Prova d'esame Analisi Matematica II - 14.06.2024

1. Integrali generalizzati

Determine whether the following integral is finite or not

$$\int_3^{+\infty} e^{\frac{1}{x^2}} \left| \sin \frac{1}{x} \right|^4 \, \mathrm{d}x$$

2. Successioni di funzioni

Determine whether the function sequence is pointwise and/or uniformly convergent

$$f_n(x) = x^n e^{-n^2 x}$$

with $x \in [0, +\infty[$

3. Calcolo in più variabili

Determine whether the following function is continuous and/or differentiable in $(0,0)$

$$f(x,y) = \begin{cases} \dfrac{x^3 y}{x^6 + y^2}, (x,y) \neq (0,0) \\ 0, (x,y) = (0,0) \end{cases}$$

4. Equazioni differenziali

Resolve the following Cauchy's Problem:

$$(PC) : \begin{cases} y'(x) + y(x) = e^x \\ y(0) = 3 \end{cases}$$