Introduction to Artificial Intelligence

Homework 4 Resolution by Dino Meng [SM3201466]

Q1. Logic

Exercise a)

i.) The assertion is valid. Without loss of generality, if we assume that $\alpha \models \gamma$ then $(a \wedge \beta) \models \gamma$ must be also true since that by assuming that $(a \wedge \beta)$ are true we have that α is true, which entails $a \models \gamma$.

ii.) Not generally valid, we have a counterexample for $\alpha = A, \beta = B, \gamma = A \wedge B$. It's certain that $(A \wedge B) \models (A \wedge B)$, but not that $A \models (A \wedge B)$.

iii.) Not generally valid, we have a counterexample for $\alpha = A, \beta = \neg A, \gamma = A$. In fact, assuming that $\alpha \models (A \vee \neg A)$ (the RHS is a tautology so it's always true) it's certain that $\alpha \models \neg\alpha$ is false.

Exercise b)

i.) True, clearly a taulogy.

ii.) Neither, it can be satisfiable. For example, when Smoke and Fire are both true.

iii.) By using some known equivalencies, we have that the formula is equivalent to Smoke $\vee \neg$ Fire which is not valid but satisfiable.

iv.) Valid, since that Fire $\vee \neg$ Fire is clearly a tautology.

v.) Valid, by transforming the LHS and the RHS with known rules we have a formula of type

$$(\neg A \vee \neg B \vee C) \iff (\neg A \vee \neg B \vee C)$$

(where A is Smoke, B is heat and C is fire) Which is obviously valid.

vi.) Likewise, by transforming the implications we have a form of type

$$(A \vee \neg A \vee \neg B \vee C) \wedge (\neg C \vee C \vee \neg A \vee \neg B)$$

which is valid, as both of the clauses are trivial (i.e. contain tautologies).

vii.) Valid, by transforming the implications we get the form Big $\vee$ Dumb $\vee\neg$ Big $\vee$ Dumb which contains a tautology.

Exercise c)

i.

$$S^{t+1} \iff \big((S^t \wedge a^t) \vee (\neg S^t \wedge b^t)\big) \wedge \big((a^t \wedge \neg b^t) \vee (b^t \wedge \neg a^t)\big)$$

The LHS (with respect to the central $\wedge$ operator) models the transition towards S^{t+1} according to the action taken, the RHS models the fact that the agent must take only one action at the time step t

We can simplify the second part by assuming that $a^t \equiv \neg b^t$, giving us the form

$$S^{t+1} \iff (S^t \wedge a^t) \vee (\neg S^t \wedge b^t)$$

ii. Using distributive and De Morgan's laws, we have the form

$$S^{t+1} \iff (S^t \vee b^t) \wedge (a^t \vee \neg S^t)$$

By doing further transformations, we obtain the final CNF composed of four clauses:

1. $\neg S^{t+1}, S^t \neg a^t$

2. $\neg S^{t+1}, \neg S^t, a^t$
3. $S^{t+1}, \neg S^t, \neg a^t$
4. S^{t+1}, S^t, a^t

Q2. FOL

iii.) (Occupation(SURGEON, EMILY) $\wedge \neg$ (Occupation(LAWYER, EMILY)) $\vee$ (Occupation(LAWYER, EMILY) $\wedge$ $\neg$ Occupation(LAWYER, EMILY)

iv.) OCCUPATION(ACTOR, JOE) $\wedge$ ($\exists$ O, O $\neq$ ACTOR $\wedge$ OCCUPATION(P, JOE))

v.) $\forall$ P, OCCUPATION(SURGEON, P) $\implies$ OCCUPATION(DOCTOR, P)

vi.) $\neg\exists$ P, OCCUPATION(LAWYER, P) $\wedge$ CUSTOMER(P, JOE)

vii. $\exists$ P, OCCUPATION(LAWYER, P) $\wedge$ BOSS(P, EMILY)

viii. $\exists$ P, OCCUPATION(LAWYER, P) $\wedge$ ($\forall$ P', CUSTOMER(P', P) $\implies$ OCCUPATION(P', DOCTOR))

ix. $\forall$ P, OCCUPATION(P, SURGEON), ($\exists$ P' OCCUPATION(P', LAWYER) $\wedge$ CUSTOMER(P, P'))

Q3. LOCAL SEARCH

a) HC

i. False, it can get stuck on local minimas which might not represent effective solutions at all

ii. False, It cannot if it's not complete then it cannot be optimal

b) SA

i. True, by the properties of the function $\exp(\Delta E/T)$ where ΔE is negative.

ii. If A or B were to be picked they would have 1.0 probability of being chosen as they have positive energy difference. C has a probability of 0.5 of being the successor if they were randomly chosen. Normalizing all of the probabilities, we obtain that A, B have probabilities of 0.4 each in being the successor, while C has the probability 0.2.

Summary: p(A)=p(B)=0.4, p(C)=0.2

iii. True, it's proven that simulated annealing is optimal

c) LBS

States A and B will be expanded before C and D, since the initial "beams" will greedily pick A,B first

d) GA

Both are true by definition of genetic algorithm.

e) GD

i. False, it can get stuck on plateaus or local minimas (unless the objective function is convex)

ii. $x_{k+1} = x_k - \alpha f'(x_k)$