A small-scale wind turbine is shown against a solid green background. The turbine has three blades and is mounted on a lattice tower. The text is overlaid on the image.

08-Off-Grid Wind Energy Conversion Systems

Off-Grid Electrical Systems in Developing Countries, 2nd Edition

Chapter 8

Preface

- These lectures slides are intended to accompany the textbook *Off-Grid Electrical Systems in Developing Countries, 2nd Edition, 2025* written by Dr. Henry Louie and published by [SpringerNature](#)
- Additional content, explanations, derivations, examples, problems, errata, and other materials are found in the book and on www.drhenrylouie.com
- To request solutions, explanations, permissions to use author-supplied images, or if you notice an error, please email the author at hlouie@ieee.org
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Learning Outcomes

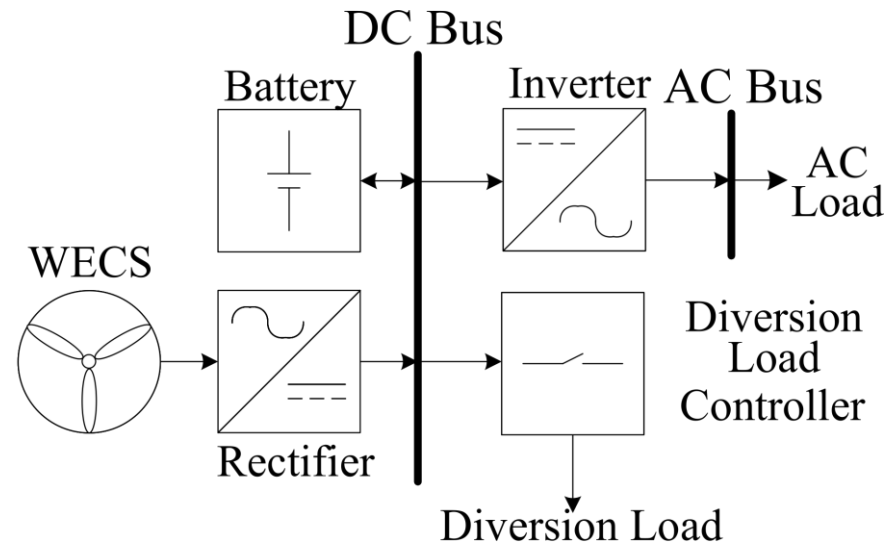
At the end of this lecture, you will be able to:

- ✓ Describe the use of wind energy conversion systems (WECS) in off-grid electrification, emphasizing the key technical, economic, environmental, and social considerations
- ✓ Explain the core principles of the wind energy conversion process and the factors that influence the operation of a WECS
- ✓ Calculate mechanical and electrical quantities relevant to WECS in off-grid applications by applying appropriate mathematical and circuit models
- ✓ Apply a WECS's power curve to estimate power and energy production from a wind resource

Introduction

- Wind energy conversion systems (WECS) can be used in off-grid systems in areas with sufficient wind resources (at least 4-6 m/s at tower height)
- Usually DC-coupled and often in hybrid systems with PV arrays
- Becoming less common due to reduced cost of solar arrays and availability of online, high-quality solar resource data

Introduction



Wind Turbine Types

- What wind types of wind turbines are there?
- Generally classified as
 - Vertical Axis Wind Turbines (VAWT)
 - Horizontal Axis Wind Turbines (HAWT)

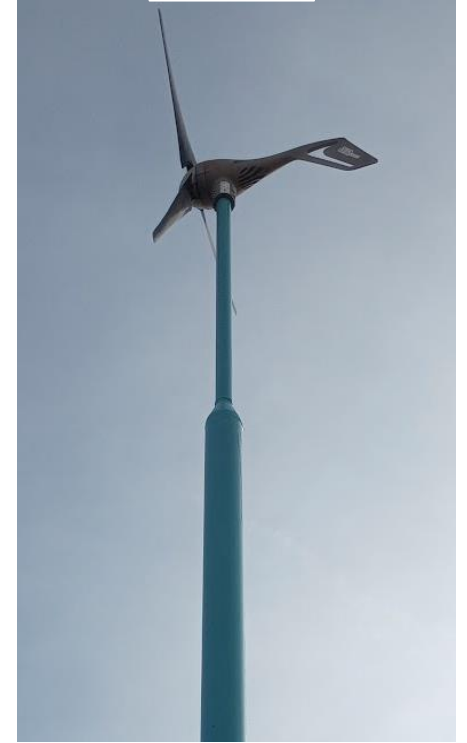
VAWT are not commonly used,
so we focus on HAWT

VAWT



Grendelkhan CC BY-SA 4.0

HAWT

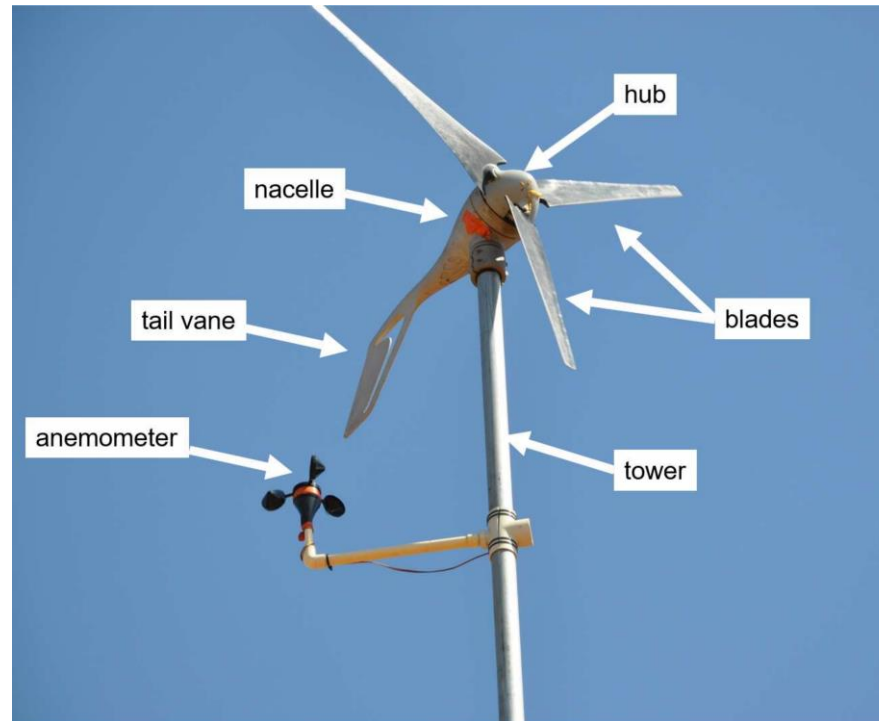


(courtesy H. Louie)

Wind Turbine Types

- Vertical Axis Wind Turbines (VAWT) are less common but have niche applications
- Advantages:
 - do not have to face the wind to harness energy
 - generator is located at the base which has mechanical advantages
- VAWTs are more expensive

Principles of Operation

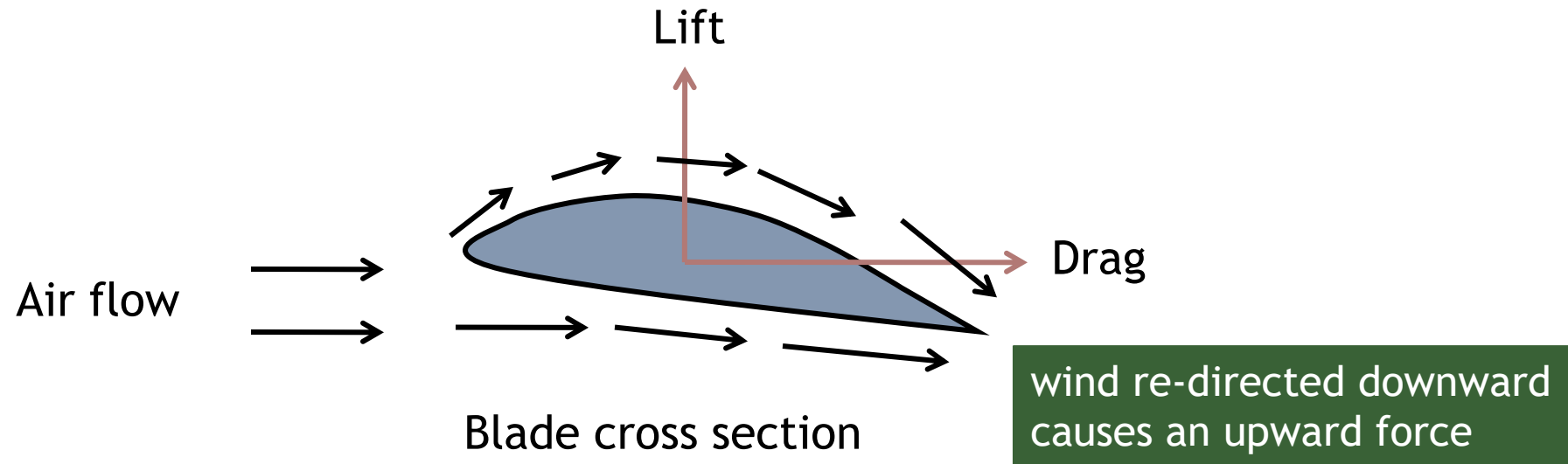


(courtesy H. Louie)

400 W three-blade
wind turbine with
blade length of 0.5 m

Principles of Operation

view: looking into a blade from the tip toward the hub

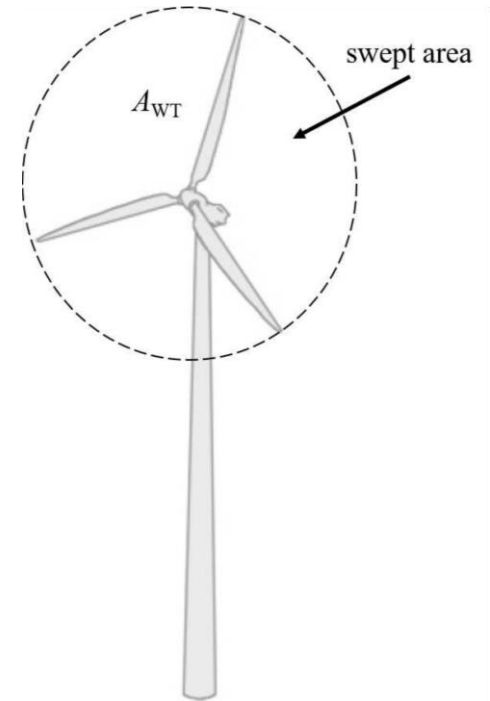


Principles of Operation

- WECS convert kinetic energy in a mass of moving air to electrical energy
- Wind turbine blades sweep out an area as they rotate

$$A_{WT} = \pi r_{WT}^2$$

- This interaction with the air creates lift on the blades, causing them to rotate



