Prioritized Sequent-Based Argumentation

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ABSTRACT

In this paper we integrate priorities in sequent-based argumentation. The former is a useful and extensively investigated tool in the context of non-monotonic reasoning, and the latter is a modular and general way of handling logical argumentation. Their combination offers a platform for representing and reasoning with maximally consistent subsets of prioritized knowledge bases. Moreover, many frameworks of the resulting formalisms satisfy common rationality postulates and other desirable properties, like conflict preservation.

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1 INTRODUCTION

Logical (or structural) argumentation is a branch of argumentation theory in which arguments have a specific structure. Among others, it has been shown useful for reasoning about knowledge, beliefs, goals and norms in agent and multi-agent systems (see, e.g., [18] for a survey and further references). In logical argumentation, arguments are expressed in terms of formal languages and acceptance of arguments is determined by logical entailments. A wealth of research has been conducted on formalizing this kind of argumentation. This includes methods that are based on Tarskian logics, like Besnard and Hunter's approach [10], in which classical logic is the deductive base (the so-called core logic). This approach was generalized to sequent-based argumentation [7], in which Gentzen's sequents [15], extensively used in proof theory, are incorporated for representing arguments, and attacks are formulated by special inference rules called sequent elimination rules. The result is a generic and modular approach to logical argumentation, in which any logic with a corresponding sound and complete sequent calculus can be used as the underlying core logic.

An important feature of reasoning in many contexts, including of course multi-agent systems, is the use of priorities, e.g. to model the agents' preferences. For many existing argumentation frameworks prioritized settings are already available, see, e.g. [2, 13, 19]. The main contribution of this paper is that we extend some of those settings to arbitrary propositional languages and logics, where arguments and the attacks among them are captured in a more moderated way. For this, we extend sequent-based argumentation frameworks with a priority function on the well-formed formulas of the core logic. By keeping the exact definition of the priority function unspecified, we are able to create a general sequent-based framework that can handle different types of preferences, specified in different languages and for various logics and purposes.

The adequacy of this prioritized version is shown by the validity, for particular attack rules, of common rationality postulates [1, 12] and by the fact that in the obtained framework conflicts are tolerated: any extension in the prioritized setting is conflict-free in the flat (i.e, the non-prioritized) case. Moreover, the use of priorities allows us to extend to the preferential case some recent results (see [6, 8]) that sequent-based argumentation frameworks provide a useful platform for representing and reasoning with maximally consistent subsets of the premises [22].

The usefulness of our approach will be demonstrated (among others) on the following toy example (to which we shall return in the conclusion of the paper), involving agents and preferences.

Example 1.1. [5, 17] An agent, representing a flat owner, negotiates the construction of a swimming pool (s), a tennis-court (t) and a private car-park (p) with other agents, representing potential tenants. It is known that any investment in two or more of these facilities will increase the rent (r), otherwise the rent will not be changed. The tenants' representatives do not have a particular preference among these options, but if they have to make a choice, they prefer not to have two sport facilities (s and t) and definitely do not want to increase the rent. Based on these inputs, that flat owner's representative needs to reach a recommendation about the facility (or facilities) to be constructed.

The remainder of the paper is organized as follows: the next section is a survey of the most important notions of sequent-based argumentation, followed by a section in which the general setting for the preferences is introduced. In Section 4 we consider some basic properties of the prioritized frameworks and show their adequacy for defeasible reasoning. Then, in Section 5, we give some representation results in terms of maximally consistent subsets of the premises. In Section 6 we consider some related approaches and conclude.

2 SEQUENT-BASED ARGUMENTATION

Throughout the paper we will consider propositional languages, denoted by $\mathcal{L}$. Atomic formulas are denoted by p, q, formulas are denoted by $\gamma, \delta, \phi, \psi$, sets of formulas are denoted by $\mathcal{S}, \mathcal{T}$, and finite sets of formulas are denoted by Γ, Δ, all of which can be primed or indexed.

Definition 2.1. A *logic* for a language $\mathcal{L}$ is a pair $\mathsf{L} = \langle \mathcal{L}, \vdash \rangle$, where $\vdash$ is a (Tarskian) consequence relation for $\mathcal{L}$, having the following properties: *reflexivity:* if $\phi \in \mathcal{S}$, then $\mathcal{S} \vdash \phi$; *transitivity:* if $\mathcal{S} \vdash \phi$ and $\mathcal{S}', \phi \vdash \psi$, then $\mathcal{S}, \mathcal{S}' \vdash \psi$; and *monotonicity:* if $\mathcal{S}' \vdash \phi$ and $\mathcal{S}' \subseteq \mathcal{S}$, then $\mathcal{S} \vdash \phi$.

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We assume that the underlying language $\mathcal{L}$ contains the following connectives:

- a $\vdash$-negation $\neg$: $p \nvdash \neg p$ and $\neg p \nvdash p$, for every atom p,
- a $\vdash$-conjunction $\wedge$: $\mathcal{S} \vdash \phi \wedge \psi$ iff $\mathcal{S} \vdash \phi$ and $\mathcal{S} \vdash \psi$.

Other connectives $\mathcal{L}$ may contain are the following:

- a $\vdash$-disjunction $\vee$: $\mathcal{S} \vdash \phi \vee \psi$ iff $\mathcal{S} \vdash \phi$ or $\mathcal{S} \vdash \psi$,
- a $\vdash$-implication $\supset$: $\mathcal{S}, \phi \vdash \psi$ iff $\mathcal{S} \vdash \phi \supset \psi$.

We shall abbreviate $(\phi \supset \psi) \wedge (\psi \supset \phi)$ by $\phi \leftrightarrow \psi$, denote by $\bigwedge \Gamma$ (respectively, by $\bigvee \Gamma$), the conjunction (respectively, the disjunction) of all the formulas in Γ, and let $\neg \mathcal{S} = \{\neg \phi \mid \phi \in \mathcal{S}\}$.

As usual in logical argumentation (see, e.g., [10, 20, 21, 23]), arguments have a specific structure based on the underlying formal language, the so-called *core logic*. In the current setting arguments are represented by the well-known proof theoretical notion of a *sequent*.

Definition 2.2. Let $\mathsf{L} = \langle \mathcal{L}, \vdash \rangle$ be a logic and $\mathcal{S}$ a set of $\mathcal{L}$-formulas.

- An $\mathcal{L}$-*sequent* (*sequent* for short) is an expression of the form $\Gamma \Rightarrow \Delta$, where Γ and Δ are finite sets of formulas in $\mathcal{L}$ and $\Rightarrow$ is a symbol that does not appear in $\mathcal{L}$.
- An L-*argument* (*argument* for short) is an $\mathcal{L}$-sequent $\Gamma \Rightarrow \psi$,[1] where $\Gamma \vdash \psi$. Γ is called the *support set* of the argument and ψ its *conclusion.*
- An L-*argument based on* $\mathcal{S}$ is an L-argument $\Gamma \Rightarrow \psi$, where $\Gamma \subseteq \mathcal{S}$. We denote by $\mathrm{Arg}_{\mathsf{L}}(\mathcal{S})$ the set of all the L-arguments based on $\mathcal{S}$.

Given an argument $a = \Gamma \Rightarrow \psi$, we denote $\mathsf{Sup}(a) = \Gamma$ and $\mathsf{Con}(a) = \psi$. We say that a' is a *sub-argument* of a iff $\mathsf{Sup}(a') \subseteq \mathsf{Sup}(a)$. The set of all the sub-arguments of a is denoted by $\mathsf{Sub}(a)$.

The formal systems used for the constructions of sequents (and so of arguments) for a logic $\mathsf{L} = \langle \mathcal{L}, \vdash \rangle$, are *sequent calculi* [15], denoted here by C. In what follows we shall assume that C is sound and complete for $\mathsf{L} = \langle \mathcal{L}, \vdash \rangle$, i.e., $\Gamma \Rightarrow \psi$ is provable in C iff $\Gamma \vdash \psi$. One of the advantages of sequent-based argumentation is that any logic with a corresponding sound and complete sequent calculus can be used as the core logic.[2] The construction of arguments from simpler arguments is done by the *inference rules* of the sequent calculus [15].

Argumentation systems contain also attacks between arguments. In our case, attacks are represented by *sequent elimination rules.* Such a rule consists of an attacking argument (the first condition of the rule), an attacked argument (the last condition of the rule), conditions for the attack (the conditions in between) and a conclusion (the eliminated attacked sequent). The outcome of an application of such a rule is that the attacked sequent is ‘eliminated’.[3] The elimination of a sequent $a = \Gamma \Rightarrow \Delta$ is denoted by $\overline{a}$ or $\Gamma \not\Rightarrow \Delta$.

Definition 2.3. A *sequent elimination rule* (or *attack rule*) is a rule $\mathcal{R}$ of the form:

$$\frac{\Gamma_1 \Rightarrow \Delta_1 \quad \ldots \quad \Gamma_n \Rightarrow \Delta_n}{\Gamma_n \not\Rightarrow \Delta_n} \quad \mathcal{R}$$

[1]Set signs in arguments are omitted.
[2]See [7] for further advantages of this approach.
[3]That is, the eliminated sequent should not be used as a condition of later applications of rules in the derivation, nor is it considered a valid conclusion of the derivation.

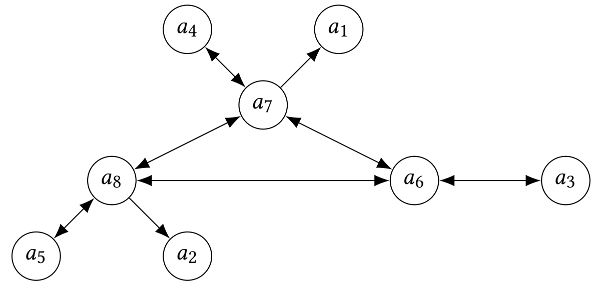

Figure 1: Part of the sequent-based argumentation graph for $\mathcal{S} = \{p, q, \neg p \vee \neg q\}$ from Example 2.6

Let $\Gamma \Rightarrow \psi, \Gamma' \Rightarrow \psi' \in \mathrm{Arg}_{\mathsf{L}}(\mathcal{S})$ and let $\mathcal{R}$ be an elimination rule. If $\Gamma \Rightarrow \psi$ is an instance of $\Gamma_1 \Rightarrow \Delta_1$, $\Gamma' \Rightarrow \psi'$ is an instance of $\Gamma_n \Rightarrow \Delta_n$ and all the other conditions of $\mathcal{R}$ are provable in C, we say that $\Gamma \Rightarrow \psi$ $\mathcal{R}$-*attacks* $\Gamma' \Rightarrow \psi'$.

