Scientific computing assignment 4 part 1

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1 Qualitative analysis of the graph

In this section I will predict the behaviour of the curves in order to check if my results are on the right track.

The four equations that describe the system are:

$$\frac{dx_1}{dt} = a_1x_1(p_1 - x_1) + a_2x_2(p_1 - x_1) \tag{1}$$

$$\frac{dx_2}{dt} = b_1x_1(p_2 - x_2) + b_2x_2(p_2 - x_2) + b_3y(p_2 - x_2) \tag{2}$$

$$\frac{dy}{dt} = c_1x_2(q - y) + c_2z(q - y) \tag{3}$$

$$\frac{dz}{dt} = d_1y(r - z) \tag{4}$$

From top to bottom they describe the infection rate of the homosexual males, the rate of infection of the bisexual males, the rate of infection of the heterosexual females and lastly the rate of infection for the heterosexual males with the parameters $a_1, a_2, b_1, b_2, b_3, c_1, c_2, d_1$ scaling the o infection rate between differnt populations.

As it is given, we start by having only a small amount of homosexual males infected, so this will be the "leading curve" which will start dragging the others. The only ones who can get infected by the homosexual men are the bisexual men so this is the curve which expected to start rising right after (can be seen on figure 1). Homosexuals and bisexuals (mem) both can get infected by their own population, but their populations are small so the first term of the first two equations will be also small and the rise will not be significant because of self mixing.

The next population which will start rising are the heterosexual women (can be seen on figure 2). The difference with the two previous populations is that they have 20 times larger population so the infection because of self mixing will be more important.

Lastly, heterosexual men, who are only getting infected by heterosexual women, will get infected and have a fast increase.
In figure 3 we see how the curves are positioned in relation to the others.

After enough time has passed all the population should be infected so I should have 100 infected heterosexual women and men and 5 bisexual and homosexual men.
For comparison I used the results of runge-kutta but it performs similarly to forward Euler. The graphs seem to agree with the differential equations.

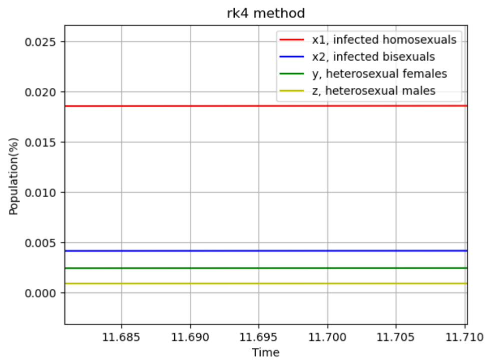

Figure 1: Populations during the phase that we start having infected bisexual males

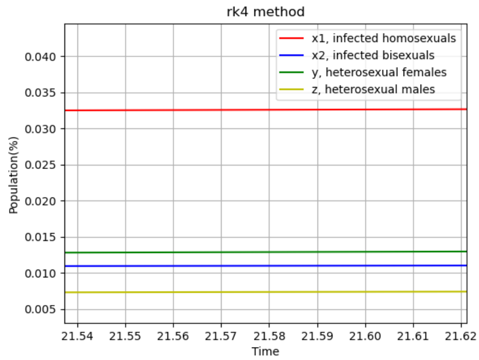

Figure 2: Populations right after infected heterosexual women are more than bisexual men

