

12-Off-Grid System AC Converters and Inverters

Off-Grid Electrical Systems in Developing Countries, 2nd Edition

Chapter 12

Preface

- These lectures slides are intended to accompany the textbook *Off-Grid Electrical Systems in Developing Countries, 2nd Edition, 2025* written by Dr. Henry Louie and published by [SpringerNature](#)
- Additional content, explanations, derivations, examples, problems, errata, and other materials are found in the book and on www.drhenrylouie.com
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Learning Outcomes

At the end of this lecture, you will be able to:

- ✓ Describe the basic function, operating principles, and application of electronic load controllers and inverters in off-grid systems
- ✓ Explain foundational concepts related to off-grid AC converters including distortion, filtering, and sinusoidal pulsewidth modulation
- ✓ Analyze and compute the voltages, currents, and power at the input, output, and internal stages of various AC converters and inverters

Introduction

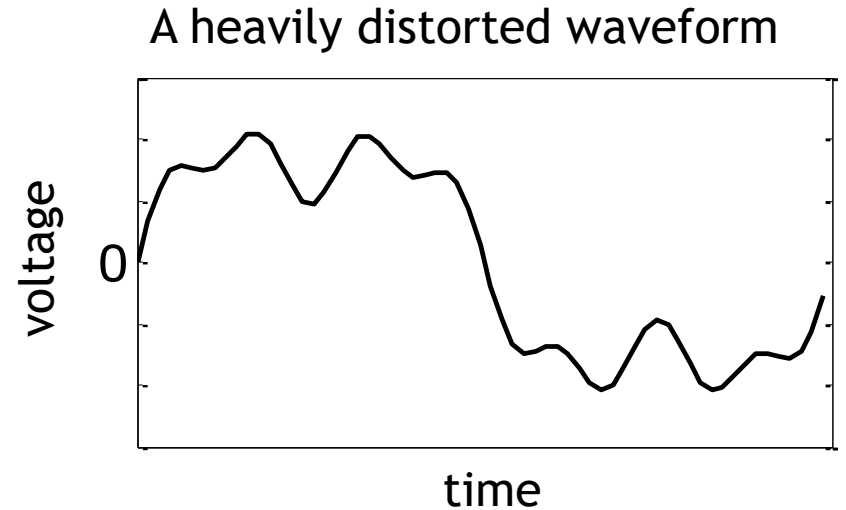
- Focus now is on AC converters
- AC converters commonly found in off-grid systems that serve AC load
- Play important role in regulating voltage and frequency of AC bus

AC Converters

Converter	Function
Electronic Load Controller	Controls power to ballast load to regulate frequency
Inverter	Converts DC to AC
Solar Inverter	Converts DC from PV sources to AC
Wind Inverter	Converts DC or variable frequency AC from WECS to fixed frequency AC
Grid Tied Inverter	Converts DC to AC and synchronizes with AC bus
Bi-Directional Converter, Inverter/Charger	Allows power to be exchanged between the DC and AC Buses
Hybrid Inverter	Combines charge controller, maximum power point tracker, charger and inverter functions

Fundamental Concepts: Distortion and Filtering

- AC voltage produced by inverters is not purely sinusoidal
- Distortion describes the deviation of the waveform from the desired sinusoid



Distortion

- Waveforms can be decomposed to a sum of sinusoids with different magnitudes F_k and phases δ_k at multiple integers of the fundamental frequency (harmonics)

$$f(t) = \sum_{k=1}^{\infty} F_k \sin(k\omega_0 t + \delta_k)$$

Total Harmonic Distortion

Total Harmonic Distortion (THD): measure of the distortion of the AC output of an inverter

THD compares the amplitudes of the harmonics to that of the fundamental

$$THD = \frac{\sqrt{\sum_{k=2}^{\infty} F_k^2}}{F_1}$$

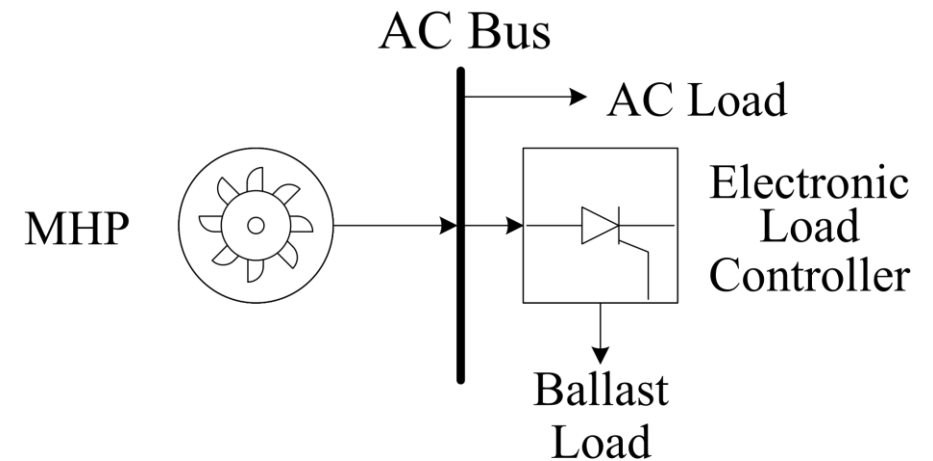
Lower THD is desirable
(amplitudes of the harmonics should be small compared to the amplitude of the fundamental)

Distortion and Filtering

- THD less than 5% is generally desired for off-grid systems
- Higher distortion may be acceptable if the load is resistive
- Low-pass filtering can be used to remove higher order harmonics

Electronic Load Controller (ELC)

- ELC is a “demand side” approach used in MHP to regulate AC bus frequency
- Balance changes in load by increasing/decreasing power to ballast load (usually a resistor) while allowing the power produced by the MHP to be constant



Electronic Load Controller

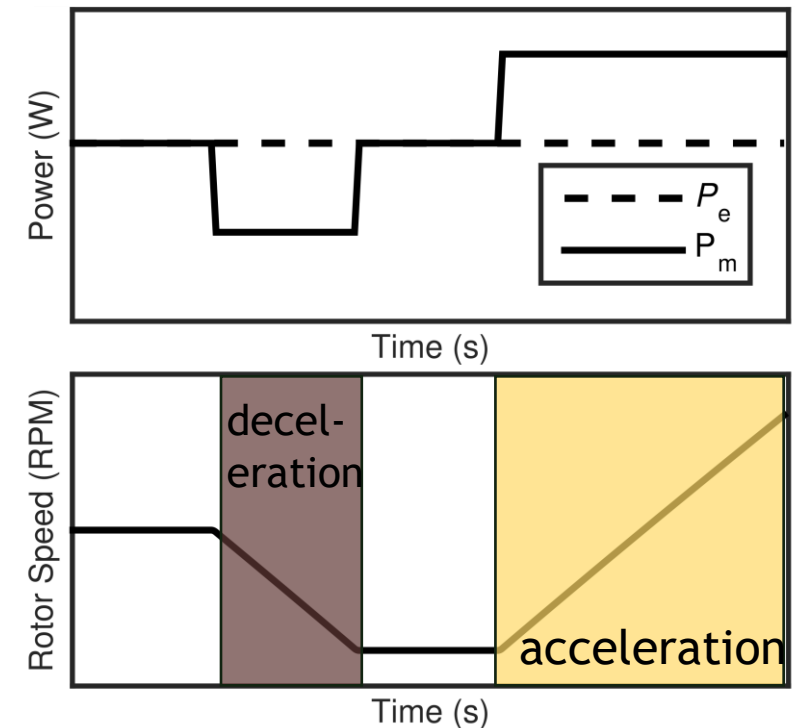
ELC have advantages over mechanically adjusting input power (water) to hydro turbine

- quickly respond to rapid changes in load
- less complex
- dissipated power can be put to productive use, for example, heating water
- require little maintenance
- less expensive

Recall: Electrical Generator Rotational Dynamics

The electrical load (P_e) on the generator must be equally matched by the mechanical power (P_m) provided by prime mover (assuming the generator is electrically and mechanically lossless); otherwise, the rotating shaft will speed up or slow down

This is a consequence of the law of conservation of energy



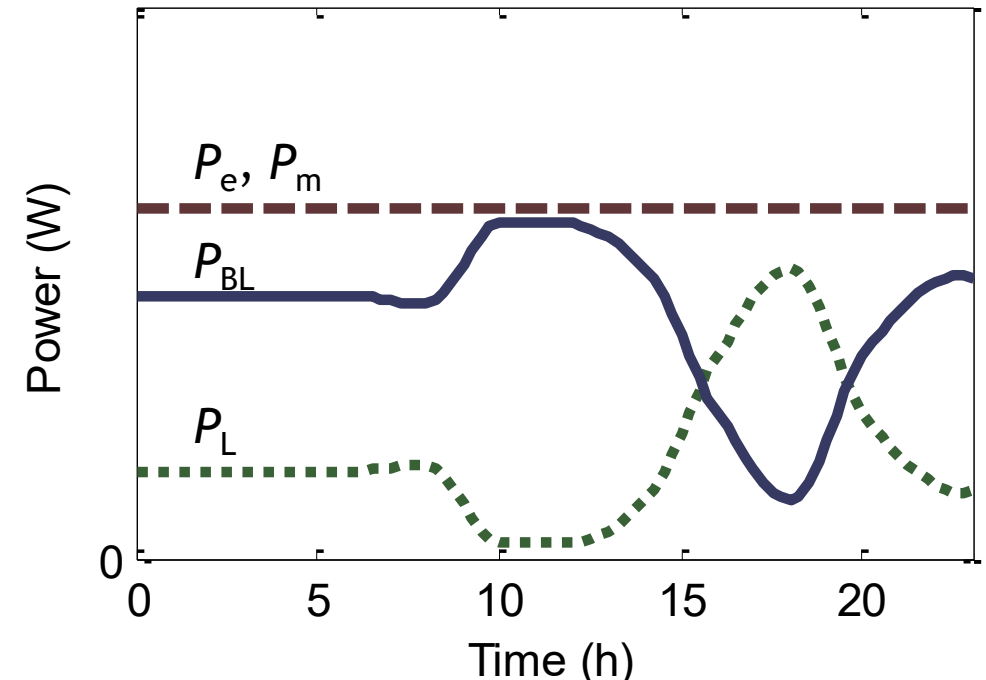
Electronic Load Controllers

- Electrical frequency regulation approach
- Adjust ballast load power so electric power is constant and equal to mechanical power

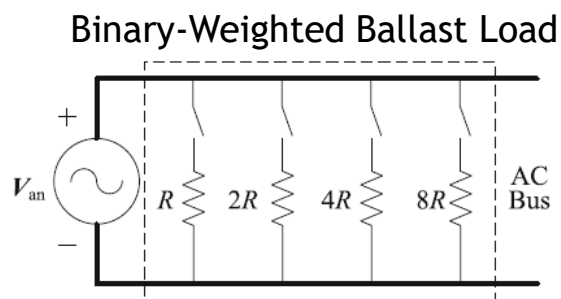
$$P_e = P_L + P_{BL} = P_m$$

P_L : power supplied to load (plus losses)

P_{BL} : power to ballast load

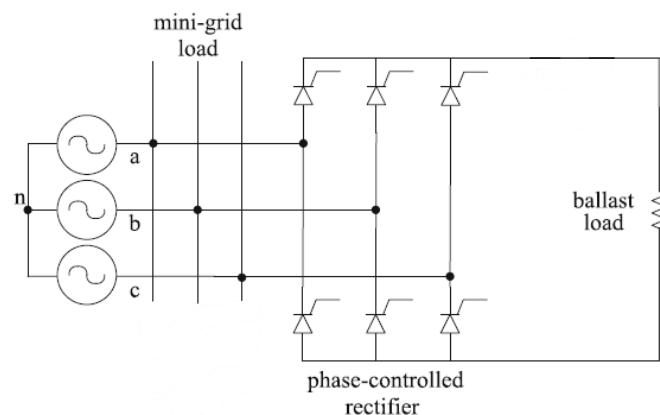


Electronic Load Controllers



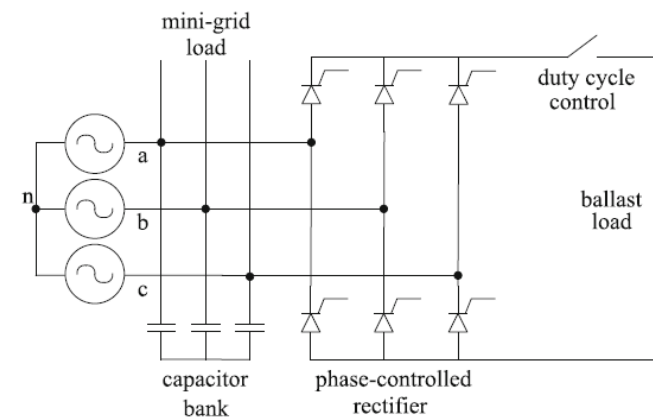
Open/close switches to achieve equivalent resistance needed to regulate electric power

Phase-Angle Controlled Rectifier



Adjust firing angle to regulate electric power

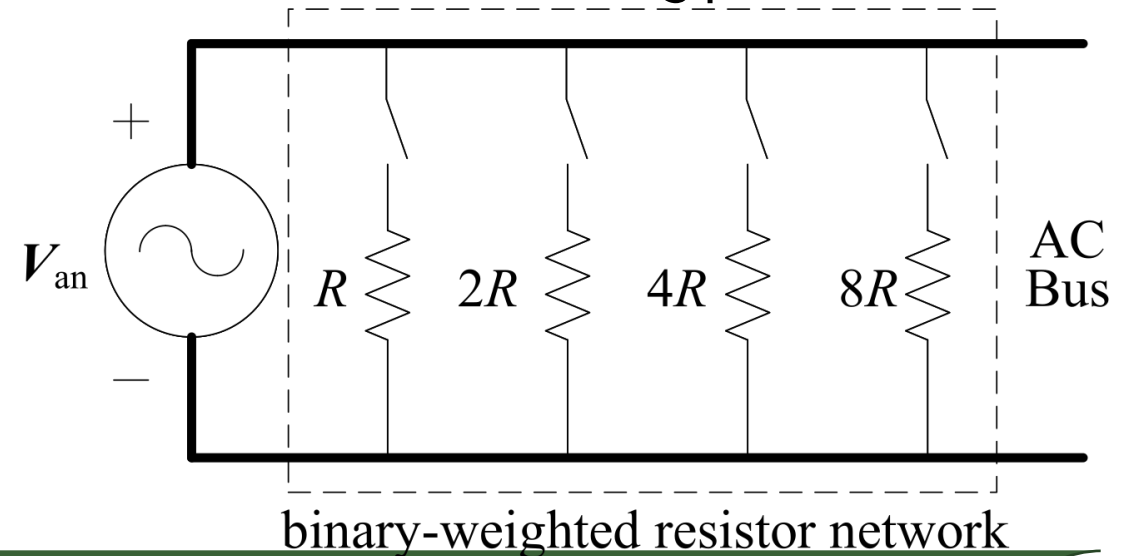
“Impedance Controller”



Adjust firing angle and switch duty cycle to regulate electric power

Example 12.1

Consider a binary weighted resistor network used in an ELC where $R = 10\ \Omega$. The network has four resistors in each phase. A three-phase MHP generator is connected to the AC bus. The line-to-line AC bus voltage is 208 V. Determine the maximum and minimum power consumed by the network assuming that at least one resistor is consuming power.

