

17-Off-Grid Distribution System Design

Off-Grid Electrical Systems in Developing Countries, 2nd Edition
Chapter 17

Preface

- These lectures slides are intended to accompany the textbook *Off-Grid Electrical Systems in Developing Countries, 2nd Edition, 2025* written by Dr. Henry Louie and published by [SpringerNature](#)
- Additional content, explanations, derivations, examples, problems, errata, and other materials are found in the book and on www.drhenrylouie.com
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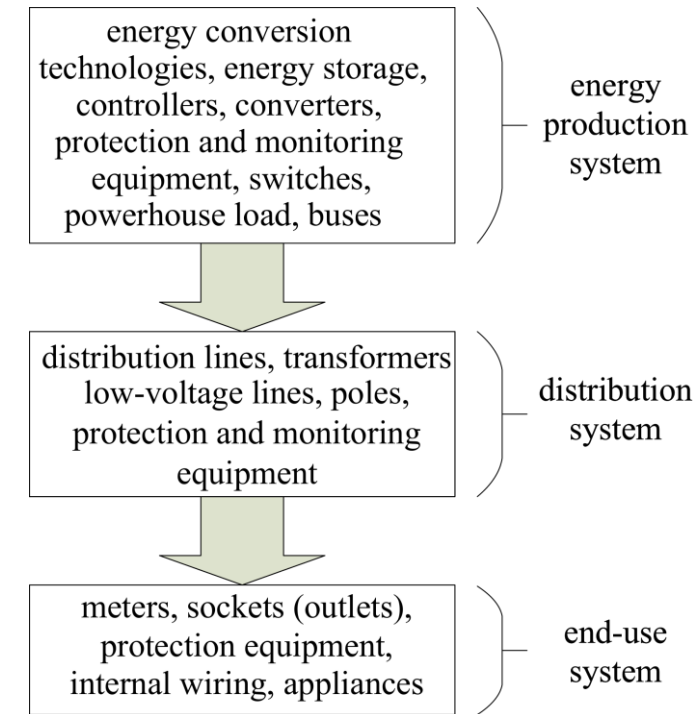
Learning Outcomes

At the end of this lecture, you will be able to:

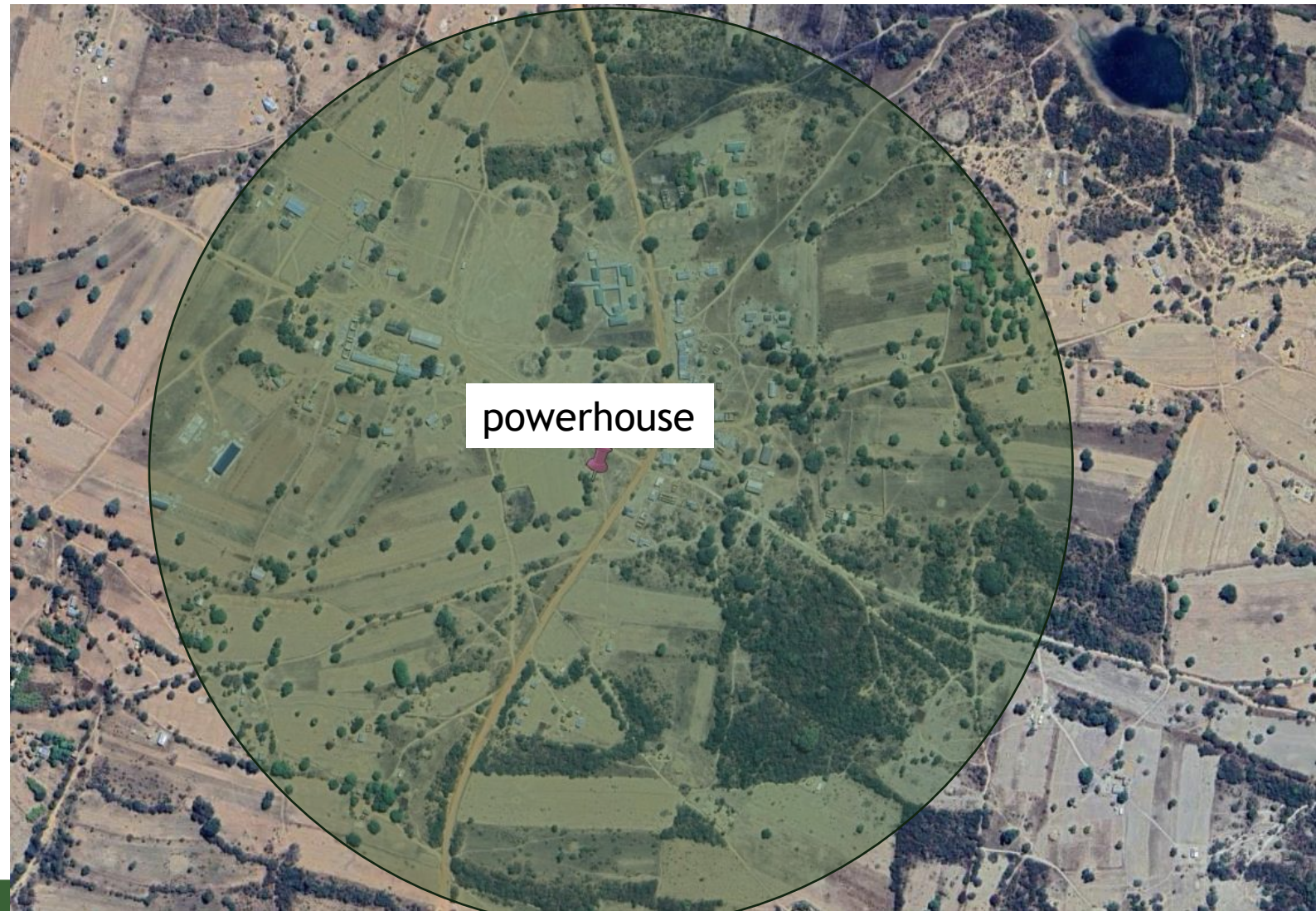
- ✓ Compare and contrast different distribution system voltage types (AC or DC), methods (over-head or underground), topologies (hub-and-spoke or trunk-and-branch), and configurations (single-phase, split-phase, three-phase delta and wye)
- ✓ Describe considerations and approaches to routing power lines in off-grid communities
- ✓ Perform engineering calculations for distribution systems including determining the voltage drop and power loss along distribution lines

