for an annuity due, equation 2.7 to arrive at the interest factor; multiply this by the regular payment and you will arrive at the correct answer. This is as follows:

n = 5   r = 0.12   R = ₦1,000

$$S_{ndue} = ₦1{,}000\left[\frac{(1.12)^6 - 1.12}{0.12}\right]$$

= ₦1,000 x 7.1152

= ₦7,115.19

A simple use of your calculator will arrive at the answer as we have it here.

- Ascertain how much will be in the account of Mr. Obi at the end of two (2) years if he deposits ₦20,000 at the beginning of every month during the two-year period and interest is paid at the rate of 9% compounded monthly?

o ₦527,697.68

Summary of Study Session 2

The basic formulae for various interest factors that were discussed in this study session are recapitulated here to aid your learning.

1. The basic compound value interest factor equation.

$P_n = P_0\,(1+r)^n$ ………………… (2-2)

Where P_n = the compound value amount at the end of n years

P_o = beginning principal or initial amount

r = compound interest rate.

n = number of years in the future

2(a) If a nominal interest rate, r, is compounded m times per year, then the effective interest rate is:

$$\text{Effective rate} = \left(1.0 + \frac{r}{m}\right)^m - 1.0$$

(b) For continuous compounding

$P_n = P_0 e^{rn}$ where e = 2.71828, the base of the natural logarithm.

Eff. Rate = $e^r - 1.0$

3(a) The compound (future) value interest factor of an ordinary annuity was derived to be

$$S_n = R\left[\frac{(1+r)^n - 1.0}{r}\right] \text{..........} \qquad \text{..................} \qquad (2.5)$$

Where S_n = Compound Sum of the annuity up to a period.

R = Periodic receipt from the annuity. Note: values shown in tables are for cases where R = ₦1.00

(b) The compound (future) value of interest factor of an annuity in advance is

$$S_{ndue} = R\left[\frac{(1+r)^{n+1} - (1+r)}{r}\right] \text{......} \qquad \text{............} \qquad (2.6)$$

Self-Assessment Questions (SAQs) for Study Session 2

Having completed this study session, you can assess how well you have achieved its Learning Outcomes by answering the following questions. You can check your answers at the end of the module.

SAQ 2.1 (tests Learning Outcome 2.1)

Why is it important to consider the time value of money concept in financial decisions?

SAQ 2.2 (tests Learning Outcomes 2.2 and 2.5)

Mr. Ade deposited ₦10,000 into a savings account at an interest rate of 9% compounded quarterly for 15 years. How much will Ade have in the account at the end of the period?

SAQ2.3 (tests Learning Outcome 2.3)

Penguin Dry Cleaning Company Ltd intends to buy a machine worth ₦1.5 million for the Company's operations. The Company has ₦0.5 million in an account paying 15% interest rate p.a. How long will the Company wait before being able to buy the equipment?

SAQ 2.4 (tests Learning Outcome 2.4)

Mrs. Nnaji, a petty trader, borrowed ₦10,000 from a Micro Finance Bank and agreed to pay ₦17,620 in 5 years time. What is the annual compound rate of interest on this loan?

SAQ 2.5 (tests Learning Outcome 2.5)

You are asked to examine the savings plan of three banks. Bank A offers interest at 12% compounded annually. Bank B offers 12% compounded semi-annually. Bank C offers 10%

compounded monthly. If a prospective customer intends to save ₦10,000 for 3 years which bank would you recommend?

SAQ 2.6 (tests Learning Outcome 2.5 and 2.6)

Caleb and Co. Ltd sets aside the sum of ₦1 million at the end of each year to create a fund for its future expansion. If the fund earns 8% compounded semi-annually, how much does it amount to at the end of 5 years?

SAQ 2.7(tests Learning Outcome 2.7)

Rose Bernard has been depositing ₦20,000 in her savings account every December starting in year 2001. Her account earns 6.5 % compounded annually. How much will she have in the account in December of 2010, including her 2010 deposit?

Should you require more explanation on this study session, please do not hesitate to contact your e-tutor via the LMS.

Are you in need of General Help as regards your studies? Do not hesitate to contact the DLI IAG Center by e-mail or phone on:

iag@dli.unilag.edu.ng
08033366677

Study Session 3: Discounting Methods

Introduction

The concept of time value of money and how it relates to future value of lump sum payments and annuities was the focus of our discussions in study session 2. This same concept and its treatment in determining the present value of lump sum and series of payments is the subject of this study session. We defined two special types of series of payments (deferred annuities and perpetuities) that occur in practice in study session 2. It is in this study session that we discuss how to analyze and solve problems on these types of series of payments that have interesting practical applications.

All you need to master the stuff in this study session is a good understanding of our discussions in study session 2. There (study session 2) we were looking at what money is worth at a particular point in time in the future. Here (study session 3) our focus is on what the money that will be received at different times in the future is worth at present (today).

Learning outcomes for Study Session 3

When you have studied this session you should be able to:

3.1Calculate the present value of a lump sum payment to be received at a particular point in the future.

3.2 Solve problems involving determination of the present value of an ordinary annuity (annuity in arrears).

3.3 Solve problems involving determination of present value of an annuity due (annuity in advance)

3.4 Compute the present value of a deferred annuity.

3.5 Solve problems on perpetuities (annuities that are paid for ever).

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3.1 DISCOUNTING METHODS – (PRESENT VALUE) OF LUMP SUM PAYMENTS

In study session 2 we discussed the techniques for calculating the compound (future) value, P_n of a sum of money P_0 deposited today. You will recall that we know P_o and we calculated P_n. In this study session we will deal with the problem of computing how much P_0 has to be deposited or invested today to have a specified amount P_n, n periods in the future. If we are offered the option of receiving a given sum in the future, and we know the appropriate interest rate, we can calculate its value today.

You can briefly flip back to equation (2.2) now to refresh your memory. If we reverse equation 2.2 making P_0 the subject by dividing both sides of the equation by $(1+r)^n$, we have:

$$P_o = \frac{P_n}{(1+r)^n} = P_n\left[\frac{1}{(1+r)^n}\right] = P_n(1+r)^{-n} \quad \text{……..} \qquad (3.1)$$

The symbols P_o, P_n, r and n have the same definitions we gave previously.
Now let us take a look at a practical example. Assuming you are offered an opportunity of ₦1.00 a year from now, or a certain sum as equivalent today. If interest rate is 10 per cent, what amount of money would make you indifferent between what is offered today and ₦1.00 in a year's time?

Equation 3.1 is useful here

$$P_o = \frac{₦1.00}{(1.10)^1} = ₦0.90909$$

In the same token, the value of ₦1.00 to be received 2 years from now if interest rate is 10% is:

$$P_o = \frac{₦1.00}{(1.10)^2} = ₦0.82645$$

In the previous study session we dealt with compounding of cash flows forward in time. What we are doing here is discounting back in time. When we calculate present value, the interest rate involved is usually referred to as the discount rate. The number $1/(1+r)^n$ or $(1+r)^{-n}$ is the present value of ₦1.00 received after n periods when interest is earned at r per cent per period. Thus $1/(1+r)^n$ is the discount factor for n periods. You will have noticed by now that a discount factor is merely a reciprocal or inverse of a number, $(1+r)^n$. Hence the table of compound (future) value of ₦1.00 (Table 1 in the Appendix) can be used to solve present value problems. This is done by deriving the reciprocal(s) of the relevant figure(s). However, due to the relative importance of the discount factor making for its frequent use, tables are also constructed for it, for various combinations of the values of r and n. An example is Table II in our Interest Factor Tables (See Appendix).
A look at the 10% column in rows 1 and 2 in Table II (Appendix) will show the same two factors we arrived at above for ₦1.00 discounted at 10% for 1 year and 2 years respectively. We will now explain the solutions to a few sample problems on discounting.

3.1.1 Sample Problems on Discounting

1. You are promised ₦100,000 fifteen years from today. What is that promise worth today if interest (discount) rate is 20 per cent?

Solution: The table of future values can be used, in simply dividing the amount promised by the compound value interest factor at 20% for fifteen years. In the alternative, we could multiply the future value promised by the present value factor in the table for 15 years at 20%. Calculators can also be used to derive any of these values by simply applying the formula.

In this problem, P_n = ₦100,000 r = 20% or 0.20; n = 15 P_o = ?

$P_o = \underline{P_n\quad}$ = $P_0(1+r)^{-n}$

$(1+r)^n$

$$P_0 = \frac{₦100,000}{(1.20)^{15}} = ₦100,000(1.20)^{-15}$$

$$P_0 = \frac{₦100,000}{15.40702} = ₦100,000(0.06491)$$

P_o = ₦6490.55

Note: The problem has been tackled in such a way as to make you see that we could either apply the compound value factor in an indirect way or the present value factor directly.

2. What is the present value of ₦1million to be received 37 years hence when the discount rate is 18 per cent per annum?

Solution: Note that the tables do not show values for 37 periods. If you still want to use the table you will note that:

$P_o = P_{37}(1.18)^{-37} = P_{37}(1.18)^{-25}(1.18)^{-12}$

Here, we calculate the present value in two steps. We first find the present value of ₦1.00 received 25 years from now at 18 per cent which is ₦0.01596. Then we let ₦0.01596 be the amount to be received 12 years later. The present value of ₦1 received 12 years later at 18 per cent is 0.13722. Therefore the present value of ₦1 to be received 37 years hence is ₦0.01596 x 0.13722 = ₦0.0219.

However a simple use of calculator to arrive at the present value of ₦1.00 at 18 per cent 37 years from now gives a quick and correct answer. That is $(1.18)^{-37}$ = ₦0.00219. so ₦1million received 37 years from now at 18 per cent discount rate is worth:

₦1,000,000 (0.00219) = ₦2189.60

- An outstanding liability of ₦2 million owed by a firm is scheduled to be paid 18 years from now. What is the present value of this liability if a factor discounts it at 20% interest rate?
  - ₦75,122.07

3.0 PICTORIAL VIEW OF THE DISCOUNTING PROCESS

The problems we solved above would have given you some insight into the way interest rates and time duration affect present value. A pictorial view of this for 0%, 8% and 15% is shown in figure 3.1 below. You will observe here that the present value of a sum to be received at some future date decreases (a) as the payment date is extended further into the future and (b) as the discount (interest) rate increases. When relatively high interest rates are involved, payments due in the future are worth very little today. On the other hand, when the discount rates are low and due date of payment is far into the future, present values of funds involved will also be small. We saw in the problem solved above, for example, that the present value of ₦1, million to be received 37 years from now at 18 percent discount rate is worth just ₦2189.60 This is just 0.22 per cent of its future value. Note that values from table II were used to construct figure 3.1.

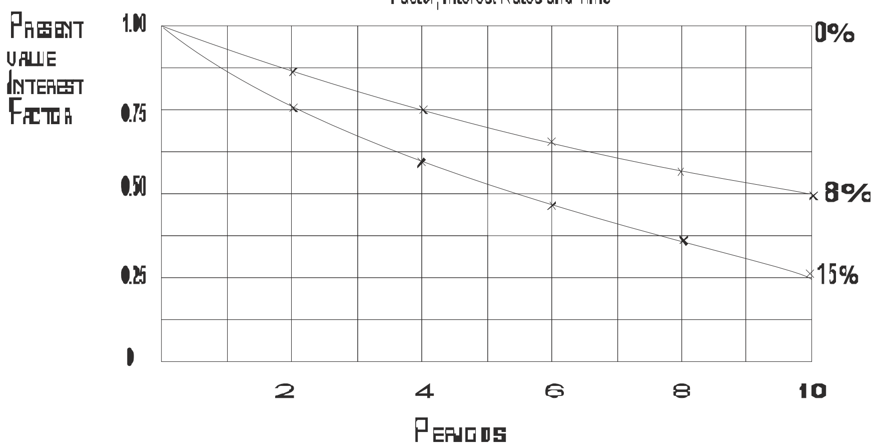


Figure 3-1 The Relationship Between Present value Interest Factor, Interest Rates and Time

3.3 PRESENT VALUE OF ORDINARY ANNUITY (IN ARREARS)

In study session 2 we defined an annuity as a series or stream of equal payments to be received at different specified time periods of equal duration in the future. Suppose you were offered an alternative:

A four year annuity of ₦100,000 a year or a lump sum payment today. Let us assume that you have no need for the money during the next four years, and when you receive it in a series you will deposit it in a savings account. How large should the lump sum be for you to be indifferent between the lump sum amount and the series of payment at 10 percent interest rate?

Before solving the problem, you should note that we can look at this situation from another point of view. This will define the concept of present value of an annuity from a different perspective. From one view point, the present value of an annuity can be defined as the current amount that, when invested at a given interest rate, will yield a specified series of equal periodic payments. Alternately, the present value of an annuity can also be defined as the current amount that could be deposited today at a given rate of interest so that equal periodic withdrawals could be made. We are saying virtually the same thing here in two slightly different ways; that will make you have a clear view of the type of problems we will try to solve when we talk of present value of an annuity.

We will now use a time line to solve the problem mentioned above, using the present value interest factor table (Table II) to start with (Remember that we introduced the use of time line in study session 2). Figure 3.2 below gives a pictorial view of the solution to the annuity problem just mentioned.

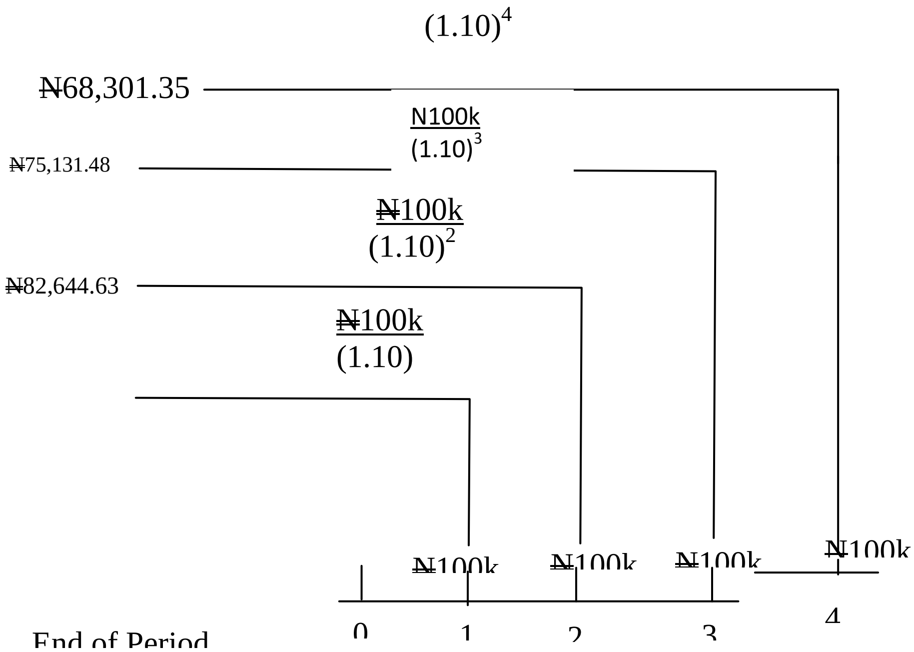


Figure 3.2: Pictorial Illustration of Present Value of an Ordinary Annuity

The simple thing we did in figure 3.2 above was to look at the present value interest factor table of ₦1.00 at 10% and then use the factors for 1, 2, 3 and 4 years respectively to multiply each of the streams of payments and thereafter add up the discounted values to arrive at ₦316,986.54 We have tables displaying the present value interest factor of an annuity of ₦1.00 that we can use alternately. An example is table VI in the interest factor tables (See Appendix). Looking at the 10 per cent column and at the row for 4 years, we can simply get the factor needed which is 3.1699 When we multiply this by ₦100,000 we arrive at the same answer (except for rounding error) that we get using four values from Table II.

You will realize that the present value of the first payment (or receipt) is R[1/(1+r)] the second is $R[1/(1+r)^2]$ the nth is $R[1/(1+r)^n]$ and so on with R defined as the regular receipts or payments from an annuity. If we define the present value of an annuity as A_n, we could generalize what we have been discussing here as follows:

$$A_n = \frac{R}{(1+r)} + \frac{R}{(1+r)^2} + \frac{R}{(1+r)^3} + \ldots + \frac{R}{(1+r)^n} \ldots\ldots\ldots\ldots(a)$$

If we multiply both sides of equation (a) by 1/(1+i), we will have

$$\frac{A_n}{(1+r)} = \frac{R}{(1+r)^2} + \frac{R}{(1+r)^3} + \ldots + \frac{R}{(1+r)^{n+1}} \ldots\ldots\ldots(b)$$

Now subtract (b) from (a) to get (c)

$$A_n - \frac{A_n}{(1+r)} = \frac{R}{(1+r)} - \frac{R}{(1+r)^{n+1}} \text{.....} \qquad \text{(c)}$$

You will realize that after the subtraction only the initial term of the first series and the last term of the second series remain. When we multiply both sides of this expression by (1+i) we have

$$A_n(1+r) - A_n = R - \frac{R}{(1+r)^n} \qquad \text{....(d)}$$

We can now simplify this to obtain

$$A_n + A_{ni} - A_n = R\left[1 - \frac{1}{(1+r)^n}\right] \quad \text{..........} \qquad \text{...} \quad \text{(e)}$$

We can rewrite this as:

$A_n r = R\ [\ 1 - (1+r)^{-n}]$ ………… ………… (f)

By dividing both sides by r we have

$$A_n = \frac{R\left[1-(1+r)^{-n}\right]}{r} \quad \text{...........} \qquad \text{..........} \qquad \text{....} \quad (3.2)$$

We have in equation 3.2 the formula for the present value interest factor of an ordinary annuity. When we assume that R = 1.0, we can find values of A_n for various combinations of the values of r and n. This is how Table VI that we referred to previously was constructed.

Let us now discuss some typical ordinary annuity present value problems that you may encounter.

3.3.1 Sample Problems on Ordinary Annuity

1. How much must be deposited now in a bank account paying 12 per cent compounded monthly to provide for 4-year monthly withdrawals of ₦7,500 each, scheduled to start in a month's time?

Solution: This is a problem in finding the present value when interest rate and periodic payment are known. The interest rate involved here and the number of periods may not be given in an annuity table. So, we need to simply use the formula we derived to calculate it.

$$\text{In this problem } A_n = R\left[\frac{1-(1+r/m)^{-mn}}{r/m}\right] \qquad \text{.......} \qquad \text{...} \quad \text{(3.2a)}$$

R = ₦7,500

$$\frac{r}{m} = \frac{0.12}{12} = 0.01;\ n = 4,\ m = 12;\ x\ \text{mxn}\ x\,12 = 48 \text{ periods}$$

Using equation (3.2a) we have

$$A_n = \frac{₦\ 7500\ [1-(1+0.01)^{-48}\ ]}{0.01}$$

A_n = ₦7500 x 37.97396 = ₦ 284,804.70

2. Tiwa systems Nigeria Ltd has been granted a loan of ₦5million by Courage Micro Finance Bank Ltd to purchase some sets of computer hardware system for his firm. What monthly payment will the company make to liquidate the loan in 5 years time at 15% interest rate compounded monthly?

Solution: This problem involves calculating the periodic payment when the present value of the annuity is known. We can do this by solving equation (3.2a) for R. We simply divide both sides of the equation by the present value interest factor of an annuity formula, for ₦1.00; which is

$$\frac{1-(1+r/m)^{-mn}}{r/m} \text{ to give us}$$

$$R = \frac{A_n}{\left(\frac{1-(1+r/m)^{-mn}}{r/m}\right)} = A_n \left(\frac{r/m}{1-(1+r/m)^{-mn}}\right)$$

For the problem here; A_n = ₦5,000,000

n = 5; m = 12 m x n = 60; $\frac{r}{m} = \frac{0.15}{12} = 0.0125$

$$R = \frac{₦\,5000{,}000}{\left(\frac{1-(1.0125)^{-60}}{0.0125}\right)} = ₦5000{,}000\left(\frac{0.0125}{1-(1.0125)^{-60}}\right)$$

$$R = \frac{₦\,5000{,}000}{42.03459} = ₦118{,}949.65 \text{ or } ₦500{,}000 \times 0.02379 = ₦118{,}949.65$$

3. Mazi Amakwe owns a parcel of land at Hometown which he can sell for ₦200,000 to be given as down payment with quarterly payments of ₦50,000 for the next 10 years. An Estate Agent offers him ₦1million cash for the property. If money can be invested at 18 per cent compounded quarterly, which is the better deal?

Solution: This problem involves finding the equivalent cash price; a situation of comparing two different sums of money that are available at two different periods of time. What we need do is to find the present value of the two different offers and choose the one with the greater present value.

For the first offer (annuity part of it) r = 0.18 n = 10 m = 4 R = ₦50,000.

$$\frac{r}{m} = \frac{0.18}{4} = 0.045 \text{ m x n} = 40$$

$$A_n = ₦\,50{,}000\left[\frac{1-(1.045)^{-40}}{0.045}\right]$$

= ₦50,000 x 18.40158

= ₦920,079.22

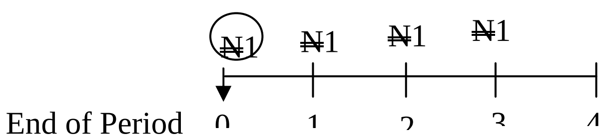
Thus the total present value of the first offer is ₦200,000 plus ₦920,079.22. This is ₦1,120,079.22. This is more than ₦1million. So the first offer is better.

- Adisco Tech network Ltd offers its internet service router for ₦10,000 down payment and charges N6,000 per month for the next 18 months. If interest rate is charged at 15% compounded monthly, find the equivalent cash value to a potential subscriber (to the nearest naira).
    - ₦106,177

3.4 PRESENT VALUE OF ANNUITY IN ADVANCE (ANNUITY DUE)

You will recall that we discussed the peculiarities of annuities due in study session 2. We will now apply the present value concept to annuities due in this section. Using a time line in the figure 3.3 below, we show the profile of the present value of an annuity in advance for four periods.

Figure 3.3 Time line for an annuity in advance for 4 periods.



Notice that except for the first circled payment, the present value time line for annuity due for 4 periods looks just like the time line for the present value of an ordinary annuity for 3 periods. We can conclude from this that a ₦1 annuity in advance for n periods has present value equal to the present value of ₦1 annuity in arrears for n – 1 periods plus ₦1 (which is the ₦1 circled above that needs no discounting since it is available right at the beginning of the series of payments). So to convert a table of present value of an ordinary annuity to one of present value of annuity due (annuity in advance), using a present value interest factor table for an ordinary annuity, take the value for n – 1 periods under the appropriate interest rate. Add 1.0 to that value. The result is the interest factor for the present value of that particular annuity due.

From first principles and using symbols already familiar to you, present value of an annuity due can be arrived at as follows:

$$A_{ndue} = R + \frac{R}{(1+r)} + \frac{R}{(1+r)^2} + \ldots\ldots + \frac{R}{(1+r)^{n-1}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

Multiply both sides by 1/(1+r)

$$\frac{A_{ndue}}{(1+r)} = \frac{R}{(1+r)} + \frac{R}{(1+r)^2} + ... + \frac{R}{(1+r)^n} \qquad …….. \qquad …… \quad (2)$$

Subtract equation (2) from (1)

$$A_{ndue} - \frac{A_{ndue}}{(1+r)} = R - \frac{R}{(1+r)^n} \qquad …..$$

Multiply both sides by (1+r)

$$A_{ndue}(1+r) - A_{ndue} = R(1+r) - \frac{R}{(1+r)^{n-1}}$$

$A_n + A_{ni} – A_n = R[(1+ r) – (1+r)^{-(n-1)}]$

$$A_{ndue} = R\left[\frac{(1+r)-(1+r)^{-(n-1)}}{r}\right] \; ….. \qquad …… \quad (3.3)$$

3.4.1 Sample Problems on Present Value of Annuity in Advance (Annuity Due)

1. Rent payable on a bed space at Kowope hostels for undergraduate students is ₦1200 per month payable at the beginning of each month. If interest rate is 12 per cent p.a. compounded monthly, what is the cash equivalent of a year's rent at the hostel?

Solution: This is a 12 period annuity due for which we want to calculate the present value.

$$A_{ndue} = R\left[\frac{(1+r/m) - (1+r/m)^{-(mn-1)}}{r/m}\right]$$

R = ₦1,200 n = 1 m = 12 r = 12%

$$\frac{r}{m} = \frac{0.12}{12} = 0.01 \qquad m \text{ x } n = 12 \text{ x } 1$$

$$A_n = ₦1,200\left[\frac{1.01-(1.01)^{-11}}{0.01}\right]$$

= ₦1200 x 11.36763
= ₦13,641.15

Alternately, we can simply check up the present value interest factor table for ordinary annuity of ₦1.00. We look at the 1 percent column and at the row for 11 periods (n - 1 = 11 in the situation we are considering). This gives a value of 10.368. We then add 1.00 to this to get 11.368 as the present value interest factor for an annuity due of ₦1.00 for 12 months at 12 percent per annum compounded monthly. This factor is then used to multiply the periodic payment of ₦1200 as above to arrive at an answer of ₦13,641.15.

2. ABC Pension Fund Ltd pays ₦150,000 to a pensioner at the beginning of every quarter under a pension plan. Calculate the cash equivalent of a year's pension if the company charges 8 per cent p.a. compounded quarterly.

Solution: R = ₦150,000 r = 0.08 n = 1 m = 4 m x n = 4 x 1= 4

$$\frac{r}{m} = \frac{0.08}{4} = 0.02$$

$$A_{ndue} = ₦150{,}000\left[\frac{1.02-(1.02)^{-3}}{0.02}\right]$$

$= ₦150,000 \times (3.88388)$

$A_{ndue} = ₦582,582.49$

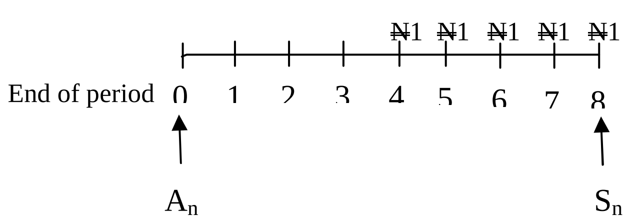
- Mr. Obioha presently occupies a room for which he pays a rent of ₦1,500 at the beginning of each month. He is considering buying a one bedroom apartment being sold by the FHA instead of paying this monthly rent for the next 5 years. Assuming he can earn 9% p.a. compounded monthly, what is the cash equivalent of his 5 years rent (to the nearest naira)?
  - ₦72,802.

3.5 DEFERRED ANUITIES AND PERPETUITIES

Before closing our discussions on present and future value concepts, we will take a look at deferred annuities and perpetuities in turn.

A deferred annuity is one whose first payment is at some later time other than the end of the first period. Figure 3.4 below shows the time line for an ordinary annuity of ₦1.00 per period for 5 periods deferred for 3 periods.

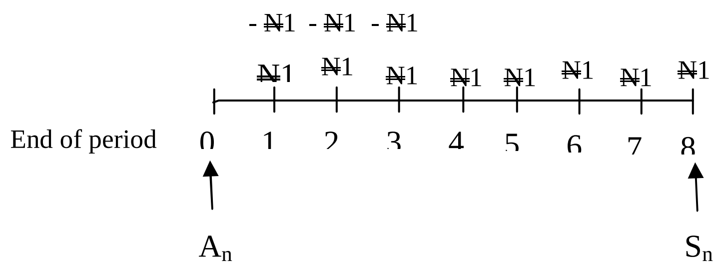
Figure 3.4: Time line for a 5-period ₦1 annuity deferred for 3 periods.



Arrows have been used to mark the time for calculating present value (A_n) and future value (S_n) of the deferred annuity. From the diagram you will realize that the future value is not affected by the deferral and it equals the future value of an ordinary annuity for 5 periods.

You will notice as shown in figure 3.5 below that the present value of an annuity of n payments deferred for d periods can be calculated by subtracting the present value of an annuity for d periods from the present value for n + d periods.

Figure 3.5 Demonstration of calculation of present value of a deferred annuity



We can summarize the procedure for calculating present value of deferred annuity from our explanation above as follows:

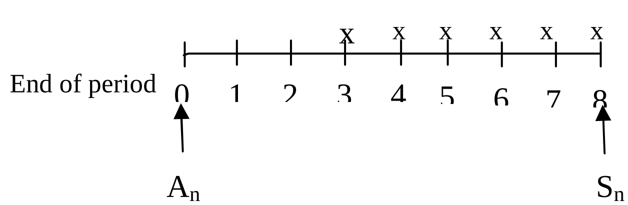
$$A_n \text{ deferred} = R\left(\left(\frac{1-(1+r)^{-n+d}}{r}\right) - \left(\frac{1-(1+r)^{-d}}{r}\right)\right)$$

3.5.1 Sample Problem on Deferred Annuities

1. On September 30th 2004 Mazi Uzodinma envisage that his only son's primary school education will cost ₦480,000 per year for six years. The child is to start school on September 30th, 2007. If interest rate is 10 per cent per annum how much should be set aside in a dedicated account for that purpose that will see the child through primary education.

Solution: This is obviously a 6-year annuity deferred for 2 years. We can use a time line to analyze the problem with 0 on the point representing September 30, 2004 and x showing when each installment will be paid.

Figure 3.6: Time Line for a 6-year annuity deferred for 2 years.



What we need do is to first calculate the present value of the annuity for 8 years, and subtract the present value of annuity of that same amount for 2 years from it to get the present value of the deferred annuity.

(a) present value (pv) of annuity for 8 years n = 8; R = ₦480,000; r = 0.10 or 10%

$$A_n = ₦480,000 \left[\frac{1-(1.10)^{-8}}{0.10}\right]$$

= ₦480,000 x5.33493

= ₦2,560,764.58

pv of annuity for 2 years, n = 2 R = ₦480,000 r = 0.10

$$A_n = ₦480,000 \left[\frac{1-(1.10)^{-2}}{0.10}\right]$$

= ₦480,000 X 1.73554

= ₦833,057.85

The present value of the 6-year annuity deferred for two years then is ₦2,560,764.58 less ₦833,057.85 = ₦1,727,706.72.

Alternately, we can calculate the present value of the 6-year annuity as if it is an the ordinary annuity. This will be the present value of the annuity 2 years from now (deferral is for 2 years). We thereafter discount back that value to period zero:

A_n deferred = ₦480,000 $[(1 - (1.10)^{-6})/ 0.10]$ $[1/ (1.10)^2]$

= ₦480,000 (4.35526) (0.82645) = ₦1,727,706.72

- Five years from now Mr. James will proceed on his retirement on a pension of ₦50,000 per month. The pension is expected to continue for 15 years. How much is this pension worth if James can earn 12% p.a. compounded monthly?
    - ₦2,293,218.90.

3.5.2 Perpetuities

Annuities can be paid for ever. Such annuities are called perpetuities. Bonds that promise payments for ever are called consoles. Such bonds are periodically issued by the British and Canadian governments. A perpetuity can be in arrears or in advance. The only difference between the two is the timing of the first payment.

Future values of perpetuities are undefined. If ₦1 is to be received at the end of every period and the discount rate is r percent, then the present value of the perpetuity is ₦1/r. We can derive this expression by the use of algebra or by observing what happens in the expression for the present value of an ordinary annuity of ₦R per payment as n, the number of payments approaches infinity.

$$A_n = \frac{R[1-(1+r)^{-n}]}{r} \quad …….. \quad ……. \quad (3.2)$$

As n approaches infinity $(1+r)^{-n} = \dfrac{1}{(1+r)^n}$

approaches zero, so that A_n approaches R(1/i). Therefore for a perpetuity we have

$$A_n = R(1/r) \qquad \ldots \qquad \ldots \qquad (3.4)$$

If the first payment of the perpetuity occurs now, the present value is R[1 + 1/r].

3.5.3 Sample Problems on Perpetuities:

1. Mrs. Chika Uba, an alumnus of Institute of Distance Learning Studies Citadel University, intends establishing an annual prize of ₦25,000 each to the best graduating student of the institute perpetually. If interest rate is 4½% p.a. how much will need be deposited in the endowment account by Mrs. Uba?

2. Mr. Ade Johnson intends instituting prizes in three departments of Own State University to be awarded to the best graduating students in these departments every year for ever. Mr. Johnson can afford only ₦1.5 million for this endowment. How much can be equally shared among the three best students p.a. (to the nearest ₦100) if interest rate is 5% per annum.

3. A state government helps a rural local government council maintain an old bridge and has agreed to pay ₦1.5 million every two years towards the expenses. The state wishes to discharge its obligation by paying a single sum to the council now in lieu of the payment due and all future payments. How much should the state pay the council if the discount rate is 8 per cent per year?

Solution 1

Annual prize to the best graduating student is ₦25,000 A_n= R [1/r], R = ₦25,000 r=0.04, n = ∞
A_n =₦25,000/0.04 = ₦ 625,000.00

Solution 2

A_n = R/r R = A_n x r, A_n = ₦1.5million r = 0.05 R =? R= ₦1,500,000x0.05=₦75,000. So each graduating student will receive ₦75,000/3=₦25,000.
Solution 3: This is a perpetuity in advance that assumes one payment at the beginning of each period forever. Here a period is two years; 8 per cent compounded once a year over two years is equivalent to a rate of $[(1.08)^2 - 1] = 0.1664$ or 16.64 per cent compounded once per two year period. Consequently, the present value of the perpetuity paid in advance every two years applying the expression discussed above is ₦1,500,000 $[1+1/[(1.08)^2-1]]$

= ₦1,500,000 [1+ 1/0.1664]
= ₦1,500,000 x 7.00962
= ₦10,514,423.08

- (i)The Sylvester Foundation plans to establish a fund that will pay ₦250,000 prize to an outstanding chartered accountant. The first prize is to be awarded one year from now, and subsequent awards will be made every two years thereafter forever. How much will the foundation need to establish the fund if interest rate is 4% compounded annually?

- (ii) Mazi Jude Ude intends instituting annual prizes to be awarded to the best graduating students in DLI Business Administration and Economics programs. Jude can only afford ₦1.5 million for this endowment. How much will each of the two best students receive p.a. (to the nearest ₦100) if interest rate is 5% compounded annually?
  - (i)₦3,186,274.51 (ii) ₦37,500

Summary of Study Session 3

Discounting problems are basically the reverse of their compounding counterparts. In this study session we have been looking at the value today of lump sum amount to be received sometime in the future and the worth today of series of cash flows over a period of time at specific interest rates. Armed with a few basic formulae whose derivation are relatively easy to understand you can solve quite a lot of these cash flow problems in financial management.
The relevant equations we encountered in this study session are as follows:

(a) $$P_0 = P_n\left[\frac{1}{(1+r)^n}\right]$$

Where P_0 = Present value at time 0
i = Interest rate
P_n = Principal value at the end of n periods.

(b) $$A_n = R\left[\frac{1-(1+r)^{-n}}{r}\right] \quad \ldots\ldots\ldots \quad \ldots\ldots\ldots\ldots \quad (3.2)$$

Where A_n = Present value of an annuity
R = Regular receipts or payments from an annuity
Note: adjustment for annuity in advance.

(c) $$A_{ndue} = R\left[\frac{(1+r)-(1+r)^{-(n-1)}}{r}\right] \quad \ldots\ldots\ldots \quad \ldots\ldots\ldots \quad (3.3)$$

$$A_n \text{ deferred} = R \left[\frac{1-(1+r)^{-n+d}}{r}\right] - \left[\frac{1-(1+r)^{-d}}{r}\right]$$

Self-Assessment Questions (SAQs) for Study Session 3

Now that you have completed study session 3, you can assess how well you have achieved its Learning Outcomes by answering the following questions. You can check your answers at the Appendix to this course text.

SAQ 3.1 (tests Learning outcome 3.1)

Your friend gave you an interest-free loan of ₦10,000 and is due to be paid back in 4 years time. What sum should your friend be willing to accept today if she is able to invest her money at 8% p.a.?

SAQ 3.2 (tests Learning Outcomes 3.2 and 3.3)

A scholarship worth ₦450,000 p.a. for four years, payable at the end of each academic session was awarded by the Education Trust Fund (ETF) to Kelechi for an undergraduate programme in Accounting at Own State University. (i) If interest rate is 8 percent p.a. what is the lump sum amount that need be deposited in an account for annual disbursement for the scholarship? (ii) What will that lump sum be if the disbursement is at the beginning of each session to Kelechi?

SAQ 3.3 (tests Learning Outcome 3.4)

Mr. Sunday Benedict has purchased an annuity to begin payment at the end of 2018 (the first payment is to be made on that date). Assume it is now the end of 2014. The annuity value is ₦2,400,000 per year and is designed to last 9 years. If the discount rate for this transaction is 12 percent, how much would Sunday pay for the annuity?

SAQ 3.4 (tests Learning Outcome 3.5)

Mr. Andrew Peters wants to establish a prize worth ₦50,000 to the best graduating student of Institute of Open and Distance Learning (IODL) every year forever. (i) How much will need be deposited in the endowment account for the prize if interest rate is 9.5% p.a. (ii) If Andrew can afford ₦500,000 only for the endowment and given interest rate of $9^{1}/_{2}$ per cent p.a. as in (i) what is the maximum annual amount that can be drawn annually from the endowment account for the for the prize?

Should you require more explanation on this study session, please do not hesitate to contact your e-tutor via the LMS.

Are you in need of General Help as regards your studies? Do not hesitate to contact the DLI IAG Center by e-mail or phone on:

iag@dli.unilag.edu.ng
08033366677

Study Session 4 Mathematics of Finance: Application

Introduction

Our preoccupation in the last two Study Sessions were compounding and discounting methods. Application of these methods is the object of this study Session. We dwell basically on three important items of application which have practical uses in the financial industry. These are sinking fund, amortization and bond valuation.

Sinking fund problems emanate from future value problems of annuities.

Amortization process on the other hand emanates from present value of an annuity problem. Our knowledge of computing the present value of a lump sum payment and an annuity is applied to valuation of bonds. All these have practical uses in the financial industry.

Learning Outcomes for Study Session 4

After studying this study session you will be able to:

4.1 Calculate the periodic payment into a sinking fund.

4.2 Set up a schedule that can be used to monitor how money grows in a sinking fund.

4.3 Compute the total periodic charge in a scenario where money was borrowed at a particular rate of interest and it is to be repaid by use of sinking fund at another rate of interest.

4.4 Compute the amount of periodic payment to amortize an interest bearing loan.

4.5 Prepare a schedule to keep track on how an interest bearing loan is gradually amortized (paid off).

4.6 Calculate the price (value) of a bond when relevant information is provided.

4.7 Determine at what level (par, premium or discount) a particular bond should be selling when relevant information are provided.

4.1 Sinking Fund

A sinking fund is a required periodic payment designed gradually to redeem bond issues, pay off debts, replace outdated equipment or provide money for purchasing new equipment. By this definition we deduce that a sinking fund is an annuity. It involves depositing a specific amount into an account at a particular rate of interest to enable the company accumulate the needed fund. Problems that this type of financial transaction entail would include finding the actual amount to be paid periodically when other information like the total amount to be accumulated, rates of interest involved and the total number of periods are known. A financial officer may also require the preparation of a sinking fund schedule in order to keep track on how much money is in the sinking fund at any point in time. Furthermore, you may want to determine the total periodic charge when money is borrowed at a particular rate of interest and it is to be paid back using a sinking fund at another rate of interest. We will illustrate how to solve some of these problems presently.

4.1.1 Determining Payment

We normally know the total amount, the time and the interest rate earned by a sinking fund. As mentioned before, it is simply an annuity (in arrears or in advance as the case may be). If S_n is the amount we want to accumulate, R is the periodic payment, n is the number of periods, and r is the interest rate per period, we can use equation 2.5 to solve problems involving determination of the periodic payment needed for a sinking fund. Now recall the formula for a compound sum of an annuity that we derived previously.

$$S_n = \frac{R\left[(1+r)^n - 1\right]}{r} \qquad \ldots.. \qquad \ldots\ldots \qquad \ldots\ldots.. \qquad (2.5)$$

To solve for R, the periodic payment, we divide both sides of equation 2.5 by the interest factor formula for ₦1.00, that is

$$\frac{\left[(1+r)^n - 1\right]}{r}$$

Thus we obtain

$$R = \frac{S_n}{\frac{\left[(1+r)^n - 1\right]}{r}} \text{ or } R = S_n \left(\frac{r}{(1+r)^n - 1} \right) \qquad \ldots. \qquad \ldots.. \qquad (2.8)$$

We have sinking fund payments tables for various combinations of values of n and r. (Table IV in the appendix). The part in brackets in equation 2.8 is used to construct such tables, assuming that S_n = ₦1.00. You will realize that this value is simply the reciprocal of the compound value interest factor of an annuity of ₦1.00. It can equally be applied to an annuity due when the adjustments mentioned previously are made (that is use the table to get the factor for n+1 periods and then subtract 1.0). Better still we can use the annuity due formula (equation 2.7). We will now illustrate with an example:

Bart and Co. Ltd estimates that ₦4 million worth of equipment will need replacement in 4 years. If money can be invested at 12 per cent compounded annually, how much should the company deposit at the end of each year to have the necessary fund to replace the equipment?

Solution:

In this problem the following are given S_n = ₦4million n = 4 r = 12% = 0.12. Use the formula in equation 2.8 or look up the factor needed in Table IV of your "Interest Factor Tables" (See Appendix)

Thus we have

$$R = ₦4{,}000{,}000\left[\frac{0.12}{(1.12)^4 - 1}\right]$$

= ₦4,000,000 x 0.20923

= ₦836,937.75

4.1.2 Setting up a Sinking Fund Schedule

We can take a step further in the problem above by setting up a sinking fund schedule so we keep track of the amount in the sinking fund at any point in time. This is illustrated in table 4-1 below.

You will notice that the fund contains no money until the end of the first year when ₦836,937.75 is deposited. Thus we enter 0 in columns 1 and 2 and ₦836,937.75 in columns 3 and 4. At the end of the second year, the fund contains the original ₦836,937.75 (column 1) plus the interest earned by the money ₦100,432.53 (12% of ₦836,937.75) shown in column 2, and an additional ₦836,937.75 installment (column 3). So the accumulated amount for the second year is addition of columns 1, 2 and 3. Starting amount for year 3 (period 3) is of course the accumulated amount for year 2. Interest at the rate of 12% is calculated on this year 3 starting amount. Arriving at the remaining figures in the schedule follows the simple explanation given here. Take a closer look at the complete table.

Table 4-1 Preparation of a Sinking Fund Schedule

Period	Starting Amount (1)	Interest Earned (2)	Periodic Payment (3)	Accumulated Amount (1) + (2) + (3)
1	7.0	0	₦836,937.75	₦836,937.75
2	₦836,937.75	₦100,432.53	₦836,937.75	₦1,774,308.02
3	₦1,774,308.02	₦212,916.96	₦836,937.75	₦2,824,162.73
4	₦2,824,162.73	₦338,899.53	₦836,937.75	₦4,000,000.00

- A firm has a ₦5 million bond issue outstanding. If it can earn 8% on its investment, (i) how much would it need to invest each year in order to retire the bond at the end of 10 years? (ii) A sinking fund is set up to monitor how money is accumulated to retire the bond mentioned in (i). What will be the accumulated amount in the fund at the end of (a) second year (b) third year?
  - (i) ₦345,147.44 (ii) (a) ₦717,906.68 (b) ₦1,120,486.66

4.1.3 Determining the Total Periodic Charge

In the example above ₦4 million was accumulated over a 4-year period and we assume this was used to obtain the needed equipment. We can look at the problem in another way. The company could have borrowed the ₦4,000,000 and set up a sinking fund to pay off the debt (interest plus principal) at the end of the 4 years. In a case like this, the company will have to meet two charges, the interest and the periodic deposits into the sinking fund. These two are separate since the interest on the debt is not paid from the sinking fund. For a total periodic charge problem interest on the loan and the amount borrowed are matured for repayment at the tail and end of the transaction. We will now discuss an illustration.

Sample Problem: Yetkem and Sons Ltd owes ₦5, million plus interest at 12% per year compounded semiannually which it must pay in 5 years time by making payments into a sinking fund. Find the amount of quarterly payment needed to pay the ₦5 million plus interest (the total periodic charge) if money in the fund earns 10% compounded quarterly.

Solution: We have to first determine how much the company must pay at the end of 5 years. We use the formula for compound value Interest Factor to determine this.

$$n = 5;\ m = 2;\ m \times n = 2 \times 5 = 10$$

$$r = 12\% \text{ or } 0.12; \frac{r}{m} = \frac{0.12}{2} = 0.06$$

Amount to be paid in 5 years is ₦5,000,000 $(1+0.06)^{10}$

= ₦5,000,000 x 1.79085
= ₦ 8,954,238.49

We now set up a sinking fund to accumulate this amount.

S_n = ₦8,954,238.49; n = 5; m = 4

m x n = 4 x 5 = 20 r = 10% or 0.10

$$\frac{r}{m} = \frac{0.10}{4} = 0.025$$

Using the formula in equation 2.8 or table IV we have

$$R = ₦8{,}954{,}238.49\left[\frac{0.025}{(1.025)^{20}}\right]$$

= ₦8,954,238.49 x 0.01526
= ₦136,612.79

You can now use this result to set up a sinking fund schedule as in table 4-1. Do it for purpose of practice.

- XYZ Ltd is owing ₦600,000 plus interest at 9% compounded semi-annually which it must pay in 3 years tine by making payment into a sinking fund. (i) What is the total periodic charge? (ii) Find the amount of semi-annual payment needed to pay the ₦600,000 plus interest (the total periodic charge) if money in the fund earns 8% compounded semi-annually?

- (i)₦781,356.08 (ii) ₦117,798.73

4.2 Amortization

Amortization is the process of repaying a loan (principal and interests) by installments. We saw this, in passing, when we solved problems concerning determination of periodic payment in an annuity. You need to take a closer look at this however so that you will be familiar with the peculiarities of amortization problems. It is in fact one of the most important applications of annuities to repayment of interest bearing debts.

To start with, you should note the difference between sinking fund and amortization. The one, that is, sinking fund, deals with making periodic deposits which is used at a future date to pay the principal of a debt, while the other, amortization, is the making of periodic payments that cover the outstanding interest and part of the principal.

One familiar example of amortization is the purchasing of a car or other items by making a series of periodic payments to liquidate the loan taken to make the purchase and interest on it. Another example is monthly payments made for 20 to 30 years, or more in some cases on a loan used to buy a house. A good comprehension of how loans are amortized and what costs are involved would help you make intelligent choice of a lender and the repayment plan.

