Homework 4 Report

Mamdani Fuzzy Inference System for Nonlinear Function Approximation

Introduction

This report presents the implementation and evaluation of a Mamdani Fuzzy Inference System (FIS) to approximate two complex nonlinear functions. The Mamdani approach was chosen due to its effectiveness in modeling uncertain and nonlinear systems without requiring precise mathematical models. The modeling was performed in two modes: crisp output (weighted average defuzzification) and fuzzy output (centroid defuzzification). The implementation was done in Python, and model accuracy was assessed using the FVU and PCC metrics.

Target Functions

The two functions to be approximated are defined as follows:

1. Function F1 (sinc-based): $F_1(x_1, x_2) = \sqrt{2\left(\frac{\sin x_1}{x_1}\right)^2 + 3\left(\frac{\sin x_2}{x_2}\right)^2}, 1 \leq x_1, x_2 \leq 10$
2. Function F2 (polynomial): $F_2(x_1, x_2) = (1 + x_1^{-2} + x_2^{-1.5})^2, 1 \leq x_1, x_2 \leq 10$

Implementation Methodology

- **Data Generation:** 800 random samples uniformly distributed in the interval [1, 10] for each input.
- **Data Split:** 70% for training (560 samples) and 30% for testing (240 samples).
- **Input Partitioning:** 7 triangular membership functions for each input variable.
- **Rule Extraction:** Wang-Mendel method (data-driven rule generation based on maximum membership degree and output averaging per grid cell).
- **Inference and Defuzzification:**
  - Crisp mode: Weighted average of rule consequents.
  - Fuzzy mode: Aggregation of output triangular membership functions followed by centroid defuzzification.

Libraries used: NumPy for computations, Matplotlib for visualization, and scikit-learn for data splitting.

Evaluation Metrics

The model performance was evaluated using:

$$FVU = \frac{\sum_{i=1}^{k}(y^e(x_i) - y(x_i))^2}{\sum_{i=1}^{k}(y(x_i) - \bar{y})^2}, \bar{y} = \frac{1}{k}\sum_{i=1}^{k} y(x_i)$$

$$PCC = \frac{\sum_{i=1}^{k}(y_i - \bar{y}) \times (y_i^e - \bar{y}^e)}{\sqrt{\sum_{i=1}^{k}(y_i - \bar{y})^2 \times \sum_{i=1}^{k}(y_i^e - \bar{y}^e)^2}}$$

Numerical Results

Function F1

- Crisp mode: FVU = 0.1272, PCC = 0.9519
- Fuzzy mode: FVU = 0.1456, PCC = 0.9477