Introduction

- Distribution systems facilitate the flow of power from energy production system to end-use system
- Mini-grids, metro-grids, and mesh/swarm grids have a distribution (“reticulation”) system
 - focus on mini-grid distribution system
 - low-voltage wires (typically <415 V)
 - usually less than 1 km in length
- Concepts from grid extension (Chap. 3) apply



approximate “reach” of a
low-voltage distribution system
(1 km)



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Distribution System Characteristics

Voltage Type: AC or DC

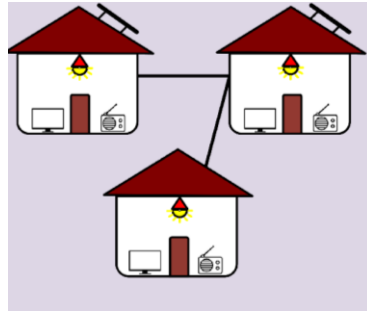
Method: overhead or underground

Topology: hub-and-spoke or trunk-and-branch

Configuration: single-phase, split-phase, or three-phase

Voltage Type: AC vs DC

- Distribution system can be AC or DC
- AC is the default choice for off-grid systems
- DC is found in smaller-capacity systems and those with only DC generation, storage, and loads. Usually low voltage
 - mesh- and swarm-grids



Advantages of AC Distribution

- ✓ Maturity of technology
- ✓ Availability of components (including load appliances)
- ✓ Capability for voltage transformation (step-up, step-down transformers)
- ✓ Compatibility with rotating machines (pumps, motors)
- ✓ Existing standards and codes (installers are more familiar with AC codes)

Advantages of DC Distribution

- ✓ Compatibility with solar power and batteries
- ✓ Compatibility with electronics
- ✓ Reduction in line impedance (no inductive component)
- ✓ Ease of control (no need for frequency regulation)
- ✓ Ease of measurement and metering

Distribution Method

- Overhead: conductors are hung overhead on utility poles
- Buried: conductors are buried in trenches 0.5 to 1 m beneath surface

Advantages of Buried Lines

- ✓ Protection from exposure and aboveground animals
- ✓ Difficult to tamper with
- ✓ Preserves community aesthetics
- ✓ No need for poles

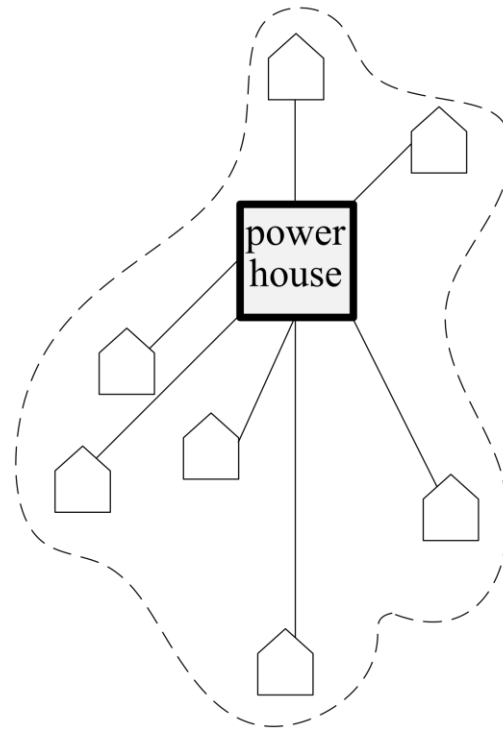
Challenges of Buried Lines

- × Difficult to inspect and locate faults
- × Difficult to expand (add connections)
- × Specialized insulation needed
- × Not practical in all terrain