There are a few notable problems of amortization that we will discuss. The first is the determination of periodic payment which we discussed in sample problem of section 3.3. You can flip back to take a look at how we solved the problem. You will notice that this is simply the division of the present value interest factor of an annuity into the present value of the annuity (A_n), to arrive at the periodic payment. If you want to apply interest factor tables to solve such problems, you can go about it in one of two ways. One way is to look into the table for the present value interest factor of an annuity under the appropriate interest rate and number of periods (Table VI in our Interest factor Tables in the appendix). The value extracted from the table is then divided into the present value amount to arrive at the periodic payment (amortization value). The other way is to use the value from Table V in the appendix (Annuity Rate Present Value of ₦1) for the appropriate interest rate and period to multiply the annuity present value amount (A_n) to give the amortization value. The values in Table V, you will notice through a cursory check, are the reciprocals of the ones under corresponding interest rates and periods in Table VI making it easy to apply. It is advisable, however, that you know the formula for present value interest factor of an annuity at your finger tips, so you can easily use calculators to compute factors for periods and or interest rates not given in the tables.

Another problem we can look at is preparation of an amortization schedule in which we decompose each periodic payment to liquidate a loan into the portion for interest and portion for principal. Larger proportion of earlier payments goes for interests, with the proportion for principal increasing, later taking the larger proportion with later installments, when payment periods are relatively long say 30-40 years. We will now discuss a sample problem.

4.2.1 Sample Problem on Amortization

Uba and Co. ltd borrowed ₦6 million that is to be paid back by making 6 semi annual payments at 14% p.a. compounded semi-annually. [questions (i) to (vi) then follows before the solution).

(i) Find the amount of each payment?
(ii) What portion of the first payment represents interest?
(iii) What is the new balance after the first payment?
(iv) What portion of the second payment represents interest?
(v) What is the new balance after the second payment?
(vi) Prepare an amortization schedule for the loan.

Solution:
From our discussions above:

$$R = \frac{A_n}{\left[\frac{1-(1+r/m)^{-mn}}{r/m}\right]} = A_n\left[\frac{r/m}{1-(1+r/m)^{-mn}}\right]$$

(i) The present value (A_n) = ₦6,000,000
n = 3, m = 2, n x m = 6 R = ?

$$r = 16\% \text{ or } 0.16 \qquad \frac{r}{m} = \frac{0.14}{2} = 0.07$$

$$R = \frac{₦6,000,000}{\left[\frac{1-(1.07)^{-6}}{0.07}\right]} = ₦6000,000\left[\frac{0.07}{1-(1.07)^{-6}}\right]$$

$$= \frac{₦6,000,000}{4.76654} = ₦6,000,000x0.20980$$

R = ₦1,258,774.80

You should take note of how we solve the problem here. The figure we used in dividing A_n can be obtained by a look up at Table VI (Column for 7%) at the row for 6 periods), while its reciprocal that we thereafter used in multiplying A_n can be obtained by a look up of Table V as previously indicated. A simple use of a calculator, once the present value interest factor of an annuity formula is known, will save you the trouble of carrying a table. It will also give you a more accurate computation since the table figures are usually rounded.

(ii) To find what portion of the first payment is interest we use the simple interest formula I = P x r x t
P = ₦6000,000
r = 14%

t = ½(6 months)
I = ₦6000,000 x 0.14 x ½
= ₦420,000

(iii) From our computations in (i) and (ii) above, we know that ₦420,000 out of ₦1,258,774.80 payment is interest. Hence the principal is reduced by: ₦1,258,774.80 – ₦420,000 = ₦838,774.80. The new balance is ₦6000,000 – ₦838,774.80 = ₦5,161,225.20

(iv) The amount still being owed is ₦5,161,225.20. Interest rate is 14% for 6 months. So interest amount is I = p x r x t = ₦5,161,225.20.46 x 0.14 x ½ = ₦361,285.76

(v) Since ₦361,285.76 of the next installment payment of ₦1,258,774.80 is interest, the ₦5,161,225.20 principal is reduced by ₦1,258,774.80 – ₦361,285.76 = ₦897,489.03. The new balance after this reduction is ₦5,161,225.20 – ₦897,489.03 = ₦4,263,736.17.

(vi) The computations we did in (i) – (v) above show the method of computing the values that go into an amortization schedule. These are seen in the table below. You should try your hands at computing other figures whose derivation are not shown here.

Table 4.2: Preparation of an Amortization Schedule

Time (1)	**Payment (₦) (2)**	**Interest (₦) (3)**	**Principal Reduction (₦) (4)**	**Balance (₦) (5)**
0	-	-	-	6,000.000.00
1	1,258,774.80	420,000,00	838,774.80	5,161,225.20
2	1,258,774.80	361,285.76	897,489.03	4,263,736.17
3	1,258,774.80	298,461.53	960.313.27	3,303,422.90
4	1,258,774.80	231,239.60	1,027,535.20	2,275,887.71
5	1,258,774.80	159,312.14	1,099,462.66	1,176,425.05
6	1,258,774.80	82,349.75	1,176,425.05	0.0

- Ben borrowed ₦500,000 from his bank and agreed to amortize his debt, principal and interest at 10% rate of interest by equal payments over the next 12 years the first due in a year's time. (i) What is the annual cost of his debt? (ii) In setting up an amortization schedule for the repayment of the loan Ben took, what will be the outstanding balance of the loan (a) after the second instalmental payment? (b) after the third installmental payment? (iii) If Ben is interested in knowing the outstanding principal amount of his ₦500,000 loan after the 10^{th} payment, what is this amount?
  - (i)₦73,381.66 (ii) (a) ₦471,630.89 (b) ₦454,844.93 (iii) ₦127,356.63

4.3 Bond Valuation

We will first discuss a few vital concepts that will aid our grasp of how bonds are valued.

A bond is a written promise made by a company or government to repay a sum of money at a specified time. The par value of a bond is the stated face value of the bond and, it is usually set at ₦1,000 or ₦100 or ₦500. The *par value* is usually the amount of money that the firm or government borrows and promise to repay at some future date.

Coupon Interest Rate: The bond will normally state that the issuer (borrower) will pay a specified amount of money as interest each year (or more usually every six months). When the coupon payment as it is called is divided by the par value, the result is the coupon interest rate. For example, if a ₦1,000 bond par value pay ₦150 interest each year, the Bond's coupon rate is 15 per cent. This represents the yearly "rent" or interest on the money.

Maturity Date: Bonds generally mature at a specified date. At maturity, the par value is repaid to each bond holder.

If a bond is not secured by any particular asset, it is called a debenture and if it is backed by a specific property it is called a mortgage bond. When bonds become a perpetual means of financing a company it is known as irredeemable bond. Another example are the bonds issued by the British government to finance the second world war. These bonds have not been retired. They are now referred to as a consol.

4.3.1 Sample Problem on Bond Valuation

Let us now see an example to illustrate how to compute bond value. Assume that a particular bond that will mature in 8 years pay interest in the amount of ₦170 every year. If the par value is ₦1,000 each, what is the bond worth if the appropriate market interest rate is (a) 17 per cent (b) 14 per cent (c) 19 per cent.

Solution: (a) You will note from the explanation given earlier that you are buying an annuity plus a lump sum of ₦1,000, so we simply find the present value (PV) of the cash flow from the investment. If the relevant interest rate is (a) 17%

periodic payment (R) = ₦170, for n = 8years . We can then look at table VI for the present value interest factor of an annuity to be used or we calculate it.

$$A_n = R\left[\frac{1-(1+r)^{-n}}{r}\right] \quad \ldots\ldots \quad \ldots\ldots\ (3.2)$$

$$\text{PV (annuity part)} = \text{₦}170\left[\frac{1-(1.17)^{-8}}{0.17}\right]$$

= ₦170 x 4.20716

= ₦715.22

Next, we calculate the present value of the ₦1,000 to be received 8 periods from now at 17% rate of interest (use table II or calculate) i.e. $P_0 = P_n(1+r)^{-n}$…………..(3.1)

$$PV = \frac{₦1000}{(1.17)^8} = ₦1000(1.17)^{-8}$$

$$\frac{₦1000}{3.51145} \quad -$$

We then add present values of the two components of the cash inflow of the investment together to derive the total present value. That is, the value of the Bond = ₦715.22 + ₦284.78 = ₦1 ,000.00.

(b) The procedure for this is same with the solution to (a). The only difference is the relevant interest rate which is 14%

The solution is as follows

(i) PV of annuity part:

$$= N170\left[\frac{1-(1.14)^{-8}}{0.14}\right]$$

₦

= ₦170 x 4.63886 = ₦788.61

(ii) PV of ₦1000 8years from now at 14% rate of interest i.e. PV of lump sum = ₦1,000$[1.14]^{-8}$ = ₦1000 x 0.35056 = ₦350.56

Total present value of the investment at 14% per annum is ₦788.61 + ₦350.56 = ₦1139.17

(c) Again, the procedure for computation here is as for (a) and (b) since discounting is once in a year) For the annuity part.

$$\mathrm{PV} = \mathrm{N}170\left[\frac{1\text{-}(1.19)^{-8}}{0.19}\right]$$

₦

= ₦170 x 3.95437

= ₦672.24

The present value of the lump sum received at the end of the period is

₦1,000$[1.19)^{-8}$

= ₦1000 x 0.24867

= ₦248.67

Present value of the total cash in flow of investment in the bond is ₦672.24 + ₦248.67 = ₦920.91

We can combine the two-step procedure for calculating present value (or price) of a bond in a standard formula as follows:

$$PV = \sum_{r=0}^{n} \frac{\frac{I_t}{m}}{(1+\frac{kr}{m})^{mn}} + \frac{P}{(1+\frac{kr}{m})^{mn}}$$

Where PV = Present value of the security
I_t = Nominal interest income
K_i = Nominal interest rate
m = Number of times interest is paid per year
P = Par value of the bond
n = Duration of the bond (in years).

You will arrive at the same answers if you use this concise formula.

The results we have above need explanations as you would have seen that the value (or price) of the bond fluctuates with the change in the on-going rate of interest in the economy. You will have observed that the bond is worth less when the going rate of interest is 19% than when it is 17%. At a price of ₦920.91 the bond provides an annual rate of return of 19% and when the price is ₦1000 it provides an annual return of 17%.

Similarly, the bond is worth more than ₦1,000 when the going rate of interest is less than 17 per cent because it offers a yield higher than the going rate. The price at 14% going rate of interest is ₦1139.17. So when interest rates in the economy decline, the prices of outstanding bonds rise and vice versa.

The interest rates we are discussing here is termed the bond's yield to maturity (YTM). The assumption, for example, is that if the initial interest rate is 17 per cent per annum, this will continue till maturity giving a yield to maturity of 17%. If the interest rate on bond of similar risk and maturity change however, the yield to maturity of the particular bond will change accordingly. The yield to maturity for a bond that sells at par consists entirely of an interest yield, but if the bond sells at (Premium or discount) yield to maturity consisted of a positive or negative capital gains yield plus the interest yield.

The experience in many economies of the world in the past many years is for regime of interest rates to go up due to inflation and devaluation of currencies. This will normally lead to decline in the value of bonds with fixed coupon rates. The consequent expectation of decline in value would make such bonds less attractive to investors. To attract investors to buy bonds, companies and governments who issue them nowadays quote floating rates of interest tied to the Central Bank of Nigeria's minimum rediscount rate (CBN MRR). An example is the Lagos State of Nigeria ₦60 million floating rate bond issued in November 1988. It is quoted as attracting a floating interest rate of 2 ¾% above CBN MRR with a minimum interest rate of 13% and a maximum of 19%. The reason indicated earlier supports the view that floating rate bonds has become a normal feature of the bond market. However, yield-to-maturity can only be applied to a particular price at a particular date based on the coupon rate, par value and remaining period before maturity.

- The Bolingo Hotel Chain has a ₦1,000 par value bond outstanding that pays N90 coupon interest p.a. The current yield to maturity on such bonds in the market is 12%. (i) What is the price of the bond if it will mature in 15 years time? (ii) At what rate of interest will the bond be selling at par?

  - (i)₦507.13 (ii) Since the coupon interest rate is (₦90/₦1000) = 9%; the bond should be selling at par when the prevailing interest rate is 9%.

Summary of Study Session 4

Our focus in this study session has been application of compounding and discounting methods for series of payments and lump sum payments. Our discussion on sinking fund is basically an application of compound (future) value of an annuity while amortization emanates from present value of an annuity. Bond valuation is rooted in a combination of present value of an annuity and that of lump such payments. We recapitulate below the equations (formula) encountered in our discussions in this study session to aid your quick recall of our discussions.

1 The value of periodic payment into a sinking fund (i.e reciprocal of interest factor for an ordinary annuity) is as follows

$$R = \frac{S_n}{\left[\dfrac{(1+r)^n - 1}{r}\right]}$$

$$R = S_n\left[\frac{r}{(1+r)^n - 1}\right] \quad \text{.........} \qquad \text{........} \qquad \textbf{(2.8)}$$

The necessary adjustment to the formula will be needed for setting up sinking funds for an annuity in advance. Note also that the values shown in sinking fund tables are derived under the assumption that

S_n = ₦1.00

2. For amortization the value of periodic payment to pay back a loan when compounding is more than once a year is:

$$R = \frac{A_n}{\left[\dfrac{1-(1+r/m)^{-mn}}{r/m}\right]} = A_n\left[\frac{r/m}{1-(1+r/m)^{-mn}}\right]$$

3. Value of the annuity part of bond's cash flow i.e. value of the coupon interest payments is the present value (PV)) of an ordinary annuity, pv $= R\,\frac{[(1-(1+r)^{-n}]}{r}$

The lump sum payment at the tail end of the transaction is $P_o = P_n\ (I + r)^{-n}$

The standard formula that combines these two steps of calculating bond values is

$$PV = \sum_{r=0}^{n} \frac{\frac{I_t}{m}}{(1+\frac{kr}{m})^{mn}} + \frac{P}{(1+\frac{kr}{m})^{mn}}$$

Self Assessment Questions (SAQs) for Study Session 4

Having completed study session 4, you can now assess how well you have achieved its Learning Outcomes by answering the following questions. You can check your answers with the Notes on the Self Assessment Questions at the Appendix to the course text.

SAQ 4.1 (tests Learning Outcomes 4.1 and 4.2)

Smith and Co. Ltd is establishing a sinking fund to repay a loan of ₦4million in 4 years time (a) What is the annual payment that will accumulate this amount if interest rate is 10%?

The sinking fund schedule for this problem is shown below. Fill in the gap of the missing figures in the schedule i.e. (a); (b); (c) and (d)?

Period	Starting amount	Interest earned	Periodic payment	Accumulated amount
	(1) ₦	(2) ₦	(3) ₦	(1) + (2) + (3) ₦
1	0	0	(a)	(a)
2	(a)	86,188.32	(a)	1,809954.75
3	809954.1, 75	(b)	(a)	(c)
4	(d)	285283.34	(a)	4,000,000

SAQ 4.2 (tests Learning Outcomes 4.1, 4.2 and 4.3)

Jones and Sons Ltd is owing ₦750,000 plus interest at 10 percent p.a. compounded quarterly which it must pay back in four years' time by making payments into a sinking fund. The company wants to set up a sinking fund into which semi-annual payments will be made at 11% p.a. compounded semi-annually in order to pay off the loan plus interest on it (the total periodic charge) at the end of 4 years. (i) What is the total periodic charge? (ii) Calculate the amount of semi-annual payment the company will need to make in order to pay the ₦750,000 plus interest. (iii) How much will be in the sinking fund after the second semi-annual payment has been made?

SAQ 4.3 (tests Learning Outcomes 4.4 and 4.5)

Darocha & Co. Ltd borrowed ₦5million that it wants to pay back (amortize) by making five (5) annual payments at $7^{1}/_{2}$% p.a.

(i)Calculate the amount of each payment? (ii) What portion of the first payment represent interest? (iii)What is the new balance after the first payment? (iv)What portion of the second payment represents interest?

SAQ 4.4 (tests Learning Outcomes 4.6 and 4.7)

Boripe Local Government bond will mature in 5 years time and pay interest of ₦65 every six months. If the bond's par value is ₦1000 each, what is the bond worth when interest rate is (a) 11% (b) 15% (c) At what rate of interest will the bond be selling at par (hint: all interest rates quoted are annual rates and compounding is semi annual).

Should you require more explanation on this study session, please do not hesitate to contact your e-tutor via the LMS.

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iag@dli.unilag.edu.ng
08033366677

Study Session 5 Mathematics of Finance: Further Applications

Introduction

Further application of time value of money concept is our object of discussion in this study session. We are particularly concerned with valuation of common shares using a model that is premised on cash flow pattern assumptions for investment of this nature. In this task we are definitely getting into the realm of uncertainty that is a feature of investment in the stock market. However with relevant assumptions that are not far from reality the developed model (Gordon's model) can and is used to arrive at expected value (price) of common shares.

Apart from relevance in valuing common shares present value of growing perpetuities is another dimension to the use of Gordon's model that we discuss in this study session. We will tackle the growing perpetuity issue after our discussion on present value of growing annuities. All these have practical and interesting applications in various financial transactions.

Learning Outcomes for Study Session 5

When you have studied this session, you should be able to:
5.1 Compute the value (price) of common stocks (shares).
5.2 Determine the expected rate of return on a common stock (shares).
5.3 Calculate the present value of a growing annuity.
5.4 Determine the present value of a perpetuity growing at a constant rate.

5.1 Valuation of Common Stocks (Shares)

When investments are made on shares, what the investors expect and the things that determine the value of such common shares are usually in two forms (a) cash dividends and (b) capital gains (there may be capital loss though) from the sale of such common shares. Let us define a few terms for clarity of our discussions.

We will represent the price of the shares at the end of period t as P_t. Note that t can take on whatever integer value we desire. So P_0 is the price of the share today being the end of period 0. P_1 is the price at end of year 1 and so on.

We will also define D_t as the dividend the shareholders expect to receive at the end of year t. D_0 is the most recent dividend which we assume is already paid while D_1 is the next expected dividend and so on.

The symbol g is used to represent the expected rate of growth in stock price (rate of capital gain). We are assuming that g is also the expected rate of growth in earnings and dividends.

Finally, we will use the symbol K_e to represent the expected rate of return on the stock. We could have continued the use of the symbol r of the last three units. The fact of some elements of risks in investments in common stock, and the consideration of returns available on similar investments prompt the distinguishing symbol (K_e).

Current Price: The rate of return that investors expect from the share at the end of the present period is defined as the expected dividend per share D_1 plus the expected price appreciation per share $P_1 - P_0$, all divided by the price at the start of the year (P_0). We have it in the following equation.

$$\text{Expected return} = K_e = \frac{D_1 + P_1 - P_o}{P_o} \qquad \ldots\ldots\ldots \qquad .. \qquad (5.1)$$

This return that is expected by shareholders is often called the market capitalization rate.

From equation 5-1 we could derive P_0, the current price of the common stock in question, by making P_0 the subject of the equation.

$$\text{Price} = P_o = \frac{D_1 + P_1}{1 + K_e} \qquad \ldots\ldots\ldots \qquad \ldots\ldots\ldots \qquad (5.2)$$

We expect the stock price to grow at the rate of g per cent per year. This implies that $P_1 = P_0 (1+g)$. When we substitute this into equation (5-2) we have

$$P_o = \frac{D_1 + P_o(1+g)}{1+K_e} \qquad ....(5\text{-}2a)$$

$$P_o(1+k_e) = D_1 + P_o(1+g) \qquad .....(5\text{-}2b)$$

$$P_o(1+k_e - 1 - g) = D_1 \qquad ....(5\text{-}2c)$$

$$P_o = \frac{\mathrm{D}_1}{\mathrm{k}_\mathrm{e} - g} \qquad ....(5\text{-}3)$$

This formula is useful only when g the anticipated growth rate is less than k, the expected rate of return, or discount rate. As g approaches k, the stock price becomes infinity (since k-g will approach 0).

5.1.1 Sample Problem on Stock Valuation

We will now give an illustration. Assume that Benedict and Co. Ltd paid a dividend of ₦1.50 per share recently and that dividends will grow at a rate of 8 per cent a year. What price would you expect to pay for Benedict's shares if the market capitalization rate (discount rate) is 12 per cent?

Solution: It is given that the last dividend was ₦1.50 and this is expected to grow by 8 per cent, so you estimate the dividend you will receive as follows:

$D_1 = D_0$ (1+g) = ₦1.50 (1.08) = ₦1.62.

With the appropriate discount rate given as 12 per cent the price of Benedict Shares would be

$$P_o = \frac{D_1}{K_e - g}$$

$$= \frac{₦1.62}{0.12 - 0.08} = ₦40.50$$

We can call this price the equilibrium price of Benedict shares since a lower price will make rational investors who will sense the undervaluation to buy more of it, so driving up the price. The effect of over valuation will be in the reverse, pulling down the price to equilibrium level.

- Jackson & Co. Ltd paid a dividend of ₦2.20 recently. It is expected that dividends of the company will grow at the rate of 5% p.a. What price will you expect to pay for Jackson's shares if the company's capitalization rate is 14 per cent?
  - ₦25.67

5.1.2 Estimation of Expected Rate of Return of Common Shares

We calculated the price you will be expected to pay for Benedict's shares above. We can reverse the problem to one in which we calculate the rate of return the investor can expect if the shares are purchased at the current price of ₦40.50. We can start from equation 5-2a.

$$P_o = \frac{D_1 + P_o(1+g)}{1+k_e} \quad \ldots\ldots. \quad \ldots\ldots \quad (5.2a)$$

We will now solve for k_e in the above equation

$$P_o + P_o k_e = D_1 + P_o(1+g) \quad \ldots(5\text{-}4a)$$

$$P_o k_e = D_1 + P_o + P_o g - P_o \quad \ldots.(5\text{-}4b)$$

$$k_e = \frac{D_1}{P_0} + g \quad \ldots\ldots \quad \ldots\ldots. \quad (5.5)$$

We can apply equation (5-5) to our previous problem on Benedict's shares. The expected rate of return seen above is the expected dividend yield plus the capital gains. So for our example,

$$K_e = \frac{₦1.62}{₦40.50} + \frac{₦3.24}{₦40.50}$$

$$= 0.04 + 0.08 = 0.12 \text{ or } 12\%$$

We can explain this situation thus: You are expecting a dividend of ₦1.62 and you also expect the price of the shares to increase by 0.08 x ₦40.50 = ₦3.24. So your expected total return is 12 per cent comprising of 4 per cent expected dividend yield and 8 percent expected capital gains yield.

- Emem enterprises Ltd desire to calculate its cost of common shares. During the next 12 months, the company expects to pay dividends (D_1) of ₦2.40 per share. The current price of its common stock is ₦40 per share. The expected growth rate of dividends is 7.5%. What is the company's expected cost (return) on common shares?
  - 13.5%

5.1.3 Valuation of Stocks for Multiple Periods

The stock valuation model we have been discussing, here is applicable mainly to a single period. We will now have a brief discussion on multi-period model.

The prices of shares are normally determined as the present value of a stream of cash flows. If we assume that an investor buys shares of stock with the intention of holding it in the family forever (in actual practice in Nigeria it has been found that this is not far from the truth since many shareholders don't trade their shares, but only buy for keeps).

In this case all the shareholder receives is a stream of dividends, and the value of the stock is calculated as the present value of an infinite stream of dividends.

Value of stock now = P_0 = PV of future dividends.

$$P_o = \frac{D_1}{(1+k_e)} + \frac{D_2}{(1+k_e)^2} + \frac{D_3}{1+k_e)^3} + ... + \frac{D\infty}{1+ke)^{\infty}} \qquad .... \qquad (5.6)$$

The question can arise of the more general case where you expect to hold the stock for a future period and then it is sold. How would the value of P_0 be determined then? We can still use equation 5-6 to determine the value of the stock. You need to realize that for any individual investor cash flow consist of dividend plus the sale price of the stock, but for all present and future investors in total, expected cash flow consist only of future dividends, except in the case of bankruptcy or liquidation or sell out to other owners. This is why equation 5-6 is alright for valuing streams of dividends.

Let us look at the equation from another point of view. Suppose Da. Rocha bought a stock expecting to hold it for 3 years. He will receive dividends in years 1, 2, and 3 and the value P_3 when he sells the stock at the end of the third year. How do we determine the value of P_3? It will be determined as the present value of a stream of dividend during years 4 through to n plus a still more distant future price P_n which in turn will be determined as the present value of another set of future dividends and even more distant stock price. We can continue the process ad infinitum, showing that equation 5-6 is the ultimate result. In fact equation 5.3 may seem to be a one-period model based on explanation we gave in this section. In actual sense it is a model for the present value of a perpetuity that grows at a constant rate. We will discuss this in the next section.

5.2 ANNUITIES WITH GROWING PAYMENTS

In our discussions on annuity in study sessions 2, 3, and 4 we assumed that annuity payments were constant through time. In line with the discussion in section 5-1 we now want to consider the case where the payments are assumed to be growing at a constant rate. This is a more realistic assumption if for example we are looking at a model of the growing dividends paid out by a firm as previously discussed. For general purpose and use we will redefine K_e, the return on common stock we used previously, as r, which is simply a symbol for rate of interest. We will also redefine P_0, the current price of a share today as PV for the same reason of general purpose and use. Assume that D_0, the current dividend per share was paid just yesterday so that it does not enter into the present value computations.

The stream of growing dividends to be received starts with the first end-of-year dividend,

$D_1 = D_0\ (1+g)$. The dividend at the end of the second year is

$D_2 = D_0 (1+g)^2$. The stream of payments is assumed to grow at a constant rate for n years, therefore its present value, PV, is

$$PV = \frac{D_1}{1+r} + \frac{D_2}{(1+r)^2} + \frac{D_3}{(1+r)^3} + \quad + \frac{D_n}{(1+r)^n} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$= \frac{D_0(1+g)}{1+r} + \frac{D_0(1+g)^2}{(1+r)^2} + \frac{D_0(1+g)^3}{(1+r)^3} + \frac{D_0(1+g)^n}{(1+r)^n}$$

If we let u = (1+g)/(1+r) the expression we have above can be rewritten as

$$PV = D_0u + D_0u^2 + D_0u^3 + \ldots\ldots + D_0u^n$$
$$= uD_0 (1 + u + u^2 + \ldots.. + u^{n-1}) \ldots\ldots.. (5\text{-}7)$$

By multiplying equation (5-7) by u and subtracting the result from equation (5-7), we have PV – uPV = uD_0 (1-u$^{n)}$

When we solve for the present value of the growing annuity, we have

$$PV = \frac{uD_0 (1-u^n)}{1-u}$$

Substituting back the value of u gives us

$$PV = \frac{\left(\frac{1+g}{1+r}\right)D_0\left(1-\left(\frac{1+g}{1+r}\right)^n\right)}{1-\left(\frac{1+g}{1+r}\right)}$$

By rearranging terms and recalling that D_0 (1+g) = D_1, we obtain

$$PV = \frac{D_1\left(1-\left(\frac{1+g}{1+r}\right)^n\right)}{r-g} \quad \ldots\ldots \quad \ldots.. \quad (5\text{-}8)$$

Equation (5-8) is the present value of n annuity payments which start at a level of D_0 and grow at a constant rate g.

5.2.1 Sample problem on Growing Annuity

We will now look at an example to illustrate the subject matter of this section.

Mr. Ben Obi took up appointment with Maxi Bites fast foods ltd. He expects his salary to grow steadily at 3%
per year for the next 10 years when he expect to retire. His current salary which has just been paid is ₦1.8
million. If Ben can earn 10 per cent on his investments, what is the present value of his remaining salaries?

Solution: First, we need to compute next year's salary. Since it will be 3 percent greater than this year's salary then D_1 = ₦1,800,000 x 1.03 = ₦1,854,000. Since r > g, to compute the present value of the growing annuity we use equation (5-8).

$$PV = \frac{₦1,854,000}{0.10-0.03}\left\{1 - \left(\frac{1.03}{1.10}\right)^{10}\right\}$$

$$= ₦12,762,460.80$$

What will be the present value of Ben's future salaries if he can earn only 3 per cent annually on her investments?

In this case r = g so we must use equation 5-10.

$$PV = \frac{₦1,854,000 \times 10}{1.03}$$

$$= ₦18,000,000$$

We can interpret these present values as follows. If Ben can earn 10 per cent on his money, then with ₦12,762,460.80 invested today he could withdraw annual amounts equal to his future salaries and have nothing left after the tenth withdrawal. However if Ben can earn only 3 per cent on his money, he needs to invest ₦18 million today to be able to make annual withdrawals equal to his future salaries and have nothing left over at the end. In effect the lower rate of interest must be compensated for by investing a greater amount now.

- Lucy works at Brillia and Co. Ltd and expects her annual salary to grow steadily at the rate of $2\frac{1}{2}$% p.a. for the next 15 years when she expects to retire. Her current salary which has just been paid is ₦840,000. Lucy can earn 8% interest rate on her investments. What is the present value of Lucy's salaries for the next 15 years?
    - ₦8,507,243.82

5.3 GROWING PERPETUITIES (GORDON'S GROWTH MODEL)

It should be noted that if the number of payments that we were discussing in section 5.2 is infinite, we can obtain a finite present value if we assume that the growth rate in dividends (or annuity payment as the case may be), g is less than the time value of money, r. If g < r, then the second term in the numerator of equation 5-8 goes to zero in the limit as n approaches infinity.

$$\lim_{n \to \infty} \left(\frac{1+g}{1+r}\right)^n = 0, \text{ iff } g < r$$

Therefore, the present value of an infinite number of growing dividends or payments is:

$$\lim_{n \to \infty} PV = \frac{D_1}{r - g} \qquad \text{.......} (5\text{-}9)$$

Equation 5-9 is the same with equation 5-3 and it is called the Gordon's growth model. It provides us with an estimate of the present value of an infinite annuity (or of a share of common stock) where the stream of periodic payments (or dividends) received are assumed to grow at a constant rate which is assumed to be less than the discount rate (which in the case of common stock would be the cost of equity capital, K_e).
It should be noted that equation (5-8) is valid not only when r > g, but also when r < g. This is in contrast to the growing perpetuity formula, which is valid only for r > g. However equation (5-8) cannot be used in the special situation in which r = g. In that case, to find the present value of the growing annuity, we have to make use of the foolproof method of adding all the present values of the individual cash flows of the growing annuity. This gives:

$$PV = \frac{D_1}{1+r} + \frac{D_2}{(1+r)^2} + \dots + \frac{D_n}{1+r^n}$$

$$= \frac{D_1}{1+r}\left\{1 + \frac{1+g}{1+r} + \dots + \frac{1+g^{n-1}}{1+r}\right\}$$

Since r = g by assumption, each term in brackets is equal to 1. Since there are n such terms within the brackets, the formula for a growing annuity when the discount rate equals the cash flow growth rate is given by:

$$PV = \frac{D_1 n}{1+r} \qquad \text{.........} \qquad (5\text{-}10)$$

5.3.1 Sample Problem on Growing Perpetuities

The Young Elites Social Club of Umuofia is to set up an endowment to fund the annual inter-house sports event of Umuofia Community Grammar School. The event costs ₦450,000 this year and this sum has just been made available by the club. The cost of the annual event is expected to grow at an average rate of 4 percent p.a. If interest rate is 13% p.a. how much will the club need to set up the endowment so the event can be funded perpetually?

Solution: A direct application of Gordon's growth model in equation 5-9 is what we use:

Next year's cost will be
₦450,000 x 1.04 = ₦468,000

$$PV = \frac{₦468{,}000}{0.13 - 0.04}$$

$$= \underline{₦5{,}200{,}000.00}$$

Mazi Adindu plan to sponsor Umuokoro Grammar School annual inter-house sports competition to the tune of ₦550,000 p.a. perpetually. It is envisaged that the cost of the event will increase at a constant rate of 3.5% per annum. Adindu has just remitted the amount for this year's event. How much will he need to deposit once and for all in an endowment account for this obligation if fund in the account attract $10\frac{1}{2}$ % interest p.a.?

₦8,132,142.86

Summary of Study Session 5

Our focus in this study session has been application of compounding and discounting methods we discussed earlier to the valuation of common stock, growing annuities and growing perpetuities. We here recapitulate the formulae and models that we encountered in the study session.

(i) Common shares are valued using the following model (Gordon's model)

$$Po = \frac{D_1}{ke - g}$$

Where

D_o = the most recent dividend which we assume has just been paid

D_1 = the expected dividend at the end of period 1

g = expected growth rate in stock price

K_e = expected rate of return on the stock

It is to be noted that Gordon's model also fits into valuing perpetuities that grow at a constant rate.

We also note that Gordon's growth model is valid only where r < g.

(ii) The expected rate of return of a stock is calculated using the following formula

$$k_e = \frac{D_1}{P_0} + g \qquad \ldots\ldots \qquad \ldots\ldots. \qquad (5.5)$$

(iii) Annuities with growing payments are valued using the following formula

$$PV = \frac{uD_0\,(1-u^n)}{1-u}$$

Substituting back the value of u gives us

$$PV = \frac{\left(\frac{1+g}{1+r}\right)D_0\left(1-\left(\frac{1+g}{1+r}\right)^n\right)}{1-\left(\frac{1+g}{1+r}\right)}$$

By rearranging terms and recalling that $D_0\,(1+g) = D_1$, we obtain

$$PV = \frac{D_1\left(1-\left(\frac{1+g}{1+r}\right)^n\right)}{r-g} \qquad \ldots\ldots \qquad \ldots.. \qquad (5\text{-}8)$$

(iv) In the special case of valuing an annuity
Where r = g the relevant formula is

$$PV = \frac{D_1 n}{1+r} \qquad \ldots\ldots\ldots \qquad (5\text{-}10)$$

Self-Assessment Questions (SAQs) for Study Session 5

Having completed study session 5, it is appropriate that you assess how well you have achieved the Learning Outcomes by answering the questions that now follows. Write your answers with full workings in your Study Notebook. You can check your answers with the notes on Self-Assessment Questions at the index to the course text.

SAQ 5.1 (tests Learning Outcomes 5.1 and 5.4)

XYZ Ltd paid a dividend of ₦2.25 recently. It is estimated that dividends for the company will grow at a rate of 6 per cent a year. What price would you pay for the company's shares if the market capitalization (discount rate) is 14 per cent?

SAQ 5.2 (tests Learning Outcome 5.2)

In a year's time, Nathanael Plc expects to pay a dividend of ₦1.70 per share. The current price of the company's common share is ₦14.60 per share. Expected growth rate of dividends is 5.5% p.a. What is the company's expected returns on common shares?

SAQ 5.3 (tests Learning Outcome 5.3)

Mary works at Ude and Co. ltd and expects her annual salary to grow steadily at 2.5 per cent per year for the next 12 years when she expect to retire. Her current salary which has just been paid is ₦2.2 million.

Mary can earn 8% interest rate on her investments. What is the present value of Mary's salaries for the next 12 years?

SAQ 5.4 (tests Learning Outcome 5.4)

Mazi Peter Nkem intends setting up an endowment account to fund the annual Speech and Prize Giving Day of Umudim Community Grammar School perpetually. Annual cost of the ceremony is ₦250,000. This sum has just been paid to fund this year's ceremony. The cost is expected to grow at an average rate of $4\frac{1}{2}\%$ per year. If interest rate is 12% p.a. how much will Peter need to set up the endowment account? (give your answer to the nearest ₦100)

Should you require more explanation on this study session, please do not hesitate to contact your e-tutor via the LMS.

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iag@dli.unilag.edu.ng
08033366677

Study Session 1 Capital Budgeting Under Certainty I

Introduction

We are starting a new module in this study session. In the previous module, study sessions 1 to 5, we laid the foundation for the course in Business Finance. Apart from the first study session that introduced you to the course, we discussed basic mathematical computations that we should encounter in Business Finance. This module on capital budgeting is the core of our course in Business Finance. This study session and the three that follows discuss capital budgeting under the conditions of certainty of economic parameters like interest rates and predictability of financial market indicators. We ignore the issue of risks (not totally though) in capital budgeting in the four study sessions. We will take up the issue of risks and uncertainty in the two units that round off this module.

This study session introduces you to the capital budgeting or investment decision in finance. We will first take a look at the nature of the investment/capital budgeting decision. We thereafter discuss how cash flows of an investment project are estimated before taking a look at various investment appraisal (selection) techniques that do not use discounted cash flows (DCF) in their application. Taking particular note of the features and application of each technique is what you need to grasp the stuff in this study session. Recognizing the merits and demerits of each technique is also important.

Learning Outcomes for Study Session 1

After studying this session you should be able to:

1.1 List and describe the various types of projects that can be undertaken under the capital budgeting
decision scenario.

1.2 Estimate and compute the cash flows for an investment project.

1.3 List the steps that should be taken in the capital budgeting process and the features of a good investment appraisal technique.

1.4 Describe the features, application, merits and demerits of the Ranking by Inspection Method for appraising project cash flows.

1.5 Describe the features, application, merits and demerits of the Pay Back Period Method for appraising project cash flows.

1.6 Describe the features, application, merits and demerits of the Accounting Rate of Return (ARR) Method for appraising project cash flows.

1.1 Types and Nature of Investment Decisions and Projects

The valuation of securities that we discussed in study session 5 is very similar to real investment decisions of business organizations known as capital budgeting. The similarities derive first from the fact that management has to estimate the expected cash flows from a given project including the value of the asset at a specified terminal date, which is akin to estimation of future dividends and prices of stocks. Secondly, an appropriate discount rate has to be chosen, which is called the cost of capital at which estimated cash flows are to be discounted. Thirdly, the expected cash flows have to be discounted in both cases and put on a present value basis in order to derive an estimate of the value of the asset to the organization. Fourthly, the derived value is compared to the cost of the project. If the asset's present value exceeds its cost, the project should be accepted as we do in case of cash flows from securities. Finally, the effect of the capital budgeting process on the value of the firm is considered, since identification of a security whose price is lower than its true value increases value of an investor's portfolio. Similarly, when a firm identifies a project whose net present value is positive, the value of the firm is increased by the investment as will be reflected in earnings per share and price of its stock in the capital market. Having seen these similarities we will now discuss further on the nature and types of investment decisions.

By definition, a capital budget outlines the planned expenditures on fixed assets used in productive efforts of a business, while capital budgeting is the total process of analyzing projects and deciding on their suitability for inclusion in the capital budget. Generally, a firm's capital budgeting decision would include addition, disposal, modification and replacement of fixed assets. We will now briefly discuss various types of projects that fall under the capital budgeting decision range.

(i) **Automation of a Process:** A business firm may want to automate its information processing system by installing various computer hardware systems. Assume for example that the computer hardware will cost ₦820,000 including costs of training the data processing staff for its use. The expected annual cost savings by the use of the computer is ₦120,000 for a period of ten years. This is an investment decision of comparing the expenditure of ₦820,000 now to annual savings and consequent increases in earnings from operations of ₦120,000, for 10 years. The company would want to explore the viability of such an investment.

(ii) **Replacement Decision:** This involves the replacement of an existing equipment with a new model. We can have an example, such that the new machine costs ₦1.75 million and has an annual operating expenses of ₦150,000 p.a. for 10 years. The old machine is valued at ₦255,000 now and has annual operating expenses of ₦270,000 for 10 years. The company would want to evaluate the costs involved in the two options to see which is more profitable.

(iii) **Lease or Buy:** This is a decision on whether to buy an equipment costing say ₦2.5 million and it is expected to be obsolete in five years versus renting the equipment for ₦65,000 per month. The options have to be analyzed to decide whether or not the firm should buy or lease the equipment.

(iv) **Expansion of Existing Products or Markets:** This involves expenditures to increase output of existing products or to expand the outlets or distribution facilities in markets now being served. For example, a new machine is needed to expand output at a cost of ₦2 million. The machine will generate a sales revenue of ₦750,000 per year for 5 years and has an operating expenses of ₦150,000. p.a. Evaluation will have to be done to examine how valuable it is to invest ₦2 million now and make a net profit of ₦600,000 for five years.

(v) **Introduction of a New Product or new Market Development:** These are expenditures needed to produce a new product or to expand into a geographical area not being presently served. Assume, for instance that the Research and Development Unit of a company has developed a new product that will cost ₦17million to introduce. After adequate market analyses, it was estimated that the company will make a net profit of ₦2.5 million from the project p.a. for 8 years. Should the company venture into the new line of business?

These are various types of decisions which are under the range of capital budgeting or investment decisions. Each of these involves comparison of different courses of action to ensure choosing what will keep the business afloat and profitable. It should be noted that these decisions involve long term assets as opposed to current assets known as working capital. However, the decision to invest in fixed assets would invariably be tied up with investments in current assets like inventories and account receivables.

The importance of the capital budgeting decision for business organizations cannot be over emphasized. This is because of the implications to the profitability and long term survival of a business that such decisions constitute. Once capital is committed to a project and it is in place, the organization is stuck to it for a length of time. Investment proposals also affect the risk profile of business organizations. If new projects makes the business more risky (e.g an organization in a viable food business now getting into real estate or furniture lines in a bid to expand and diversify) the valuation of the firm in the capital market will be affected and this would reflect in the price of its shares. Since investment proposals get appraised through evaluation of cash flows we will now turn to the discussion on cash flow estimation.

- (i)List three (3) types of investment projects known to you. (ii) Briefly describe the nature of one of these types of investment projects
  - (i)a. Automation of a process b. Replacement decision c. Lease or buy decision
  - (ii) Lease or buy is a type of investment in which a decision has to be made on which is more beneficial: outright purchase of an equipment or leasing (renting) it for a period of time.

1.2 Estimation of Cash Flows

Measurement of cash inflows and outflows is vital to capital budgeting decisions. The magnitude and timing of cash flows largely influence whether or not an investment project should be undertaken. The estimation of cash flows is usually made on a differential or incremental basis. That is to say that what we measure is the additional cash inflows and cash outflows that a project generates during its life. This is what matters; since we are aiming at deciding whether or not we should carry out the project.

We will define a few terms before we continue the discussion. We denote Revenue as R, Expenses as E; Income tax as T; the tax rate as t; and the Depreciation charge from a project for a particular year as D.

The operating cash flow from a project before tax considerations is:

$$R - E \qquad \text{….(6-1)}$$

The taxes on the operating cash flows are the tax rate t, multiplied by revenues less expenses and depreciation, that is: Tax on operating cash flow = t (R-E-D) …(6-2)

The cash flow for capital budgeting purpose then is:

Estimated cash flow = (R-E) – t (R-E-D) …. (6.3)

We can rewrite this after expansion as:

(R-E) – tR + tE + tD …(6.4)

= (R-E) (1-t) + tD … (6.5)

You will realize from this that the accounting definition of net income is different from what we have in expression (6.5) above. What we have in (6.5) is the correct cash flow because it fully takes care of inflow and outflow of funds incidental to the project under consideration after tax. We will later see how this equation works out in a practical situation. Before then, we will discuss important considerations in cash flow estimation.

In arriving at the incremental cash flows, separate estimates of cash inflows and cash outflows are normally made. The beginning cash outlay of a project will include 'the basic costs of the equipment plus installation costs, less the realized value of equipment that has to be sold (if any) to make room for the new project. We considered expenses in our derivation of an expression for the cash flow above. These cash outflows are normally adjusted against cash inflows of the same year to get the net cash flows. The net cash flow will be negative in a period when the cash outflow for that period exceeds inflows; the sign of the net cash flow will be positive otherwise.

Another consideration is the salvage value of the asset. Salvage value is the value of a capital asset at the end of a specified period. It is the current market price of an asset being considered for replacement in a capital budgeting problem. The salvage value of a new asset increases cash inflows of the last period while the disposal of an old asset decreases the initial cash outflow of the new project to be embarked upon to replace the old. The salvage value of the old asset at the end of a normal life would decrease the cash outflow of the last period of the life of the asset. An important consideration is the investment tax credit allowed in some situations whereby a certain percentage (say 20% or 25%) of the basic cost of the asset is deductable in computing profit after tax of the company, usually in the year of purchase. This is added to depreciation and normally deducted as a non-cash expense before tax is calculated.

The issue of whether or not depreciation is a source of funds to a firm could be confusing to finance students. In arriving at net income, depreciation (a non-cash expenditure) has to be subtracted from gross profit. To determine the actual cash flow from operations, depreciation is added back to net profit after tax liability has been determined. Depreciation represents an attempt to allocate the initial cost of an asset over its useful life. In essence, we attempt to match the annual expense of plant and equipment ownership against the revenue being produced. The charging of depreciation is purely an accounting entry and does not involve the movement of funds.

Finally, you would have noted from your previous studies in accounting that there are various methods of calculating depreciation charge for each year of the life of a project. You can review your study of accounting in this area to refresh your memory. What should be noted here is accelerated depreciation method like the sum of the years' digit (SYD) or the Double Declining Balance (DDB) methods that lead to reduced cash outflows in the first few years of a project in contrast with the traditional methods of straight line or reducing balance. We will now discuss a sample problem in estimation of a project's cash flow.

1.2.1 Sample Problem on Cash flow Estimation

The Karrie-go and Co. Ltd is considering a new investment which costs ₦12,000. It can be depreciated for five years using the SYD method. It is expected to have no salvage value at the end of its life. The equipment is expected to result in an increase in annual revenues (we assume all sales are in cash) of ₦7,000 and additional annual costs requiring cash outlays of ₦3,000 (not including depreciation of the equipment). The income tax rate is 50%. What is the expected cash inflow from the project?

Solution: The first step is to compute the annual income tax that the project will generate during its lifetime. This will then aid in computation of net annual cash proceeds.

Table 1-1 computation of Net Cash Flow

Year (1)	*Revenue (2)*	*Other Costs (3)*	*Depreciation (Net) (4)*	*Income Tax (5)*	*Cash Inflow (6)*
1	₦7,000	₦3,000	₦4,000	₦0	₦4,000
2	₦7,000	₦3,000	₦3,200	₦400	₦3,600
3	₦7,000	₦3,000	₦2,400	₦800	₦3,200
4	₦7,000	₦3,000	₦1,600	₦1,200	₦2,800
5	₦7,000	₦3,000	₦800	₦1,600	₦2,400

We can use the equation we derived previously to verify our computation above. We will see how we arrive at the computation for year 2. Using equation (6.5) = (R-E)(1+t) + tD, after-tax proceeds for that year = (₦7000 - ₦3000) (1-0.50) + 0.50 x 3,200.

= ₦4000 x 0.50 + 0.50 x ₦3,200

= ₦2000 + ₦1600 = ₦3,600

It is after we have derived the cash inflows and the cash outflows of a project that we now evaluate these net flows using various criteria (that we shall consider later in this unit and in the next) to decide whether or not the project is viable.

▪ (i)What is the basis of estimating cash flows of a new capital investment project of a firm? (ii) What are the cash flow effects of (a) salvage value of new asset (b) salvage value of an old asset?

o (i) Differential or Incremental Basis (ii) a. Salvage value of a new asset increases cash inflows of the last period of the life of the asset. b. Salvage value of an old asset decreases the initial cash outflow of the new project that replaces the old.

1.3 The Capital Budgeting Procedure

The process of capital budgeting is quite complex. It takes serious thinking to generate good investment proposals. It requires careful planning and large financial outlays for research and development.