Example 2.4. We refer to [7, 24] for a definition of many sequent elimination rules. Below are three of them (assuming that $\Gamma_2 \neq \emptyset$):

Undercut (Ucut): $$\frac{\Gamma_1 \Rightarrow \psi_1 \quad \Rightarrow \psi_1 \leftrightarrow \neg \bigwedge \Gamma_2 \quad \Gamma_2, \Gamma_2' \Rightarrow \psi_2}{\Gamma_2, \Gamma_2' \not\Rightarrow \psi_2}$$

Direct Ucut (DUcut): $$\frac{\Gamma_1 \Rightarrow \psi_1 \quad \Rightarrow \psi_1 \leftrightarrow \neg \gamma \quad \gamma, \Gamma_2' \Rightarrow \psi_2}{\gamma, \Gamma_2' \not\Rightarrow \psi_2}$$

Consistency Ucut (ConUcut): $$\frac{\Rightarrow \neg \bigwedge \Gamma_2 \quad \Gamma_2, \Gamma_2' \Rightarrow \psi}{\Gamma_2, \Gamma_2' \not\Rightarrow \psi}$$

A sequent-based framework is now defined as follows:

Definition 2.5. A *sequent-based argumentation framework* for a set of formulas $\mathcal{S}$ based on a logic $\mathsf{L} = \langle \mathcal{L}, \vdash \rangle$ and a set AR of sequent elimination rules, is a pair $\mathcal{AF}_{\mathsf{L},\mathsf{AR}}(\mathcal{S}) = \langle \mathrm{Arg}_{\mathsf{L}}(\mathcal{S}), \mathcal{AT} \rangle$, where $\mathcal{AT} \subseteq \mathrm{Arg}_{\mathsf{L}}(\mathcal{S}) \times \mathrm{Arg}_{\mathsf{L}}(\mathcal{S})$ and $(a_1, a_2) \in \mathcal{AT}$ iff there is an $\mathcal{R} \in \mathsf{AR}$ such that a_1 $\mathcal{R}$-attacks a_2.

In what follows, to simplify notation, we will omit the subscript L and/or AR, when this is known or arbitrary.

Example 2.6. Let $\mathcal{S} = \{p, q, \neg p \vee \neg q\}$ and let $\mathcal{AF}_{\mathsf{L},\{\mathsf{Ucut}\}}(\mathcal{S})$ be a framework for $\mathcal{S}$, induced by classical logic CL, its corresponding sound and complete sequent calculus LK, and Ucut as the only attack rule. Some of the arguments are:

$a_1 = p \Rightarrow p$ $\qquad$ $a_4 = p \Rightarrow \neg((\neg p \vee \neg q) \wedge q)$
$a_2 = q \Rightarrow q$ $\qquad$ $a_5 = q \Rightarrow \neg((\neg p \vee \neg q) \wedge p)$
$a_3 = \neg p \vee \neg q \Rightarrow \neg p \vee \neg q$ $\qquad$ $a_6 = p, q \Rightarrow p \wedge q$
$a_7 = \neg p \vee \neg q, q \Rightarrow \neg p$ $\qquad$ $a_8 = \neg p \vee \neg q, p \Rightarrow \neg q$

See Figure 1 for a graphical representation of these arguments and the attacks between them.

Given a (sequent-based) framework, Dung-style semantics [14] can be applied to it to determine what combinations of arguments (called *extensions*) can collectively be accepted from it.

Definition 2.7. Let $\mathcal{AF}_{\mathsf{L}}(\mathcal{S}) = \langle \mathrm{Arg}_{\mathsf{L}}(\mathcal{S}), \mathcal{AT} \rangle$ be an argumentation framework and $\mathsf{S} \subseteq \mathrm{Arg}_{\mathsf{L}}(\mathcal{S})$ a set of arguments.

- S *attacks* an argument a if there is an $a' \in \mathsf{S}$ such that $(a', a) \in \mathcal{AT}$;
- S *defends* an argument a if S attacks every attacker of a;

- S is *conflict-free* if there are no arguments $a_1, a_2 \in \mathrm{S}$ such that $(a_1, a_2) \in \mathcal{AT}$;
- S is *admissible* if it is conflict-free and it defends all of its elements.

An admissible set that contains all the arguments that it defends is a *complete extension* of $\mathcal{AF}_{\mathsf{L}}(\mathcal{S})$. Below are definitions of some other extensions of $\mathcal{AF}_{\mathsf{L}}(\mathcal{S})$:

- a *preferred extension* of $\mathcal{AF}_{\mathsf{L}}(\mathcal{S})$ is a maximal (with respect to $\subseteq$) complete extension of $\mathrm{Arg}_{\mathsf{L}}(\mathcal{S})$;
- a *stable extension* of $\mathrm{Arg}_{\mathsf{L}}(\mathcal{S})$ is a complete extension of $\mathrm{Arg}_{\mathsf{L}}(\mathcal{S})$ that attacks every argument not in it;
- the *grounded extension* of $\mathcal{AF}_{\mathsf{L}}(\mathcal{S})$ is the minimal (with respect to $\subseteq$) complete extension of $\mathrm{Arg}_{\mathsf{L}}(\mathcal{S})$.

In what follows we shall refer to either complete (cmp), grounded (grd), preferred (prf) or stable (stb) semantics as *completeness-based semantics.* We denote by $\mathsf{Ext}_{\mathsf{sem}}(\mathcal{AF}_{\mathsf{L}}(\mathcal{S}))$ the set of all the extensions of $\mathcal{AF}_{\mathsf{L}}(\mathcal{S})$ under the semantics sem $\in$ {cmp, grd, prf, stb}. The subscript is omitted when this is clear from the context.

Definition 2.8. Given a sequent-based argumentation framework $\mathcal{AF}_{\mathsf{L}}(\mathcal{S})$, the semantics as defined in Definition 2.7 induces corresponding (nonmonotonic) *entailment relations*:

- *Skeptical entailment:* $\mathcal{S} \mid\!\sim^{\cap}_{\mathsf{L},\mathsf{sem}} \phi$ iff for every extension $\mathcal{E} \in \mathsf{Ext}_{\mathsf{sem}}(\mathcal{AF}_{\mathsf{L}}(\mathcal{S}))$, there is $\Gamma \Rightarrow \phi \in \mathcal{E}$ for $\Gamma \subseteq \mathcal{S}$
- *Credulous entailment:* $\mathcal{S} \mid\!\sim^{\cup}_{\mathsf{L},\mathsf{sem}} \phi$ iff for some extension $\mathcal{E} \in \mathsf{Ext}_{\mathsf{sem}}(\mathcal{AF}_{\mathsf{L}}(\mathcal{S}))$, there is $\Gamma \Rightarrow \phi \in \mathcal{E}$ for $\Gamma \subseteq \mathcal{S}$
- *Weakly skeptical entailment:* $\mathcal{S} \mid\!\sim^{\Cap}_{\mathsf{L},\mathsf{sem}} \phi$ iff there is an $a \in \mathrm{Arg}_{\mathsf{L}}(\mathcal{S})$ with $\mathrm{Con}(a) = \phi$ such that $a \in \mathcal{E}$ for every $\mathcal{E} \in \mathsf{Ext}_{\mathsf{sem}}(\mathcal{AF}_{\mathsf{L}}(\mathcal{S}))$.[4]

Example 2.9. Consider again the framework of Example 2.6. It holds that $\mathcal{S} \mid\!\sim^{\cup}_{\mathsf{CL},\mathsf{prf}} p$ and $\mathcal{S} \mid\!\sim^{\cup}_{\mathsf{CL},\mathsf{prf}} \neg p$, while $\mathcal{S} \not\mid\!\sim^{\star}_{\mathsf{CL},\mathsf{prf}} p$ and $\mathcal{S} \not\mid\!\sim^{\star}_{\mathsf{CL},\mathsf{prf}} \neg p$ for $\star \in \{\cap, \Cap\}$. Moreover, $\mathcal{S} \mid\!\sim_{\mathsf{CL},\mathsf{grd}} \psi$ if and only if ψ is a tautology in classical logic. On the other hand, it is easy to see that $\mathcal{S} \cup \{r\} \mid\!\sim_{\mathsf{CL},\mathsf{grd}} r$ and $\mathcal{S} \cup \{r\} \not\mid\!\sim_{\mathsf{CL},\mathsf{grd}} \neg r$, since e.g. $r \Rightarrow r$ is in the grounded extension of $\mathcal{S} \cup \{r\}$.

3 PREFERENCE FUNCTIONS AND PRIORITIZED ARGUMENTATION

We now formulate a general setting for prioritized sequent-based argumentation, allowing to make preferences among different arguments.

Definition 3.1. A *priority function* for a language $\mathcal{L}$ is a function $\pi : \mathcal{L} \mapsto \mathbb{N}^+$. Given a set of $\mathcal{L}$-formulas $\mathcal{S}$, we denote: $\pi(\mathcal{S}) = \{\pi(\phi) \mid \phi \in \mathcal{S}\}$.