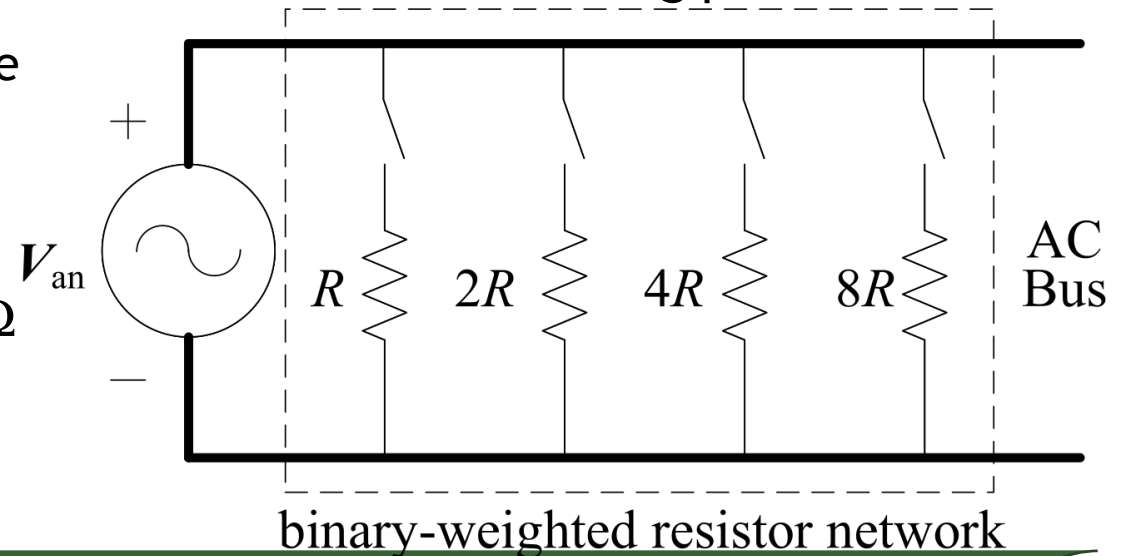


Example 12.1

Consider a binary weighted resistor network used in an ELC where $R = 10\ \Omega$. The network has four resistors in each phase. A three-phase MHP generator is connected to the AC bus. The line-to-line AC bus voltage is 208 V. Determine the maximum and minimum power consumed by the network assuming that at least one resistor is consuming power.

We will consider one phase of the resistor network. The maximum power is consumed when all resistors are connected. Since the resistors are all in parallel, the equivalent resistance is

$$R_{eq} = \frac{1}{\frac{1}{R} + \frac{1}{2R} + \frac{1}{4R} + \frac{1}{8R}} = \frac{1}{\frac{1}{10} + \frac{1}{20} + \frac{1}{40} + \frac{1}{80}} = 5.3\ \Omega$$

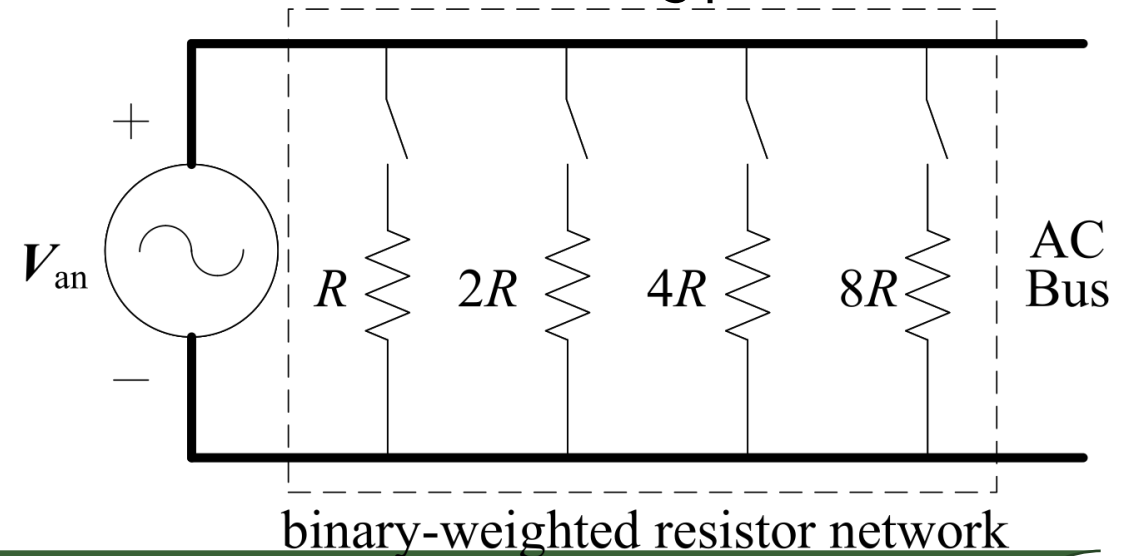


Example 12.1

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The power consumed by all three phases is

$$P_{BL} = 3 \frac{\left(\frac{V_{ll}}{\sqrt{3}} \right)^2}{R_{eq}} = 3 \frac{120^2}{5.33} = 8.1 \text{ kW}$$

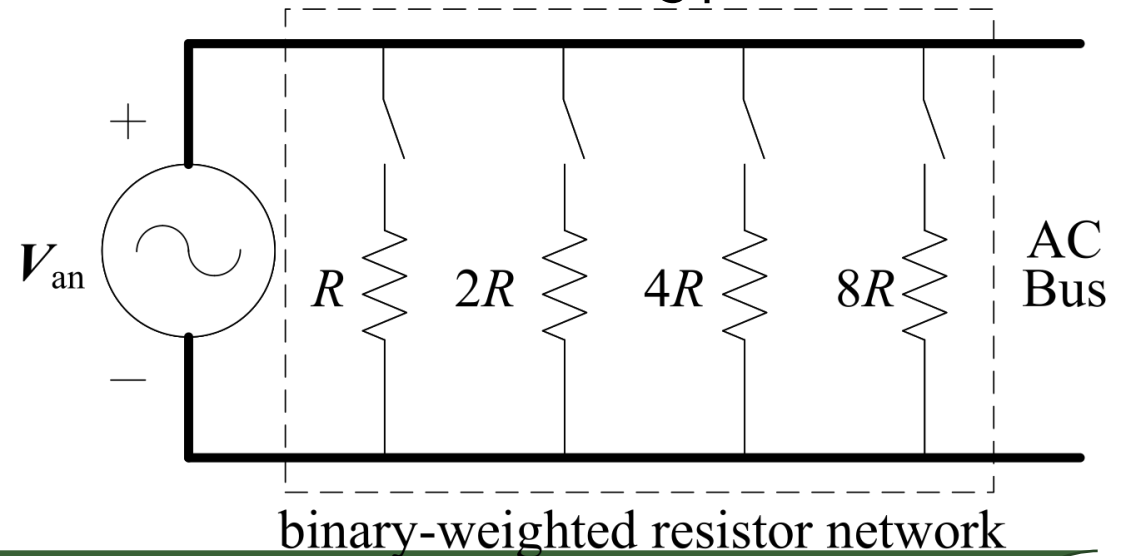


Example 12.1

Consider a binary weighted resistor network used in an ELC where $R = 10\ \Omega$. The network has four resistors in each phase. A three-phase MHP generator is connected to the AC bus. The line-to-line AC bus voltage is 208 V. Determine the maximum and minimum power consumed by the network assuming that at least one resistor is consuming power.

The minimum power consumed by the ballast load is when only the $80\ \Omega$ resistor is connected. The corresponding power is

$$P_{BL} = 3 \frac{120^2}{80} = 540.8\text{ W}$$

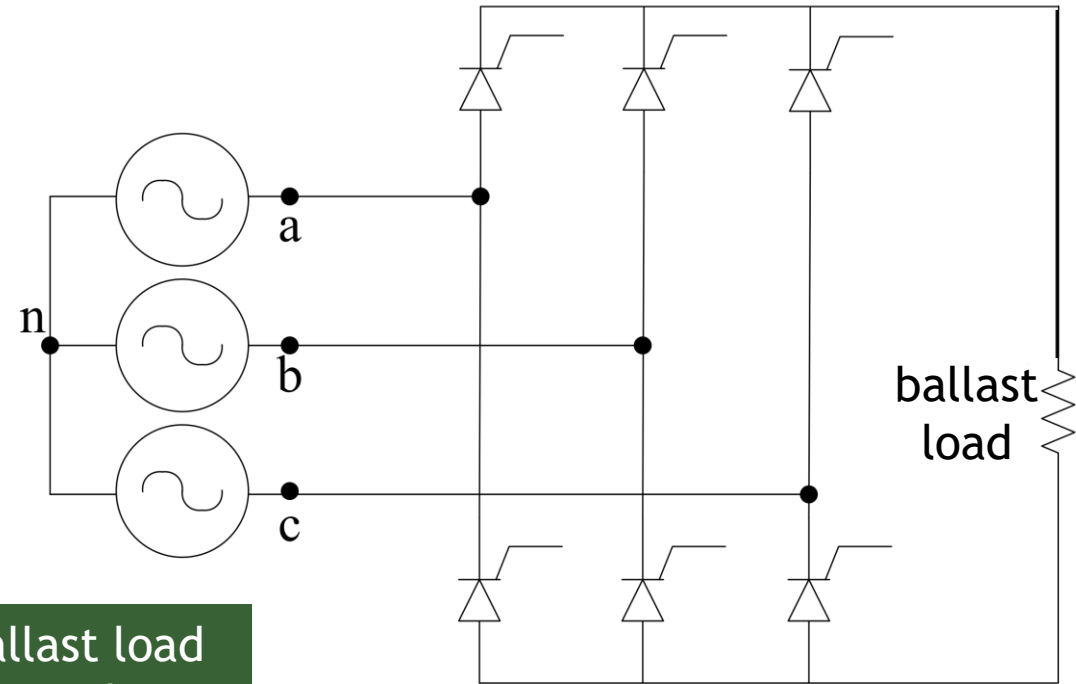


Electronic Load Controller: Phase-controlled Rectifier

- Thyristor firing angles α controlled to achieve desired power to ballast load
- Similar circuit as used in AVRs from last chapter
- Recall that voltage across output is

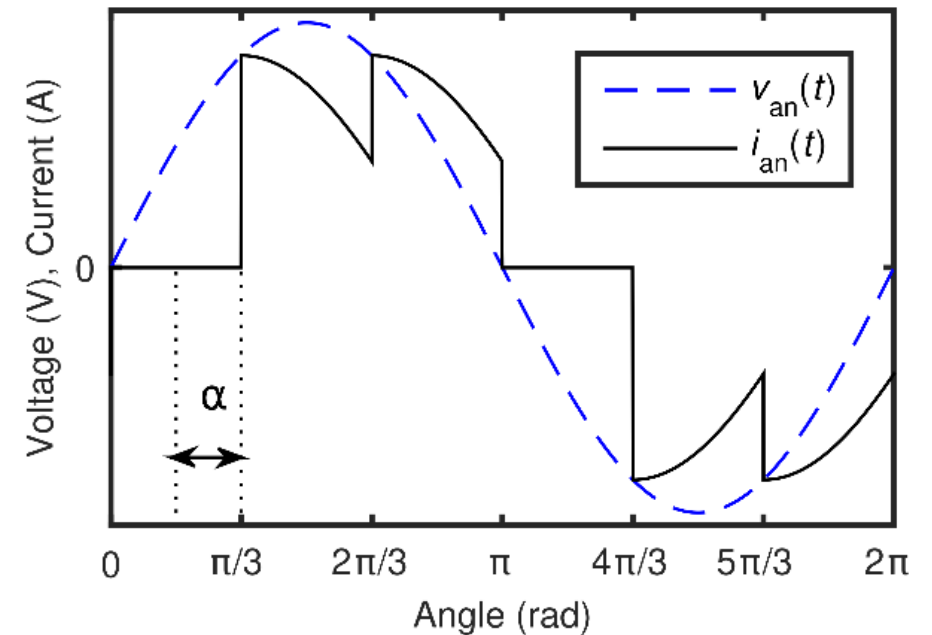
$$V_{DC} \approx 0.955 V_{\ell\ell}^{\max} \cos(\alpha)$$

real power to ballast load varies with firing angle



Phase-controlled Rectifier

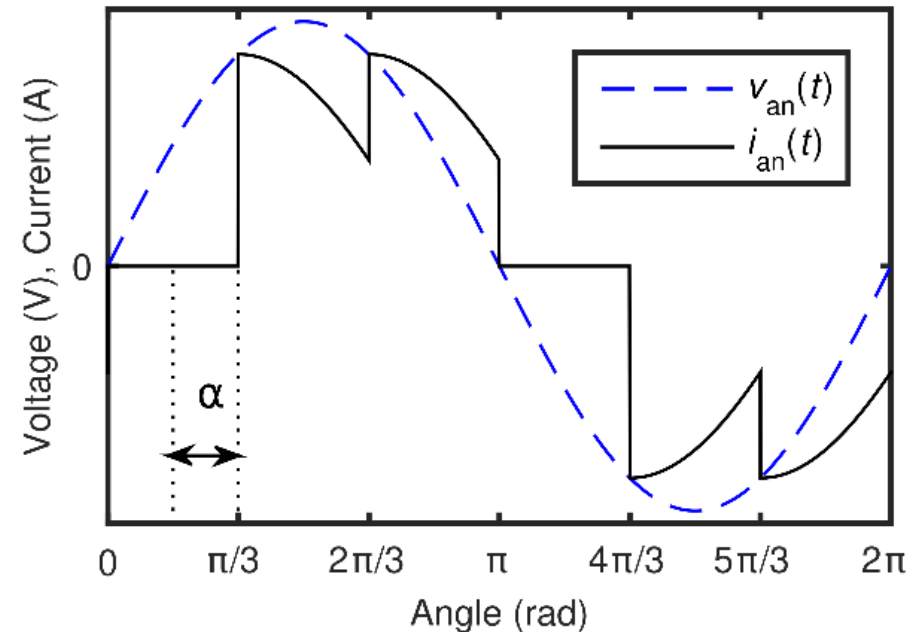
- Problem: as firing angle increases, the current from each phase of the AC bus begins to lag its voltage
- May exceed reactive power capability of MHP generator and decrease bus voltage and increase losses



Phase-Angle Controlled Rectifier

- Power factor not unity
- Non-unity power factor can occur with resistive loads if current or voltage is non-sinusoidal

$$PF = \frac{P}{I_{\text{RMS}} V_{\text{RMS}}} = \frac{\frac{1}{T} \int_0^T i(t)v(t)dt}{\sqrt{\frac{1}{T} \int_0^T i^2(t)dt} \sqrt{\frac{1}{T} \int_0^T v^2(t)dt}}$$



Phase-Angle Controlled Rectifier

PF of three phase phase-angle controlled rectifier is

$$PF = \frac{I_{RMS,1}}{I_{RMS}} \cos(\alpha)$$



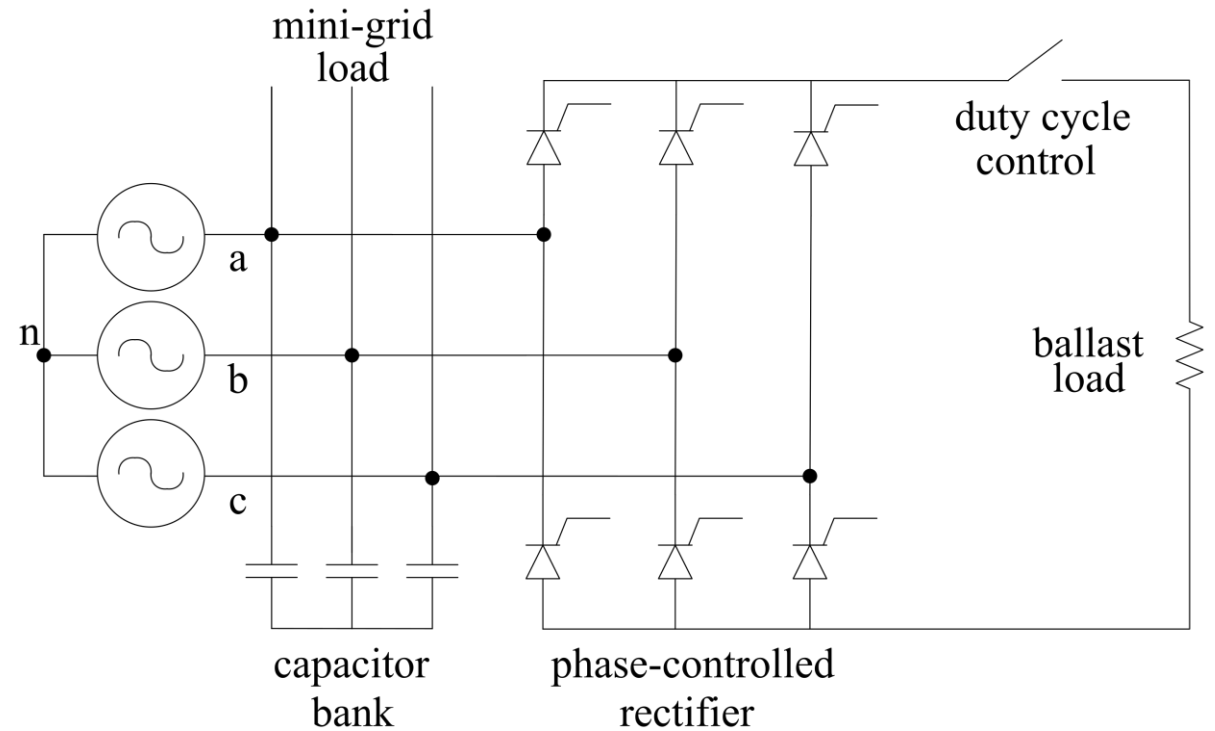
first harmonic of the
current

Impedance Controller

- Firing angle affects both the real and reactive power in phase-angle controlled rectifiers
- Additional degree-of-freedom can allow for better control