Power in the Air

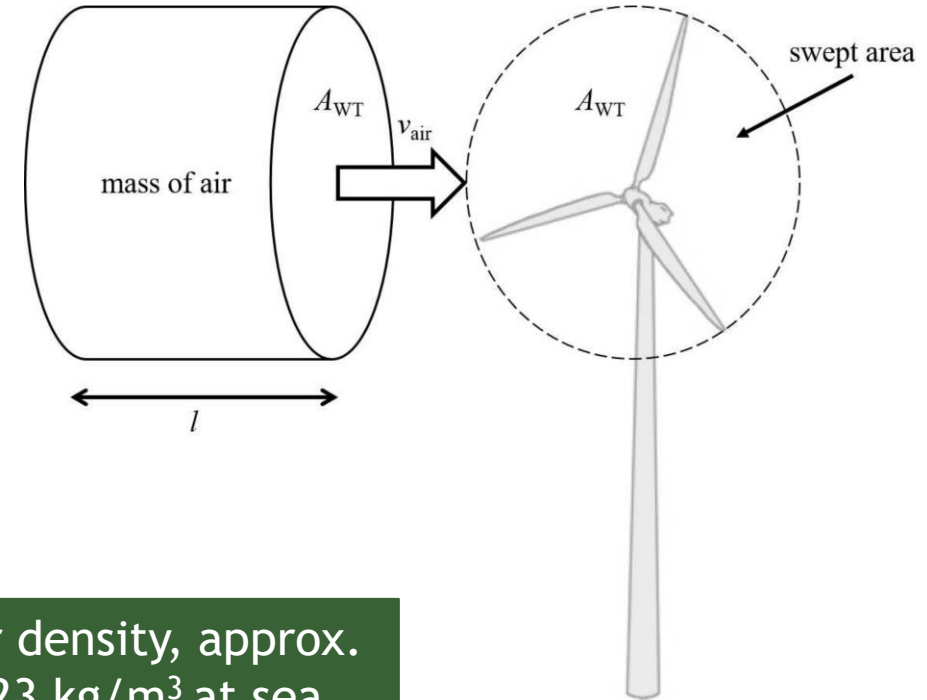
- WECS interacts with a cylinder of air through the swept area
- Mass of air

$$m_{\text{air}} = A_{\text{WT}} \times l \times \rho_{\text{air}} = A_{\text{WT}} \times v_{\text{air}} \times t \times \rho_{\text{air}}$$

$$l = v_{\text{air}} \times t$$

length of the cylinder is equal to the velocity of the air multiplied by time

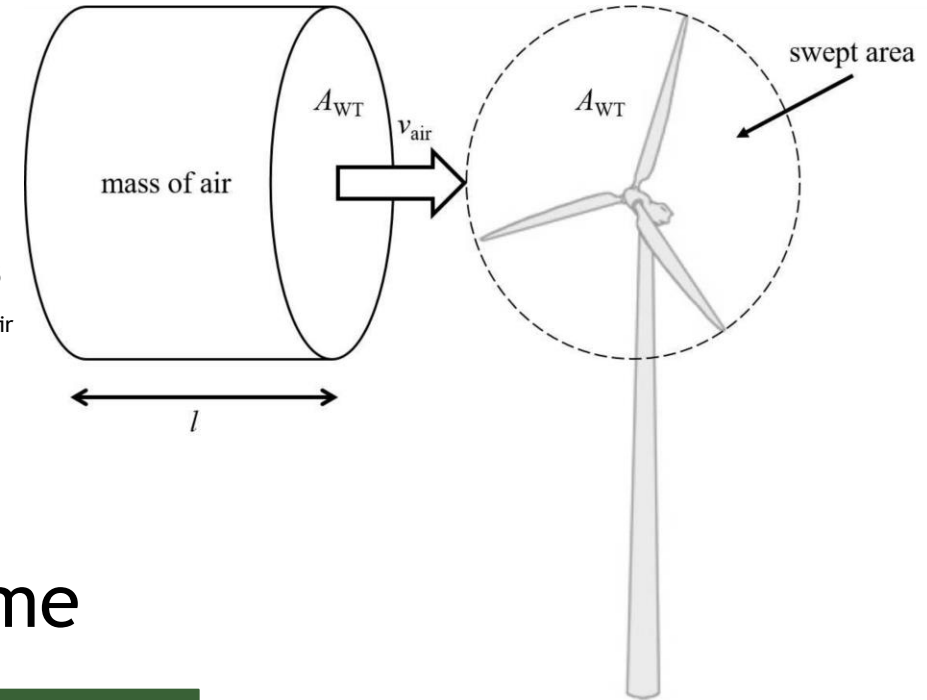
air density, approx. 1.23 kg/m³ at sea level



Power in the Air

Kinetic energy in moving air:

$$E_{\text{air}} = \frac{1}{2} m_{\text{air}} v_{\text{air}}^2 = \frac{1}{2} \underbrace{A_{\text{WT}} l \rho_{\text{air}}}_{\text{volume}} v_{\text{air}}^2 = \frac{1}{2} A_{\text{WT}} v_{\text{air}} t \rho_{\text{air}} v_{\text{air}}^2 = \frac{1}{2} A_{\text{WT}} t \rho_{\text{air}} v_{\text{air}}^3$$



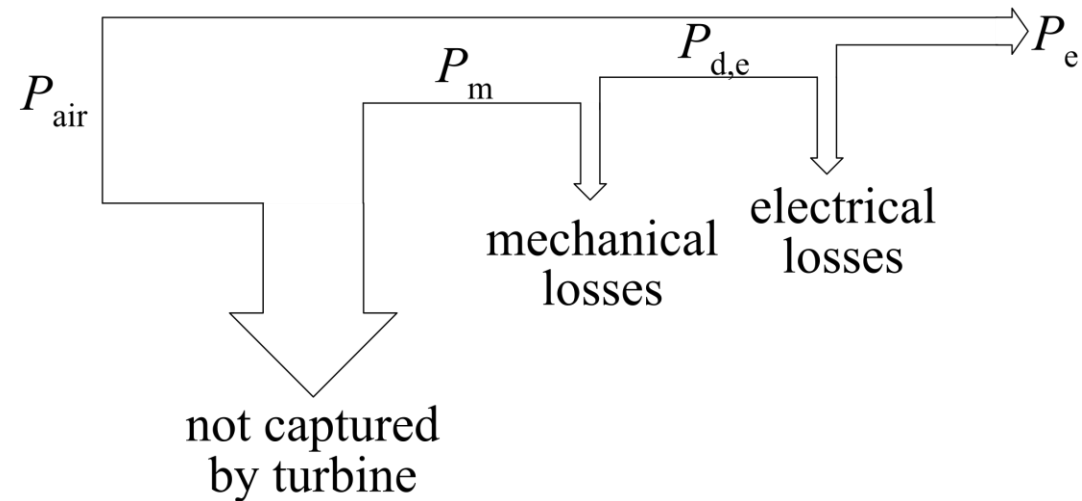
Power is the derivative of energy w.r.t. time

$$P_{\text{air}} = \frac{dE_{\text{air}}}{dt} = \frac{1}{2} A_{\text{WT}} \rho_{\text{air}} v_{\text{air}}^3$$

power increases with cube of wind speed!

Power in the Air

Not all the power in the air is captured or converted by the WECS



Mechanical Power

- A portion of the power in the air is converted by the turbine into mechanical power (rotating the generator's shaft)

$$P_m = C_p P_{\text{air}} = \frac{1}{2} C_p A_{\text{WT}} \rho_{\text{air}} v_{\text{air}}^3$$

power coefficient

- C_p is usually between 0.2 and 0.4

higher C_p means more
efficient energy conversion

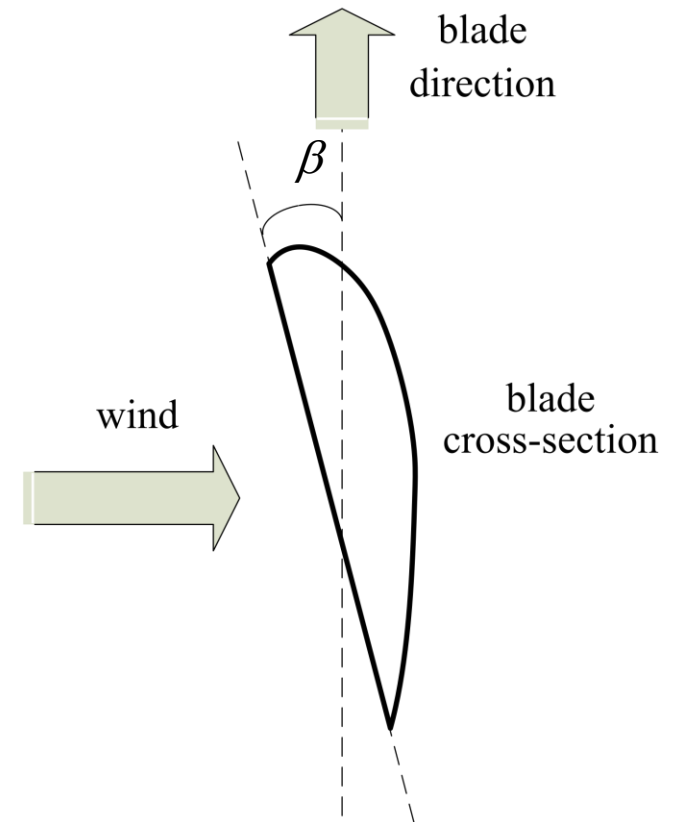
Mechanical Power

Power coefficient is not constant and varies with

- pitch angle of blades β
- tip speed ratio (TSR) λ
- number of blades
- other factors

Pitch

- Pitch: angle between the plane of rotation and chord of the blade
- Changing β affects the aerodynamics of the turbine and hence C_p
- Pitch is fixed in most small-scale turbines, but is often adjustable in larger turbines



Tip Speed Ratio (TSR)

- TSR (λ): ratio of the tangential velocity of the tip of the blade to the velocity of the incident air

$$\lambda = \frac{U}{v_{\text{air}}}$$

← speed of the tip of the blade

- Tip speed depends on blade length and rotation speed

$$U = \omega_m r_{\text{WT}} = 2\pi N_m r_{\text{WT}} \times \frac{1}{60}$$

← speed in RPM

Example 8.1

A small wind turbine has a blade length of 0.5 m. The wind turbine is rotating at 800 RPM when the wind speed is 8 m/s. Compute the power in the air incident to the wind turbine and the TSR.

Example 8.1

A small wind turbine has a blade length of 0.5 m. The wind turbine is rotating at 800 RPM when the wind speed is 8 m/s. Compute the power in the air incident to the wind turbine and the TSR.

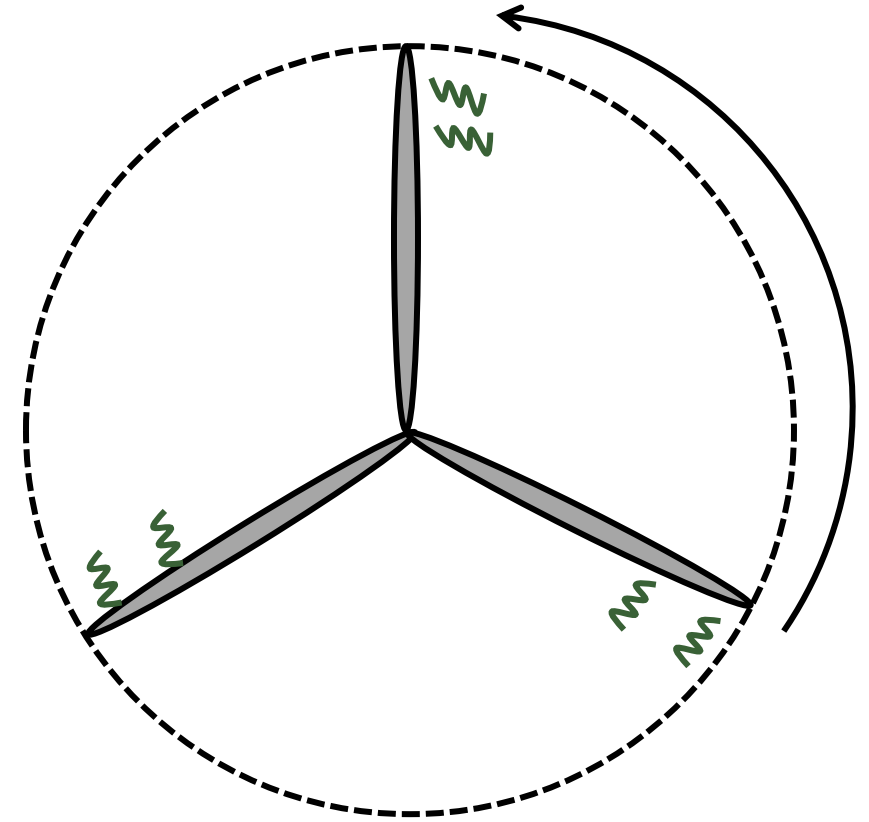
$$A_{\text{WT}} = \pi \times 0.5^2 = 0.79 \text{ m}^2$$

$$P_{\text{air}} = \frac{1}{2} A_{\text{WT}} \rho_{\text{air}} v_{\text{air}}^3 = \frac{1}{2} \times 0.79 \times 1.23 \times 8^3 = 247.3 \text{ W}$$

$$U = 2\pi N_{\text{m}} r_{\text{WT}} \times \frac{1}{60} = 2\pi \times 800 \times 0.5 \times \frac{1}{60} = 41.9 \text{ m/s}$$