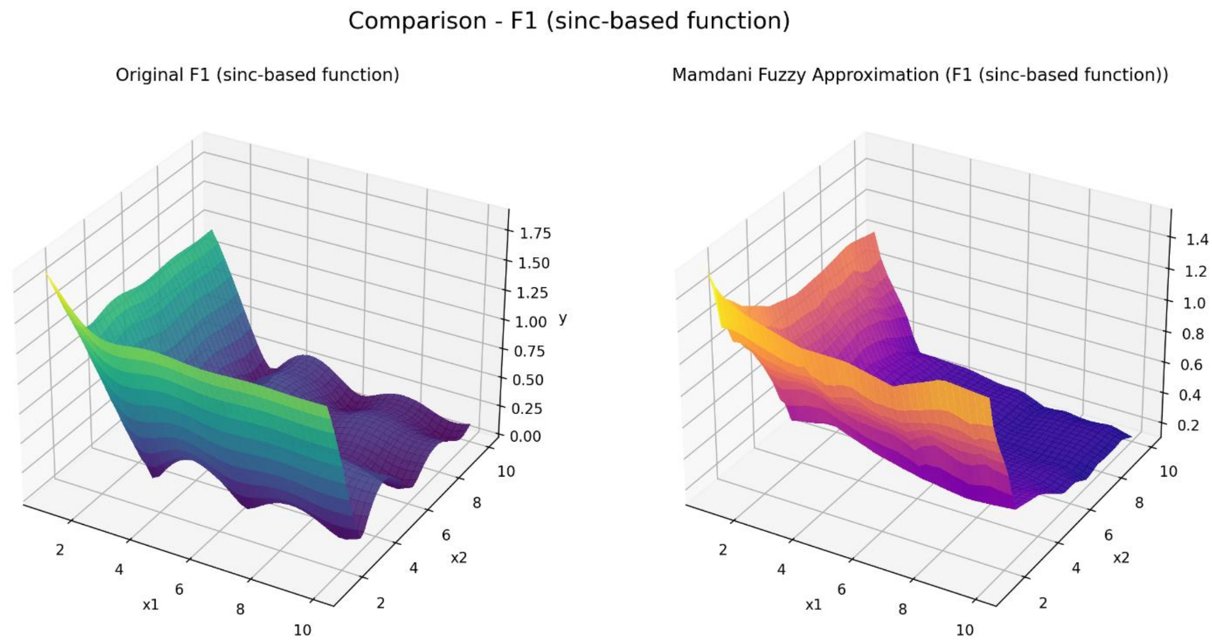
Function F2

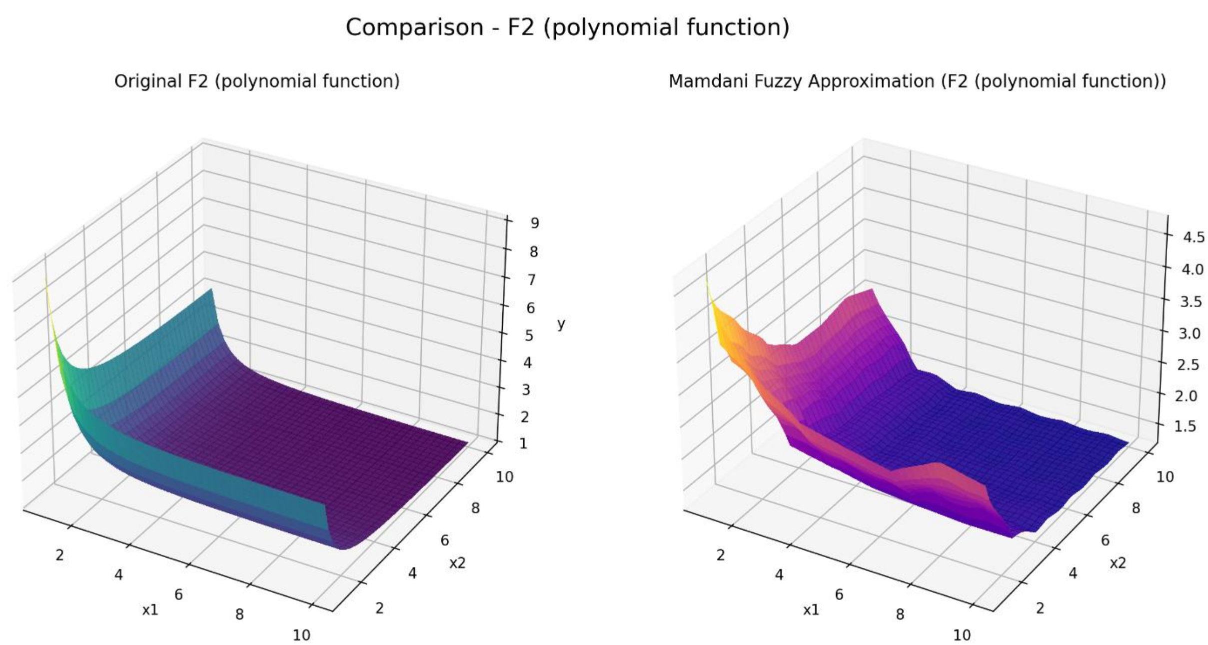
- Crisp mode: FVU = 0.1047, PCC = 0.9488
- Fuzzy mode: FVU = 0.1630, PCC = 0.9437

Analysis: The crisp mode consistently outperforms the fuzzy mode in both functions, likely due to simpler and less error-prone defuzzification. All PCC values above 0.94 demonstrate strong linear correlation between predicted and actual outputs.

Visual Comparison

Three-dimensional surface plots were generated to visually compare the original functions with the fuzzy model approximations.





The fuzzy model successfully captures the overall shape and trends of both functions, though some smoothing is observed in rapidly changing regions (especially in F1 due to sinc oscillations).

Conclusion

The Mamdani fuzzy system with 7 partitions per input effectively approximated both nonlinear functions. The crisp output mode is recommended for practical applications due to its higher accuracy and computational simplicity. Future improvements could include experimenting with different partition counts or incorporating optimization techniques (e.g., genetic algorithms) for rule tuning.