Assignment 4 part 2

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1 Introduction

In this assignment we are solving 2 coupled partial differential equations, in particular the reaction-diffusion equations.

$$p_t = D_p \Delta p + p^2 q + C - (K+1)p \tag{1}$$

$$q_t = D_q \Delta q - p^2 q + Kp \tag{2}$$

The method we are applying is the finite difference approximation, which means that the derivatives will be approximated.

In our two equations, the only derivatives we have is in the Laplacian so we will calculate by taking the differences of the adjacent sites of the one that we are calculating.

$$\Delta p[i,j] = \frac{(p[i-1,j] + p[i+1,j] + p[i,j-1] + p[i,j+1] - 4p[i,j]}{dxdy} \tag{3}$$

where dx and dy are the spatial step sizes of the grid which are considered to be 1

2 No-flux Neumman boundary conditions

We have to make sure that my matrix has no flux from the boundaries which translates in having zero first derivative. To do so I introduced one extra column and row in each end which have the same value as the second to last row and column.

3 Solving the equations

The equations are solved for different values of K ranging from 7 to 12. The simulation with Neumann's boundary conditions run for 2000 time steps depict the values of q and p at t=2000. As initial conditions I have a square in the middle of my matrix where q has 4.6 and q has values ranging from 1,75 to 2,87 depending on the value of K.

4 Comment on the results

In both equations the last terms can be explained as something that p gives to q in every step and the second to last as something that q gives to p. These two terms make our system more complex than just having a diffusion and make our equations communicate.

When changing K the initial condition change, where larger K means largest initial q. When q and K are larger the last two terms become more significant.

Overall, an increased value of K seems to make the system evolve faster and more complex because the last two terms become more significant. This can also be seen on the graphs that follow, where for larger K we have somewhat more complex patterns emerging.