In some firms **investment proposals** of any type can originate at any level from top management to factory hands. For example, the proposal to add a new product could come from the marketing division's experience in the sales field. Suggestions for replacement of an old machine or greater automation of the production process may come from the factory. These ideas are necessary to ensure that viable projects continue to be embarked upon by the firm.

After generating the proposals there must be rigorous **appraisal** of the sales and costs associated with particular projects. This would have to be done for several years into the future in many instances. For objectivity purpose, it is better that the **benefit/cost appraisal** be done by an independent and impartial group of experts in the firm. One would expect a group with a preponderance of marketing officers for the appraisal of a new product suggested by them to be overly optimistic in the projections of costs and revenue cash flows. On the other hand, production officers would be generally interested in acquiring the most modern equipment, not minding the fact that productivity may not improve. Objective appraisal will ensure that cash flow estimates are as accurate as possible taking cognizance of risks involved in the projects.

The next step in the capital budgeting process is **project selection**. The procedure for this will differ from firm to firm and for various types of projects. The capital budgeting process is very significant since large expenditures are normally involved. Besides, it affects the firm's operations on long term basis and is generally irreversible, once fund is committed to it. These reasons do often prompt top management input into the selection aspect of the capital budgeting procedure. However, projects are screened at various levels before top management sanction or reject it. Top management can also delegate authority for project approval giving commitment limits to various cadres of management.

Finally, funds are committed for capital expenditures after the approval. This step is the **project execution** stage. The plan for fund appropriation for various projects in a firm is what we call the capital budget. Project execution has to be monitored to ensure that funds are disbursed according to plan. Monitoring will also, at every stage, compare projections with actual cash flows to see areas where forecasting techniques have not been accurate enough. This will improve those techniques for future budgeting exercise.

We need to discuss further the selection criteria for capital projects, being a crucial step in the capital budgeting process. There are many criteria in use to select capital projects. However a good **investment appraisal method or technique** should have the following features:

(a) Ability to classify projects as acceptable or unacceptable
(b) Should provide the ranking of projects in order of the viability or preference
(c) Should make choosing among projects relatively easy
(d) Should be applicable to a wide range of projects, and
(e) Should discriminate favourably towards projects with larger benefits or higher earlier benefits.

By way of preview, two classes of investment selection criteria are discussed – These are the non-discounting and discounting techniques:
The non-discounting techniques are:

(a) Ranking by inspection
(b) Payback period, and
(c) Average (Accounting) rates of return method

The discounted cash flow (DCF) techniques are:
(a) Net present value method (NPV)
(b) Internal Rate of Return method (IRR)
(c) Profitability Index Method (PI)

We will proceed in the section that follows after this to discuss the non-discounted cash flow methods in turn. The discounted cash flow methods would be the subject of our discussions in study session 2 of this module.

- What are the first three steps in the capital budgeting process?
  - (i)Investment Proposal (ii) Benefit/Cost Appraisal (iii) Project Selection.

1.4 Ranking by Inspection

This is a rule of Thumb method and is applicable in a few limited cases. This is where project(s) to be selected are determined by mere cursory inspection of their cash flow patterns. We will discuss two basic cases where this is applicable.

(i) If two investments have identical cash flows each year but one of them still generates cash flow for more years or periods than the other, the project with the longer period of cash inflow is chosen.

Example: Let us assume that projects A and B have the following cash flows

Cash Flow In Naira For Periods 0 to 5						
Projects	0	1	2	3	4	5
A	-1,000	+650	+450	+320	…	…
B	-1,000	+650	+450	+320	+200	+100

Which of the two projects will you choose if we apply ranking by inspection?

Solution: Simple inspection reveal that both projects have identical cash flow pattern up to year 3 of the project. But project B still has cash inflows in years 4 and 5 which project A lacks, having terminated in year 3. So project B is superior to project A using the ranking by inspection criterion.

(ii) A second circumstance where ranking by inspection can be used is when cash flows are identical except for the timing. Assume that two investments have the same initial outlay and the same earning life and earn the same total proceeds. If cash proceeds are virtually identical but a project has greater cash inflow in at least one year than the other, that project is preferred.

Example: The cash flows of projects E and F are as shown below. Which of the two will be preferred using the ranking by inspection method?

Cash Flow In Naira For Periods 0 to 5						
Projects	0	1	2	3	4	5
E	-800	300	250	250	200	0
F	-800	0	250	300	250	200

In the above example, we discover that both projects have the same initial cash out lay and same total cash in flows. However the cash inflow for project E comes in earlier than that for project F. So project E is chosen as the more viable project.

The merit of the ranking by inspection method is that it is easy to use to rank the order of preference of two or more projects. It does not require any sophisticated calculations to use it. The demerit lies in the fact that choosing project by this cursory inspection of cash flows could lead to execution of unviable projects since time values of cash flows are not taken into consideration. In addition, the method can only be used in situations where cursory inspection would reveal differences in cash flow patterns. It does not have wide application in project appraisal.

- Indicate one circumstance under which Ranking by Inspection method can be used to choose which one out of two or more investment projects to execute?
  - It can be used if two investments have identical cash flows in each year or period, but one of them still generates cash flows for more years or periods than the other.

1.5 Payback Period Method

This method of investment appraisal is widely applied in industry. The payback period is defined as the length of time required for the stream of cash flows generated by an investment to equal the original cash outlay required by the investment. If the same amount of cash flow is generated year by year, the payback period can be determined by dividing the total original cash outlay by the amount of the annual cash proceeds expected. Then the payback period is

$$\text{Payback Period} = \frac{\text{Initial Outlay}}{\text{Annual Cash Flow}} \qquad \ldots (6\text{-}6).$$

Example: The initial cash outlay for project Q is ₦42,000 and has an annual cash flow of ₦14,000 for 6 years. Calculate the payback period for the project.

Solution: The payback period for the project is simply.

$$\frac{₦42{,}000}{₦14{,}000} = 3 \text{ Years}$$

When the cash flows are unequal the payback period can be found by adding up cash inflows until the total is equal to the initial cash outlay.

Example 2: Determine the payback period for a project which requires a cash outlay of ₦25,000 and produces the annual cash flows of ₦8,000; ₦9,000; ₦6,000; ₦6,000.

Solution: The addition of the cash inflow for the first 3 years amount to ₦23,000. It remains ₦2,000 generated in the fourth year to arrive at the figure of ₦25,000. We assume that the cash inflows are spread evenly during the year. Therefore the time required to recover ₦2,000 in the fourth year is (₦2000/₦6000) x 12 months = 4 months. The payback period for the project then is 3 years and 4 months.

Management would normally set the maximum payback period required for an acceptable project. All investment proposals for which the payback period is longer than this maximum are rejected. For instance if a firm is faced with the two projects in the examples above and maximum payback period is put at 2 years, none of the two projects will be accepted.

The payback period method can also be used to rank projects in order of preference. Here, highest ranking is given to a project with the shortest payback period while lowest rank is given to the projects with the longest payback period. This type of ranking is done for projects A to E in the illustration in table 6.2 below.

Table 1-2 Ranking of Independent Projects Using Payback Period

Project	Payback Period (Years)	Rank
A	1.5	2
B	1.0	1
C	2.0	4
D	3.3	5
E	1.8	3

Example 3: The following projects have the same initial cash outlay of ₦15,000 each. Calculate the payback period for each of them.

Cash Inflows for Projects				
Year	Project R	Project S	Project T	Project U
1	₦4,000	₦6,500	₦1,000	₦7,000
2	₦4,000	₦5,000	₦3,000	₦4,000
3	₦4,000	₦3,500	₦4,000	₦3,000
4	₦3,000	-	₦7,000	₦1,000
5	₦5,000	-		
6	₦6,000	-		

Solution: A simple addition of the cash inflow for each project from year to year shows that the projects have the payback periods shown in the table below:

Project	R	S	T	U
Payback Period	4 Years	3 Years	4 Years	4 Years

Merits of the Payback Period Method

The payback period method has some good features to recommend it for project selection. These include:

(i) It is simple to understand and easy to calculate when compared with other methods. This is why it is widely used in business organizations.

(ii) It is a good technique to employ in a capital shortage situation or when there is high degree of risk.

Demerits of the Payback Period Method

Though simple to use, the payback, period has some serious shortcomings making for analysts to go for other methods. These are

1.The method does not consider the total cash flows generated by a project. Consequently, it is not an appropriate measure of profitability. Using the projects in example 3 as illustration Project S will be accepted as having shortest payback period as compared with R; whereas R would be more profitable since it continues to generate more cash flows beyond the 3 years for project S.

2. The method fails to consider the pattern of cash flows; that is, the timing and the magnitude. By way of illustration, projects T and U in example 3 would be equally ranked using the payback period. But project U should be preferred since it generates greater cash flows in the earlier years.

a. There is no way to determine whether or not the wealth of shareholders are being maximized when the payback period is used.

b. There may be difficulty of determining the maximum payback period to use since it is more of a rule of thumb decision.

c. It does not take into account the time value of money.

Despite its demerits, the payback period method is still popular among financial managers. There is need for caution in its use, however. If the same cut off point is used regardless of project life, many short lived projects would tend to be accepted. This has the advantage of minimizing the risk of the business since short lived projects are generally less risky. However, the company may have to sacrifice future growth for current earnings in such situation.

- Why is the payback period method popular among financial managers despite its demerits?
  - It is simple to understand and calculate; it is useful in capital shortage situations or when there is a high degree of risk.

1.6 Average (Accounting) Rate of Return (ARR) Method

This method is also known as average return on book value. As the name implies the method uses accounting information as shown in the statements of accounts to measure the return on investment proposals. The ARR is found by dividing the average income after taxes by the average investment. The average investment would be equal to the initial outlay or investment plus salvage value (if any) divided by two (2). The ARR is an average rate and is determined by:

$$ARR = \frac{\text{Average Income}}{\text{Average Investment}} \times \frac{100}{1} \qquad \ldots (6\text{-}7)$$

Example: Thomas and Co. Limited has an investment proposal that will cost ₦500,000. The project will last for 5 years and depreciation is on straight line basis. The equipment will have zero scrap value at the end of the period. The project will generate annual cash operating revenues of ₦180,000; ₦200,000; ₦140,000, ₦180,000 and ₦160,000 for the 5 years. If tax rate is 50%, calculate the accounting rate of return (ARR) of the project.

Calculation of Accounting Rate of Return (Figures in ₦'000)						
Years	1	2	3	4	5	Avg
Operating Revenue ₦'000	180	200	140	180	160	172
Less Depreciation ₦'000	100	100	100	100	100	100
Earnings before Taxes	80	100	40	80	60	72
Taxes at 50%	40	50	20	40	30	36
Net Earnings after Tax	40	50	20	40	30	36

The average book value of the investment is:

$$\frac{₦500{,}000 + ₦0}{2} = ₦\,250{,}000$$

Average profit after tax is ₦36,000. The Average (Accounting) rate of return is

$$\frac{₦36{,}000}{₦250{,}000} \times 100 = 14.4\%$$

The project will be undertaken if the company's desired rate of return is less than 14.4%. So the ARR method will accept all those projects whose ARR is higher than the minimum rate set by management and reject those that have ARR less than the minimum rate.

The following strong points recommend the ARR technique: it is very simple to understand and use. Accounting information from which it can be calculated are readily available. It also uses the entire stream of income in calculating the average rate.

This criterion, however, suffers from several serious demerits. First, since it considers only the average return on book investment, there is no allowance for the fact that immediate receipts are more valuable than distant ones. Whereas the payback period method gives no weight to the more distant flows, the ARR method gives them too much weight. Secondly, the method does not depend on project cash flows, rather it depends on accounting income. Both are different since the accountant's decision on what constitutes depreciation charge are quite arbitrary and depends on what the accountant labels as capital investments, which are depreciated and operating expenses which are deducted immediately from each year's income. Hence the ARR method cannot fully determine cash flow and so should not be used as accept or reject rule. Besides, the method does not allow for the fact that the profits can be reinvested. Finally, it is not compatible with a firm's objective of maximizing the market value of shares since it ignores the opportunity cost of money.

- Write out ONE merit and ONE demerit of the ARR method.
  - Merit: It is very simple to understand and use; Demerit: Since the method considers only the average return on book investment, there is no allowance for the fact that immediate receipts are more valuable than distant ones.

Summary of Study Session 1

We first focused on the various types and nature of decisions a manager would make in planning (budgeting) on a firm's line(s) of investment in this unit. These include decision involving whether or not to (a) automate a process (b) replace an existing equipment, (c) lease or buy an equipment, (d) expand production of an existing product and or target market and (e) introduce a new product into the market and or develop a new market for the firm's products. These decisions involve making choices that determine the survival of the firm.

A fundamental step in making an investment decision is estimation of cash flows of the alternative lines of investment that should be taken. By way of recapitulation, we arrived at the correct cash flow for an investment action through the use of the following expression:

Cash flow (for capital budgeting purpose) =

$$(R-E)\ (1 - t) + tD \qquad \ldots\ldots\ldots\ldots \qquad (6\text{-}5)$$

Where: R = Revenue

E = Expenses

T = Income tax

t = tax rate

D = depreciation charge for a project in a particular year.

The four steps in the capital budgeting process are (a) investment proposal (b) benefit/cost appraisal (c) project selection and (d) project execution. The third step on project selection is vital in financial management. It is important that a good investment appraisal method or technique have features that make it possible to choose viable projects with larger net benefits or higher earlier net benefits.

Important equations/formulae for the non-discounted cash flow investment appraisal techniques (ranking by inspection, payback period method and accounting rate of return method) that we discussed in this unit are.

$$\text{Pay back period} = \frac{\text{initial outlay}}{\text{Annual cash flows}} \quad \text{….. 6-6}$$

This formula is useful only when same amount of cash flow is generated year by year while the project lasts.

$$\text{ARR} = \frac{\text{Average income}}{\text{Average investment}} \times \frac{100}{1} \quad (6\text{-}7)$$

Self-Assessment Questions (SAQs) for Study Session 1

With your completion of this study session, you can now assess how well you have achieved its Learning Outcomes by answering the following questions. You can check your answers with the Solutions to the Self Assessment questions in the appendix to this course text.

SAQ 1.1 (tests Learning Outcome 1.1)

. Briefly explain why the investment decision is vital to business organizations?

SAQ 1.2 (tests Learning Outcome 1.2)

The Caxton and Co. Ltd is contemplating buying an ice-cream machine at a cost of ₦25,000. The ice-cream machine is expected to last five years. Sales that will be realized over the life of the machine are expected to be as follows:

Year	1	₦12,000
	2	₦15,000
	3	₦15,000
	4	₦16,000
	5	₦15,000

All sales are in cash. There will be 10% commission on sales per year. The scrap value of the machine is expected to be ₦2,500. Other operating expenses for each year apart from depreciation and commissions are expected to be as follows:

Year	1	₦3,000
	2	₦3,750
	3	₦3,750
	4	₦4,200
	5	₦5,500

The ice cream machine is to be depreciations over the life of the project on a straight line basis. The company is liable to pay 50% tax on the profit of each year. Estimate the cash flows of the project.

SAQ 1.3 (tests Learning Outcome 1.3)
List four (4) characteristics of a good investment appraisal method.
SAQ 1.4 (tests Learning Outcome 1.4)
The following data relate to two independent investments projects.

Cash flow in ₦ for period 0-5						
Project	0	1	2	3	4	5
K	-25,000	7500	6500	6500	8000	5,000
L	-25,000	7500	7500	6500	8500	5000

Rank the two projects by inspection. Give reason(s) for your ranking.

SAQ 1.5 (tests Learning Outcomes 1.5 and 1.6)
A company is considering an investment proposal costing ₦50,000. The facility has a use expectancy of 5 years and no salvage value. The company's tax rate is 45% and no investment allowance is allowed.
The firm uses straight line depreciation. The estimated cash flows before tax (CFBT) from the proposed investment proposal are as follows:

Year	CFBT
1	₦10,000
2	₦11,000
3	₦14,000
4	₦15,000
5	₦25,000

Compute (i) Payback period (ii) Average rate of return for the project.

REFERENCES

Khan. M. Y and Jain, P.K. 1992 Financial Management Text and Problems Tata McGraw-Hill Publishing Company Ltd. New Delhi 2nd edition pp 171-199.

Olaoluniyi, O. 2010 Elements of Business Finance Panaf Publishing Inc. Abuja 2nd edition pp. 81-98.

Pandy, S. M. 1999 Financial Management Vikas Publishing House PVF Ltd New Delhi 8th ed. pp. 391-484.

Should you require more explanation on this study session, please do not hesitate to contact your e-tutor via the LMS.

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iag@dli.unilag.edu.ng
08033366677

Study Session 2 Capital budgeting under certainty II

Introduction

Our previous immediate past Study Session (1) served as introduction to the capital budgeting topic which we mentioned to be the core of the Business Finance I course. We ended that Study Session with discussions on features of various non-discounted cash flow investments appraisal techniques. Our attention in this Study Session is on features of the discounted cash flow (DCF) investment appraisal techniques. These techniques are in conformity with the conventional wealth maximization objective of financial management. This is with respect to application of the time value of money concept. The use of interest rate compounding, and discounting computations that we discussed in the first module of this course text come into focus prominently in this and subsequent Study Sessions. Knowledge and application of these simple calculations is all you need to work through this and subsequent Study Sessions. In addition, it is important that you grasp the characteristics, merits and demerits of each of the DCF appraisal technique.

Learning Outcomes for Study Session 2

When you have studied this study session you should be able to:

2.1 Describe the features, application, merits and demerits of the Discounted Payback Period method for appraising investment projects cash flows.

2.2 Describe the features, application, merits and demerits of the Net Present Value (NPV) method for appraising investment projects cash flows; plus describe how the NPV method can be used to select between mutually exclusive projects or rank independent projects in order of viability for implementation.

2.3 Explain the features, application, merits and demerits of the Internal Rate of Return (IRR) method for appraising investment projects cash flows; plus describe how the IRR method can be used to select between mutually exclusive projects or rank independent projects in order of viability for implementation.

2.4 Describe the features, application, merits and demerits of the Profitability Index (PI) method for appraising investment projects cash flows; plus describe how the PI method can be used to select between mutually exclusive projects or rank independent projects in order of viability for implementation

2.1 Discounted Payback Period Method

In some firms, the cash flows from investment proposals are discounted before calculating the payback period. This discounted payback rule seeks to find out the number of periods the project have to last in order to pay back the present value of the initial outlay. This adjustment to the payback rule overcomes the objection under the ordinary payback rule that equal weight is given to all flows before the cut-off date. However the discounted payback rule still takes no account of any cash flows after the cut-off date.

Let us discuss an illustration. Assume we have two mutually exclusive projects X and Y. Each requires ₦10 million initial out lay and is expected to generate equal stream of cash flows starting in year 1. The cash flow for investment X is ₦3.5 million and lasts for 7 years. The cash flow for investment Y is ₦3 million and lasts for 12 years. We will assume that the appropriate discount rate for each project is 10 per cent. It is obvious from the given information that since X has larger cash receipts than Y in each year of its life it, would have the shorter discounted payback. Investment X gives the shorter discounted payback of a little less than 4 years since the present value of ₦3.5 million for 4 years at 10 percent is ₦11.094 million. The discounted payback of Y is a little more than 4 years since the present value of ₦3 million for 4 years at 10 percent is ₦9.51 million (we simply use the formula for present value of ordinary annuity equation here, to compute the figures).

We can deduce however that investment Y is better since the present value of its cash inflow for 12 years, which is ₦20.4 million is higher than the present value of the cash inflow for X for its 7 years life which is ₦17.04 million.

We see in this example that discounted payback method is a bit better than undiscounted payback. It gives expression to the fact that a naira at the beginning of the payback period is worth more than a naira at the end of the payback period. However, the discounted payback method still depends on the choice of an arbitrary cut off date and also ignores all cash flows after that date.

- Why is the Discounted Pay Back Period Method a bit better than the undiscounted Pay Back Period Method?
  - The Discounted Pay Back Period Method seeks to find out the number of periods the project have to pay back the present value of the initial outlay. This adjustment to the Pay Back rule overcome the objection under the undiscounted pay back rule that equal weights are given to all cash flows before the cut off date.

2.2 Net Present Value (NPV) Method

You will recall that we learned how to calculate the present value of a stream of expected future cash flows in module 1, study session 3. This concept is what we are applying in appraising investment projects in using the discounted cash flow techniques.

Capital investment projects normally involve an initial outlay of funds that is later followed by a stream of future cash inflows.

The Net present value is a project's net contribution to wealth - present value of cash inflow minus initial investment. The Net present value (NPV) method is applied to investment appraisal by subtracting the present value of cash outflows from the present value of cash inflows using an appropriate discount rate. Let us discuss an illustration.

An investment of ₦200 million will produce a cash inflow of ₦120 million in a year's time, a further ₦110 million in year 2 and ₦80 million in year 3. What is the net present value of this investment if a return of 12 per cent is required?

We can set up a table as shown below to facilitate clear layout of the calculations that we need to do, to arrive at the net present value.

Table 2.1: Computation of Net Present Value of Project P

Period (1)	Cash Flows (2) ₦'m	Present Value Factor at 12% (3)	Present Value (4) = (2) x (3)
0	-200	1.000	-200
1	120	0.8929	107.15
2	110	0.7972	87.69
3	80	0.7118	**56.94**
			NPV = 51.78

With the above, we can now formulate a general symbolic definition of net present value. If we represent cash flow in period t as C_t, the required rate of interest as k. In symbols:

$$NPV = \sum_{t=0}^{n} \frac{C_t}{(1+k)^t} \qquad \ldots \qquad \ldots \qquad (2\text{-}1)$$

Note that we are summing from t = 0 thereby including the initial cash outlay. You should further note that C_t could take on positive or negative values depending on whether it is a net cash inflow or net cash outflow respectively.

You should note the following formulae for a more practical application of the net present value.

(a) $$NPV = C_0 + \sum_{t=1}^{n} \frac{C_t}{(1+k)^t} \qquad \ldots \qquad (2\text{-}1a)$$

This is applicable when cash flow is a constant amount.

(b) $$NPV = C_0 + \frac{C_1}{(1+k)} + \frac{C_2}{(1+k)^2} + \ldots + \frac{C_n}{(1+k)^n} \qquad \ldots\ldots\ldots\ldots \qquad (2\text{-}1b)$$

This second formula is applicable when cash flows are not a constant sum but vary from one period to the other.

2.2.1 Application of the NPV Method

When applied to capital budgeting decisions, the NPV rule simply states that: Accept an investment project if its net present value is greater than zero when evaluated at the appropriate required rate of interest. Reject the project otherwise. The investment project analyzed in table 2-1 above will consequently be accepted as viable since its net present value is positive. The market value of a firm's share is expected to increase if projects which generate positive net present values are implemented.

The NPV method can hence be used to choose between mutually exclusive projects by taking the one with the highest NPV. It can also be used to rank projects in order of the magnitude of the net present values, that is, first rank will be given to the project with the highest net present value and so on.

2.2.2 Merits and Demerits of the NPV Method

The NPV method has some advantages that highly recommend it as an investment appraisal technique. This includes:

1. It recognizes the time value of money
2. It considers all cash flows over the entire life of the project to arrive at the net present value.
3. The NPV method is consistent with the objective of maximizing the wealth of the firm's shareholders.

The NPV method has some limitations which are:

1. It is relatively difficult to use and there may be need to use costly and sophisticated computers for appraisal of complex projects using the NPV method.
2. The calculation of the net present value assumes that the discount rate which is usually the firm's cost of capital is readily known. But the cost of capital is quite a difficult concept to grasp and measure in practice.
3. It may not give an optimal result when the projects being compared involve different amounts of investment. The project with the higher net present value may not be desirable if it also requires a large investment.

- ▪ List the two ways the NPV method can be used in appraising investment projects.
- o The NPV method can be used (i) to choose between mutually exclusive projects by taking the one with the highest NPV (ii) it can also be used to rank projects in order of the magnitude of net present values i.e. first rank will be given to the project with the highest net present value and so on.

2.3 Internal Rate of return (IRR) Method

This is another discounted cash flow technique which takes account of the magnitude and timing of cash flow. The internal rate or return (IRR) is that discount rate which equates the present value of cash inflows with the present value of the cash outflows. This implies a rate at which the present value of the investment is zero. This means that to find the IRR for an investment project lasting t years, we must solve for IRR (that is r) in the following expression.

$$NPV = \frac{C_1}{(1+r)} + \frac{C_2}{(1+r)^2} + ... + \frac{C_t}{(1+r)^t} \quad .... \quad .... \text{ (2-1c)}$$

The actual calculation of IRR usually involve trial and error. Let us see an illustration:

Adonis and Co. Ltd is considering an investment in new equipment costing ₦50,000. The project will generate ₦14,000 cash flow per year for five years. Calculate the internal rate of return of the project.

Solution: We have to carry out a trial and error calculation using various rates of interest to discount. In the first instance we will select a rate of 10% and 11% and calculate the NPV as follows:

Table 2-2: Trial and Error Computation of IRR I

Period	Cash Flow ₦	PV Factor at 10%	PV at 10% ₦	PV factor at 11%	PV at 11% ₦
0	-50,000	1.00	-50,000.00	1.00	-50,000.00
1	14,000	0.9091	12,727.27	0.9009	12,612.61
2	14,000	0.8264	11,570.25	0.8116	11,362.71
3	14,000	0.7513	10,518.41	0.7312	10,236.68
4	14,000	0.6830	9,562.19	0.6587	9,222.23
5	14,000	0.6209	**8,692.90**	0.5935	8,308.32
			3,071.01		1,742.56

Since we have not arrived at our target of ₦ 0.0 NPV at any of these rates, and since NPV is positive at each of the two rates, we will try higher rates of interest. The NPV at rates of 12% and 13% is shown in table 2-3 below.

Table 2-4: Trial and Error Computation of IRR II

Period	Cash Flow ₦	PV Factor at 12%	PV at 12% ₦	PV factor at 13%	PV at 13% ₦
0	-50,000	1.00	-50,000.00	1.00	-50,000.00
1	14,000	0.8929	12,500.00	0.8850	12,389.38

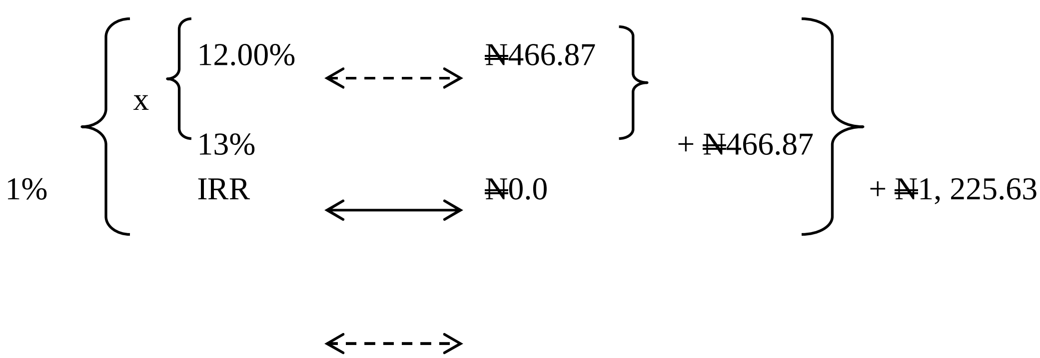
2	14,000	0.7972	11,160.71	0.7831	10,964,.05
3	14,000	0.7118	9,964.92	0.6931	9,702.70
4	14,000	0.6355	8,897.25	0.6133	8,586.48
5	14,000	0.5674	**7,943.98**	0.5428	7,598.64
		NPV	**466.87**		-758.76

From our calculations here it is evident that the rate of interest at which NPV = 0 lies between 12% and 13% since NPV is positive at the former rate and it is negative at the later. To zero in on the actual internal rate of return we may need to try out the calculations at other rates like 12.75%, 12.50% and so on. This could be cumbersome and we may not even be able to arrive at the actual rate, more so if that rate is to an odd two places of decimal like 12.08%, 23.69%, etc. as we would find it to be in this example. However, there is a way out. Once we are able to get a range of two values between which the internal rate of return lies as in our example here, we can interpolate between these two sets of values to arrive at the rate at which NPV is ₦0.0 which is the internal rate of return. Let us see how this works out in the example we have been discussing here.

2.3.1 Interpolation to Arrive at IRR

Since the IRR lies between the 12% and 13% rates of interest, the desired rate at which NPV is ₦ 0.0 is the same fraction of the way from 12% to 13% as ₦0.0 is from ₦466.87 to – ₦758.76. This is made clear from the diagram below

(Figure 2-1: Illustration of Interpolation to arrive at Actual IRR)



We now construct a proportion from the illustration to solve for x

$$\frac{x}{1.0} = \frac{+466.87}{+1225.63}$$

$x = 0.3809$

The desired IRR is 12% + 0.38% = 12.38%

More concisely the interpolation can be calculated as follows to arrive at IRR:

$$12\% + \frac{₦466.87}{₦466.87 - (-₦758.76)} \left(13\% - 12\% \right)$$

$$= \quad 12.38\%$$

By way of recapitulation of the steps involved in calculating the IRR, the approach is to select any rate of interest to compute the net present value of cash flows. If the calculated net present value of the expected cash flows is positive, a higher rate should be tried, if negative, a lower rate should be tried. Once you arrive at two rates between which the IRR should fall (that is one at which the NPV is positive and the other at which NPV is negative for same cash flows), interpolation as above can then be used to arrive at the exact internal rate of return. The trial and error way of computation are built into some financial calculators and it can arrive at the actual rate in less than a minute once the relevant data are keyed in. Your sure bet is to understand the interpolation method. Using a programmable financial calculator, the IRR for the problem we have been illustrating was given as 12.3762% which essentially tallies with what we arrived at using interpolation.

The IRR method can be applied to investment project appraisal in one of two ways. It can be used to appraise a single project when its IRR is compared to the minimum required rate of return. The project is accepted if its IRR is greater than the minimum required rate of return and it is rejected otherwise. Secondly, the internal rate of return of various projects can be calculated and used to rank such projects with the one with the highest IRR being given the highest ranking.

2.3.2 Merits and Demerits of the Internal Rate of Return Method

The internal rate of return method has operational merits and demerits which you should do well to note. Some of its merits are:

(a) Similar to the NPV method; it considers the time value of money.

(b) It takes into consideration cash flows over the entire life of the project.

(c) It gives the rate of return on capital for each individual project that it appraises.

(d) We do not need to first derive the cost of capital for a firm to use IRR. The IRR shows the return for the project and gives a good idea of how profitable each project is.

(e) It conforms to the objective of maximizing shareholders' wealth to a large extent.

The IRR method has the following operational limitations to its use:

(i)It requires complicated computation and it is not easy to understand in practice.

(ii)It assumes that the intermediate cash flows generated by the project are reinvested at the internal rate of return of the project. This is far from the truth since IRR is most likely to be different from the prevailing interest rates in the economy.
(iii)The IRR method may come up with results inconsistent with the NPV if the projects differ in the expected duration or magnitude of cash outlays or timing of cash flows.
(iv)A unique answer may not be derivable under the IRR method. Results may be negative in some situations while multiple rates may be derived in some other situations. We will touch more on some of these problems and the way around them in Study Session 3 of this module.

- List two (2) limitations to the use of the IRR method for project appraisal.
  - Two operational limitations of the IRR method are:(i) It requires complicated computation and it is not easy to understand in practice (ii) The IRR method may come up with results inconsistent with the NPV if the projects differ in the expected duration or magnitude of cash outlays or timing of cash flows.

2.4 Profitability Index Method

This method, also called the benefit/cost ratio is another method that applies the concept of the time value of money. The profitability index (PI) is the ratio of the present value of future cash benefits, at the required rate of return to the initial cash outflow of investment. It may be a gross or net ratio. It is net when we subtract the initial outlay from the present value of the cash flows or 1.0 from the gross index. The net index is also termed the excess value project index (EVPI). We can define the profitability index symbolically as follows:

$$PI(B/C) = \frac{PV}{C_0} \qquad \text{……..} \qquad (2.2)$$

Where PV stands for the present value of a project's expected cash inflow, while C_0 is the initial cash outlay.

More explicitly we could define the formula for profitability index in terms of what we considered previously as

(a) $$PI = \sum_{t=1}^{n} \frac{\frac{C_1}{(1+k)^t}}{C_0} \qquad \text{………} \qquad (2.2a)$$

This is the Gross Profitability Index, and

(b) $$PI = C_0 - \sum_{t=1}^{n} \frac{\frac{C_1}{(1+k)^t}}{C_0} \qquad \text{………} \qquad (2.2b)$$

The second formula is the net profitability index.

The PI or B-C ratio will accept a project if the project's profitability index (PI) is greater than one (1). Such project will have a positive net present value. Let us now discuss an illustration.

2.4.1 Sample Problem on the PI Method

The Charisma Investment Company is considering a project with initial cash outlay of ₦400,000. This project will generate ₦130,000, ₦190,000 and ₦260,000 in 3 years of its life. Calculate the profitability Index (PI) of the project. Assume a 13 percent cost of capital (required rate of return).

The calculation for this is shown in table 2.4 below:

Table 2-4: Calculation of Profitability Index

Year	**Cash inflows ₦**	**Discount Factor at 13%**	**Present Value** *N*
1	130,000	0.8850	115,050.00
2	190,000	0.7831	148,789.00
3	260,000	0.6931	**180,206.00**
			444,045.00
		Less initial outlay	<u>400,000.00</u>
		NPV.	44,045.00

$$PI(gross) = \frac{444{,}045}{400{,}000} = 1.11$$

The profitability Index is similar to the NPV and IRR and it is a conceptually sound method of appraising investment projects. It gives due consideration to the time value of money. Administratively it requires more computation than the traditional methods but less than the IRR method. Projects can be ranked in accordance with the magnitude of the PI with the highest rank being given to the project with the highest PI, while lowest rank is given to the project having the lowest PI.

When compared with the Net Present Value Method PI would in most cases arrive at the same accept or reject rule since PI would be greater than 1.0 when the project's net present value is positive. However there may be conflict between PI and NPV in deciding between mutually exclusive projects. If, for example, Charisma Investment Company has a project B that is mutually exclusive to the one we considered above with an initial cash outlay of ₦800,000 and cash inflows of ₦360,00 at the end of each of 3 years. Let us now evaluate the NPV and PI of this project at 13% required rate of return for Charisma Investment Company. The calculations are shown in table 7.5.

Table 2.5: Calculation of Profitability Index

Year	Cash inflows ₦	Discount Factor at 13%	Present Value(₦)
1	360,000	0.8850	318,600.00
2	360,000	0.7831	281,916.
3	360,000	0.6931	249,516.00
			850,032.00
		LESS INITIAL OUTLAY	800,000.00
		NPV	50,032.00

$$PI = \frac{850,000}{800,000} = 1.063$$

The NPV method would recommend project B while the PI method would recommend project A. But the question is, which one do we accept? Except in situation of capital rationing we should always accept the project with the higher net present value.

- A Finance Director has calculated the Profitability Index (PI) for a new proposal to be 1.24. The proposal's initial outlay is ₦9000. Find out the proposal's annual cash inflow if it has a life of 5 years and the required rate of return is 8 per cent.
  - ₦2,795

2.5 Net Terminal Value Method

This method assumes that the project's cash inflows are reinvested as they are received at the firm's required rate of return. The terminal value of the inflows are hence calculated and the terminal value of the initial cash outlay is deducted from it to arrive at the net terminal value (NTV). So, what we do in the case of net terminal value method is to compound the cash flows forward to the end of the duration of the project rather than discounting that we do under the present value rule.

The accept or reject rule under the net terminal value method then is; accept any project with positive net terminal value. When all cash flows are compounded forward at the reinvestment rate the NTV method will always make management arrive at the same decision as the NPV method. You could verify this by applying compounding to the terminal period, the cash flows in some of the examples we have discussed in this chapter at the respective rates of interest.

- Compare the NTV with the NPV method as to their similarities and differences.

- The NTV method compounds the cash flow forward to the end of the duration of the project. In contrast the NPV method discounts cash flows to the present. When all cash flows are compounded forward at the re-investment rate, the NTV method will always make management arrive at the same decision as the NPV method.

Summary of Study Session 2

We have been discussing the capital budgeting decision process and project appraisal methods in this chapter. We saw that the basic steps in the capital budgeting process are:

(i) Project generation (ii) Project evaluation (iii) Project selection and (iv) Project execution.

The evaluation step is very crucial since its outcome will determine whether or not sound investment decisions are made. A good evaluation technique should be able to adequately help a financial manager simply make the decision to accept or reject a project, it should rank independent projects in order of preferences and should be biased towards projects that bring in more net benefits as opposed to less.

Some basic formulas for applying the appraisal techniques discussed in this chapter are:

1. NPV = $-C_0 \sum_{t=1}^{n} \frac{C_1}{(1+K)^t}$ ….. (2-1a)

2. To derive IRR we solve for k in NPV =

$$C_0 + \frac{C_1}{(1+K)} + \frac{C_2}{(1+K)^2} + \ldots + \frac{C_1}{(1+K)^t} = 0 \qquad \text{….. (2-1b)}$$

3. PI or B/C ratio = PV/C_0 ……. (2-2)

Self Assessment Questions (SAQs) for Study Session 2

Having completed this study session, you can now assess how well you have achieved its Learning Outcomes by answering the following questions. You can check your answers with the Solutions to the Self Assessment questions in the appendix to this course text.

SAQ 2.1 (tests Learning Outcome 2.2).

A project provides annual net cash flow of ₦15,000 for 5years. What is the project NPV if its initial cost is ₦50,000 and cost of capital is 10%: (a) ₦56,862

(b) ₦6,862 (c) ₦6,862 (d) ₦41,575 (e) ₦91,575

SAQ 2.2 (test Learning Outcome 2.2)

Assuming the net present values of a project at three different discount rates are as follows, determine the project's IRR.

Discount rate Should you require more explanation on this study session,

please do not hesitate to contact your e-tutor via the LMS.

Are you in need of General Help as regards your studies? Do not hesitate to contact the DLI IAG Center by e-mail or phone on:

iag@dli.unilag.edu.ng

08033366677

NPV

%

₦'000

10

500

20 200

30 -100

(a) 27% (b) 26% (c) 18% (d) 18.3% (e) none of the above

Each of the following mutually exclusive projects require an initial outlay of ₦200,000.

SAQ 2.4 (tests Learning Outcomes 2.2, 2.3 and 2.4)

The company's required rate of return is 10%.

	Cash flow at time period (₦'000)				
	1	2	3	4	5
Project A	50	50	50	50	190
Project B	80	80	80	30	0
Project C	100	100	10	-	-

(i) Rank each project applying net present value, internal rate of return and profitability index.

(ii) Explain why the three capital budgeting appraisal techniques yield conflicting answers.

(iii) Recommend the project to be adopted and give reasons.

Study Session 3 Net Present Value (NPV) and Internal Rate of Return (IRR)

Introduction

We discussed the discounted cash flow (DCF) techniques of investment appraisal in the last study session. The more prominent techniques of the ones discussed are the NPV and IRR techniques. Writers and practitioners in the field of financial management have discovered that these techniques, though similar in their features and usefulness do not in some cases of mutually exclusive projects arrive at same selection decision. They do in fact give conflicting decisions in deciding between mutually exclusive projects when we have differences (a) in the scale of the projects, (b) pattern of cash flows, (c) duration of the projects and (d) re-investment rate. Our focus in this study session is on how to tackle such conflicts and arrive at the right decision in investment project selection. Bear it in mind that while various avenues can be employed to resolve these conflicts, the NPV technique have been found to be more useful in virtually all situations.

Learning Outcomes for Study Session 3

After studying this session you should be able to:

3.1 Use a graph to determine the IRR of an independent project, and so determine that IRR and NPV give same answer in such situation.

3.2 Explain the use of incremental cash flow analysis to solve scale problem on choice between two mutually exclusive projects with different initial cash outlays or difference in cash flow patterns that could create conflicts of choice between NPV and IRR investment appraisal techniques.

3.3 Resolve issue of conflict of choice between NPV and IRR techniques in cases of mutually exclusive projects with different lives.

3.4 Explain why the NPV technique is superior to the IRR technique when we consider the reinvestment rate

assumption and mutually exclusive projects with loan type cash flows

3.5 Explain why the NPV technique is superior to the IRR technique when we consider projects with multiple IRR

and projects with no IRR.

3.1 Independent Investments: Accept or Reject Decisions

There are many situations in industry when the investment decision to be made is whether or not to accept or reject a project when the cash flows of the project do not affect the cash flows of other projects. This type of project is referred to as an independent investment. The accept or reject rule for the internal rate of return (IRR) method as we saw previously is to accept an independent investment if it's IRR is greater than some minimum acceptable discount rate. We will illustrate this with an example.

We have an investment N with assumed initial cash outlay at period 0 of ₦112,000 and cash inflows of ₦40,000; ₦50,000 and ₦60,000 at end of period 1, 2, and 3 respectively. A way to calculate the IRR is to plot some combinations of NPV and discount rate on a graph like we do for this illustration in figure 3-1. We then connect the points with a smooth curve and read off the discount rate at which NPV = 0.

In our diagram for this illustration the IRR is found to be 15 per cent. If the minimum acceptable rate of discount is less than 15 per cent, the net present value is positive. With rates greater than 15 per cent, the net present value is negative. The project is hence accepted if the required rate of discount is less than or equal to15 per cent.

When we compare the required rate of interest with the IRR on a project, we are in effect asking whether the project has a positive NPV. This is true in all cases. We find that the IRR rule will give the same answer as the net present value rule whenever the investment consists of one or more periods of cash outlays followed only by periods of cash inflows. Since most independent investments have cash flow patterns that meet this specifications, it is fair to conclude that IRR and NPV methods would give the same recommendations for independent investments. In such situations, the NPV of a project is a smoothly declining function of the discount rate as in the illustration shown in figure 3-1.

- Under what condition will the IRR and NPV methods give the same recommendation for independent investments?
    - When the investment consist of one or more periods of cash outlay followed only by periods of cash inflows.

3.2 Mutually Exclusive Investments

Investment projects that are substitutes to one another are mutually exclusive investments. Examples include alternate projects that use the same plant size, decisions as to transportation provisions for an oil company – pipelines or tankers; projects that call for using a single non-duplicatable asset such as a given stock of managerial talent etc. Mutually exclusive investment alternatives are found to be common in practice.

The internal rate of return (IRR) method has been found to give less correct recommendations for mutually exclusive investments than those that result from the application of the NPV method, because it reflects the average rather than the incremental cash flow. We will discuss some categorical circumstances where this occur:

These are situations where we have:

1.Differences in the scale of projects

2. Differences in the pattern of cash flows

3. Differences in the duration of the projects and

4. Differences in re-investment rate

We will now discuss some of these and show how to resolve the conflicting evaluation results when we use NPV and IRR techniques in these situations.

Discount Rate (%)	0	5	10	15	20	25	30	35	40
NPV (₦'000)	38	23.3	10.8	0.0	-9.2	-17.3	-24.3	-30.6	-36.1

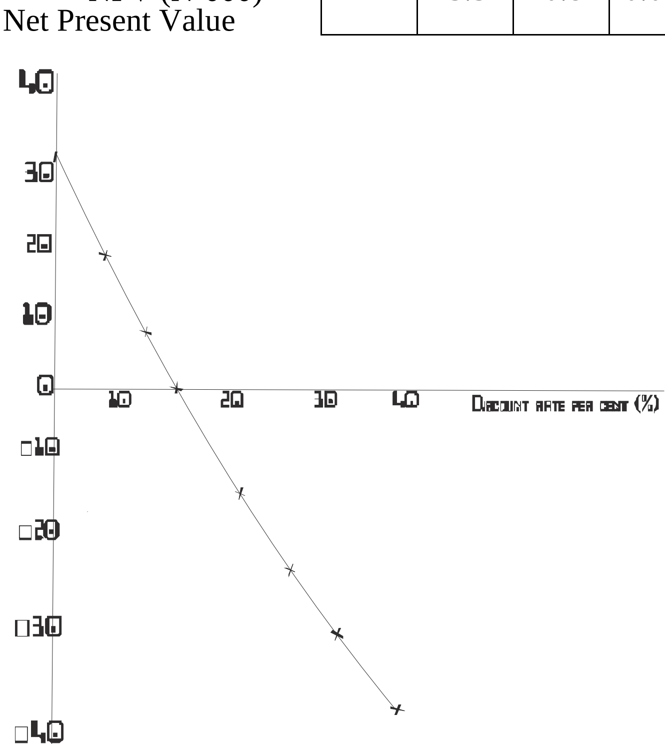


Figure 6-1 A project's NPV graphical profile

3.2.1 The Scale Problem

Consider two mutually exclusive investment projects, P and Q with the cash flows shown below:

Cash Flows (₦'000s)

Project	C_0	C_1	C_2	C_3	IRR Percent	NPV at 13%
P	-400	130	190	260	18.68	44.04
Q	-800	360	360	360	16.65	50.01

The data given in the illustration show that both projects are good. A quick conclusion using the IRR rule would be that project P is more desirable, since it has the greater IRR. But this answer is wrong. The NPV rule shows that investment Q is better since it gives a greater net present value of cash flow. To buttress this, we can consider that a rate of return of say 200 per cent on a naira investment is a poor substitute for a rate of 20 per cent on ₦2,000 if only one of the investments can be undertaken. So we may have problem with the IRR rule when mutually exclusive projects of different magnitude of initial outlay and consequent cash inflow are under consideration.

We can make the IRR useful in this type of situation, however. We do this by looking at the internal rate of return on the incremental flows. In doing this we first consider the smaller project (P in our example). It has an IRR of 18.68 per cent when the required rate of return for the company is 13 per cent, so it is acceptable. We then ask whether or not it is worthwhile making the additional ₦400,000 investment in Q. The incremental flows from undertaking Q rather than P are as follows:

Cash Flows (₦'000s)						
Project	C_0	C_1	C_2	C_3	IRR Percent	NPV at 13%
P-Q	-400	230	170	100	14.01	5.98

The IRR on the incremental investment is 14 per cent which is still greater than the 13 per cent required rate of return. This will make you to prefer project Q.

We see in this illustration that except you consider incremental cash flows, IRR is unreliable in ranking projects of different patterns of cash flow over time. We will now illustrate this last aspect next.