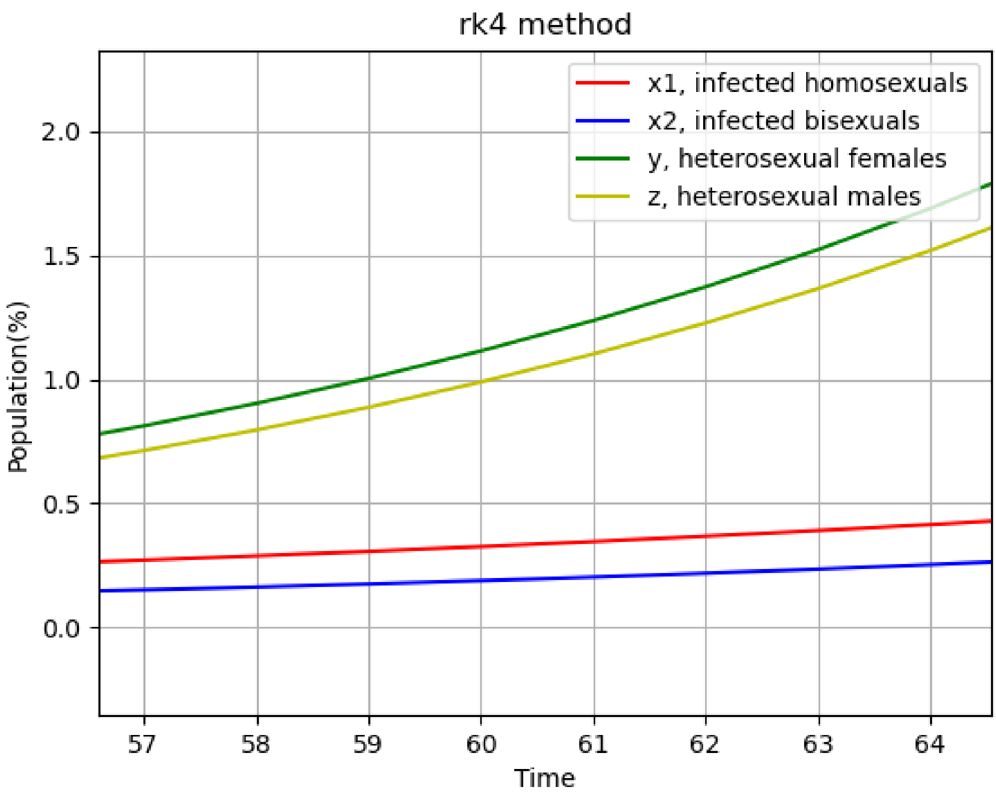

Figure 3: Infected populations right when they start rising

2 Introducing the effects of blood transfusions

With the inclusion of blood transfusion, we make all our populations being mixed no matter the sexual orientation. We expect a high value for e will make every population rise faster. For the calculation of e I used the given expression

$$e = 0.01 * frac \tag{5}$$

$$frac = \frac{x_1 + x_2 + y + z}{p_1 + p_2 + q + r} \tag{6}$$

The effects of the $e = 0.01 * frac$ parameter are not instantly obvious, since it is a fairly small value. However, upon increasing the value constant that scales the frac variable we see that the effects become visible.In the graph I used a high enough value that it will be visible ($e = 20 * frac$)

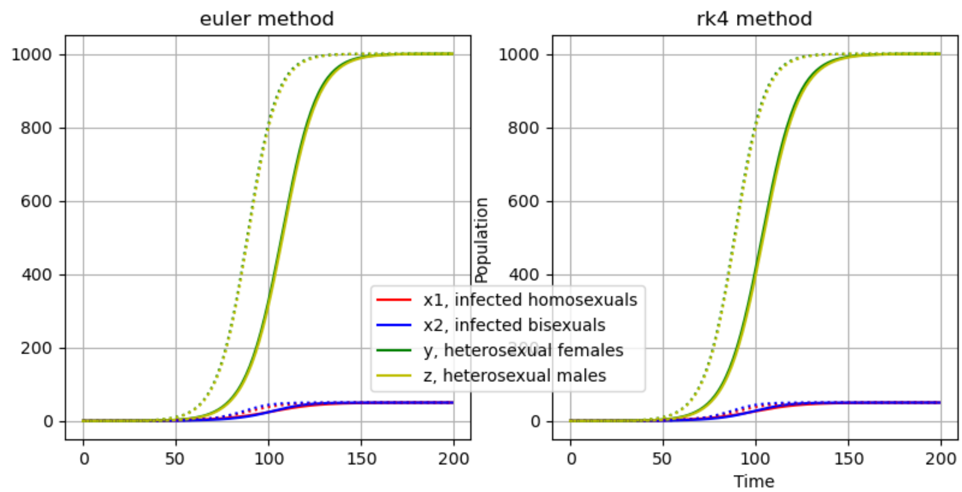

Figure 4: Dotted line corresponds to the model with the blood transussion

It is apparent that unpon increase of the transfusion parameter, the rate of infection becomes bigger, as the virus spreads faster because another way of infecting it is introduced and its contribution parameter is being increased. almost at the same time z, and finally x1. This is a significant difference between the previous case and this. That 'critical' value of e signifies the point at which blood transfusions become a more significant way of transmission of the virus than sexual contact, since the group which was leading the infection rate is now increasing more or less in a similar way with the other groups. A more step by step increase of the transfusion parameter around that specific value is shown below, using Euler's method.

3 Removal due to death

Finally, with the introduction of the removal parameters, the transmission of infection becomes lower and lower, as the parameters are increased. In extreme cases (r = 10) we see that there is a significant rise in the number of casualties.

