Notebook DM1

November 17, 2022

[3]:
```
# import the necessary libraries
import pandas as pd
import numpy as np
from random import *
from math import *
```

[4]:
```
# import the data
df=pd.read_table('acc_usines.txt', header=None, names=["num_acc"])
df.head()
```

[4]:

	num_acc
0	4
1	5
2	4
3	1
4	0

[5]:
```
y=np.array(df.num_acc) # convert the data into array
T=len(y)  # length of data
```

y contains our data. In the next cells, we will be using the following notations in code :

l denotes λ, le point de changement

d: denotes δ, the poisson parameter if t>l

g: denotes γ, the poisson parameter if t<=l

1 Utility Functions

$S_k = \sum_{t=1}^{k} y_t$

[41]:
```
# for l=k, calculate the sum of y_t from 1 til k
#this sum appears in the conditional distribution of l
#please refer to the excercice solution
def s(k:int):
    s=0
    for i in range(k+1):
        s+=y[i]
    return s        # returns a sum
```

$\bar{S}_k = \sum_{k+1}^{T} y_t$

[42]:
```
# for l=k, calculate the sum of y_t from k+1 til T
#this sum appears in the conditional distribution of l
# please refer to the exercice solution
def s_barre(k:int):
    s_barre=0
    for i in range(k+1,(len(y))):
        s_barre+=y[i]
    return s_barre             # returns a sum
```

$\mathbb{P}\left(\lambda = k \mid \gamma, \delta, y\right) = \frac{\exp(-k(\gamma-\delta)\gamma^{S_k}\delta^{\bar{S_k}}}{\sum_{k=1}^{T-1}\exp(-k(\gamma-\delta)\gamma^{S_k}\delta^{\bar{S_k}}}$

[10]:
```
# la loi de probabilité marginale de lambda(l) sachant gamma(g), delta(d) et y
# c'est la posteriori de l sachant les autres paramètres
def P(l,d,g):
    nom=0
    for j in range(1,len(y)):
        nom+=(np.exp(-j*(g-d)))*(g**(s(j)))*(d**(s_barre(j)))

    p=(np.exp(-l*(g-d)))*(g**(s(l)))*(d**(s_barre(l)))
    proba_cond_lambda=(p/nom)

    return proba_cond_lambda
```

[12]:
```
_lambda=range(1,len(y))         # the range the possible values of lmabda(l)
```

[13]:
```
# retourne une liste de probalités de chaque valeurs de lambda sachant g,d et y
# utile pour la simulation de la loi de l
def list_proba_cond_lambda(d,g):
    list_proba_cond_lambda=[]
    for e in _lambda:
        list_proba_cond_lambda.append(P(e,d,g))
    return list_proba_cond_lambda
```

[14]:
```
# retourne un entier entre 1 et T-1
# la focntion simule la loi de l
def simul(valeurs,probas):
    assert len(valeurs)==len(probas)
    nb=random()
    curseur=0
    for i in range(len(valeurs)):
        if curseur<=nb and nb<curseur+probas[i]:
            return valeurs[i]
        else:
            curseur+=probas[i]
```

2 Algorithme de gibbs pour la simulation des lois posteriori

$\gamma \mid \delta, \lambda, y \sim G(a_1 + S_\lambda, \lambda + a_2)$

$\delta \mid \gamma, \delta, y \sim G(d_1 + \bar{S_\lambda}, T - \lambda + d_2)$

$\mathbb{P}(\lambda = k \mid \gamma, \delta, y) = \frac{\exp(-k(\gamma-\delta)\gamma^{S_k}\delta^{\bar{S_k}}}{\sum_{k=1}^{T-1}\exp(-k(\gamma-\delta)\gamma^{S_k}\delta^{\bar{S_k}}}$

[15]:
```
def gibbs_sampling(g_0,d_0,l_0,a_1,a_2,d_1,d_2,n_iter):
    sample=np.empty((n_iter,3), dtype=float)
    l=l_0
    for k in range(n_iter):
        L=[]
        g_k=np.random.gamma(a_1+s(l), (1/(l+a_2)))
        d_k=np.random.gamma(d_1+s_barre(l),(1/(T-l+d_2)))
        l_k=simul(_lambda,list_proba_cond_lambda(d_k,g_k))
        L.append(g_k)
        L.append(d_k)
        L.append(l_k)
        sample[k]=L
        l=l_k
    return sample
```