We now use π for defining a preference relation $\leq_\pi$ on $\mathcal{L}$-sequents. The next example illustrates some ways of doing so. We shall write $a_1 \leq_\pi a_2$ to intuitively indicate that the sequent a_1 is at least as preferred as the sequent a_2.[5]

Example 3.2. The following are possible conditions for letting $a_1 \leq_\pi a_2$:[6]

(1) $\min(\pi(\mathsf{Sup}(a_1))) \leq \min(\pi(\mathsf{Sup}(a_2)))$. In this case only the most preferred formulas in the support of the sequents are compared.
(2) $\max(\pi(\mathsf{Sup}(a_1))) \leq \max(\pi(\mathsf{Sup}(a_2)))$. Here, for every formula in the support of a_2 there is a more preferred formula in the support of a_1.
(3) $\max(\pi(\mathsf{Sup}(a_1))) \leq \min(\pi(\mathsf{Sup}(a_2)))$. In this case all the formulas in the support of a_1 are at least as preferred as the formulas in the support of a_2.
(4) $\min(\pi(\mathsf{Sup}(a_1) \setminus \mathsf{Sup}(a_2))) \leq \min(\pi(\mathsf{Sup}(a_2) \setminus \mathsf{Sup}(a_1)))$. Like in the first item, the most preferred formulas are compared, but now only the formulas that are not part of the support of the other argument.
(5) $f(\mathsf{Sup}(a_1)) \leq f(\mathsf{Sup}(a_2))$, where f is an aggregation function on $\mathsf{Sup}(a_i)$ (like the average, median, summation of the π-values of the supports, or the max/min function on the support, as in the previous items).
(6) $\mathsf{Sup}(a_1) \leq_s \mathsf{Sup}(a_2)$ if either $\mathsf{Sup}(a_1) = \emptyset$ or $\mathsf{Sup}(a_1) = \mathsf{Sup}(a_2)$ or there is an $i \in \mathbb{N}$, such that:
   - $\{\psi \in \mathsf{Sup}(a_1) \mid \pi(\psi) = i\} \supsetneq \{\psi \in \mathsf{Sup}(a_2) \mid \pi(\psi) = i\}$,
   - $\{\psi \in \mathsf{Sup}(a_1) \mid \pi(\psi) = j\} = \{\psi \in \mathsf{Sup}(a_2) \mid \pi(\psi) = j\}$ for every $j < i$.
(7) $\mathsf{Sup}(a_1) \leq_c \mathsf{Sup}(a_2)$ if either $\mathsf{Sup}(a_1) = \emptyset$ or there is an $i \in \mathbb{N}$ such that:
   - $|\{\psi \in \mathsf{Sup}(a_1) \mid \pi(\psi) = i\}| > |\{\psi \in \mathsf{Sup}(a_2) \mid \pi(\psi) = i\}|$,
   - $|\{\psi \in \mathsf{Sup}(a_1) \mid \pi(\psi) = j\}| = |\{\psi \in \mathsf{Sup}(a_2) \mid \pi(\psi) = j\}|$ for every $j < i$,

   or for every $i \in \mathbb{N}$:
   $|\{\psi \in \mathsf{Sup}(a_1) \mid \pi(\psi) = i\}| = |\{\psi \in \mathsf{Sup}(a_2) \mid \pi(\psi) = i\}|$.

Remark 1. The last two items of Example 3.2 are inspired by Brewka's approach to reasoning with preferred theories [11]. This approach is adjusted to our case by viewing the arguments' support sets as stratified theories, where each stratification consists of the formulas with the same π-value. Accordingly $\leq_s$ is a subset-inclusion comparison, and $\leq_c$ is a comparison by cardinality.

Remark 2. The Items 1, 2, 4, 6 and 7 of Example 3.2 are *pre-orders*, that is: $\leq_\pi$ is reflexive ($a \leq_\pi a$) and transitive (if $a \leq_\pi b$ and $b \leq_\pi c$ then $a \leq_\pi c$). Whether the relation in Item 5 is a pre-order depends on the function f.

The orders in Items 1, 4, 6 and 7 and their strict counterparts are also *left monotonic*: if $a \leq_\pi b$ (resp. $a <_\pi b$) and $\mathsf{Sup}(a) \subseteq \mathsf{Sup}(a')$ then $a' \leq_\pi b$ (resp. $a' <_\pi b$).

Example 3.3. In Example 2.6, let $\pi(p) = 1$, $\pi(q) = 2$ and $\pi(\neg p \vee \neg q) = 3$. Consider each of the seven instances for $\leq_\pi$ from Example 3.2:

(1) When the most preferred supports are compared we have that $a_1 <_\pi a_2 <_\pi a_3$, $a_1 <_\pi a_7$, $a_8 <_\pi a_2$, $a_6 <_\pi a_3$, and $a_6 = a_8$.
(2) When the least preferred supports are compared we still have $a_1 <_\pi a_2 <_\pi a_3$, $a_1 <_\pi a_7$ and $a_6 <_\pi a_3$, but now $a_2 <_\pi a_8$ and only $a_6 <_\pi a_8$.
(3) The max-min-comparison yields again $a_1 <_\pi a_2 <_\pi a_3$, $a_1 <_\pi a_7$ and $a_6 <_\pi a_3$, but this time a_2 and a_6 are $\leq_\pi$-incomparable with a_8.

[4] Since the grounded extension is unique, $\mid\!\sim^{\cap}_{\mathsf{L},\mathsf{grd}}$, $\mid\!\sim^{\cup}_{\mathsf{L},\mathsf{grd}}$ and $\mid\!\sim^{\Cap}_{\mathsf{L},\mathsf{grd}}$, are the same, and will be denoted by $\mid\!\sim_{\mathsf{L},\mathsf{grd}}$.

[5] When $a_1 \leq_\pi a_2$ we shall sometimes write $a_1 = a_2$ and $a_1 <_\pi a_2$ to indicate, respectively, that $a_2 \leq_\pi a_1$ and that $a_2 \not\leq_\pi a_1$.

[6] We let $\min(\emptyset) = \max(\emptyset) = f(\emptyset) = 0$.

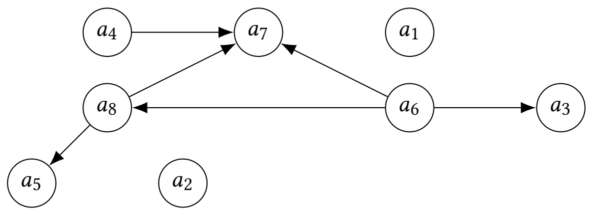

Figure 2: Part of the sequent-based argumentation graph for $\mathcal{S} = \{p, q, \neg p \vee \neg q\}$, the prioritized case from Example 3.5

(4) Clearly, when the comparison takes place on restricted support sets, $a_1 <_\pi a_2 <_\pi a_3$, $a_1 <_\pi a_7$, $a_8 <_\pi a_2$ and $a_6 <_\pi a_3$, since the restriction on the support set has no effect here and thus is the comparison the same as the first item. However, $a_6 <_\pi a_8$, since $\pi(p) < \pi(\neg p \vee \neg q)$,
(5) If $f(\Gamma) = \frac{1}{|\Gamma|} \sum_{\phi \in \Gamma} \pi(\phi)$, then $a_2 = a_8$ and $a_6 <_\pi a_8$.
(6) According to $\leq_s$, we have that $a_8 <_\pi a_2$ and $a_6 <_\pi a_8$.
(7) Similarly, according to $\leq_c$, $a_8 <_\pi a_2$ and $a_6 <_\pi a_8$.

Definition 3.4. Let $\mathcal{R}$ be a sequent elimination rule as in Definition 2.3 and let $\leq_\pi$ be a preference order on L-arguments. We say that $\mathcal{R}$ is $\leq_\pi$*-applicable* if it is applicable in the standard (non-prioritized) case and the instance a_2 of the attacked argument that is obtained by the application is not $<_\pi$-smaller than (i.e., not $<_\pi$-preferred over) the instance a_1 of the attacking argument that is obtained by the same application. In this case we say that a_1 $\mathcal{R}_{\leq_\pi}$-*attacks* a_2.[7,8]

Remark 3. Note that the first item of Example 3.2 might lead to some counter-intuitive situations. Consider for example the set $\mathcal{S} = \{p, \neg p, q\}$ where $\pi(q) = 1$, $\pi(\neg p) = 2$ and $\pi(p) = 3$. Then $\neg p \Rightarrow \neg p$ attacks $p \Rightarrow p$, which cannot defend itself, but $q, p \Rightarrow p$ attacks $\neg p \Rightarrow \neg p$, since q is preferred over p and $\neg p$. A possible solution would be to only consider *compact arguments*: an $\mathcal{S}$-argument $\Gamma \Rightarrow \phi$ is *compact* iff there is no $\mathcal{S}$-argument $\Gamma' \Rightarrow \phi$ for some $\Gamma' \subset \Gamma$. However, this places a restriction on the arguments of the framework. Another solution is to restrict the parts of the support that determine the strength of an argument, such as in Item 4 of Example 3.2.

Example 3.5. Consider again Example 2.6. Figure 2 depicts a prioritized version of Figure 1 for the π-assignment from Example 3.3 and the priority ordering in Item 6 of Example 3.2. In this case a_1 and a_4 are no longer attacked, and while a_4 and a_7 $\mathcal{R}$-attack each other in the original framework, in the prioritized setting a_4 $\mathcal{R}_{\leq_\pi}$-attacks a_7 but not vice versa. Indeed, $\{p\} = \{\psi \in \mathsf{Sup}(a_4) \mid \pi(\psi) = 1\} \supsetneq \{\psi \in \mathsf{Sup}(a_7) \mid \pi(\psi) = 1\} = \emptyset$.

Remark 4. Since we assume that an argument with empty support has always priority value 0, according to each of the attack rules in Example 2.4, sequents with empty support are maximally strong: attacks by such sequents are always successful.

The definition of a sequent-based argumentation framework, now with a priority function, is very similar to the one given in Definition 2.5.

[7] When π is clear from the context it will be omitted from $\leq_\pi$.
[8] Attacks that are based on priorities are sometimes called *defeats*, to distinguish them from 'pure' attacks.

Definition 3.6. Let $\mathsf{L} = \langle \mathcal{L}, \vdash \rangle$ be a core logic, C a corresponding sound and complete sequent calculus, AR a set of attack rules, π a priority function on $\mathcal{L}$, and $\leq_\pi$ a preference order on $\mathcal{L}$-sequents. The *prioritized sequent-based argumentation framework* for the set $\mathcal{S}$ of formulas (induced by L, C, AR, and $\leq_\pi$), is a triple: $\mathcal{AF}^{\leq_\pi}_{\mathsf{L},\mathsf{AR}}(\mathcal{S}) = \langle \mathrm{Arg}_\mathsf{L}(\mathcal{S}), \mathcal{AT}, \leq_\pi \rangle$, where $\mathcal{AT} \subseteq \mathrm{Arg}_\mathsf{L}(\mathcal{S}) \times \mathrm{Arg}_\mathsf{L}(\mathcal{S})$ and $(a_1, a_2) \in \mathcal{AT}$ iff a_1 $\mathcal{R}_{\leq_\pi}$-attacks a_2 for some $\mathcal{R} \in \mathsf{AR}$.

Like before, we will omit the subscripts L, AR and/or π if these are known or arbitrary.