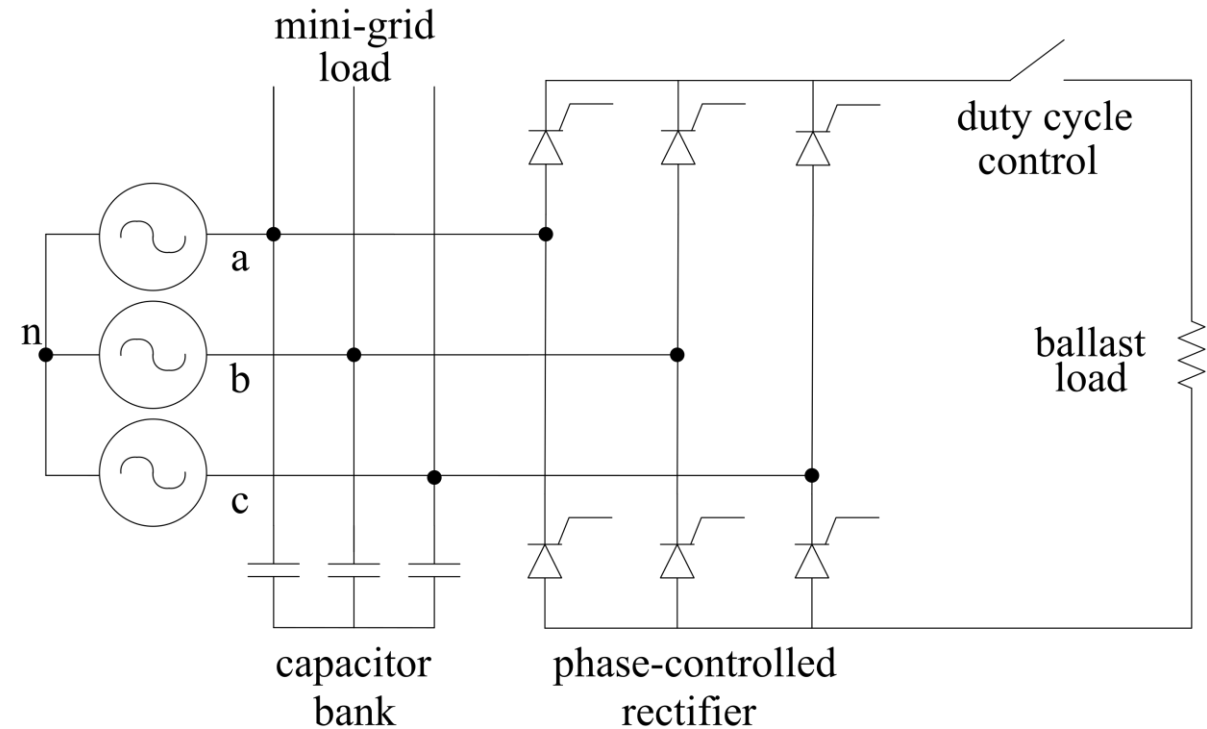
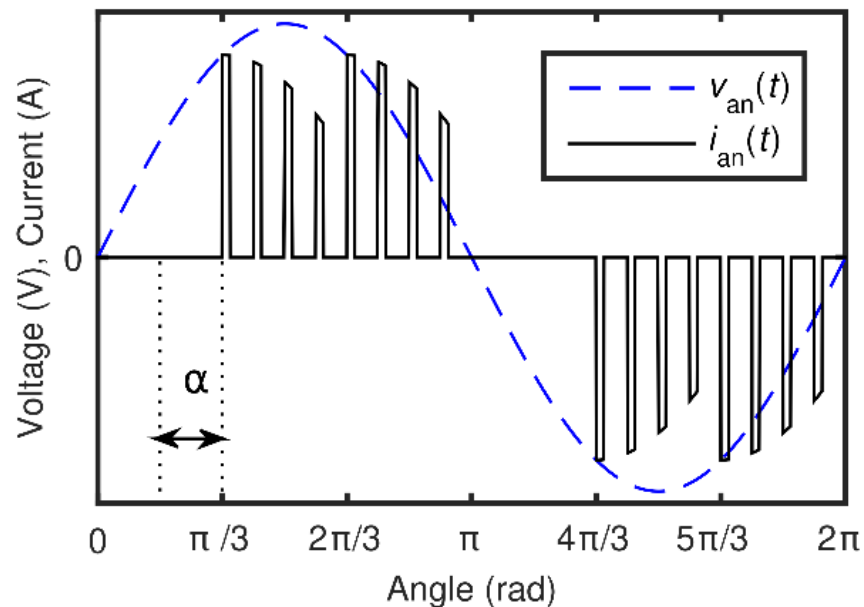
Impedance Controller

- Add controllable chopper switch
- Add capacitor bank to offset reactive power consumption



Impedance Controller

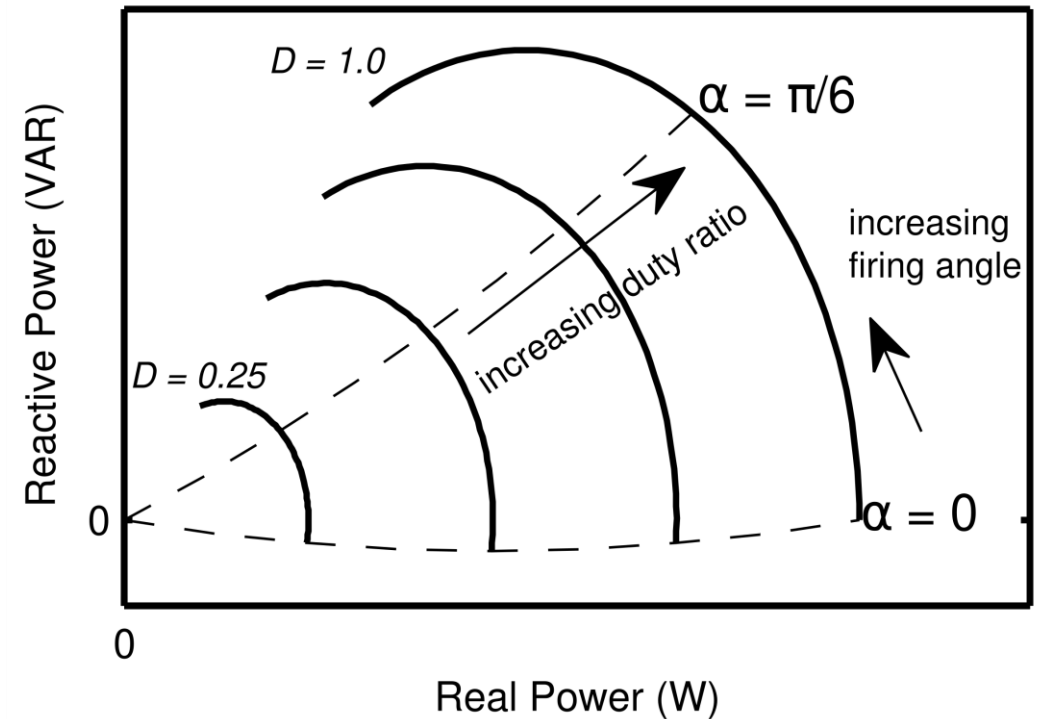
Resulting waveform



Impedance Controller

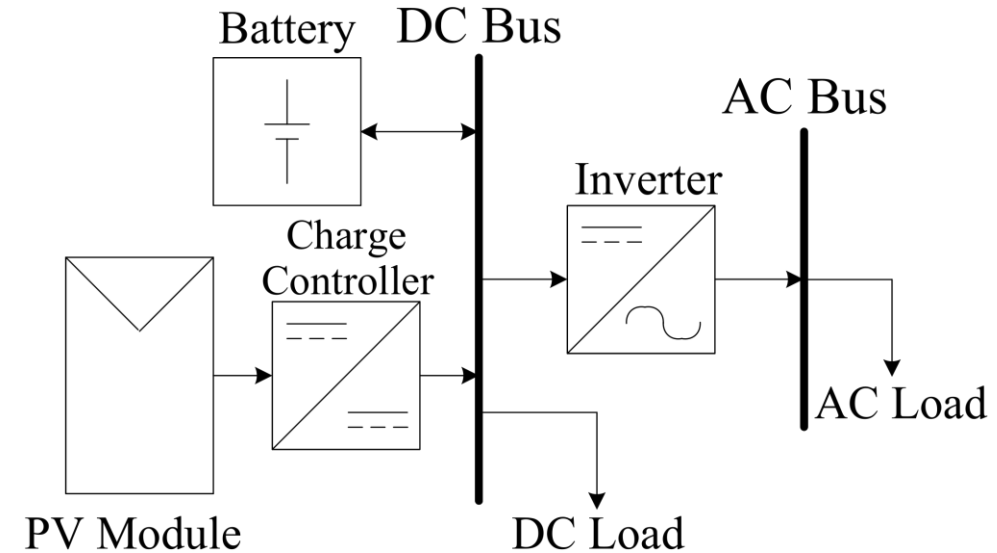
- Increasing duty ratio increases real power
- Increasing firing angle increases reactive power
- P , Q not completely decoupled
- Capacitor offsets reactive power consumed by the ballast load

$$Q_{IC} = Q_{BL}(\alpha, D) - Q_{cap}$$



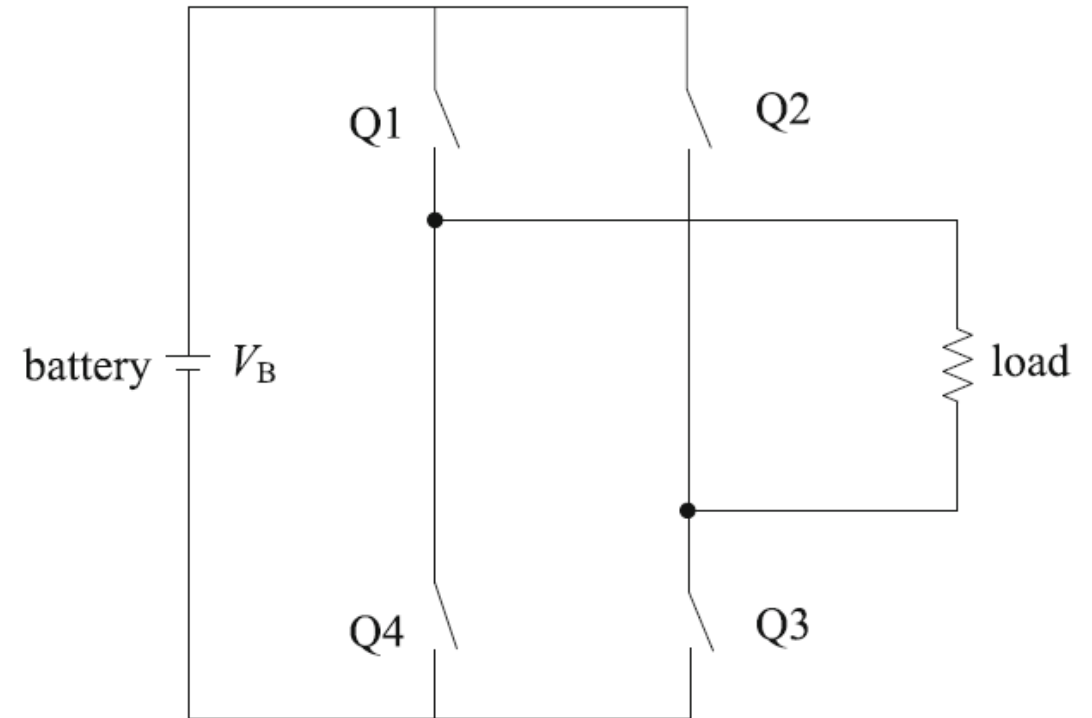
Inverters

- Inverter: converts DC electricity to AC electricity
- Very common in off-grid systems that serve AC load
- Couples DC bus with AC bus

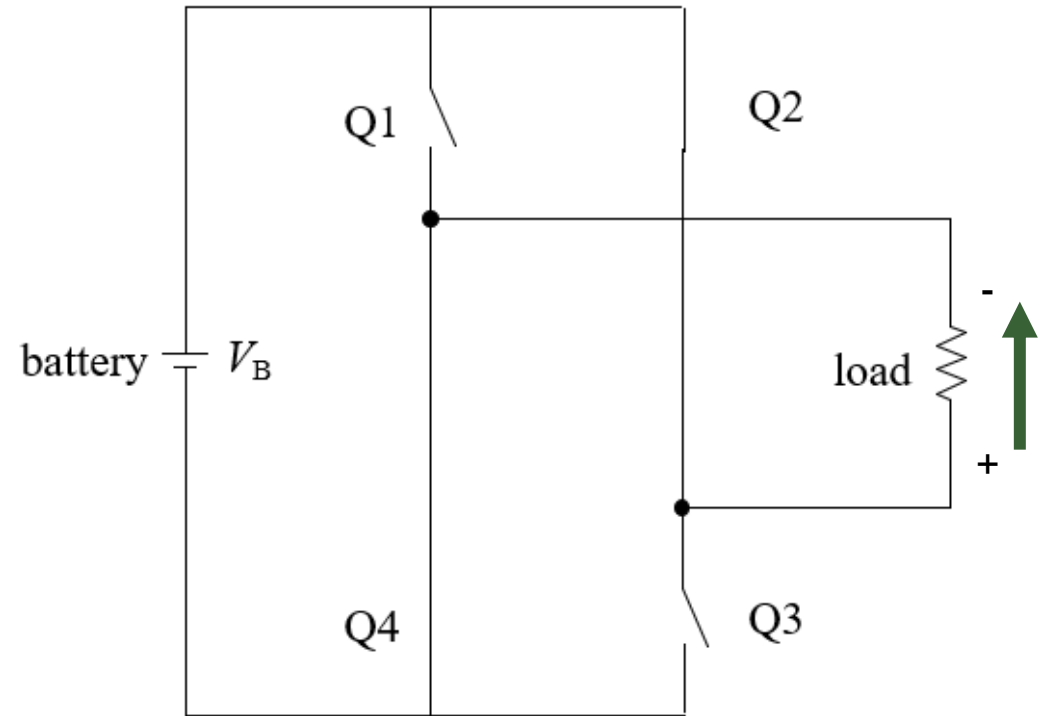
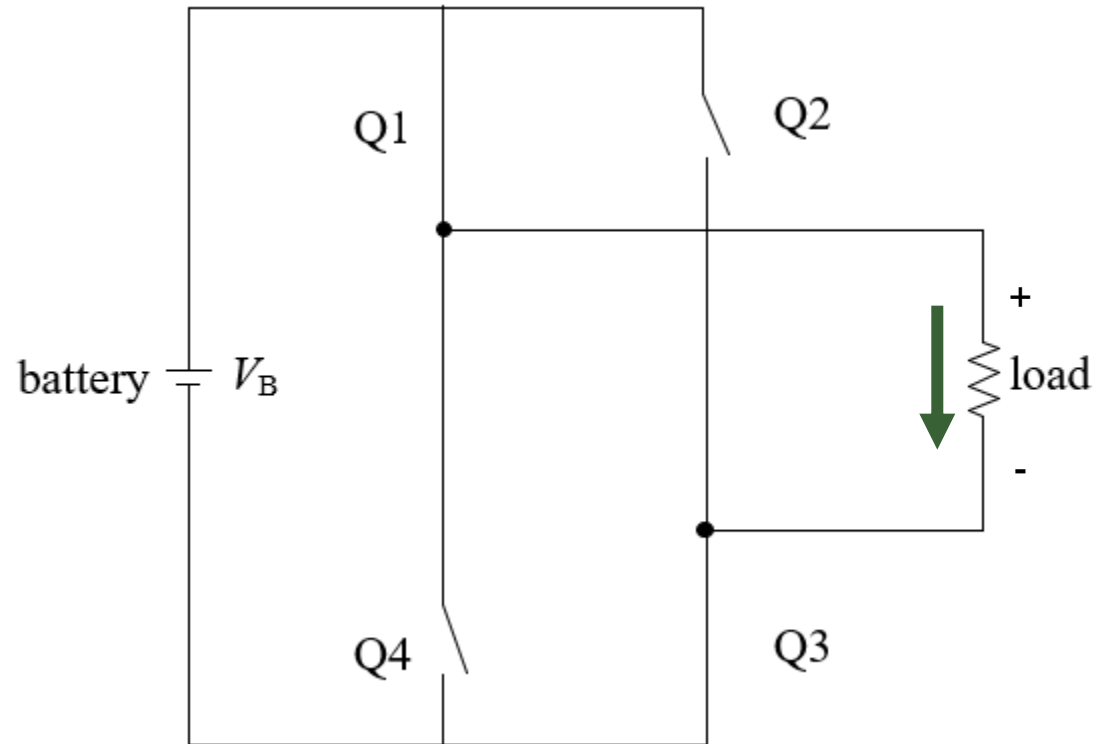