Optimal TSR

- Desirable for the turbine blades to interact with as much air as possible (high rotational speed)
- Rotating too fast causes each blade to encounter turbulence left by the blade “ahead” of it
 - turbulence dissipates more quickly with shorter blades and faster air speed
 - as air speed increases, the turbine can rotate faster without encountering turbulence
 - this implies that the turbine should rotate faster as wind speed increases



Optimal TSR

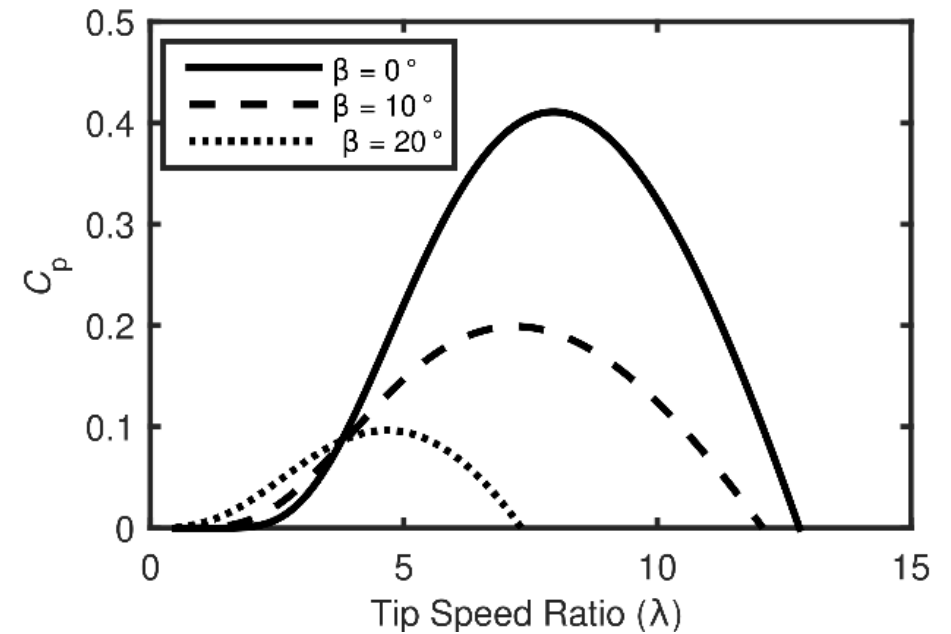
- Constant TSR means that tip speed (and hence rotational speed) increase with velocity of wind speed, which is desirable

$$\lambda = \frac{U}{v_{\text{air}}}$$

- Optimal TSR for 3-bladed turbines typically 6-8, depending on pitch angle

C_p -curve

- Power coefficient varies with TSR and pitch
- Smaller pitch β leads to higher maximum C_p
- Rotating too fast (large TSR) or too slow (small TSR) reduces C_p
- Best to operate at optimal TSR



Speed-Mechanical Power Curve

- Rotational speed in RPM at a given TSR

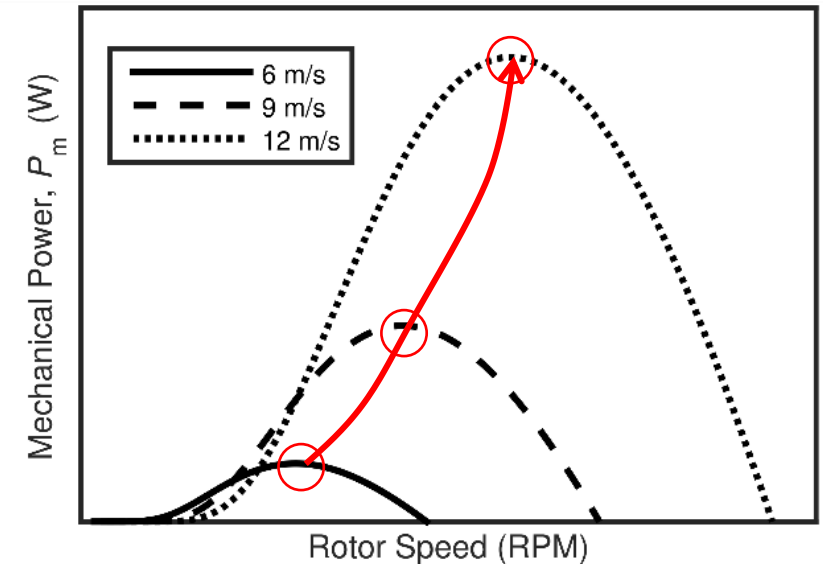
$$N_m = \lambda \times v_{\text{air}} \times 60 \times \frac{1}{2\pi r_{\text{WT}}}$$

- Mechanical power of turbine

$$P_m(N_m, v_{\text{air}}, \beta) = \frac{1}{2} C_p(N_m, v_{\text{air}}, \beta) A_{\text{WT}} \rho_{\text{air}} v_{\text{air}}^3$$

C_p depends on rotational speed, air velocity, and pitch

rotor speed must increase with wind speed to maximize P_m



Example 8.2

An energy kiosk uses a wind turbine to recharge lead-acid batteries. The blade length is 3.2 m. The turbine has three blades, and C_p is maximized when the TSR is 6.84. Compute the required rotor speed of the generator, in RPM, for the turbine to operate at optimum efficiency when the wind speed is 12 m/s. Compute the frequency of the produced AC voltage, assuming the turbine is directly coupled to an eight-pole synchronous generator.

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The speed that the tip of the rotor blade must travel to maximize the performance coefficient is

$$U = v_{\text{air}} \lambda = 12 \times 6.84 = 82.08 \text{ m/s}$$

Example 8.2

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Since the blade length is 3.2 m, the turbine must rotate at

$$\omega_m = \frac{U}{r_{WT}} = \frac{82.08}{3.2} = 25.65 \text{ rad/s}$$

Example 8.2

An energy kiosk uses a wind turbine to recharge lead-acid batteries. The blade length is 3.2 m. The turbine has three blades, and C_p is maximized when the TSR is 6.84. Compute the required rotor speed of the generator, in RPM, for the turbine to operate at optimum efficiency when the wind speed is 12 m/s. Compute the frequency of the produced AC voltage, assuming the turbine is directly coupled to an eight-pole synchronous generator.

Converting to RPM

$$N_m = 25.65 \times \frac{60}{2\pi} = 245 \text{ RPM}$$

Example 8.2

An energy kiosk uses a wind turbine to recharge lead-acid batteries. The blade length is 3.2 m. The turbine has three blades, and C_p is maximized when the TSR is 6.84. Compute the required rotor speed of the generator, in RPM, for the turbine to operate at optimum efficiency when the wind speed is 12 m/s. Compute the frequency of the produced AC voltage, assuming the turbine is directly coupled to an eight-pole synchronous generator.

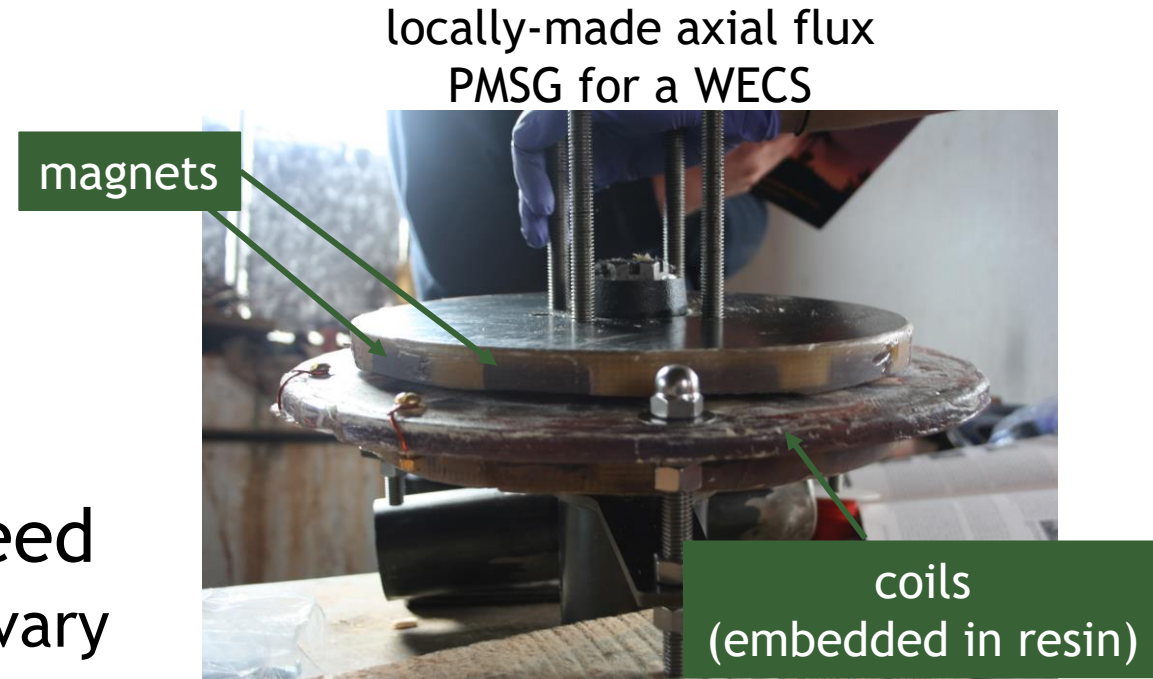
The frequency of the voltage of an eight-pole synchronous generator is

$$f_e = 245 \times \frac{8}{2 \times 60} = 16.33 \text{ Hz}$$

This is not compatible with most electrical equipment

Electrical Power

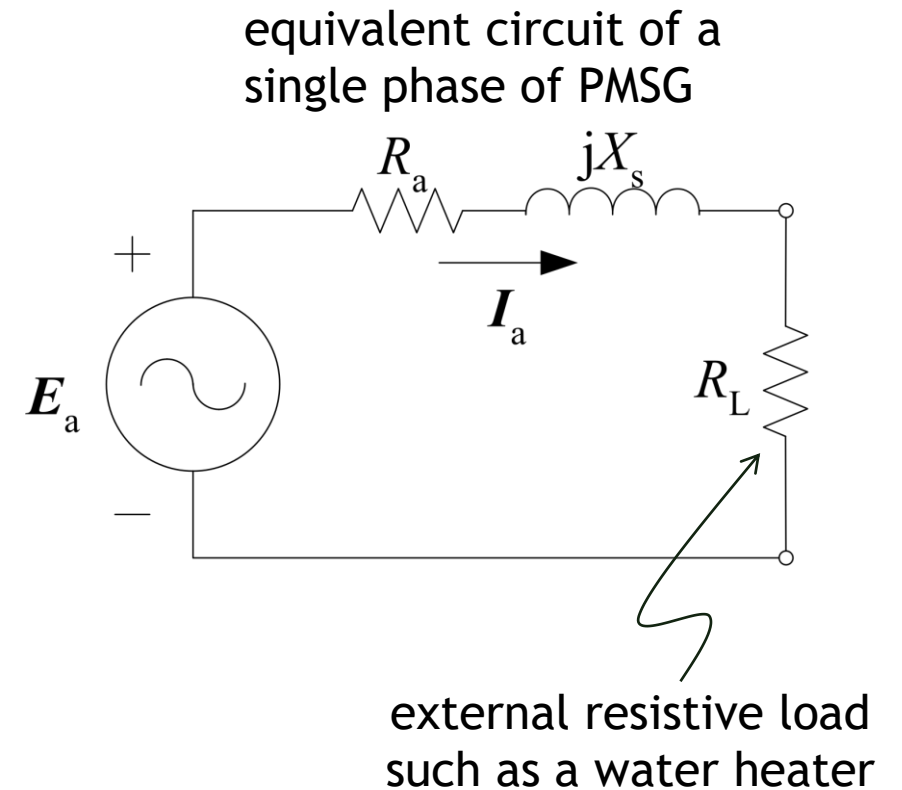
- Small-scale WECS usually use permanent magnet synchronous generators (PMSG)
- Output voltage frequency and magnitude varies with rotation speed
 - power supplied to the load also will vary with rotation speed
 - cannot be used to form AC bus



