Q1. Logic

(a) Prove, or find a counterexample to, each of the following assertions:

(i) If $\alpha \vDash \gamma$ or $\beta \vDash \gamma$ (or both) then $(\alpha \wedge \beta) \vDash \gamma$

The assertion is valid. Without loss of generality, if we assume that a entails g is true then surely (a and b) entails g since that if a and b are true so is a which entails gamma.

(ii) If $(\alpha \wedge \beta) \vDash \gamma$ then $\alpha \vDash \gamma$ or $\beta \vDash \gamma$ (or both).

Counterexample: a = A, b=B, g=A and B. It's certainly true that (A and B) entails (A and B), but it's not surely true that A entails (A and B).

(iii) If $\alpha \vDash (\beta \vee \gamma)$ then $\alpha \vDash \beta$ or $\alpha \vDash \gamma$ (or both).

Counterexample: b=-A, g=B. Assuming that a entails (A or -A), it's not true that A entails -A as a variable cannot be both true and false

(b) Decide whether each of the following sentences is valid, unsatisfiable, or neither.

(i) $Smoke \implies Smoke$

Valid, A => A is a tautology

(ii) $Smoke \implies Fire$

Neither, it is satisfiable

(iii) $(Smoke \implies Fire) \implies (\neg Smoke \implies \neg Fire)$

(iv) $Smoke \vee Fire \vee \neg Fire$

Valid, as Fire v -Fire is a tautology

(v) $((Smoke \wedge Heat) \implies Fire) \iff ((Smoke \implies Fire) \vee (Heat \implies Fire))$

Valid, by transforming the implications into its logical equivalent form we have a formula of type (-A v -B v C) iff (-A v -B v C)

(vi) $(Smoke \implies Fire) \implies ((Smoke \wedge Heat) \implies Fire)$

Valid, by transforming the implications we have a form of type (A v -A v -B v C) & (-C v C v -A v -B) which contains two tautologies

(vii) $Big \vee Dumb \vee (Big \implies Dumb)$

Valid, by transforming the implication we get the formula A v B v -A v B which contains a tautology

(c) Suppose an agent inhabits a world with two states, S and $\neg S$, and can do exactly one of two actions, a and b. Action a does nothing and action b flips from one state to the other. Let S^t be the proposition that the agent is in state S at time t, and let a^t be the proposition that the agent does action a at time t (similarly for b^t).

(i) Write a successor-state axiom for S^{t+1}.

AND (S^t XOR not S^t) AND (a^t XOR b^t)

The first "clause" describes the transition from S^t to S^(t+1) according to the action, the second one describes that the agent must take only one of the two actions

(ii) Convert the sentence in the previous part into CNF.

Using distributive law and De Morgan's laws is sufficient to convert it into a CNF

Q2. First Order Logic

Consider a vocabulary with the following symbols:

- *Occuption*(*p*, *o*): Predicate. Person *p* has occuption *o*.

- *Customer*(*p*1, *p*2): Predicate. Person *p*1 is a customer of person *p*2.

- *Boss*(*p*1, *p*2): Predicate. Person *p*1 is a boss of person *p*2.

- *Doctor*, *Surgeon*, *Lawyer*, *Actor*: Constants denoting occupations.

- *Emily*, *Joe*: Constants denoting people.

Use these symbols to write the following assertions in first-order logic:

(iii) Emily is either a surgeon or a lawyer.

(Occupation(SURGEON, EMILY) and not (Occupation(LAWYER, EMILY)) OR (Occupation(LAWYER, EMILY) and not Occupation(LAWYER, EMILY)

(iv) Joe is an actor, but he also holds another job.

OCCUPATION(ACTOR, JOE) AND (EXISTS O, O!=ACTOR AND OCCUPATION(P, JOE)

(v) All surgeons are doctors.

FORALL P, OCCUPATION(SURGEON, P) => OCCUPATION(DOCTOR, P)

(vi) Joe does not have a lawyer (i.e., is not a customer of any lawyer).

NOT EXISTS P, OCCUPATION(LAWYER, P) AND CUSTOMER(P, JOE)

(vii) Emily has a boss who is a lawyer.

EXISTS P, OCCUPATION(LAWYER, P) AND BOSS(P, EMILY)

(viii) There exists a lawyer all of whose customers are doctors.

EXISTS P, OCCUPATION(LAWYER, P) AND (FORALL P', CUSTOMER(P', P) AND OCCUPATION(P', DOCTOR))

(ix) Every surgeon has a lawyer.

FORALL P, OCCUPATION(P, SURGEON), (EXISTS P' OCCUPATION(P', LAWYER) AND CUSTOMER(P, P'))
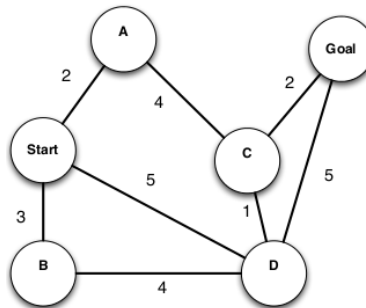
Q3.   Local Search

(a) Hill Climbing

    (i) Hill-climbing is complete. ☐ True ☒ False

    (ii) Hill-climbing is optimal. ☐ True ☒ False

(b) Simulated Annealing

    (i) The higher the temperature T is, the more likely the randomly chosen state will be expanded. ☒ True ☐ False

    (ii) In one round of simulated annealing, the temperature is 2 and the current state S has energy 1. It has 3 successors: A with energy 2; B with energy 1; C with energy 1-ln 4. If we assume the temperature does not change, What's the probability that these states will be chosen to expand after S eventually? **2/5, 2/5, 1/5**

    (iii) On a undirected graph, If T decreases slowly enough, simulated annealing is guaranteed to converge to the optimal state. ☒ True ☐ False

(c) Local Beam Search

The following state graph is being explored with 2-beam graph search. A state's score is its accumulated distance to the start state and lower scores are considered better. Which of the following statements are true?

☒ States A and B will be expanded before C and D.

☐ States A and D will be expanded before B and C.

☐ States B and D will be expanded before A and C.

☐ None of above.

(d) Genetic Algorithm

    (i) In genetic algorithm, cross-over combine the genetic information of two parents to generate new offspring.
☒ True ☐ False

    (ii) In genetic algorithm, mutation involves a probability that some arbitrary bits in a genetic sequence will be flipped from its original state.
☒ True ☐ False

(e) Gradient Descent

    (i) Gradient descent is optimal. ☐ True ☒ False

    (ii) For a function $f(x)$ with derivative $f'(x)$, write down the gradient descent update to go from x_t to x_{t+1}. Learning rate is α.

$$x\_(k+1) = x\_k - a\ f'(x\_k)$$