The Dung-style semantics from Definition 2.7 are defined equivalently for $\mathcal{AF}^{\leq}_{\mathsf{L}}(\mathcal{S}) = \langle \mathrm{Arg}_\mathsf{L}(\mathcal{S}), \mathcal{AT}, \leq \rangle$, now with respect to both $\mathcal{AT}$ and $\leq$. Based on this, we define the entailment relations for $\mathcal{AF}^{\leq}_{\mathsf{L}}(\mathcal{S})$ with respect to the different semantics as in Definition 2.8. For a given semantics sem and $\star \in \{\cup, \cap, \Cap\}$, the relation is denoted by $\mid\!\sim^{\star,\leq}_{\mathsf{L},\mathsf{sem}}$ (super/subscripts are omitted when they are clear from the context).

Example 3.7. The flat case (without priorities) of $\mathcal{AF}^{\leq}_{\mathsf{CL}}(\mathcal{S})$ with $\mathcal{S} = \{p, q, \neg p \vee \neg q\}$ and Ucut as the sole attack rule is the same as the framework of Example 2.6. The grounded extension only contains sequents with empty support sets, since there are complete extensions that contain only two of the arguments a_1, a_2 and a_3. When considering the priority function π from Example 3.3, in any of the definitions for $\leq_\pi$ from Example 3.2, a_1 cannot be attacked. Thus $\mathcal{S} \mid\!\sim^{\leq}_{\mathsf{CL},\mathsf{grd}} p$. For q the result depends on the choice of $\leq_\pi$.

- When using the first instance of $\leq_\pi$ from Example 3.2, $a_8 \leq_\pi a$, for any $a \in \{\Gamma \Rightarrow \psi \mid \emptyset \subset \Gamma \subseteq \mathcal{S}\}$. Moreover, a_6 and a_8 attack each other, one can therefore construct two different admissible sets, one in which a_6 defends a and one in which it does not. Therefore, $\mathcal{S} \not\mid\!\sim^{\leq}_{\mathsf{CL},\mathsf{grd}} q$.
- According to the fourth and sixth instance of $\leq_\pi$ from Example 3.2, a_8 does not attack a_6, thus a_6 is no longer attacked, and so it defends a_2. Hence $\mathcal{S} \mid\!\sim^{\leq}_{\mathsf{CL},\mathsf{grd}} q$ in this case.

4 SOME BASIC PROPERTIES

Next, we consider some basic properties of prioritized argumentation frameworks and the entailment relations induced by them.

4.1 Conservativity

For every preferential ordering from Example 3.2, prioritized reasoning is a conservative extension of the flat case:

Proposition 4.1. *If $\leq_\pi$ is degenerated (i.e., π is uniform) then $\mid\!\sim^{\star}_{\mathsf{L},\mathsf{sem}}$ and $\mid\!\sim^{\star,\leq}_{\mathsf{L},\mathsf{sem}}$ are the same for every $\star \in \{\cap, \cup, \Cap\}$ and* $\mathsf{sem} \in \{\mathsf{grd}, \mathsf{cmp}, \mathsf{prf}, \mathsf{stb}\}$.

Proof. (Sketch) Immediate from Definition 3.4 of $\mathcal{R}_{\leq_\pi}$-attacks since no arguments are $\leq_\pi$-preferred over others, thus $\mathcal{R}_{\leq_\pi}$-attacks coincide with $\mathcal{R}$-attacks. □

4.2 Rationality postulates

Caminada and Amgoud [1, 12] propose several postulates for argumentation reasoning. Below we consider those postulates using the next definitions.

Definition 4.2. Let $\mathsf{L} = \langle \mathcal{L}, \vdash \rangle$ be a propositional logic and $\mathcal{S}$ a set of $\mathcal{L}$-formulas.

- The *transitive closure* of $\mathcal{S}$ with respect to the logic L is the set $\mathsf{CN_L}(\mathcal{S}) = \{\psi \mid \mathcal{S} \vdash \psi\}$.
- $\mathcal{S}$ is L-*consistent* if there is no $\Gamma \subseteq \mathcal{S}$ such that $\vdash \neg\bigwedge\Gamma$.
- A subset $\mathcal{T} \subseteq \mathcal{S}$ is an L-*minimal conflict* of $\mathcal{S}$, if it is not L-consistent, but $\mathcal{T} \setminus \{\psi\}$ *is* L-consistent for every $\psi \in \mathcal{T}$.
- $\mathsf{Free}(\mathcal{S})$ is the set of formulas that are not part of any minimal conflict of $\mathcal{S}$.

Definition 4.3. [1, 12] The postulates below refer to a prioritized sequent-based argumentation framework $\mathcal{AF}^{\leq}_{\mathsf{L}}(\mathcal{S}) = \langle \mathrm{Arg_L}(\mathcal{S}), \mathcal{AT}, \leq_\pi \rangle$, a semantics sem of it (i.e., one of those in Definition 2.7), every extension $\mathcal{E} \in \mathsf{Ext_{sem}}(\mathcal{AF}^{\leq}_{\mathsf{L}}(\mathcal{S}))$, and arbitrary argument $a \in \mathrm{Arg_L}(\mathcal{S})$.

- *Closure of extensions:* $\mathsf{Con}(\mathcal{E}) = \mathsf{CN_L}(\mathsf{Con}(\mathcal{E}))$.
- *Closure under sub-arguments:* if $a \in \mathcal{E}$ and $b \in \mathsf{Sub}(a)$ then $b \in \mathcal{E}$.
- *Weak Closure under sub-arguments:* if $a \in \mathcal{E}$, $b \in \mathsf{Sub}(a)$ and $b \leq a$, then $b \in \mathcal{E}$.
- *Consistency:* $\mathsf{Con}(\mathcal{E})$ is consistent.
- *Exhaustiveness:* if $\mathsf{Sup}(a) \cup \{\mathsf{Con}(a)\} \subseteq \mathsf{Con}(\mathcal{E})$, $a \in \mathcal{E}$.
- *Weak exhaustiveness:* if $\mathsf{Sup}(a) \subseteq \bigcup_{b \in \mathcal{E}} \mathsf{Sup}(b)$, $a \in \mathcal{E}$.
- *Free precedence:* $\mathrm{Arg_L}(\mathsf{Free}(\mathcal{S})) \subseteq \mathcal{E}$.

Below, we shall consider these postulates under the following assumptions:

1. The core logic is *non-trivial* (there is no ϕ such that both $\vdash \phi$ and $\vdash \neg\phi$) and *contrapositive* ($\Gamma \vdash \neg \bigwedge \Delta$ implies that $(\Gamma \setminus \Gamma'), \Delta' \vdash \neg \bigwedge((\Delta \setminus \Delta') \cup \Gamma')$, for $\Delta' \subseteq \Delta$ and $\Gamma' \subseteq \Gamma$).
2. The preferential order $\leq_\pi$ and its strict counterpart $<_\pi$ are *left monotonic*: If $a \leq_\pi b$ (resp. $a <_\pi b$) and $\mathsf{Sup}(a) \subseteq \mathsf{Sup}(a')$ then $a' \leq_\pi b$ (resp. $a' <_\pi b$).
3. Given a priority function π for $\mathcal{L}$, we will consider preference orders $\leq_\pi$ on sets of $\mathcal{L}$-formulas for which $\leq_\pi$ and $<_\pi$ are monotonic, reflexive and transitive relations (as in Items 1, 4, 6, and 7 in Example 3.2). We lift $\leq_\pi$ to sequents as follows: $a \leq_\pi b$ iff $\mathsf{Sup}(a) \leq_\pi \mathsf{Sup}(b)$.

Proposition 4.4. *Let $\mathcal{AF}^{\leq}_{\mathsf{L}}(\mathcal{S})$ be a prioritized framework in which the core logic and the preferential order satisfy the three conditions specified above. Suppose also that DUcut is the attack rule. Then, for every completeness-based semantics, $\mathcal{AF}^{\leq}_{\mathsf{L}}(\mathcal{S})$ satisfies closure of extensions, weak closure under sub-arguments, consistency, and weak exhaustiveness. When ConUcut is also an attack rule, $\mathcal{AF}^{\leq}_{\mathsf{L}}(\mathcal{S})$ satisfies free precedence as well.*

The following lemmas are required to prove Proposition 4.4.

Lemma 4.5. *Let $\mathcal{E}$ be a complete extension of $\mathcal{AF}^{\leq}_{\mathsf{L}}(\mathcal{S})$. If (1) $a = \Delta \Rightarrow \delta \in \mathcal{E}$ and $b = \Gamma \Rightarrow \gamma \in \mathcal{E}$, (2) $a < b$, (3) $\Gamma' \subseteq \Gamma$, (4) $\Delta, \Gamma' \vdash \psi$, and (5) $\Delta \cup \Gamma'$ is consistent, then $\Gamma', \Delta \Rightarrow \psi \in \mathcal{E}$.*

Proof. Suppose that $d = \Theta \Rightarrow \phi$ attacks $c = \Gamma', \Delta \Rightarrow \psi$. Since $\Gamma' \cup \Delta$ is consistent (by Condition (5)), d DUcut attacks c. Hence $c \not< d$. By left monotonicity $a \not< d$. Assume for a contradiction that $b < d$. Since $a < b$ (by Condition (2)), also $a < d$, which is a contradiction. Thus, $b \not< d$. Since d DUcut attacks c, there is a $\beta \in \Gamma' \cup \Delta$ for which both $\gamma \Rightarrow \neg\beta$ and $\beta \Rightarrow \neg\gamma$ are derivable. If $\beta \in \Delta$ then d DUcut attacks a. If $\gamma \in \Gamma'$ then d DUcut attacks b. By the admissibility of $\mathcal{E}$ there is an $f \in \mathcal{E}$ that attacks d, and so f defends c from d. ◻

Lemma 4.6. *If $a \not< b$ and $\mathsf{Sup}(b) \subseteq \mathsf{Sup}(b')$, then $a \not< b'$.*

Proof. Suppose that $a \not< b$ and $\mathsf{Sup}(b) \subseteq \mathsf{Sup}(b')$. Assume for a contradiction that $a < b'$. By reflexivity, $b \leq b$, and so by left monotonicity, $b' \leq b$. By transitivity, $a < b$, which contradicts our supposition. ◻

Lemma 4.7. *Let $\mathcal{E}$ be a complete extension of $\mathcal{AF}^{\leq}_{\mathsf{L}}(\mathcal{S})$. If $\Gamma \Rightarrow \gamma$ and $\Delta \Rightarrow \delta$ are in $\mathcal{E}$ and $\Gamma, \Delta \Rightarrow \phi \in Arg_{\mathsf{L}}(\mathcal{S})$, then $\Gamma, \Delta \Rightarrow \phi$ is also in $\mathcal{E}$.*

Proof. We start with assuming that DUcut is the only attack rule. Suppose that $a = \Theta \Rightarrow \tau \in \mathrm{Arg_L}(\mathcal{S})$ attacks $b = \Gamma, \Delta \Rightarrow \phi$. Then $\Rightarrow \tau \leftrightarrow \neg\tau'$ is derivable in C for some $\tau' \in \Gamma \cup \Delta$ and $\Gamma \cup \Delta \not<_\pi \Theta$. By left monotonicity, also $\Gamma \not<_\pi \Theta$ and $\Delta \not<_\pi \Theta$. Thus $b \not<_\pi a$, $\Delta \Rightarrow \delta \not<_\pi a$, and $\Gamma \Rightarrow \gamma \not<_\pi a$. Suppose, without loss of generality, that $\tau' \in \Delta$. Then a attacks $\Delta \Rightarrow \delta$. Since $\mathcal{E}$ is admissible there is some $a' \in \mathcal{E}$ that attacks a. This shows that $\mathcal{E}$ defends b and since $\mathcal{E}$ is complete, $b \in \mathcal{E}$.