Inverters: Basic Circuit

- Use solid-state switching to alternate polarity of voltage applied to a load
- Switch in pairs
 - Q1, Q3 open and close at same time
 - Q2, Q4 open and close at the same time
- Voltage output can be increased via DC–DC converter at the input or transformer at the output (or both)

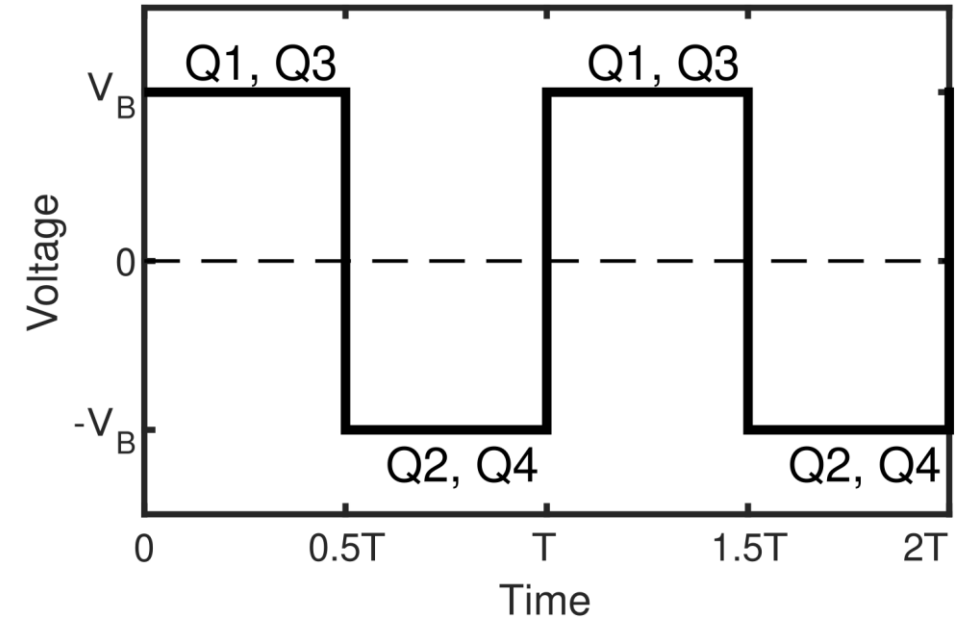


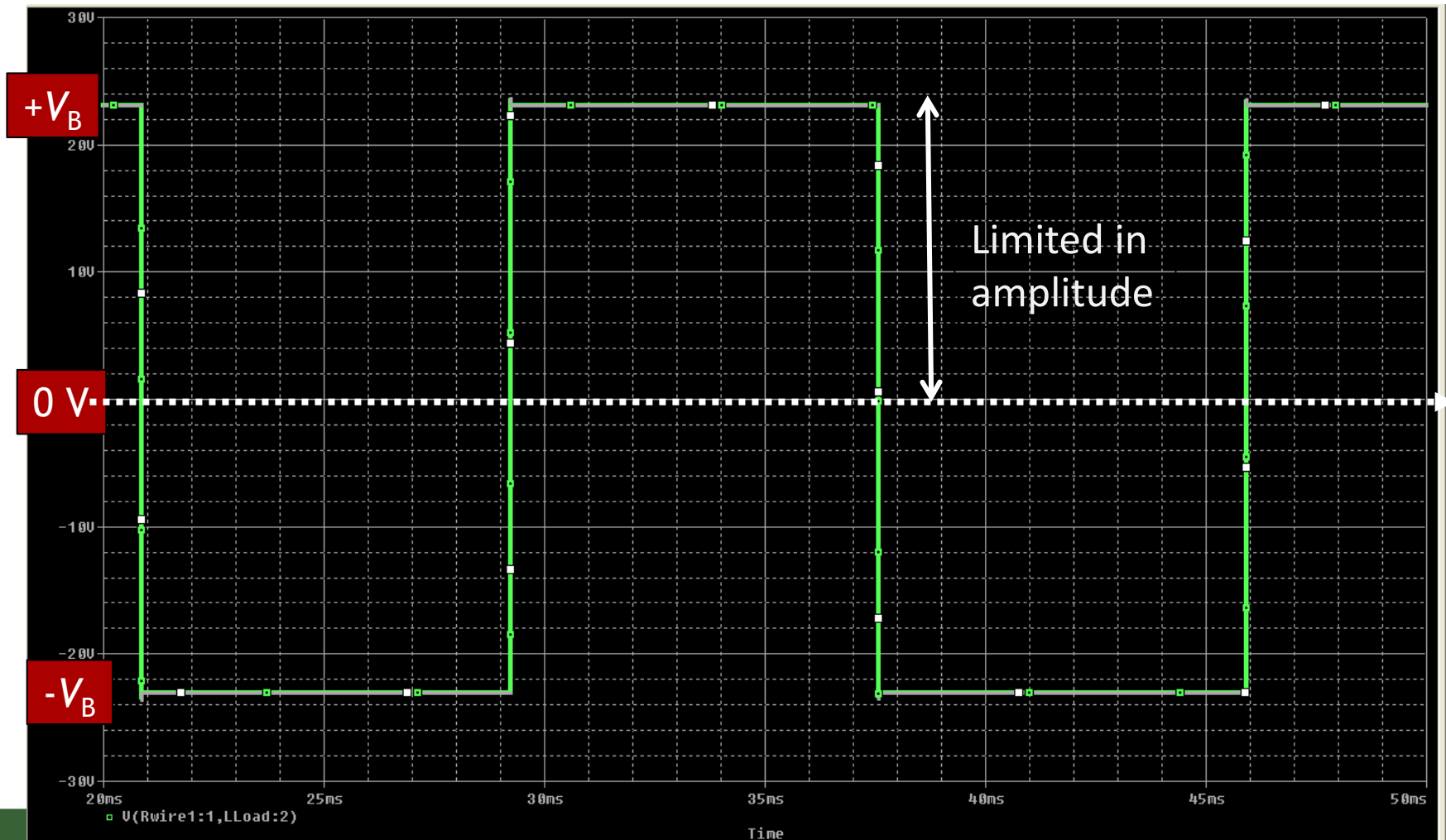
Inverters: Basic Circuit



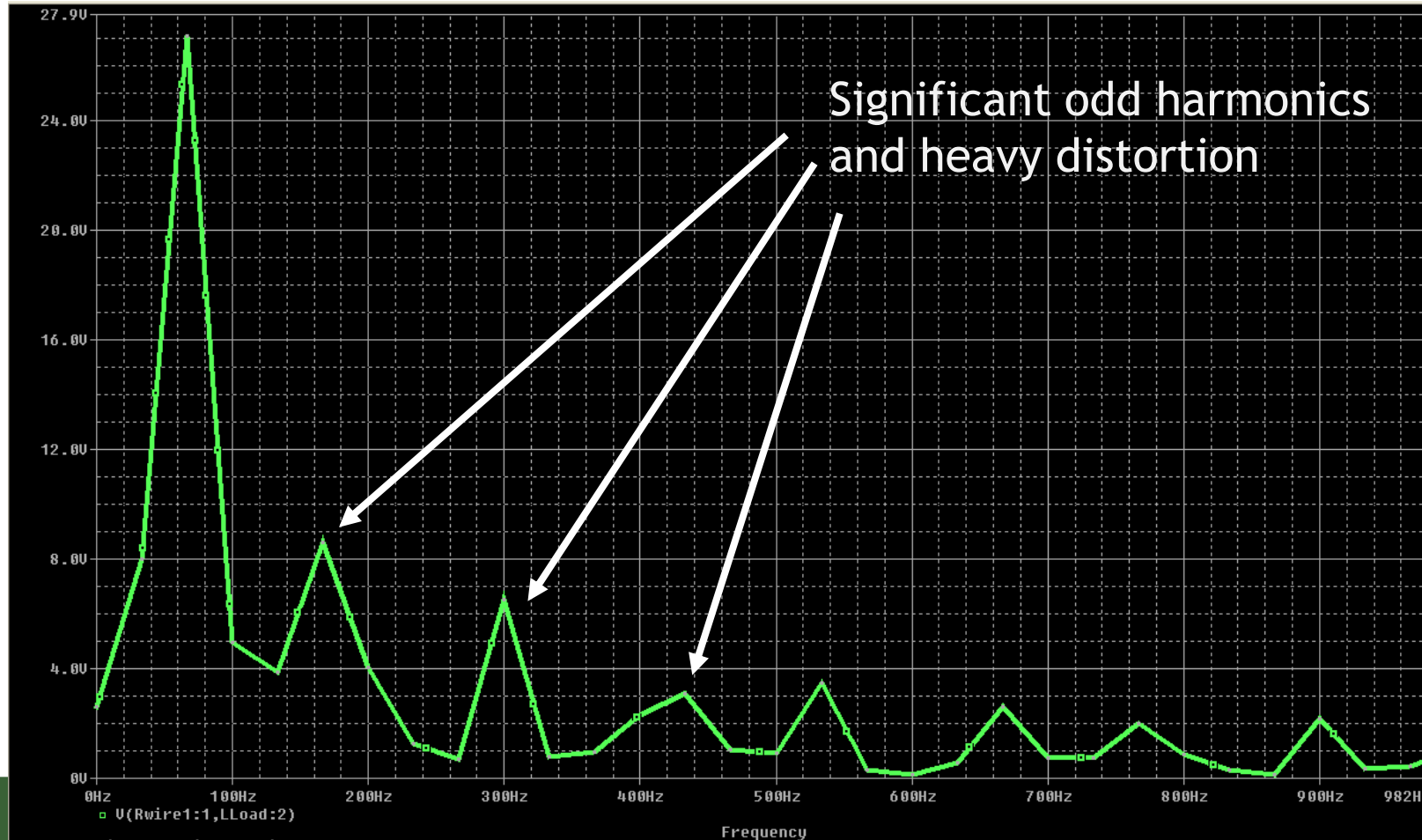
Output Signal

- Waveform is square
- Centered at zero



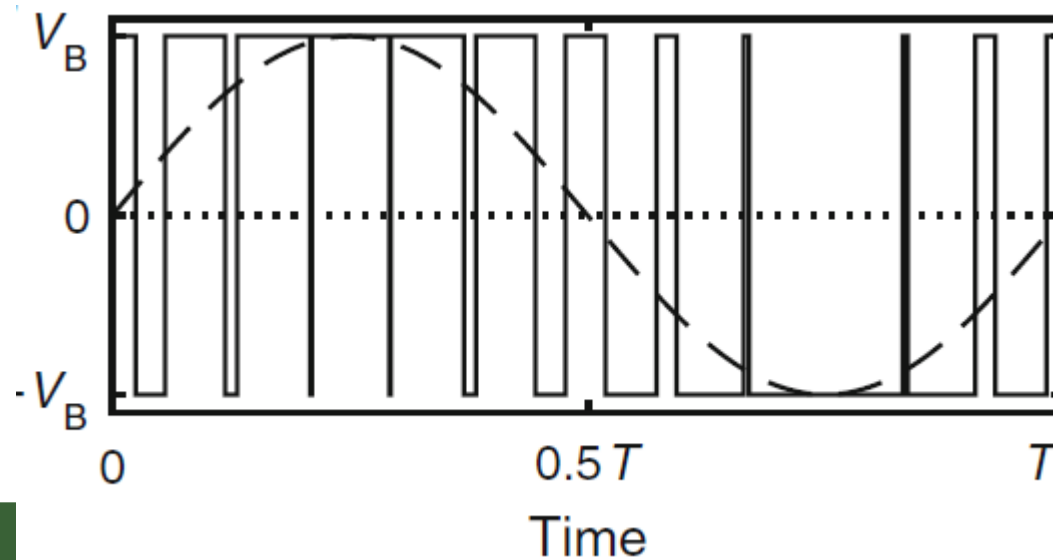


Frequency Sweep of Output Waveform



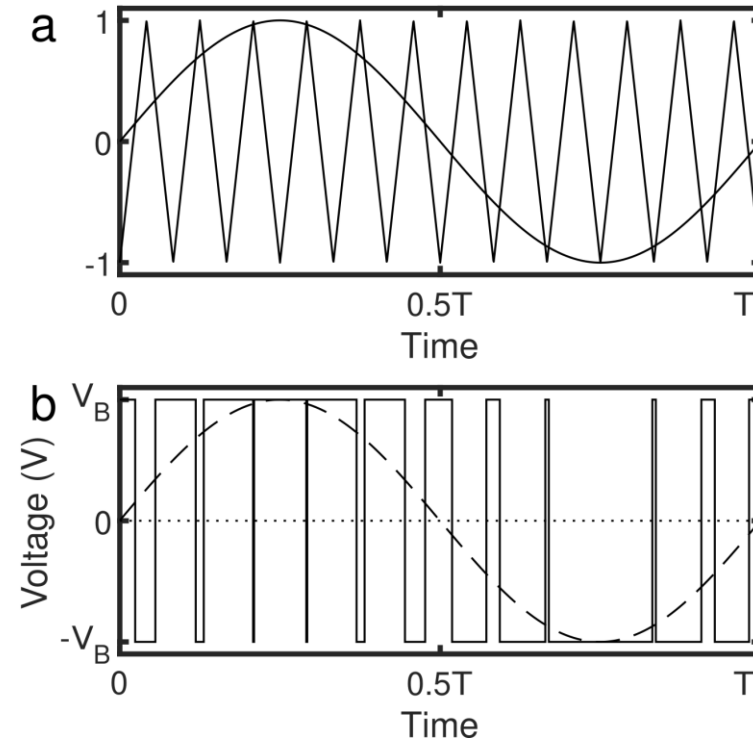
Sinusoidal Pulse Width Modulation (SPWM) Inverters

