Complex Physics Assignment 2

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October 2022

1 Avalanche size for L=25 grid

The steady state condition means that on average one avalanche has to provide sufficiently many toplings to transport one grain to the boundary and subsequently out of the system. This means that the mass in the grid stays constant with additional grains of rice added to the system . The red line corresponds to the mean value of the mass on each tile which value fluctuates around 0.67. This steady state is reached after 450-500 grains of sand added to the system. 1

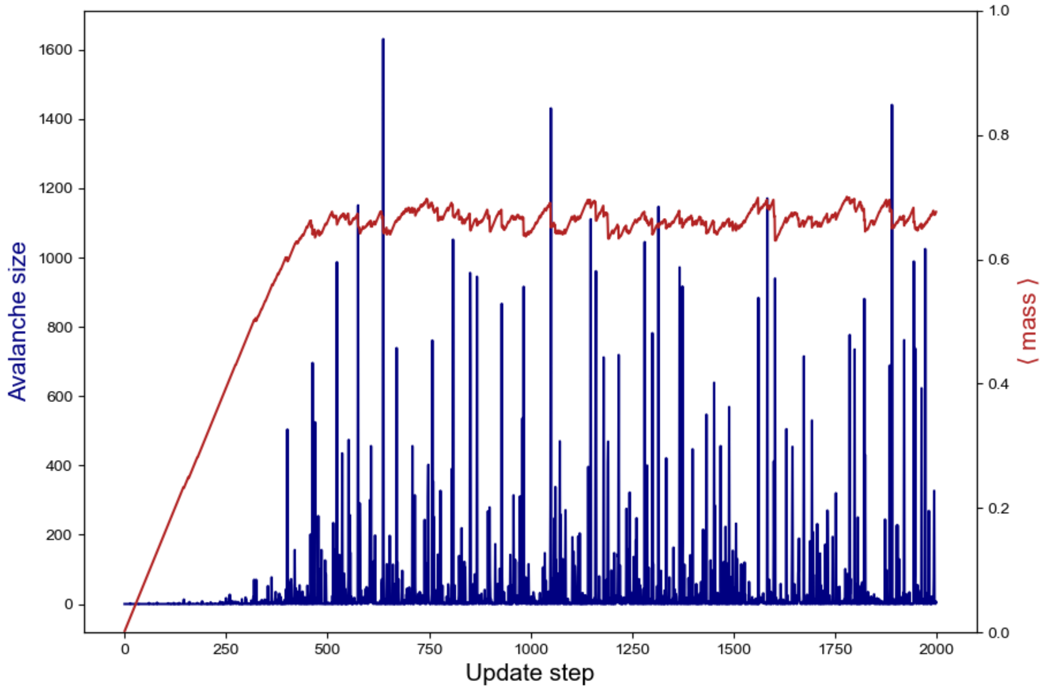

Figure 1: Avalanche size as a function of time

2 Avalanche size distribution

The size distribution follows a combination of exponential and power law

$$P(s) = \frac{1}{s^\tau} e^{s/L^D} \tag{1}$$

The exponential part becomes important only for very big size avalanches so the flat part can be described only by the power law. I used 1 million grains of sand randomly distributed in the system. In order to calculate how many updates I should to reach a steady state I repeated question 1 for all the sizes. After plotting and fitting the power law I get a value for $\tau = 1.21$ which is the negative of the slope.