Now, suppose that ConUcut is one of the attack rules and assume for a contradiction that $\Gamma, \Delta \Rightarrow \phi$ is ConUcut-attacked. In this case $\Gamma \cup \Delta$ is inconsistent. We that have either $\Delta \not< \Gamma$ or $\Gamma \not< \Delta$. Without loss of generality we suppose $\Delta \not< \Gamma$.

Since $\Gamma \cup \Delta$ is inconsistent, there is a maximal $\Delta' \subseteq \Delta$ for which $\Gamma \cup \Delta'$ is consistent. (Note for this that Γ is consistent since otherwise $\Gamma \Rightarrow \gamma$ is ConUcut attacked and cannot be defended, which is impossible since $\Gamma \Rightarrow \gamma \in \mathcal{E}$.) Thus, there is a $\delta' \in \Delta \setminus \Delta'$ for which $\Gamma, \Delta' \vdash \neg\delta'$. Let $d = \Gamma, \Delta' \Rightarrow \neg\delta' \in \mathrm{Arg_L}(\mathcal{S})$ and $a = \Delta \Rightarrow \delta$. Note that $a \not< d$ by Lemma 4.6. So, d DUcut attacks a.

Hence, there is a $e = \Theta \Rightarrow \psi \in \mathcal{E}$ that defends a from this attack by attacking d. Note that e does not ConUcut-attack d since $\Gamma \cup \Delta'$ is consistent. Hence, $d \not< e$ and there is a $\beta \in \Gamma \cup \Delta'$ for which both $\psi \Rightarrow \neg\beta$ and $\beta \Rightarrow \neg\psi$ are derivable.

We have two cases: (a) $\beta \in \Gamma$ and (b) $\beta \in \Delta'$. We now show that both lead to a contradiction.

If (a) holds, then by left monotonicity $b = \Gamma \Rightarrow \gamma \not< e$ which means that e attacks b in contradiction to the conflict-freeness of $\mathcal{E}$.

If (b) holds, then $a < e$, since otherwise e DUcut attacks a in contradiction to the conflict-freeness of $\mathcal{E}$. Since $\Theta \vdash \neg\beta$ there is a maximal $\Theta' \subseteq \Theta$ for which $\Delta \cup \Theta'$ is consistent and for which $\Delta \cup \Theta' \vdash \neg\alpha$ for some $\alpha \in \Theta \setminus \Theta'$. By Lemma 4.5, $a' = \Delta, \Theta' \Rightarrow \neg\alpha \in \mathcal{E}$. By left monotonicity $a' < e$. But then a' attacks e in contradiction to the conflict-freeness of $\mathcal{E}$. ◻

Proof of Proposition 4.4. We show each postulate:

Weak Closure under sub-argument: Suppose that $a = \Gamma \Rightarrow \psi \in \mathcal{E}$ and let $b = \Delta \Rightarrow \psi \in \mathsf{Sub}(a)$ and $b \leq a$. Thus, $\Delta \subseteq \Gamma$. Suppose some c attacks b. Thus, $b \not< c$ and since $b \leq a$, also $a \not< c$. Hence, c also attacks a. Since $\mathcal{E}$ defends a, it attacks c and so also b is defended by $\mathcal{E}$. Since $\mathcal{E}$ is complete, $b \in \mathcal{E}$.

Closure of extensions: $\mathsf{Con}(\mathcal{E}) \subseteq \mathsf{CN_L}(\mathsf{Con}(\mathcal{E}))$ holds by the reflexivity of L. For $\mathsf{Con}(\mathcal{E}) \supseteq \mathsf{CN_L}(\mathsf{Con}(\mathcal{E}))$, let $\phi \in \mathsf{CN_L}(\mathsf{Con}(\mathcal{E}))$. Since L is finitary, there are $\phi_1, \ldots, \phi_n \in \mathsf{Con}(\mathcal{E})$ for which $\phi_1, \ldots, \phi_n \Rightarrow \phi$ is derivable in C. Thus, there are $a_1 = \Gamma_1 \Rightarrow \phi_1, \ldots, a_n = \Gamma_n \Rightarrow \phi_n \in \mathcal{E}$. By n applications of cut, $\Gamma_1, \ldots, \Gamma_n \Rightarrow \phi \in \mathcal{E}$. Hence $\phi \in \mathsf{Con}(\mathcal{E})$.

Consistency: Suppose that $\mathsf{Con}(\mathcal{E})$ is inconsistent. Thus, there are $\phi_1, \ldots, \phi_n \in \mathsf{Con}(\mathcal{E})$ for which $\Rightarrow \neg \bigwedge_{i=1}^{n} \phi_i$ is derivable in

C. Since there are $a_1 = \Gamma_1 \Rightarrow \phi_1, \ldots, a_n = \Gamma_n \Rightarrow \phi_n \in \mathcal{E}$, by Lemma 4.7 $\Gamma_1, \ldots, \Gamma_n \Rightarrow \bigwedge_{i=1}^{n} \phi_i \in \mathcal{E}$. By Contraposition $\bigwedge_{i=1}^{n} \phi_i \Rightarrow$ is derivable in C. Now, by Cut and Lemma 4.7, $\Gamma_1, \ldots, \Gamma_n \Rightarrow\ \in \mathcal{E}$. By Contraposition, Monotonicity and Lemma 4.7, $b = \Gamma_1, \ldots, \Gamma_n \Rightarrow \neg\gamma \in \mathcal{E}$ where $\gamma \in \Gamma_1$. Clearly, b attacks a_1, which contradicts the conflict-freeness of $\mathcal{E}$.

Weak exhaustiveness: Suppose that $a = \Delta \Rightarrow \psi \in \mathrm{Arg}_{\mathsf{L}}(\mathcal{S})$ such that $\Delta \subseteq \bigcup_{b \in \mathcal{E}} \mathrm{Sup}(b)$. Since Δ is finite, there are $b_1, \ldots, b_n \in \mathcal{E}$ such that $\Delta = \mathrm{Sup}(b_1) \cup \ldots \cup \mathrm{Sup}(b_n)$. By $n-1$ applications of Lemma 4.7, $a \in \mathcal{E}$.

Free precedence: Assume that ConUcut is part of the attack rules as well. Let $a = \Gamma \Rightarrow \phi$ where $\Gamma \subseteq \mathrm{Free}(\mathcal{S})$. In particular, Γ is consistent, and so a cannot be ConUcut-attacked. Suppose that $b = \Delta \Rightarrow \delta$ attacks a. Then $\Rightarrow \delta \leftrightarrow \neg\gamma$ is derivable in C for some $\gamma \in \Gamma$. By Cut, $\Delta \Rightarrow \neg\gamma$ is also derivable and Cut and Contraposition again show that $\Rightarrow \neg \bigwedge(\Delta \cup \{\gamma\})$ is derivable in C. Since γ is not a member of a minimally inconsistent subset of $\mathcal{S}$, there is a $\Theta \subsetneq \Delta$ for which $c = \Rightarrow \neg \bigwedge \Theta$ is derivable in C. Thus, b is attacked by c. Since c has no attackers, $c \in \mathcal{E}$. Thus, $\mathcal{E}$ defends a and thus $a \in \mathcal{E}$ by the completeness of $\mathcal{E}$. □

Some negative results are reported next:

(1) Exhaustiveness is not satisfied by every framework that satisfies the requirements of Proposition 4.4 (just Weak exhaustiveness is satisfied):

Example 4.8. Let $\mathcal{S} = \{p \wedge q, q, s, \neg s, t \wedge (\neg s \vee \neg q), \neg t\}$ and assume $\pi(p \wedge q) = 1$, $\pi(q) = 3$, $\pi(s) = \pi(\neg s) = \pi(t \wedge (\neg s \vee \neg q)) = \pi(\neg t) = 2$ where $\leq_\pi$ is as in Example 3.2 Items 1, 4, 5, or 6. Here, $\mathcal{E} = \{\Rightarrow \phi \mid \vdash \phi\} \cup \{p \wedge q \Rightarrow \phi \mid p \wedge q \vdash \phi\}$ is a complete extension. Note that $q \Rightarrow q \notin \mathcal{E}$. The reason is that $s, t \wedge (\neg s \vee \neg q) \Rightarrow \neg q$ attacks $q \Rightarrow q$, while no argument in $\mathcal{E}$ attacks $s, t \wedge (\neg s \vee \neg q) \Rightarrow \neg q$. Moreover $\mathcal{E}$ does not defend any other argument in $\mathrm{Arg}_{\mathsf{L}}(\mathcal{S}) \setminus \mathcal{E}$.

(2) Consistency does not hold for Undercut:

Example 4.9. Consider the flat framework $\mathcal{AF}_{\mathsf{CL}}(\mathcal{S})$ of Example 2.6, for $\mathcal{S} = \{p, q, \neg p \vee \neg q\}$. It can be shown that $\mathrm{S} = \{a_1, a_2, a_3, a_4, a_5\}$ is admissible in $\mathcal{AF}_{\mathsf{CL}}(\mathcal{S})$, however, Con(S) is inconsistent.