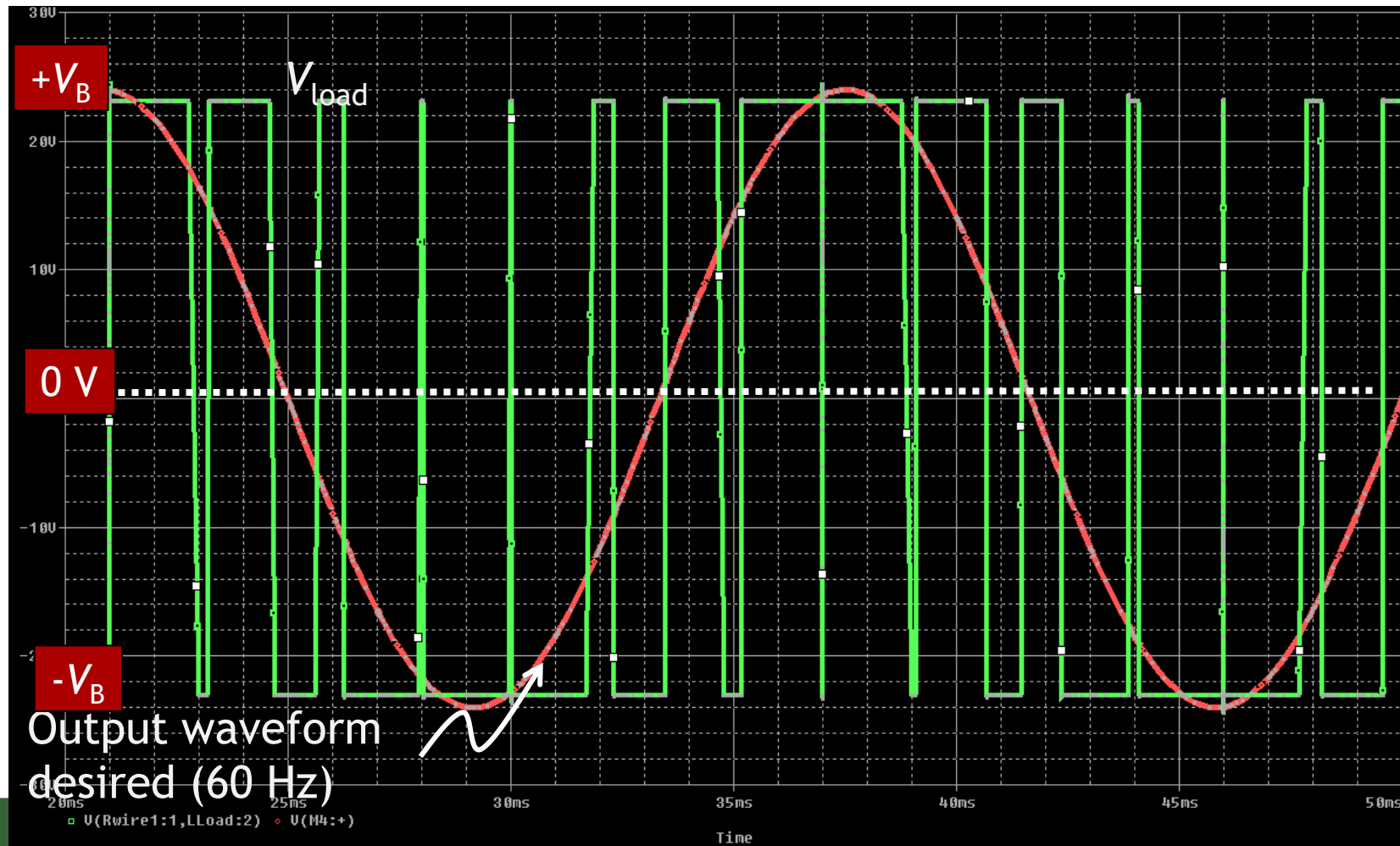
- Distortion in output can be reduced using SPWM
- Basic idea: vary duty cycle over the course of the output

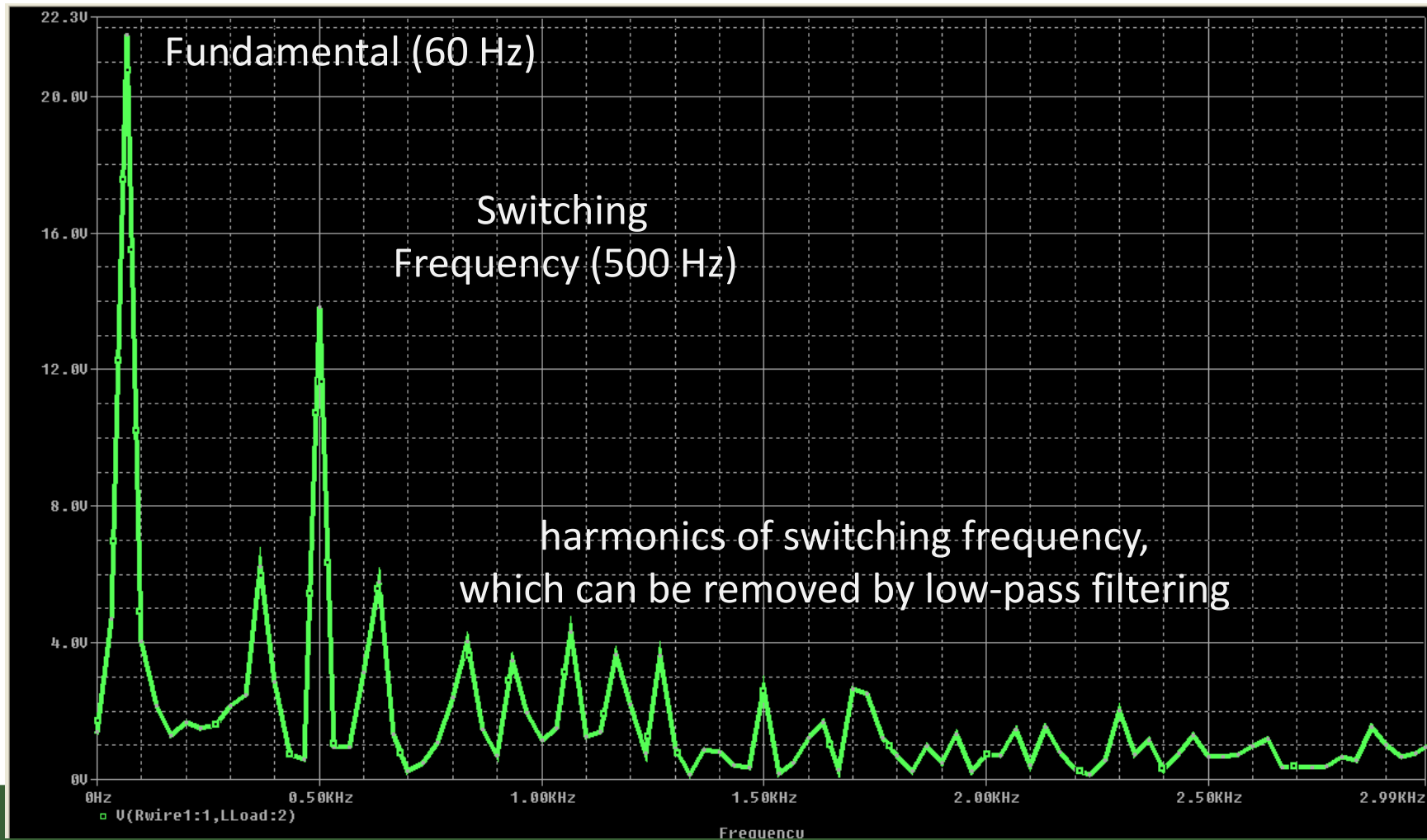


Sinusoidal Pulse Width Modulation

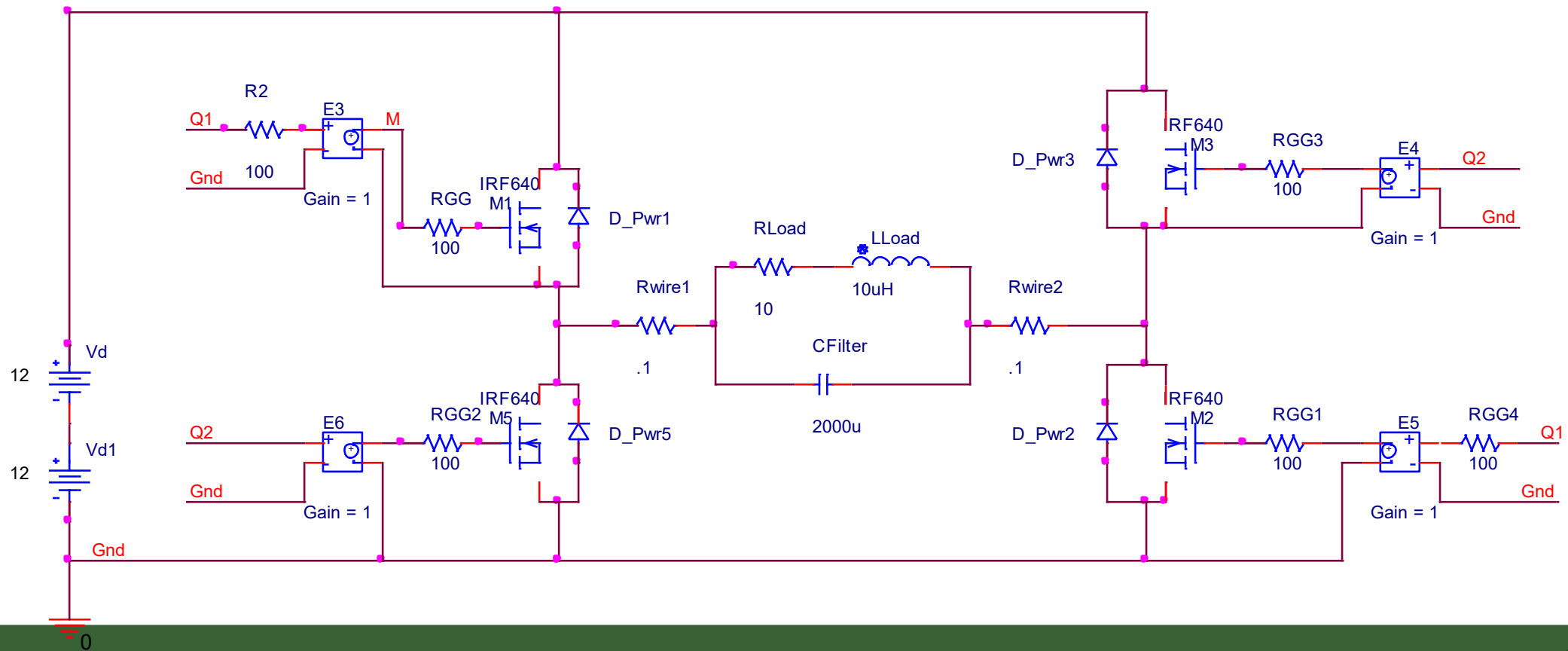
- Modulating signal $f_m(t)$: sinusoidal signal whose frequency is the desired output frequency (e.g. 50 Hz)
- Carrier signal $f_\Delta(t)$: triangle shaped signal whose frequency is usually kilohertz
- Duty cycle of switches controlled based on logic
 - Q1, Q3 closed: $f_m(t) > f_\Delta(t)$
 - Q2, Q4 closed: $f_m(t) \leq f_\Delta(t)$



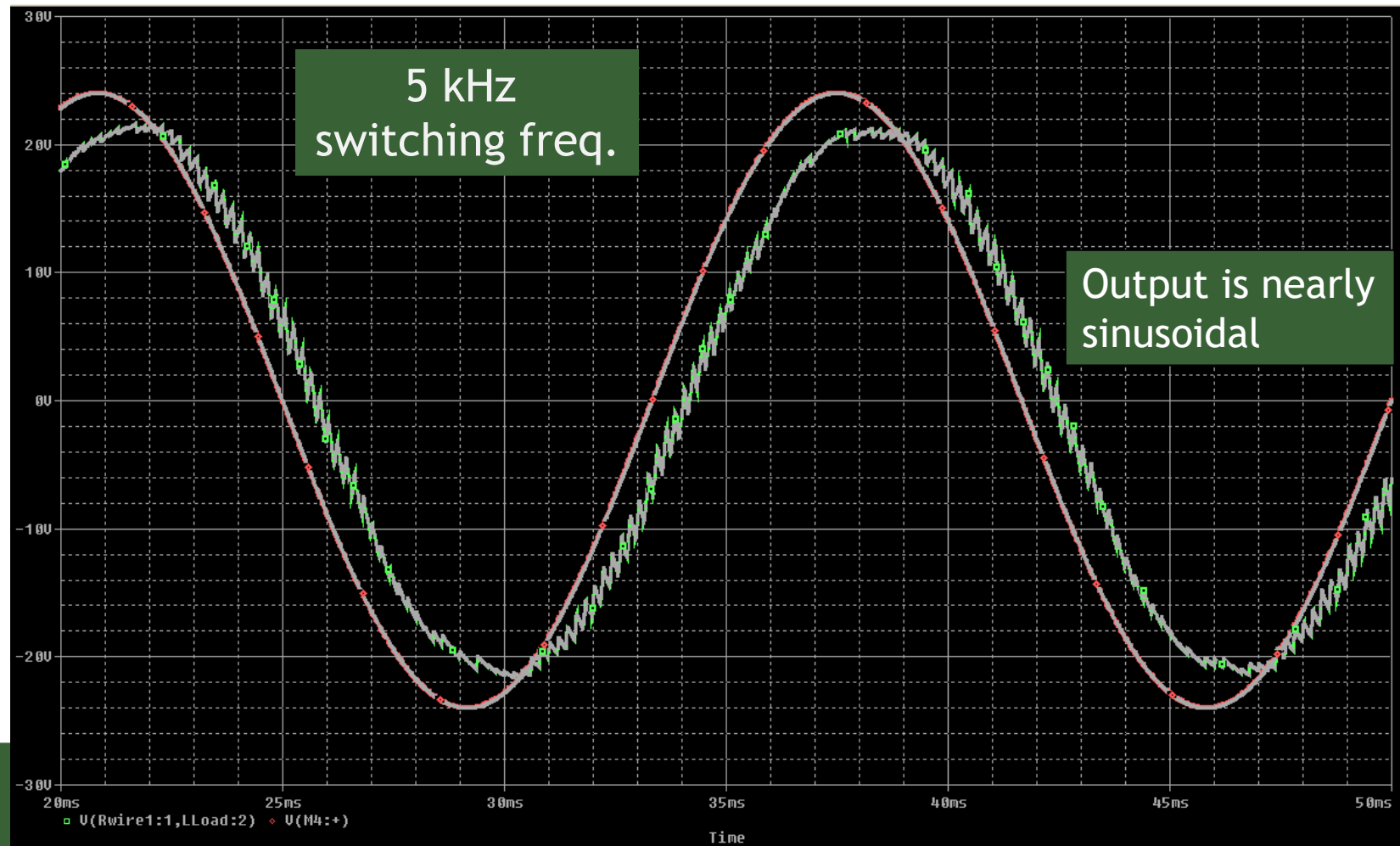




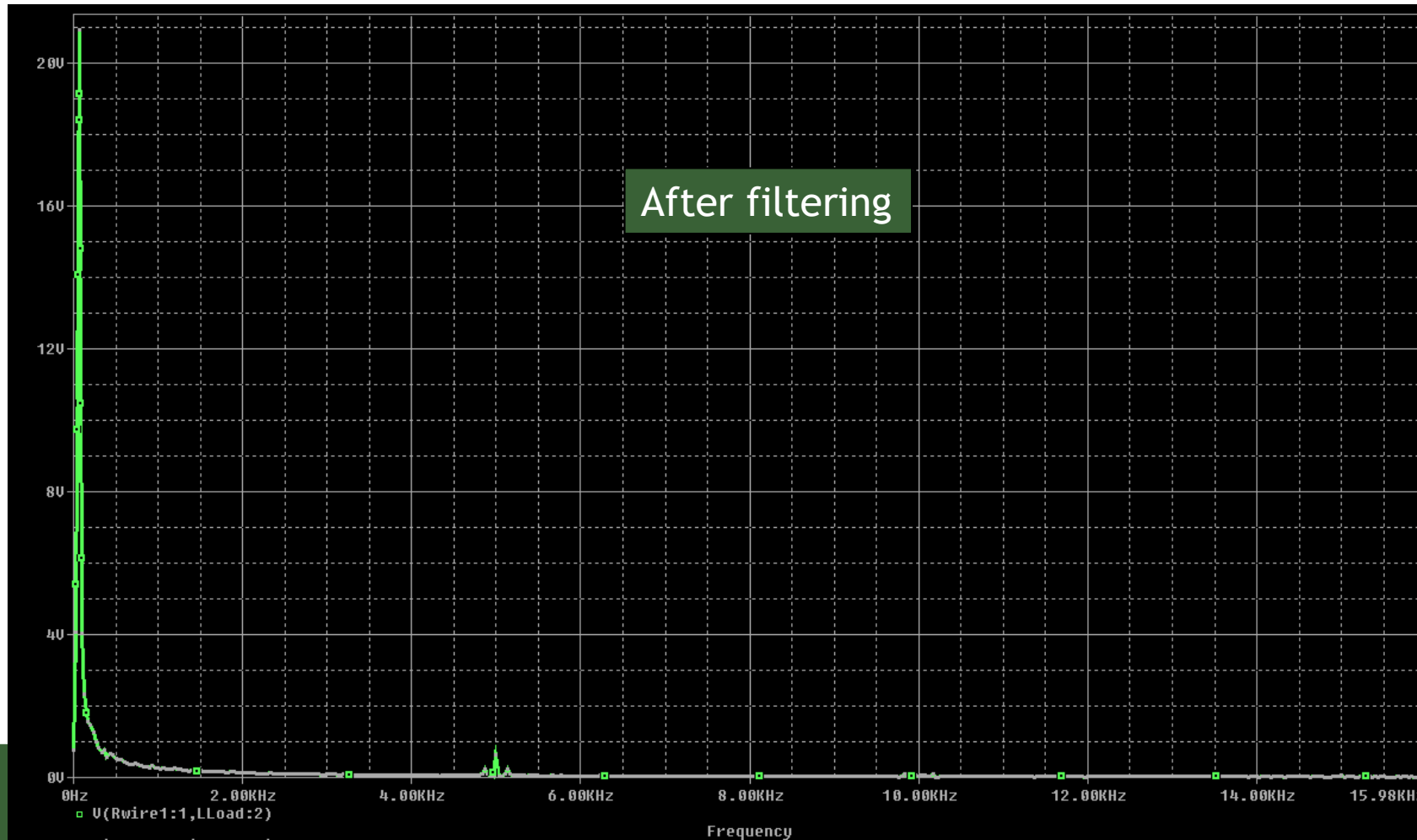
PWM Inverter



PWM Inverter



PWM Inverter



SPWM

Magnitude of output voltage can be controlled by adjusting the amplitude of the modulating and carrier signals