5 Results

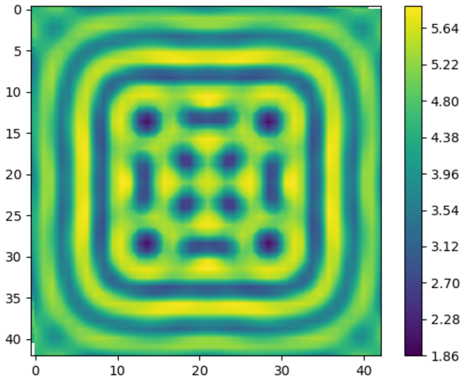

Figure 1: p for K=7

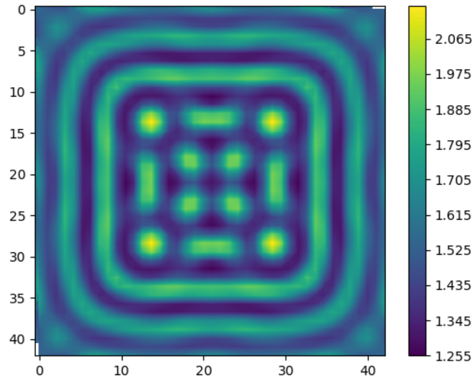

Figure 2: q for K=7

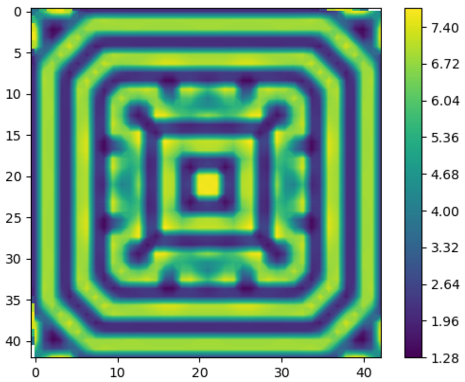

Figure 3: p for K=8

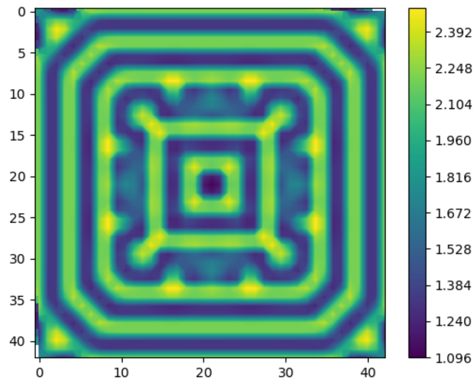

Figure 4: q for K=8

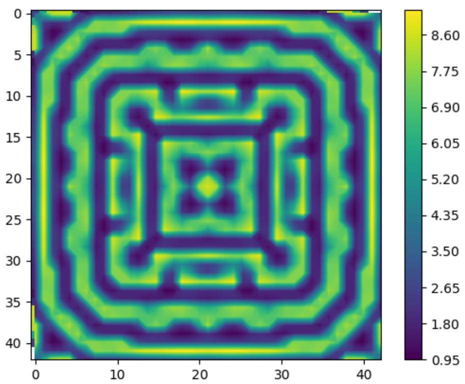

Figure 5: p for K=9

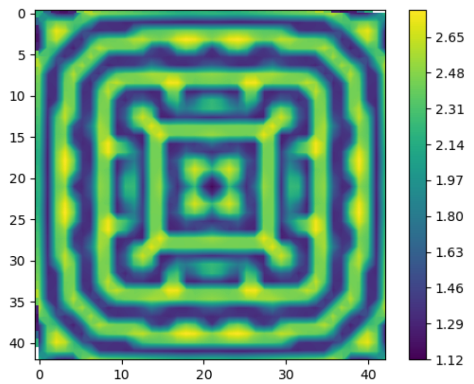

Figure 6: q for K=9

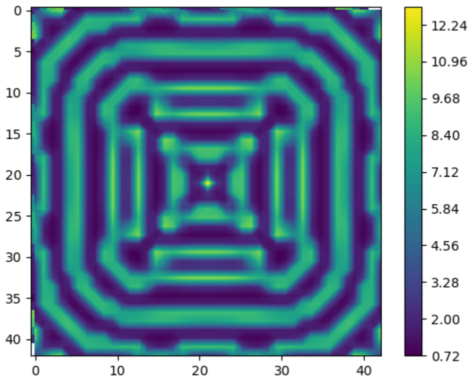

Figure 7: p for K=10

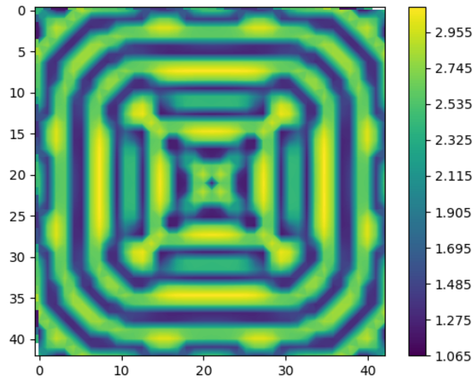

Figure 8: q for K=10

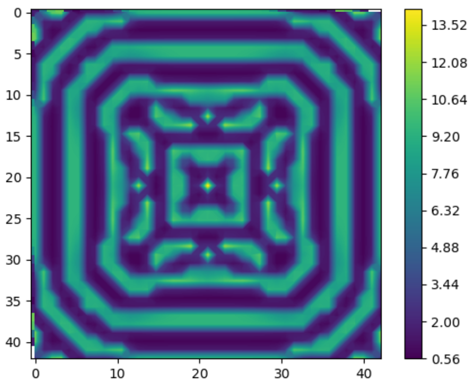

Figure 9: p for K=11

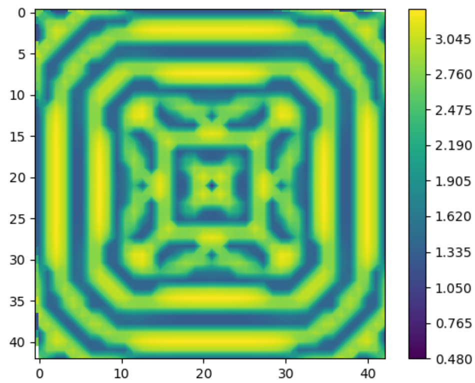

Figure 10: q for K=11

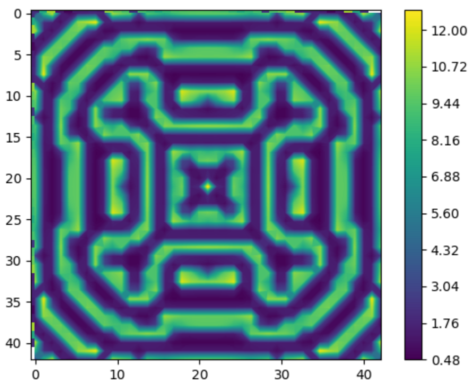

Figure 11: p for K=12

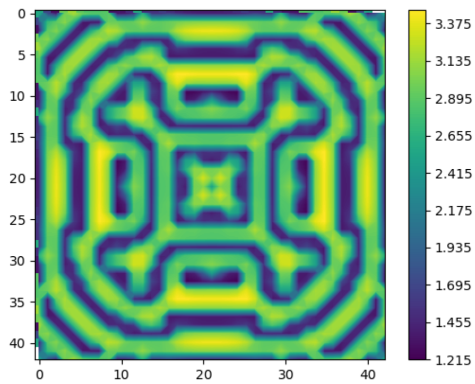

Figure 12: q for K=12