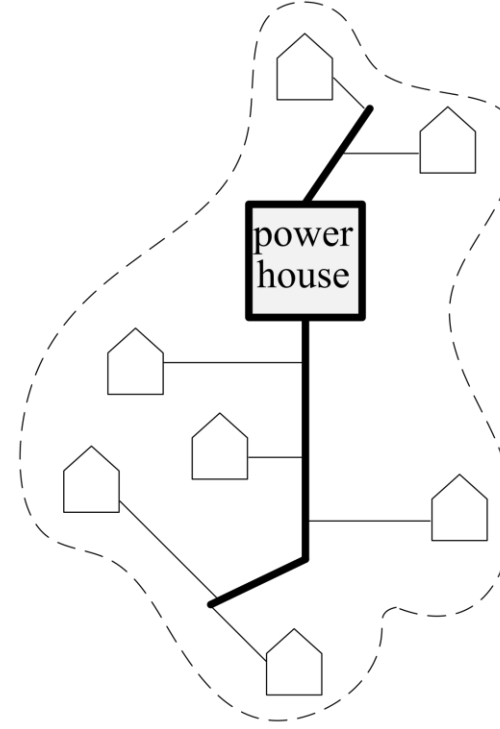
Overhead Lines

- ✓ Easy to inspect
- ✓ May not need to be insulated
- ✓ May be less expensive

Distribution Topology

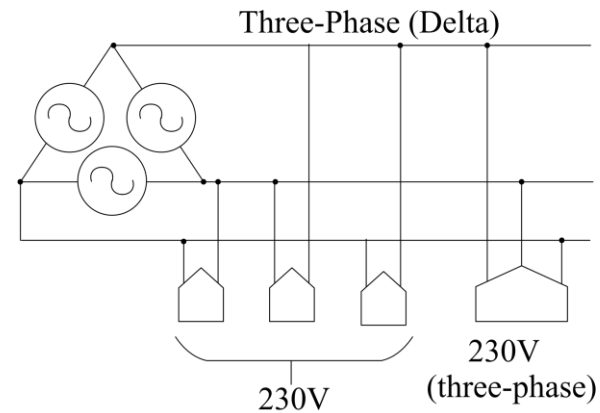
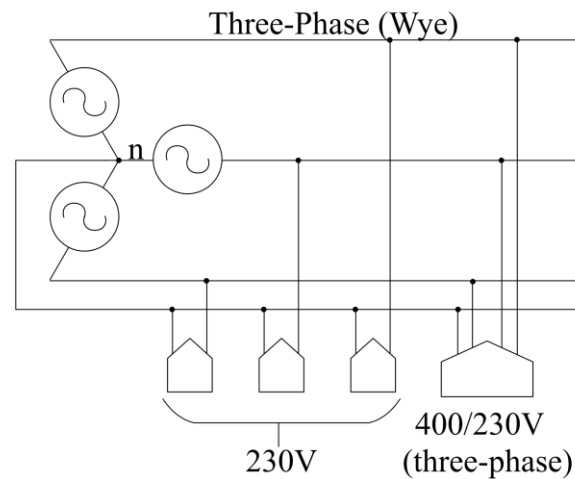
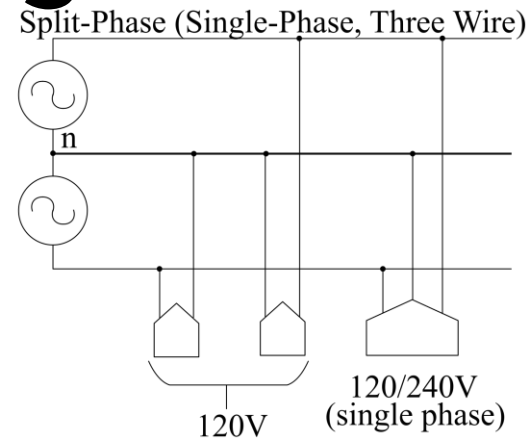
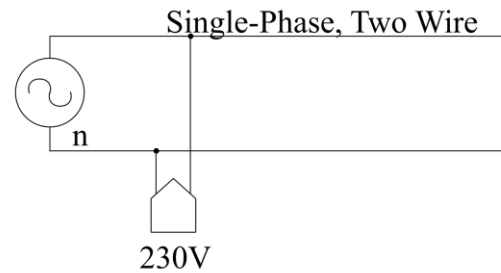


hub-and-spoke



trunk-and-branch

AC Distribution Configurations



split-phase is not common outside North/South America

Distribution Line Design Considerations

Thermal Limit

- Resistance causes distribution line to heat due to current (power) within it
- Overheating can cause failure
- Heating is inefficient (losses)
- Larger conductors reduce power loss