$$m_a = \frac{a_m}{a_{\Delta}}$$

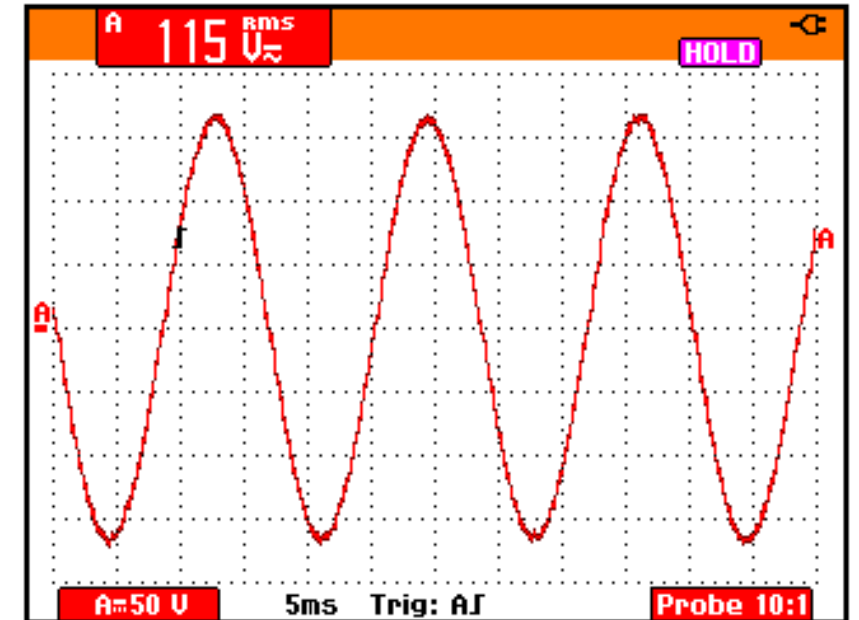
so that $V_{AC}^{\max} = m_a V_{DC}$

Inverter

Inverters are able to output waveforms with very little distortion

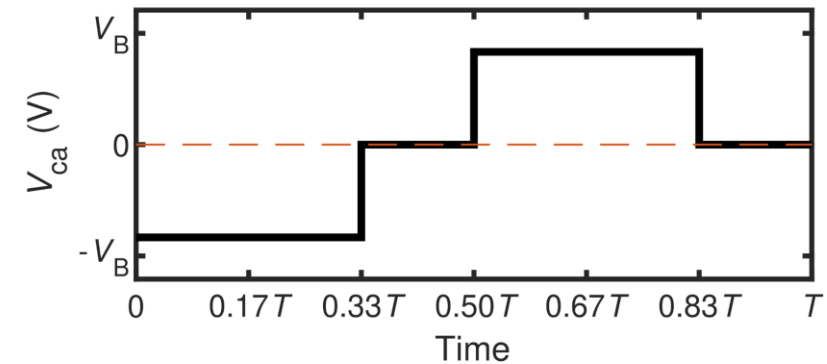
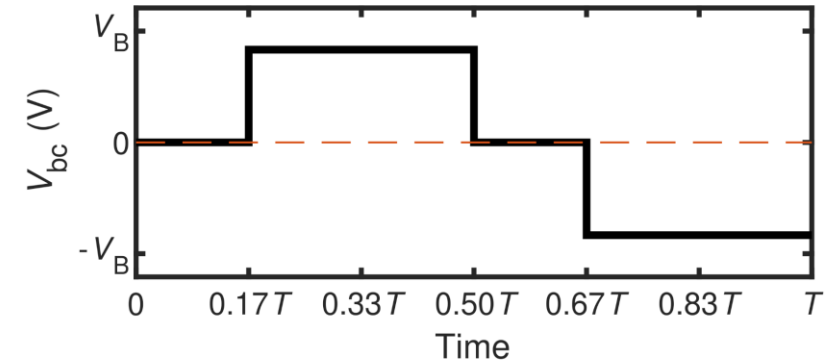
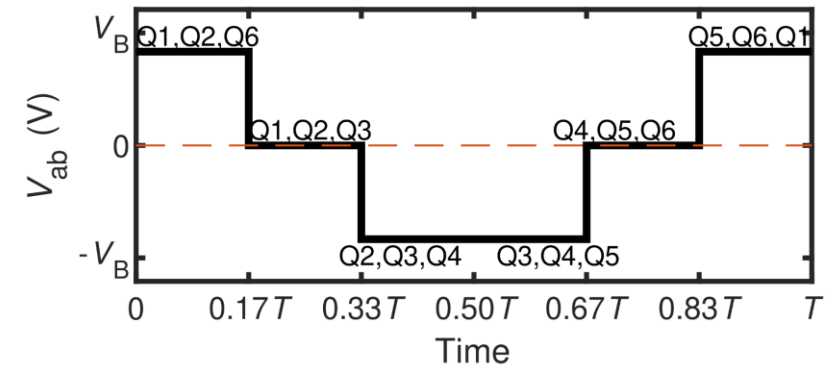
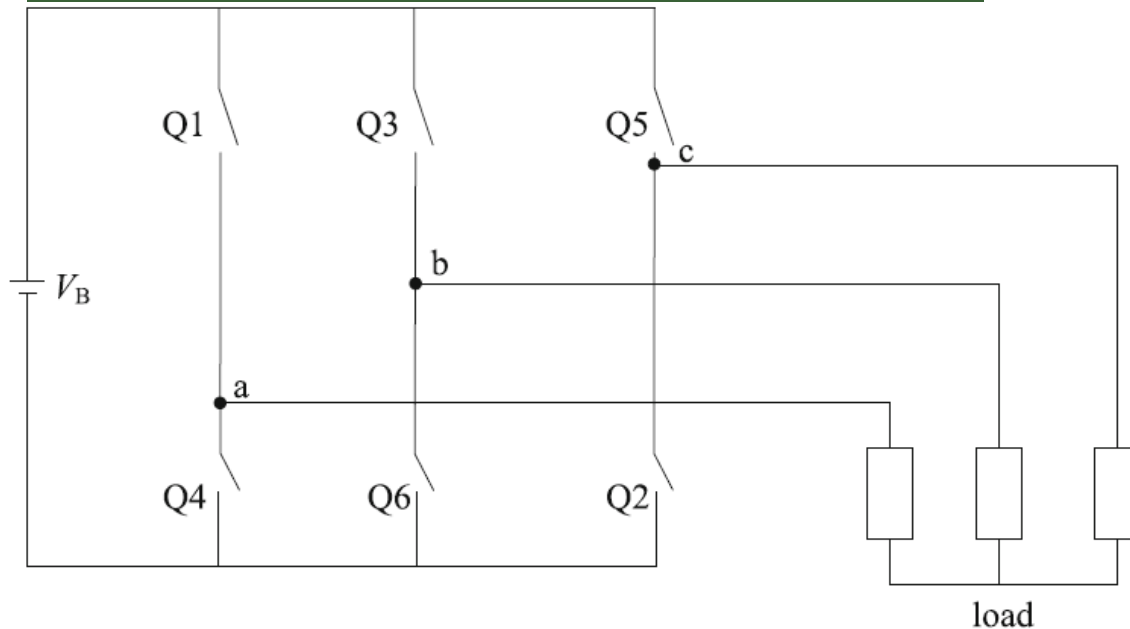


(courtesy H. Louie)



Three-Phase Inverters

Three-phase inverters follow same basic principles as single-phase inverters, and output balanced 3-phase voltage



Inverter Efficiency

- Input and output relationship for inverters assuming lossless conditions

$$P_{\text{inv,in}} = V_{\text{DC}} I_{\text{DC}} = P_{\text{inv,out}} = |V_{\text{inv}}| |I_{\text{inv}}| \cos(\Phi)$$

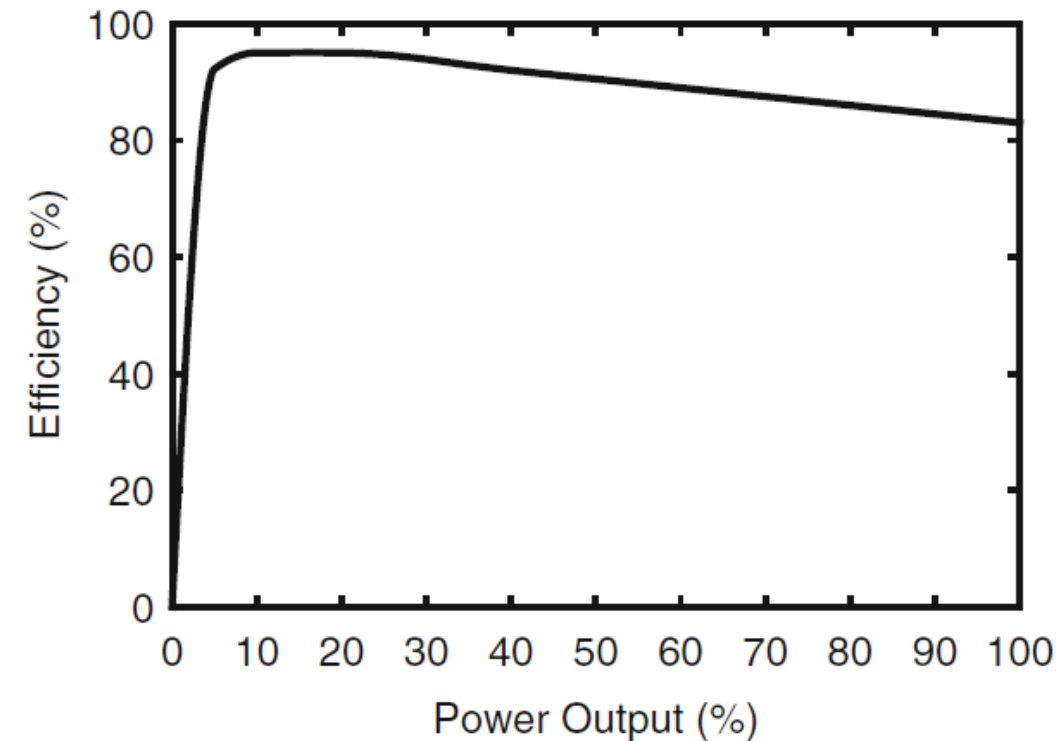
- Including the efficiency of the inverter:

$$P_{\text{inv,out}} = \eta_{\text{inv}} P_{\text{inv,in}}$$

Inverter Efficiency

- Inverter efficiency is not constant, and is non-linear
- Low efficiency at low-loading
- Inverter efficiency is sometimes reported as the “European” efficiency, which considers different operating points

$$\eta_{\text{inv,Euro}} = 0.03\eta_{\text{inv}}(0.05) + 0.06\eta_{\text{inv}}(0.10) + 0.13\eta_{\text{inv}}(0.20) \\ + 0.10\eta_{\text{inv}}(0.30) + 0.48\eta_{\text{inv}}(0.50) + 0.20\eta_{\text{inv}}(1.00)$$

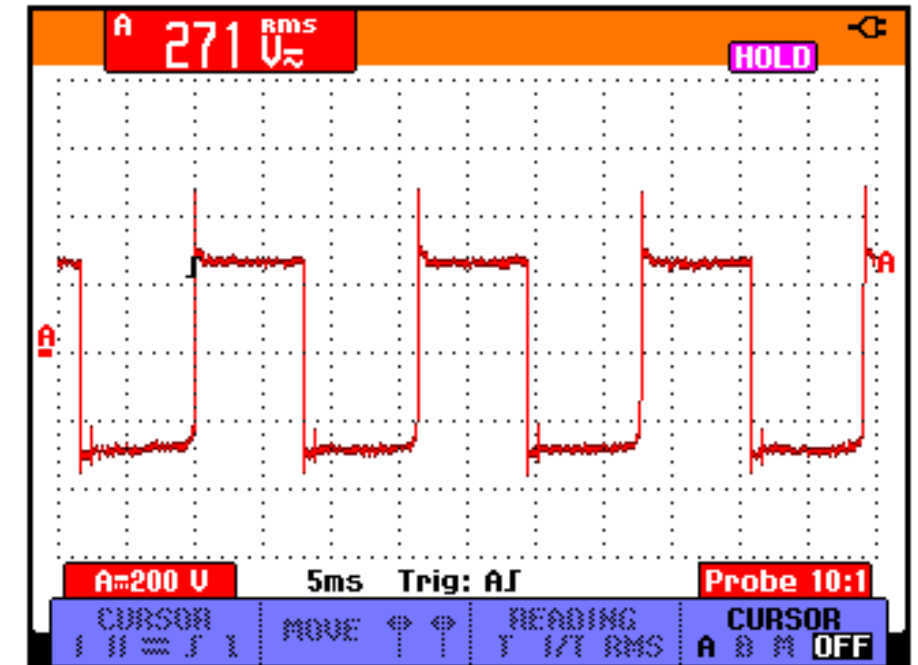


Modified Sinewave (Squarewave) Inverter

Inexpensive, but has high distortion and poor voltage regulation

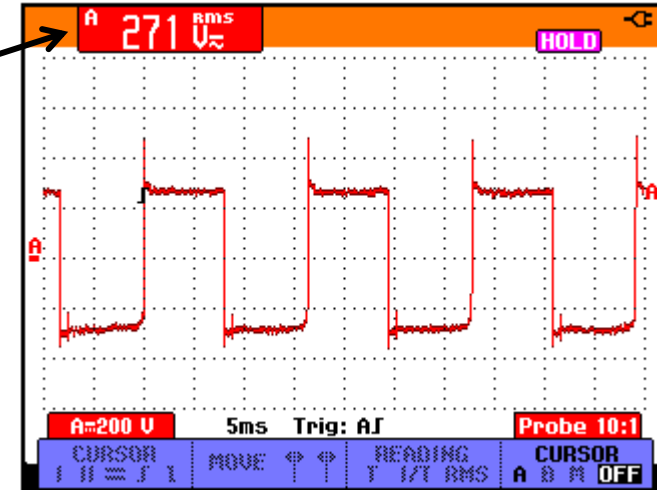


(courtesy H. Louie)