(source: H. Louie)

Equivalent Circuit

- Equivalent circuit of PMSG consists of voltage source, winding resistance, synchronous reactance
- Voltage source depends on:
 - rotational speed
 - physical characteristics of the generator
- Synchronous reactance depends on rotational speed



Equivalent Circuit

Solving for the current

$$E_a = k_g \omega_m \angle 0^\circ$$

machine constant

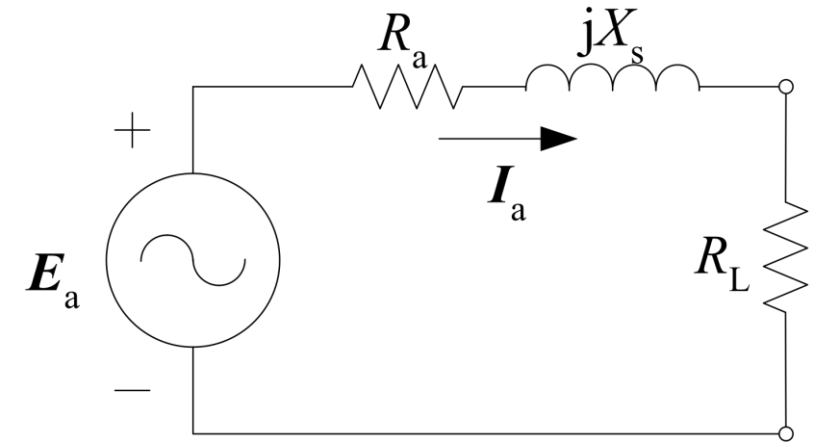
$$E_a = I_a (R_a + j\omega_e L + R_L)$$

$$k_g \omega_m = I_a \left(R_a + j \frac{p}{2} \omega_m L + R_L \right)$$

number of poles

$$I_a = \frac{k_g \omega_m}{R_a + j \frac{p}{2} \omega_m L + R_L}$$

equivalent circuit of a single phase of PMSG



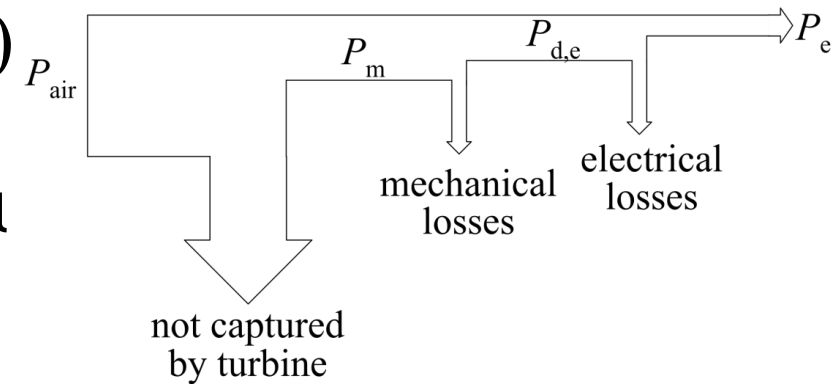
Electrical Power

Real power produced by the a three-phase WECS generator when connected to a resistive load

$$P_e = 3 |I_a|^2 R_L = 3 \frac{k_g^2 \omega_m^2}{\left| \left(R_a + j \frac{p}{2} \omega_m L + R_L \right) \right|^2} R_L = 3 \frac{k_g^2 \omega_m^2}{(R_a + R_L)^2 + \frac{p^2}{4} \omega_m^2 L^2} R_L$$

Developed Power

- Developed electrical power is the power generated by the induced voltage E_a (times three)
- Developed power is equal to
 - electrical power output by the generator PLUS electrical losses
 - mechanical power if mechanical losses are ignored



$$P_{d,e} = P_e + P_{e,loss} = 3 |I_a|^2 (R_L + R_a)$$

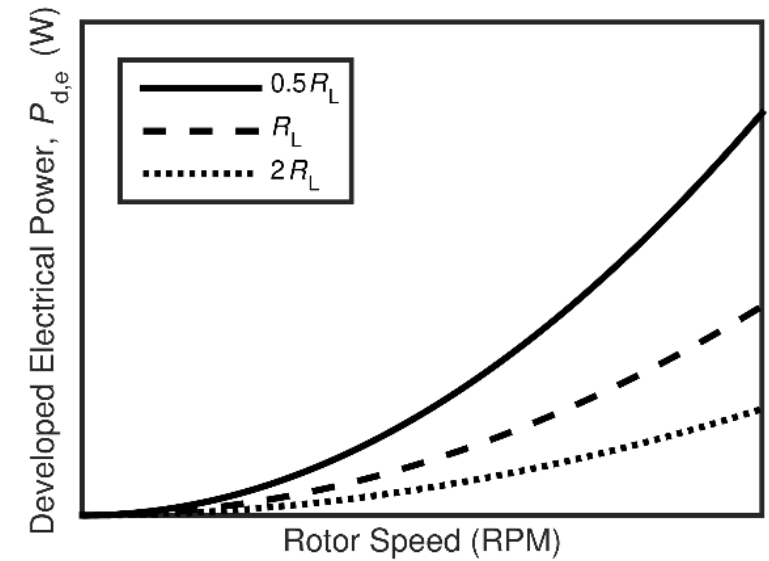
$$P_{d,e} = 3 \frac{k_g^2 \omega_m^2}{(R_a + R_L)^2 + \frac{p^2}{4} \omega_m^2 L^2} (R_L + R_a)$$

Developed Power as Function of Rotor Speed

- Developed power depends on rotational speed
 ω_m appears in numerator and denominator

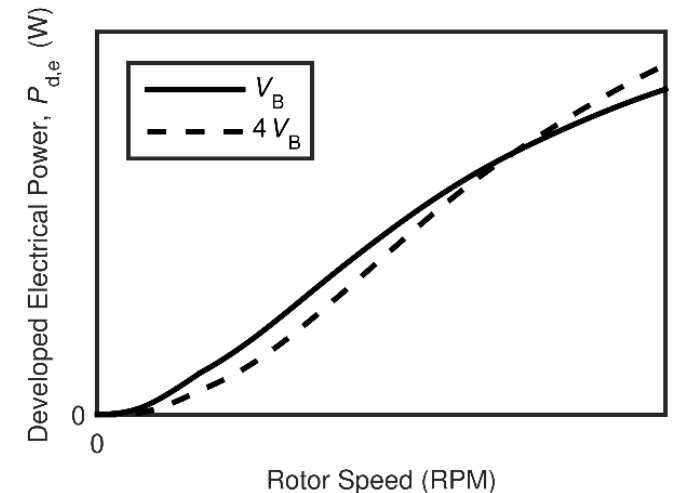
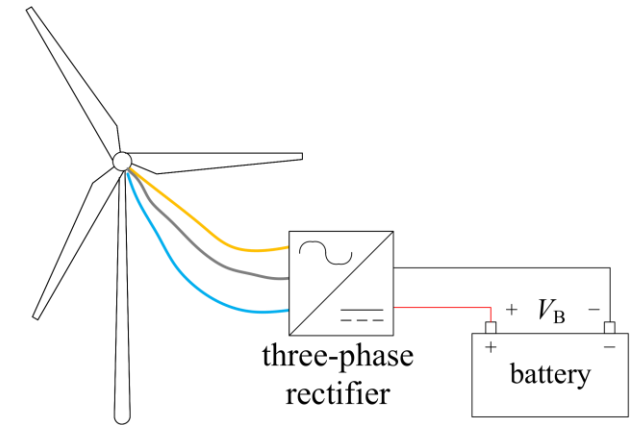
$$P_{d,e} = 3 \frac{k_g^2 \omega_m^2}{(R_a + R_L)^2 + \frac{p^2}{4} \omega_m^2 L^2} (R_L + R_a)$$

- As speed increases, so does the voltage induced in the windings (increases power) but the inductive reactance also increases (reduces current and hence, power)



Battery Charging

- WECS in off-grid systems are typically DC-coupled and used to charge batteries through a 3-phase rectifier
- Power varies with rotational speed and the voltage of the battery
 - output voltage must exceed battery voltage before charging can begin
 - higher battery voltage results in more power developed by the WECS at higher speeds



Steady-State Operation

- What determines the rotational speed at which a WECS operates?
- Recall from Chap. 6 the shaft will accelerate or decelerate until the mechanical torque (from the wind turbine) matches the electrical torque (from the generator)

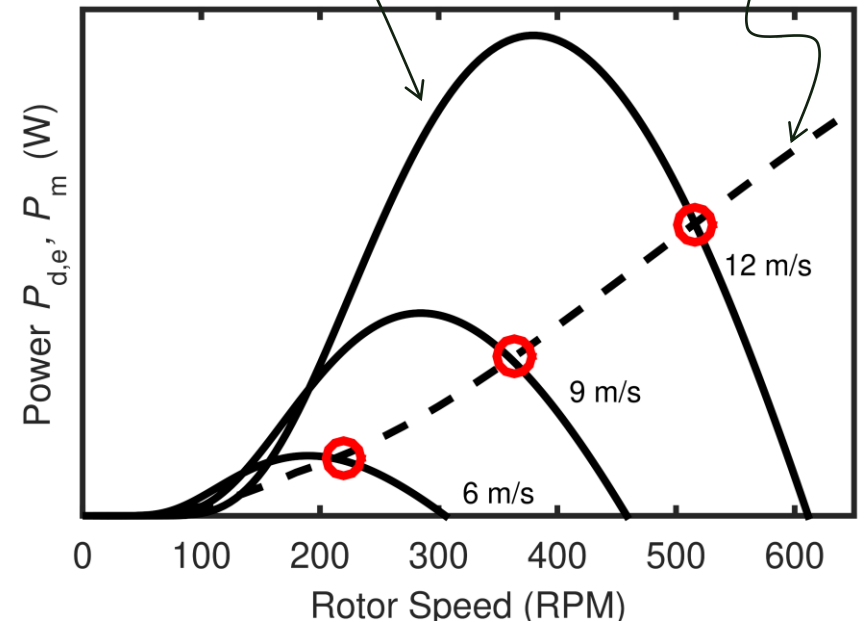
Net Torque	Condition	Result
$T_{\text{net}} > 0$	$T_{\text{m}} > T_{\text{e}}, P_{\text{m}} > P_{\text{e}}$	Rotor accelerates
$T_{\text{net}} = 0$	$T_{\text{m}} = T_{\text{e}}, P_{\text{m}} = P_{\text{e}}$	Rotor speed constant
$T_{\text{net}} < 0$	$T_{\text{m}} < T_{\text{e}}, P_{\text{m}} < P_{\text{e}}$	Rotor decelerates