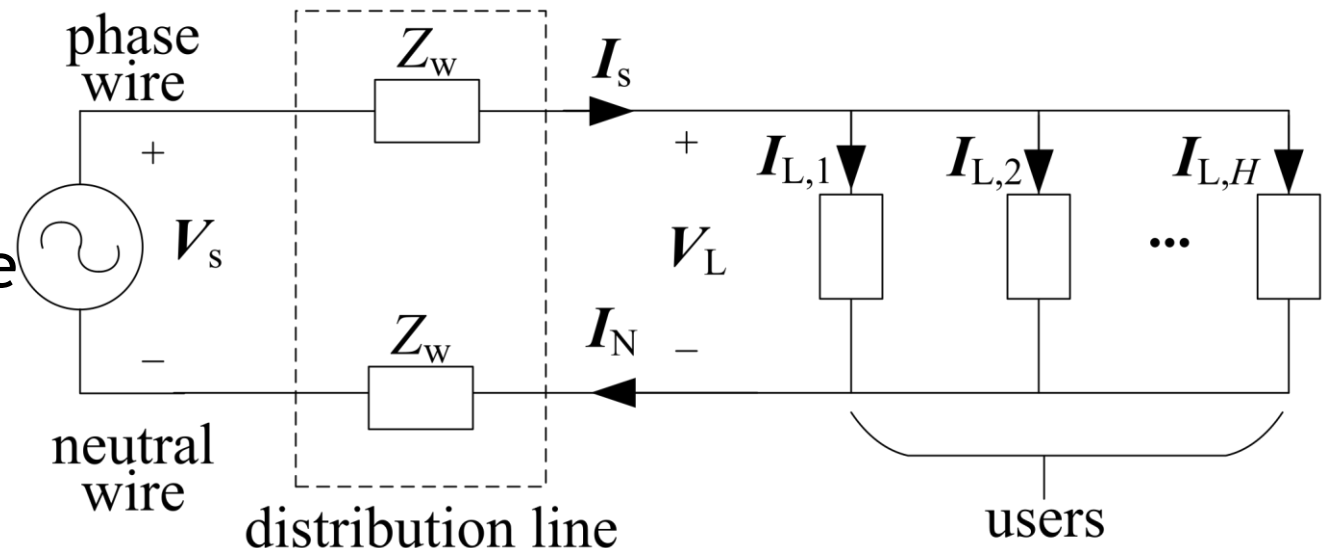
Distribution Line Design Considerations

Voltage Drop Limit

- Impedance causes a reduction in voltage along distribution line
- Voltage drop increases with current (power) flow
- Limit voltage drop to less than 5-10%
- Larger conductors reduce voltage drop

Voltage Drop: Single Phase Distribution

- Consider a line serving H users
- Assume:
 - all users consume the same power at the same power factor
 - users are located at the end of the line

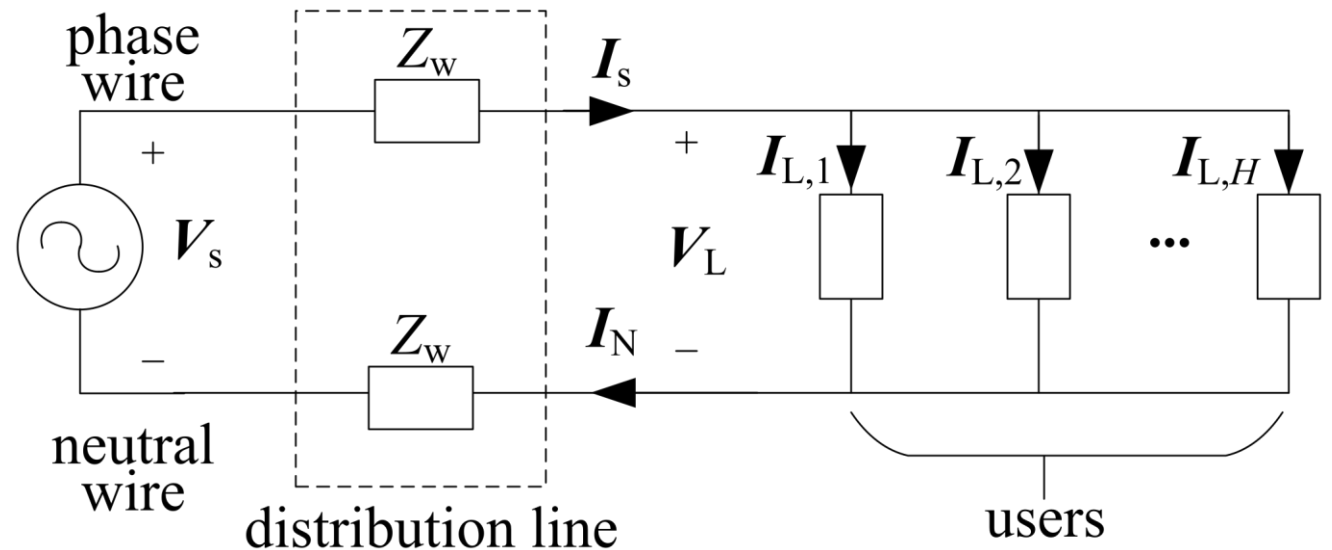


Voltage Drop: Single Phase Distribution

- By KCL:

$$I_s = \sum_{h=1}^H I_{L,h} = I_L H = I_N$$

- Neutral current I_s is equal to phase wire current I_N



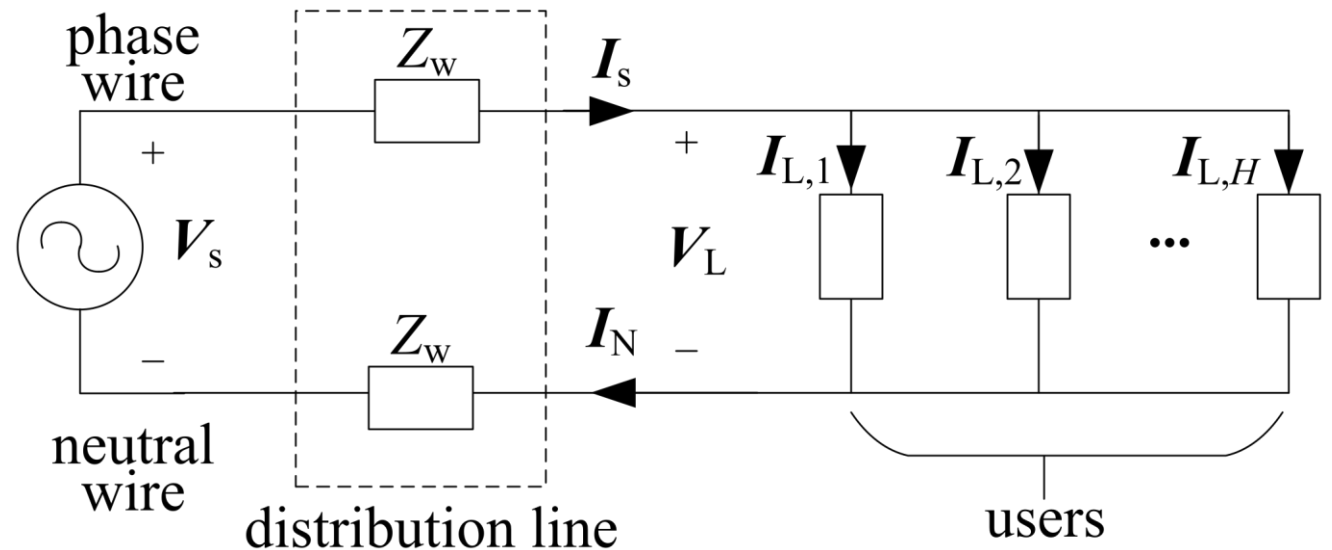
Voltage Drop: Single Phase Distribution

- By KVL:

$$V_s = I_s Z_w + V_L + I_N Z_w = 2I_s Z_w + V_L$$

- Voltage drop is:

$$V_{\text{drop}} = |V_s - V_L| = 2|I_s Z_w| = 2|I_L|H|Z_w|$$



Exercise

Consider a distribution line serving 20 houses. The aggregate peak load is expected to be 2000 W with a power factor of 0.85. The houses are supplied by a low-voltage line at 230 V, with an impedance of

$$Z_W = 0.75 + j0.31 \, \Omega$$

Determine the voltage drop along the line.

Exercise

Line current is:

$$|I_s| = \frac{S_{load}}{|V_\phi|}$$

$$|I_s| = \frac{2000 / 0.85}{230} = 10.23 \text{ A}$$

$$V_{drop} = |V_s - V_L| = 2 |I_s Z_w|$$

$$V_{drop} = 2 |I_s Z_w| = 15.67 \text{ V}$$

If this exceeds the requirement, then use a cable with lower impedance (larger cross section)

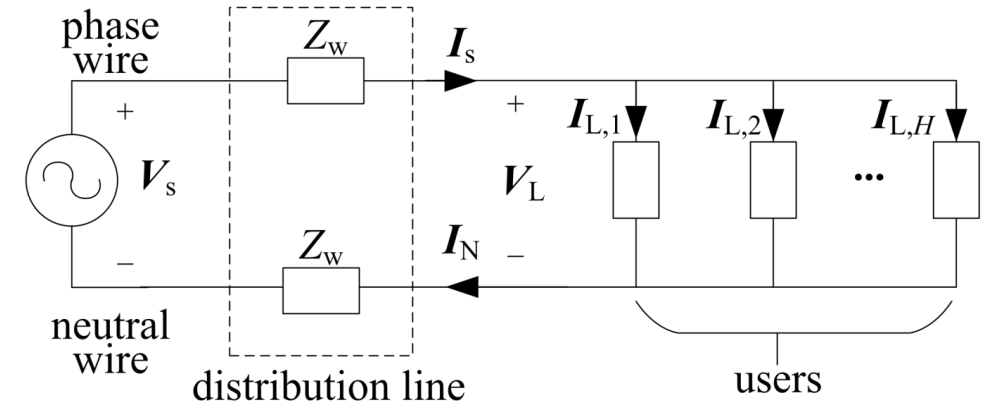
Power Loss: Single Phase Distribution

- Power loss is:

$$P_{\text{Loss}} = |I_s|^2 R_w + |I_N|^2 R_w = 2 |I_s|^2 R_w = 2 |I_L|^2 H^2 R_w$$