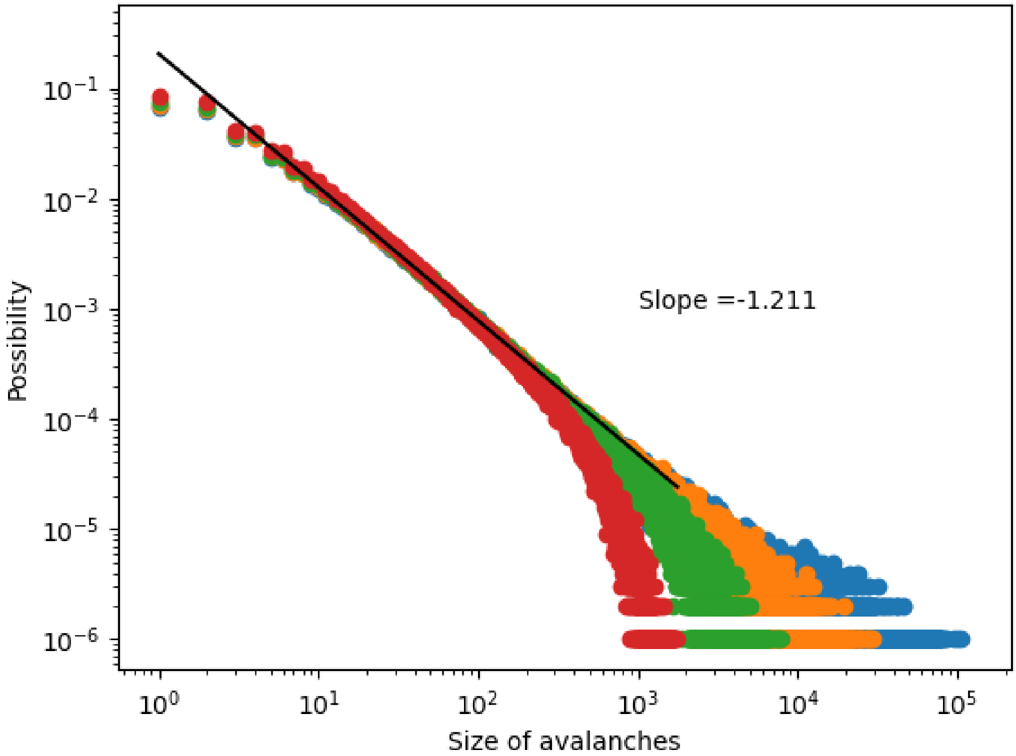

Figure 2: Probability of an avalanche to occur. Red is size 25, green is size 50, orange is size 100, blue is 200

We can see clearly that for smallest size grids the plot behaves better. That is because for largest sizes we have bigger avalanches so we need way more data to fill the distribution. In order to calculate the cut-off point I used a different plot so it will be easier to visualize that point. That graph has for the x-axis x/L^D and for y-axis yx^τ and is plotted on figure 3. For the x-axis I used the dimension around 2.4. The cut-off point is where the plot starts going down and we can find the size avalanche size that corresponds to that point by solving the system of its coordinates.