Steady-State Operation

- The torques balance at the intersection of the developed electrical power and mechanical power curves
- Steady-state speed depends on wind speed and the load
- Rotational speed will increase with wind speed under most circumstances

mechanical power curve for given wind speed, pitch, TSR

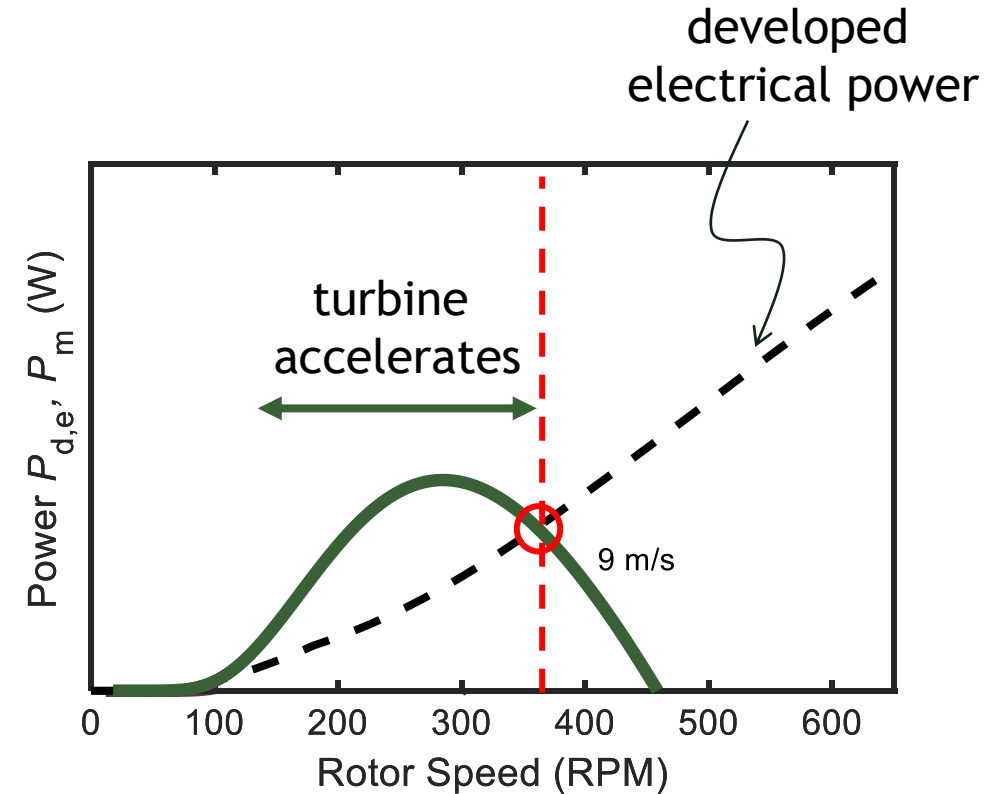
developed electrical power



Note: this load is not well matched for the wind turbine at 9 and 12 m/s wind speeds since the intersection points are far below the maximum C_p for those wind speeds

Steady-State Operation

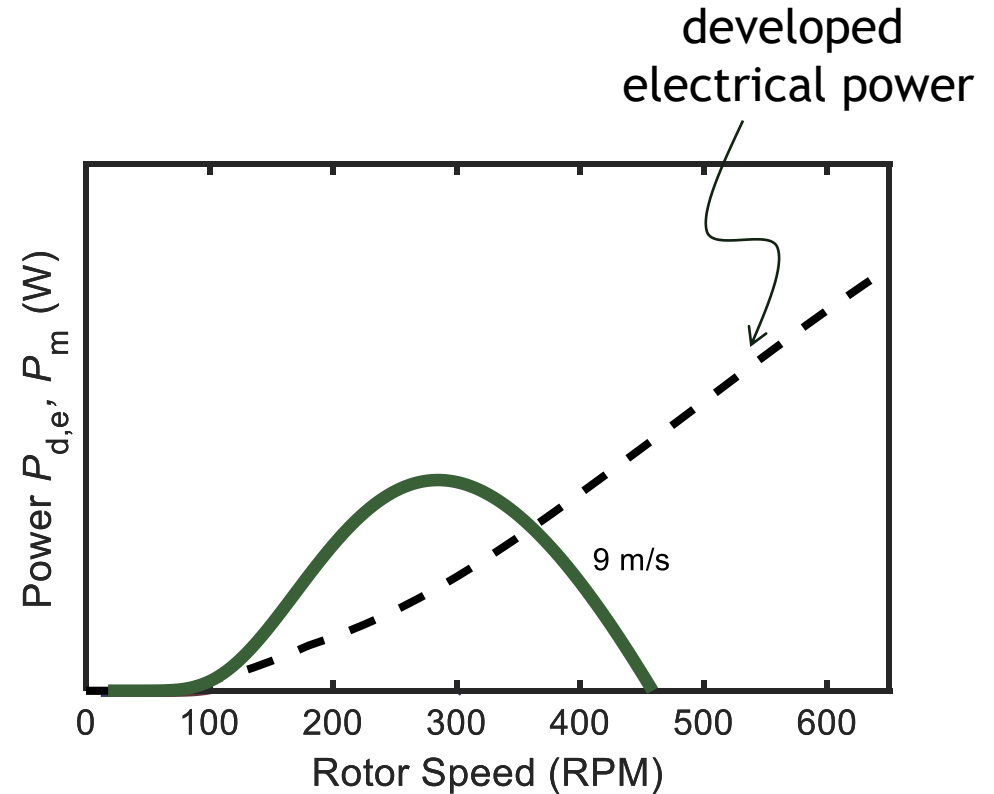
If the turbine is rotating at less than 375 RPM (speed at which the mechanical power and developed electrical power intersect), the turbine will accelerate until it reaches 375 RPM



Exercise

If the turbine is rotating at 500 RPM, what will happen to its speed?

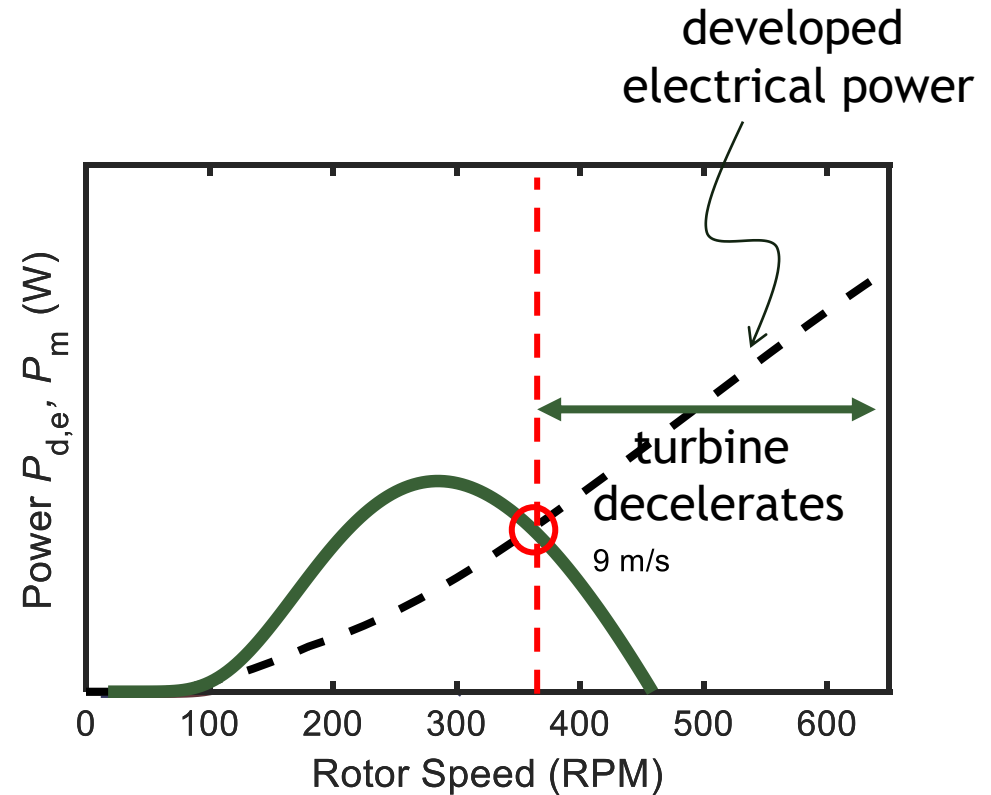
- A. Accelerate
- B. Stay at 500 RPM
- C. Decelerate
- D. Not enough information



Exercise

If the turbine is rotating at 500 RPM, what will happen to its speed?

- A. Accelerate
- B. Stay at 500 RPM
- C. Decelerate
- D. Not enough information



WECS Control

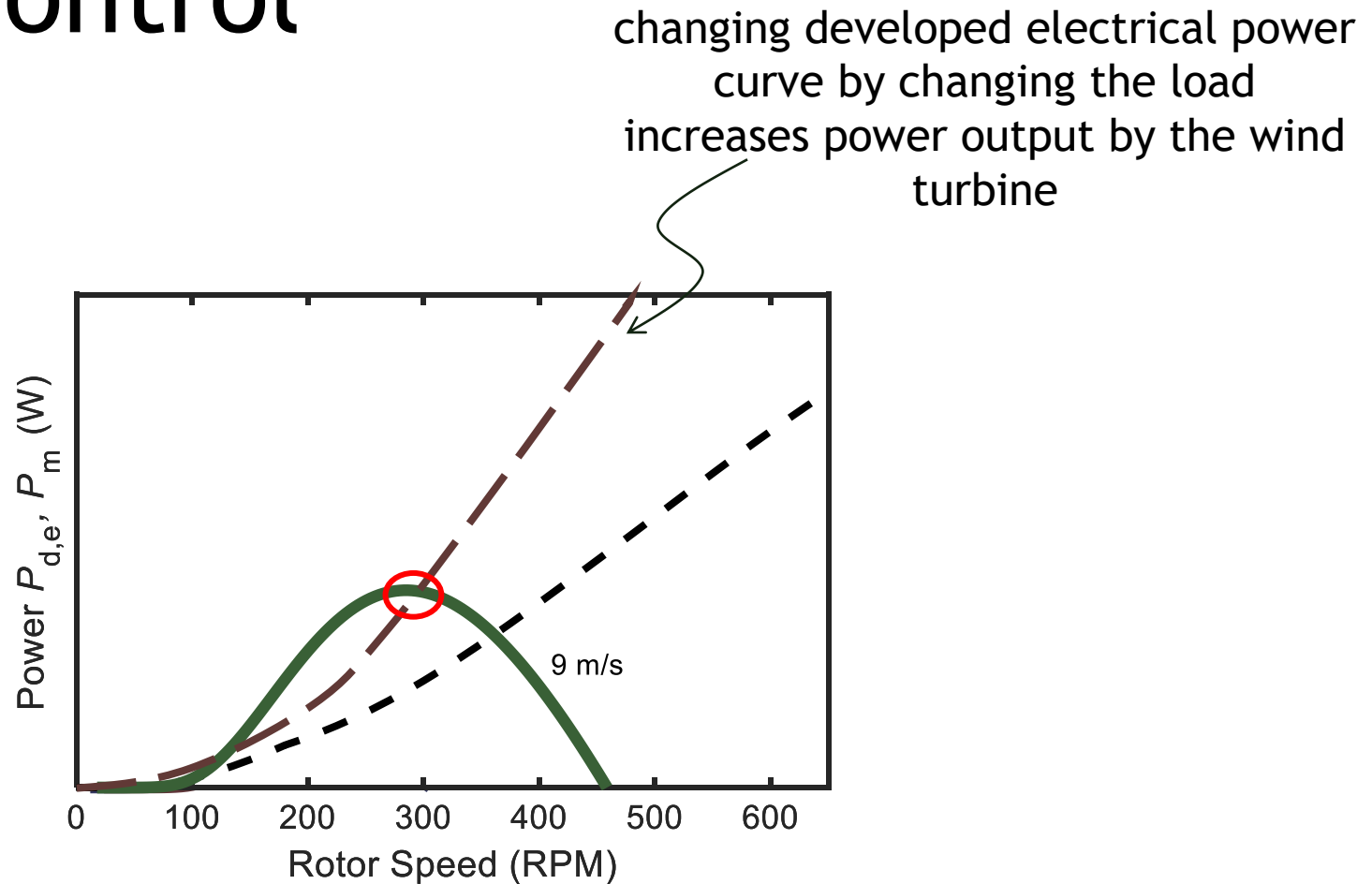
Objectives of WECS control

- improve power production by tracking optimal TSR as load and wind speed change
- avoid damage caused by over-speed operation
- avoid damage caused by power production in excess of the rated power