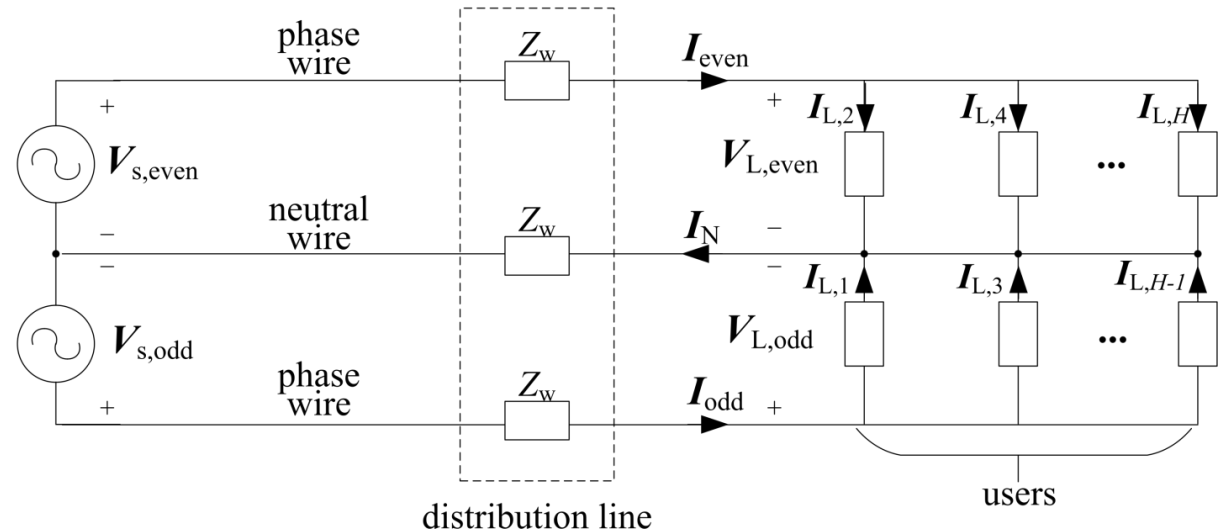
- For the last exercise

$$P_{\text{Loss}} = |I_s|^2 R_w + |I_N|^2 R_w = 2 |I_s|^2 R_w = 146.5 \text{ W}$$



Split-Phase (single-phase, three wire)

- Voltage sources out of phase by 180°
- Assume
 - users divided into two equal groups
 - equal load
 - users at end of the line
- Load connection to both phase wires possible (but not considered here)



Split-Phase (single-phase, three wire)

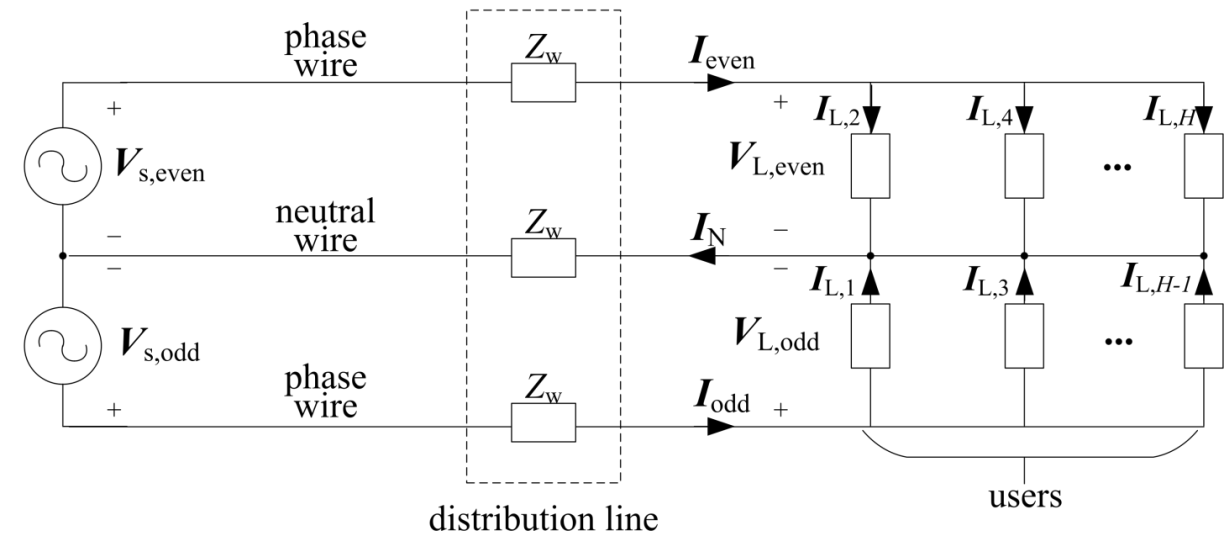
- Let $V_{s,\text{even}} = -V_{s,\text{odd}}$
- Currents in the phase wires are:

$$I_{\text{even}} = \sum_{h:\text{even}} I_{L,h} = \frac{H}{2} I_{L,\text{even}}$$

$$I_{\text{odd}} = \sum_{h:\text{odd}} I_{L,h} = \frac{H}{2} I_{L,\text{odd}}$$

- Neutral wire current

$$I_N = I_{\text{even}} + I_{\text{odd}}$$



Split-Phase (single-phase, three wire)