(3) Sub-argument closure for complete extensions does not hold when ConUcut is part of the system:

Example 4.10. Let $\mathcal{S} = \{p \wedge q, s, r, r \supset (\neg p \wedge \neg s)\}$ and assume $\pi(p \wedge q) = 1$, $\pi(r) = \pi(r \supset (\neg p \wedge \neg s)) = 2$ and $\pi(s) = 3$ where $\leq_\pi$ is as in Example 3.2 Item 1. Note that there is a complete extension with $a = p \wedge q, s \Rightarrow s$ but without $b = s \Rightarrow s$. This follows since the only attacker of a is $c = p \wedge q, r, r \supset (\neg p \wedge \neg s) \Rightarrow \neg s$, but c is ConUcut-attacked and thus cannot be defended.[9]

4.3 Conflict preservation

An attack in a sequent-based argumentation framework $\mathcal{AF}_{\mathsf{L}}(\mathcal{S})$ will not always be successful in the prioritized argumentation framework $\mathcal{AF}^{\leq}_{\mathsf{L}}(\mathcal{S})$, because the attacked argument might be $\leq_\pi$-stronger than the attacking argument. This way, it might be that attacks and conflicts are lost. This is sometimes avoided by requiring that attacks are always symmetric (see, e.g., [16]) or by reversing the attacks and rejecting the attacking argument instead of the attacked argument (see, e.g., [13]). In the ASPIC$^+$ framework [19] this is handled taking the structure of arguments into account.

The next proposition shows that argumentation frameworks with priorities are conflict preserving: extensions of the prioritized framework are conflict-free in the non-prioritized case.

Proposition 4.11. *Let $\mathcal{AF}^{\leq}_{\mathsf{L}}(\mathcal{S}) = \langle Arg_{\mathsf{L}}(\mathcal{S}), \mathcal{AT}, \leq \rangle$ be a prioritized sequent-based argumentation framework with Ucut and/or DUcut that satisfies the requirements of Proposition 4.4 and let $\mathcal{AF}_{\mathsf{L}}(\mathcal{S}) = \langle Arg_{\mathsf{L}}(\mathcal{S}), \mathcal{AT} \rangle$ be the corresponding flat (i.e., preference-free) sequent-based framework. For any completeness-based semantics* sem *(Definition 2.7), we have that:*

(1) any $\mathcal{E} \in \mathsf{Ext}_{\mathsf{sem}}(\mathcal{AF}^{\leq}_{\mathsf{L}}(\mathcal{S}))$ is conflict-free in $\mathcal{AF}_{\mathsf{L}}(\mathcal{S})$,
(2) any $\mathcal{E} \in \mathsf{Ext}_{\mathsf{sem}}(\mathcal{AF}_{\mathsf{L}}(\mathcal{S}))$ is conflict-free in $\mathcal{AF}^{\leq}_{\mathsf{L}}(\mathcal{S})$.

Proof. Let sem $\in \{\mathsf{cmp}, \mathsf{grd}, \mathsf{prf}, \mathsf{stb}\}$.

(1) Let $\mathcal{E} \in \mathsf{Ext}_{\mathsf{sem}}(\mathcal{AF}^{\leq}_{\mathsf{L}}(\mathcal{S}))$. Suppose that there are $a = \Gamma \Rightarrow \gamma$ and $b = \Delta \Rightarrow \delta$ in $\mathcal{E}$ such that $(a, b) \in \mathcal{AT}$. Assume first that DUcut is the attack rule. By Lemma 4.7 and the monotonicity of L we have that $a' = \Gamma, \Delta \Rightarrow \gamma \in \mathcal{E}$. Since $(a', b) \in \mathcal{AT}^{\leq}$ (by left monotoncity of $\leq$) this is a contradiction to $\mathcal{E} \in \mathsf{Ext}_{\mathsf{sem}}(\mathcal{AF}^{\leq}_{\mathsf{L}}(\mathcal{S}))$.
Suppose now that Ucut is the attack rule. Then $\Rightarrow \gamma \leftrightarrow \neg \bigwedge \Delta'$ is derivable in C for some $\Delta' \subseteq \Delta$. By Cut $\Gamma \Rightarrow \neg \bigwedge \Delta'$ is derivable in C and by Contraposition and Monotonicity also $b' = \Delta \Rightarrow \neg \bigwedge \Gamma$ and $a' = \Gamma \Rightarrow \neg \bigwedge \Delta$ are derivable. Since a' (resp. b') has the same attackers as a (resp. b), also $a', b' \in \mathcal{E}$ by the completeness of $\mathcal{E}$. It is easy to see that either $(a', b') \in \mathcal{AT}^{\leq}$ or $(b', a') \in \mathcal{AT}^{\leq}$, a contradiction to $\mathcal{E} \in \mathsf{Ext}_{\mathsf{sem}}(\mathcal{AF}^{\leq}_{\mathsf{L}}(\mathcal{S}))$.

(2) This follows immediately from the fact that every $\mathcal{R}_{\leq}$-attack is in particular an $\mathcal{R}$-attack. □

By Proposition 4.11 any completeness-based extension of the prioritized framework is still conflict-free in the flat case, and so no conflicts are lost, although, as shown in Example 3.7, the extensions in both frameworks are not the same.

An example, discussed in [19] for the ASPIC$^+$ framework, is the following:

Example 4.12. Let $\mathcal{AF}^{\leq}_{\mathsf{CL}}(\mathcal{S}) = \left\langle \mathrm{Arg}_{\mathsf{CL}}(\mathcal{S}), \mathcal{AT}, \leq_\pi \right\rangle$ be a prioritized sequent-based argumentation framework based on classical logic as the core logic, the attack rules DUcut and ConUcut, and the formulas $\mathcal{S} = \{p, q, \neg p\}$, such that $\pi(q) = 1$, $\pi(\neg p) = 2$, $\pi(p) = 3$. Some of the arguments of $\mathrm{Arg}_{\mathsf{CL}}(\mathcal{S})$ are the following:

$a_1 = p \Rightarrow p$	$a_2 = q \Rightarrow q$	$a_3 = \neg p \Rightarrow \neg p$
$a_4 = p, q \Rightarrow p$	$a_5 = p, q \Rightarrow q$	$a_6 = p, q \Rightarrow p \wedge q$
$a_7 = \neg p, q \Rightarrow \neg p$	$a_8 = \neg p, q \Rightarrow q$	$a_9 = \neg p, q \Rightarrow \neg p \wedge q$

The preference-based argumentation frameworks (PAFs) of [3, 4] result in a stable extension that contains both a_3 and a_6, and so consistency is not preserved by PAFs. In our case, when e.g., $\leq_\pi = \leq_s$ (the sixth item in Example 3.2) is taken as the preference ordering, this problem is avoided, since every stable extension that contains

[9]Given the result of Proposition 5.6, this is not a problem for preferred and stable semantics.

a_3 contains also a_7, thus all the sequents whose supports sets are $\{p\}$ or even $\{q, p\}$ are attacked by the latter.[10]

5 REASONING WITH MAXIMALLY CONSISTENT SUBSETS

A well-known method for handling inconsistent sets of formulas is by taking the maximally consistent subsets of this set [22]. In the flat case, corresponding entailments are defined as follows:

Definition 5.1. Let $\mathsf{L} = \langle \mathcal{L}, \vdash \rangle$ be a propositional logic and $\mathcal{S}$ a set of $\mathcal{L}$-formulas. We denote by $\mathsf{MCS}_\mathsf{L}(\mathcal{S})$ the set of all the maximally consistent subsets of $\mathcal{S}$ (with respect to $\subseteq$).

- $\mathcal{S} \mid\!\sim_{\mathsf{mcs}} \psi$ if and only if $\psi \in \mathsf{CN}_\mathsf{L}(\bigcap \mathsf{MCS}_\mathsf{L}(\mathcal{S}))$,
- $\mathcal{S} \mid\!\sim_{\cap\mathsf{mcs}} \psi$ if and only if $\psi \in \bigcap_{\mathcal{T} \in \mathsf{MCS}_\mathsf{L}(\mathcal{S})} \mathsf{CN}_\mathsf{L}(\mathcal{T})$,
- $\mathcal{S} \mid\!\sim_{\cup\mathsf{mcs}} \psi$ if and only if $\psi \in \bigcup_{\mathcal{T} \in \mathsf{MCS}_\mathsf{L}(\mathcal{S})} \mathsf{CN}_\mathsf{L}(\mathcal{T})$.

It has been shown that sequent-based argumentation is a useful platform for representing and reasoning with maximally consistent subsets [6, 8]. Here we extend these results to the prioritized case.

We continue to use $\preceq_\pi$ as a preference order, determined by π, on sets of $\mathcal{L}$-formulas. In what follows we shall abbreviate $\preceq_\pi$ [resp. $\npreceq_\pi$] by $\preceq$ [resp. $\npreceq$], and write $\mathcal{T} \prec \mathcal{S}$ to denote that $\mathcal{T} \preceq \mathcal{S}$ and $\mathcal{S} \npreceq \mathcal{T}$. Accordingly, the set of the $\preceq$-most preferred maximally consistent subsets of an $\mathcal{S}$ is defined as follows:

Definition 5.2. $\mathsf{MCS}^{\preceq}_\mathsf{L}(\mathcal{S}) = \{\mathcal{T} \in \mathsf{MCS}_\mathsf{L}(\mathcal{S}) \mid \nexists \mathcal{T}' \in \mathsf{MCS}_\mathsf{L}(\mathcal{S})$ such that $\mathcal{T}' \prec \mathcal{T}\}$.

Example 5.3. Consider again the set $\mathcal{S} = \{p, q, \neg p \vee \neg q\}$ from Example 2.6 and the priority assignment π from Example 3.3 on $\mathcal{S}$. We have that: $\mathsf{MCS}_\mathsf{CL}(\mathcal{S}) = \{\{p, q\}, \{p, \neg p \vee \neg q\}, \{q, \neg p \vee \neg q\}\}$. When $\preceq_\pi$ is the preference order as in Items 2, 4, 5 (when e.g. f is the average function), 6 and 7 of Example 3.2, we get: $\mathsf{MCS}^{\preceq_\pi}_\mathsf{CL}(\mathcal{S}) = \{\{p, q\}\}$. When $\preceq_\pi$ is as in Item 1 of Example 3.2 we have that $\mathsf{MCS}^{\preceq_\pi}_\mathsf{CL}(\mathcal{S}) = \{\{p, q\}, \{p, \neg p \vee \neg q\}\}$.

Now we can consider the prioritized versions of the entailment relations from Definition 5.1.