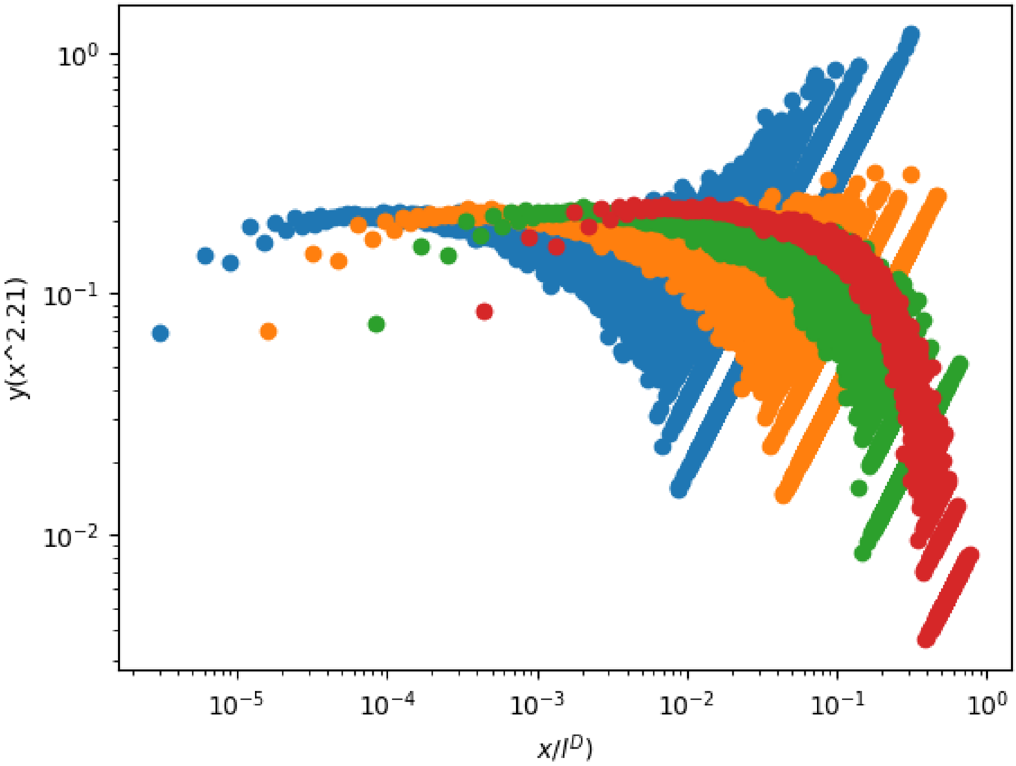

Figure 3: Probability of an avalanche to occur. Blue is size 200, orange is size 100, green is size 50, red is 25

3 Dimension calculation

In that particular model the dimension is how much the size of the avalanche has grown in order to expand to a certain region on the grid. For that reason, using a small grid will not be any good to our calculations, so I used the biggest one that my laptop could handle in a reasonable amount of time. In order to calculate the dimension I fitted a power law. The grid I used is with L=100.

The result of D = 2.41 agrees with the formula that connects the τ and the dimension of the avalanches.

$$\tau = 2 - \frac{2}{D} \tag{2}$$

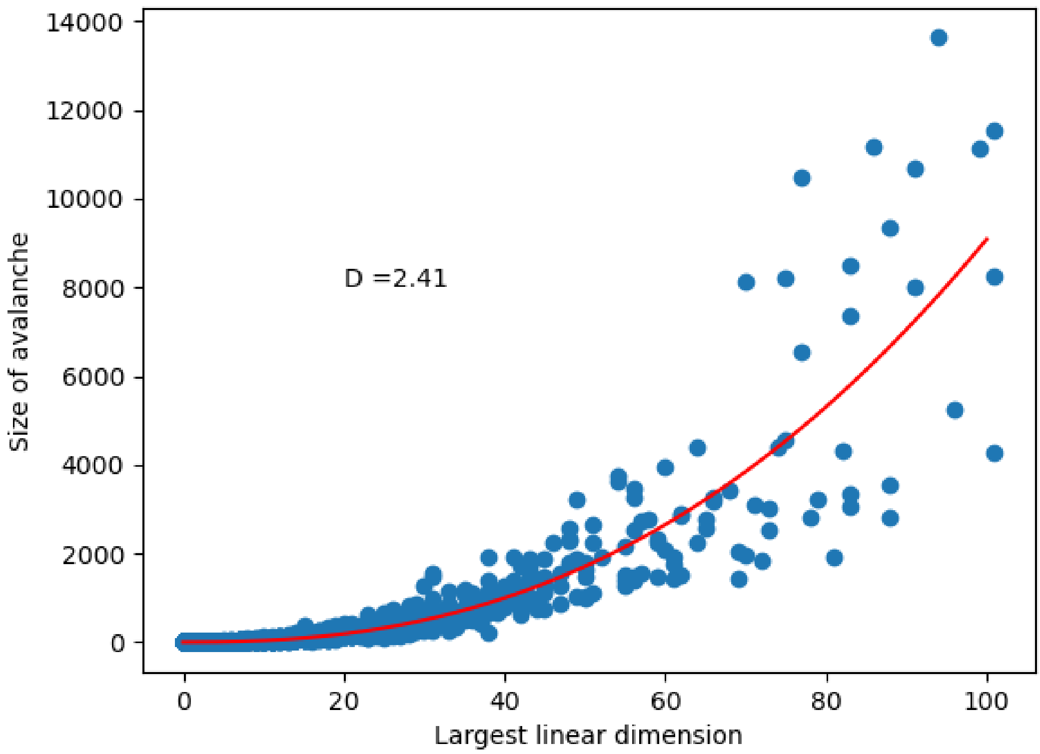

Figure 4: Avalanches largest linear dimension as a function of the size

4 Border filling

This time we simulate the system where the grains are dropped randomly on the border of the grid. In this model the grains will be exiting the grid way faster than the previous one because they are right next to the border. I expect to have more smaller probability for big avalanches and larger probability for small avalanches. Considering that we expect a bigger value for τ in the power law. Following exactly the same steps as question 2 I came up with the following plot shown: I found a value of $\tau = 1.743$ as can be seen from the slope of the straight part of the plot which agrees with my hypothesis