WECS Control Types

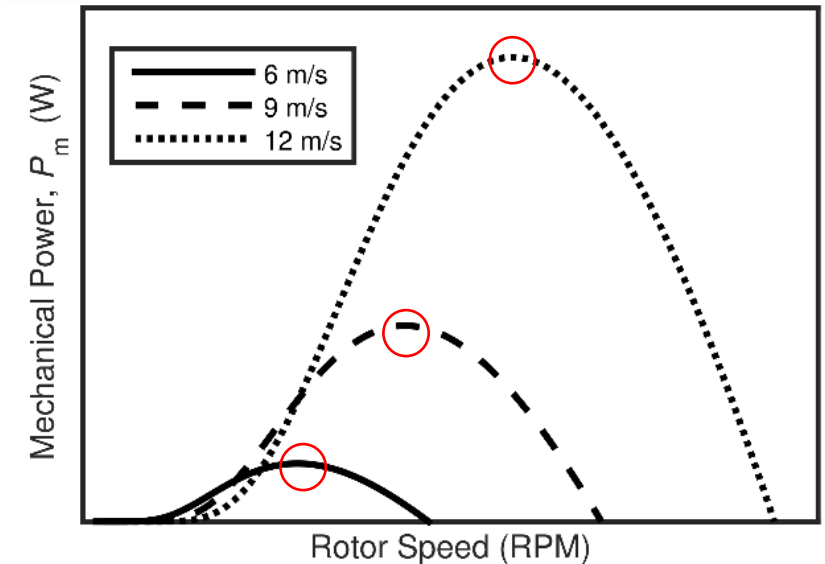
- Yaw: rotate the “face” of the wind turbine away from the wind to reduce the power of the air it interacts with
- Pitch: adjust the blade pitch β to increase or decrease the power coefficient C_p (not common in small-scale WECS)
- Generator Control: change the speed-developed power curve so state-steady operating point changes (this is done in WECS maximum power point trackers)

Generator Control

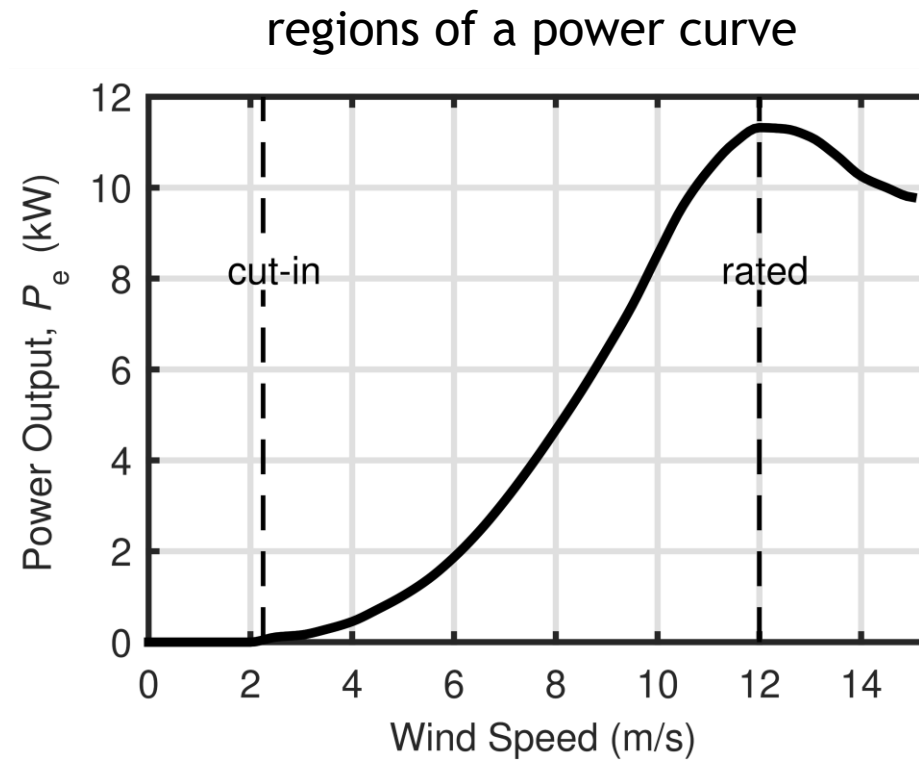


Power Curve

- Power curve: graphically expresses the relationship between wind speed and power production capability of WECS
 - assumes the WECS is connected to a load or is controlled to maximize power developed at each wind speed
 - this assumption often does not apply to WECS in off-grid systems unless a WECS maximum power point tracker is used
 - still useful in understanding the power production potential of a WECS



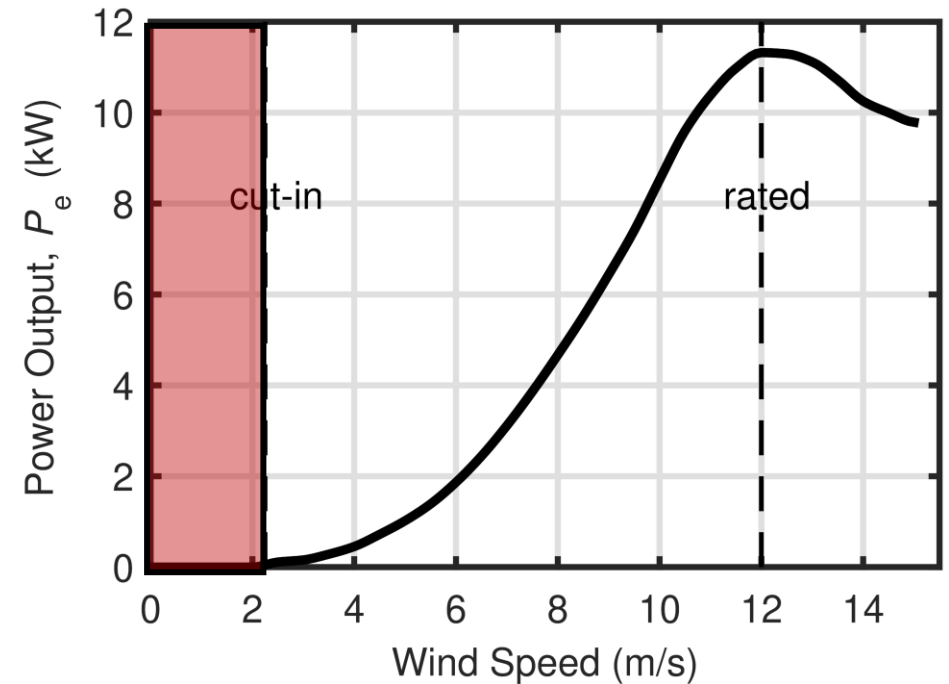
Power Curve



Power Curve

Below cut-in wind speed

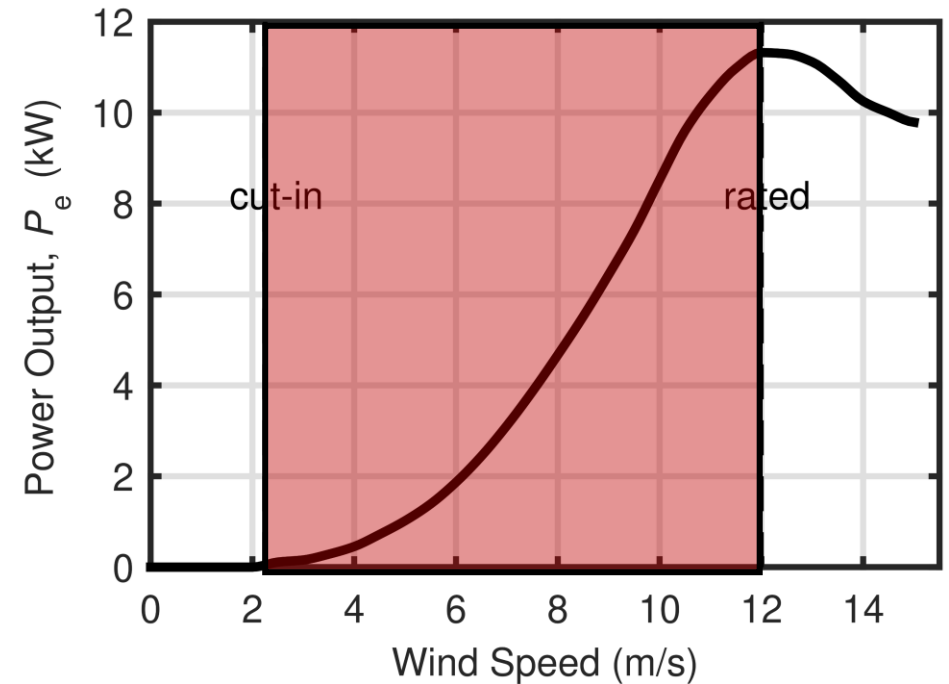
- mechanical power converted by wind turbine is insufficient to produce meaningful electricity
- turbine may or may not be rotating



Power Curve

Between cut-in and rated wind speeds

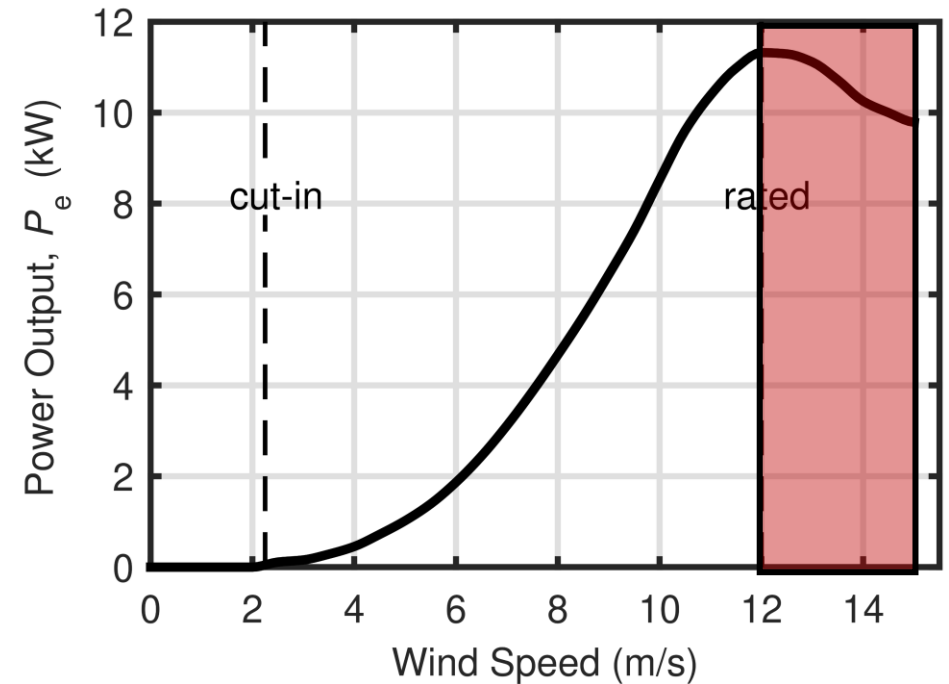
- WECS produces power
- WECS is controlled to maximize C_p