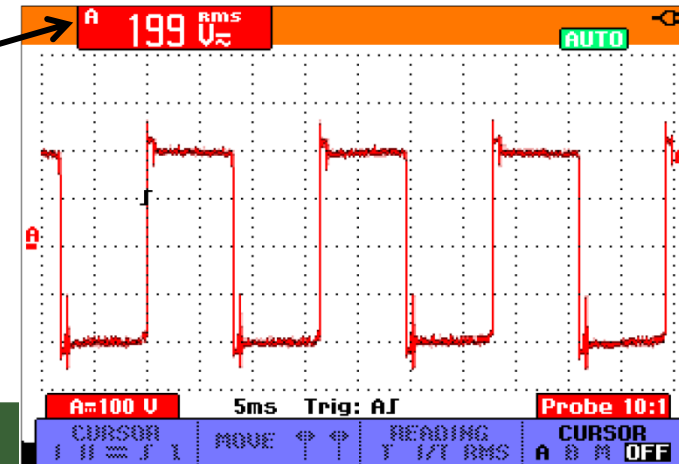


Squarewave Inverter

$$V_B = 12.4 \text{ V}$$



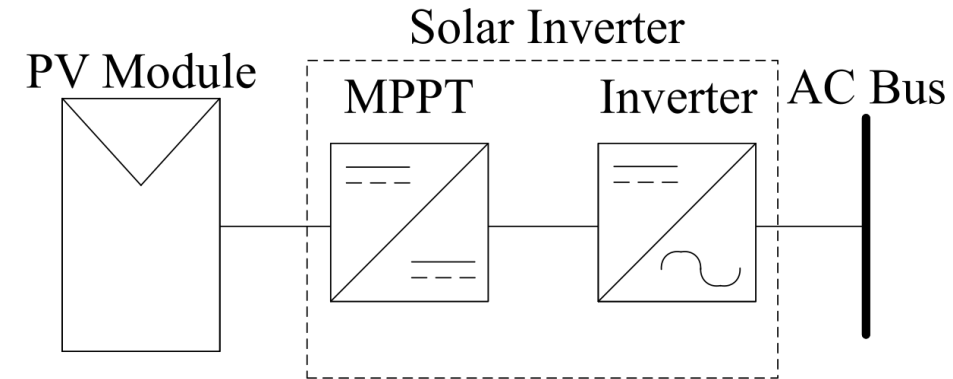
$$V_B = 9 \text{ V}$$



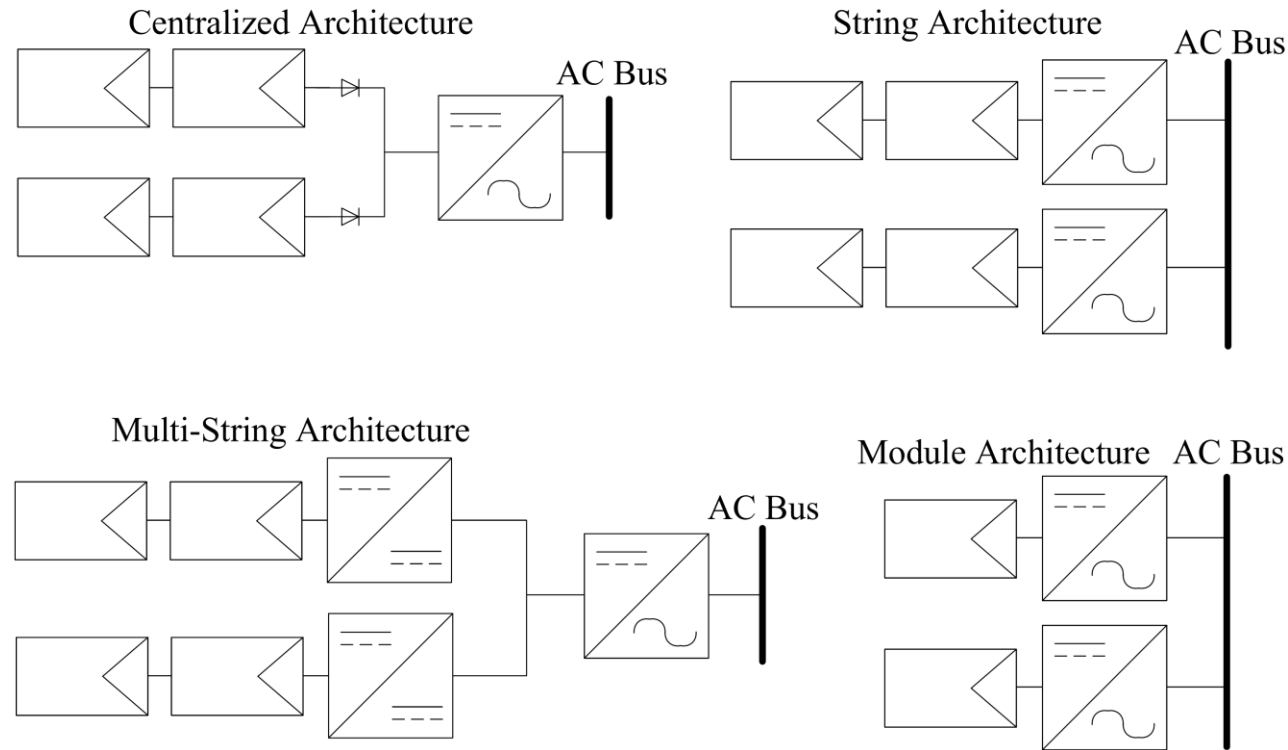
Solar Inverters

Solar inverters (or “PV inverters”) couple a PV module, string, or array directly to AC bus

- AC-coupled architecture
- DC bus is internal
- another generation source must form the AC bus
- MPPT integrated into inverter

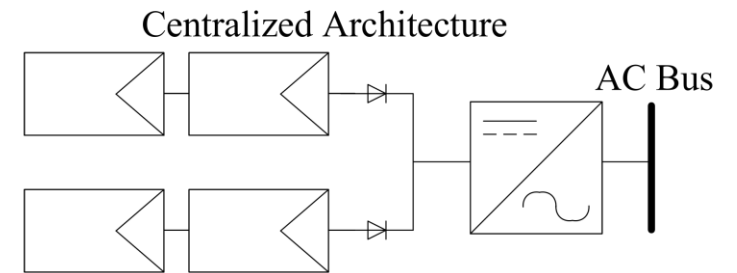


Solar Inverter Architectures



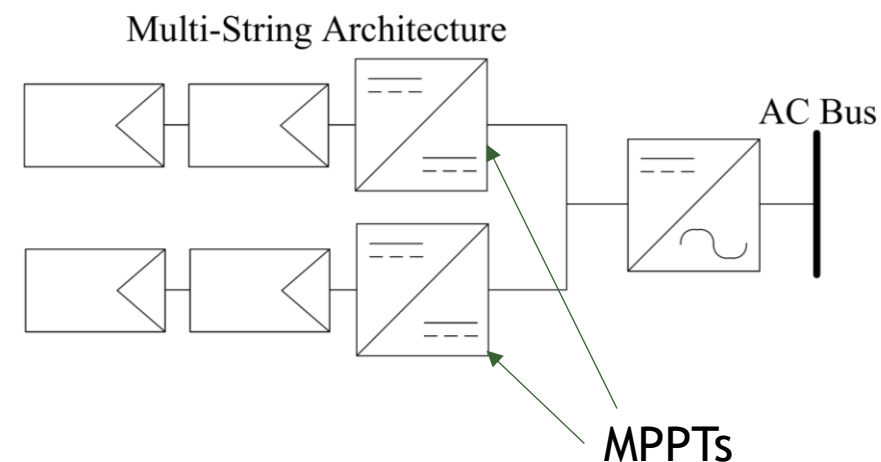
Centralized Architecture

- Several PV modules connect to one inverter
- Lowest cost-per-kilowatt architecture
- Highest mismatch losses (MPPT performed for array, rather than individually)
- Requires blocking diodes if there are multiple strings
- Single point of failure



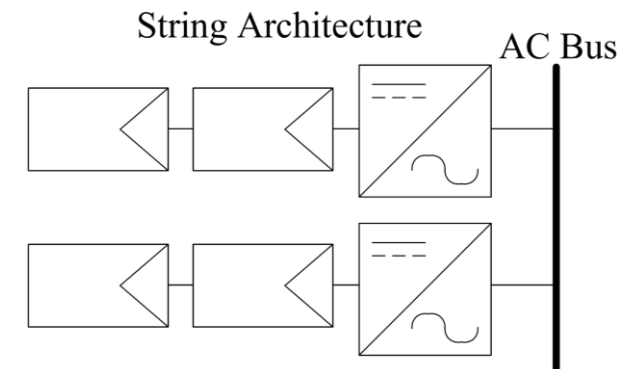
Multi-String Architecture

- Each string connected to its own MPPT
 - lowers mismatch losses (MPPT performed for each string)
- Single DC/AC converter
- No need for blocking diodes
- Increased cost-per-kilowatt
- Single point of failure (inverter)



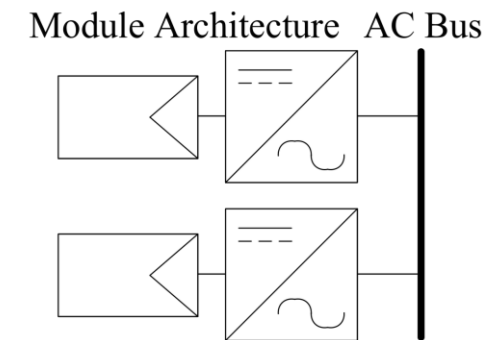
String Architecture

- Each string connected to own solar inverter
 - same mismatch losses as multi-string architecture
- Increased cost-per-kilowatt
- Modularity allows for easy expansion of additional strings



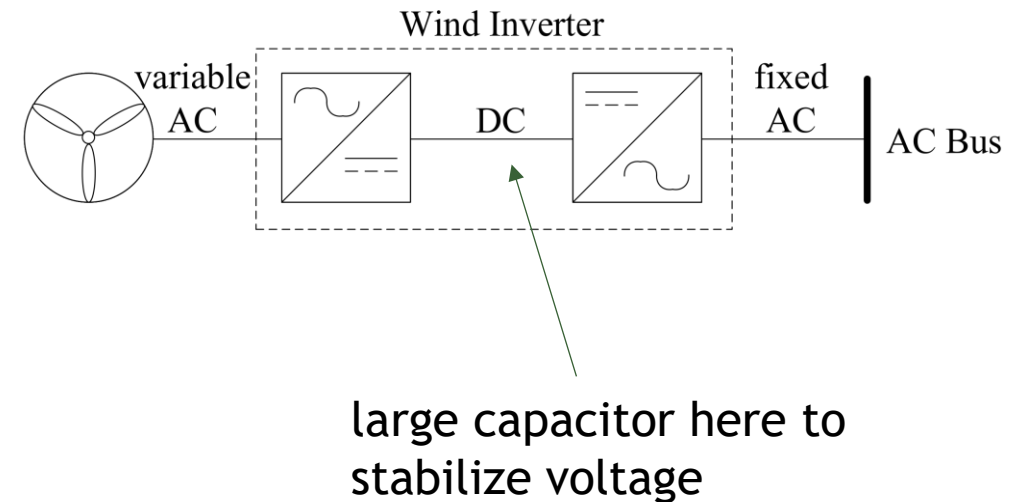
Module Architecture

- Each module connected to own solar inverter
 - lowest mismatch losses
- Highest cost-per-kilowatt
- Modularity allows for easy expansion of additional modules



Wind Inverters

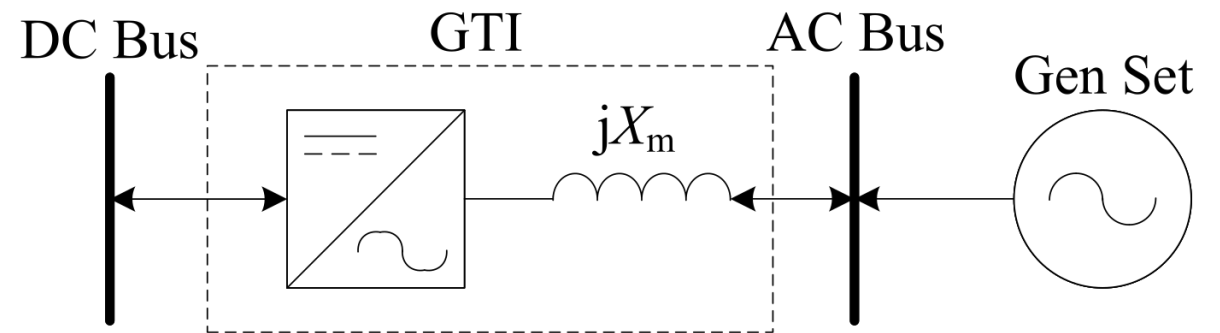
- WECS are usually DC-coupled due to their variable voltage frequency output
- Wind inverters used to AC couple WECS
 - variable frequency from WECS is converted to DC, then to fixed frequency AC that is synchronized to AC bus voltage
- Another generation source is needed to form AC bus voltage



Grid Tied Inverters (GTI)

Certain inverters can be coupled to AC Bus's with other AC coupled sources

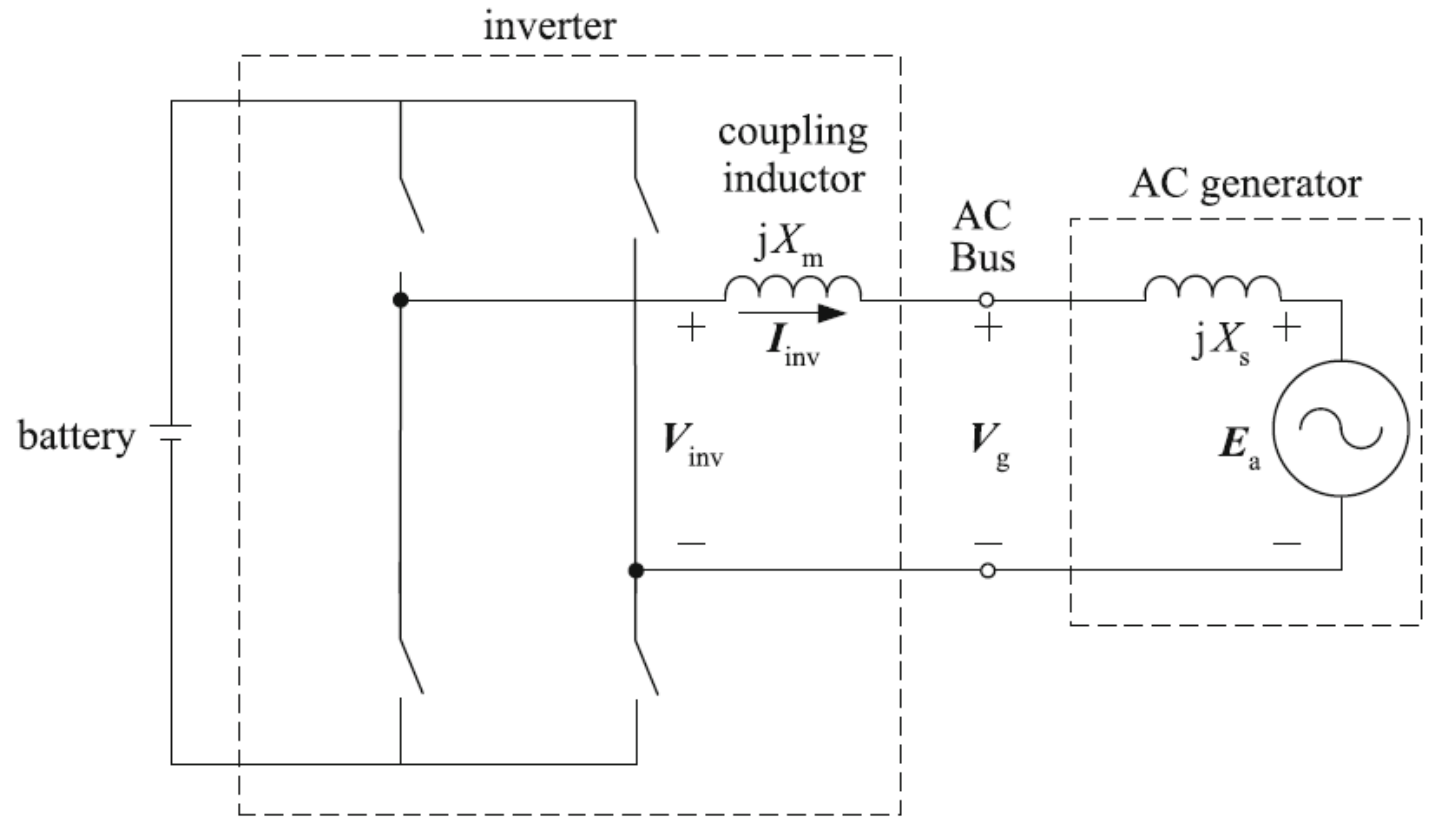
Note that “grid tied” does not necessarily mean the national grid is on site, rather it means there is an AC bus that has been formed by another generation source



