Introduction to Artificial Intelligence

Homework 3 Resolution by Dino Meng [SM3201466]

Q1. Searching with Heuristics

A1. $C = h^*(S)$, by direct calculations we can estimate $h'(X)$ as

$$h'(X) = \frac{1}{2}h(X) \leq \frac{1}{2}h^*(X) \leq h^*(X)$$

Meaning that h' is admissible thus gives an optimal solution

A2. $C = h^*(S)$, since that we can similarly estimate $h'(X)$ as before:

$$h'(X) = \frac{h(X) + h^*(X)}{2} \leq \frac{2h^*(X)}{2} = h^*(X)$$

Proving again that h' is admissible.

A3. $C \geq h^*(S)$, as $h'(X)$ is not admissible. By estimating h' again, we get the bounds

$$h'(X) \leq h(X) + h^*(X) \leq 2h^*(X)$$

Meaning that in certain cases it could be that $2h'(X) \geq h'(X) > h'(X)$, which does not guarantee admissibility for the heuristic.

A4. $C \geq h^*(S)$, as h' is admissible. To prove that it is admissible, we first can observe that that $h'(X) - h(X) \leq 0$ for any node; the idea is that h' will be lower than h as it tries to find a node of a lower cost in its "network" of neighbouring nodes. We will prove it by induction on h'. - Base case: We assume that $K(X)$ empty, then $h'(X) \leq h(X)$ which is $h'(X) - h(X) \leq 0$ - Inductive Case: Assume that for each $Y \in K(X) \neq \emptyset$ we have that $h'(Y) - h(Y) \leq 0$. Then calculating $h'(X) - h(X)$ we have that

$$h'(X) - h(X) \leq \min_{y \in K(X)} h'(Y) - h(Y) \leq 0$$

Then by making few direct calculations on h, we can see that:

$$h'(X) \leq \begin{cases} \overbrace{\min_{Y \in K(X)} h'(Y) - h(Y)}^{\leq 0} + h(X), K(X) \neq \emptyset \\ h(X), K(X) = \emptyset \end{cases} \leq h(X) \leq h^*(X)$$

proving that h' is admissible, concluding.

A5. $C \geq h^*(S)$, as h' is admissible. Notice that $\texttt{cost}(x, y) = h^*(x) - h^*(y)$. Then the heuristic h' can be estimated as follows:

$$\begin{aligned} h'(x) &= \begin{cases} \min_{Y \in K(X)}(h(Y) + \texttt{cost}(X, Y)), K(X) \neq \emptyset \\ h(X), K(X) = \emptyset \end{cases} && (1) \\ &\leq \begin{cases} \min_{Y \in K(X)}(h^*(Y) + h^*(X) - h^*(Y)), K(X) \neq \emptyset \\ h^*(X), K(X) = \emptyset \end{cases} && (2) \\ &= \begin{cases} h^*(X), K(X) \neq \emptyset \\ h^*(X), K(X) = \emptyset \end{cases} = h^*(X) && (3) \end{aligned}$$

Proving that h' is admissible, concluding.

A6. $C \geq h^*(S)$, as h' is admissible. By the properties of the minimum,

$$\min_{Y \in K(X)+\{X\}} h(Y) \leq h(X)$$

So then

$$h'(X) \leq h(X) \leq h^*(X)$$

proving h' is admissible.

ii. The only heuristic which is surely to dominate h is the second one, as we are taking the average between h and h^* so since that $h^* \geq h$ surely so will the average dominate h. The rest are either not admissible or are proven to be dominated by h.

B1. h' is consistent, to verify it we can make direct calculations and using the fact that h is consistent:

$$\begin{aligned}
h'(X) &= \frac{1}{2}h(X) && (4)\\
&\leq \frac{1}{2}(c(X,Y) + h(Y)) && (5)\\
&= \frac{1}{2}c(X,Y) + \frac{1}{2}h(Y) && (6)\\
&= \frac{1}{2}c(X,Y) + h'(Y) && (7)\\
&\leq c(X,Y) + h'(Y) && (8)
\end{aligned}$$

Which is the definition of consistency on h', concluding.

B2. Similarly to B1, h' is consistent and we can verify it with direct calculations:

$$\begin{aligned}
h'(X) &= \frac{h(X) + h^*(X)}{2} && (9)\\
&\leq \frac{c(X,Y) + h(Y) + h^*(X)}{2} && (10)\\
&= \frac{h^*(X) - h^*(Y) + h(Y) + h^*(X) + h^*(Y) - h^*(Y)}{2} && (11)\\
&= \frac{2c(X,Y)}{2} + h'(Y) && (12)\\
&= c(X,Y) + h'(y) && (13)
\end{aligned}$$

Concluding.

B3. $C \geq h^*(S)$: If h' is not admissible, then it cannot be consistent (as consistency implies admissibility).

B4. $C = h^*(S)$: we can see that h' is consistent. To prove it, we can see that by the properties of the minimum that

$$\forall Y \in K(X), h(X) \leq h'(Y) - h(Y) + h(X)$$

which implies

$$h'(Y) \leq h(X) - h(Y) \leq c(X,Y)$$

So

$$h'(Y) \leq c(X,Y) + h(X)$$

Which implies h' is consistent, concluding.

B5. $C = h^*(S)$: we can see that h' is consistent. Before proving its consistency, let us observe that if h is consistent then $h' \geq h$: in fact we have that

$$h(X) - h(Y) \leq c(X,Y) \iff h(X) \leq c(X,Y) + h(Y)$$

Then applying it for $Y = \min\{Y \in K(X) : h(Y) + c(X,Y)\}$, we have that

$$h(X) \leq h'(X)$$

Now let us prove the consistency. By definition, if we have that $K(X) \neq 0$ then

$$h'(X) \leq h(Y) + c(X,Y), \forall Y \in K(X)$$

For dominance we have that

$$h'(X) \leq h'(Y) + c(X,Y)$$

concluding the proof.

B6. We cannot be sure that it is consistent, in fact we have a counterexample for the graph $G = \{S, A, B, G\}$ with weights $SA = 4, AB = 2, SB = 8, BG = 42$.

ii. h' is consistent and dominant over h for **B2** and **B5**.

C1. Bob's conclusion is wrong, because h'' can be inadmissible for $X_0 = G$ as it becomes negative for $h''(X_0)$.

Q2. Iterative Deepening Search

a)

- i) $+\infty$
- ii) False
- iii) new_limit
- iv) cutoff
- v) min(new_limit, f[child_node])
- vi) True
- vii) new_limit

b) The number of times that iterative deepening A* expands a node is greater than or equal the number of times A* will expand a node, because the variant runs the original algorithms more times for each different limit value, so it tends to re-explore certain nodes