3.2.2 Difference In Cash flow pattern

Consider a comparison between project R whose cash flow pattern is shown below and project P that we discussed previously in Section 8.3 as mutually exclusive projects. The cash flows, IRR and NPV values for project R is shown below:

Project	C_0	C_1	C_2	IRR %	NPV at 13%
R	-400	220	310	19.73	37.47

Here again one can easily jump into a conclusion that project R is better if IRR is considered. But project P has a higher NPV value and should be preferred. The scale problem may not be easily identified here but it is there all the same. It is basically one of timing of the cash inflow since the cash outflows are the same, but project P has cash inflow in year 3 which project R has not. It is to be noted that the cash inflow for project R is greater than for P in the two years that it lasts. However, to see why project P is better consider the terminal value of the cash inflows of projects P and R as at the end of the third year, at 13 per cent compounded rate: For project P this will be ₦130 $(1.13)^2$ + ₦190 (1.13) + ₦260 = ₦640.70 while for R this is ₦220 $(1.13)^2$ + ₦310 (1.13) = ₦631.22. So the fund available is higher if project P is accepted.

We find that the IRR rule ignores the fact that idle funds must be invested at the cost of capital. If you construct a net present value graph for projects P and R (you could try this out as previously explained), you will discover that at lower rates of interest NPV of P is higher while NPV of R is higher at greater rates of interest. The total cash inflow of P is greater but tends to occur later. Hence the NPV rule gives the right decision of which project to choose in this situation.

- Briefly explain how conflict between IRR and NPV can be resolved in choosing between two mutually exclusive projects when we have scale problem.
    - By consideration of incremental cash flows.

3.3 Projects with Different Lives

Yet another cause of conflict between NPV and IRR is in situations where we want to choose between long-lived and short-lived projects, that are mutually exclusive. We will consider an illustration. Assume that projects U and V whose cash flows are shown below are mutually exclusive. Project U lasts for 6 years while V lasts for 3. The NPV of both projects at 13% and their IRR are shown in the table.

Cash Flows (₦'000)									
Project	C_0	C_1	C_2	C_3	C_4	C_5	C_6	IRR Percent	NPV at 13%
U	-30	6	8	8	10	12	12	18.63	5.53
V	-30	10	15	20	-	-	-	20.61	4.46

Using the NPV rule one will want to choose project U, while the IRR rule will rather choose V. There is a conflict. This can be resolved, however. You should note that project V has to be replaced 3 years before the end of project U. There is hence a future investment decision contingent on a choice today of U or V. A way out is to assume that project V is replaced in the last year of its life with an identical project, that ends in year 6. This will make V readily comparable to U. The cash flow of project V assuming such a policy is shown below.

Cash Flows (₦'000)									
Project	C_0	C_1	C_2	C_3	C_4	C_5	C_6	IRR %	NPV at 13%
V(Replicated)	-30	10	15	-10	10	15	20	20.61	7.55

It is clear as shown here that project V is the best choice in the situation when we assume constant replication.

There may be need to replicate both projects when the life of the long-lived one is not just an exact multiple of the short-lived project. This would make handling problems of projects with different lives more tedious than seen here. This solution can always be used to solve this type of problem however.

- Briefly explain how to resolve conflict between IRR and NPV in the case of two mutually exclusive projects with different lives.
    - We assume constant replication of the projects with the shorter life.

3.4 The Reinvestment Rate Assumption

The reinvestment rate is identical with the opportunity cost of capital. The IRR and NPV rule make implicit assumptions about the reinvestment rate. The NPV rule assumes that the shareholders can reinvest their money at the opportunity cost of capital. This is 13 per cent in our example. This rate is the market-determined opportunity cost of capital at which investments of similar risks with the ones we are considering should be discounted. The IRR rule also assumes that the investors can reinvest their money at the IRR for each project. In our examples above the internal rates of return are above and different from the opportunity cost of capital. It sound absurd to assume that some investors could reinvest at 13% cost of capital while some others could reinvest at higher rates for projects of similar risk. This shows that the implicit reinvestment rate assumption of the IRR rule is not logical. Though the IRR rule does discount cash flows, it does not discount them at the opportunity cost of capital.

We can however swing the argument in the opposite direction; looking at the fact that we really don’t need to know the cost of capital before we can calculate the IRR. In addition we find that the IRR will be the same for projects of identical cash flows whether or not the cash flows are assumed to be reinvested. So, we find that the reinvestment rate assumption, though quite in order, may not necessarily impair the usefulness of the IRR rule.

3.4.1 Loan Type Cash Flows

Not all cash flow streams have the property that NPV declines as the discount rate increases, which are the normal investment type of cash flows. Consider the following projects S and T.

Cash Flow (₦’000)				
Project	C_0	C_1	IRR Percent	NPV at 15%

S	-100	+140	40	21.74
T	+100	-140	40	-21.74

Each project has an IRR of 40 per cent, that is

$$-100 + \frac{140}{1.40} = 0 \quad \text{and} \quad +100 - \frac{140}{1.40} = 0$$

Does this mean that the two projects are equally attractive? Definitely not. Since in the case of S where the firm is initially paying out ₦100,000, it is lending money at 40 per cent. When we lend money we would desire a high rate of return and when we borrow money we would require a low rate of return. If the NPV graph of project T is plotted against various rates of interest you will discover that the NPV increases as the discount rate increases. It is clear that the internal rate of return rule as we stated previously would not work in this case. We would prefer an IRR that is higher than the required cost of capital.

Let us take a look at the case of project Y whose cash flow structure is shown below:

Cash Flows (₦'000)

Project	C_0	C_1	C_2	C_3	IRR %	NPV at 13%
Y	-100	425	-576	252	20	-0.34

The cash flow for project Y is not as straight forward as those we have been seeing so far. It is in part like lending money because we pay out money now and we receive money in the first period and pay out again in the second. The question is whether we should accept or reject the project. We are in a good position to decide when we look at the net present value. When we calculate this at 13% required rate of return for the firm we discover that the NPV is negative at that rate and so we reject the project.

- Why is it that the re-investment rate assumption, which though is not in order, may not impair the usefulness of the IRR rule?
    - We really don't need to know the cost of capital before we can calculate IRR, and IRR will be the same for projects of identical cash flows whether or not the cash flows are assumed to be re-invested.

3.5 Projects With Multiple Rates of Return

There are instances when a project may not have a unique IRR. This happens generally when there is more than one change in the sign of the cash flows. Consider for example project Z with the cash flow data given below:

Cash flows (₦'000)

Project	C_0	C_1	C_2	IRR %	NPV at 13%

Z	-100	230	-120	20 and 50	9.56

The project cost ₦100,000 and generates cash inflow of ₦230,000 in year 1; then in the second year the firm has to pay out ₦120,000. (You should note that some projects have terminal cash outflows, especially in cases like in the mining industry when after minerals have been mined funds need be expended to reclaim the land for other future use). There are two discount rates at which the NPV = 0 for project Z. In other words, each of the following arithmetic statements are true of project Z.

$$\frac{230}{1.20} - \frac{120}{(1.20)^2} = 0 \text{ and}$$

$$-100 + \frac{230}{1.50} - \frac{120}{(1.50)^2} = 0$$

Put in other words the investment has an IRR of both 20 and 50 per cent. The reason for this is the double change in the sign of the cash flow stream. There can be as many different internal rates of return for a project as there are changes in the sign of the cash flows.

There are also situations where for a project, no internal rates of return exists! This for example is the case for project W whose cash flow is shown below:

Cash Flows (₦'000s)					
Project	C_0	C_1	C_2	IRR %	NPV at 13%
W	-42	90	-50	None	-₦1.51

Some writers suggest a number of adaptations of the IRR rule to deal with abnormal situations of multiple internal rates of returns. These suggestions have been found not to be all adequate. The simple solution is to apply the NPV rule to these situations.

- (i)Under what condition can a project fail to have a unique IRR? (ii) How do we determine whether or not to embark on such projects?
    - (i)When there is more than one change in the sign of the cash flows (ii) Apply the NPV rule.

Summary of Study Session 3

We have considered situations that will cause the NPV and IRR methods of investment appraisal to agree or disagree in ranking of investment projects in this chapter. It was established that in situations of conventional projects of one or more periods of initial cash outlays followed only by periods of net cash inflows till the termination of the project, both criteria will arrive at the same decision. There will be conflict in the ranking of projects by the two techniques under the following circumstances.

1.When we have projects that differ substantially in their magnitude of cash outlay and consequent magnitude of cash inflow involved (The scale effect).

2. When the timing of cash flows differ (timing effect)

3. When the projects have unequal lives or duration; and

4. Situations where the projects have multiple internal rates of return, or none.

We considered a few adaptation techniques to make the IRR perform like the NPV.

This includes consideration of incremental cash flows and application of net terminal value rule to the projects. Another technique that we applied for cases of unequal lives is assumption of replication of the projects, to aid appraisal for equal number of years.

It is to be noted that a simple solution is to apply the NPV rule to avoid the inconsistencies of the IRR method.

Self-Assessment Questions (SAQs) for Study Session 3

Now that you have completed this study session, you can assess how well you have achieved its Learning Outcomes by answering the following questions. You can check your answers in the Appendix to the course text.

SAQ 1 (tests Learning Outcome 3.1)
Consider the following mutually exclusive projects

Cash flow (₦)				
Project	C_0	C_1	C_2	C_3
A	10,000	N 2000	4,000	11,784
B	10,000	10,000	3,000	2,830

Calculate the NPV for each project assuming interest rates of 0, 5, 10, 20, 30 and 40 per cent.

Draw the NPV graph for the projects to determine their IRR, and

Show calculations of IRR for each project to confirm results in (ii). State the project you would recommend and why?

SAQ 3.2 (tests Learning Outcome 3.2)

Consider projects X and Y with the following cash flows.

Cash flows ₦' 000			
Project	C_0	C_1	C_2
X	-400	241	293
Y	-200	131	172

What is the IRR of each of these two projects?

The finance manager is tempted to undertake the project with the higher IRR. Explain to him why this may not be the best option, and show how to adapt the IRR rule to choose the best project.

SAQ 3.3 (tests Learning Outcomes 3.3, 3.4 and 3.5)

Assume that projects M and N whose cash flows are shown below are mutually exclusive. Project L lasts for 6 years while M lasts for 3 years. (i) What is the IRR of each of the two projects (ii) A financial manager has the inclination to choose the project with the highest IRR. Show how you will use the NPV to decide on the best project of the two.

Cash Flow ₦'000							
Project	C0	C1	C2	C3	C4	C5	C6
M	-40	8	11	11	14	16	16
N	-40	14	20	25	-	-	-

REFERENCES

Bierman, H. & Smidt, S. *The Capital Budgeting Decision* 5th Edition, Macmillan Publishing Co. N.Y. (1980).

Brealey, R. & Myers, S. Principles of Corporate Finance. McGraw Hill Book Company (1981).

Copeland, T.E. & Weston, J.F. *Financial Theory and Corporate Policy*. Addison-Wesley Publishing Company Reading Massachusetts (1979). Should you require more explanation on this study session, please do not hesitate to contact your e-tutor via the LMS.

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iag@dli.unilag.edu.ng
08033366677

Study Session 4 Capital Budgeting: Replacement Projects, Capital Rationing and Inflation.

Introduction

Practical investment scenario that we discussed in study section 1 of this module, often arise in firms, bringing up need to decide whether to replace an equipment or not. Such replacement decisions is our first focus in this study session where we explain the uses of works sheet(s) to analyze incremental cash flows discounting to appraise replacement projects. An alternate less cumbersome technique for choosing between mutually exclusive projects with unequal life span, equivalent annuity (or infinite life) method, rounds off our discussions on replacement project appraisal.

Scarcity of fund for all viable projects is often experienced in firms due to internal and external factors. We will discuss resolution of such budget constraints for one-period scenario by use of profitability index ranking.

We previously alluded to the fact of inflationary pressures that is prevalent in many economies when we discussed bond valuation in study session 5 of module 1. Practical investment appraisal would often need incorporation of inflation rates that affect various components of a project's cash flow. We rounded off this study session with how this issue of inflation can be tackled in capital budgeting exercises. You will discover that a good grasp of contents of previous study sessions is all that is needed to comprehend this present session with relative ease.

Learning Outcomes for Study Session 4

After studying this session you should be able to:

4.1 Appraise a replacement project by use of a worksheet to determine and discount incremental cash flows involved.

4.2 Explain and demonstrate how equivalent annual annuity (or infinite life) method can be used to choose between two mutually exclusive projects with unequal life span.

4.3 Explain why investment funds may need be rationed in a firm and hence demonstrate the resolution of one – period capital rationing problem through the use of profitability index (PI) to rank available projects.

4.4 Explain why and how facts of inflation would be incorporated into appraisal of investment projects of a firm.

4.1 Evaluation of Replacement Projects

The point at which an equipment is replaced normally reflects economic considerations, rather than total physical collapse. Decision must be made when to replace. This will rarely come from the state of the machine. We will discuss an example to demonstrate how replacement decisions are made in capital budgeting.

4.1.1 Sample Problem on Replacement Project

Sarkin Limited is considering replacing a machine with a more efficient one. The old machine has a book value of ₦150,000 and could be sold for ₦70,000. The old machine is being depreciated on a straight line basis with zero salvage value over the next five years. The new machine costs ₦400,000 and has an expected life of five years, after which it can be sold for ₦80,000. The new machine is expected to reduce labour and raw materials usage sufficiently to cut operating costs from ₦170,000 to ₦80,000 per annum, before depreciation and taxes.

Assuming straight line depreciation, a 45% marginal tax rate, and a minimum required rate of return of 15 per cent, should the machine be bought? An investment tax credit of 10 per cent of the purchase price can be used if the new machine is acquired.

Solution to this is shown in Table 4.1 where a worksheet is used to analyze the project.

Table 4-1: Worksheet for Analysis of Replacement Project

	Net Outflow at the time investment is made t = 0	Amount Before tax ₦'000 (1)	Amount After tax ₦'000 (2)	Year Event occurs (3)	PV factor at 15% (4)	PV ₦'000 (5)
1	Cost of New Machine	400	400	0	1.0	400
2	Salvage value of old machine	(70)	(70)	0	1.0	(70)
3	Tax effect of Sale of old machine	(80)	(36)	0	1.0	(36)
4	Increase in working capital (if	-		0	1.0	-
5	applicable)	(40)	(40)	0	1.0	(40)
6	Investment tax credit					254
	Total initial outflows (PV of costs)					

Net Inflows Over the Project’s Life t = 1-5					
Decrease in costs	90	49.5	1-5	3.3522	165.93
Depreciation on new machine	64				
Depreciation on old machine	(30)				
Net change in tax savings from depreciation	34	15.30	1-5	3.3522	51.29
Depreciation	80	80	5	0.4972	39.77
Estimated Salvage value of new machine	-	-	-	-	-
Return of working capital (if applicable)					
Decrease in costs	90	49.5	1-5	3.3522	165.93
Depreciation on new machine	64				
Depreciation on old machine	(30)				
Net change in tax savings from depreciation	34	15.30	1-5	3.3522	51.29
Depreciation	80	80	5	0.4972	39.77
Estimated Salvage value of new machine	-	-	-	-	-
Return of working capital (if applicable)					

1 3	Total present value of cash inflows					257.00

NPV = PV of Inflows less PV of costs

= ₦257.00 - ₦254.00 = ₦3.00 that is ₦3,000

NPV as calculated in the table is ₦3,000. Since the value of the firm will increase by this amount if the replacement is made, the old machine should be replaced.

An explanation of each line of the worksheet now follows:

1. Purchase cost of the new machine

2. Amount received for sale of old machine. The brackets denote that these amounts are deducted when finding the net cash flow.

3. Since the old machine is to be sold at a loss, this reduces taxable income which in turn affects company's income. The tax savings is equal to:

Loss x tax = (₦80)(0.45) = ₦36. If a profit is realized on the sale (i.e sales price exceed book value) line 3 would have shown taxes paid, a positive cash outflow. In the actual sense the machine is to be sold at a loss, so no taxes were paid. In effect the company receives a tax credit of ₦36.

4. If the project had required additional working capital, the amount would have been entered here. The before tax and after tax amounts would be equal. Inventories might have been necessary to operate the new machine for example.

5. The investment tax credit is given as 10 per cent of the purchase price.

6. Line 6 shows the net cash outflow at the time the replacement is made. The company doles out ₦400,000 to pay for the machine, but this is partially offset by the items on lines 2, 3 and 5.

7. The cost reductions are shown in column 1. This amount is multiplied by (1-t) to obtain the after-tax benefits. Thus ₦90 (1 – 0.45) = ₦90(0.55) = ₦49.5. This benefit occurs in years 1-5, so the PV of this annuity is equal to ₦49.5(3.3522) = ₦165.93, the amount shown in the last column. Note also that had the replacement resulted in increase in sales, in addition to the reduction in costs, then this amount would also be reported in line 7, alternatively, a separate line could be added.

8-10 Lines 8 and 9 show the depreciation on the new and old machines while line 10 shows the net addition to depreciation and the tax savings from this additional depreciation. Depreciation in itself is not a cash inflow, but the increase in depreciation reduces the company's taxes. The amount of the taxes saved by the increased depreciation is ₦34(t) = ₦34(0.45) = ₦15.30. This figure is shown in column 2. The savings occur in years one through five, and the present value of this 5-year annuity is ₦15.30 (3.3522) = ₦51.29. We show this amount in column 5. Note that the relevant cash flow is the tax savings on the net increase in depreciation. Replacement decisions are based on incremental cash flows.

11. The estimated salvage value of the new machine at the end of its five-year life is ₦80.00. This is a return of capital, so no taxes are involved. The PV of the salvage value is ₦80 (0.4972) = ₦39.78).

12. In this example no additional working capital is required; had additional working capital, say inventories been required, then the investment in this item would have been recovered in year 5.

13. This line shows the present value of the benefits over the new machine's five-year life. Note the other comments at the foot of table 4-1 showing why the replacement should be made.

- (i)What type of cash flows are used in appraising replacement projects? (ii) What effect does proceeds from the sale of an existing asset have on the cash outflow of a replacement project?
  - (i)Incremental cash flows (ii) It reduces the cash outflow for the replacement project.

4.2 Equivalent Annual Annuity Method

We have assumed here that the new machine has a life equal to the remaining life of the existing one. This is to simplify the issues. If we must choose between two mutually exclusive replacement alternatives that have different lives, the constant replication procedure would be applied. We discussed this in study session 3 of this module where we used it to get around problems posed by projects of different lives causing conflicts in ranking by NPV and IRR. You will notice in that example that the replication procedure requires setting up a series of replacement chains extending out to the "least common multiple" year i.e. the year in which both alternatives would require replacement. If, for example, project A has a life span of 7 years and B 10 years, the replacement chain will extend to 70 years – ten replacements for A and seven for B, It could be quite an involving and complex arithmetic. We might hence need another method of dealing with uneven asset lives. This is the Equivalent Annual Annuity method which we consider next to round off our discussion of appraisal of replacement projects.

The Equivalent Annual Annuity method is also referred to as the infinite life method. Under this method, the net present value of each project is found assuming continuous replacement chains up to infinity. The following is the procedure involved in this method using projects U and V of section 3.3 of this module as illustration.

(i) We first find each project's NPV over its original life. This is ₦5530 for project U and ₦4460 for project V.

(ii) Divide the original NPV of each project by the present value annuity factor for the project's original life to obtain the equivalent annual annuity.

Equivalent annual annuity for project U = NPV_u/A_n = ₦5530/3.998 = ₦1383.35.

Equivalent annual annuity for Project V = NPV_v/A_n = ₦4460/2.361 = ₦1889.03. This implies that project U provides an equivalent annuity of ₦1383.35 per year for six years – such an annuity would be worth ₦1383.35 (3.998) = ₦5530.00. Project V on the other hand provides an equivalent annuity of ₦1889.03 per year for three years.

(iii) We thereafter assume infinite replacement i.e. that these equivalent annual annuities will continue on to infinity. This implies that we assume they evolve into perpetuities. Recall from our previous discussion in study session 4 of module 1 that the value of a perpetuity is $A_n = R\ (1/r)$. We can with this, compute the present value of the infinite annuities that projects U and V can provide.

Infinite NPV_u = ₦1383.35/0.13 = ₦10,641.13

Infinite NPV_v = ₦1889.03/0.13 = ₦14,531.00

Since the infinite horizon NPV of project V exceeds that of project U, project V should be accepted. This shows that the equivalent annual annuity method leads to the same decision rule as the simple replication method-accept project V.

- What basic assumption under lie the use of the Equivalent Annual Annuity method?
  - Assumption of continuous replacement chains up to infinity.

4.3 Capital Budgeting under Capital Rationing

We have implicitly assumed, so far, that the firm can obtain unlimited funds to execute its capital projects as long as it is willing to pay the current market interest rate. We will be relaxing this assumption here. This will lead us to considering the consequences for capital budgeting that limited fund poses. We will first see how and why this phenomenon occurs in a firm. But first let us define some terms.

The term capital rationing or capital constraints are normally applied to situations where the supply of funds to the firm is limited. It is a situation where the firm cannot obtain necessary funds to invest in all projects with positive NPV. This can arise due to factors external to the firm or it could be due to management action. We will describe these situations of external and internal factors leading to capital rationing in turn.

External factors that cause capital rationing may occur on account of the imperfections of the capital market. Imperfections, in turn, may be due to deficiencies in market information or by rigidities of attitude that hamper the free flow of capital. There could also be restrictions in bank lending due to government regulations. Depression in the capital market could also lead to capital rationing. So also are other factors that make it difficult for a firm to obtain the necessary capital to finance its profitable investment opportunities due to fear of losing control of the business. Besides, prospective investors may not be convinced of the viability of proposed projects of a firm. In such a case needed fund to finance projects with positive net present values would not be forth coming. This does not, however, negate the use of the NPV rule.

Internal capital rationing is often caused by self-imposed restrictions by a firm's management. For example, management may decide not to obtain additional funds by incurring debt. This may be a part of the firm's conservative financial policy to reduce leverage. Management can also fix an arbitrary limit to the amount of funds that can be committed to investment projects by divisional managers. Management could also require a minimum rate of return on projects to be invested in, thereby restricting the number of profitable projects embarked upon by the firm.

It is not always easy to justify internal capital rationing. It is however, generally used as a means of financial control. In a firm with divisional organization; divisional managers may overstate their investment requirements. One way of forcing them to carefully assess their investment opportunities and set priorities is to put upper limits to their capital expenditures. Similarly, a company may limit funding of investment projects if it finds itself incapable of coping with the strains and organizational problems of a fast growth, say due to short supply of managerial talents or technical personnel.

4.3.1 Resolution of One period Capital rationing   Problems

There is no simple solution to the investment decision when capital rationing is present, especially in a multi-period capital constraint scenario. There are however, suggested solutions to one-period capital rationing situations. This is under the assumption that any investment opportunity not undertaken in the period of capital rationing will be lost to the firm forever.

A number of methods can be applied to seek solution to one period capital rationing problems. We will consider the use of profitability ratio. This is a method of ranking with reference to the ratio that present value of a project's net cash flow bears to the scarce fund they require. We already discussed this method of project appraisal and ranking in study session 2 of this module.

We can also rank the projects according to their relative profitabilities by the use of the Excess Value Project Index (EVPI).

$$\text{EVPI} = \frac{\text{NPV}}{\text{Outlay}}$$

This is an alternate method to the profitability Index Method. EVPI will rank projects in the same order as PI. Here, we will discuss an example of how to use any of these methods to select projects to execute in a capital constraints setting. We will first assume that projects are divisible i.e. we can undertake say 70 per cent of a project and receive 70 per cent of the net present value in return.

4.3.2 Sample Problem on Resolution of One-Period Capital Rationing Problem

The Crystal Pharmaceutical Plc has ₦80,000 allocated for capital expenditures, which of the following projects should the company accept to stay within the ₦80,000 budget? The firm's cost of capital is 18%.

Cash Flows (₦'000)					
Years	0	1	2	3	4
Project A	-30	9	13.5	18	19
Project B	-25	7	9.5	14	14.5

Project C	-14	3	5	7	9
Project D	-35	12	12	12	12
Project E	-20	5	8	12	18
Project F	-21	5	9	11	15

Solution: What we need do is to find the profitability index (PI) or Excess Value Project Index (EVPI) of each project and use this to rank them accordingly. Projects are then chosen starting with the one having the highest PI or EVPI until the available fund is exhausted. This is shown in the table below.

Table 4-2: Ranking Projects Under Capital Rationing

Project (1)	Initial outlay (₦ '000) (2)	**PV of Cash Inflow (₦'000) (3)**	NPV (₦'000) (4)	Profitability Index (PI) (5)	EVPI (6)	Ranking (7)
A	-30	38.0782	8.08	1.2693	0.2693	2
B	-25	28.7549	3.76	1.1502	0.1502	4
C	-14	15.0355	1.04	1.0740	0.0740	5
D	-35	32.2807	-2.72	0.9223	-0.078	6

E	-20	26.5707	6.57	1.3285	0.3285	1
F	-21	25.1329	4.13	1.1968	0.1968	3

Project D will be rejected because it has a negative net present value. The ranking shows that the firm should implement projects E, A, F and 36 per cent of project B. This portfolio yields a total net present value of ₦20,133.56.

Under a second assumption that the projects are indivisible, we could look at possible combinations of projects bearing in mind the maximum fund available. As we compare the total net present values of such combinations, a choice of the best combinations can be made. This is shown in the following table.

Table 4-3: Project Combinations with Indivisible assumption under capital rationing

Possible Project combination	*Total cost ₦'000*	*Total NPV ₦'000*
ABC	69	12.88
ABE	75	18.41
ABF	76	15.97
ACF	65	13.25

AEF	71	18.78
BCEF	80	15.50

From the table here, with an assumption that the projects are not divisible, projects A, E and F with a total cost of ₦71,000 will be the acceptable projects. This is because of the fact that of all possible project combinations, the A, E, and F combination gives the highest total net present value of ₦18,780. There will also be a surplus fund of ₦9,000 besides.

The internal rate of returns (IRR) method can be used in ranking the projects. It has been found, however that the use of the Profitability Index (PI) or EVPI is better. Use can also be made of the linear programming technique, which we will not go into here.

- Mention one external factor and one internal factor that could cause capital rationing in a firm.

  - An external factor that can cause capital rationing is that there could be restrictions in bank lending due to government regulation. An internal factor that could cause capital rationing could be management decision not to obtain additional funds by incurring debt.

4.4 Capital Budgeting and Inflation

We are all aware of the reality of inflationary pressures on prices in most nations, particularly developing ones. So consideration must be given to it in capital budgeting analyses. We will consider the two most frequently used methods of dealing with inflation here. These are:

1. Explicit adjustment
2. Implicit adjustment

Investors do recognize the fall in value of investments with the passage of time and so incorporate expectations about inflation into the required rate of return. For example, assume that investors seek a real rate of return (R_r) of 9 per cent on an investment with a given degree of risk. Suppose further that they anticipate an annual rate of inflation (i) of 5 per cent. Then in order to end up with the 9% real rate of return, the nominal rate of return must be a value such that

$$1 + R_n = (1 + R_r)(1 + i)$$

$$R_n = (1 + R_r)(1 + i) - 1$$

$$= R_r + i + R_r i$$

(R_n = nominal rate of return)

In our example,

$$R_n = 0.09 + 0.05 + (0.09)(0.05)$$

$$= 0.1445 \text{ or } 14.45\%.$$

If the investor earns a nominal return of 14.45% on a ₦100 investment, the ending value in real terms will be ₦100 (1.1445)/(1.05) = ₦114.45/1.05 = ₦109. This produces the required 9% real rate of return. Typically, the cross product is ignored because (1) it is small and (2) given the estimates used in the analyses, accuracy to four decimal places is not necessary. Thus the discount rate used in the analysis is:

$$R_n = R_r + i = 9\% + 5\% = 14\%$$

A project's NPV in the absence of inflation where $R_r = R_n$ is calculated as follows:

$$NPV = \sum_{t=1}^{n} \frac{C_{tt}}{(1+R_r)t} \quad \text{“t”}$$

Now, suppose we expect both sales prices and input costs to rise at the rate i, the same inflation rate is built into the estimated cost of capital. In this event, C_t, the actual net cash flows in any year t, will increase annually at the rate i per cent producing:

$$C_t = \text{Actual Cashflow}_t$$

$$= \text{Inflation-free cashflow}_t \times (1+i)^t$$

For example, if we expect a net cash flow of ₦100 in year 5 in the absence of inflation, then with a 6% rate of inflation,

$$C_5 = ₦100\ (1.06)^5 = ₦133.82.$$

Now, if net cash flows increase at the rate i per year, and if this same inflation factor is built into the cost of capital, then

$$NPV = \sum_{t=1}^{n} \frac{C_t (1+i)^t}{(1+R_r)^t (1+i)^t} - C_0$$

Since the $(1+i)^t$ terms in the numerator and denominator cancel out, we are left with

$$NPV = \sum \frac{C_t}{(1+R_r)^t} - C_0$$

Thus, whenever costs and prices are both expected to rise at the same inflation rate that investors have built into the cost of capital, then the inflation adjusted NPV is identical to the inflation-free NPV. It stands to reason therefore, that where inflation rates differ between costs and prices, respective estimated flows have to be worked out in analyzing the capital budgeting decision.

4.4.1 Sample Problem on Capital Budgeting Under Inflation

We will now discuss a capital budgeting decision with consideration for inflation. This will make for a better feel of how inflation problems are dealt with in capital budgeting:

A company is considering a five-year project which will cost ₦9,000. Annual cash flows are estimated at ₦3,000 while fixed costs (excluding depreciation) will be ₦300 and variable expenses will be ₦1200. Because of inflation, annual revenues and variable expenses are expected to increase at 8 per cent and 6 per cent respectively. The company's corporate tax rate is 40%. The company's cost of capital is 8%. Inflation rate for the cost of capital is estimated at 4 percent per annum. The project is expected to be depreciated on a straight line basis with no salvage value. Should the company embark on the project?

In tackling this problem, it is proper first to arrive at the inflation adjusted cash flows for the project, before applying a technique to evaluate it. The analysis and workings are shown in the table below:

Table 4.4: Worksheet For Project Evaluation Under Inflation

	Year 1	Year 2	Year 3	Year 4	Year 5

	₦	₦	₦	₦	₦
1. Cash inflow (unadjusted)	3000	3000	3000	3000	3000
2. Inflation adjustment factor	(1.08)	$(1.08)^2$	$(1.08)^3$	$(1.08)^4$	$(1.08)^5$
3. Adjusted cash flow	3240	3499	3779	4081	4408
4. Tax Savings from depreciation	720	720	720	720	720
5. Total Adjusted Cash flow	3960	4219	4499	4801	5128
6. Variable Costs unadjusted	1200	1200	1200	1200	1200
7. Inflation Adjustment factor for 6	(1.06)	$(1.06)^2$	$(1.06)^3$	$(1.06)^4$	$(1.06)^5$
8. Adjusted Variable Cost	1272	1348	1429	1515	1606
9. Fixed cost	300	300	300	300	300
10. Total Cash outflow	1572	1648	1729	1815	1906
11. Net cash flow (5 less 10)	2688	2571	2770	2266	3222
12. PV factor (adjusted @ 12%)	0.8929	0.7972	0.7118	0.6355	0.5674
13. PV of Net Cash Flow	2400	2050	1972	1440	1828

NPV = - ₦9000 + (₦2400 + ₦2050 + ₦1972 + ₦1440 + ₦1828)

= - ₦9000 + ₦9690 = ₦690

Since the analysis shows a positive net present value, the company should embark on the project. Having incorporated inflationary factors and we still have a positive NPV, the project should be accepted.

- What are the two methods that can be used to deal with inflationary trends as it pertain to capital budgeting decisions?
  - The two methods are (i) Explicit adjustment and (ii) Implicit adjustment

Summary of Study Session 4

Our focus in this has been considerations a little beyond straight appraisal of new investment projects. In real life, there will often be need to replace obsolete or inefficient equipments with new, more efficient models. Firms will not always have all the funds needed for profitable capital investments, which calls for serious attention to the how of selecting more profitable projects. Furthermore, inflationary pressures are real in everyday life. All these call for incorporation of replacement issues, capital rationing considerations and the fact of inflation into our capital budgeting discussions.

We used a worksheet to analyse replacement decision using incremental cash flow principles. We thereafter highlighted the reason for internal and external factors making for capital constraints in firms. Solving capital rationing problems require the use of profitability index or Excess Value Project Index (EVPI) methods to rank projects and this aids more judicious use of scarce funds. By way of recapitulation.

$$EVPI = \frac{NPV}{Outlay}$$

We rounded off by discussing how inflation affects estimated cash flows and real interest rates expected by investors and how this can be accommodated in our capital budgeting analyses.

What we discussed in this study session is a greater adaptation of capital budgeting analyses to the real world situation where uncertainty is the norm. More explicit focus on this will be our next preoccupation.

Self-Assessment Questions (SAQs for Study Session 4

Now that you have completed this study session, you can assess how well you have achieved its Learning Outcomes by answering the following questions. You can check your answers in the Appendix to the course text.

SAQ 4.1 (tests Learning Outcome 4.1)

Peacock Textiles Ltd is considering the purchase of a new machine tool to replace an old one. The old machine has both a tax book value and a market value of zero. It is in good working order and will last for at least ten additional years. The proposed machine will perform the operations much more efficiently and Peacock's engineers estimate that labour, material and other direct costs of the operation will be reduced by ₦5,800 a year if the new machine is installed. The new machine cost ₦25,000 delivered and installed, and its economic life is estimated to be ten years with a zero salvage value. The firm's cost of capital is 18 percent. Tax rate is 50 per cent, and the firm uses straight line depreciation. Should Peacock Textiles buy the new machine? (Ignore investment tax credit).

SAQ 4.2 (tests Learning Outcome 4.2)

Projects D and E whose cash flows are shown below are mutually exclusive. Through the use of the Equivalent Annual Annuity Method show which of the two projects should be chosen for execution.

Cash Flow ₦						
Project	Co	C1	C2	C3	IRR%	NPV at 10%
D	-10,000	7,500	7,500	-	32	3,013
E	-10,000	2,000	4,000	12,000	27	4,134

SAQ 4.3 (tests Learning Outcome 4.3)

The Gloryland Company is considering identified projects for the current year as follow

Project	A	B	C	D	E	F

Initial outlay (₦)	30,000	25,000	40,000	35,000	22,000	50,000
Annual Net cash flows (₦)	12,000	9,000	15,000	10,000	8,000	21,000

Each project is scheduled to have a life of 5 years. The company hope to be able to raise ₦90,000 out of the ₦160,000 budgeted for capital expenditure. The cost of capital is 16%. Which combination of projects will maximize the value of the firm assuming (a) that the projects are divisible (b) projects are indivisible?

SAQ 4.4 (tests Learning Outcome 4.4)

Nexus International has an investment proposal of ₦40,000 which is expected to yield benefits over a five-year period. Annual cash inflows of ₦90,000 and annual cash outflows of ₦75,000 are expected, excluding taxes and depreciation tax shelter. The tax rate is 45% and the cost of capital is 8%. Nexus uses straight line depreciation. After the first year, inflows are expected to increase at 4% and outflows at 6 per cent per year. The annual rate of inflation is expected to be about 6 per cent. Should Nexus undertake the project?

REFERENCES

1. Brigham, E.F. *Financial Management:* Theory and Practice. The Dryden Press Hinsdale Illinois (1979).

2. Brealey, R. and Myers, S. *Principles of Corporate Finance* McGraw Hill Book Company N.Y. (1981).

3. Pandey, I.M. *Financial Management* Vikas Publishing House PVT Ltd New

Delhi (1990).

Should you require more explanation on this study session, please do not hesitate to contact your e-tutor via the LMS.

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iag@dli.unilag.edu.ng
08033366677

Study Session 5 Capital Budgeting Under Uncertainty 1

Introduction

Our pre-occupation in the first four study sessions of this module has been capital budgeting under certainty. Since real life capital budgeting decisions often involve risks and uncertainty of cash flows, we will in this and the next study session discuss treatment of this phenomenon (uncertainty and risk) in Financial Management.

The capital investment under capital budgeting analysis states that there is risk or uncertainty involved in capital investment decision because net benefits derived from a project would accrue in the future that may be subject to unknown chance effects. The estimates of future revenues of a project are based on the following: market size, market share and selling prices etc., although none of these items are known with certainty they may be estimated as likely to fall within a range of values. Thus, in all, investment decision involves some risk or uncertainty because it is impossible to say precisely the future event surrounding it. Techniques for dealing with risks and uncertainty in capital budgeting will hence be the focus of our discussions in this study session and the next. The approach as in the techniques of capital budgeting under certainty is to describe the features of each method and demonstrate how they are applied through worked examples. In addition, the advantages and disadvantages of each method are highlighted so the extent of their usefulness can be noted.

Learning Outcomes for Study Session 5

When you have studied this session, you should be able to:

5.1 Define risk and uncertainty; list the types and elements of risk and in addition itemise the various methods of handling risk in financial management.

5.2 Explain the features, application, advantages and disadvantages of the discounted payback period method for appraising investment projects cash flows; plus describe how the discounted payback period method can be used to determine whether or not a proposed project should be selected for execution.

5.3 Explain the features, application, advantages and disadvantages of the Risk Adjusted Discount Rate Method

for appraising investment projects cash flows; plus describe how the Risk Adjusted Discount Rate Method

can be used to determine whether or not a proposed project should be selected for execution.

5.4 Explain the features, application, advantages and disadvantages of the Certainty Equivalent Cash Flow method

for appraising investment projects cash flows; plus describe how the Certainty Equivalent Cash Flow method

can be used to determine whether or not a proposed project should be selected for execution.

5.1 Overview of Uncertainty and Risk

Uncertainty can be defined as a situation where a decision maker may or may not identify such future states of nature that may affect his/her decision parameters. It can also be defined as a situation where the outcome cannot be predicted with any degree of confidence from knowledge of past or existing events, so that no probability estimates for various possible outcomes are available. Ordinarily the term risk and uncertainty are used inter-changeably to imply the same thing for instance when a businessman views his prospect in the future, he often does so with some fear of the unknown because the future is not only uncertain but also entails some element of risk.

5.1.1 Risk

The term risk can be defined as a situation in which there are known probabilities (subjective or objective) for possible outcomes. It can also be defined as the variability that is likely to be associated with future return from a project. Risk is a situation where the future outcome is unknown with certainty but the likelihood of various possible future outcomes may be assessed with some degree of confidence based on the knowledge of past or existing events. In other words, probabilities of various possible outcomes can be estimated.

The problem of risk is more pronounced with capital budgeting because of the time period involved, which is normally more than one year.

5.1.2 Types of Risk

The various types of risk that can be encountered are as follows:

(i) Business Risk: is caused by variability in operating earnings or earnings before interest and taxes as a result of changes in demand, cost of goods, selling price and a firm's management decision on the amount of fixed cost assets deployed etc. In other words, business risk is the potential variability of earnings due to the nature and type of business operations.

(ii) Financial Risk: This is the risk that is due to the variability in operating earnings arising from a firm to meet its fixed interest payment obligations on debts used to finance its operations. It is the risk associated with the method of financing employed by the firm.

(iii) Portfolio Risk: It is the risk arising due to the variability in operating earnings as a result of the level of efficient diversification in a firm's portfolio of asset.

(iv) Investment risk: Is the risk due to the variability in operating earnings as a result of the level of efficient diversification in a firm's portfolio of assets.

(v) Interest Rate Risk: it is the risk due from the variability in the value of an asset as interest rate changes.

5.1.3 Elements in Business Risk

In risky situations, the decision maker is assumed to be aware of all possible future state of nature that may occur and that affect his/her decision parameters and is able to assign probability values on occurrence of each state of nature. Since our focus is on investment decisions, we centre our mind on business risk. Business risk, as defined above, is the variability in operating earnings or earnings before interest and taxes (EBIT) as a result of nature and type of business operations.

The following are regarded as elements in business risk: (i) The factors affecting the economy e.g. political; monetary and fiscal policies; etc. (ii) The inherent risk of the industry or market itself; and (iii) Company risk - this includes: (a) Change in management, strike, fire, earthquake, storm, etc affecting the company directly.

(b) Operating risk i.e. the proportion of fixed cost to total cost. The higher the fixed cost the greater the sales volume that has to be attained in order to breakeven, and hence the higher the business risk, and vice-versa.

(c) Product stage level: Every product has four stages namely: introduction, growth, maturity and decline. An investment made in a product at its introductory phase is risky, because the product may have a very short market life span due to non acceptability of the product. Furthermore, an investment made in a product in the declining stage is also risky because of rapid decline in sales / revenue.

5.1.4 Methods of Handling Risk

The following are the various methods of handling risk in capital budgeting: (i) Discounted payback period. (ii) Risk adjusted discount rate approach. Certainty equivalent cash flow method. (iii) Sensitivity analysis method. (iv) Probability based approach. (v) Standard deviation. (vi)Simulation. (vii) Decision tree Method.

(i)Define Uncertainty. (ii) What do you understand by the term 'Risk'?

(i)Uncertainty can be defined as a situation where the outcome cannot be predicted with any degree of confidence from either the knowledge of past or existing events, so that no probability estimates can be attached

(i)List the types of risk you know. (ii) Risk can be defined as the variability that is likely to be associated with future returns from a project. It can also be defined as situations in which there are known probabilities (subjective or objective) for potential outcome.

5.2 Discounted Payback Period Method

The first method of appraising project under uncertainty is Discounted Payback Period. This method ascertains the viability of a project by a firm. It is used to ensure minimization of the risk on a capital project by setting maximum payback period for a project before it can be accepted.

The following are the two ways of applying a payback time limit: (i) The project should show a positive NPV and

(ii) The project should payback in discounted cash flow terms within a certain time period.

5.2.1 Sample Problem on Discounted Payback Period Method

Governor Ltd, is to spend N3.2 million on a project that has a life span of 10 years and also to generate annual cash flows of N800,000. The return expected by the Company is 15%, while the payback in discounted cash flow term is 5years. Advise the company on this project.

Solution: Life span of the project: 10 years.

Year	cash flow (N)	CDF 15%	Present Value (N)
0	(3,200,000)	1.0000	(3,200,000)
1-10	800,000	5.0188	4,015,040
Net Present Value	(NPV)		815,040

Considering the life span of the project the Net Present Value (NPV) is positive therefore the project is worthwhile.

Considering the payback period policy of the company which is 5 years:

Year	cash flow (N)	CDF 15%	Present Value(N)
0	(3,200,000)	1,0000	(3,200,000)
1-5	800,000	3,3522	2,681,760
Net Present Value	(NPV)		(518,240)

Considering the payback period of the project the Net Present Value (NPV) is negative therefore the project should be rejected, it is not worthwhile, it will reduce the shareholder’s wealth.

We now calculate the exact payback period for the project as follows:

$$3,200,000 = 800,000 \frac{(1-(1.15)^{-n})}{0.15}$$

Cross multiply by 0.15

$$480,000 = 800,000\ (1-(1.15)^{-n})$$

Divide both sides by 800,000

$0.6= (1-(1.15)^{-n})$

$1- (1.15)^{-n} = 0.6$

$- (1.15)^{-n} = 0.6-1$

$- (1.15)^{-n} = - 0.4$

Multiply both sides -1

Take ln of both sides.

-n1n1.15 = 1n 0.4

$$-n = \frac{1n.04}{1n\ 1.15}$$

$-n = - 6.56$

Multiply both sides by -1

n = 6.56.

n = 6.56 years.

The exact payback period is 6.56 years.

5.2.2Advantages and Disadvantages of Payback Period Method

Advantages: (i) It is simple to understand and calculate. It is objectively based because it makes use of project cash flow rather than accounting profit. (ii) It favours quick return of the projects which produce faster growth and enhance liquidity. (iii) It is used by most business firms. (iv) It is inexpensive to use. (iv) It can constitute an overriding consideration when there are large differences between the various options.

Disadvantages: (i) It is a rough measure of liquidity. (ii)It provides a crude measure of the timing of project's cash flow. (iii) The cash flows outside payback period are ignored. It does not consider the time value of money. Companies' target payback period is subjectively determined when the method is in use.

- What is discounted payback period?
  - Payback period is used to reduce the risk on a capital project by setting maximum period for a project to recoup or recover its initial outlay before it can be accepted.

5.3 Risk Adjusted Discount Rate Method

Another method of appraising project under uncertainty is Risk Adjusted Discount Rate. For a long time the economic theorists have assumed that to allow for risk, the business man requires a premium over and above an alternative which is risk free. The more uncertain the returns in the future, the greater the risk, and the greater the premium required. In other words, the management adds an agreed margin to the company's cost of capital, and hence allows for risk by building in risk premium to the discount rate used in evaluating capital investment projects. The risk premium reflects the attitude of investors toward risk which varies among investors.

Formula for rate used to appraise a project is expressed thus: $R_A = R_f + R_p$ where:

R_A= Risk Adjusted discount rate

R_f = Risk Free rate

R_p= Risk Premium.

Risk free rate is derived from those securities that are regarded as riskless; for example the ones issued by governments be it Federal, State or Local like treasury bills, treasury certificates, development loan bonds or stocks etc. This is because regardless of what happen to those in power, the financial obligations of these securities will continue to be met.

5.3.1 Sample Problem on Risk Adjusted Discount Rate Method

Governor limited is embarking on a project costing N150,000. The project life span is four (4) years and is to generate the following net cash flow:

Year 1 Net cash flow (N)

1	72,000
2	63,000
3	30,000
4	27,000

The risk free rate is 10% while the company requires a risk premium of 7% for the riskiness of this project.

Required: Advise the company on this Project

Solution: Using the Risk Free Rate.

GOVERNOR LTD.

Year	Cash flow (N)	DF at 10%	Present Value (N)
0	(150,000)	1.0000	(150,000)
1	72,000	0.9091	65,455
2	63,000	0.8264	52,063
3	30,000	0.7513	22,539
4	27,000	0.6830	18,441
Net Present Value			8,498

Using risk free rate the Net Present Value is positive.

Using risk adjusted discount rate.

$R_A = R_F + R_P$

Risk Free Rate = R_f = 10%

Risk Premium = R_P = 7%

Risk Adjusted Discount Rate = R_A =?

R_A = 10% + 7%

R_A = 17%

Year	Cash flow (N)	DF at 17%	Present value (N)
0	(150,000)	1.0000	(150,000)
1	72,000	0.8547	61,538
2	63,000	0.7305	46,022
3	30,000	0.6244	18,732
4	27,000	0.5337	14,410
	Net Present Value		(9,298)

Using risk adjusted discount rate the Net Present Value is negative.

Comment: The project although has a positive NPV at the risk free rate, it has a negative NPV using the risk adjusted discount rate. This means that the project is risky and unacceptable. It is therefore not worthwhile, it will reduce the shareholder's wealth; therefore the project should be rejected.

5.3.2Advantages and Disadvantages of Risk-Adjusted Discount Rate

Advantages: (i) It is simple to understand and calculate. (ii) It incorporates risk aversion towards uncertainty. (iii) It is easily understandable and more useful to decision makers. (iv) It is intuitively correct since investors do require a higher expected return on riskier investments. (iv) It acknowledges the fact that risky investments should earn a higher return as a way for compensating for risk taken.