Definition 5.4. For a propositional logic $\mathsf{L} = \langle \mathcal{L}, \vdash \rangle$, a set $\mathcal{S}$ of $\mathcal{L}$-formulas, and a priority function π on $\mathcal{L}$, we define:

- $\mathcal{S} \mid\!\sim^{\preceq}_{\mathsf{L},\mathsf{mcs}} \phi$ if and only if $\phi \in \mathsf{CN}_\mathsf{L}(\bigcap \mathsf{MCS}^{\preceq}_\mathsf{L}(\mathcal{S}))$;
- $\mathcal{S} \mid\!\sim^{\preceq}_{\mathsf{L},\cap\mathsf{mcs}} \phi$ if and only if $\phi \in \bigcap_{\mathcal{T} \in \mathsf{MCS}^{\preceq}_\mathsf{L}(\mathcal{S})} \mathsf{CN}_\mathsf{L}(\mathcal{T})$;
- $\mathcal{S} \mid\!\sim^{\preceq}_{\mathsf{L},\cup\mathsf{mcs}} \phi$ if and only if $\phi \in \bigcup_{\mathcal{T} \in \mathsf{MCS}^{\preceq}_\mathsf{L}(\mathcal{S})} \mathsf{CN}_\mathsf{L}(\mathcal{T})$

Example 5.5. In Example 5.3 we have that $\mathcal{S} \mid\!\sim_{\cup\mathsf{mcs}} \psi$ for every $\psi \in \mathcal{S}$, but $\mathcal{S} \mid\!\sim_\star \phi$ when $\star \in \{\mathsf{mcs}, \cap\mathsf{mcs}\}$ only if ϕ is a CL-tautology (since $\bigcap \mathsf{MCS}_\mathsf{CL}(\mathcal{S}) = \emptyset$). In the prioritized case, when $\preceq_\pi$ is as defined in Items 2, 4, 5 (for e.g. the average function), 6 and 7 of Example 3.2, we have that $\mathcal{S} \mid\!\sim^{\preceq_\pi}_\star \phi$ for every $\star \in \{\mathsf{mcs}, \cap\mathsf{mcs}, \cup\mathsf{mcs}\}$ and $\phi \in \{p, q\}$. If $\preceq_\pi$ is as in Item 1 of Example 3.2, then $\mathcal{S} \mid\!\sim^{\preceq}_\star p$ for $\star \in \{\mathsf{mcs}, \cap\mathsf{mcs}\}$ and $\mathcal{S} \mid\!\sim^{\preceq}_{\cup\mathsf{mcs}} \phi$ for $\phi \in \mathcal{S}$.

The main result of this section is given in the next proposition. It extends the results in [6, 8] to the prioritized case.

Proposition 5.6. *Let $\mathsf{L} = \langle \mathcal{L}, \vdash \rangle$ be a contrapositive propositional logic, $\mathcal{S}$ a finite set of $\mathcal{L}$-formulas, and π a priority relation on $\mathcal{L}$. Let $\preceq$ be a monotonic and transitive preference relation on sets of formulas that is induced by π, and let $a \leq b$ iff $\mathsf{Sup}(a) \preceq_\pi \mathsf{Sup}(b)$ be the induced preference relation on arguments. Denote by $\mathcal{AF}^{\preceq}_\mathsf{L}(\mathcal{S}) = \langle Arg_\mathsf{L}(\mathcal{S}), \mathcal{AT}, \leq \rangle$ the corresponding prioritized framework where $\mathcal{AT}$ is based on the rules DUcut and ConUcut. Then:*

(1) $\mathcal{S} \mid\!\sim^{\cap,\preceq}_{\mathsf{L},\mathsf{stb}} \phi$ iff $\mathcal{S} \mid\!\sim^{\cap,\preceq}_{\mathsf{L},\mathsf{prf}} \phi$ iff $\mathcal{S} \mid\!\sim^{\preceq}_{\mathsf{L},\cap\mathsf{mcs}} \phi$,
(2) $\mathcal{S} \mid\!\sim^{\Cap,\preceq}_{\mathsf{L},\mathsf{grd}} \phi$ iff $\mathcal{S} \mid\!\sim^{\Cap,\preceq}_{\mathsf{L},\mathsf{stb}} \phi$ iff $\mathcal{S} \mid\!\sim^{\Cap,\preceq}_{\mathsf{L},\mathsf{prf}} \phi$ iff $\mathcal{S} \mid\!\sim^{\preceq}_{\mathsf{L},\mathsf{mcs}} \phi$,
(3) $\mathcal{S} \mid\!\sim^{\cup,\preceq}_{\mathsf{L},\mathsf{stb}} \phi$ iff $\mathcal{S} \mid\!\sim^{\cup,\preceq}_{\mathsf{L},\mathsf{prf}} \phi$ iff $\mathcal{S} \mid\!\sim^{\preceq}_{\mathsf{L},\cup\mathsf{mcs}} \phi$.

We sketch here the proof of the first item (the proofs of the other items are similar). First, some lemmas.

Lemma 5.7. *If $\mathcal{T} \in \mathsf{MCS}^{\preceq}_\mathsf{L}(\mathcal{S})$ and $\mathcal{S}' \subseteq \mathcal{S}$ is a consistent set, then $\mathcal{S}' \nprec \mathcal{T}$.*

Proof. Since $\mathcal{S}'$ is consistent, there is a $\mathcal{S}'' \in \mathsf{MCS}_\mathsf{L}(\mathcal{S})$ such that $\mathcal{S}' \subseteq \mathcal{S}''$. By the left monotonicity of $\preceq$, $\mathcal{S}'' \preceq \mathcal{S}'$. Since $\mathcal{T} \in \mathsf{MCS}^{\preceq}_\mathsf{L}(\mathcal{S})$, $\mathcal{S}'' \nprec \mathcal{T}$ and by the transitivity of $\preceq$ also $\mathcal{S}' \nprec \mathcal{T}$. □

Lemma 5.8. *If $\mathcal{S}$ is finite and $\mathcal{T} \in \mathsf{MCS}^{\preceq}_\mathsf{L}(\mathcal{S})$, then $Arg_\mathsf{L}(\mathcal{T}) \in \mathsf{Stb}(\mathcal{AF}^{\preceq}_\mathsf{L}(\mathcal{S}))$.*

Proof. Suppose that $\mathcal{T} \in \mathsf{MCS}^{\preceq}_\mathsf{L}(\mathcal{S})$ and $\mathcal{E} = \mathrm{Arg}_\mathsf{L}(\mathcal{T})$. We show that $\mathcal{E}$ is stable.

Assume for a contradiction that there are $a = \Gamma \Rightarrow \gamma$ and $b = \Delta \Rightarrow \delta$ in $\mathcal{E}$ such that a attacks b. Then $\Rightarrow \gamma \leftrightarrow \neg\delta'$, where $\delta' \in \Delta$, is derivable in C. But then $\Rightarrow \neg \bigwedge(\Gamma \cup \{\delta'\})$ is derivable in C by Cut and Contraposition. Since $\Gamma \cup \{\delta'\} \subseteq \mathcal{T}$ this is a contradiction to the consistency of $\mathcal{T}$.

Suppose that $b = \Theta \Rightarrow \tau \in \mathrm{Arg}_\mathsf{L}(\mathcal{S}) \setminus \mathcal{E}$. Thus, $\Theta \setminus \mathcal{T} \neq \emptyset$. Suppose first that Θ is inconsistent. Then $\Rightarrow \neg \bigwedge \Theta$ is derivable in C, it attacks b and it is in $\mathcal{E}$ since it has no attackers.

Now suppose that Θ is consistent. By Lemma 5.7, $\Theta \nprec \mathcal{T}$. Let $\tau \in \Theta \setminus \mathcal{T}$. Thus, there is a finite $\Gamma' \subseteq \mathcal{T}$ for which $\Gamma' \Rightarrow \neg\tau$ is derivable. By monotonicity, $a = \mathcal{T} \Rightarrow \neg\tau \in \mathcal{E}$.[11] Since $\Theta \nprec \mathcal{T}$, a attacks b.

Thus, whether Θ is consistent or not, we have shown that $\mathcal{E}$ attacks any argument in $\mathrm{Arg}_\mathsf{L}(\mathcal{S}) \setminus \mathcal{E}$, and so $\mathcal{E}$ is stable. □

Lemma 5.9. *If $\mathcal{E} \in \mathsf{Cmp}(\mathcal{AF}^{\preceq}_\mathsf{L}(\mathcal{S}))$, there is a $\mathcal{T} \subseteq \mathcal{S}$ for which $\mathcal{E} = \mathrm{Arg}_\mathsf{L}(\mathcal{T})$.*

Proof. Let $\mathcal{T} = \bigcup_{a \in \mathcal{E}} \mathsf{Sup}(a)$, $\Gamma \subseteq \mathcal{T}$, and $b = \Gamma \Rightarrow \phi \in \mathrm{Arg}_\mathsf{L}(\mathcal{S})$. By weak exhaustiveness (Proposiotion 4.4), $b \in \mathcal{E}$. Thus, $\mathcal{E} = \mathrm{Arg}_\mathsf{L}(\mathcal{T})$. □

Lemma 5.10. *If $\mathcal{S}$ is finite and $\mathcal{E} \in \mathsf{Prf}(\mathcal{AF}^{\preceq}_\mathsf{L}(\mathcal{S}))$, there is some $\mathcal{T} \in \mathsf{MCS}^{\preceq}_\mathsf{L}(\mathcal{S})$ for which $\mathcal{E} = Arg_\mathsf{L}(\mathcal{T})$.*

Proof. By Lemma 5.9 there is a $\mathcal{T} \subseteq \mathcal{S}$ for which $\mathcal{E} = \mathrm{Arg}_\mathsf{L}(\mathcal{T})$. Assume first for a contradiction that $\mathcal{T}$ is inconsistent. Thus, there is a $\Gamma \subseteq \mathcal{T}$ for which $a = \Rightarrow \neg \bigwedge \Gamma$ is derivable in C. By weak exhaustiveness (Proposition 4.4), $b = \Gamma \Rightarrow \bigwedge \Gamma \in \mathcal{E}$. b is attacked by a and cannot be defended, which is a contradiction to the fact that $\mathcal{E}$ is admissible. Thus, $\mathcal{T}$ is consistent.