Power Curve

Between rated and cut-out wind speed

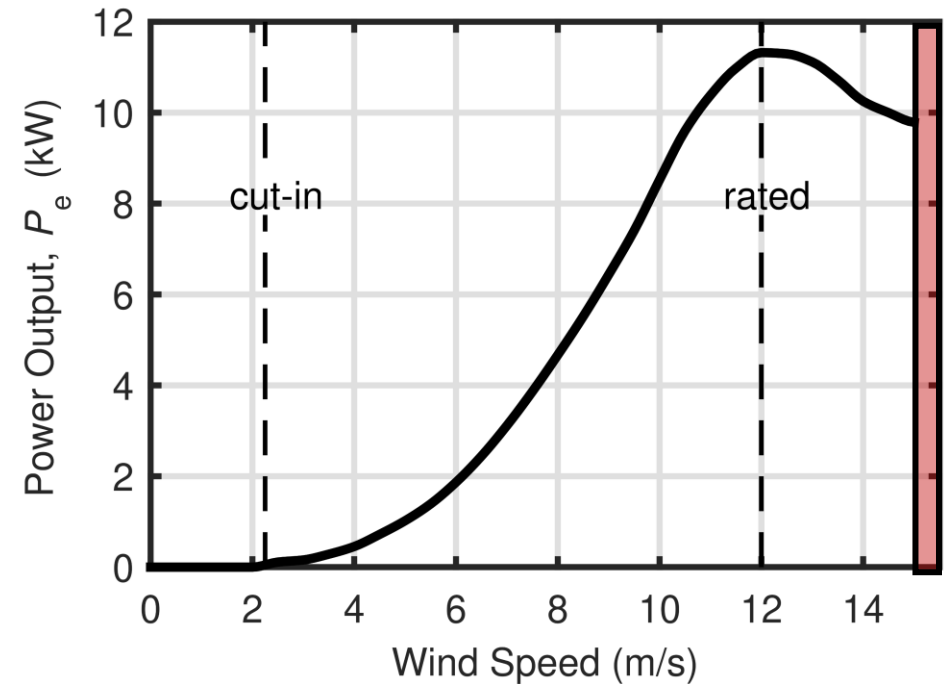
- WECS power is reduced (or held constant)
- power production is limited by the generator (to avoid overheating or damaging it)
- controlled to reduce C_p (“spilling” wind) as the power in the air increases with speed



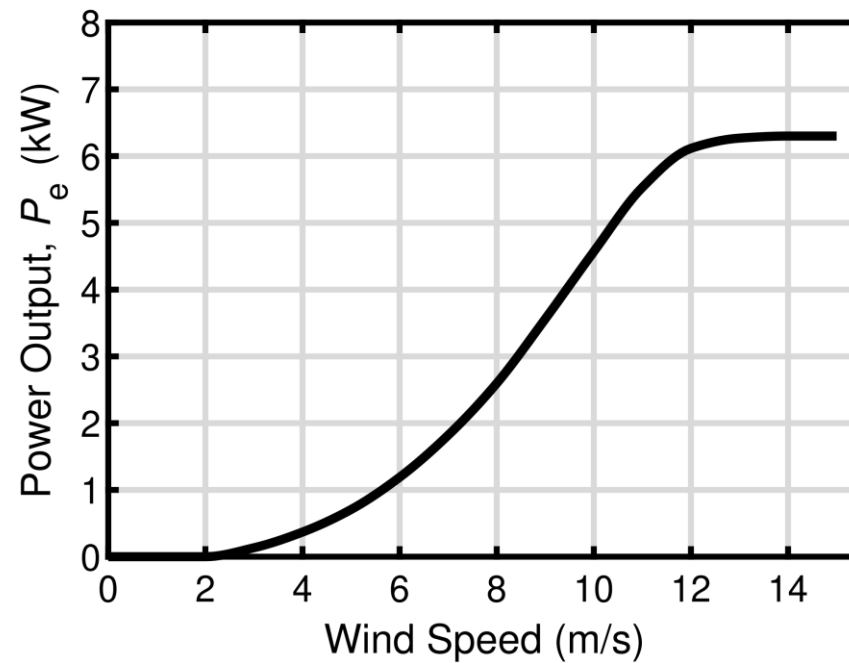
Power Curve

Above cut-out

- high wind speed can damage a WECS
- WECS power is reduced to zero
- can be accomplished by shorting the output (electromagnetic braking)



Power Curve



some WECS are able to tightly maintain rated power above the rated windspeed

Power Curve Model

Mathematically

$$P_e(v_{\text{air}}) \begin{cases} 0 : v_{\text{air}} \leq v_{\text{cut-in}} \\ \eta_{\text{gen}} \frac{1}{2} C_p(\lambda, \beta) A \rho v_{\text{air}}^3 : v_{\text{cut-in}} < v_{\text{air}} \leq v_{\text{rated}} \\ P_{\text{rated}} : v_{\text{rated}} < v_{\text{air}} \leq v_{\text{cut-out}} \\ 0 : v_{\text{air}} > v_{\text{cut-out}} \end{cases}$$

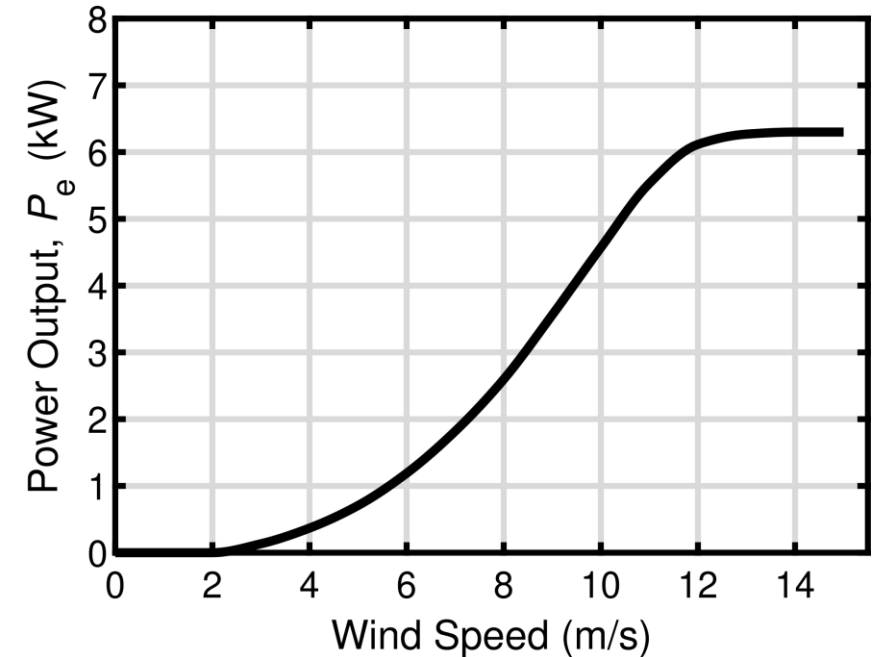
Exercise

A WECS powers a standalone off-grid system. Use the power curve to estimate the energy production if the wind speed is:

1 m/s for 8 hours per day

6 m/s for 12 hours per day

12 m/s for 4 hours per day



Exercise

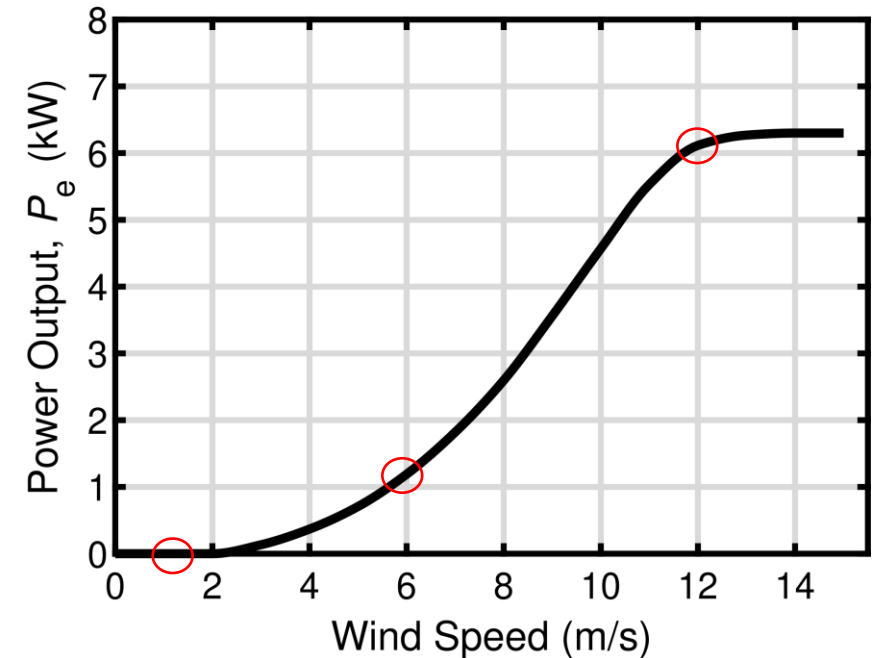
A WECS powers a standalone off-grid system. Use the power curve to estimate the energy production if the wind speed is:

1 m/s for 8 hours per day: $0 \text{ kW} \times 8 \text{ h} = 0 \text{ kWh}$

6 m/s for 12 hours per day: $1.25 \text{ kW} \times 12 \text{ h} = 15 \text{ kWh}$

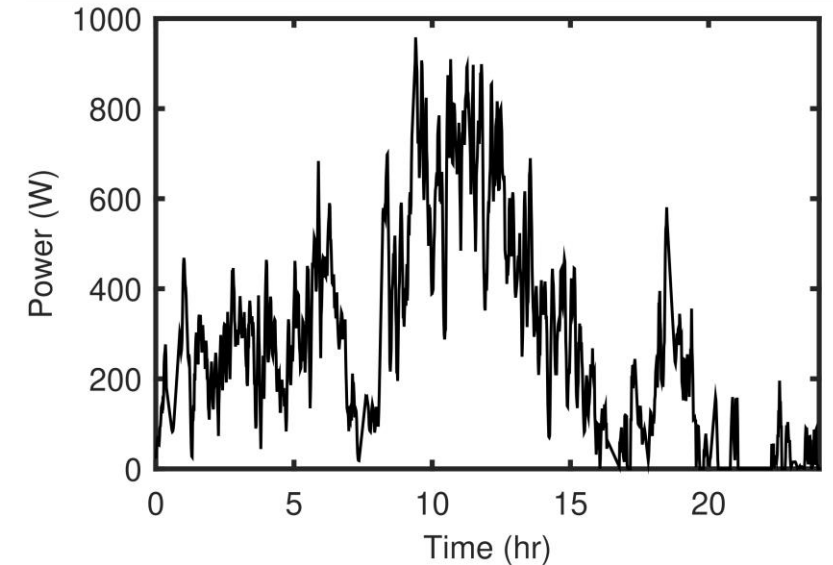
12 m/s for 4 hours per day: $6.25 \text{ kW} \times 4 \text{ h} = 25 \text{ kWh}$