[16]:
```
# taille de l'échantillon 200
# initialisation des paramètres
sample_=gibbs_sampling(0,0,1,0,0,0,0,200)
```

[44]:
```
# as we know, les MCMC convergent vers la vraie distribution quand n_iter tend␣
 ↪vers l'infini
# ici, on a pris les 100 derniers échantillons
sample=sample_[100:]
```

[29]:
```
# représentation des échantillons sous forme d'un dataframe for easy␣
 ↪manipulations
dd=pd.DataFrame(sample,columns=['g','d','l'])
dd.head()
```

[29]:

	g	d	l
0	3.479424	0.931949	35.0
1	3.528910	0.937035	35.0
2	3.616757	0.858208	41.0
3	3.201354	0.824543	41.0
4	2.945311	0.857086	38.0

[30]:
```
# génération des statistiques
dd.describe()
```

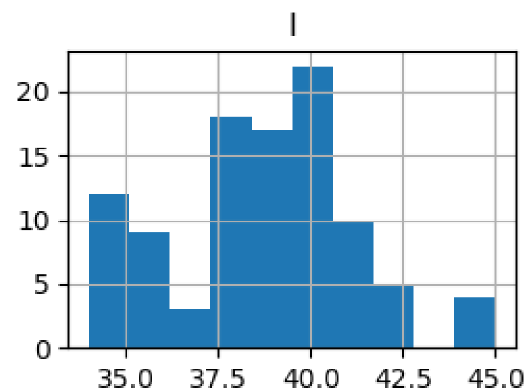
[30]:

	g	d	l
count	100.000000	100.000000	100.000000
mean	3.209230	0.893896	38.780000
std	0.307484	0.106770	2.389286
min	2.462608	0.604916	34.000000
25%	2.987881	0.823685	38.000000
50%	3.199654	0.894752	39.000000
75%	3.412206	0.962149	40.000000
max	4.211965	1.117734	45.000000

3 Représentation graphiques des postérioris

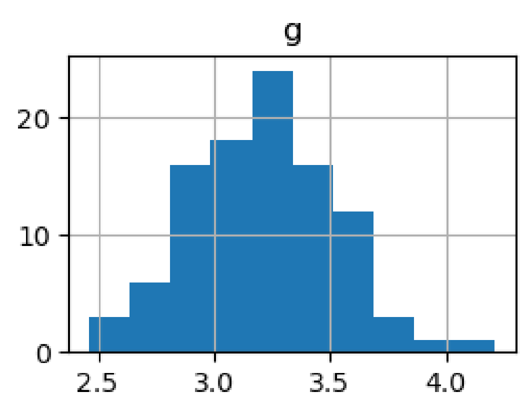
3.1 Postériori de λ

[46]:
```
hist=dd.hist(column='l',figsize=(3,2))
```



3.2 Postériori de γ

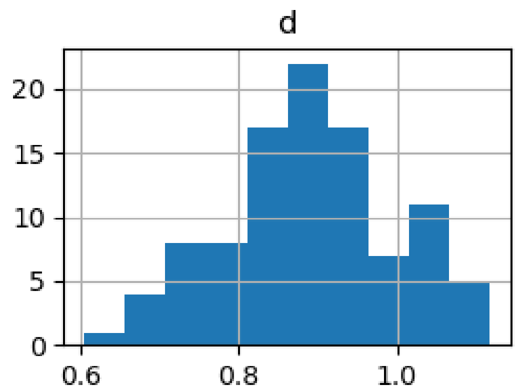
[39]:
```
dd.hist(column='g',figsize=(3,2))
```



3.3 Postériori de δ

[40]:
```
dd.hist(column='d',figsize=(3,2))
```

[40]:



Calcul du nombre d'accidents avant et aprés ce changement ?

[23]:
```
s(39)
```

[23]: 125

[24]:
```
s_barre(39)
```

[24]: 66

[25]:
```
s(41)
```

[25]: 128

[26]:
```
s_barre(41)
```

[26]: 63

[27]:
```
s(37)
```

[27]: 120

[28]:
```
s_barre(37)
```

[28]: 71

4 Interprétation & Conclusion

D'après les statistiques issues des lois a postériori, le point de changement λ(mean=39, std=2) est situé entre 1888 et 1892. le nombre d'accident avant le point de changement est entre 120 et 128. Ainsi, le nombre d'accident après le point de changement a diminué et situé entre 63 et 71.