Disadvantages: (i) There is difficulty in determining the risk adjusted discount rate of a project i.e. risk premium is determined subjectively; it is based on the assessment of risk by each individual or company. (ii) It is difficult for the layman without financial knowledge to calculate the risk-adjusted discount rate. (ii) It does not make any risk adjustment in the numerator for the cash flows that are forecasted over the future. (iii) It is based on the assumption that investors are risk- averse. (iv) It is difficult to the non-finance literate to determine and apply the risk-adjusted discount rate of a project even when risk premium is known. (v) The value of the risk premium varies with the riskiness of the project.

- What is the risk-adjusted discount rate method?
  - This is a technique based on the assumption that investors require a premium over and above the risk-free rate to compensate for the risk borne on an investment.

5.4 Certainty Equivalent Approach

Another way of handling risk in capital budgeting is through the use of Certainty Equivalent Approach/Method. It is a capital budgeting technique used in assessing the viability of a project by adjusting the cash flow using the Certainty Equivalent Factor or Risk-Adjusting Factor to take account of risk before calculating the Net Present Value of the Project. In using this technique to calculate the Net Present Value of the project the risk free rate is used to discount the cash flow.

5.4.1 Sample Problem on Certainty Equivalent Approach

Governor limited is to spend N9,500 on a project with the following cash flow estimates.

Year	1	2	3	4	5
Cash Flow N	5250	4500	4800	4200	3000

The following Certainty Equivalent Factors (∞) has also been estimated as per the project.

Year	0	1	2	3	4	5
∞	1	0.9	0.8	0.6	0.5	0.4

If the risk free rate is 12%, advise on the viability of the project.

Solution: Governor Limited.

Year	Cash flow (N)	∞	DF at 12%	Present Value (N)
0	(9,500)	1.0	1.0000	(9,500)
1	5,250	0.9	0.8929	4,218.95
2	4,500	0.8	0.7972	2,869.92
3	4,800	0.6	0.7118	2,049.98
4	4,200	0.5	0.6355	1,334.55
5	3,000	0.4	0.5674	680.88
	Net Present Value			1,654.28

DECISION: the project has a positive NPV; it implies that the project is worthwhile, it adds to the shareholder's wealth and therefore should be accepted.

5.4.2 Advantages and Disadvantages of Certainty Equivalent Approach

Advantages: (i) It is simple and clear to understand. (ii) It recognizes risk of an investment. (iii) The lower the Certainty Equivalent Coefficient, the higher the perceived risk of project.

Disadvantages: (i) The main problem with the method is the ability to accurately determine certainty equivalent factors which are subjectively determined by management. (ii) Managers might be tempted to inflate their cash flow forecast in order to positively influence the viability of their investment proposal. (iii) The process for reducing the forecasted cash flows is likely to be inconsistent from one investment to another.

Mention one (1) advantage and one (1) disadvantage of certainty equivalent approach.

The following are the advantages of certainty equivalent approach:

(i) It recognizes risk of an investment.

(ii) The lower the certainty equivalent coefficient, the higher the perceived risk of project.

(iii)It is simple and clear to understand.

Summary of Study Session 5

In Study Session 5, you have learnt that:

(i) Uncertainty is a situation where the future outcome cannot be predicted with any degree of confidence from knowledge of past or existing events.

(ii) Risk occurs where though the future outcome is not known but the various possible outcomes can be predicted with some degree of confidence either through the knowledge of the past or existing events.

(iii) Discounted Payback Period is the number of years it takes a project or investment to recoup or recover its initial outlay. This technique is the oldest, popular and widely used.

(iv) Risk Adjusted Discount Rate is used in accessing the viability of an investment by adjusting the discount rate to take cognizance of risk before using it to calculate the NPV of the project.

(v) Lastly, the Certainty Equivalent Cash Flow takes care of risk by adjusting the cash flows of a project using certainty equivalent factor before determining the NPV of the project or investment using risk free rate.

Self- Assessment Questions (SAQs) for Study Session 5

Now that you have completed this study session, you can assess how well you have achieved its Learning Outcomes by answering the following questions. Write your answers in your Study Diary and discuss them with your Tutor at the next Study Support Meeting. You can check your answers with the Notes on Answers to Self-Assessment Questions in Appendix at the end of this course text.

SAQ 5.1 (test Learning Outcome 5.1)

(i)What do you understand by the term “uncertainty?” (ii) List the various methods of handling risk in capital budgeting.

SAQ 5.2 (test Learning Outcomes 5.2)

Define discounted pay back method.

SAQ 5.3 (test Learning Outcomes 5.3)

ABC Ltd. is considering a project with the following cash flow estimates:

Year	Cash Flow (N)
0	(30,000)
1	10,000
2	16,000
3	18,000
4	12,000

The risk – free rate is 10% while the risk premium for thus project is 5%.

Required:

Compute the NPV of the project using risk adjusted discount rate approach.

SAQ 5.4 (test Learning Outcomes 5.4)

Mention the advantages and disadvantages of Certainty equivalent

Solution to SAQ 5.1

Uncertainty can be defined as a situation where the outcome cannot be predicted with any degree of confidence from knowledge of past or existing events, so that no probability estimates for various possible outcomes are available.

Solution to SAQ 10.2

The following are the various methods of handling risk in capital budgeting: Discounted payback period. Risk adjusted discount rate approach. Certainty equivalent cash flow approach Sensitivity analysis. Probability based approach. Standard deviation. Simulation. Decision tree

Solution to SAQ 10.3

Discounted Pay back method is the period the initial outlay of the project is recovered from discounted cash flow term.

. Put differently, it is used to reduce the risk on a capital project by setting maximum payback period for a project before it can be accepted.

Solution to SAQ 10.4

Using risk adjusted discount rate.

$R_A = R_F + R_P$

Risk Free Rate = R_f = 10%

Risk Premium = R_P = 5%

Risk Adjusted Discount Rate = R_A =?

R_A = 10% + 5%

R_A = 15%

Year	Cash flow (N)	DF at 15%	Present value (N)
0	(30,000)	1.0000	(30,000)
1	10,000	0.8696	8,696
2	16,000	0.7561	12,098
3	18,000	0.6575	11,835
4	12,000	0.5718	6,862
	Net Present Value		9,491

Decision

Based on the risk adjusted discount rate the NPV is N9,491 which is positive it thus adds to the shareholder's wealth. Therefore, the project should be accepted.

Solution to SAQ 5.5

The following are the advantages and dis-advantages of Certainty Equivalent.

Advantages of certainty equivalent approach

(i) It is simple and clear to understand.

(ii) It recognizes risk of an investment.

(iii) The lower the certainty equivalent coefficient, the higher the perceived risk of project.

Disadvantages

(i) The main problem is the ability to accurately determine certainty equivalent factors which are subjectively determined by management.

(ii) Managers might be tempted to inflate their cash flow forecast in order to affect the viability of their investment proposal.

(iii) The process for reducing the forecasted cash flows is likely to be inconsistent from one investment to another.

REFERENCES

Harold, B. Jr. and Seymour, S. (1980), The Capital Budgeting Decision, Macmillan Publishing Co., Inc., United State.

Olaoluniyi, O. (2010), Elements of Business Finance, Panaf Publishing Inc., Abuja.

Olowe, R. F. (2008) Financial Management: Concepts, Analysis and Capital Investments, Brierly Jones Nigeria Limited, Lagos.

Owualah .S.I.(2003) Principles of Financial Management, G-Mag Investments Ltd. Lagos.

Owualah .S.I.(2003) Understanding Business Finance, Second Edition, G-Mag Investments Ltd. Lagos.

Pandy, I.M. (1999) Financial Management 2nd Edition, Vikas Publishing House PVT Ltd., New Delhi. Should you require more explanation on this study session, please do not hesitate to contact your e-tutor via the LMS.

Are you in need of General Help as regards your studies? Do not hesitate to contact the DLI IAG Center by e-mail or phone on:

iag@dli.unilag.edu.ng
08033366677

Study session 6 Capital Budgeting under Uncertainty 2

Introduction

In Study Session 5 you learnt about risk, uncertainty, discounted payback method, risk adjusted discount rate approach and certainty equivalent cash flow. We discussed in the previous study session (5) how we incorporate risks into the capital budgeting decision through the use of these techniques. In this Study Session the other methods of measuring risk are extensively discussed with illustrations (sample problems). The same approach adopted in our treatment of the techniques discussed previously has been used for each of these techniques of appraising projects under condition of risk and uncertainty.

Learning Outcomes for study Session 6

When you have studied this session, you should be able to:

6.1 On Sensitivity analysis; have an in-depth knowledge, know the advantages and disadvantages and also solve problems using the method.

6.2 On probability; have an in-depth knowledge, know the advantages and disadvantages and also solve problems using the method.
6.3 On Standard Deviation; have an in-depth knowledge, know the advantages and disadvantages and also solve problems using the method.
6.4 On Simulation; have an in-depth knowledge, know the advantages and disadvantages and also solve problems using the method. And lastly,
6.5 On Decision Tree; have an in-depth knowledge, know the advantages and disadvantages and also solve problems using the method.

6.1 Sensitivity Analysis
This is another method of handling risk in capital budgeting by varying the values of the key factors such as, life span of the project, cost of capital, cost per unit, sales volume and selling price. This makes the management to determine the factors that affect the project most or rather the factors that are sensitive.
Put differently, it's a minimum tolerable unfavorable change in the key variables factors affecting the project under consideration.

It is calculated thus:

$$\frac{\text{Net Present Value of the Project}}{\text{Present value of the key factor}} \times \frac{100}{1}$$

6.1.1 Sample Problem on Sensitivity Analysis
Governor limited is contemplating on a project with the following cash flows:

Year	Purchase of Plant (N)	Operating costs (N)	Cash Revenue (N)
	500,000		
		240,000	440,000
		250,000	480,000
		270,000	550,000
		280,000	580,000

If the firm's cost of capital is 15%.

Required
Measure the sensitivity of the project to change in the levels of expected operating costs and revenues.

Solution: GOVERNOR LIMITED

Year	DCF at 15%	present values Plant cost	present value Operating costs	PV Revenue	PV Net cash flow
0	1.0000	(500,000)			(500,000)
1	0.8696		(208,704)	382,624	173,920
2	0.7561		(189,025)	362,928	173,903

3	0.6575		(177,525)	361,625	184,100
4	0.5718		(160,104)	331,644	171,540
		500,000	(735,358)	1,438,821	~~203,463~~

Plant cost:

$$\frac{\text{NPV of the Project}}{\text{PV Plant cost}} \times \frac{100}{1}$$

$$= \frac{203,\ 403}{500,000} \times \frac{100}{1}$$

= 40.68%.

Implication: It means plant cost can rise by 40.68% and the project will still be worthwhile, beyond this 40.68% the Net Present Value of the project becomes negative thus making the project unviable.

Operating Cost:

$$\frac{\text{NPV of the project}}{\text{PV of Operating Cost}} \times \frac{100}{1}$$

$$= \frac{203,403}{735,481} \times \frac{100}{1}$$

= 27.65578 or 27.66%

Interpretation

It means PV of operating cost can increase by 27.66% and the project is still viable above this percentage i.e. 27.66%, it will result to negative net present value thus making the project not worthwhile.

Revenue / Sales.

$$= \frac{\text{NPV of the project}}{\text{PV of sales/revenue}}$$

$$= \frac{203,403}{1,438,821} \times \frac{100}{1}$$

= 14.13678 or 14.14%

Interpretation: The revenue will need to fall or rather the price can fall by almost 14.14% and the project will still be worthwhile, beyond this percentage PV of the project becomes negative making the project not viable

6.1.2 Advantages and Disadvantages of Sensitivity Analysis

Advantages of Sensitivity Analysis: (i)It identifies areas crucial to success of the project.

(ii) It is a less complicated to understand. (iii) Information is presented to management to facilitate decision making. (iv) Simple and understandable in principle. (iv) The analysis shows the effect on project outcome by varying the value of the elements such as cost, sales, and other key variables.

Disadvantages of Sensitivity Analysis: (i) Only one variable is altered at a time, which is unrealistic. Thus, it fails to focus on the interrelationship between variables. (ii) While it recognizes the problem of uncertainty, it does not attempt directly to measure the level of uncertainty. (iii) Where the decision is between mutually exclusive projects, additional sensitivity analysis would be necessary for the purpose of ranking and decision making.
(iv) It involves certain amount of computations. (v) It gives no indication of the likelihood of a variable occurring.
Mention the disadvantages of sensitivity Analysis.
The following are the disadvantages of the Sensitivity Analysis: (i) Only one variable is altered at a time, which is unrealistic. Thus, it fails to focus on the interrelationship between variables. (ii) While it recognizes the problem of uncertainty, it does not attempt directly to measure the level of uncertainty. (iii) Where the decision is between mutually exclusive projects, additional sensitivity analysis would be necessary for the purpose of ranking and decision. (ii) It involves certain amount of computations. (iii) It gives no indication of the likelihood of a variable occurring.

6.2 Probability Based Method / Technique.
Statistical techniques are valuable analytical tools in handling risk in capital budgeting. One of such is the use of probability assignment. This is a capital budgeting techniques used in assessing the viability of a risky project by classifying cash flow into three namely: The high, the low and the best guess estimates or the Optimistic, most likely and pessimistic estimates, and then attaching probability before determining the NPV of the project.

6.2.1 Sample Problem on Probability Based Method / Technique.
Governor limited is considering the following project. The possible cash flows and the associated probabilities for the one year projects are as follows:

Event	Cash flow (N)	Probability
A	4,250	0.6
B	3,750	0.3
C	3000	0.1

If the initial outlay of the project is N3,250 and the discount rate is 12%.
Calculate the expected value of the NPV of the project.

Solution: GOVERNOR LIMITED

Event	Cash flow (C_{jt})	Probability(P_{jt})	Expected Value (CjtPjt)
A	4,250	.6	2,550
B	3,750	.3	1,125
C	3,000	.1	300
Expected Monetary Value			3,975

Projects expected value of the NPV

$$NPV = \frac{3,975}{1.12} - 3250$$

= 3549.107 – 3250 = N299.107

Decision: Since the expected value of NPV is positive, the one-year project is worthwhile and should be accepted.
The same principle applies if the project has a life span of more than a year and the probability estimates are given for each year, then the expected value of cash flow would be calculated for each year. The expected net present value would then be obtained by discounting the expected value of cash flow at the discount rate which is normally the risk free rate.

6.2.2 Sample Problem 2 on Probability Based Method / Technique.
The cash flows of a project and their probability distributions are given below.

Year 1		Year 2		Year 3	
Cash flow (N)	Prob.	Cash flow N	Prob	Cash flow N	Prob
14,000	0.25	12,000	.20	10,000	.15
17,000	0.30	16,000	.35	16,000	.35
18,000	0.35	20,000	.15	19,000	.25
2,000	0.10	24,000	.30	22,000	.25

The project is expected to cost N 15,000 while the risk free rate is 14%.
Required:
Calculate the projects expected net present value.

Solution
Year 1

Cash flow (N)	Probability	Expected value(N)
14,000	.25	3,500
17,000	.30	5,100
18,000	.35	6,300
2,000	.10	200
		15, 100

Year 2

Cash flow (N)	Probability	Expected value (N)
12,000	.20	2,400
16,000	.35	5,600
20,000	.15	3,000
24,000	.30	7,200
		18,200

Year 3

Cash flow	Probability	Expected value
10,000	.15	1,500
16,000	.35	5,600
19,000	.25	4,750
22,000	.25	5,500
		17,350

Expected Value of Net Present Value (NPV) = (N 15,000) + $\frac{15,100}{(1.14)} + \frac{18,200}{(1.14)^2} + \frac{17,350}{(1.14)^3}$

= (N 15,000) + N 13,245.61 + 14,004.31 + 11,710.76

= N 8,960.68

Decision

Since the expected value of NPV is positive, the project is worthwhile and should be embarked upon.

6.2.1 Advantages and Disadvantages of Probability Based Approach

Advantages: (i) It is simple to calculate and understand. (ii) It takes account of the expected variability of all the outcomes. (iii) It recognizes the time value of money. (iv) The net present value approach is consistent with the objective of maximizing the wealth of the shareholders. (v) It considers all the cash flows.

Disadvantages: (i) The actual outcome may never equate to expectation. (ii) Assignment of probabilities to various outcomes is based on management guesses i.e. subjective rather than be objective. (iii)nThe concept of expectation may provide an excuse for poor performance, the actual result being attributed to one of the possible outcomes. (iv) It is difficult to use, there may be need to use costly and sophisticated computers to evaluate complex projects.

Describe the probability based approach of handling risk.

This is a capital budgeting technique of handling risk by adjusting the cash flows with the probability to take cognizance of risk before determining the Net Present Value of the project.

6.3. Standard Deviation of Cash Flows

Another method of determining the viability of investment under uncertainty is Standard deviation. Though the expected value calculation incorporates risk into the capital investment analysis, risk analysis will be appreciated if the dispersion of cash flow is computed.

Standard deviation is a measure of dispersion and it is also relevant in the measurement of risk. The following are the steps taken to calculate standard deviation:

Calculate the expected cash flows as shown above.

Deduct the expected cash flow from each possible cash flow to arrive at a set of deviations about the expected cash flow.

Deviation = C_{jt}-C_t.

Square each deviation and sum these products to obtain the variance.

$S^2 = \sum^n (C_{jt} - C_t)^2 P_{jt}$

The standard deviation is found by calculating the square root of the variance.

Standard deviation S = $(cjt – ct)^2 P_{jt}$.

6.2.1 Sample Problem on Standard Deviation of Cash Flow

A company XYZ has determined the following probabilities for net cash flows generated by project XYZ.

Year 1		Year 2		Year 3	
Net cash flow (N)	Prob.	Net cash flow (N)	Prob.	Net cash flow (N)	Prob.
4,000	.2	6,000	.2	8,000	.2
6,000	.3	8,000	.3	10,000	.3
8,000	.5	10,000	.5	12,000	.5

If the cost of project is N15,000 and the risk free rate is 12%.

You are required to calculate the following: (a) Expected value of net present value of the project. (b) Standard deviation of the project. (c)Coefficient of variation of the project.

Solution: Company XYZ

YEAR 1

Cash flow N	Prob.	Expected Cash flow N	A_{jt}-EV	$(Ajt\text{-}EV)^2$ Pjt N
4,000	.2	800	(2,600)	1,352,000
6,000	.3	1800	(600)	108,000
8,000	.5	4000	1,400	980,000
EV Expected Value		6,600		S^2= 2,440,000

YEAR 2

Cash flow Cjt	Prob. Pjt	Expected cash flow Cjt Pjt	Ajt - Ev.	$(Ajt\text{-}EV)^2 P_{jt}$ N
6,000	.2	1,200	(2,600)	1,352,000
8000	.3	2,400	(600)	108,000
10,000	.5	5,000	1,400	980,000
Expected Value (EV)		8,600		S^2 = 2,440,000

YEAR 3

Cash flow Ajt	Prob. P_{jt}	Expected cash flow Cjt Pjt	Ajt-Ev	$(Ajt\text{-}Ev)^2 p_{jt}$
8,000	.2	1,600	(2,600)	1,352,000
10,000	.3	3,000	(600)	108,000
12,000	.5	6,000	1,400	980,000
EV = Expected value		10,600		S^2 = 2,440,000

$$\text{EV of NPV} = (N\ 15{,}000) + \frac{6{,}600}{(1.12)^1} + \frac{8{,}600}{(1.12)^2} + \frac{10{,}600}{(1.12)^3}$$

=(N 15,000) + 5892.86 + 6855.87 + 7544.87

= (N 15,000) + 5892.86 + 6855.87 + 7544.87

= N5,293.6

SOLUTION TO ILLUSTRATION (ii)

STANDARD DEVIATION OF THE PROJECT.

$$S = \sqrt{\sum_{t=1}^{n} \frac{S^2t}{(1+r)^{2t}}}$$

$$= \sqrt{\frac{2{,}4000{,}000}{(1.12)^2} + \frac{2{,}400{,}0000}{1.12^4} + \frac{2{,}400{,}000}{1.12^6}}$$

$= \sqrt{4,731,892}$

$= \underline{\underline{2,175}}$

SOLUTION TO ILLUSTRATION (iii)

(c) Co-efficient of Variation= $\dfrac{\text{Standard Deviation of the Project}}{\text{Expected value of NPV of the Project}}$

$$\frac{2,175}{5,293.6} \times \frac{100}{1}$$

$$= \underline{\underline{41.09\%}}$$

6.3.2 Advantages and Disadvantages of Standard Deviation

Advantages It is used in finance to measure the volatility of a business. It is used to represent the risk associated with financial securities. Standard deviation is higher when the data is widely dispersed, and vice versa. It gives better picture of data than just the mean value.

Disadvantages: Both the profit and loss have the same volatility. Complex calculation requires the use of computer that becomes difficult for illiterate person to calculate. Less scattered data always results to mean that do not reflect the reality of the market. It is affected by outlier’s number i.e. rare numbers e.g. much smaller or larger figures which can lead to skewed graph.

Define standard deviation.

Standard deviation is a measure of dispersion and it is relevant in the measurement of risk for the purpose of appraising investment.

6.4 Simulation Models

Lastly, the last method which can be used to evaluate capital budgeting under uncertainty is Simulation. It is fashioned after Monte-Carol simulation that is based on the idea of taking random samples from mathematical models which represent a real life situation. This method involves establishing a probability distribution for each of the variables and then select at random individual values that will eventually be used for appropriate decision. In short it is a process of experimenting or using a model for investment appraisal and noting the results which occur. It is used where analytical techniques are not available or would be complex

The following are steps involving simulation problem: Define the problem. Introduce the variables associated with the problems. Attach the probability distribution to each variable. Develop a computer programme that compute the probability distribution of each variable and use the values to calculate the project net present value.

IN TEXT QUESTION

What is simulation?

IN TEXT ANSWER

Simulation is a process of experimenting or using a model for investment appraisal and noting the results which occur.

ILLUSTRATION

Governor Limited has prepared estimates for a proposed investment project to be

	Year	Probability	Amount (N)
Cost of equipment	0	1.0	200,000
Cash revenue per annum	1-5	.2	200,000
		.35	250,000
		.25	275,000
		.20	300,000
Operating expenses per annum	1-5	.15	125,000
		.35	150,000
		.30	175,000
		.20	200,000

If the risk free rate is 12%.
Assuming the following random numbers are generated: 478630017649.
Require:
Calculate the project's NPV using simulation model.

SOLUTION TO ILLUSTRATION

	Revenue		Operating expenses	
Calculation	Random Number	Value per annum	Random Number	Value per annum
1	47	250,000	86	200,000
2	30	250,000	01	125,000
3	76	275,000	49	150,000

SET 1

Year	Cash flow (N)	DF @ 12%	Present Value (N)
0	(200,000)	1.0000	(200,000)
1-5	250,000	3.6048	901,200
1-5	(200,000)	3.6048	(720,960)
	Net Present Value		(19,760)

SET 2

Year	Cash flow (N)	DF @ 12%	Present Value (N)
0	(200,000)	1.0000	(200,000)
1-5	250,000	3.6048	901,200
1-5	(125,000)	3.6048	(450,600)
	Net Present Value		250,600

SET 3

Year	Cash flow (N)	DF @ 12%	Present Value (N)
0	(200,000)	1.0000	(200,000)
1-5	275,000	3.6048	991,320
1-5	(150,000)	3.6048	(540,720)
	Net Present Value		250,600

EXPECTED NPV BASED ON THE THREE SETS OF SIMULATION:

SET	NPV
	(19,760)
	250,600
	250,600
Total	481,440

$$\text{Average} = \frac{N481{,}440}{3} = N160{,}480$$

Advantages and Disadvantages of Simulation

Advantages

Some problems are too complex for analytical method thus facilitates the use of simulation technique.

The use of the technique enables various alternatives to be deeply examined.

It considers risk in a project by examining variances and co-efficient of various alternatives.

The basic principle is simple and also attractive to non-qualitative experts.

It provides the opportunity to study the interactive effect of individual variables in order to determine which are important.

It considers time value of money.

It takes into account all the cash flows on the entire life span of the projects.

It is in line with the objective of maximization of shareholder's wealth.

Disadvantages

It does not produce the optimal result but only offers "good –enough" solution.

Too much managerial and technical time are involved.

It is cheaper than tampering with the real system.

It does not produce answers by itself. Managers and others must generate conditions and constraints for solutions they are examining.

It involves the use of computers which can be above the reach of many firms/ companies.

Summary of Study Session 6

In study Session 6 you have learned that:

1. Sensitivity analysis can be used to appraise the sensitivity of key factors, such as; cost of the plant, project life span, revenue, operating cost, etc.
2. Probability is an assessment of a decision maker about the likelihood that an event will occur; once the probability has been assigned to future cash flow, risk can be assessed by calculating the expected net present value. Furthermore, where there is sufficient information available, a useful method of assessing risk is the probabilities-based method.

3. Standard deviation is a measure of dispersion and it is also relevant in the measurement of risk in the for the purpose of appraising investment project.

4. Simulation is the process of experimenting i.e. using a model and noting the results that occur. It is a technique that can be applied in complex analysis of risk and uncertainty. And lastly

5. Decision tree is a pictorial method of showing a sequence of inter-related decisions and outcomes. Decision tree is used in appraising investment project that requires capital outlays over several years.

Self- assessment Questions (SAQS) for Study Session 6

Now that you have completed this study session, you can assess how well you have achieved its Learning Outcomes by answering the following questions. Write your answers in your Study Diary and discuss them with your Tutor at the next Study Support Meeting. You can check your answers with the Notes on the Self-Assessment Questions at the end of this Module.

SAQ 6.1 (test Learning Outcomes 6.1)

Governor Limited is considering a project having the following cash flow:

Year	Outlay (N)	Revenue (N)	Running Cost (N)
0	(200,000)		
1		84,000	28,000
2		112,000	49,000
3		168,000	70,000
4		210,000	84,000

Assuming the cost of capital is 10%.

Required:

Calculate the project's sensitivity to the following:

Outlay of the project.

Revenue

Running cost

Life span of the project.

SAQ .2 (test Learning Outcomes 6.2)

The cash flows of a XYZ project and their probability distributions are given bellows.

Year 1		Year 2		Year 3	
Cash flow (N)	Prob.	Cash flow N	Prob	Cash flow N	Prob
15,000	0.20	9,000	.20	11,000	.15
18,000	0.35	13,000	.35	17,000	.30
19,000	0.35	27,000	.25	20,000	.25
3,000	0.10	29,000	.20	23,000	.30

The project is expected to cost N 330,000 while the risk free rate is 15%.

Required:

Calculate the projects expected net present value.

SAQ 6.3 (test Learning Outcomes 6.3)

Highlight the steps taken in solving standard deviation.

SAQ 6.4 (test Learning Outcomes 6.4)

ABC Limited is considering the following probability estimates prepared for a proposed investment project.

	Year	Probability	N
Initial Outlay	0	1.00	(80,000)

Revenue per annum	1-4	.15	80,000
		.40	100,000
		.30	110,000
		.15	120,000
Running cost per annum	1-4	.40	50,000
		.25	60,000
		.35	70,000

Assuming company's cost of capital is 12% while the random numbers generated are 30,71,20,00,79,94.

Required:
Calculate the net present value of the project.
Using the random numbers given at the end the question to prepare the NPV simulation of the project.

SAQ 6.5 (test Learning Outcomes 11.5)
Highlight the advantages and disadvantages of Decision Tree.

SOLUTIONS TO SELF ASSESSMENT QUESTIONS (SAQS)

Solution to SAQ 6.1
Governor Limited is considering a project having the following cash flow:

Year	DCF@ 10%	P/Value	P/Value	P/Value
O		Outlay (N)	Revenue (N)	Running Cost (N)
0	1.0000	(200,000)		
1	0.9091		76,364	(25,456)
2	0.8264		92,557	(40,494)
3	0.7513		126,218	(52,591)
4	0.6830		143,430	(57,372)
		(200,000)	438,569	(175,913)

Plant cost:

$$\frac{\text{NPV of the Project}}{\text{PV Plant cost}} \times \frac{100}{1}$$

$$= \frac{62,657}{200,000} \times \frac{100}{1}$$

= 31.3285% or 31.33%

IMPLICATION

It means plant cost can rise by 31.33% and the project will still be worthwhile, beyond this 31.33% the Net Present Value of the project becomes negative thus making the project unviable.

Operating Cost:

$$\frac{\text{NPV of the project}}{\text{PV of Operating Cost}} \times \frac{100}{1}$$

$$= \frac{62,657}{175,912} \times \frac{100}{1}$$

= 35.618377 or 35.62%

INTERPRETATION

It means PV of operating cost can increase by 35.62% and the project is still viable above this percentage i.e. 35.62%, it will result to negative net present value thus making the project not worthwhile.

REVENUE / SALES.

$$= \frac{\text{NPV of the project}}{\text{PV of sales/revenue}}$$

$$= \frac{62,657}{438,569} \times \frac{100}{1}$$

= 14.28669 or 14.29%

INTERPRETATION

The revenue will need to fall or rather the price can fall by almost 14.29% and the project will still be worthwhile, beyond this percentage PV of the project becomes negative making the project not viable.

Solution to SAQ 6.2

SOLUTION TO ILLUSTRATION

Year 1

Cash flow (N)	Probability	Expected value(N)
15,000	.20	3,000
18,000	.35	6,300
19,000	.35	6,650
3,000	.10	300
		16, 250

Year 2

Cash flow (N)	Probability	Expected value (N)
9,000	.20	18,000
13,000	.35	4,550
27,000	.25	6,750

29,000	.20	5,800
		35,100

Year 3

Cash flow	Probability	Expected value
11,000	.15	1,650
17,000	.30	5,100
20,000	.25	5,000
23,000	.30	6,900
		18,650

Expected Value of Net Present Value (NPV) = $(N\ 35{,}000) + \frac{16{,}250}{(1.15)} + \frac{35{,}100}{(1.15)^2} + \frac{18{,}650}{(1.15)^3}$

= (N 35,000) + N 14,131 + 26,539 + 12,262

= N 17,932

DECISION

Since the expected value of NPV is positive, the project is worthwhile i.e. it will add to the shareholder's wealth and therefore should be embarked upon.

Solution to SAQ 6.3

Standard deviation is a measure of dispersion and it is relevant in the measurement of risk. The following are the steps taken to calculate standard deviation:

Calculate the expected cash flows as shown above.

Deduct the expected cash flow from each possible cash flow to arrive at a set of deviations about the expected cash flow.

Deviation = C_{jt}-C_t.

Square each deviation and sum these products to obtain the variance.

$S^2 = \sum^n (C_{jt} - C_t)^2 P_{jt}$

The standard deviation is found by calculating the square root of the variance.

Standard deviation S = $(cjt - ct)^2 P_{jt}$.

Solution to SAQ 6.4

REVENUE

PROBABILTY	CASH-FLOW (N)
.20	200,000
.55	250,000
.80	275,000
1.00	300,000

RUNNING COST

PROBABILTY	CASH-FLOW (N)
.15	125,000
.50	150,000
.80	175,000
1.00	200,000

REVENUE

YEAR	RANDOM NUMBER	VALUE PER ANNUM (N)
1	30	100,000
2	20	100,000
3	79	110,000

RUNNING COST

YEAR	RANDOM NUMBER	VALUE PER ANNUM
1	71	70,000
2	00	50,000
3	94	70,000

SET 1

YEAR	CASHFLOW (N)	CDF @ 10%	PRESENT VALUE (N)
0	(80,000)	1.0000	(80,000)
1-4	100,000	3.1699	316,990
1-4	(70,000)	3.1699	(221,893)
			15,097

SET 2

YEAR	CASHFLOW (N)	CDF @ 10%	PRESENT VALUE (N)
0	(80,000)	1.0000	(80,000)
1-4	100,000	3.1699	316,990
1-4	(50,000)	3.1699	(158,495)
			78,495

SET 3

YEAR	CASHFLOW (N)	CDF @ 10%	PRESENT VALUE (N)
0	(80,000)	1.0000	(80,000)
1-4	110,000	3.1699	348,689
1-4	(70,000)	3.1699	(221,893)
			46,796

EXPECTED NET PRESENT VALUE BASED ON THE THREE SETS OF SIMULATION

SET	NPV
1	15,097
2	78,495
3	46,796
TOTAL	140,388

$$\text{AVERAGE} = \frac{140,388}{3}$$

$$= N46,796$$

Solution to SAQ 6.5

Please find below the advantages and dis-advantages of decision tree.

Advantages

a) It is easy to understand, simple and also easy to draw.

b) It enables error to be easily located and traced.

c) It gives the picture of the whole data of distribution at a glance.

c) It gives a clearer overview of investment information to be obtained.

Dis-advantages

a) It does not show more information than could be shown in a tabular form.

b) Decision tree problems comprise investments spread over a number of years.

c) Decision tree suffers from the same dis-advantages as expected value because the tree is evaluated using Expected Values.

REFERENCES

Damilola, D. A. (2011), Business Finance, High Rise Associate Ventures, Lagos.

Harold, B. Jr. and Seymour, S. (1980), The Capital Budgeting Decision, Macmillan Publishing Co., Inc., United State.

ICAN Study Pack (2006) Management Accounting, VI Publishing Limited, Nigeria.

Lucey, T. (1996), Quantitative Techniques, Asford Colour Press, Gosport, GreatBritain.

Olaoluniyi, O. (2010), Elements of Business Finance, Panaf Publishing Inc., Abuja.

Olowe, R. F. (2008) Financial Management: Concepts, Analysis and Capital Investments, Brierly Jones Nigeria Limited, Lagos.

Owualah .S.I.(2003) Principles of Financial Management, G-Mag Investments Ltd. Lagos.

Owualah .S.I.(2003) Understanding Business Finance, Second Edition, G-Mag Investments Ltd. Lagos.

Pandy, I.M. (1999) Financial Management 2nd Edition, Vikas Publishing House PVT Ltd., New Delhi.

Should you require more explanation on this study session, please do not hesitate to contact your e-tutor via the LMS.

Are you in need of General Help as regards your studies? Do not hesitate to contact the DLI IAG Center by e-mail or phone on:

iag@dli.unilag.edu.ng

08033366677

Study Session 1 Cost of Capital 1

Introduction

Cost of capital have been used to discount the future cash inflows and then subtracting the cost in order to determine the viability or profitability of a project or an investment using net present value (NPV) criteria. It follows then that cost of capital can be called the appropriate discount rate or required return expected on a project or investment by the supplier of firm's funds. It is a basic fact that firm's uses debenture capital, preference shares capital and equity capital to make up their firm's capital structure. Thus, the cost of capital will be the mixture of the returns needed to compensate the provider of debenture capital, preference shares capital and equity capital.

Learning outcome for study session 1

At the end of this study session, you should be able to;

1.1 Demonstrate a good understanding why companies need to know their cost of capital

1.2 Understand the fundamental of cost of capital

1.3 Define cost of debt and solve application problem involves cost of debt

1.4 Define cost of preference share and solve application problem involves cost of preference share

1.5 Define cost of equity and solve application problem involves cost of equity

1.6 Evaluate the various component of cost of capital

1.1 DEFINITION AND CONCEPT OF COST OF CAPITAL:

a) **Definition**

Cost of capital can be defined as the cost of company funds. It represented the minimum rate of return that must be earned on the capital invested. The cost of capital of a company reflects the expectations of various investors that contribute money or funds to the company. (For instance; Provider of a company equity capital or ordinary shares capital, Provider of a company preference shares capital and Provider of debenture stock capital).

The cost of capital is the rate of return that company capital could be expected to earn in an alternative investment of an equal risk.

N.B

* The providers of equity capital also known as an ordinary shares capital or common stock capital are regarded as the owner of the company. They are entitled to an unlimited dividend declared out of that company's residual income, that is, profit after all other claims (provider of preference shares capital and debenture stock capital) have been settled.

* The providers of preference shares capital are not regarded as permanent part owner of the company since their dividend are at fixed rates, but entitles them to a prior claim on the company's profit and assets before the provider of equity capital or ordinary shares capital.

Preference share can be redeemable (For instance, years to maturity is known with certainty) or irredeemable (For instance, years to maturity is unknown)

- The provider of debenture stock capital received fixed interest rate on their investment. The interest on the debenture stock is normally against the company's revenue and not on share of the distributable profit.

  Debenture stock can be redeemable (For instance, years to maturity is known with certainty) or irredeemable (For instance, years to maturity is unknown).

b) **Representation**

A company cost of capital is usually represented by a capital letter K in all cases. For instance ABC Plc cost of capital is 20%, then K is 20%; XYZ Ltd cost of capital is 10%, then K is 10%. You should also know that K varies from one company to another but there must be only one K for a particular company

The basic element of cost of capital can be expressed as stated below:

$K = R_f + P_{br} + P_{fr}$

R_f = a risk free rate of return

P_{br} = a premium for business risk

P_{fr} = a premium for financial risk

Note: Olowe (1997) has defined business risk as the risk associated with uncertainty about the future and a firm's business prospects while a financial risk is the risk associated with the introduction of debt into a firm's capital structure.

- One of the following statements is not true about cost of capital.

  a. Cost of funds raised by firms. b. Discount rate of returns for discounting a project's cash flows. c. Minimum rate of return expected by the provider of a company funds. d. Maximum rate of return expected by the provider of a company funds. e. None of the above.

- The element of cost of capital is among other the following.

  a. Risk free rate. b. Premium for financial risk c. Premium for insurance risk d. a and b e. None of the above

- .... is the risk associated with introduction of debt into a company capital structure.

  a. Business risk. b. Specific risk. c. Interest rate risk d. Financial risk e. none of the above.

- The providers of preference share capital receive .... on their investment.

  a. Fixed interest. b. Fixed dividend. c. Residual dividend. d. Residual interest. e. None of the above.

- ☐ d. Maximum rate of return expected by the provider of a company funds.
- ☐ d. a and b
- ☐ d. Financial risk.
- ☐ b. Fixed dividend

1.2 MEASURING COMPONENT OF COST OF CAPITAL

1.2.1 COST OF DEBT (I.E. COST OF DEBENTURE DENOTED AS Kd)

Definition;

In line with the definition of cost of capital, the cost of debt or debenture is the minimum rate of return expected by the providers of debenture capital of a company funds.

The cost of debt can be also defined as the discount rate of return that equates the current market price of debt or debenture to the discounted future proceeds from the debt or debenture.

There are two ways or methods of calculating cost of debt or debenture;

(a) Redeemable debenture issued either at discount or premium.

The cost of debt or debenture for redeemable debenture issued at discount or premium can be calculated by using the formula stated below.

It is given by $P_o = \sum_{t=2}^{n} \frac{R(1-T)}{(1+kd)^t} + \frac{PT}{(1+kd)^n}$

Where

P_o = Current market price of debt or debenture.

K_d = Cost of debt or debenture

R = Annual interest payment

P_T = Terminal value or maturity value of debt or debenture

n = Year to maturity

T = Corporate tax rate

NOTES;

(1) The current market price of debt or debenture (P_O) is at ex-interest value.(after paying annual fixed interest on debt or debenture).

(2) The formula above can only be used for debt or debenture issued at premium or discount.

(3) If $P_o < P_T$, then the debt or debenture stock is issued at discount

(4) If $P_o > P_T$, then the debt or debenture stock is issued at premium

A few examples are present below to illustrate the application of cost of debenture in solving finance problem

HINT;

The knowledge of computation of internal rate of return (IRR) is very essential to the computation of cost of debenture (Kd) and this have been covered extensively in module 1 study session ….. Please revise this very well before you work through the examples below. If you still have problem please consult your lecturer.

Example; Redeemable Debt or Debenture issued at discount;

ABC Plc has an issue of 18% N1, 000 debenture that are currently selling at N850. Calculate the cost of debt if the year to maturity is 5 years and the corporate tax rate is 35%.

Solution

Using the available information as provided in the question where;

Current market Price of Debenture = P_o = N850

Maturity value of Debenture = P_T = N1, 000

Annual interest payment = R = 0.18 X N1, 000 = N180

Year to maturity = n = 5 years

Corporate tax rate = T = 0.35

Using the formula for calculating cost of debt for redeemable debenture

$$P_o = \sum_{t=2}^{n} \frac{R(1-T)}{(1+kd)^t} + \frac{PT}{(1 + kd)^n}$$

$$850 = \sum_{t=1}^{5} \frac{180(1-0.35}{(1+kd)t} + \frac{1000}{(1+kd)^5}$$

$$850 = \frac{117}{(1+kd)} + \frac{117}{(1+kd)^2} + \frac{117}{(1+kd)^3} + \frac{117}{(1+kd)4} + \frac{117}{(1+kd)5} + \frac{1000}{(1+kd)5}$$

Transfer this equation into a table as shown below

Year	Cash flow	DCF@20%	Present value	Year	Cash flow	DCF@10%	Present value
0	(850)	1.0000	(850)	0	(850)	1.0000	(850)
1-5	117	2.9906	349.90	1-5	117	3.7908	443.52
5	1000	0.4019	401.9	5	100	0.6209	620.9

NPV = (98.2) NPV 214.42

After obtaining two distinct values of NPV as shown in the table above,, then apply the concept of internal rate of return (IRR) formu

$$IRR = DCF_+ + \frac{NPV_+}{NPV_+ - NPV_-}(DCF_- - DCF_+)$$

$$Kd = 0.10 + \frac{214.42}{214.42+98.2}(0.2 - 0.1)$$

Kd = 0.10 + 0.069

Kd = 0.169

Kd = 16.9%

The cost of debt (Kd) of the company is 16.9% or approximately 17%

Example; Redeemable Debt or Debenture issued at a premium

XYZ Plc has an issue of 18.75% N100 debenture that are currently selling at N106. Calculate the cost of debt if the debenture is redeemable at par in 5years time and corporate tax rate is 35%.

Solution

Using the available information as provided in the question

Current market price of the Debenture = P_o = N106

Maturity value of the Debenture = P_T = N100

Annual interest payment = R = 0.1875 X N100 = N18.75

Year to maturity = n = 5 years

Corporate tax rate = T = 0.35

Using the formula for calculating cost of debt for redeemable debenture

$$P_o = \sum_{t=2}^{n} \frac{R(1-T)}{(1+kd)^t} + \frac{PT}{(1 + kd)^n}$$

$$106 = \sum_{t=1}^{5} \frac{18.75(1-0.35)}{(1+kd)t} + \frac{100}{(1+kd)^5}$$

$$106 = \frac{12.19}{(1+kd)} + \frac{12.19}{(1+kd)^2} + \frac{12.19}{(1+kd)^3} + \frac{12.19}{(1+kd)4} + \frac{12.19}{(1+kd)5} + \frac{100}{(1+kd)5}$$

Transfer this equation into a table as shown below

Year	Cash flow	DCF@20%	Present value	Year	Cash flow	DCF@10%	Present value
0	(106)	1.0000	(106)	0	(106)	1.0000	(106)
1-5	12.19	2.9906	36.46	1-5	12.19	3.7908	46.21

5	100	0.4019	40.19	5	100	0.6209	62.09

NPV = (29.35) NPV 2.30

After obtaining two distinct values of NPV as shown in the table above, then apply the concept of IRR formula;

$$IRR = DCF_+ + \frac{NPV_+}{NPV_+ - NPV_-}(DCF_- - DCF_+)$$

$$Kd = 0.10 + \frac{2.30}{2.30 + 29.35}\ (0.2 - 0.1)$$

Kd = 0.10 + 0.007

Kd = 0.107

Kd = 10.7%

The cost of debt (Kd) of the company is 10.7% or approximately 11%

(b) PERPETUAL OR IRREDEEMABLE DEBT OR DEBENTURE

The formula for cost of debt for a perpetual or irredeemable debt or debenture is given as follows:

$$Kd = \frac{R(1-t)}{Po}$$

Where

P_o = Current market price of debt or debenture.

K_d = Cost of debt

R = Annual interest payment

T = Corporate tax rate

Example:

LIZZY Plc has an issued 18% N1, 000 irredeemable debentures currently selling at N1, 150. Calculate the cost of debt, if the corporate tax rate is 35%.

Solution;

Using the available information as provided in the question

Current market price of debenture = Po = N1, 150

Annual interest payment = R = 0.18 X N1, 000 = N180

Corporate tax rate = T = 0.35

Using the irredeemable debenture formula for cost of debt as shown below

$$Kd = \frac{R(1 - T)}{Po}$$

$$Kd = \frac{180(1-0.35)}{1150}$$

$$Kd = 0.102$$

$$Kd = 10.2\%$$

The cost of debt (Kd) of the company is 10.2% or approximately 10%

- One of these is correct about cost of debenture stock.

  a. Cost of funds raised by firms. b. Discount rate of return that equate terminal value of debt with the proceeds from the debt. c. Discount rate of return that equate current market value of debt with the proceeds from the debt. d. It has a fixed dividend return. e. It does not involve corporate tax.

- Kindly determine P_o, PT, T, n and R from the following information;

  Ade plc has an issue of 21% N1, 000 debenture currently selling at N1,230 and the year to maturity is five years. The current corporate tax rate is 35%

- Johnson plc has in issue 14% N1, 000 irredeemable Debenture currently selling at N950. Determine the cost of debt if the corporate tax rate is 35%.

  a. 10% b. 19% c. 20% d. 14% e. None of the above

- The providers of debenture capital received .... on their investment.

  a. Fixed dividend b. Fixed interest c. Fixed stock d. nominal dividend. e. high interest.

☐ C = Discount rate of return that equate current market value of debt with the proceeds from the debt.

☐ P_o = N1,230, PT = N1,000, T = 35% = 0.35, n = 5, R = 0.21 X N1,000 = N210.

☐ $$Kd = \frac{R(1-T)}{P_o}$$

$$= \frac{0.14 \text{ X N1, 000 } (1 - 0.35)}{\text{N950}}$$

$$= \frac{\text{N91}}{\text{N950}}$$

= 0.096 = 10% = option

☐ b. = Fixed interest

1.2.2 COST OF PREFERENCE SHARE (denoted by Kp)

Definition

In line with the definition of cost of capital above, Cost of preference share is the minimum rate of return required by the providers of preference share capital. The computation of cost of preference share is close to that of debenture as their characteristics look similar. The only difference between a preference share and a debenture is that the dividends payments on preference shares are not allowable for tax adjustments. Therefore, all the formula used in calculating cost of debenture is applicable to cost of preference share except that there is no need for tax adjustments.

There are two ways or methods of calculating cost of preference share (Kp).