Total energy: $0 + 15 + 25 = 40 \text{ kWh/day}$



Energy Production

- Challenges with WECS:
 - variable power: power produced by WECS varies with wind speed
 - uncertain power: difficult to predict future power production
- Estimating the energy produced is crucial to designing off-grid systems
- Wind resource must be accurately modelled

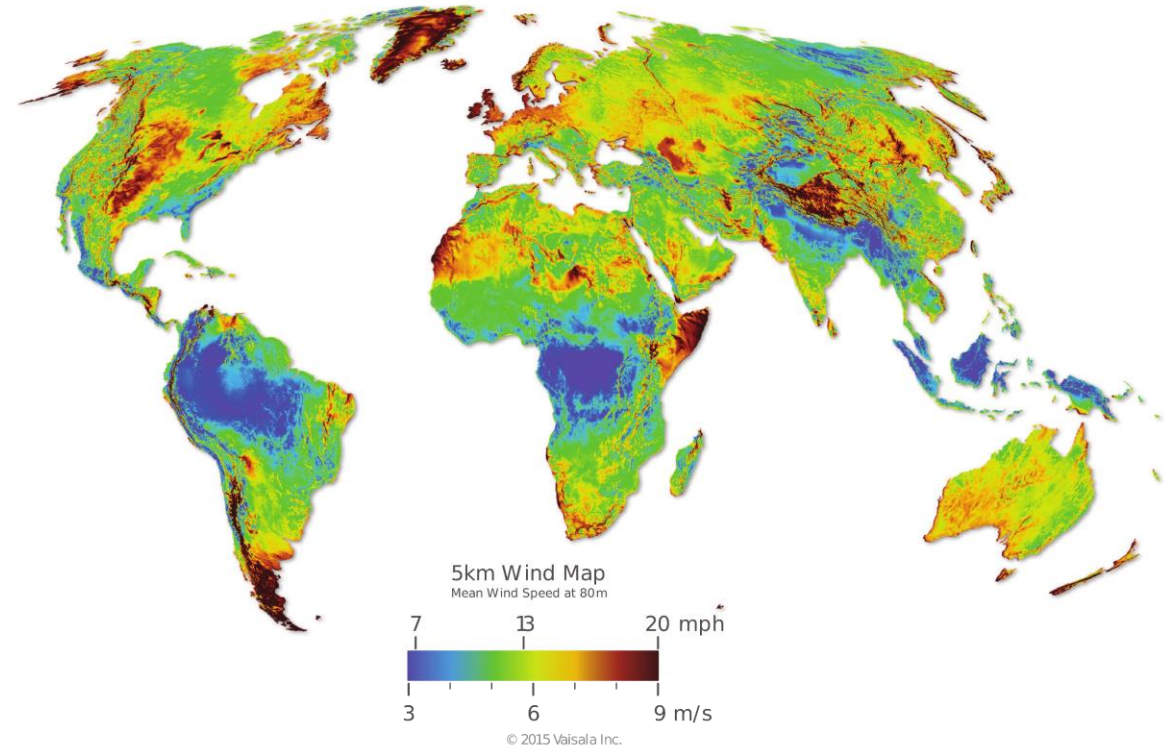


power produced by WECS can vary rapidly

Wind Resource

VAISALA

Wind resource maps can be used to determine general locations where the wind resource is adequate. Pay close attention to the height at which the map applies (small-scale WECS are usually 10 - 20 m from the ground, much lower than utility-scale WECS)



A wind resource map shows the average wind speed at a certain height above ground (courtesy of Viasala, Copyright (c) 2017 Vaisala)

Effect of Height

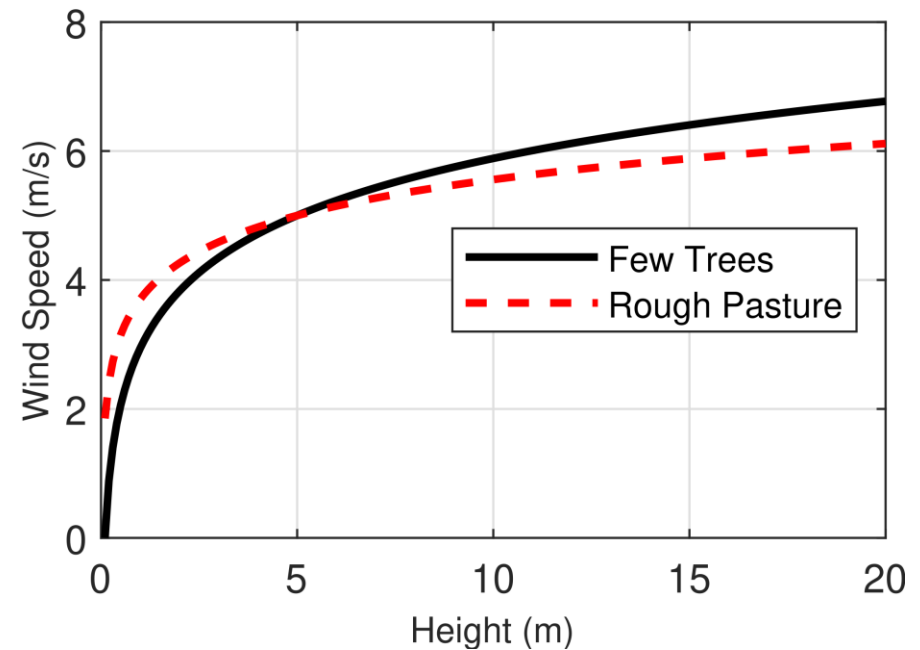
- Wind speed increases with height above surface (wind shear)
- Empirical model to relate speeds at two different heights

The diagram shows the logarithmic wind profile equation: $v_1 \ln\left(\frac{z_2}{z_0}\right) = v_2 \ln\left(\frac{z_1}{z_0}\right)$. Hand-drawn arrows and labels identify the variables: 'height 2' points to z_2 , 'height 1' points to z_1 , 'velocity at height 1' points to v_1 , 'velocity at height 2' points to v_2 , and 'roughness length' points to z_0 .

$$v_1 \ln\left(\frac{z_2}{z_0}\right) = v_2 \ln\left(\frac{z_1}{z_0}\right)$$

Effect of Height

Terrain	Roughness Length (m)
Snow surface	0.003
Rough pasture	0.010
Fallow field	0.03
Crops	0.05
Few trees	0.10
Forest and woodlands	0.50



Towers

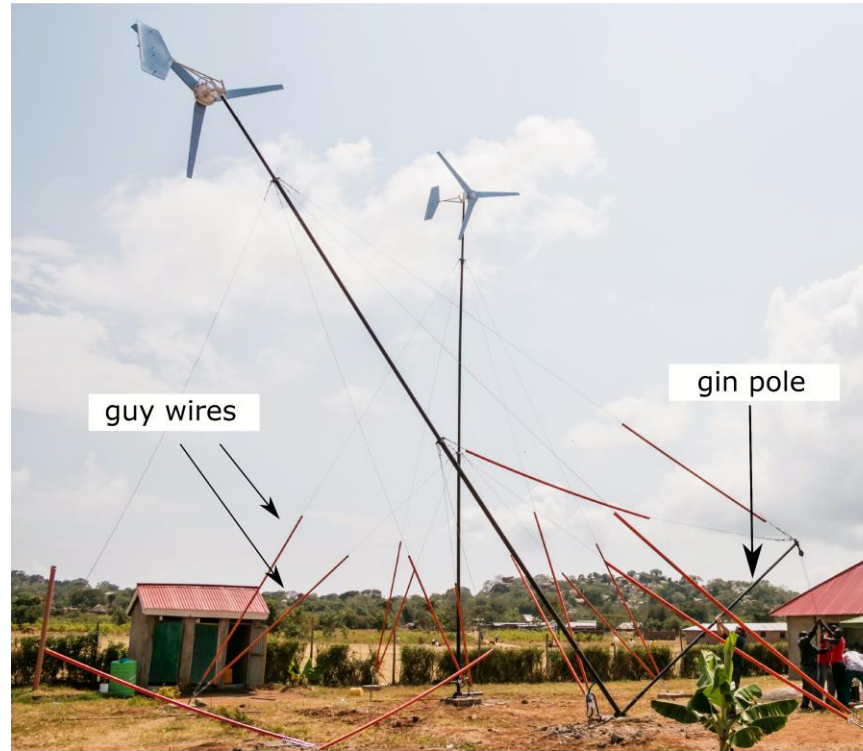
- WECS should be mounted atop tall towers to capture higher, more consistent wind speeds (typically 10-20 m tall)
- Tower must be properly engineered and maintained
- Tower can cost as much as the WECS

Tower Types



(courtesy H. Louie)

Free standing



(courtesy E. Patten)

Tilt-Up

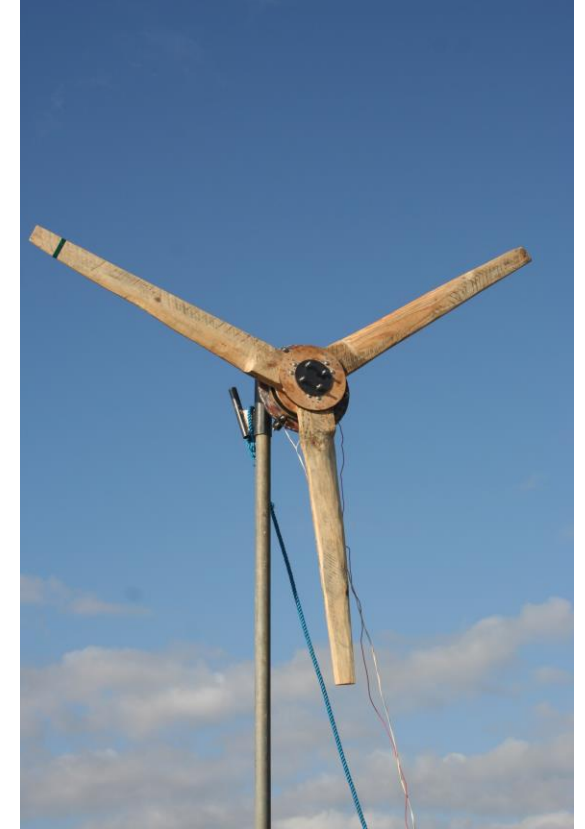


(courtesy H. Louie)

Lattice

Economic Considerations

- Capital cost: high capital and installation costs, typically US\$2000-US\$4000/kW, also require batteries, load controllers, etc.
- Fuel cost: zero
- Operation and maintenance cost: periodic skilled/semi-skilled maintenance of both tower and turbine required
- Energy cost: depends on wind resource, but can exceed US\$1/kWh
- Other: WECS can be manufactured locally, which can substantially reduce the cost



(courtesy H. Louie)

Environmental Considerations

- Resource availability: not many communities have sufficient wind resources, and it requires on-site data collection to know if resource is adequate
- No emissions or water use
- Noise: can be noisy depending on blade design and TSR
- Land use: tower footprint can be large and disruptive
- Wildlife: bird and bat strikes, lightning strike risk

Social Considerations

- Community benefit: WECS can be used to power large off-grid systems and stand-alone systems, large upfront cost and design requirements make it unlikely for a single home to use WECS
- Community burden: WECS can attract unwanted attention to a community
- Health & Safety: danger if tower is not well designed or maintained; wind storms present threat of tower failure
- Community involvement: possible to be constructed and maintained locally (with training) and materials

Summary

- WECS convert kinetic energy in moving air to electrical energy
 - power varies with cube of wind speed
- Three-blade horizontal axis most common
- Developed electrical power depends on pitch, TSR, rotational speed and load characteristics
- Power curve expresses the relationship between power output potential and wind speed
 - power produced is variable and uncertain
- Tower siting, design, and cost are important considerations