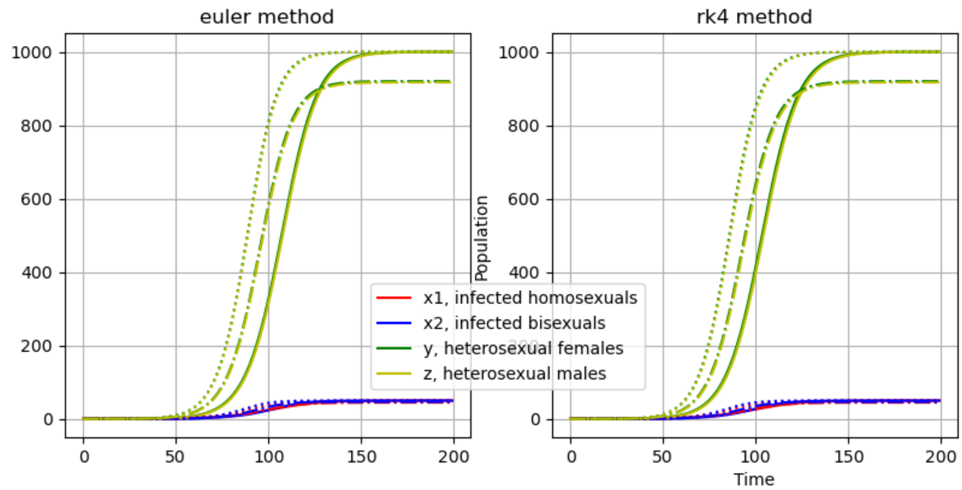

Figure 5: Dashed lines correspond to the model which includes removal because of game over as well as the blood transfussion and the dotted lines corresponds to the model with only blood transussion

But there is something wrong with this graph, and that is that although the populations are still infected they have stopped dying. That is because by just adding the extra term on the equation we don't update the populations (they are reduced because of the casualties) and the equations think that there is still uninfected population. What we see is the equilibrium where we have an infinite supply of people that fill the space of the gone ones. In reality after a sufficient amount of time all the populations should be extinct because there is nothing stopping them from dying. The only exception is if the death rate is high enough that there is not enough time to infect everyone.

In order to make the curves more realistic I added 4 extra differential equations that describe the reduction of the population because of the deaths.

$$\frac{dp_1}{dt} = p_1 - (r_1 x_1) \tag{7}$$

$$\frac{dp_2}{dt} = p_2 - (r_2 x_2) \tag{8}$$

$$\frac{dq}{dt} = q - (r_3 y) \tag{9}$$

$$\frac{dr}{dt} = r - (r_4 z) \tag{10}$$

The final graphs are shown in figure 6 and we can see that indeed the populations are disappearing because there is no function that makes the people cured.

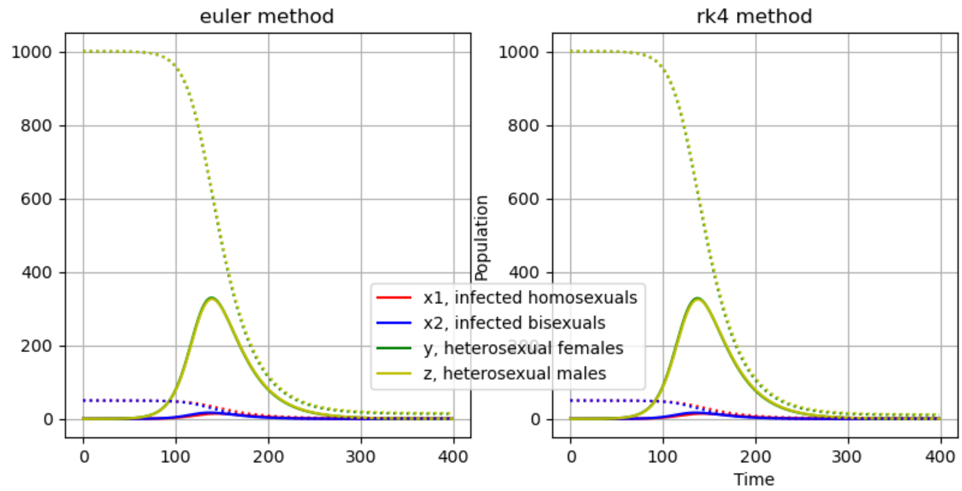

Figure 6: In this graph I have included the reduction of the populations depicted with the dotted line

In the figure 6 I used $e = 20 * frac$ and $r = 50$ and we can see that there are some survivors that did not get infected. For even bigger values of r (or lower value of e) I will have even more survivors.

4 Final thoughts

In every model I created I used both of the euler and rk4 method as can be seen from the graphs. In all of them there is not a noticeable difference as long as we have a small enough timestep(I used timestep of 0.001). Forward euler starts diverging from the correct solution if we make the step 0.01 or bigger where Runge-Kutta still has respectable results. If I had to point out a difference this would be that rk4 method rises faster than the backward Euler probably because it takes into account the curvature of the graph.