(a) Redeemable preference share issued either at discount or premium;

The cost of preference share for redeemable preference share issued either at discount or premium can bse determine by the formula present below

$$P_o = \sum_{t=1}^{n} \frac{D_t}{(1+k_p)^t} + \frac{P_t}{(1+k_p)^n}$$

Where

P_o = Current market price of a preference share

P_T = Maturity value of a preference share

n = Year to maturity of the preference share

D_t = Annual fixed dividend payment on a preference share

K_p = Cost of preference share

Notes;

i. The current market price a preference share is stated in the above formula as ex-dividend (after the annual fixed dividend has been paid).
ii. The above formula can only be used for preference share issued either at premium or discount.
iii. If $P_o < P_T$, then the preference share is issued at discount
iv. If $P_o > P_T$, then the preference share is issued at premium

HINT;

The knowledge of computation of internal rate of return (IRR) is very essential to the computation of the cost of preference share (Kp) and this have been covered extensively in module 1, study session ….. Please revise this before you work through the examples below. If you have problem please consult your lecturer.

EXAMPLE: Preference share issued at discount

NANCY LTD has in issue a 13% N10.00 preference share with a maturity of 5 years. Calculate the cost of preference share if the current market value is N9.50.

Solution

Using the formula for calculating cost of preference share for redeemable preference share

Po = $$P_o = \sum_{t=1}^{n} \frac{D_t}{(1+k_p)^t} + \frac{P_t}{(1+k_p)^n}$$

Where;

Current market price of preference share = P_o = N9.50

Terminal value of preference share = P_t = N10.00

Annual fixed dividend payment on preference share = Dp = 0.13 X N10.00 = N1.30

Years to maturity = n = 5

$$9.50 = \sum_{t=1}^{5} \frac{1.30}{(1+k_p)^t} + \frac{10}{(1+k_p)^n}$$

Transfer the above equation to a table as shown below

Year	Cash	DCF@20%	Present	Year	Cash	DCF@10%	Present

	flow		value		flow		value
0	(9.50)	1.0000	(9.50)	0	(9.50)	1.0000	(9.50)
1-5	1.30	2.9906	3.89	1-5	1.30	3.7908	4.93
5	10	0.4019	4.02	5	10	0.6209	6.21

NPV= (1.59) NPV = 1.64

After obtaining two distinct values of NPV as shown in the table above, then apply the concept of IRR formula;

$$IRR = DCF_+ + \frac{NPV_+}{NPV_+ - NPV_-}(DCF_- - DCF_+)$$

$$Kp = 0.1 + \frac{1.64}{1.64 + 1.59}(0.2 – 0.1)$$

$$Kp = 0.1 + \frac{1.64}{3.23}(0.1)$$

$$Kp = 0.1 + 0.051$$

$$Kp = 0.151$$

$$Kp = 15.1\%$$

The cost of preference share (Kp) of the company is 15.1% or approximately 15%

EXAMPLE: Preference share issued at premium

JOHN BULL LTD has in issue a 13% N10.00 preference share with a maturity of 5 years. Calculate the cost of preference share if the current market value is N10.50.

Solution

Using the formula for calculating cost of preference share for redeemable preference share

$$P_o = \sum_{t=1}^{n} \frac{D_t}{(1+k_p)^t} + \frac{P_t}{(1+k_p)^n}$$

Where;

Current market price of preference share = P_o = N10.50

Terminal value of preference share = P_t = N10.00

Annual fixed dividend payment on preference share = Dp = 0.13 X N10.00 = N1.30

Years to maturity = n = 5

$$10.50 = \sum_{t=1}^{5} \frac{1.30}{(1+k_p)^t} + \frac{10}{(1+k_p)^n}$$

Transfer the above equation to a table as shown below

Year	Cash flow	DCF@20%	Present value	Year	Cash flow	DCF@10%	Present value
0	(10.50)	1.0000	(10.50)	0	(10.50)	1.0000	(10.50)
1-5	1.30	2.9906	3.89	1-5	1.30	3.7908	4.93
5	10	0.4019	4.02	5	10	0.6209	6.21

NPV= (2.59) NPV = 0.64

After obtaining two distinct values of NPV as shown in the table above, then apply the concept of IRR formula; $\frac{NPV_+}{NPV_+ - NPV_-}(DCF_- - DCF_+)$

IRR = DCF_+ +

$$Kp = 0.1 + \frac{0.64}{0.64 + 2.59}(0.2 - 0.1)$$

$$Kp = 0.1 + \frac{0.64}{3.23}(0.1)$$

Kp = 0.1 + 0.02

Kp = 0.12

Kp = 12%

The cost of preference share (Kp) of the company is 12%

(b) PERPETUAL OR IRREDEEMABLE PREFERENCE SHARE

The formula for cost of preference share for a perpetual or irredeemable preference share is given as follows:

$$K_p = \frac{D_t}{P_o}$$

Where

P_o = Current market price of a preference share

D_t = Annual fixed dividend on a preference

K_p = Cost of preference share

Example ;

KAREEM LTD has in issue a 13% N10.00 irredeemable preference share currently selling at N14.75. Calculate the cost of preference share.

Solution

Using the available information as provided in the question where

Current market price of preference share = Po = N14.75

Annual fixed dividend payment on preference share = 0.13 X N10.00 = N1.3

Using the formula for cost of preference for irredeemable preference share as;

$$Kp = \frac{Dt}{Po}$$

$$Kp = \frac{0.13 \text{ X } N10.00}{N14.75}$$

$$Kp = \frac{1.3}{14.75}$$

$$Kp = 0.088$$

$$Kp = 8.8\%$$

The cost of preference share (Kp) of the company is 8.8% or approximately 9%

- Cost of preference share is best define as the…………rate of return expected by the provider of preference share capital.

  a. Maximum b. Highest c. Biggest d. Apex e. Minimum

- Kindly determine P_o, PT, n and Dt from the following information;

  Dede Plc has an issue of 15% N10 preference share currently selling at N15 and the year to maturity is 5 years.

- Johnson plc has in issue 14% N100, 000 irredeemable preference share currently selling at N95,000. Determine the cost of preference share.

  a.10% b.19% c.20% d.14% e. None of the above

  - ☐ e. = Minimum
  - ☐ P_o = N15, PT = N10, , n = 5, Dt = 0.15 X N10 = N15.
  - ☐ $Kd = \frac{Dt}{P_o}$

$$= \frac{0.14 \text{ X N}100,000}{\text{N}95,000}$$

$$= \frac{\text{N}14,000}{\text{N}95,000}$$

= 0.143 = 14% = option a

1.2.3 COST OF EQUITY (denoted by K_e)

Definition

In line with the definition of cost of capital, Cost of equity is the minimum rate of returns required by the provider of ordinary share capital. The cost of equity is more difficult to calculate as equity does not pays a set of returns to its investors. The cost of equity could be estimated by means of the dividend valuation model (DVM). The Dividend valuation model relate the markets value of equity to expected future dividends on the equity.

Assuming an initial dividend D_1 and a constant payout ratio, the Dividend valuation model is given as:

$$P_o = \sum_{t=1}^{n} \frac{D_{tt}}{(1+k_p)^t}$$

$$= \frac{D_1}{(1+k_e)} + \frac{D_2}{(1+k_e)^2} + \ldots \frac{D-0}{(1+k_e)^e}$$

The model assume a constant dividend growth rate g per year into indefinite future

$$= \frac{D_o+(1+g)}{1+k} + \frac{D_o+(1+g)^2}{(1+k)^2} + \ldots. \frac{D_o+(1+g)^2}{(1+k)^2}$$

$$P_o = \frac{D_o+(1+g)}{k_e-g} = \frac{D_1}{k_e-g}$$

$$K_e = \frac{D_o + (1+g)}{P_o} + g \text{ or } \frac{D_1}{P_o} + g \text{………………………………..(M)}$$

Equation (M) can be used to compute the cost of equity.

Where

K_e = cost of equity

P_o = current market price of equity

D_o = Dividend per share that has been declared or existing dividend per share

D_1 = Proposed dividend per share or dividend to be declared at the end of the current year

g = Annual dividend growth rate

Example:

JANE Plc has just paid a dividend of 15k per share, the duration of JANE Plc have the opinion that dividend growth rate at 8% per annum, if the current market price of the share is N1.50. Calculate the cost of equity.

Solution:

Using the cost of equity formula

$$k_e = \frac{D_O(1+g)}{P_O} + g$$

Where;

Existing dividend per share = D_o = N0.15

Annual dividend growth rate = g = 0.08

Current market price of equity = P_o = N1.50

k_e = $\underline{0.15\ (1.08)}$ + 0.08

1.50

$k_e = 0.108 + 0.08$

$k_e = 0.188$

$k_e = 18.8\%$

The cost of equity (Ke) of the company is 18.8% or approximately 19%

Example;

JOHN Plc is financed with 2,000,000 ordinary shares of N1.00 each. The firm's dividends have been growing at the rate of 7% annually and this trend is expected to continue. A divided of 32k per share is expected to paid in a year's time while the securities are currently quoted on the stock market at N2,52 per each ordinary shares. Calculate cost of equity.

Solution

Using the cost of equity formula

$$K_e = \frac{D_1}{P_o} + g$$

Where

Propose dividend per share = D_1 = N0.32

Annual dividend growth rate = g = 0.07

Current market price of equity = P_o = N2.52

$$k_e = \frac{0.32}{2.52} + 0.07$$

$k_e = 0.127 + 0.07$

$k_e = 0.197$

$k_e = 19.7\%$

The cost of equity (Ke) of the company is 19.7% or approximately 20%

Note

i. If the annual dividend growth rate (g) is not given, the formula shown below can be used to determine g.

$$g = \sqrt[n-1]{\frac{\text{Latest dividend}}{\text{Earliest dividend}}} - 1$$

Where;

g = annual dividend growth rate

n = number of year's growth

Example;

NENE Plc is a company with 1,000,000 ordinary shares of N1 each currently quoted at N41.70 as at 31st December 2011. The summary of dividend in the last 5years is as follows;

YEAR	DIVIDEND (N)
2007	397, 000
2008	410, 000
2009	438, 000
2010	463, 000
2011	480, 000

REQUIRED

Compute the annual dividend growth rate for the company.

Solution

Using the formula for annual dividend growth rate (g)

$$g = \sqrt[n-1]{\frac{\text{Latest dividend}}{\text{Earliest dividend}}} - 1$$

Where;

Latest dividend = N480, 000

Earliest dividend = N397, 000

n = 5-1 = 4

$$g = \sqrt[4]{\frac{480{,}000}{397{,}000}} - 1$$

$g = (1.2091)^{1/4} - 1$

$g = (1.2091)^{0.25} - 1$

g = 1.05 - 1

g = 0.05

g = 5%

The annual dividend growth rate of the company is 5%

- Cost of equity can be evaluated using the …………..

a. Dividend valuation model b. Dividend valuation growth model. c. Dividend growth rate model

d. Valuation growth model e. None of the above

- Do and D1 simply means .... and .... respectively.

  a. Propose dividend and existing dividend. b. Propose dividend per share and existing dividend per share. c. Existing earnings and propose earnings. d. existing dividend per share and propose dividend per share. e. current dividend and propose dividend.

- Which of these is not relevant in computing cost of equity?

a. Propose dividend per share. b. Existing dividend per share. c. Corporate tax rate. d. current market price of equity. e. annual dividend growth rate.

- The providers of equity capital received .... on their investment.

a. Fixed return b. Nominal return c. Residual dividend d. fixed dividend e. fixed income..

- ☐ b = Dividend valuation growth model.
- ☐ c. = existing dividend per share and propose dividend per share
- ☐ d = corporate tax rate.
- ☐ e = residual dividend.

1.2.4 COST OF RETAINED EARNINGS:

Retained earnings represent part of company earnings (earnings after interest and tax) available for distribution to shareholders by way of dividends. When cash from retained earnings is invested in a new project, such projects are assumed to be generally similar to the normal activities of the company.

SUMMARY OF THE STUDY SESSION 1;

In this session, you have learnt about cost of capital, cost of debt or debenture, cost of preference share and cost of equity. How to determine the various component of cost of capital and you can solve simple finance problems using the formula of Cost of debenture, cost of preference shares and cost of equity. If there is any problem, you can go back to the session again because the knowledge here is a pre-requisite for the immediate next study session.

Self Assessment Questions (SAQs) for Study Session 1

SAQ 1.1 (tests Learning Outcomes 1.1 and 1.2)

a. Explain the term cost of capital.

b. Mention and explain basic elements of cost of capital.

SAQ 1.2 (tests Learning Outcomes 1.3, 1.4 and 1.5)

Write short notes on the following;

(a) Cost of preference share

(b) Cost of equity

(c) Cost of debt

SAQ 1.3 (tests Learning Outcomes 1.3, 1.4, 1.5 and 1.6)

Oluwatobi Plc has the following information;

i. 2 million ordinary shares at N1.00 each currently selling at N1.20.

ii. The total ordinary dividend of N500, 000 was deducted at the last annual general meeting.
iii. Besides the above, the company expects a stable annual growth rate of 6% on its ordinary dividend.
iv. The debentures 18.75% N600, 000, which are currently selling at N1, 080 per N1,000 debentures; are currently selling at par in five years time.
v. The preference share 15% N1,000,000, which are currently selling at N1.80 per N1.00 preference share are irredeemable
vi. Preference dividend and debenture interest have just been paid
vii. Corporate tax rate is currently 30%.

REQUIRED

a. Compute the company’s cost of debt
b. Compute the company’s cost of preference share
c. Compute the company’s cost of equity

Answer to Study Sessions Self Assessment Questions (SAQs) for Module 3

Study Session 1

1a. Cost of capital is the cost of fund raised by a company. It represents the minimum rate of return that must be earned on the capital invested. It is the minimum rate of return expected by the provider of ordinary share capital, preference share capital and debenture capital.

b. The basic element of Cost of capital (K) can be expressed as:

$K = R_f + P_{br} + P_{fr}$

Where K = cost of capital

R_f = a risk free rate of return

P_{br} = a premium for business risk

P_{fr} = a premium for financial risk

2a. Cost of preference share is the minimum rate of return required by the provider of preference share capital.

b. Cost of equity is the minimum rate of return required by the provider of ordinary share capital.

c. Cost of debt is the minimum rate of return required by the provider of debenture stock capital.

3a. Using the formula for calculating cost of debt for redeemable debenture

$$P_o = \sum_{t=2}^{n} \frac{R(1-T)}{(1+kd)^t} + \frac{PT}{(1+kd)^n}$$

$$1{,}080 = \sum_{t=1}^{5} \frac{187.5(1-0.3)}{(1+kd)t} + \frac{1{,}000}{(1+kd)^5}$$

$$1{,}080 = \frac{131.25}{(1+kd)} + \frac{131.25}{(1+kd)^2} + \frac{131.25}{(1+kd)^3} + \frac{131.25}{(1+kd)4} + \frac{131.25}{(1+kd)5} + \frac{1000}{(1+kd)5}$$

Transfer this equation into a table as shown below

Year	Cash flow	DCF@20%	Present value	Year	Cash flow	DCF@10%	Present value
0	(1,080)	1.0000	(1,080)	0	(1,080)	1.0000	(1,080)

1-5	131.25	2.9906	392.52	1-5	131.25	3.7908	497.54
5	1,000	0.4019	401.90	5	1000	0.6209	620.90

NPV = (285.58) NPV 38.44

After obtaining two distinct values of NPV as shown in the table above, then apply the concept of IRR formula;

$$IRR = DCF_{+} + \frac{NPV_{+}}{NPV_{+} - NPV_{-}}(DCF_{-} - DCF_{+})$$

$$Kd = 0.10 + \frac{38.44}{38.44 + 285.58}\ (0.2 - 0.1)$$

Kd = 0.10 + 0.012

Kd = 0.112

Kd = 11.2%

The cost of debt (Kd) of the company is 11.2% or approximately 11%

3b. Using the formula for cost of preference for irredeemable preference share as;

$$Kp = \frac{Dt}{Po}$$

$$Kp = \frac{0.15 \text{ X N}1.00}{\text{N}1.80}$$

$$Kp = \frac{0.15}{1.80}$$

Kp = 0.083

Kp = 8.3%

The cost of preference share (Kp) of the company is 8.3% or approximately 8%

3c.Using the formula for cost of equity as;

$$k_e = \frac{D_O(1+g)}{P_O} + g$$

Where

Existing dividend per share = $D_o = \frac{N500,000}{2,000,000} = N0.25$

Annual dividend growth rate = g = 0.06

Current market price of equity = P_o = N1.20

$$k_e = \frac{0.25\,(1.06)}{1.20} + 0.06$$

k_e = 0.221 + 0.06

k_e = 0.227

k_e = 22.7%

The cost of equity (Ke) of the company is 22.7% or approximately 23%

REFERNCES AND FURTHER STUDIES

Ezike John (2002) Essential of Corporate Financial Management, Lagos Jaylycent pp 275 - 288

Olowe R.A (1997) Financial Management: Concept, Analysis and capital Investment. Lagos Bierly Jones Nigeria Limited pp 388 – 400

Otaniyi O. (1997) Elements of Business Finance, Panaf Publishing Incorporation. pp 119 - 126

Owualah S.I (2003) Understanding Business Finance, Lagos G-Mag Investment Ltd pp 51 - 65

Pandey I.M (2000) Financial Management India Vikas Publishing PVT Ltd pp 493 - 511

Shiro A.A (2004) Problems and Solutions in Financial Management, Lagos. El-Toda Ventures Ltd. pp 171 – 194

Should you require more explanation on this study session, please do not hesitate to contact your e-tutor via the LMS.

Are you in need of General Help as regards your studies? Do not hesitate to contact the DLI IAG Center by e-mail or phone on:

iag@dli.unilag.edu.ng
08033366677

Study Session 2 Cost of Capital II

INTRODUCTION;

Companies tend to have a mixture of the different forms of capital in their capital structure namely equity share capital, preference share capital and debenture stock capital as demonstrated in the immediate past study session.

After determine the cost of each source of capital used by company (cost of equity, cost of preference share and cost of debenture), we then multiply the proportion of each weight of the capital structure items by the corresponding costs and add them up.

The total is known as overall cost of capital or weighted average cost of capital (WACC). Weighted average cost of capital of a company must reflects the company marginal cost of capital (weighted average cost of incremental capital) before it can be use to evaluate investment projects.

Learning Outcomes for Study Session 2

After reading and understanding the contents of this unit and working through all worked examples and practice questions you should be able to;

2.1 Understand the concept of weighted average cost of capital

2.2 Evaluate various components of weighted average cost of capital

2.3 Measure weighted average cost of capital.

2.4 Know the conditions at which weighted average cost of capital can be use as a discounting rate to evaluate projects

2.5 Understand the concept of marginal cost of capital.

2.1 DEFINITION OF WEIGHTED AVERAGE COST OF CAPITAL (WACC)

Weighted Average Cost of Capital (WACC) is the overall cost of capital of a firm or a company. It is the weighted average of the various sources of funds in a company capital structure where each weight is a represented by the proportion of each sources of fund in the capital structure.

Weighted average cost of capital is always expressed as a percentage. For instance, if a company works with a weighted average cost of capital (WACC) of 16%, then this means that all projects or investments accepted or selected should have or gives a return higher than the weighted average cost of capital (WACC) of 16%.

It is possible to combine cost of debt or debenture, cost of preference shares and cost of equity to produce a single number for a company weighted average cost of capital in which each sources of funds or capital is proportionately weighted.

- WACC simply means

  a. Marginal cost and capital b. World average cost of capital c. Weighted cost of capital

  d. Weighted average cost of capital e. None of the above

- The overall cost of capital is known as ....

  a. Weighted average b.cost of capital c.weighted average cost of capital d.cost of equity e.cost of debenture

  - ☐ d=Weighted average cost of capital
  - ☐ c=.weighted average cost of capital

2.2 COMPONENT OF WEIGHTED AVERAGE COST OF CAPITAL

Weighted average cost of capital (WACC) can be computed by using the formula stated below;

$$WACC = \frac{D}{V}(Kd) + \frac{P}{V}(Kp) + \frac{E}{V}(Ke)$$

Where;

D = Value of debt or debenture = $\frac{\text{Market Price}}{\text{Nominal Price}}$ X Number of debenture issued

P = Value of Preference share = $\frac{\text{Market Price}}{\text{Nominal Price}}$ X Number of preference share issued

E = Value of Equity share = $\frac{\text{Market Price}}{\text{Nominal Price}}$ X Number of equity share issued

V == Value of the firm or company = D + P + E.

Kd = Cost of debt or debenture.

Kp = Cost of preference shares.

Ke = Cost of equity

NOTES;

1. One can calculate weighted average cost of capital (WACC) using weight either based on book value or the market value, but it is more meaningful and appropriate to use market value when the data are available.

2. Weighted average cost of capital (WACC) can only be used as the discounting rate to evaluate company's investment projects if and only if;
   a. The company capital structure is optimal.
   b. The company weighted average cost of capital (WACC) reflects the company's marginal cost of capital (MCC).
   c. The company weighted average cost of capital (WACC) reflects the company long term future capital structure.
   d. If company existing projects and the company propose projects are in the same risk classification.
   e. If new funds are raised in a way that keeps the company's capital structure unchanged.

3. If the above conditions are relaxed, then, weighted average cost of capital (WACC) can never be used as discounted rate to evaluate company projects.
   a. The company capital structure is not optimal.
   b. The company weighted average cost of capital (WACC) does not reflect the company marginal cost of capital (MCC).
   c. The company weighted average cost of capital (WACC) does not reflect the company long term future capital structure.
   d. If company existing projects and the company propose projects are not in the same risk classification.
   e. If new funds are raised in a way that the company's capital structure changes.

   A few examples are present below to illustrate the application of weighted average cost of capital (WACC) in solving finance problem.

EXAMPLE;

Goodluck Plc is trying to decide on the cost of capital discounting rate to apply to the evaluation of investment project. The company has in issued shared capital of 2,000,000 ordinary shares of N1.00 each with a current market value of N3.60 per share. It also has in issue N1.2 million 19% debenture which are redeemable at par in five years time and have a current market value of N105 per N100 debenture and N800, 000 20% irredeemable preference share currently selling at N0.96.Ordinary divided of N1, 260, 000 was recently paid and annual

Dividend growth rate is 12 percent. The corporate tax rate is 35%.

Required;

Estimate the weighted average cost of capital (WACC)

Solution;

You can start computing the weighted average cost of capital with any of the three costs (i.e cost of preference share, cost of equity and cost of debenture or debt). But, what is important is the usage of the correct formula and accuracy of your computation.

STEP 1;

CALCULATION OF COST OF PREFERENCE SHARE (Kp)

Using the formula for cost of preference share (Kp) for irredeemable preference share as shown below;

$$Kp = \frac{Dp}{Po}$$

Where

Annual fixed dividend payment $= D_p =$ N0.20

Current market price of preference share $= P_o =$ N0.96

$$Kp = \frac{0.20}{0.96}$$

$$Kp = 0.208$$

The company cost of preference share (Kp) is 20.8%

STEP 2;

CALCULATION OF COST OF EQUITY (Ke)

Using the formula for computing cost of equity (k_e) as shown below

$$Ke = \frac{Do\,(1+g)}{Po} + g$$

Where;

Existing Dividend Per Share = Do = $\frac{1,260,000}{2,000,000}$

D_o = N0.63

Current Market Price of Equity = P_o = N3.60

Annual Dividend Growth Rate = g = 0.12

$$K_e = \frac{0.63(1 + 0.12)}{3.60} + 0.12$$

k_e = 0.316

The company cost of equity (Ke) is 31.6%

STEP 3;

CALCULATION OF COST OF DEBT (K_d)

Using the formula for computing cost of debt of redeemable debenture as shown below;

$$P_o = \sum_{t=2}^{n} \frac{R(1-T)}{(1+kd)^t} + \frac{PT}{(1 + kd)^n}$$

Where

Current market price of debenture = Po = N105

Terminal price of debenture = PT = N100

Annual fixed interest payment = R = 0.19 X N100 = N19

Corporate tax rate = T = 0.35

Years to maturity = n = 5

$$105 = \sum_{t=1}^{5} \frac{19(1-0.35)}{(1+kd)t} + \frac{100}{(1+kd)^5}$$

$$105 = \frac{12.35}{(1+kd)} + \frac{12.35}{(1+kd)^2} + \frac{12.35}{(1+kd)^3} + \frac{12.35}{(1+kd)4} + \frac{12.35}{(1+kd)5} + \frac{100}{(1+kd)5}$$

Transfer the above equation to a table as shown below

Year	Cash	DC@20%	Present	Year	Cash	DCF@10%	Present

	flow		value		flow		value
0	(105)	1.0000	(105)	0	(105)	1.0000	(105)
1-5	12.35	2.9906	36.93	1-5	12.35	3.7908	46.82
5	100	0.4019	40.19	5	100	0.6209	62.09

NPV= (27.88)   NPV = 3.91

After obtaining two distinct value of NPV as show in the table above, the apply the concept of IRR formula;

$$IRR = DCF_{+} + \frac{NPV_{+}}{NPV_{+} - NPV_{-}}(DCF_{-} - DCF_{+})$$

$$K_d = 0.10 + \frac{3.91}{3.91 + 27.88}(0.2 - 0.1)$$

$$K_d = 0.10 + \frac{3.91}{31.79}(0.1)$$

$$K_d = 0.10 + 0.012$$

$$K_d = 0.112$$

The cost of debenture (Kd) is 11.2%

STEP 4;

CALCULATION OF WEIGHTED AVERAGE COST OF CAPITAL (WACC)

Using the formula for computing weighted average cost of capital as shown below

WACC = $\underline{D}$ (Kd) + $\underline{P}$ (Kp) + $\underline{E}$ (Ke)

V V V

Where

Value of debenture (D) = $\frac{N105}{N100}$ X 1,200,000 = N1, 260, 000

Value of preference share (P) = $\frac{N0.96}{N1.00}$ X 800,000 =N768, 000

Value of equity (E) = $\frac{N3.60}{N1.00}$ X 2,000,000 = N7, 200, 000

Value of the company (V) = D + P + E

= N1, 260, 000 + N768, 000 + N7, 200, 000

Value of the company (V) = N9, 228, 000

Cost of equity = $K_e = 0.316$

Cost of debtor debenture = $K_d = 0.112$

Cost of preference share = Kp = 0.208

WACC = $\frac{N1,260,000}{N9,228,000}(0.112) + \frac{N768,000}{N9,228,000}(0.208) + \frac{N7,200,000}{N9,228,000}(0.316)$

WACC =0.015 + 0.017 + 0.247

WACC = 0.279

WACC = 27.9%

The company weighted average cost of capital (WACC) is 27.9% or approximately 28%.

EXAMPLE;

The capital structure of DEDE Ltd, a quoted company is given as follows:

	N
Ordinary Shares of 50k each	2,000,000
21% preference shares (N1 per value)	1,000,000
18.75% debentures	1,500,000
Retained earnings	4,000,000
	8,500,000

The following information are relevant:

i. The debenture are currently selling at N106 per N100 debentures redeemable at par in five years time.

ii. The preference shares are currently selling at 84km per share and are irredeemable

iii. The preference dividend and debenture interest have just been paid

iv. The company ordinary shares currently have market value of N1.69 per share

v. Ordinary share dividend of N480,000 was recently paid

vi. The dividend annual growth rate is 6%

vii. Corporate tax rate is currently 35%

REQUIRED

Estimate the weighted average cost of capital

Solution;

You can start computing the weighted average cost of capital with any of the three cost(i.e cost of preference share, cost of equity and cost of debenture). But, what is important is the usage of the correct formula and accuracy of your answer.

STEP 1;

CALCULATION OF COST OF PREFERENCE SHARE (Kp)

Using the formula for cost of preference share for irredeemable preference share as shown below;

$$Kp = \frac{Dp}{Po}$$

Where

Annual fixes dividend payment = D_p = N0.21

Current market price of preference share = P_o = N0.84

$$Kp = \frac{0.21}{0.84}$$

$$Kp = 0.250$$

The company cost of preference share (Kp) is 25%

STEP 2;

CALCULATION OF COST OF EQUITY (Ke)

Using the formula for computing cost of equity (k_e) as shown below

$$Ke = \frac{Do(1 + g)}{Po} + g$$

Where

Existing dividend per share = $Do = \frac{480{,}000}{4{,}000{,}000}$

D_o = N0.12

Current market price P_o = N1.69

Annual dividend growth rate g = 0.06

$K_e = \frac{0.12(1 + 0.06)}{1.69} + 0.06$

Ke = 0.075 + 0.06

$k_e = 0.135$

The company cost of equity (Ke) is 13.5%

STEP 3;

CALCULATION OF COST OF DEBT OR DEBENTURE (K_d)

Using the formula for computing cost of debt for redeemable debenture as shown below;

$$P_o = \sum_{t=2}^{n} \frac{R(1-T)}{(1+kd)^t} + \frac{PT}{(1 + kd)^n}$$

Where;

Current market price of debenture = Po = N106

Terminal price of debenture = PT = N100

Annual fixed interest on debenture = R = 0.1875 X N100 = N18.75

Corporate tax rate = T = 0.35

Years to maturity = n = 5

$$106 = \sum_{t=1}^{5} \frac{18.75(1-0.35)}{(1+kd)t} + \frac{100}{(1+kd)^5}$$

$$106 = \frac{12.19}{(1+kd)} + \frac{12.19}{(1+kd)^2} + \frac{12.19}{(1+kd)^3} + \frac{12.19}{(1+kd)4} + \frac{12.19}{(1+kd)5} + \frac{100}{(1+kd)5}$$

Transfer the above equation to a table as shown below

Year	Cash	DCF@20%	Present	Year	Cash	DCF@10%	Present

	flow		value		flow		value
0	(106)	1.0000	(106)	0	(106)	1.0000	(106)
1-5	12.19	2.9906	36.46	1-5	12.19	3.7908	46.21
5	100	0.4019	40.19	5	100	0.6209	62.09

NPV= (29.35) NPV = 2.30

After obtaining two distinct values of NPV as shown in the table above, then apply the concept of IRR formula;

$$IRR = DCF_{+} + \frac{NPV_{+}}{NPV_{+} - NPV_{-}}(DCF_{-} - DCF_{+})$$

$$K_d = 0.10 + \frac{2.30}{2.30 + 29.35}(0.2 - 0.1)$$

$$K_d = 0.10 + \frac{2.30}{31.65}(0.1)$$

$$K_d = 0.10 + 0.007$$

$$K_d = 0.107$$

The cost of debt or debenture (Kd) is 10.7%

STEP 4;

CALCULATION OF WEIGHTED AVERAGE COST OF CAPITAL

Using the formula for computing weighted average cost of capital as shown below

$$WACC = \frac{D}{V}(Kd) + \frac{P}{V}(Kp) + \frac{E}{V}(Ke)$$

Where;

Value of debenture (D) = $\frac{N106}{N100}$ X 1,500,000 = N1, 590, 000

Value of preference share (P) = $\frac{N0.84}{N1.00}$ X 1,000,000 =N840, 000

Value of equity E) = $\frac{N1.69}{N0.50}$ X 2,000,000 = N6, 760,000

Value of the company (V) = D + P + E

= N1, 590, 000 + N840, 000 + N6, 760, 000

Value of the company (V) = N9, 190, 000

Cost of equity = K_e = 0.135

Cost of debenture = K_d = 0.107

Cost of preference share = Kp = 0.250

WACC = $\frac{N1, 590, 000}{N9, 190, 000}$ (0.107) + $\frac{N840, 000}{N9, 190, 000}$(0.250) + $\frac{N6, 760, 000}{N9, 190, 000}$(0.135)

WACC =0.019 + 0.022 + 0.099

WACC = 0.140

WACC = 14%

The company weighted average cost of capital (WACC) is 14%

ALTERNATIVE METHOD

Source	No of items issued	Po (N)	Value (N)	Proportion	Cost	
D	15,000	106	1,590,000	0.173	0.107	0.019
P	1,000,000	0.84	840,000	0.091	0.250	0.022
E	4,000,000	1.69	6,760,000	0.735	0.135	0.099
			9,190,000	1.000		0.140 → WACC

- One of the condition for using weighted average cost of capital as a discount rate for investment project appraisal is that…………………….

  a. The capital is optimal b. The capital structure is not optimal c. The new project and proposed project are not optimal d. New funds are raised in such a way that the capital structure remains unchanged e. None of the above

- One of the components of weighted average cost of capital are among others except

  a. Marginal cost of capital b. cost of debt c. cost of preference shares d. cost of equity

  e. value of the firm

- WACC simply means

  a. Marginal cost and capital b. World average cost of capital c. Weighted cost of capital

d. Weighted average cost of capital e. None of the above

- ☐ d = New funds are raised in such a way that the capital structure remains unchanged
- ☐ b = Weighted average of extra cost of capital
- ☐ d = Weighted average cost of capital

2.3 WEIGHTED AVERAGE COST OF CAPITAL VERSUS MARGINAL COST OF CAPITAL

Weighted average cost of capital has been defined as weighted average of the various sources of funds in a company capital structure where each weight is represented by the proportion of each source of fund in the company capital structure whereas marginal cost of capital is known as the weighted average cost of extra or incremental capital.

One of the condition to be met before weighted average cost of capital (WACC) of a company can be used to evaluate company investment projects is that it must reflected the company marginal cost of capital (MCC). So, therefore, the correct company cost of capital to use to evaluate the company investment projects is the marginal cost of capital (MCC) of the capital raised to finance the investment project.

Marginal cost of capital (MCC) can be computed by using the formula stated below:

$$MCC = \frac{E(Mk_e)}{V} + \frac{P(Mk_p)}{V} + \frac{D(Mk_d)}{V}$$

Where;

E = Value of equity

P = Value of preference share

D = Value of debt

V = Value of the firm = E + P + D

$$Mk_e = \text{Marginal cost of equity} = \frac{D_o}{P_i - C_i}$$

Where;

D_o = Dividend per share = Current dividend yield X Market value of share

P_i = Issue price

C_i = Cost of issue

Mk_p = Marginal cost of preference share = $\dfrac{D_p}{P_i - C_i}$

Where;

D_p = Annual fixed dividend payment on preference share

P_i = Issue price

C_i = Cost of issue of preference share

Mk_d = Marginal cost of Debt = $\dfrac{R}{P_i - C_i}$

Where;

R = Annual fixed interest on debt

P_i = Issue price of debt

C_i = Cost of issue of debt

EXAMPLE;

Adeleke Ltd has in issue, the following securities against each of which details of their present market standing are given below;

Types of securities	Nominal value	Market value	Current yield
Ordinary shares	N2million N1each	N3.08 per share	10% per annum
12% Preference shares	N1million N1each	N0.92 per share	13% per annum
14% Debenture	N2million	N120 percent	12% per annum

The company is considering raising extra capital to finance a new investment project. The company has been advised by its bank that the following items will be needed to be offered in each type of security as follow;

	Ordinary share	Preference share	Debenture

Issue price	N2 .50	N0.86	N115 percent
Cost of issue	N0.13 per share	N0.07 per share	2% of nominal value

Required;

a. Determine the company’s
    1. Marginal cost of equity.
    2. Marginal cost of preference share
    3. Marginal cost of debt
b. Determine the marginal weighted cost of capital.

Solution;

Marginal cost of equity can be computed by using the formula;

$$Mk_e = \frac{D_o}{P_i - C_i}$$

Where;

Mk_e = Marginal cost of equity?

D_o = Dividend per share = Current dividend yield X Market value of share.

= 0.1 X N 3.08

= N.0.31

P_i = Issue price = N2.80

C_i = Cost of issue N0.13

$$Mk_e = \frac{0.31}{1.50 - 0.13}$$

$$Mk_e = \frac{0.31}{2.37}$$

Mk_e = 0.131

The marginal cost of equity is 0.131

Marginal cost of preference share can be computed by using the formula;

$$Mk_p = \frac{D_p}{P_i - C_i}$$

Where;

Mk_p = Marginal cost of preference share

D_p = Annual fixed dividend payment on preference share

= 0.12 X N1

= N 0.12

P_i = Issue price = N0.86

C_i = Cost of issue of preference share = N 0.07

$$Mk_p = \frac{0.12}{0.86 - 0.07}$$

$$Mk_p = \frac{0.12}{0.79}$$

$$Mk_p = 0.152$$

Marginal cost of preference share is 0.152.

Marginal cost of debt can be computed by using the formula stated below;

$$Mk_d = \frac{R}{P_i - C_i}$$

Where;

Mk_d = Marginal cost of debt.

R = Annual fixed interest on debt

R = 0.14 X N100

R = N14

P_i = Issue price of debt = N115

C_i = Cost of issue of debt = 0.02 X N100 = N2

$$Mk_d = \frac{14}{155 - 2}$$

$$Mk_d = \frac{14}{113}$$

$Mk_d \quad = \quad 0.124$

Marginal cost of debt is 0.124

Marginal Weighted Average Cost of Capital (MCC) can be computed by using the formula as stated below;

$$MCC = \frac{E(Mk_e)}{V} + \frac{P(Mk_p)}{V} + \frac{D(Mk_d)}{V}$$

Where;

E = Value of equity = 2, 000, 000 X N3.08 =N6, 160, 000

P = Value of preference share = 1,000,000 X N0.92 = N920, 000

D = Value of debt = 20,000 X N120 = N2, 400, 000

V = Value of the firm = E + P + D

= N6, 160, 000 + N920, 000 + N2, 400, 000

= N9, 480, 000

Mk_e = Marginal cost of equity

Mk_p = Marginal cost of preference share

Mk_d = Marginal cost of Debt

The marginal weighted average cost of capital (MCC) is;

$$MCC = \frac{6,160,000(0.131)}{9,480,000} + \frac{920,000(0.152)}{9,480,000} + \frac{2,400,000(0.124)}{9,480,000}$$

$= 0.085 + 0.015 + 0.031$

$= 0.131$

The marginal weighted average cost of capital (MCC) is 13.1%

- ABC Ltd has in issue 2,000,000 ordinary share of N1.00 each with a current market value of N3.00 and current yield is 9%. Determine the marginal cost of equity.

  a.10% b.11% c.12% d.13% e.14%

- Marginal cost of capital is best defined as

  a. Weighted average cost and capital b. Weighted average of extra cost of capital

  c. Weighted average cost of discounting capital d. Minimum rate of return required by the provider of company fund e. None of the above

- Marginal Cost of equity can be computed by using the formula

$$MKe = \frac{Do}{Pi - Ci}$$

$$= \frac{0.09 \times 3.00}{3.00 - 1.00}$$

$$= 0.135$$

The marginal cost of equity = 0.14 or 14%, e is the correct answer.

- a = Marginal cost of capital

2.4 Summary of study session 2;

You have just learn about the basic component of weighted average cost of capital (WACC) which is known as the overall cost of capital of a company and marginal cost of capital in this study session, their definition and how you will solve problems involving them. Here are some assessment questions you need to do based on what you have done so far.

Self Assessment Questions (SAQs) for Study Session 2

SAQ2.1 (tests Learning Outcomes 2.1 and 2.5)

1. Distinguish between the term weighted average cost of capital and marginal cost of capital.

SAQ2.2 (tests Learning Outcome 2.4)

1. On what conditions would it be valid to use the weighted average cost of capital as the discount rate for investment evaluation?.

SAQ2.3 (tests Learning Outcomes 2.2 and 2.3)

Johnson Plc has the following information;

	Book Value	Market Value
	N	N
1 million Ordinary share at (N1.00 each)	1,000,000	1,200,000
15% Preference shares	1,000,000	1,800,000
19% Debenture	600,000	650,000

You have the following additional information.

i. The total ordinary dividend of N250, 000 was deducted at the last annual general meeting.

ii. Besides the above, the company expects a stable annual growth rate of 6% on its ordinary dividend.

iii. Annual corporate income tax of 30%

iv. The debenture has 5-year life span left.

Required

Calculate the overall cost of capital or weighted average cost of capital (WACC)

Answer to Study Sessions Self Assessment Questions (SAQs) for Module 2

1. Weighted average cost of capital is the weighted average of the various sources of funds in a company capital structure where each weight is represented by the proportion of each sources of fund in the capital structure while marginal cost of capital is defined as weighted average cost of extra or incremental capital.

2i. If the company capital structure is optimum

ii. If the company weighted average cost of capital reflects its long – term future capital structure.

iii. If new funds are raised in a way that keep the company capital structure unchanged

iv. If the weighted average cost of capital reflected marginal cost of capital

v. If existing projects and proposed project are homogeneous

3. Using the formula for calculating cost of debt for redeemable debenture

$$P_o = \sum_{t=2}^{n} \frac{R(1-T)}{(1+kd)^t} + \frac{PT}{(1 + kd)^n}$$

$$650,000 = \sum_{t=1}^{5} \frac{114,000(1-0.3)}{(1+kd)t} + \frac{600,000}{(1+kd)^5}$$

$$650,000 = \frac{79,800}{(1+kd)} + \frac{79,800}{(1+kd)^2} + \frac{79,800}{(1+kd)^3} + \frac{79,800}{(1+kd)4} + \frac{79,800}{(1+kd)5} + c$$

Transfer this equation into a table as shown below

Year	Cash flow	DCF@20%	Present value	Year	Cash flow	DCF@10%	Present value
0	(650,000)	1.0000	(650,000)	0	(650,000)	1.0000	(650,000)
1-5	79,800	2.9906	238,650	1-5	79,800	3.7908	302,506
5	600,000	0.4019	241,140	5	600,000	0.6209	372540

NPV = (170,210) NPV=25,046

After obtaining two distinct values of NPV as shown in the table above, then apply the concept of IRR formula;

$$IRR = DCF_+ + \frac{NPV_+}{NPV_+ - NPV_-}(DCF_- - DCF_+)$$

$$Kd = 0.10 + \frac{25{,}046}{25{,}046 + 170{,}210}\ (0.2 - 0.1)$$

Kd = 0.10 + 0.013

Kd = 0.113

Kd = 11.3%

The cost of debt (Kd) of the company is 11.3% or approximately 11%

. Using the formula for cost of preference share for irredeemable preference share as;

$$Kp = \frac{Dt}{Po}$$

Where;

Dp = Annual fixed dividend payment on preference share = 0.15 X 1, 000, 000 = N150, 000

Po = Current market price of preference share = N1, 800, 000

$$Kp = \frac{0.15 \text{ X N1, 000, 000}}{\text{N1, 800, 000}}$$

$$Kp = \frac{150,\ 000}{1,\ 800,\ 000}$$

Kp = 0.083

Kp = 8.3%

The cost of preference share (Kp) of the company is 8.3% or approximately 8%

. Using the formula for calculating cost of equity as;

$$k_e = \frac{D_O(1+g)}{P_O} + g$$

Where;

Existing dividend per share = $D_o = \frac{\text{Dividend declared}}{\text{No. of outstanding shares}} = \frac{N250,000}{1,000,000} = N0.25$

Annual dividend growth rate = g = 0.06

Current market price of equity = $P_o = \frac{\text{Market value}}{\text{No. of outstanding shares}} = \frac{N1,200,000}{1,000,000} = N1.20$

$$k_e = \frac{0.25\,(1.06)}{1.20} + 0.06$$

$k_e = 0.221 + 0.06$

$k_e = 0.281$

$k_e = 28.1\%$

The cost of equity (Ke) of the company is 28.1% or approximately 28%

Using the formula for computing weighted average cost of capital as shown below;

$$WACC = \frac{D}{V}(Kd) + \frac{P}{V}(Kp) + \frac{E}{V}(Ke)$$

Where;

Value of debenture (D) = N650, 000

Value of preference share (P) = ₦1, 800, 000

Value of equity (E) = N1, 200, 000

Value of the company (V) = D + P + E

= N650, 000 + N1, 800, 000 + N1, 200, 000

= N3, 650, 000

Cost of equity = K_e = 0.281

Cost of debenture = K_d = 0.113

Cost of preference share = Kp = 0.083

$$WACC = \frac{N650,000}{N3,650,000}(0.113) + \frac{N1,800,000}{N3,650,000}(0.083) + N\frac{1,200,000}{N3,650,000}(0.281)$$

WACC = 0.020 + 0.041 + 0.092

WACC = 0.153

WACC = 15.3%

The company weighted average cost of capital (WACC) is 15%

REFERENCES AND FURTHER STUDIES

Ezike John (2002) — Essential of Corporate Financial Management, Lagos Jaylycent pp 275 - 288

Olowe R.A (1997) — Financial Management: Concept, Analysis and capital Investment. Lagos Bierly Jones Nigeria Limited pp 401 – 405

Otaniyi O. (1997) — Elements of Business Finance, Panaf Publishing Incorporation. pp 119 - 126

Owualah S.I (2003) — Understanding Business Finance, Lagos G-Mag Invesment Ltd pp 51 - 65

Pandey I.M (2000) — Financial Management India Vikas Publishing PVT Ltd pp 512 - 530

Shiro A.A (2004) — Problems and Solutions in Financial Management, Lagos. El-Toda Ventures Ltd. pp 171 – 194Should you require more explanation on this study session, please do not hesitate to contact your e-tutor via the LMS.

Are you in need of General Help as regards your studies? Do not hesitate to contact the DLI IAG Center by e-mail or phone on:

iag@dli.unilag.edu.ng

08033366677

Study Session 1 Cost of Capital 1

Introduction

Cost of capital have been used to discount the future cash inflows and then subtracting the cost in order to determine the viability or profitability of a project or an investment using net present value (NPV) criteria. It follows then that cost of capital can be called the appropriate discount rate or required return expected on a project or investment by the supplier of firm's funds. It is a basic fact that firm's uses debenture capital, preference shares capital and equity capital to make up their firm's capital structure. Thus, the cost of capital will be the mixture of the returns needed to compensate the provider of debenture capital, preference shares capital and equity capital.

Learning outcome for study session 1

At the end of this study session, you should be able to;

1.1 Demonstrate a good understanding why companies need to know their cost of capital

1.2 Understand the fundamental of cost of capital

1.3 Define cost of debt and solve application problem involves cost of debt

1.4 Define cost of preference share and solve application problem involves cost of preference share

1.5 Define cost of equity and solve application problem involves cost of equity

1.6 Evaluate the various component of cost of capital

1.2 DEFINITION AND CONCEPT OF COST OF CAPITAL:

c) **Definition**

Cost of capital can be defined as the cost of company funds. It represented the minimum rate of return that must be earned on the capital invested. The cost of capital of a company reflects the expectations of various investors that contribute money or funds to the company. (For instance; Provider of a company equity capital or ordinary shares capital, Provider of a company preference shares capital and Provider of debenture stock capital).

The cost of capital is the rate of return that company capital could be expected to earn in an alternative investment of an equal risk.

N.B

* The providers of equity capital also known as an ordinary shares capital or common stock capital are regarded as the owner of the company. They are entitled to an unlimited dividend declared out of that company's residual income, that is, profit after all other claims (provider of preference shares capital and debenture stock capital) have been settled.

* The providers of preference shares capital are not regarded as permanent part owner of the company since their dividend are at fixed rates, but entitles them to a prior claim on the company's profit and assets before the provider of equity capital or ordinary shares capital.