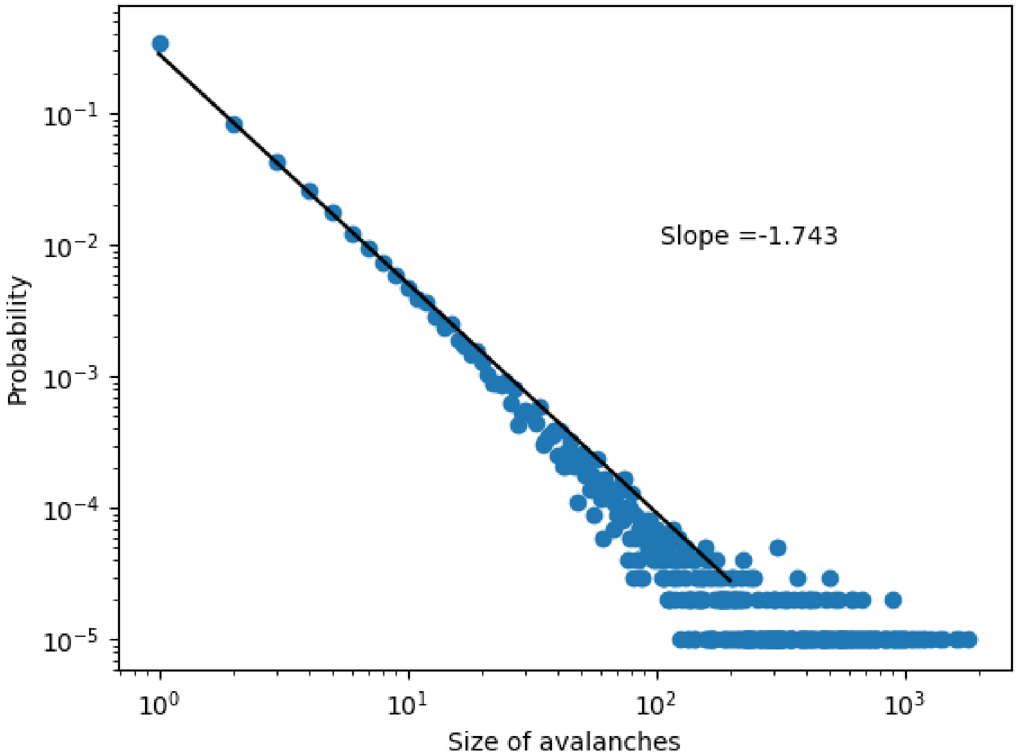

Figure 5: Avalanches largest linear dimension as a function of the size

5 Border filling

The first return for a single grain of sand dropped in the border of the grid is given by the formula:

$$\int_0^{L^2} \frac{tdt}{t^{3/2}} = L \tag{3}$$

The mean size of an avalanche can also be calculated by the sum of all the avalanche sizes multiplied by their probability.

$$\int_0^{\infty} sP(s)ds = \int_0^{L^D} s^{1-\tau} ds = L^{D(2-\tau)} \tag{4}$$

But this is equal to the first return so

$$L = L^{D(2-\tau)} \tag{5}$$

$$\tau = 2 - \frac{1}{D} \tag{6}$$

For $\tau = 1.743$ we get a dimension of 3.89 which is way bigger then the dimension we got when we placed the sand randomly in the grid. This happened because by placing the grains of sand in the edge we need way more toplings to spread over a certain region than placing the grains somewhere in the center of the grid.