- Applying KVLs at the top and bottom:

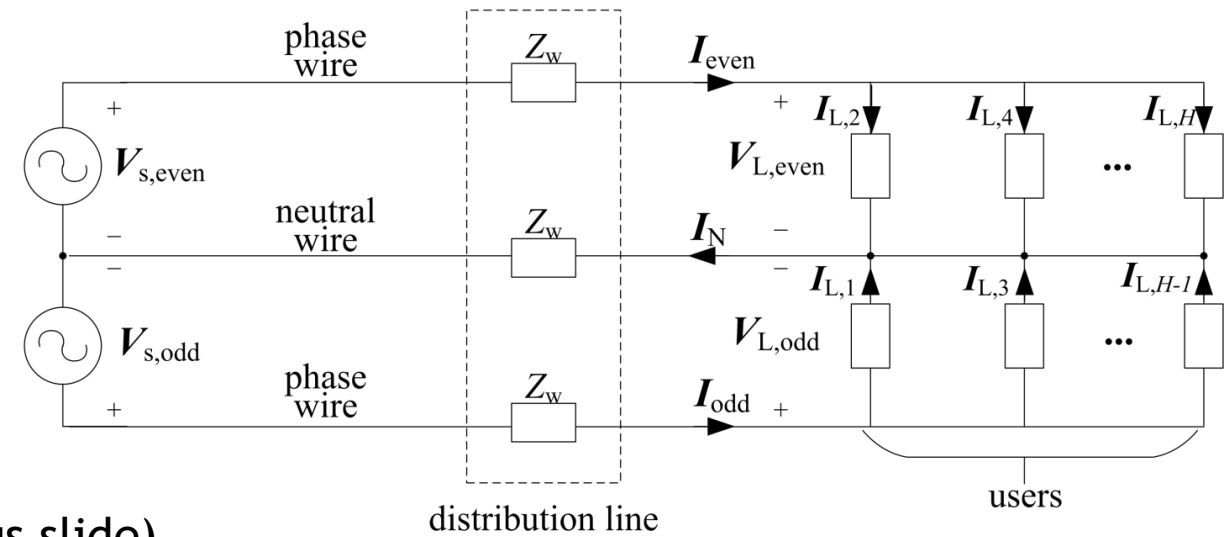
$$V_{s,\text{even}} = I_{\text{even}} Z_w + V_{L,\text{even}} + I_N Z_w$$

$$V_{s,\text{odd}} = I_{\text{odd}} Z_w + V_{L,\text{odd}} + I_N Z_w$$

- Adding the even and odd voltages

$$V_{s,\text{even}} + V_{s,\text{odd}} = (I_{\text{even}} + I_{\text{odd}}) Z_w + V_{L,\text{even}} + V_{L,\text{odd}} + 2I_N Z_w$$

$$0 = 3I_N Z_w + V_{L,\text{even}} + V_{L,\text{odd}} \quad \text{via substitution (see previous slide)}$$



Split-Phase (single-phase, three wire)

- We can show that

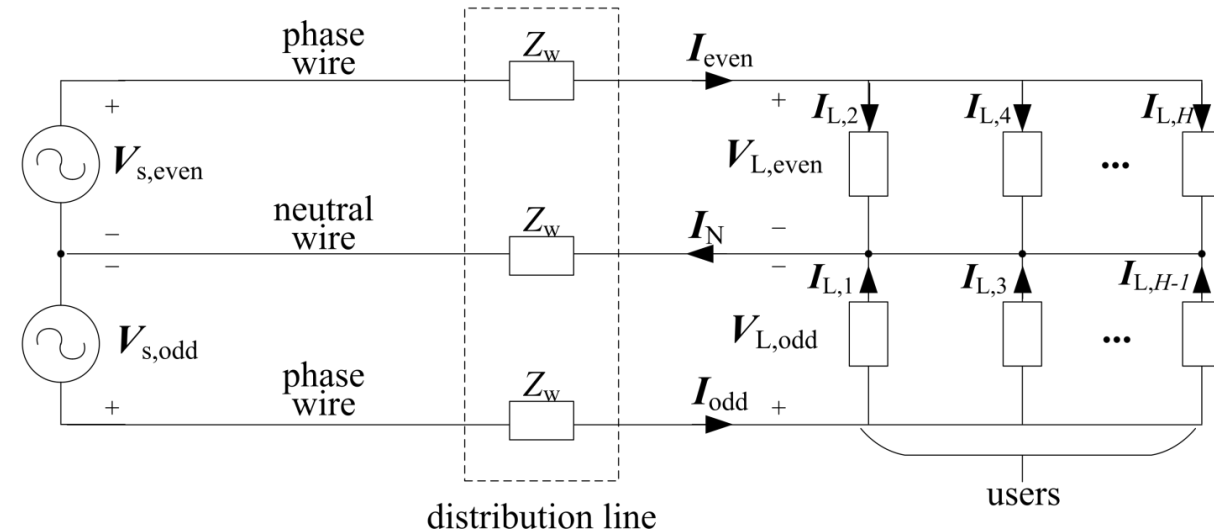
$$0 = 3I_N Z_w + V_{L,even} + V_{L,odd} \text{ means that}$$

