Assignment 1 - Inverse Problems

Linear Tomography

We shall analyze first-arrivals of acoustic waves from two sources, recorded by 12 detectors at the surface of an object. We will consider a 13×11 m vertical cross section of the object, located immediately below the detectors.
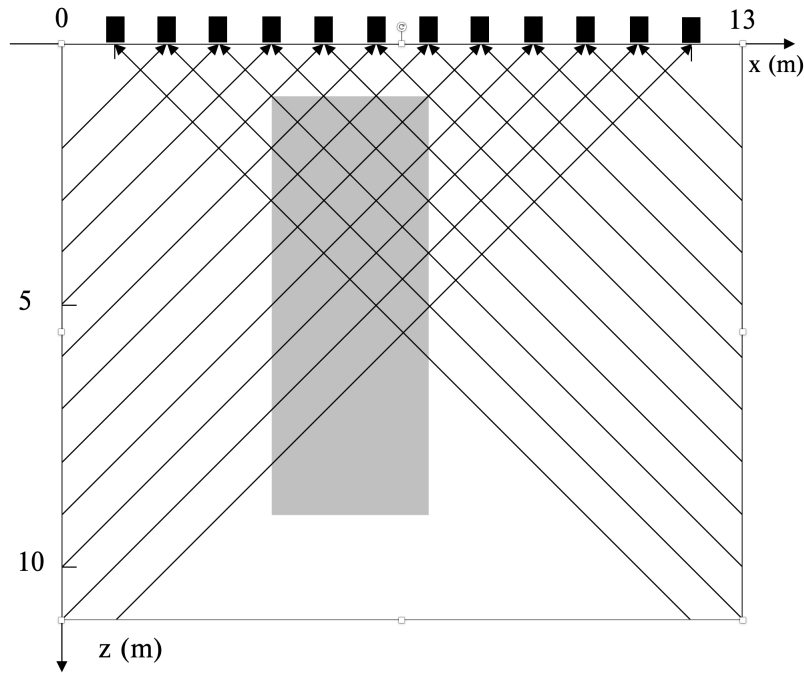

Figure 1: The geometry of the problem. 12 detectors (black squares) are deployed on a line with a 1 meter spacing. Rays from 2 sources hit the detectors with an angle of incidence of 45 degrees. Rays from source 1 come "from left", and rays from source 2 come "from right".

The direct problem (the forward problem)

Let us simplify the problem by assuming a homogeneous object outside the considered 13×11 m area, with a wave propagation velocity of $v = 5$ m/s. Inside the area, the wave propagation velocity is also 5 m/s, except in the central, rectangular zone (grey on the figure), where the velocity is 5.2 m/s. The presence of the grey zone introduces an arrival-time anomaly (a deviation of the arrival time from a situation where the entire 13×11 m area is homogeneous

with wave propagation velocity 5 m/s, that is, without the grey zone). The arrival-time anomaly t_γ for a wave propagating along a ray γ, is given by

$$t_\gamma = \int_\gamma s(u)du \tag{1}$$

where $s(u)$ is the slowness anomaly (the anomaly of the reciprocal velocity) along the ray, and u is a parameter determining the point we consider on the ray. In the following we will assume that all rays are straight lines (and hence independent of the velocity field) as shown on the figure. This assumption is reasonable when the velocity variations in the subsurface are small.

1. Calculate arrival-time anomalies for each of the 12 detectors, for each of the sources.

2. Discretize Equation (1), e.g., by subdividing the area in the figure in 13×11 squares of 1 by 1 m.

The inverse problem

We will here simulate the observed data vector $\mathbf{t}^{obs}$ by adding a noise vector $\mathbf{n}$ to the "pure" data vector, calculated in question 1:

$$\mathbf{t}^{obs} = \mathbf{t}^{pure} + \mathbf{n} \tag{2}$$

where $\mathbf{n}$ has independent, normal distributed components with mean value 0 s and

$$\|\mathbf{n}\|_2 = \frac{1}{18}\|\mathbf{t}^{pure}\|_2 . \tag{3}$$

Subscript "2" after the norm symbol means that it is the usual "Euclidian" 2-norm. We wish to simulate that the unknowns are the 13×11 slowness anomaly values in the area. These values are arranged in one long vector $\mathbf{s}$, to be estimated from the "observed" arrival-time anomalies $\mathbf{t}_{obs}$.

3. Formulate the discrete inverse problem.

4. Show that the problem is linear.

5. Determine if the solution to the problem is unique or not, by looking at the relation between data and model.

6. Find a solution to the problem using the *Tikhonov Regularization*. The solution must be physically acceptable, give the best possible resolution of the model, but, at the same time, "barely fit the data within the noise".

7. Describe how well the model is resolved by the inverse operator. Plot for instance how a delta function (zero everywhere, except in a single 1×1 m square) in the true model is reconstructed in the estimated model.

8. Discuss the differences between your solution and the true model.