Preference share can be redeemable (For instance, years to maturity is known with certainty) or irredeemable (For instance, years to maturity is unknown)

- The provider of debenture stock capital received fixed interest rate on their investment. The interest on the debenture stock is normally against the company's revenue and not on share of the distributable profit.

  Debenture stock can be redeemable (For instance, years to maturity is known with certainty) or irredeemable (For instance, years to maturity is unknown).

d) **Representation**

A company cost of capital is usually represented by a capital letter K in all cases. For instance ABC Plc cost of capital is 20%, then K is 20%; XYZ Ltd cost of capital is 10%, then K is 10%. You should also know that K varies from one company to another but there must be only one K for a particular company

The basic element of cost of capital can be expressed as stated below:

$K = R_f + P_{br} + P_{fr}$

R_f = a risk free rate of return

P_{br} = a premium for business risk

P_{fr} = a premium for financial risk

Note: Olowe (1997) has defined business risk as the risk associated with uncertainty about the future and a firm's business prospects while a financial risk is the risk associated with the introduction of debt into a firm's capital structure.

- One of the following statements is not true about cost of capital.

  a. Cost of funds raised by firms. b. Discount rate of returns for discounting a project's cash flows. c. Minimum rate of return expected by the provider of a company funds. d. Maximum rate of return expected by the provider of a company funds. e. None of the above.

- The element of cost of capital is among other the following.

  a. Risk free rate. b. Premium for financial risk c. Premium for insurance risk d. a and b e. None of the above

- .... is the risk associated with introduction of debt into a company capital structure.

  a. Business risk. b. Specific risk. c. Interest rate risk d. Financial risk e. none of the above.

- The providers of preference share capital receive .... on their investment.

  b. Fixed interest. b. Fixed dividend. c. Residual dividend. d. Residual interest. e. None of the above.

- ☐ d. Maximum rate of return expected by the provider of a company funds.
- ☐ d. a and b
- ☐ d. Financial risk.
- ☐ b. Fixed dividend

1.2 MEASURING COMPONENT OF COST OF CAPITAL

1.2.2 COST OF DEBT (I.E. COST OF DEBENTURE DENOTED AS K_d)

Definition;

In line with the definition of cost of capital, the cost of debt or debenture is the minimum rate of return expected by the providers of debenture capital of a company funds.

The cost of debt can be also defined as the discount rate of return that equates the current market price of debt or debenture to the discounted future proceeds from the debt or debenture.

There are two ways or methods of calculating cost of debt or debenture;

(a) Redeemable debenture issued either at discount or premium.

The cost of debt or debenture for redeemable debenture issued at discount or premium can be calculated by using the formula stated below.

It is given by $P_o = \sum_{t=2}^{n} \frac{R(1-T)}{(1+kd)^t} + \frac{PT}{(1 + kd)^n}$

Where

P_o = Current market price of debt or debenture.

K_d = Cost of debt or debenture

R = Annual interest payment

P_T = Terminal value or maturity value of debt or debenture

n = Year to maturity

T = Corporate tax rate

NOTES;

(5) The current market price of debt or debenture (P_O) is at ex-interest value.(after paying annual fixed interest on debt or debenture).

(6) The formula above can only be used for debt or debenture issued at premium or discount.

(7) If $P_o < P_T$, then the debt or debenture stock is issued at discount

(8) If $P_o > P_T$, then the debt or debenture stock is issued at premium

A few examples are present below to illustrate the application of cost of debenture in solving finance problem

HINT;

The knowledge of computation of internal rate of return (IRR) is very essential to the computation of cost of debenture (Kd) and this have been covered extensively in module 1 study session ….. Please revise this very well before you work through the examples below. If you still have problem please consult your lecturer.

Example; Redeemable Debt or Debenture issued at discount;

ABC Plc has an issue of 18% N1, 000 debenture that are currently selling at N850. Calculate the cost of debt if the year to maturity is 5 years and the corporate tax rate is 35%.

Solution

Using the available information as provided in the question where;

Current market Price of Debenture = P_o = N850

Maturity value of Debenture = P_T = N1, 000

Annual interest payment = R = 0.18 X N1, 000 = N180

Year to maturity = n = 5 years

Corporate tax rate = T = 0.35

Using the formula for calculating cost of debt for redeemable debenture

$$P_o = \sum_{t=2}^{n} \frac{R(1-T)}{(1+kd)^t} + \frac{PT}{(1 + kd)^n}$$

$$850 = \sum_{t=1}^{5} \frac{180(1-0.35}{(1+kd)t} + \frac{1000}{(1+kd)^5}$$

$$850 = \frac{117}{(1+kd)} + \frac{117}{(1+kd)^2} + \frac{117}{(1+kd)^3} + \frac{117}{(1+kd)4} + \frac{117}{(1+kd)5} + \frac{1000}{(1+kd)5}$$

Transfer this equation into a table as shown below

Year	Cash flow	DCF@20%	Present value	Year	Cash flow	DCF@10%	Present value
0	(850)	1.0000	(850)	0	(850)	1.0000	(850)
1-5	117	2.9906	349.90	1-5	117	3.7908	443.52
5	1000	0.4019	401.9	5	100	0.6209	620.9

NPV = (98.2) NPV 214.42

After obtaining two distinct values of NPV as shown in the table above,, then apply the concept of internal rate of return (IRR) formu

$$IRR = DCF_{+} + \frac{NPV_{+}}{NPV_{+} - NPV_{-}}(DCF_{-} - DCF_{+})$$

$$Kd = 0.10 + \frac{214.42}{214.42 + 98.2}(0.2 - 0.1)$$

Kd = 0.10 + 0.069

Kd = 0.169

Kd = 16.9%

The cost of debt (Kd) of the company is 16.9% or approximately 17%

Example; Redeemable Debt or Debenture issued at a premium

XYZ Plc has an issue of 18.75% N100 debenture that are currently selling at N106. Calculate the cost of debt if the debenture is redeemable at par in 5years time and corporate tax rate is 35%.

Solution

Using the available information as provided in the question

Current market price of the Debenture = P_o = N106

Maturity value of the Debenture = P_T = N100

Annual interest payment = R = 0.1875 X N100 = N18.75

Year to maturity = n = 5 years

Corporate tax rate = T = 0.35

Using the formula for calculating cost of debt for redeemable debenture

$$P_o = \sum_{t=2}^{n} \frac{R(1-T)}{(1+kd)^t} + \frac{PT}{(1 + kd)^n}$$

$$106 = \sum_{t=1}^{5} \frac{18.75(1-0.35)}{(1+kd)t} + \frac{100}{(1+kd)^5}$$

$$106 = \frac{12.19}{(1+kd)} + \frac{12.19}{(1+kd)^2} + \frac{12.19}{(1+kd)^3} + \frac{12.19}{(1+kd)4} + \frac{12.19}{(1+kd)5} + \frac{100}{(1+kd)5}$$

Transfer this equation into a table as shown below

Year	Cash flow	DCF@20%	Present value	Year	Cash flow	DCF@10%	Present value
0	(106)	1.0000	(106)	0	(106)	1.0000	(106)
1-5	12.19	2.9906	36.46	1-5	12.19	3.7908	46.21
5	100	0.4019	40.19	5	100	0.6209	62.09

NPV = (29.35) NPV 2.30

After obtaining two distinct values of NPV as shown in the table above, then apply the concept of IRR formula;

$$IRR = DCF_{+} + \frac{NPV_{+}}{NPV_{+} - NPV_{-}}(DCF_{-} - DCF_{+})$$

$$Kd = 0.10 + \frac{2.30}{2.30 + 29.35}\ (0.2 - 0.1)$$

Kd = 0.10 + 0.007

Kd = 0.107

Kd = 10.7%

The cost of debt (Kd) of the company is 10.7% or approximately 11%

(b) PERPETUAL OR IRREDEEMABLE DEBT OR DEBENTURE

The formula for cost of debt for a perpetual or irredeemable debt or debenture is given as follows:

$$Kd = \frac{R(1-t)}{Po}$$

Where

P_o = Current market price of debt or debenture.

K_d = Cost of debt

R = Annual interest payment

T = Corporate tax rate

Example:

LIZZY Plc has an issued 18% N1, 000 irredeemable debentures currently selling at N1, 150. Calculate the cost of debt, if the corporate tax rate is 35%.

Solution;

Using the available information as provided in the question

Current market price of debenture = Po = N1, 150

Annual interest payment = R = 0.18 X N1, 000 = N180

Corporate tax rate = T = 0.35

Using the irredeemable debenture formula for cost of debt as shown below

$$Kd = \frac{R(1 - T)}{Po}$$

$$Kd = \frac{180(1 - 0.35)}{1150}$$

$$Kd = 0.102$$

$$Kd = 10.2\%$$

The cost of debt (Kd) of the company is 10.2% or approximately 10%

- One of these is correct about cost of debenture stock.
  b. Cost of funds raised by firms. b. Discount rate of return that equate terminal value of debt with the proceeds from the debt. c. Discount rate of return that equate current market value of debt with the proceeds from the debt. d. It has a fixed dividend return. e. It does not involve corporate tax.
- Kindly determine P_o, PT, T, n and R from the following information;

Ade plc has an issue of 21% N1, 000 debenture currently selling at N1,230 and the year to maturity is five years. The current corporate tax rate is 35%

- Johnson plc has in issue 14% N1, 000 irredeemable Debenture currently selling at N950. Determine the cost of debt if the corporate tax rate is 35%.

  b. 10% b. 19% c. 20% d. 14% e. None of the above
- The providers of debenture capital received .... on their investment.

  b. Fixed dividend b. Fixed interest c. Fixed stock d. nominal dividend. e. high interest.

☐ C = Discount rate of return that equate current market value of debt with the proceeds from the debt.

☐ P_o = N1,230, PT = N1,000, T = 35% = 0.35, n = 5, R = 0.21 X N1,000 = N210.

☐ $Kd = \frac{R(1-T)}{P_o}$

$= \frac{0.14 \text{ X N1, 000 } (1 - 0.35)}{\text{N950}}$

$= \frac{\text{N91}}{\text{N950}}$

= 0.096 = 10% = option

☐ b. = Fixed interest

1.2.2 COST OF PREFERENCE SHARE (denoted by Kp)

Definition

In line with the definition of cost of capital above, Cost of preference share is the minimum rate of return required by the providers of preference share capital. The computation of cost of preference share is close to that of debenture as their characteristics look similar. The only difference between a preference share and a debenture is that the dividends payments on preference shares are not allowable for tax adjustments. Therefore, all the formula used in calculating cost of debenture is applicable to cost of preference share except that there is no need for tax adjustments.

There are two ways or methods of calculating cost of preference share (Kp).

(c) Redeemable preference share issued either at discount or premium;

The cost of preference share for redeemable preference share issued either at discount or premium can bse determine by the formula present below

$$P_o = \sum_{t=1}^{n} \frac{D_t}{(1+k_p)^t} + \frac{P_t}{(1+k_p)^n}$$

Where

P_o = Current market price of a preference share

P_T = Maturity value of a preference share

n = Year to maturity of the preference share

D_t = Annual fixed dividend payment on a preference share

K_p = Cost of preference share

Notes;

v. The current market price a preference share is stated in the above formula as ex-dividend (after the annual fixed dividend has been paid).

vi. The above formula can only be used for preference share issued either at premium or discount.

vii. If $P_o < P_T$, then the preference share is issued at discount

viii. If $P_o > P_T$, then the preference share is issued at premium

HINT;

The knowledge of computation of internal rate of return (IRR) is very essential to the computation of the cost of preference share (Kp) and this have been covered extensively in module 1, study session ….. Please revise this before you work through the examples below. If you have problem please consult your lecturer.

EXAMPLE: Preference share issued at discount

NANCY LTD has in issue a 13% N10.00 preference share with a maturity of 5 years. Calculate the cost of preference share if the current market value is N9.50.

Solution

Using the formula for calculating cost of preference share for redeemable preference share

$$Po \quad = \qquad P_o = \sum_{t=1}^{n} \frac{D_t}{(1+k_p)^t} + \frac{P_t}{(1+k_p)^n}$$

Where;

Current market price of preference share = P_o = N9.50

Terminal value of preference share = P_t = N10.00

Annual fixed dividend payment on preference share = Dp = 0.13 X N10.00 = N1.30

Years to maturity = n = 5

$$9.50 = \sum_{t=1}^{5} \frac{1.30}{(1+k_p)^t} + \frac{10}{(1+k_p)^n}$$

Transfer the above equation to a table as shown below

Year	Cash flow	DCF@20%	Present value	Year	Cash flow	DCF@10%	Present value
0	(9.50)	1.0000	(9.50)	0	(9.50)	1.0000	(9.50)

1-5	1.30	2.9906	3.89	1-5	1.30	3.7908	4.93
5	10	0.4019	4.02	5	10	0.6209	6.21

NPV= (1.59) NPV = 1.64

After obtaining two distinct values of NPV as shown in the table above, then apply the concept of IRR formula;

$$IRR = DCF_{+} + \frac{NPV_{+}}{NPV_{+} - NPV_{-}}(DCF_{-} - DCF_{+})$$

$$Kp = 0.1 + \frac{1.64}{1.65 + 1.59}(0.2 - 0.1)$$

$$Kp = 0.1 + \frac{1.64}{3.23}(0.1)$$

$$Kp = 0.1 + 0.051$$

$$Kp = 0.151$$

$$Kp = 15.1\%$$

The cost of preference share (Kp) of the company is 15.1% or approximately 15%

EXAMPLE: Preference share issued at premium

JOHN BULL LTD has in issue a 13% N10.00 preference share with a maturity of 5 years. Calculate the cost of preference share if the current market value is N10.50.

Solution

Using the formula for calculating cost of preference share for redeemable preference share

$$P_o = \sum_{t=1}^{n} \frac{D_t}{(1+k_p)^t} + \frac{P_t}{(1+k_p)^n}$$

Where;

Current market price of preference share = P_o = N10.50

Terminal value of preference share = P_t = N10.00

Annual fixed dividend payment on preference share = Dp = 0.13 X N10.00 = N1.30

Years to maturity = n = 5

$$10.50 = \sum_{t=1}^{5} \frac{1.30}{(1+k_p)^t} + \frac{10}{(1+k_p)^n}$$

Transfer the above equation to a table as shown below

Year	Cash flow	DCF@20%	Present value	Year	Cash flow	DCF@10%	Present value
0	(10.50)	1.0000	(10.50)	0	(10.50)	1.0000	(10.50)
1-5	1.30	2.9906	3.89	1-5	1.30	3.7908	4.93
5	10	0.4019	4.02	5	10	0.6209	6.21

NPV= (2.59) NPV = 0.64

After obtaining two distinct values of NPV as shown in the table above, then apply the concept of IRR formula;

$$IRR = DCF_+ + \frac{NPV_+}{NPV_+ - NPV_-}(DCF_- - DCF_+)$$

$$Kp = 0.1 + \frac{0.64}{0.65 + 2.59}(0.2 - 0.1)$$

$$Kp = 0.1 + \frac{0.64}{3.23}(0.1)$$

$$Kp = 0.1 + 0.02$$

$$Kp = 0.12$$

$$Kp = 12\%$$

The cost of preference share (Kp) of the company is 12%

(d) PERPETUAL OR IRREDEEMABLE PREFERENCE SHARE

The formula for cost of preference share for a perpetual or irredeemable preference share is given as follows:

$$K_p = \frac{D_t}{P_o}$$

Where

P_o = Current market price of a preference share

D_t = Annual fixed dividend on a preference

K_p = Cost of preference share

Example ;

KAREEM LTD has in issue a 13% N10.00 irredeemable preference share currently selling at N14.75. Calculate the cost of preference share.

Solution

Using the available information as provided in the question where

Current market price of preference share = Po = N14.75

Annual fixed dividend payment on preference share = 0.13 X N10.00 = N1.3

Using the formula for cost of preference for irredeemable preference share as;

$$Kp = \frac{Dt}{Po}$$

$$Kp = \frac{0.13 \times N10.00}{N14.75}$$

$$Kp = \frac{1.3}{14.75}$$

$$Kp = 0.088$$

$$Kp = 8.8\%$$

The cost of preference share (Kp) of the company is 8.8% or approximately 9%

- Cost of preference share is best define as the…………rate of return expected by the provider of preference share capital.

a. Maximum b. Highest c. Biggest d. Apex e. Minimum

- Kindly determine P_o, PT, n and Dt from the following information;

  Dede Plc has an issue of 15% N10 preference share currently selling at N15 and the year to maturity is 5 years.

- Johnson plc has in issue 14% N100, 000 irredeemable preference share currently selling at N95,000. Determine the cost of preference share.

  a.10% b.19% c.20% d.14% e. None of the above

  - ☐ e. = Minimum
  - ☐ P_o = N15, PT = N10, , n = 5, Dt = 0.15 X N10 = N15.
  - ☐ $Kd = \frac{Dt}{P_o}$

$$= \frac{0.14 \times N100,000}{N95,000}$$

$$= \frac{\underline{N14,000}}{N95,000}$$

$= 0.143 = 14\% = \text{option a}$

1.2.3 COST OF EQUITY (denoted by K_e)

Definition

In line with the definition of cost of capital, Cost of equity is the minimum rate of returns required by the provider of ordinary share capital. The cost of equity is more difficult to calculate as equity does not pays a set of returns to its investors. The cost of equity could be estimated by means of the dividend valuation model (DVM). The Dividend valuation model relate the markets value of equity to expected future dividends on the equity.

Assuming an initial dividend D_1 and a constant payout ratio, the Dividend valuation model is given as:

$$P_o = \sum_{t=1}^{n} \frac{D_{tt}}{(1+k_p)^t}$$

$$= \frac{D_1}{(1+k_e)} + \frac{D_2}{(1+k_e)^2} + \ldots \frac{D-0}{(1+k_e)^e}$$

The model assume a constant dividend growth rate g per year into indefinite future

$$= \quad \frac{D_o+(1+g)}{1+k} + \frac{D_o+(1+g)^2}{(1+k)^2} + \ldots. \frac{D_o+(1+g)^2}{(1+k)^2}$$

$$P_o = \frac{D_o+(1+g)}{k_e-g} = \frac{D_1}{k_e-g}$$

$$K_e = \frac{D_o+(1+g)}{P_o} + g \text{ or } \frac{D_1}{P_o} + g \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(M)$$

Equation (M) can be used to compute the cost of equity.

Where

K_e = cost of equity

P_o = current market price of equity

D_o = Dividend per share that has been declared or existing dividend per share

D_1 = Proposed dividend per share or dividend to be declared at the end of the current year

g = Annual dividend growth rate

Example:

JANE Plc has just paid a dividend of 15k per share, the duration of JANE Plc have the opinion that dividend growth rate at 8% per annum, if the current market price of the share is N1.50. Calculate the cost of equity.

Solution:

Using the cost of equity formula

$$k_e = \frac{D_O(1+g)}{P_O} + g$$

Where;

Existing dividend per share = D_o = N0.15

Annual dividend growth rate = g = 0.08

Current market price of equity = P_o = N1.50

$$k_e = \frac{0.15\ (1.08)}{1.50} + 0.08$$

k_e = 0.108 + 0.08

k_e = 0.188

$k_e \;\; = 18.8\%$

The cost of equity (Ke) of the company is 18.8% or approximately 19%

Example;

JOHN Plc is financed with 2,000,000 ordinary shares of N1.00 each. The firm's dividends have been growing at the rate of 7% annually and this trend is expected to continue. A divided of 32k per share is expected to paid in a year's time while the securities are currently quoted on the stock market at N2,52 per each ordinary shares. Calculate cost of equity.

Solution

Using the cost of equity formula

$$K_e = \frac{D_1}{P_o} + g$$

Where

Propose dividend per share = D_1 = N0.32

Annual dividend growth rate = g = 0.07

Current market price of equity = P_o = N2.52

$$k_e = \frac{0.32}{2.52} + 0.07$$

$k_e \quad = 0.127 + 0.07$

$k_e \quad =0.197$

$k_e \quad =19.7\%$

The cost of equity (Ke) of the company is 19.7% or approximately 20%

Note

vi. If the annual dividend growth rate (g) is not given, the formula shown below can be used to determine g.

$$\sqrt{\frac{\text{Latest dividend}}{\text{Earliest dividend}}} - 1$$

g = n-1

Where;

g = annual dividend growth rate

n = number of year's growth

Example;

NENE Plc is a company with 1,000,000 ordinary shares of N1 each currently quoted at N41.70 as at 31st December 2011. The summary of dividend in the last 5years is as follows;

YEAR	DIVIDEND (N)
2007	397, 000
2008	410, 000
2009	438, 000
2010	463, 000
2011	480, 000

REQUIRED

Compute the annual dividend growth rate for the company**.**

Solution

Using the formula for annual dividend growth rate (g)

$$g = \sqrt[n-1]{\frac{\text{Latest dividend}}{\text{Earliest dividend}}} - 1$$

Where;

Latest dividend = N480, 000

Earliest dividend = N397, 000

n = 5-1 = 4

$$g = \sqrt[4]{\frac{480,000}{397,000}} - 1$$

$g = (1.2091)^{1/4} - 1$

$g = (1.2091)^{0.25} - 1$

$g = 1.05 - 1$

$g = 0.05$

$g = 5\%$

The annual dividend growth rate of the company is 5%

- Cost of equity can be evaluated using the …………..

  a. Dividend valuation model b. Dividend valuation growth model. c. Dividend growth rate model
  d. Valuation growth model e. None of the above

- Do and D1 simply means .... and .... respectively.

  b. Propose dividend and existing dividend. b. Propose dividend per share and existing dividend per share. c. Existing earnings and propose earnings. d. existing dividend per share and propose dividend per share. e. current dividend and propose dividend.

- Which of these is not relevant in computing cost of equity?

  b. Propose dividend per share. b. Existing dividend per share. c. Corporate tax rate. d. current market price of equity. e. annual dividend growth rate.

- The providers of equity capital received .... on their investment.

b. Fixed return b. Nominal return c. Residual dividend d. fixed dividend e. fixed income..

☐ b = Dividend valuation growth model.
☐ c. = existing dividend per share and propose dividend per share
☐ d = corporate tax rate.
☐ e = residual dividend.

1.2.5 COST OF RETAINED EARNINGS:

Retained earnings represent part of company earnings (earnings after interest and tax) available for distribution to shareholders by way of dividends. When cash from retained earnings is invested in a new project, such projects are assumed to be generally similar to the normal activities of the company.

SUMMARY OF THE STUDY SESSION 1;

In this session, you have learnt about cost of capital, cost of debt or debenture, cost of preference share and cost of equity. How to determine the various component of cost of capital and you can solve simple finance problems using the formula of Cost of debenture, cost of preference shares and cost of equity. If there is any problem, you can go back to the session again because the knowledge here is a pre-requisite for the immediate next study session.

Self Assessment Questions (SAQs) for Study Session 1

SAQ 1.1 (tests Learning Outcomes 1.1 and 1.2)

c. Explain the term cost of capital.

d. Mention and explain basic elements of cost of capital.

SAQ 1.2 (tests Learning Outcomes 1.3, 1.4 and 1.5)

Write short notes on the following;

(d) Cost of preference share

(e) Cost of equity

(f) Cost of debt

SAQ 1.3 (tests Learning Outcomes 1.3, 1.4, 1.5 and 1.6)

Oluwatobi Plc has the following information;

i. 2 million ordinary shares at N1.00 each currently selling at N1.20.

ii. The total ordinary dividend of N500, 000 was deducted at the last annual general meeting.

iii. Besides the above, the company expects a stable annual growth rate of 6% on its ordinary dividend.

iv. The debentures 18.75% N600, 000, which are currently selling at N1, 080 per N1,000 debentures; are currently selling at par in five years time.

v. The preference share 15% N1,000,000, which are currently selling at N1.80 per N1.00 preference share are irredeemable

vi. Preference dividend and debenture interest have just been paid

vii. Corporate tax rate is currently 30%.

REQUIRED

d. Compute the company's cost of debt

e. Compute the company's cost of preference share

f. Compute the company's cost of equity

Answer to Study Sessions Self Assessment Questions (SAQs) for Module 3

Study Session 1

1a. Cost of capital is the cost of fund raised by a company. It represents the minimum rate of return that must be earned on the capital invested. It is the minimum rate of return expected by the provider of ordinary share capital, preference share capital and debenture capital.

b. The basic element of Cost of capital (K) can be expressed as:

$K = R_f + P_{br} + P_{fr}$

Where K = cost of capital

R_f = a risk free rate of return

P_{br} = a premium for business risk

P_{fr} = a premium for financial risk

2a. Cost of preference share is the minimum rate of return required by the provider of preference share capital.

d. Cost of equity is the minimum rate of return required by the provider of ordinary share capital.

e. Cost of debt is the minimum rate of return required by the provider of debenture stock capital.

3a. Using the formula for calculating cost of debt for redeemable debenture

$$P_o = \sum_{t=2}^{n} \frac{R(1-T)}{(1+kd)^t} + \frac{PT}{(1 + kd)^n}$$

$$1{,}080 = \sum_{t=1}^{5} \frac{187.5(1-0.3)}{(1+kd)t} + \frac{1{,}000}{(1+kd)^5}$$

$$1{,}080 = \frac{131.25}{(1+kd)} + \frac{131.25}{(1+kd)^2} + \frac{131.25}{(1+kd)^3} + \frac{131.25}{(1+kd)4} + \frac{131.25}{(1+kd)5} + \frac{1000}{(1+kd)5}$$

Transfer this equation into a table as shown below

Year	Cash flow	DCF@20%	Present value	Year	Cash flow	DCF@10%	Present value
0	(1,080)	1.0000	(1,080)	0	(1,080)	1.0000	(1,080)

1-5	131.25	2.9906	392.52	1-5	131.25	3.7908	497.54
5	1,000	0.4019	401.90	5	1000	0.6209	620.90

NPV = (285.58) NPV 38.44

After obtaining two distinct values of NPV as shown in the table above, then apply the concept of IRR formula;

$$IRR = DCF_+ + \frac{NPV_+}{NPV_+ - NPV_-}(DCF_- - DCF_+)$$

$$Kd = 0.10 + \frac{38.44}{38.44 + 285.58}\ (0.2 - 0.1)$$

Kd = 0.10 + 0.012

Kd = 0.112

Kd = 11.2%

The cost of debt (Kd) of the company is 11.2% or approximately 11%

3b. Using the formula for cost of preference for irredeemable preference share as;

$$Kp = \frac{Dt}{Po}$$

$$Kp = \frac{0.15 \text{ X N}1.00}{\text{N}1.80}$$

$$Kp = \frac{0.15}{1.80}$$

Kp = 0.083

Kp = 8.3%

The cost of preference share (Kp) of the company is 8.3% or approximately 8%

3c.Using the formula for cost of equity as;

$$k_e = \frac{D_o(1+g)}{P_o} + g$$

Where

Existing dividend per share = $D_o = \frac{N500,000}{2,000,000} = N0.25$

Annual dividend growth rate = g = 0.06

Current market price of equity = P_o = N1.20

$$k_e = \frac{0.25\,(1.06)}{1.20} + 0.06$$

k_e = 0.221 + 0.06

k_e = 0.227

k_e = 22.7%

The cost of equity (Ke) of the company is 22.7% or approximately 23%

REFERNCES AND FURTHER STUDIES

Ezike John (2002) Essential of Corporate Financial Management, Lagos Jaylycent pp 275 - 288

Olowe R.A (1997) Financial Management: Concept, Analysis and capital Investment. Lagos Bierly Jones Nigeria Limited pp 388 – 400

Otaniyi O. (1997) Elements of Business Finance, Panaf Publishing Incorporation. pp 119 - 126

Owualah S.I (2003) Understanding Business Finance, Lagos G-Mag Investment Ltd pp 51 - 65

Pandey I.M (2000) Financial Management India Vikas Publishing PVT Ltd pp 493 - 511

Shiro A.A (2004) Problems and Solutions in Financial Management, Lagos. El-Toda Ventures Ltd. pp 171 – 194Should you require more explanation on this study session, please do not hesitate to contact your e-tutor via the LMS.

Are you in need of General Help as regards your studies? Do not hesitate to contact the DLI IAG Center by e-mail or phone on:

iag@dli.unilag.edu.ng
08033366677

Study Session 2 Cost of Capital II

INTRODUCTION;

Companies tend to have a mixture of the different forms of capital in their capital structure namely equity share capital, preference share capital and debenture stock capital as demonstrated in the immediate past study session.

After determine the cost of each source of capital used by company (cost of equity, cost of preference share and cost of debenture), we then multiply the proportion of each weight of the capital structure items by the corresponding costs and add them up.

The total is known as overall cost of capital or weighted average cost of capital (WACC). Weighted average cost of capital of a company must reflects the company marginal cost of capital (weighted average cost of incremental capital) before it can be use to evaluate investment projects.

Learning Outcomes for Study Session 2

After reading and understanding the contents of this unit and working through all worked examples and practice questions you should be able to;

2.1 Understand the concept of weighted average cost of capital

2.2 Evaluate various components of weighted average cost of capital

2.3 Measure weighted average cost of capital.

2.4 Know the conditions at which weighted average cost of capital can be use as a discounting rate to evaluate projects

2.5 Understand the concept of marginal cost of capital.

2.1 DEFINITION OF WEIGHTED AVERAGE COST OF CAPITAL (WACC)

Weighted Average Cost of Capital (WACC) is the overall cost of capital of a firm or a company. It is the weighted average of the various sources of funds in a company capital structure where each weight is a represented by the proportion of each sources of fund in the capital structure.

Weighted average cost of capital is always expressed as a percentage. For instance, if a company works with a weighted average cost of capital (WACC) of 16%, then this means that all projects or investments accepted or selected should have or gives a return higher than the weighted average cost of capital (WACC) of 16%.

It is possible to combine cost of debt or debenture, cost of preference shares and cost of equity to produce a single number for a company weighted average cost of capital in which each sources of funds or capital is proportionately weighted.

- WACC simply means

  a. Marginal cost and capital b. World average cost of capital c. Weighted cost of capital

  d. Weighted average cost of capital e. None of the above

- The overall cost of capital is known as ....

  b. Weighted average b.cost of capital c.weighted average cost of capital d.cost of equity e.cost of debenture

  - ☐ d=Weighted average cost of capital
  - ☐ c=.weighted average cost of capital

2.2 COMPONENT OF WEIGHTED AVERAGE COST OF CAPITAL

Weighted average cost of capital (WACC) can be computed by using the formula stated below;

$$WACC = \frac{D}{V}(Kd) + \frac{P}{V}(Kp) + \frac{E}{V}(Ke)$$

Where;

D = Value of debt or debenture = $\frac{\text{Market Price}}{\text{Nominal Price}}$ X Number of debenture issued

P = Value of Preference share = $\frac{\text{Market Price}}{\text{Nominal Price}}$ X Number of preference share issued

E = Value of Equity share = $\frac{\text{Market Price}}{\text{Nominal Price}}$ X Number of equity share issued

V == Value of the firm or company = D + P + E.

Kd = Cost of debt or debenture.

Kp = Cost of preference shares.

Ke = Cost of equity

NOTES;

4. One can calculate weighted average cost of capital (WACC) using weight either based on book value or the market value, but it is more meaningful and appropriate to use market value when the data are available.

5. Weighted average cost of capital (WACC) can only be used as the discounting rate to evaluate company's investment projects if and only if;
    f. The company capital structure is optimal.
    g. The company weighted average cost of capital (WACC) reflects the company's marginal cost of capital (MCC).
    h. The company weighted average cost of capital (WACC) reflects the company long term future capital structure.
    i. If company existing projects and the company propose projects are in the same risk classification.
    j. If new funds are raised in a way that keeps the company's capital structure unchanged.

6. If the above conditions are relaxed, then, weighted average cost of capital (WACC) can never be used as discounted rate to evaluate company projects.
   f. The company capital structure is not optimal.
   g. The company weighted average cost of capital (WACC) does not reflect the company marginal cost of capital (MCC).
   h. The company weighted average cost of capital (WACC) does not reflect the company long term future capital structure.
   i. If company existing projects and the company propose projects are not in the same risk classification.
   j. If new funds are raised in a way that the company's capital structure changes.

A few examples are present below to illustrate the application of weighted average cost of capital (WACC) in solving finance problem.

EXAMPLE;

Goodluck Plc is trying to decide on the cost of capital discounting rate to apply to the evaluation of investment project. The company has in issued shared capital of 2,000,000 ordinary shares of N1.00 each with a current market value of N3.60 per share. It also has in issue N1.2 million 19% debenture which are redeemable at par in five years time and have a current market value of N105 per N100 debenture and N800, 000 20% irredeemable preference share currently selling at N0.96.Ordinary divided of N1, 260, 000 was recently paid and annual

Dividend growth rate is 12 percent. The corporate tax rate is 35%.

Required;

Estimate the weighted average cost of capital (WACC)

Solution;

You can start computing the weighted average cost of capital with any of the three costs (i.e cost of preference share, cost of equity and cost of debenture or debt). But, what is important is the usage of the correct formula and accuracy of your computation.

STEP 1;

CALCULATION OF COST OF PREFERENCE SHARE (Kp)

Using the formula for cost of preference share (Kp) for irredeemable preference share as shown below;

$$Kp = \frac{Dp}{Po}$$

Where

Annual fixed dividend payment $= D_p =$ N0.20

Current market price of preference share $= P_o =$ N0.96

$$Kp = \frac{0.20}{0.96}$$

$$Kp = 0.208$$

The company cost of preference share (Kp) is 20.8%

STEP 2;

CALCULATION OF COST OF EQUITY (Ke)

Using the formula for computing cost of equity (k_e) as shown below

$$Ke = \frac{Do\,(1+g)}{Po} + g$$

Where;

Existing Dividend Per Share = Do = $\frac{1,260,000}{2,000,000}$

$D_o =$ N0.63

Current Market Price of Equity = P_o = N3.60

Annual Dividend Growth Rate = g = 0.12

$$K_e = \frac{0.63(1 + 0.12)}{3.60} + 0.12$$

k_e = 0.316

The company cost of equity (Ke) is 31.6%

STEP 3;

CALCULATION OF COST OF DEBT (K_d)

Using the formula for computing cost of debt of redeemable debenture as shown below;

$$P_o = \sum_{t=2}^{n} \frac{R(1-T)}{(1+kd)^t} + \frac{PT}{(1 + kd)^n}$$

Where

Current market price of debenture = Po = N105

Terminal price of debenture = PT = N100

Annual fixed interest payment = R = 0.19 X N100 = N19

Corporate tax rate = T = 0.35

Years to maturity = n = 5

$$105 = \sum_{t=1}^{5} \frac{19(1-0.35)}{(1+kd)t} + \frac{100}{(1+kd)^5}$$

$$105 = \frac{12.35}{(1+kd)} + \frac{12.35}{(1+kd)^2} + \frac{12.35}{(1+kd)^3} + \frac{12.35}{(1+kd)4} + \frac{12.35}{(1+kd)5} + \frac{100}{(1+kd)5}$$

Transfer the above equation to a table as shown below

Year	Cash	DC@20%	Present	Year	Cash	DCF@10%	Present

	flow		value		flow		value
0	(105)	1.0000	(105)	0	(105)	1.0000	(105)
1-5	12.35	2.9906	36.93	1-5	12.35	3.7908	46.82
5	100	0.4019	40.19	5	100	0.6209	62.09

NPV= (27.88) NPV = 3.91

After obtaining two distinct value of NPV as show in the table above, the apply the concept of IRR formula;

$$IRR = DCF_{+} + \frac{NPV_{+}}{NPV_{+} - NPV_{-}}(DCF_{-} - DCF_{+})$$

$$K_d = 0.10 + \frac{3.91}{6.91 + 27.88}(0.2 - 0.1)$$

$$K_d = 0.10 + \frac{3.91}{31.79}(0.1)$$

$$K_d = 0.10 + 0.012$$

$$K_d = 0.112$$

The cost of debenture (Kd) is 11.2%

STEP 4;

CALCULATION OF WEIGHTED AVERAGE COST OF CAPITAL (WACC)

Using the formula for computing weighted average cost of capital as shown below

WACC = $\underline{D}$ (Kd) + $\underline{P}$ (Kp) + $\underline{E}$ (Ke)

V V V

Where

Value of debenture (D) = $\frac{N105}{N100}$ X 1,200,000 = N1, 260, 000

Value of preference share (P) = $\frac{N0.96}{N1.00}$ X 800,000 =N768, 000

Value of equity (E) = $\frac{N3.60}{N1.00}$ X 2,000,000 = N7, 200, 000

Value of the company (V) = D + P + E

= N1, 260, 000 + N768, 000 + N7, 200, 000

Value of the company (V) = N9, 228, 000

Cost of equity = $K_e = 0.316$

Cost of debtor debenture = $K_d = 0.112$

Cost of preference share = Kp = 0.208

WACC = $\frac{N1,260,000}{N9,228,000}(0.112) + \frac{N768,000}{N9,228,000}(0.208) + \frac{N7,200,000}{N9,228,000}(0.316)$

WACC =0.015 + 0.017 + 0.247

WACC = 0.279

WACC = 27.9%

The company weighted average cost of capital (WACC) is 27.9% or approximately 28%.

EXAMPLE;

The capital structure of DEDE Ltd, a quoted company is given as follows:

	N
Ordinary Shares of 50k each	2,000,000
21% preference shares (N1 per value)	1,000,000
18.75% debentures	1,500,000
Retained earnings	4,000,000
	8,500,000

The following information are relevant:

i. The debenture are currently selling at N106 per N100 debentures redeemable at par in five years time.

ii. The preference shares are currently selling at 84km per share and are irredeemable

iii. The preference dividend and debenture interest have just been paid

iv. The company ordinary shares currently have market value of N1.69 per share

v. Ordinary share dividend of N480,000 was recently paid

vi. The dividend annual growth rate is 6%

vii. Corporate tax rate is currently 35%

REQUIRED

Estimate the weighted average cost of capital

Solution;

You can start computing the weighted average cost of capital with any of the three cost(i.e cost of preference share, cost of equity and cost of debenture). But, what is important is the usage of the correct formula and accuracy of your answer.

STEP 1;

CALCULATION OF COST OF PREFERENCE SHARE (Kp)

Using the formula for cost of preference share for irredeemable preference share as shown below;

$$Kp = \frac{Dp}{Po}$$

Where

Annual fixes dividend payment = D_p = N0.21

Current market price of preference share = P_o = N0.84

$$Kp = \frac{0.21}{0.84}$$

$$Kp = 0.250$$

The company cost of preference share (Kp) is 25%

STEP 2;

CALCULATION OF COST OF EQUITY (Ke)

Using the formula for computing cost of equity (k_e) as shown below

$$Ke = \frac{Do(1 + g)}{Po} + g$$

Where

Existing dividend per share = $Do = \frac{480,000}{4,000,000}$

D_o = N0.12

Current market price P_o = N1.69

Annual dividend growth rate g = 0.06

$$K_e = \frac{0.12(1 + 0.06)}{1.69} + 0.06$$

$$Ke = 0.075 + 0.06$$

$$k_e = 0.135$$

The company cost of equity (Ke) is 13.5%

STEP 3;

CALCULATION OF COST OF DEBT OR DEBENTURE (K_d)

Using the formula for computing cost of debt for redeemable debenture as shown below;

$$P_o = \sum_{t=2}^{n} \frac{R(1-T)}{(1+kd)^t} + \frac{PT}{(1 + kd)^n}$$

Where;

Current market price of debenture = Po = N106

Terminal price of debenture = PT = N100

Annual fixed interest on debenture = R = 0.1875 X N100 = N18.75

Corporate tax rate = T = 0.35

Years to maturity = n = 5

$$106 = \sum_{t=1}^{5} \frac{18.75(1-0.35)}{(1+kd)t} + \frac{100}{(1+kd)^5}$$

$$106 = \frac{12.19}{(1+kd)} + \frac{12.19}{(1+kd)^2} + \frac{12.19}{(1+kd)^3} + \frac{12.19}{(1+kd)4} + \frac{12.19}{(1+kd)5} + \frac{100}{(1+kd)5}$$

Transfer the above equation to a table as shown below

Year	Cash	DCF@20%	Present	Year	Cash	DCF@10%	Present

	flow		value		flow		value
0	(106)	1.0000	(106)	0	(106)	1.0000	(106)
1-5	12.19	2.9906	36.46	1-5	12.19	3.7908	46.21
5	100	0.4019	40.19	5	100	0.6209	62.09

NPV= (29.35) NPV = 2.30

After obtaining two distinct values of NPV as shown in the table above, then apply the concept of IRR formula;

$$IRR = DCF_{+} + \frac{NPV_{+}}{NPV_{+} - NPV_{-}}(DCF_{-} - DCF_{+})$$

$$K_d = 0.10 + \frac{2.30}{2.30 + 29.35}(0.2 - 0.1)$$

$$K_d = 0.10 + \frac{2.30}{31.65}(0.1)$$

$$K_d = 0.10 + 0.007$$

$$K_d = 0.107$$

The cost of debt or debenture (Kd) is 10.7%

STEP 4;

CALCULATION OF WEIGHTED AVERAGE COST OF CAPITAL

Using the formula for computing weighted average cost of capital as shown below

$$WACC = \frac{D}{V}(Kd) + \frac{P}{V}(Kp) + \frac{E}{V}(Ke)$$

Where;

Value of debenture (D) $= \dfrac{N106}{N100} \times 1{,}500{,}000 = N1,590,000$

Value of preference share (P) $= \dfrac{N0.84}{N1.00} \times 1{,}000{,}000 = N840,000$

Value of equity E) $= \dfrac{N1.69}{N0.50} \times 2{,}000{,}000 = N6,760{,}000$

Value of the company (V) = D + P + E

= N1, 590, 000 + N840, 000 + N6, 760, 000

Value of the company (V) = N9, 190, 000

Cost of equity = K_e = 0.135

Cost of debenture = K_d = 0.107

Cost of preference share = Kp = 0.250

$$WACC = \frac{N1,590,000}{N9,190,000}(0.107) + \frac{N840,000}{N9,190,000}(0.250) + \frac{N6,760,000}{N9,190,000}(0.135)$$

WACC =0.019 + 0.022 + 0.099

WACC = 0.140

WACC = 14%

The company weighted average cost of capital (WACC) is 14%

ALTERNATIVE METHOD

Source	No of items issued	Po (N)	Value (N)	Proportion	Cost	
D	15,000	106	1,590,000	0.173	0.107	0.019
P	1,000,000	0.84	840,000	0.091	0.250	0.022
E	4,000,000	1.69	6,760,000	0.735	0.135	0.099
			9,190,000	1.000		0.140 → WACC

- One of the condition for using weighted average cost of capital as a discount rate for investment project appraisal is that…………………….

  a. The capital is optimal b. The capital structure is not optimal c. The new project and proposed project are not optimal d. New funds are raised in such a way that the capital structure remains unchanged e. None of the above

- One of the components of weighted average cost of capital are among others except

  a. Marginal cost of capital b. cost of debt c. cost of preference shares d. cost of equity

  e. value of the firm

- WACC simply means

  a. Marginal cost and capital b. World average cost of capital c. Weighted cost of capital

d. Weighted average cost of capital e. None of the above

- ☐ d = New funds are raised in such a way that the capital structure remains unchanged
- ☐ b = Weighted average of extra cost of capital
- ☐ d = Weighted average cost of capital

2.3 WEIGHTED AVERAGE COST OF CAPITAL VERSUS MARGINAL COST OF CAPITAL

Weighted average cost of capital has been defined as weighted average of the various sources of funds in a company capital structure where each weight is represented by the proportion of each source of fund in the company capital structure whereas marginal cost of capital is known as the weighted average cost of extra or incremental capital.

One of the condition to be met before weighted average cost of capital (WACC) of a company can be used to evaluate company investment projects is that it must reflected the company marginal cost of capital (MCC). So, therefore, the correct company cost of capital to use to evaluate the company investment projects is the marginal cost of capital (MCC) of the capital raised to finance the investment project.

Marginal cost of capital (MCC) can be computed by using the formula stated below:

$$MCC = \frac{E(Mk_e)}{V} + \frac{P(Mk_p)}{V} + \frac{D(Mk_d)}{V}$$

Where;

E = Value of equity

P = Value of preference share

D = Value of debt

V = Value of the firm = E + P + D

$$Mk_e = \text{Marginal cost of equity} = \frac{D_o}{P_i - C_i}$$

Where;

D_o = Dividend per share = Current dividend yield X Market value of share

P_i = Issue price

C_i = Cost of issue

Mk_p = Marginal cost of preference share = $\dfrac{D_p}{P_i - C_i}$

Where;

D_p = Annual fixed dividend payment on preference share

P_i = Issue price

C_i = Cost of issue of preference share

Mk_d = Marginal cost of Debt = $\dfrac{R}{P_i - C_i}$

Where;

R = Annual fixed interest on debt

P_i = Issue price of debt

C_i = Cost of issue of debt

EXAMPLE;

Adeleke Ltd has in issue, the following securities against each of which details of their present market standing are given below;

Types of securities	Nominal value	Market value	Current yield
Ordinary shares	N2million N1each	N3.08 per share	10% per annum
12% Preference shares	N1million N1each	N0.92 per share	13% per annum
14% Debenture	N2million	N120 percent	12% per annum

The company is considering raising extra capital to finance a new investment project. The company has been advised by its bank that the following items will be needed to be offered in each type of security as follow;

	Ordinary share	Preference share	Debenture

Issue price	N2 .50	N0.86	N115 percent
Cost of issue	N0.13 per share	N0.07 per share	2% of nominal value

Required;

c. Determine the company's
   4. Marginal cost of equity.
   5. Marginal cost of preference share
   6. Marginal cost of debt
d. Determine the marginal weighted cost of capital.

Solution;

Marginal cost of equity can be computed by using the formula;

$$Mk_e = \frac{D_o}{P_i - C_i}$$

Where;

Mk_e = Marginal cost of equity?

D_o = Dividend per share = Current dividend yield X Market value of share.

= 0.1 X N 3.08

= N.0.31

P_i = Issue price = N2.80

C_i = Cost of issue N0.13

$$Mk_e = \frac{0.31}{1.51 - 0.13}$$

$$Mk_e = \frac{0.31}{2.37}$$

Mk_e = 0.131

The marginal cost of equity is 0.131

Marginal cost of preference share can be computed by using the formula;

$$Mk_p = \frac{D_p}{P_i - C_i}$$

Where;

Mk_p = Marginal cost of preference share

D_p = Annual fixed dividend payment on preference share

= 0.12 X N1

= N 0.12

P_i = Issue price = N0.86

C_i = Cost of issue of preference share = N 0.07

$$Mk_p = \frac{0.12}{0.86 - 0.07}$$

$$Mk_p = \frac{0.12}{0.79}$$

$$Mk_p = 0.152$$

Marginal cost of preference share is 0.152.