Suppose now for a contradiction that there is a $\mathcal{T}' \in \mathsf{MCS}^{\preceq}_\mathsf{L}(\mathcal{S})$ for which $\mathcal{T}' \prec \mathcal{T}$. By Lemma 5.8, $\mathrm{Arg}_\mathsf{L}(\mathcal{T}') \in \mathsf{Stb}(\mathcal{AF}^{\preceq}_\mathsf{L}(\mathcal{S}))$.

[10] As noted in [19], in ASPIC[+] this problem is avoided as well.

[11] Note that, if $\mathcal{S}$ would be infinite, $\mathcal{T}$ might be infinite as well, in which case a would not be a valid sequent.

Since $\mathcal{E} \in \mathsf{Prf}(\mathcal{AF}^{\leq}_{\mathsf{L}}(\mathcal{S}))$, $\mathrm{Arg}_{\mathsf{L}}(\mathcal{T}) \setminus \mathrm{Arg}_{\mathsf{L}}(\mathcal{T}') \neq \emptyset$, thus there is a $\gamma \in \mathcal{T} \setminus \mathcal{T}'$. Then there is a $\Delta \subseteq \mathcal{T}'$ for which $\Delta \Rightarrow \neg\gamma$ is derivable in C. By monotonicity, also $c = \mathcal{T}' \Rightarrow \neg\gamma$ and $d = \mathcal{T} \Rightarrow \gamma$ are derivable in C (note that $\gamma \Rightarrow \gamma$ is derivable as well). Since $\mathcal{T}' \prec \mathcal{T}$, c attacks d. Thus, there is a $e = \Theta \Rightarrow \theta \in \mathcal{E}$ which attacks c. Hence, $\mathcal{T}' \not\prec \Theta$. By left monotonicity and since $\Theta \subseteq \mathcal{T}$, $\mathcal{T} \leq \Theta$. By transitivity, $\mathcal{T}' \leq \Theta$, which is a contradiction. Altogether, this shows that $\mathcal{T} \in \mathsf{MCS}^{\leq}_{\mathsf{L}}(\mathcal{S})$. □

Now we can show Proposition 5.6:

Proof.

- (⇐) Suppose that $\mathcal{S} \mid\!\sim^{\leq}_{\mathsf{L},\cap\mathsf{mcs}} \phi$ and let $\mathcal{E} \in \mathsf{Prf}(\mathcal{AF}^{\leq}_{\mathsf{L}}(\mathcal{S}))$. By Lemma 5.10, there is a $\mathcal{T} \in \mathsf{MCS}^{\leq}_{\mathsf{L}}(\mathcal{S})$ for which $\mathcal{E} = \mathrm{Arg}_{\mathsf{L}}(\mathcal{S})$. By the assumption $\mathcal{T} \vdash \phi$, hence $\mathcal{T} \Rightarrow \phi \in \mathcal{E}$. This shows that $\mathcal{S} \mid\!\sim^{\cap,\leq}_{\mathsf{L},\mathsf{prf}} \phi$, which also implies that $\mathcal{S} \mid\!\sim^{\cap,\leq}_{\mathsf{L},\mathsf{stb}} \phi$.
- (⇒) Suppose that $\mathcal{S} \mid\!\sim^{\cap,\leq}_{\mathsf{L},\mathsf{stb}} \phi$ and let $\mathcal{T} \in \mathsf{MCS}^{\leq}_{\mathsf{L}}(\mathcal{S})$. By Lemma 5.8, $\mathcal{E} = \mathrm{Arg}_{\mathsf{L}}(\mathcal{T}) \in \mathsf{Stb}(\mathcal{AF}^{\leq}_{\mathsf{L}}(\mathcal{S}))$. Thus, there is a $\Delta \Rightarrow \phi \in \mathcal{E}$ for which $\Delta \subseteq \mathcal{T}$. Hence $\mathcal{T} \vdash \phi$, which shows that $\mathcal{S} \mid\!\sim^{\leq}_{\mathsf{L},\cap\mathsf{mcs}} \phi$. □

Remark 5. Lemma 5.10 (and so Proposition 5.6) does not hold for infinite sets (for instance, for the orderings in Example 3.2, Items 6 and 7). Here is an example: let $\mathcal{AF}^{\leq}_{\mathsf{CL}}(\mathcal{S}) = \langle \mathrm{Arg}_{\mathsf{CL}}(\mathcal{S}), \mathcal{AT}, \leq \rangle$ be a prioritized sequent-based argumentation framework, with CL as core logic, DUcut and ConUcut as attack rules and $\mathcal{S} = \{p_i \mid i \geq 1\} \cup \{q, \neg q\}$ where $\pi(p_i) = 1$ for all $i \geq 1$, $\pi(q) = 2$ and $\pi(\neg q) = 3$. We have two MCSs, $\mathcal{T} = \{p_i \mid i \geq 1\} \cup \{q\}$ and $\mathcal{T}' = \{p_i \mid i \geq 1\} \cup \{\neg q\}$ where $\mathcal{T} \prec \mathcal{T}'$. Nevertheless, $\mathrm{Arg}(\mathcal{T}')$ is a stable extension of $\mathcal{AF}^{\leq}_{\mathsf{CL}}(\mathcal{S})$.[12]

6 CONCLUSION

Sequent-based argumentation frameworks provide general and modular formalisms for representing arguments and reasoning with them, using different kinds of languages, logics, and attacks.[13] The goal of this work is to carry these formalisms a step forward and to incorporate external information in the form of priorities that the reasoner might want to introduce for properly choosing the arguments that can be mutually accepted. Once the priorities have been decided, different orders may be defined for making preferences among the underlying arguments, and accordingly applying entailment relations for drawing conclusions from a given set of assertions.

Clearly, the entailment relations that are induced by a prioritized framework depend on many factors, among which are the choice of the core logics, the attack relations, and the preferences among the arguments. We have shown that in many cases these choices provide the reasoner with a robust framework, satisfying rationality postulates and enjoying other desirable properties, like conflict preservation and strong links to reasoning with the most preferred maximally consistent subsets of the premises, a well-studied approach for handling inconsistent information.

There are several other formalisms for supporting prioritized data in the context of argumentation systems. A detailed comparison to some of these formalisms will be provided in the full version of this work. Here we only mention one of them, the ASPIC$^+$ framework [19], which also provides a general setting for (prioritized) logical argumentation. Apart of the different representations of objects (like arguments and attacks) in the two frameworks, a primary difference from ASPIC$^+$ is that our approach is more proof theoretically oriented, using tools and methods (like sequents and their derivations by proof systems) from proof theory. Among others, in future work we plan to strengthen this characteristic of our approach and provide dynamic proof systems [9] for non-monotonically reasoning with the prioritized data in a proof-like manner.

We conclude this paper by exemplifying some of the advantages of our approach using the puzzle given in the introduction (Example 1.1).

Example 6.1 (Example 1.1 continued). Recall the flat owner negotiating with potential tenants about the construction of a swimming pool (s), a tennis-court (t) and a private car-park (p). The consideration that the rent (r) increases if more than one facility is constructed can be represented by the formula $\psi_1 = r \leftrightarrow ((s \wedge t) \vee (s \wedge p) \vee (t \wedge p))$. The preferences of the tenants not to increase the rent and not to have two sport facilities are modeled by $\neg r$ and by $\psi_2 = s \supset \neg t$ and $\psi_3 = t \supset \neg s$, respectively.

This situation may be represented by a prioritized sequent-based framework $\mathcal{AF}^{\leq}_{\mathsf{CL}}(\mathcal{S}) = \langle \mathrm{Arg}_{\mathsf{CL}}(\mathcal{S}), \mathcal{AT}, \leq \rangle$, induced by classical logic, Ucut and ConUcut as attack rules, and set of formulas $\mathcal{S} = \{s, t, p, \neg r, \psi_1, \psi_2, \psi_3\}$, where $\pi(\neg r) = 1$, $\pi(\psi_1) = \pi(\psi_2) = \pi(\psi_3) = 2$ and $\pi(s) = \pi(t) = \pi(p) = 3$. We take the preference relation by the $\leq_s$ comparison (Item 6 of Example 3.2).

- We have that $\neg r \leftrightarrow \big(\neg(s \wedge t) \wedge \neg(s \wedge p) \wedge \neg(t \wedge p)\big)$ classically follows from ψ_1, which implies that $\neg r, \psi_1 \Rightarrow \neg(x \wedge y)$ is in $\mathrm{Arg}_{\mathsf{CL}}(\mathcal{S})$ for every distinct $x, y \in \{p, s, t\}$. It follows that every argument of the form $x, y \Rightarrow x \wedge y$ for such x, y (suggesting to construct two facilities) is Ucut-attacked by a more preferred argument.
- Arguments such as $s \Rightarrow s$ and $t \Rightarrow t$, which suggest to construct a swimming pool and a tennis court are respectively attacked by the more preferred arguments $t, \psi_3 \Rightarrow \neg s$ and $s, \psi_2 \Rightarrow \neg t$.
- The argument $a = p \Rightarrow p$, suggesting to construct a car park, is attacked by $b = \neg r, s, \psi_1 \Rightarrow \neg p$. However, the argument $a' = p, \neg r, \psi_1, \psi_2, \psi_3 \Rightarrow p$ for the same conclusion is not attacked by b since $a' < b$. In fact, a' is only attacked by arguments whose support set is classically inconsistent, for instance $\mathcal{S} \Rightarrow \neg p$. These attacks are counter ConUcut attacked by the tautological argument $\Rightarrow \neg \bigwedge \mathcal{S}$ and so a' is defended.

From the above considerations it follows that the only sequents of the form $\Gamma \Rightarrow x$ for some $\Gamma \subseteq \mathcal{S}$ and $x \in \{s, t, p\}$ that belong to the grounded extension of the prioritized sequent-based argumentation framework under consideration, are those in which $x = p$. That is, based on the considerations and the preferences stated above, according to the grounded semantics of the framework, the flat owner should decide to build only a parking lot.

[12] Note that for every $s = \Gamma, q \Rightarrow \phi \in \mathrm{Arg}_{\mathsf{CL}}(\mathcal{S}) \setminus \mathrm{Arg}_{\mathsf{CL}}(\mathcal{T}')$ (where $\Gamma \subseteq \{p_i \mid i \geq 1\}$) the argument $t' = \Gamma, p_k, \neg q \Rightarrow \neg q \in \mathrm{Arg}_{\mathsf{CL}}(\mathcal{T}')$ (where $p_k \notin \Gamma$) attacks s.

[13] For a discussion of the advantages of this approach we refer to [6–8]

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