GTI

- Inverters have large coupling inductors X_m
 - Allow the inverter voltage V_{inv} to be different from the AC bus voltage V_g
- Inverter current:

$$I_{inv} = \frac{V_{inv} - V_g}{jX_m}$$



Inverter Power

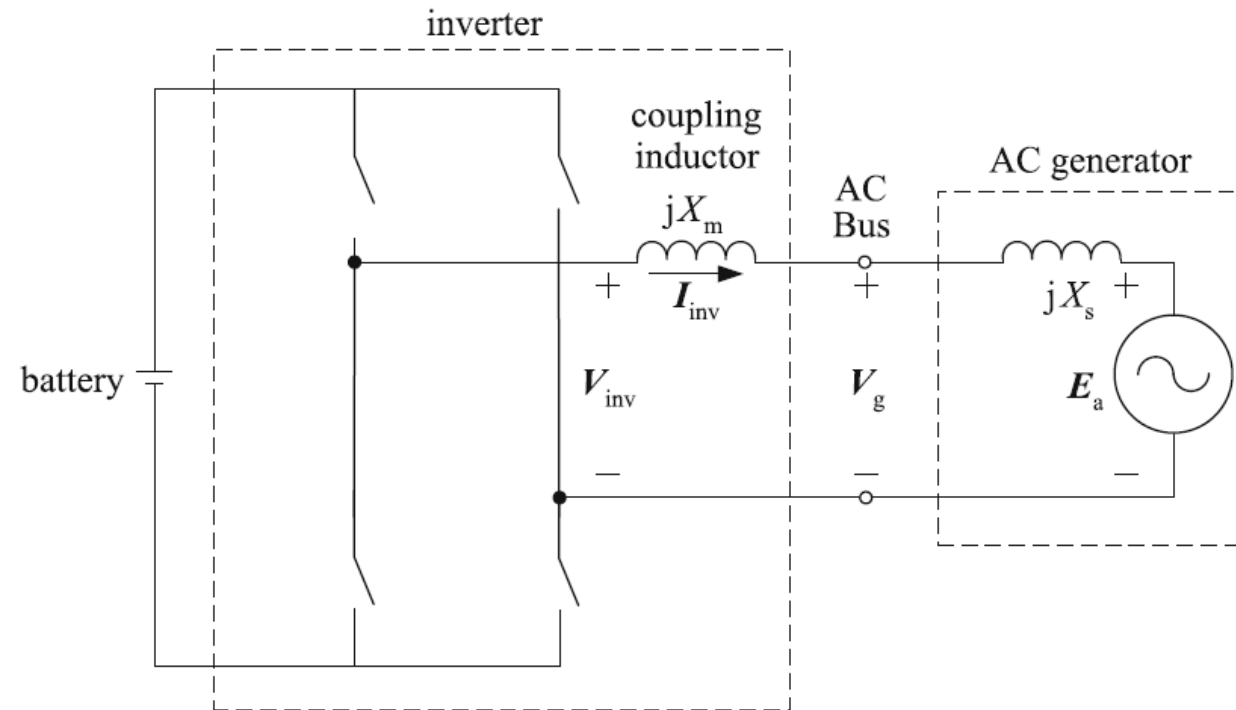
$$P_{\text{inv}} = \text{Re} \left\{ \mathbf{V}_{\text{inv}} I_{\text{inv}}^* \right\} = \text{Re} \left\{ \mathbf{V}_{\text{inv}} \left(\frac{\mathbf{V}_{\text{inv}} - \mathbf{V}_g}{jX_m} \right)^* \right\}$$

$$P_{\text{inv}} = \text{Re} \left\{ \frac{|\mathbf{V}_{\text{inv}}|^2}{-jX_m} - \frac{\mathbf{V}_{\text{inv}} \mathbf{V}_g}{-jX_m} \right\}$$

$$P_{\text{inv}} = \text{Re} \left\{ \frac{|\mathbf{V}_{\text{inv}}|^2}{-jX_m} - \frac{|\mathbf{V}_{\text{inv}}| |\mathbf{V}_g| \cos(\delta) + j |\mathbf{V}_{\text{inv}}| |\mathbf{V}_g| \sin(\delta)}{-jX_m} \right\}$$

$$P_{\text{inv}} = \frac{|\mathbf{V}_{\text{inv}}| |\mathbf{V}_g| \sin(\delta)}{X_m}$$

Power supplied by the inverter can be controlled by adjusting the inverter's voltage magnitude $|\mathbf{V}_{\text{inv}}|$ and phase δ , but is more sensitive to the phase



Inverter Reactive Power

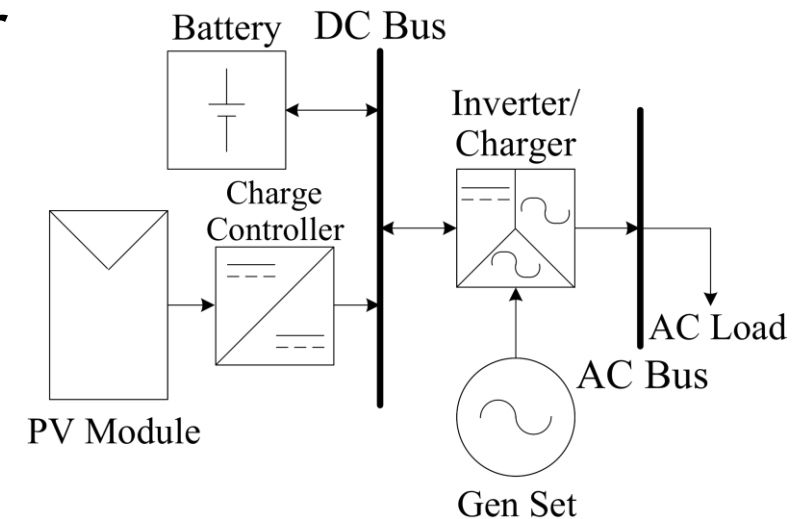
- Reactive power supplied by the inverter is

$$Q_{\text{inv}} = \frac{|V_{\text{inv}}|^2}{X_m} - \frac{|V_{\text{inv}}| |V_g| \cos \delta}{X_m}$$

- includes reactive power consumed by the coupling inductor
- Reactive power is more sensitive to the inverter's voltage magnitude than the angle δ

Bi-Directional Converters (Inverter/Chargers)

- Some inverters allow bi-directional power flow, typically to charge batteries
- Often called “inverter/charger” or “bidirectional converters”
- Allow power to/from AC and DC busses



Example 12.3

A lossless single-phase inverter/charger is connected to an AC bus. A single-phase gen set is also connected to the AC bus. The real portion of the AC load is 13 kW. The gen set supplies 10 kW. The inverter output reactance is $0.5 \, \Omega$; let the generator voltage be 230 V and the inverter voltage be 232 V. Compute the required power from the inverter and the associated angle of its voltage δ . Compute the reactive power from the inverter.

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The real power required from the inverter is $P_{\text{inv}} = 13 - 10 = 3 \text{ kW}$. The angle δ is computed as

$$\sin(\delta) = P_{\text{inv}} \frac{X_m}{|V_{\text{inv}}| |V_g|} = 3000 \frac{0.5}{230 \times 232} = 0.0281 \text{ rad}$$

$$\sin^{-1}(0.0281) = 0.0281 \text{ rad} = 1.61^\circ$$

Example 12.3

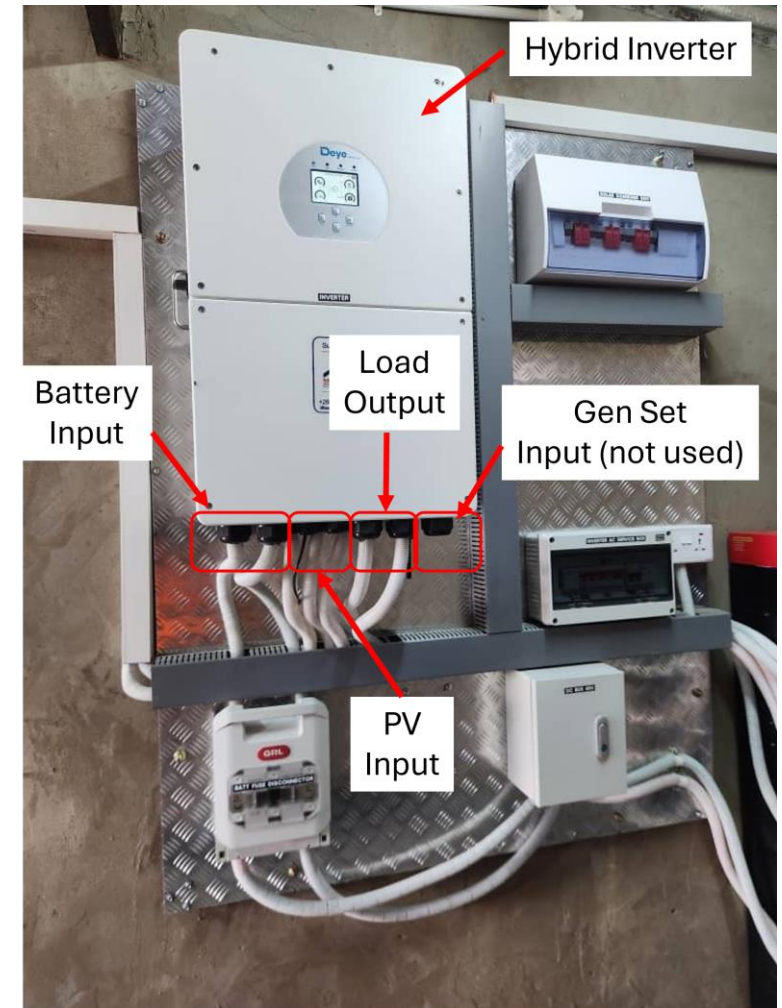
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The reactive power of the inverter is

$$Q_{\text{inv}} = \frac{|V_{\text{inv}}|^2}{X_m} - \frac{|V_{\text{inv}}| |V_g| \cos \delta}{X_m} = \frac{232^2}{0.5} - \frac{232 \times 230 \times \cos(1.61^\circ)}{0.5} = 970.2 \text{ VAR}$$

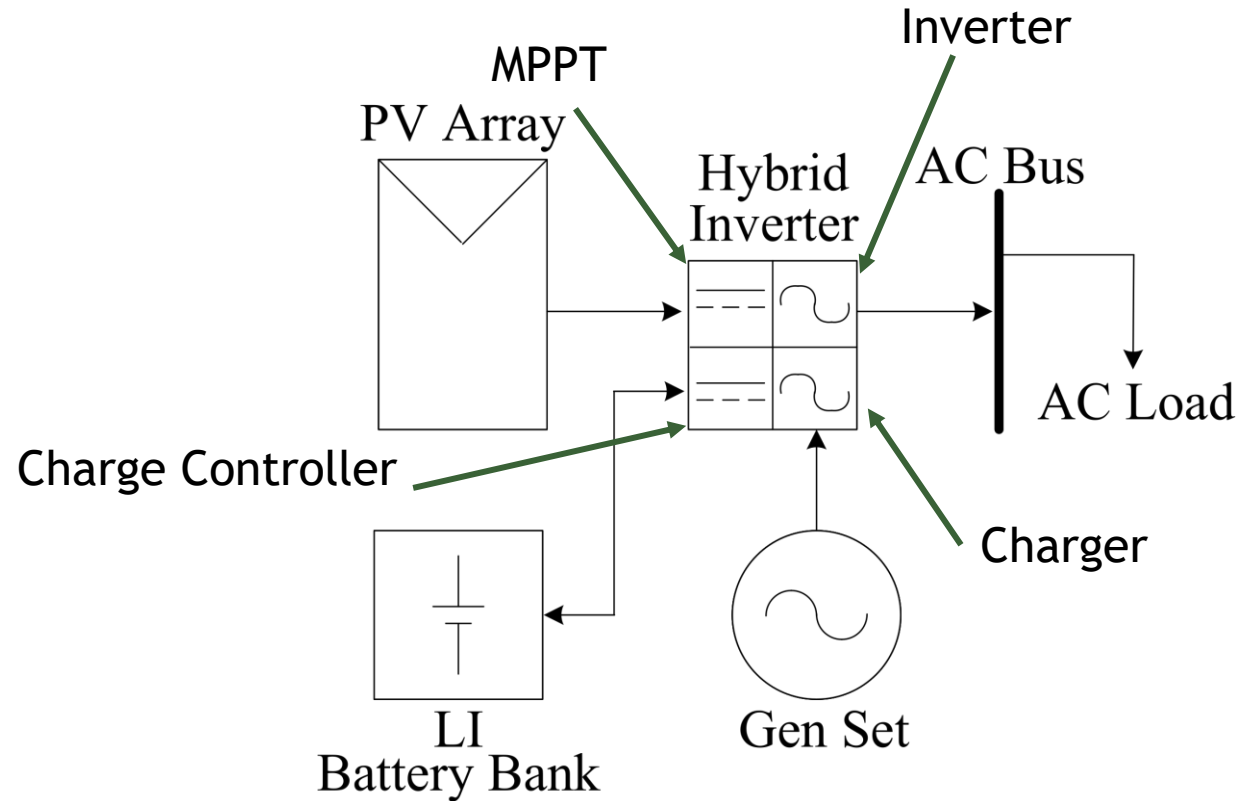
Hybrid Inverters

- Hybrid inverters the functions of several components into one device
 - maximum power point tracker
 - inverter
 - charger
 - charge controller
- Can reduce installation time and complexity, and lower cost
- Single point of failure



(courtesy D. Nausner)

Hybrid Inverters



Practical Considerations

Higher-quality inverters feature:

- output sinusoidal voltage with little distortion and constant frequency and magnitude;
- good voltage regulation;
- high efficiency at low loading;
- insensitivity to changes in input voltage;
- short-term increased surge capacity;
- low-voltage disconnect capability;
- can be configured in the field;
- include data logging and diagnostic features

Summary

- **Electronic Load Controller (ELC):** used for speed control in MHP. Power to a ballast load is controlled to counteract changes in load. Types include binary-weighted resistor networks, phase angle control, impedance control
- **Inverter:** converts DC to AC
- **Modified Sinewave Inverter:** inexpensive inverter that outputs a heavily distorted waveform. They should not be used with sensitive loads

Summary

- **Solar Inverter:** converts DC output of a PV module, string, or array to AC. They often incorporate MPPT.
- **Wind Inverter:** used AC couple WECS by converting variable frequency AC to DC, then to fixed frequency AC that is synchronized to the AC bus
- **Grid Tied Inverter:** an inverter that can synchronize its voltage to the AC bus. Usually capable of supplying real and reactive power

Summary

- **Bi-Directional Converter (Inverter/Charger):** inverter that allows power to pass through in either direction (AC to DC or DC to AC)
- **Hybrid Inverter:** combines several functions into a single unit, usually charge controller, MPPT, inverter, and charger