Marginal cost of debt can be computed by using the formula stated below;

$$Mk_d = \frac{R}{P_i - C_i}$$

Where;

Mk_d = Marginal cost of debt.

R = Annual fixed interest on debt

R = 0.14 X N100

R = N14

P_i = Issue price of debt = N115

C_i = Cost of issue of debt = 0.02 X N100 = N2

$$Mk_d = \frac{14}{155 - 2}$$

$$Mk_d = \frac{14}{113}$$

$Mk_d \quad = \quad 0.124$

Marginal cost of debt is 0.124

Marginal Weighted Average Cost of Capital (MCC) can be computed by using the formula as stated below;

$$MCC = \frac{E(Mk_e)}{V} + \frac{P(Mk_p)}{V} + \frac{D(Mk_d)}{V}$$

Where;

E = Value of equity = 2, 000, 000 X N3.08 =N6, 160, 000

P = Value of preference share = 1,000,000 X N0.92 = N920, 000

D = Value of debt = 20,000 X N120 = N2, 400, 000

V = Value of the firm = E + P + D

= N6, 160, 000 + N920, 000 + N2, 400, 000

= N9, 480, 000

Mk_e = Marginal cost of equity

Mk_p = Marginal cost of preference share

Mk_d = Marginal cost of Debt

The marginal weighted average cost of capital (MCC) is;

$$MCC = \frac{6{,}160{,}000(0.131)}{9{,}480{,}000} + \frac{920{,}000(0.152)}{9{,}480{,}000} + \frac{2{,}400{,}000(0.124)}{9{,}480{,}000}$$

$$= \quad 0.085 \quad + \quad 0.015 \quad + \quad 0.031$$

$$= \quad 0.131$$

The marginal weighted average cost of capital (MCC) is 13.1%

- ABC Ltd has in issue 2,000,000 ordinary share of N1.00 each with a current market value of N3.00 and current yield is 9%. Determine the marginal cost of equity.

  a.10% b.11% c.12% d.13% e.14%

- Marginal cost of capital is best defined as

  a. Weighted average cost and capital b. Weighted average of extra cost of capital

  c. Weighted average cost of discounting capital d. Minimum rate of return required by the provider of company fund e. None of the above

- Marginal Cost of equity can be <u>computed</u> by using the formula

$$MKe = \frac{Do}{Pi - Ci}$$

$$= \frac{0.09 \text{ X } 3.00}{3.00 - 1.00}$$

$$= \quad 0.135$$

The marginal cost of equity = 0.14 or 14%, e is the correct answer.

- a = Marginal cost of capital

Summary of study session 2;

You have just learn about the basic component of weighted average cost of capital (WACC) which is known as the overall cost of capital of a company and marginal cost of capital in this study session, their definition and how you will solve problems involving them. Here are some assessment questions you need to do based on what you have done so far.

Self Assessment Questions (SAQs) for Study Session 2

SAQ2.1 (tests Learning Outcomes 2.1 and 2.5)

2. Distinguish between the term weighted average cost of capital and marginal cost of capital.

SAQ2.2 (tests Learning Outcome 2.4)

1. On what conditions would it be valid to use the weighted average cost of capital as the discount rate for investment evaluation?.

SAQ2.3 (tests Learning Outcomes 2.2 and 2.3)

Johnson Plc has the following information;

	Book Value	Market Value
	N	N
1 million Ordinary share at (N1.00 each)	1,000,000	1,200,000
15% Preference shares	1,000,000	1,800,000
19% Debenture	600,000	650,000

You have the following additional information.

v. The total ordinary dividend of N250, 000 was deducted at the last annual general meeting.

vi. Besides the above, the company expects a stable annual growth rate of 6% on its ordinary dividend.

vii. Annual corporate income tax of 30%

viii. The debenture has 5-year life span left.

Required

Calculate the overall cost of capital or weighted average cost of capital (WACC)

Answer to Study Sessions Self Assessment Questions (SAQs) for Module 2

1. Weighted average cost of capital is the weighted average of the various sources of funds in a company capital structure where each weight is represented by the proportion of each sources of fund in the capital structure while marginal cost of capital is defined as weighted average cost of extra or incremental capital.

2i. If the company capital structure is optimum

vii. If the company weighted average cost of capital reflects its long – term future capital structure.

viii. If new funds are raised in a way that keep the company capital structure unchanged

ix. If the weighted average cost of capital reflected marginal cost of capital

x. If existing projects and proposed project are homogeneous

3. Using the formula for calculating cost of debt for redeemable debenture

$$P_o = \sum_{t=2}^{n} \frac{R(1-T)}{(1+kd)^t} + \frac{PT}{(1+kd)^n}$$

$$650,000 = \sum_{t=1}^{5} \frac{114,000(1-0.3)}{(1+kd)t} + \frac{600,000}{(1+kd)^5}$$

$$650,000 = \frac{79,800}{(1+kd)} + \frac{79,800}{(1+kd)^2} + \frac{79,800}{(1+kd)^3} + \frac{79,800}{(1+kd)4} + \frac{79,800}{(1+kd)5} + \frac{600,000}{(1+kd)5}$$

Transfer this equation into a table as shown below

Year	Cash flow	DCF@20%	Present value	Year	Cash flow	DCF@10%	Present value

0	(650,000)	1.0000	(650,000)	0	(650,000)	1.0000	(650,000)
1-5	79,800	2.9906	238,650	1-5	79,800	3.7908	302,506
5	600,000	0.4019	241,140	5	600,000	0.6209	372540

NPV = (170,210) NPV=25,046

After obtaining two distinct values of NPV as shown in the table above, then apply the concept of IRR formula;

$$IRR = DCF_+ + \frac{NPV_+}{NPV_+ - NPV_-}(DCF_- - DCF_+)$$

$$Kd = 0.10 + \frac{25,046}{25,046 + 170,210}\ (0.2 - 0.1)$$

Kd = 0.10 + 0.013

Kd = 0.113

Kd = 11.3%

The cost of debt (Kd) of the company is 11.3% or approximately 11%

. Using the formula for cost of preference share for irredeemable preference share as;

$$Kp = \frac{Dt}{Po}$$

Where;

Dp = Annual fixed dividend payment on preference share = 0.15 X 1, 000, 000 = N150, 000

Po = Current market price of preference share = N1, 800, 000

Kp = <u>0.15 X N1, 000, 000</u>

N1, 800, 000

$$Kp = \frac{150,000}{1,800,000}$$

Kp = 0.083

Kp = 8.3%

The cost of preference share (Kp) of the company is 8.3% or approximately 8%

. Using the formula for calculating cost of equity as;

$$k_e = \frac{D_O(1+g)}{P_O} + g$$

Where;

Existing dividend per share = $D_o = \frac{\text{Dividend declared}}{\text{No. of outstanding shares}} = \frac{N250,000}{1,000,000} = N0.25$

Annual dividend growth rate = g = 0.06

Current market price of equity = $P_o = \frac{\text{Market value}}{\text{No. of outstanding shares}} = \frac{N1,200,000}{1,000,000} = N1.20$

$$k_e = \frac{0.25\ (1.06)}{1.20} + 0.06$$

k_e = 0.221 + 0.06

k_e = 0.281

k_e = 28.1%

The cost of equity (Ke) of the company is 28.1% or approximately 28%

Using the formula for computing weighted average cost of capital as shown below;

$$WACC = \frac{D}{V}(Kd) + \frac{P}{V}(Kp) + \frac{E}{V}(Ke)$$

Where;

Value of debenture (D) = N650, 000

Value of preference share (P) = ₦1, 800, 000

Value of equity (E) = N1, 200, 000

Value of the company (V) = D + P + E

= N650, 000 + N1, 800, 000 + N1, 200, 000

= N3, 650, 000

Cost of equity = K_e = 0.281

Cost of debenture = K_d = 0.113

Cost of preference share = Kp = 0.083

$$WACC = \frac{N650,000}{N3,650,000}(0.113) + \frac{N1,800,000}{N3,650,000}(0.083) + \frac{N1,200,000}{N3,650,000}(0.281)$$

$$WACC = 0.020 + 0.041 + 0.092$$

$$WACC = 0.153$$

WACC = 15.3%

The company weighted average cost of capital (WACC) is 15%

REFERENCES AND FURTHER STUDIES

Ezike John (2002) Essential of Corporate Financial Management, Lagos Jaylycent pp 275 - 288

Olowe R.A (1997) Financial Management: Concept, Analysis and capital Investment. Lagos Bierly Jones Nigeria Limited pp 401 – 405

Otaniyi O. (1997) Elements of Business Finance, Panaf Publishing Incorporation. pp 119 - 126

Owualah S.I (2003) Understanding Business Finance, Lagos G-Mag Invesment Ltd pp 51 - 65

Pandey I.M (2000) Financial Management India Vikas Publishing PVT Ltd pp 512 - 530

Shiro A.A (2004) Problems and Solutions in Financial Management, Lagos. El-Toda Ventures Ltd. pp 171 – 194

Should you require more explanation on this study session, please do not hesitate to contact your e-tutor via the LMS.

Are you in need of General Help as regards your studies? Do not hesitate to contact the DLI IAG Center by e-mail or phone on:

iag@dli.unilag.edu.ng
08033366677

Study Session 3 Analysis of Leverage I

Introduction

Our focus in this study session and the next which round off this first course in Business Finance, is Leverage. Leverage provides the framework for financing decisions of a firm. It may be defined as the employment of an asset or source of fund for which the firm has to pay a fixed cost, or fixed return i.e. the use of fixed costs in the operations of a firm and the use of fixed return securities in financing a firm. In other words, operating and financial leverage respectively. Leverage that is associated with asset acquisition/investment activities is referred to as operating leverage. We will take a look at how to analyze and measure this in the present study session. We will be employing the tool of Break-even analyses to give proper understanding of operating Leverage and its implications on the revenue accruing to a firm. You should be able to follow through this study session as you endeavor to grasp our background analyses of the various types of costs that features in the operations of a firm.

Learning Outcomes for Study Session 3

When you have studied this session, you should be able to:

3.1 Define basic terms on Leverage like 'Operating Leverage', 'Financial Leverage', Business Risk' and 'Financial Risk'

3.2 Explain and solve problems on break-even analysis

3.3 Explain and solve problems on calculating the Operating Leverage of a firm once relevant data on fixed cost, variable cost and selling price of its products are given.

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3.1 Concept of Leverage

To a physicist leverage implies the use of a lever to raise a heavy object with a small force. To the ordinary man on the street, if a person has leverage, say in politics, a word or action from him can accomplish much. In our context, a high degree of leverage implies that a relatively small change in sales results in a large change in profit. There are two types of leverage that we will be discussing in this chapter: Operating and Financial Leverage.

Operating leverage derives from the extent to which fixed costs are used in a firm's operation. Break-even analysis is used to measure the extent to which operating leverage is employed in a firm. If a high percentage of a firm's total costs are fixed costs, then the firm is said to have a high degree of operating leverage. This implies that a relatively small change in sales results in a large change in operating income.

Financial leverage refers to the use of fixed income securities – debt and preferred shares. It is measured by the ratio of long-term debt to long term debt plus equity to put it simply. We will show in our discussions in the next study session and in further studies in this field that financial leverage affects a firm's expected earnings per share, the riskiness of these earnings and consequently, the price of the firm's shares or stock.

We need to distinguish between financial risk and business risk. Business risk is the basic risk in a firm's operations. This is the inherent uncertainty or variability of expected pre-tax returns on the firm's portfolio of assets. Financial risk as noted above is the risk that results in using debt in a firm. Business and financial risk together form a firm's total corporate risk.

- What is operating leverage?

  - **Operating leverage** is the ability to use fixed operating costs to magnify the effect of changes in sales on the operating profits (EBIT) of a firm.

3.2 Break-Even Analysis

Break-even analysis is a technique for studying established relationship between costs, volume and profits. It shows the level of sales at which costs and revenues are in equilibrium. The equilibrium point is normally known as the break-even point (B-E-P). The break-even point can be defined as that point of sales volume at which total revenue is equal to total costs. It is a no-profit, no-loss point. Break-even analysis is however concerned, more with the effects of changes in costs, volume and prices on profits.

The possibility of separating costs into fixed and variable components is a pre-requisite for carrying out break-even analyses. If a firm's costs are all variable, the problem of break-even volume would seldom arise since no profit, no loss situation will arise at zero sales volume. Profit in such situation would vary proportionately with sales. Since a firm would normally have some fixed, and some variable costs, it must suffer losses until a given volume has been reached.
Two approaches can be used to compute the break-even point: (a) The algebraic approach and (b) the chart approach. Besides, the break-even-point can be computed in terms of units of production, or in terms of naira (money) value. We will presently go into these discussions in turn.

Illustration: Find the break-even point and the profit from sales of 40,000 units of product Z when selling price is ₦5; variable cost is ₦3; fixed cost is ₦40,000.

The break-even quantity is defined as that volume of output at which revenue is just equal to total costs (fixed costs plus variable costs).

If	P	=	Price per unit
	Q	=	Quantity produced and sold
	F	=	Fixed costs
	V	=	Variable costs per unit

Then PQ = F + VQ
PQ – VQ = F

$$Q(b.e.p) = \frac{F}{P - V} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.1)$$

This gives us the level of production (units) at which we break-even. For our illustration:

₦

₦ ₦3

$$Q(b.e.p) = \frac{N40{,}000}{N5 - N3} = 20{,}000 units$$

What we have here can be graphed into a break-even chart as we take the following steps:

1.Sales volume is plotted on the horizontal axis. This may be expressed in Naira, units or percentage of capacity. Equal distances are cut along the horizontal line to show sales volume at different activity levels.

2. Vertical axis is used to represent revenue, fixed costs, and variable costs. The vertical line is also spaced in equal parts. A parallel vertical line may be drawn on the right of the chart to complete the square.

3. The fixed cost line parallel to the horizontal axis, can be drawn through the fixed cost point (₦40,000).

4. We assume that the budgeted sales/production levels is 40,000 units. This brings total sales and total costs to ₦200,000 and ₦160,000 respectively. To draw the total sales line, the zero sales point is connected with the sales budget point (₦200,000) on the right-hand vertical line. Similarly, total costs line can be drawn by connecting fixed costs point (₦40,000) with the total costs budget point (₦160,000) on the right hand vertical line.

The point of intersection between the total sales and the total cost lines is the break-even point. In our present illustration, it occurs at a sales volume of ₦100,000 (20,000 units). The area to the left of the break-even point is the loss area and represents the uncovered fixed costs, while to the right of it there is the profit area. As illustrated in figure 3.1, the variable cost is represented by the gap between the total cost and the fixed cost.

Firms also have interest in calculating break-even points on the basis of naira sales rather than basis of units of output. This method makes it easy to determine a general break-even point for a firm that sells many products at varying prices. Furthermore the procedure does not require much data. Only three are needed: sales, fixed costs and variable costs. These data are readily available for firms.

We arrive at the required formula as follows:

1.At the break-even point, Sales (S_B) are equal to fixed cost (FC) plus variable cost (VC): (no loss no profit point)

S_B = FC + VC

2.Because both the sales price and the variable cost per unit are assumed to be constant in break-even analysis, the ratio VC/S for any level of sales is also constant and may be found from the annual income statement.

Since variable cost is a constant percentage of sales we can rewrite equation 3.1 as follows:

$$S_b = FC + \frac{VC}{S}(S_b)$$

$$S_b(1 - \frac{VC}{S}) = FC$$

$$S_b = \frac{FC}{1 - \frac{VC}{S}} \qquad \text{.................................(3.2)}$$

where S_b =Sales volume at break-even point.

Any sales level can be taken in the circumstances with the related data to determine the break-even point.
From the illustration we have been using, let us assume that 40,000 units were actually produced and sold. Sales in naira will then be ₦200,000 while variable cost is ₦120,000. Fixed cost is given as ₦40,000.

$$\text{Break-even point} = \frac{N40{,}000}{1-\dfrac{120{,}000}{200{,}000}}$$

$$= \frac{₦40{,}000}{0.4} = ₦100{,}000$$

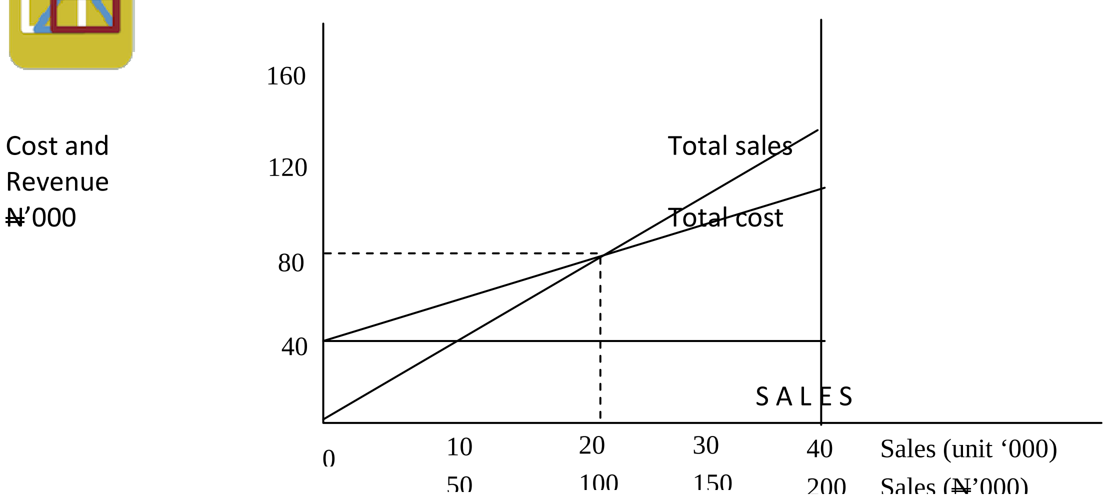
A further analysis of equation 3.2 provides an insight into the mechanism of the break-even point. When the variable costs (per unit or total) are divided by sales (selling price or total), we get variable cost ratio. Thus the 0.60 or 60% (₦120,000 ÷ ₦200,000) derived above shows that variable costs are 60% of sales or in other words 60 kobo of every naira sales is required to cover the variable costs. We can derive what is called the profit-volume (P/V) ratio or the marginal income or contribution ratio when the variable cost ratio is subtracted from 1.
Thus the denominator in equation (3.2).

$$= 1-\frac{\text{variable costs}}{\text{sales}}$$

gives us the P/V or contribution ratio. This is 0.40 in our illustration and shows that 40% of sales are available to cover fixed costs and generate profits. This means that 40 kobo of each naira sales is available to cover fixed costs and earn a profit. Since profit at break-even point is zero, dividing the fixed costs by the P/V ratio gives the sales volume that is necessary to recover total fixed costs (₦40,000 ÷ 0.4 = ₦100,000). Thus we can rewrite equation 3.2 as follows:

$$\text{Break-even point (sales)} = \frac{\text{Fixed Cost}}{\text{P/V ratio or contribution ratio}} \quad ..(3.3)$$

Figure 3.1 BREAK-EVEN CHART



- (i)Explain the break-even point. (ii) Royal Grace Farms Ltd sells a product at ₦4 per unit and the variable cost of the product is ₦2 per unit. The current fixed cost is ₦30, 000.00. What is the break-even point at which the company can produce?
    - The **break-even point (BEP)** is the point at which the cost of producing a product or providing a service exactly matches the revenue gained from selling that product or service. For example, if a firm's total annual costs are million naira and in the same year it generates million naira of revenue, then the firm is said to have broken-even, as it hasn't made any more or less than it has invested:
    - (ii) You can apply the following formula:

= Fixed costs / (selling price per unit) –Variable cost pr unit)

$$\text{BEP (QTY)} = \frac{FC}{SP-VC}$$

$$= \frac{30{,}000}{4-2}$$

=15, 000 UNIT

3.3 Operating Leverage

The degree of operating leverage is said to be high in a firm if it employs a greater amount of fixed costs and a small amount of variable costs. The profits of a highly leveraged firm will increase at a faster rate than the increase in sales. However, if the sales fall, the firm with a high degree of operating leverage will suffer a grater loss than a firm with low or moderate degree of operating leverage.

Illustration: We will be illustrating the significance of the degree of operating leverage using the following example:

Firms P, Q and R manufacture the same product. The fixed costs for each of them respectively are ₦8,000, ₦20,000 and ₦36,000. The selling price of a unit of the product is ₦10, while the variable costs per unit for each firm are ₦8, ₦6 and ₦4 respectively. We will carry out break-even analysis for each firm in a bid to determine the impact on their profits of changes in units sold. The basic data is as follows to calculate the break-even points for each of the three firms.

Firm P:

B/E point = $\frac{\text{₦}8,000}{\text{₦}10 - \text{₦}8}$ = 4,000 units

Firm Q:

B/E point = $\frac{\text{₦}20,000}{\text{₦}10 - \text{₦}6}$ = 5,000 units

Firm R:

B/E point = $\frac{\text{₦}36,000}{\text{₦}10 - \text{₦}4}$ = 6,000 units

Further analysis to show costs and profits at various sales capacity levels are shown in table 10.1 and the discussion that follows:

Table 3.1 Effect of Operating Leverage

		Firm P		**Firm Q**		**Firm R**	
Units Sold	**Sales (₦)**	**Costs (₦)**	**Profit (₦)**	**Costs (₦)**	**Profit (₦)**	**Costs (₦)**	**Profit (₦)**
4,000	40,000	40,000	0	44,000	(4,000)	52,000	(12,000)
5,000	50,000	48,000	2,000	50,000	0	56,000	(6,000)
6,000	60,000	56,000	4,000	56,000	4,000	60,000	0
8,000	80,000	72,000	8,000	68,000	12,000	68,000	12,000
10,000	100,000	88,000	12,000	80,000	20,000	76,000	24,000
12,000	120,000	104,000	16,000	92,000	28,000	84,000	36,000

If we take the 10,000 units as a reference point, we can deduce from the table that a 20 per cent increase in sales to 12,000 units will make for increase in profits in firm P, Q and R of 33⅓%, 40% and 50% respectively, showing the effect of the low, moderate and high leverage used in the respective firms.

The same percentage decrease in profit is observed if units sold decrease from 10,000 to 8,000 units. So the firm with the highest fixed cost (R) is observed to be most vulnerable to changes in sales while firm P with the lowest fixed cost is least vulnerable.

We can define operating leverage more precisely in terms of the way a given change in volume affects profits. For this purpose, the degree of operating leverage (DOL) is defined as the percentage change in profits or operating income resulting from a percentage change in units sold. We express this algebraically as follows:

$$DOL = \frac{\textit{Percentage change in Profit}}{\textit{Percentage change in sales}} \quad \text{…………………(3.4)}$$

$$= \frac{\Delta \text{Profit}}{\text{Profit}} \Big/ \frac{\Delta Q}{Q} = \frac{\Delta EBIT}{EBIT} \div \frac{\Delta SALES}{SALES}$$

Where Δ (called delta) is change and EBIT is earnings before interest and taxes and is equivalent to operating profit.

For firm R:

$$DOL = \frac{\dfrac{₦36000 - ₦24000}{₦24000}}{\dfrac{₦120{,}000 - ₦100{,}000}{₦100{,}000}}$$

$$= \frac{₦12000}{₦24000} / \frac{₦20000}{₦100000} = \frac{50\%}{20\%}$$

The operating leverage of 2.5 indicates that if the sales change by 100 per cent, operating profits will change by 250 per cent. In other words a 1 time change in sales will lead to 2.5 times change in profits. We can also apply the formula in (3.4) to find the operating leverage for firms P and Q at the same level of activity. When we do this we determine the operating leverage for firm P to be 1.67 and for firm Q to be 2.0. You can try this out on your own for the purpose of practice.

We can further develop equation 3.4 algebraically as follows:

When P = price per unit; V = variable cost and ΔQ = change in units sold. Since fixed costs are constant, change in profits can be defined symbolically as ΔQ(P-V). The initial profit is Q(p-v) – F. Hence the percentage change in profit is:

$$\frac{\Delta Q(P-V)}{Q(P-V)-F}$$

The percentage change in output is ΔQ/Q. So the ratio of the change in profits to the change in output is:

$$DOL = \frac{\dfrac{\Delta Q(P-V)}{Q(P-V)-F}}{\dfrac{\Delta Q}{Q}}$$

$$= \frac{\Delta Q(P-V)}{Q(P-V)-F} \bullet \frac{Q}{\Delta Q}$$

$$DOL = \frac{Q(P-V)}{Q(P-V)-F} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.5)$$

When we realize that PQ = Sales (S) and VQ = Total variable costs (VC), and F is fixed cost, we can rewrite equation 10.5 in terms of total naira figures as follows:

$$DOL = \frac{S - VC}{S - VC - F} \quad \text{...(3.6)}$$

We can apply the formulae that we have in (3.5) and (3.6) to the calculations we made above. We will of course, arrive at the same result. Try this out. By way of illustrations then we will apply here equations 3.5 and 3.6 to the data we have above for firms P and Q respectively.

Degree of operating leverage for firm P applying formula in (3.5) at 10,000 units level of operation.

Q	=	10,000
P	=	₦10
V	=	₦8
F	=	₦8,000

$$DOL = \frac{Q(P - V)}{Q(P - V) - F}$$

$$DOL = \frac{10,000(N10 - N8)}{10,000(N10 - N8) - N8000}$$

$$= \frac{₦20,000}{₦12,000} = 1.67$$

Degree of operating leverage for firm Q applying formula in (3.6), at 10,000 units level of operation.

S	=	₦10,000
V	=	₦60,000
F	=	₦20,000

$$DOL = \frac{S - VC}{S - VC - F}$$

$$= \frac{₦100,000 - ₦60,000}{₦100,000 - ₦60,000 - ₦20,000} = \frac{₦40,000}{₦20,000} = 2.0$$

We can derive an alternate formula for DOL. From equation (3.4)

DOL = <u>% change in operating profit</u>
% change in sales

or DOL = <u>ΔEBIT</u> ÷ <u>Δsales</u>

EBIT sales

For a given level of fixed costs, the change in operating profit (EBIT) will be equal to change in contribution. Note that

$$\frac{\Delta EBIT}{EBIT} = \frac{\Delta contribution}{EBIT} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(i)}$$

Using equation (i) the formula for DOL can be also written as follows:

$$DOL = \frac{\Delta contribution}{EBIT} \div \frac{\Delta sales}{sales} \quad \ldots\ldots\ldots\ldots\ldots\text{(ii)}$$

Since Δcontribution = contribution ratio multiplied by Δsales, we can rewrite equation (ii) as

$$\frac{(\text{contribution ratio})\ \Delta sales}{EBIT} \times \frac{sales}{\Delta sales} \quad \ldots\ldots\ldots\ldots\ldots\text{(iii)}$$

From this expression: $DOL = \dfrac{\text{contribution ratio x sales}}{EBIT}$

$$DOL = \frac{contribution}{EBIT} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(3.7)}$$

- Olu Photos Nig Ltd has have invested N0.5 million into development and marketing for its latest film, and will sells for N45 per copy. Each film costs the company N5 to sell. Sales volume reaches one million. Calculate the degree of operational leverage.

  - Q=1,000,000 Units V=N5, P=N45, F=10,000

$$DOL = \frac{1{,}000{,}000\ (45-5)}{1{,}000{,}000(45-5)-500{,}000}$$

$$= \frac{40{,}000{,}000}{39{,}500{,}000}$$

$$=1.013$$

Summary of Study Session 3

Our concern in this study session has been the effect of Operating Leverage on the earnings of a firm. Break-even analysis served as our basic premise to the discussion on operating leverage to aid our understanding. This show the effect of using fixed costs when the fortune of a firm changes for one reason or the other.

The results of analyzing leverage effect can be quite interesting and revealing as to the financial and corporate risk a firm is exposed to.

By way of recapitulation, we need take a look at the various formulae we developed and used in the study session:

(1) Break even analysis

(a) Quantity at which a firm break-even

$$Q = \frac{F}{P - V} \quad \text{...}(3.1)$$

(b) Sales level at which we have break-even point

Or $\underline{\qquad FC \qquad}$ .. ……..…………(3.3)

$$S_B = \frac{FC}{1 - VC/S} \quad \text{...}(3.3)$$

PV ratio or contribution ratio

(2) Operating Leverage

$$DOL = \frac{Q(P - V)}{Q(P - V) - F} \quad \text{…..……………………………}(3.5)$$

$$or \ DOL = \frac{S - VC}{S - VC - F} \quad \text{...}(3.6)$$

$$DOL = \frac{\text{contribution}}{\text{EBIT}} \quad \text{……………………………...}(3.7)$$

Self-Assessment Questions (SAQs) for Study Session 3

Having completed this study session, you can assess how well you have achieved its Learning Outcomes by answering the following questions. You can check your answers at the appendix to the course text.

SAQ 3.1(tests Learning Outcomes 3.1, 3.2 and 3.3

The comparative income statements for two companies :ROYAL GRACE FARMS LTD and QUALIFOODS NIG LTD as at 31st December 2008 are given below:

	RGFL (N'000)		QNL (N'000)	
Sales		2,000		3,500
Less Variable Cost	300		500	
Fixed Cost	250	550	350	850
Net Income before Tax		1450		2,650
Less interest on debt		325.5		-
Net Income before Tax		1,124.5		2,650
Less Company Tax (40%)		449.8		1,060
Net Income after Tax		674.7		1,590

Calculate (i) the B-E-P (Sales) for each of the two companies (ii) the operating Leverage of the two companies

Answer to Study Sessions Self Assessment Questions (SAQs) for Module 3 session 3

Solution 2:

	RGFL (N'000)	QNL (N'000)
Sales	2,000	3,500
Less Variable cost	300	500
CONTRIBUTION	1,700	3,000
Less Fixed cost	250	350
EBIT	1,450	2,650
LESS Interest on debt	325.5	0
	1,124.5	2,650
Less Tax	449.8	1,060
Net Income	674.7	1,590

Degree of Operating leverage DOL:	RGFL (N'000)	QNL (N'000)
DOL = <u>Contribution</u>	<u>1,700</u>	<u>3000</u>

EBIT	1.450	2,650

Should you require more explanation on this study session, please do not hesitate to contact your e-tutor via the LMS.

Are you in need of General Help as regards your studies? Do not hesitate to contact the DLI IAG Center by e-mail or phone on:

iag@dli.unilag.edu.ng
08033366677

Study Session 4 Analyses of Leverage II

Introduction

We started discussing Analysis of Leverage in our last study session. Our focus then was on Operating Leverage. In this study session that round up our course in Business Finance I, we will be taking a look at Financial Leverage. As the nomenclature imply, Financial Leverage is related to the financing activities of a firm. We will see in our discussions that Financial Leverage results from the presence of fixed financial charges. Such expenses do not vary with the operating profits (EBIT). They have to be paid regardless of the amount of EBIT available to pay them. After paying them, what remains of the EBIT belongs to the shareholders. Hence Financial Leverage is concerned with the effect of changes in EBIT on the earnings available to the shareholders (EPS). Financial Leverage can then be defined as the ability of a firm to use fixed financial charges to magnify the effect of changes in EBIT on EPS.

Finally, we will be taking a look at the combined effect of Operating and Financial Leverage to end this module. Combined Leverage is the product of Operating and Combined Leverage. It indicates the effect that changes in sales have on EPS.

Learning Outcomes for Study Session 4

After studying this session, you should be able to:

4.1 Explain the features and solve problems on measuring of Financial Leverage of a firm once relevant information on the components of the firm's financial structure is given.

4.2 Solve problems on measuring and computing the Combined Leverage of a firm.

4.1 Financial Leverage

While operating leverage refers to the use of fixed operating assets financial leverage refers to the use of fixed interest securities – debt and preference shares. Financial leverage can also be described as trading on equity. The use of the term "trading on equity" is derived from the fact that it is the owners' equity that is used as a basis to raise debt i.e. it is the equity that is traded upon. The supplier of debt has limited participation in the company's profits and therefore will insist on protection in earnings and protection in values represented by ownership equity. The rate of interest on debt is usually fixed irrespective of the company's rate of return on assets. The company is legally bound to pay interest on debt. The rate of preference dividend is also often fixed, but the preference dividends are usually paid when the firm earns profits. The common shareholders are entitled to the remaining income. The earnings after interest and taxes and preference dividends belong to common shareholders. What they receive depend on what the firm earned as income and the dividend policy of the company.

Companies have the choice of using common equity only, or a mixture of common equity and fixed income securities to finance their projects. It is often seen in practice that the value of a firm (measured in terms of shareholders fund) that has no debt first rises as it substitutes debt for equity. But it may decline as its use of debt becomes excessive. This is financial leverage. If for example a firm borrows ₦1000 at 14 percent interest (₦140 per annum), and the firm succeeds in investing it at a rate of 20 per cent interest (₦200 per annum), the balance of ₦60 after payment of interest belongs to the shareholders and constitutes profit from financial leverage. If on the other hand the company could only earn a return of 10 percent on the investment (₦100 per annum), the loss to the shareholders would be ₦40 per annum. This example illustrates the fact that financial leverage has the potential of increasing the shareholders earnings and at the same time create the risks of loss to them. It could be described consequently, as a double edged sword.

4.1.1 Earnings Per Share (EPS)

The basic aim of a company in using financial leverage is to enlarge the returns to shareholders under favorable conditions. This is based on the assumption that the fixed-charges funds (bonds and preference shares) can be obtained at a cost lower than the company's return on its total assets. Earnings per share will then increase when the difference between the earnings generated by assets financed by fixed-charges funds and costs of such funds are distributed to shareholders.

There will be a fall in earnings per share however if the fixed-charges funds were obtained at a cost higher than the rate of return on the company's assets.

Earnings per share (EPS) can be calculated by dividing earnings after taxes (EAT) or net income (NI) by the number of shares outstanding. To derive EAT, we first deduct the interest on debt (I) from earnings before interest and taxes (EBIT) or net operating income (NOI). Taxes are then computed on and subtracted from earnings before taxes (EBT) to arrive at the Figure for earnings after taxes. We can represent this by a formula:

$$EPS = \frac{(EBIT - I)(1-t)}{N} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4.1)$$

Where t = the corporate tax rate

N = the number of common shares outstanding.

When a company does not use any debt then the formula would simply be.

$$EPS = \frac{EBIT(1-t)}{N} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4.2)$$

If a company uses preference capital then EPS can be calculated as follows:

$$EPS = \frac{(EBIT - I)(1-t) - DIVP}{N} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4.3)$$

Where DIVP = Dividends to preference shareholders.

It is to be noted that dividends on preference shares are not tax deductible. We can illustrate the effect of alternate financial plans on EPS with an example as follows:

Naija Company (Plc) wants to raise ₦1 million for a new factory. The factory is expected to annually yield a before tax earnings of ₦250,000. The financial director of Naija is considering the following options:

Issuing 100,000 common shares at ₦10 each

Issuing 60,000 common shares at ₦10 each and 18 percent debenture to raise ₦400,000 or

Common shares of 50,000 units at ₦10 each, with 16% preference share capital of ₦200,000 and 18% debentures of ₦300,000. What are the effects of the three plans respectively on the shareholders earnings? Assume corporate tax rate of 40 per cent.

Table :4.1 EFFECT OF FINANCIAL PLAN ON EPS

		I ₦	II ₦	III ₦
1	Earnings before interest & taxes (EBIT)	250000	250000	250,000
2	Less Interest (I)	0	72,000	54,000
3	Earnings before taxes	250,000	178,000	196,000
4	Less taxes t(EBIT-I)	100,000	71,300	78,400
5	Profit after taxes (I-t)(EBIT-I)	150,000	106,800	117,600
6	Less Preference dividend		0	32,000
7	Total earnings of investors (2)+(5)+(6)	150,000	178,800	171,600
8	Number of common shares	100,000	60,000	50,000
9	EPS = (EBIT-I)(1-t)/N	1.50	1.78	1.71

Effect of financial leverage is clearly seen in table 10.2. When 40 per cent debt is used to finance the investment, the firm earns ₦1.78 per share, 18.7 per cent more than the ₦1.50 per share earned when there is no leverage. Under the third alternate financing scheme of 50 per cent fixed charges financing scheme, ₦1.71 EPS was earned, 14 per cent above the no leverage option.

We have these results for the reason already explained, that is, the project's ability to earn above the fixed charges cost of debt and preference shares financing under alternatives II and III. The total assets earns 25 per cent while funds are obtained at a lower rate of 16% and 18% for preference shares and debt financing respectively. The surplus earnings after fixed securities charges are paid accrues to the common shareholders.

In our example here the effect of leverage will be unfavourable if the firm is earning say 15 per cent on its assets. Shareholders will have to bear the shortfall of earnings needed to pay the interest to fixed charges securities holders. This will result in a drastic fall in EPS after the preference share dividend and debenture holders interest charges are paid.

4.1.2 Degree of Financial Leverage

Our discussions so far show that financial leverage affects the earnings per share of a firm. When economic conditions are favourable and a company's EBIT is increasing, its EPS increase faster with more fixed charges financing (debts) in the capital structure.

We can as with operating leverage, define financial leverage more precisely, as a relationship between EPS and EBIT as follows: The degree of financial leverage is defined as the percentage change in earnings available to common shareholders (EPS) due to a given percentage change in earnings before interest and taxes (EBIT), that is:

$$DFL = \frac{\%\text{ change in EPS}}{\%\text{ change in EBIT}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4.4)$$

We will take a closer look at this to develop a more precise formula to aid us in calculating the degree of financial leverage for any given level of EBIT and interest charges (I)

As shown previously $EPS = \frac{(EBIT - I)(1-t)}{N}$

Note that I is a constant, so ΔEPS, the change in EPS is

$$\Delta EPS = \frac{\Delta EBIT(1-t)}{N}$$

The percentage increase in EPS is the change over the original EPS or

$$\frac{\dfrac{\Delta EBIT(1-t)}{N}}{\dfrac{(EBIT - I)(1-t)}{N}} = \frac{\Delta EBIT}{EBIT - I}$$

The degree of financial leverage is the percentage change in EPS over the percentage change in EBIT, so

$$DFL = \frac{\dfrac{\Delta EBIT}{EBIT - I}}{\dfrac{\Delta EBIT}{EBIT}} = \frac{EBIT}{EBIT - I} \quad \ldots\ldots(4.5)$$

Using the illustrations we have in table (4.1) we can use the formula in (4.5) to calculate the degree of financial leverage under the second financial alternative when 60 per cent equity and 40 per cent debt was used by the Company.

$$DFL = \frac{₦\ \text{N}250{,}000}{₦250{,}000 - ₦72{,}000} = 1.40$$

The implication of this is that a 100 per cent increase in EBIT would result in a 140 per cent increase in earnings per share and vice-versa in the case of a decrease in EBIT.

The degree of financial leverage for the first plan is of course 1.0 since this is an all equity financing with no fixed interest securities.

For the third financing option, it is good to note that preference dividend (a fixed interest payment) is involved. Note that in equation 10.12 the amount of interest used is the before tax figure just as EBIT (as the name imply) is a before-tax parameter. For us to see the effect of preference share dividend as in the third financing option of our illustration we have to calculate the before-tax amount of preference dividend so it will fit into our formula for degree of financial leverage. We arrive at this as follows:

We represent after tax cost of debt or cost of fixed interest financing plan (preference dividend) as Y; while X is the before-tax cost of debt, and t is the tax rate.

$$Y = (1 - t)\,X$$

$$X = \frac{Y}{1 - t}$$

$$\frac{0.16}{1 - 0.4} = 0.2667 \text{ or } 26.67\%$$

So, the before tax rate of the preference dividend in our example above is.

To incorporate the effect of preference dividend on the EPS we modify our formula in (4.5) as follows:

$$DFL = \frac{\text{EBIT}}{\text{EBIT - I - (DIVP) /(1 - t)}} \quad \text{.......................................}(4.5a)$$

Where DIVP as previously noted is the dividend on preference shares.

So the degree of financial leverage for the third financing option of our illustration using (4.5a) is

$$DFL = \frac{₦250,000}{₦250,000 - ₦54,000 - ₦32,000/(1-0.4)}$$

$$= \frac{₦250,000}{250,000 - 107,333.33} = 1.75$$

This imply that a 100 per cent increase in EBIT would result in 175 per cent increase in earnings per share, and vice versa in the case of a decrease in EBIT.

Advantages and Disadvantages of Financial Leverage

Advantages: Leverage is good as long as it is increasing earnings per share or brings share capital appreciation. The following are the advantages of financial leverage. (i)It enables the firm to finance its projects (ii)We use it to measure the borrowing position of the company (iii) It indicates the capacity of the firm to meet fixed financial charges.

Disadvantages: (i) High leverage results into high interest to the company. (ii) Earnings due to the company will be reduced (iii) Reduced net income due to the shareholders Persistency can lead to insolvency.

- What is leverage ratio?
  - A leverage ratio summarizing the effect a particular amount of financial leverage has on a company's earnings per share (EPS). Financial leverage involves using fixed costs to finance the firm, and will include higher expenses before interest and taxes (EBIT). The higher the degree of financial leverage, the more volatile EPS will be, all other things remaining the same. The formula is as follows:

$$DFL = \frac{\%\ \text{Change in EPS}}{\%\ \text{Change in EBIT}}$$

4.2 Combined Leverage

the effect of operating leverage is that a change in sales volume could have a magnified effect on EBIT. If financial leverage is superimposed on operating leverage, changes in EBIT will have a magnified effect on earnings per share (EPS). Therefore, if a firm uses a considerable amount of both operating leverage and financial leverage, even small changes in the level of sales will produce wide fluctuations in EPS.

Formulae in (3.5) for the degree of operating leverage can be combined with the one in (4.5) for financial leverage to show the total leverage effect of a given change in sales on earnings per share. Degree of combined leverage (DCL) will be formula in (10.5) multiplied by formula in (4.5). We can make these compatible as follows:

Note that EBIT = Q(P-V)-F. with this we can rewrite equation 4.5 as:

$$\frac{EBIT}{EBIT - \mathrm{I}} = \frac{Q(P - V) - F}{Q(P - V) - F - 1}$$

$$DCL = \frac{Q(P - V)}{Q(P - V) - F} \times \frac{Q(P - V) - F}{Q(P - V) - F - I}$$

$$DCL = \frac{Q(P - V)}{Q(P - V) - F - I} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.6)$$

We can convert the formula in (4.6) to one that makes use of total sales, and variable costs values as noted previously.

Illustration: Benosa Company Plc has a fixed cost of ₦60,000 in producing 50,000 bottles (liquid content only), of Elixvite, a brand of malt drink. Benosa sells a bottle of Elixvite for ₦6 but incurs a variable cost of ₦3 per bottle, of the drink. The company's capital structure is made up of ₦300,000 equity shares and ₦200,000 13 per cent, debenture. Calculate the company's operating, financial and combined leverage.

Solution:

$$\mathrm{DOL} = \frac{\mathrm{QP - V)}}{\mathrm{Q(P - V) - F}}$$

$$= \frac{₦50{,}000(₦6 - ₦3)}{₦50{,}000(6 - 3) - ₦60{,}000} = 1.67$$

$$\mathrm{DFL} = \frac{\mathrm{Q(P - V) - F}}{\mathrm{Q(P - V) - F - I}}$$

$$= \frac{₦50{,}000(₦6 - ₦3) - ₦60{,}000}{₦50{,}000(6 - 3) - ₦60{,}000 - ₦26{,}000} = 1.41$$

$$DCL = \frac{Q(P-V)}{Q(P-V)-F-I}$$

$$= \frac{50{,}000\text{₦}(6-3)}{50{,}000\text{₦}(6-3) - 60{,}000\text{₦} - 26{,}000\text{₦}} = 2.34$$

- Combine leverage is defined as:

  a. Sales x EBIT b. EPS X DOL c. DFL x EBIT d. DOL x DFL

  - DOL × DFL

Summary of Study Session 4

Our concern in this chapter has been the effect of financial leverage and combined leverage on the earnings of a firm. Financial leverage borders on the effect that the use of fixed interest securities in financing a firm has on the proportion of earnings that is ultimately available to shareholders. It was shown that the more of such securities are used at a relatively low cost the better for the shareholders. We also considered the effect of total or combined leverage which is a product of operating and financial leverage.

The results of analyzing leverage effect can be quite interesting and revealing as to the financial and corporate risk a firm is exposed to. More on this will be considered in our further study of financial management.

$$EPS = \frac{(EBIT - I)(1-t) - DIVP}{N} \qquad (4.3)$$

Where we also have preference capital in the capital structure.

Combined Leverage

$$DFL = \frac{EBIT}{EBIT - I} \qquad (4.5)$$

$$DFL = \frac{EBIT}{EBIT - I - DIVP(\frac{1}{1-t})} \qquad (4.5a)$$

$$DCL = \frac{Q(P-V)}{Q(P-V)-F-I} \qquad (4.6a)$$

$$\text{or} \quad \mathrm{DCL} = \frac{\mathrm{S - VC}}{\mathrm{S - VC - F - I}} \qquad (4.6b)$$

Self-Assessment Questions (SAQs) for Study Session 4

Now that you have completed this study session, you can assess how well you have achieved its Learning Outcomes by answering the following questions. You can check your answers in the Appendix to the course text.

SAQ 4.1 (tests Learning Outcomes 4.1 and 4.2)

Campack Ltd has a fixed cost of ₦60, 000.00 in producing 48,000 units of Cartons to produce Chalk in Lagos. It sells a unit for ₦6.00 and in cures a variable cost of ₦4.00 per bottle of the product. The company's Capital structure is made up of Four thousand naira equity share and ₦0.4million 10% Debenture.

Calculate the company's:

1 Degree of Operational leverage

2 Degree of Financial leverage

3. Degree of combined leverage

SAQ 4.2 (tests Learning Outcomes 4.1 and 4.2)

Royal Grace Farms Nig Ltd has a fixed cost of ₦600,000.00 in producing 480000 units of Cartons to produce Topee Herbal Balm in Lagos. It sells a bottle for ₦60.00 and in cures a variable cost of ₦40.00 per bottle of the balm. The company Capital structure is made up of Four million naira equity share and N2.4million 10% Debenture.

Calculate the company's:

1 Degree of Financial leverage

2.. Degree of combined leverage

Answer to Study Sessions Self Assessment Questions (SAQs) for session 4

SOLUTION2:

	N '000
Sales of 480000 @ N60.00	28,800
Less	
Variable cost of 480000 @ N40.00	19,200
Contribution	9,600

Less	
Fixed Cost	600
EBIT	9,000
Less Interest	240
Net Income	8,760

1. DOL $= \dfrac{\text{Contribution}}{\text{EBIT}}$

$$= \frac{9600}{9000}$$

$$= 1.07$$

2. DFL $= \dfrac{\text{EBIT}}{\text{EBIT-Interest}}$

$$= \frac{9000}{9000 - 240}$$

$$= 1.03$$

3 Combined Leverage = DOL x DFL

$$= \frac{9600}{9000} \times \frac{9000}{9000-240}$$

$$= 1.10$$