$$I_N = 0$$

and

$$I_{odd} = -I_{even}$$

- No current on the neutral, and so there is no voltage drop



Split-Phase Voltage Drop

Voltage drop for the “odd” homes

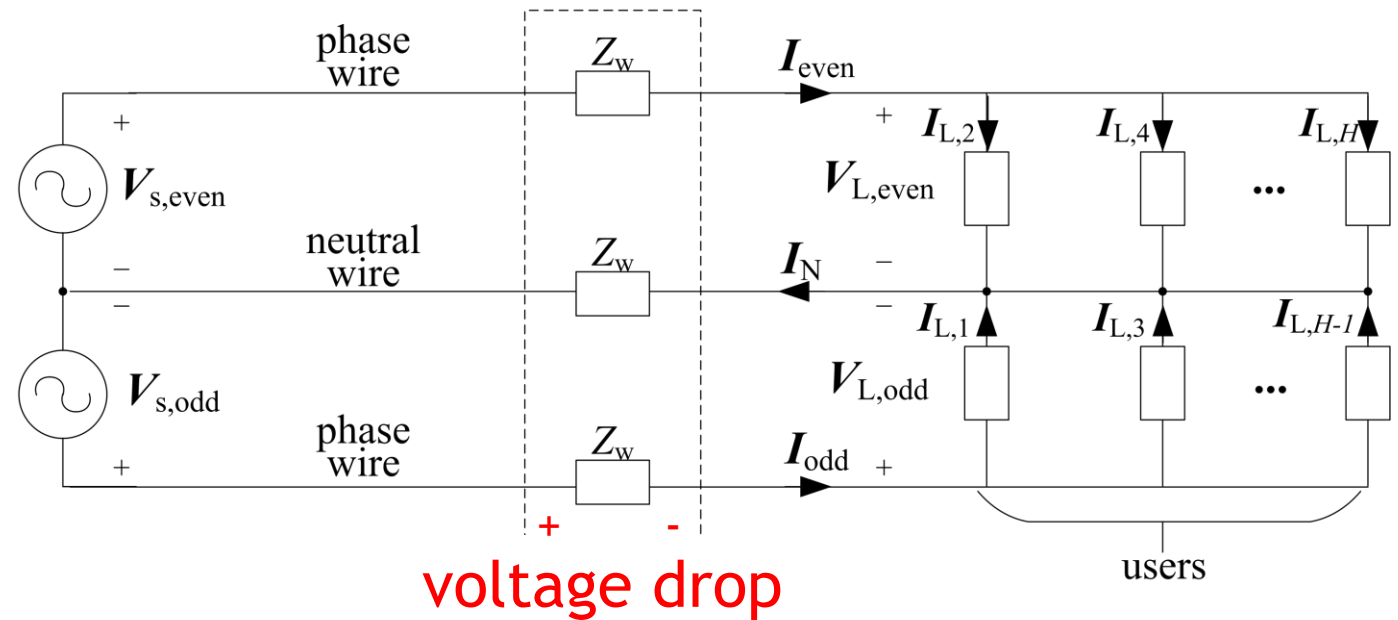
$$V_{\text{drop}} = |V_{s,\text{odd}} - V_{L,\text{odd}}|$$

$$V_{\text{drop}} = |(I_{\text{odd}}Z_w + V_{L,\text{odd}} + I_N Z_w) - V_{L,\text{odd}}|$$

$$= |I_{\text{odd}}Z_w + I_N Z_w|$$

$$= |I_{\text{odd}}Z_w| = \frac{H}{2} |I_L| |Z_w|$$

same result for “even” homes



Split-Phase Voltage Drop

For the same number of homes and load, split-phase configuration results in one-fourth the voltage drop compared to single-phase

$$V_{\text{drop}} = 2 |I_L| H |Z_w|$$

single-phase

$$V_{\text{drop}} = |I_{\text{odd}} Z_w| = \frac{H}{2} |I_L| |Z_w|$$

split-phase

reason: phase conductors carry half the current and there is no voltage drop on the neutral

Split-Phase Power Loss

- Power loss along the distribution line is

$$P_{\text{loss}} = |I_{\text{s,odd}}|^2 R_w + |I_{\text{s,even}}|^2 R_w = 2 \left(\frac{H}{2} |I_L| \right)^2 R_w$$

- Lower loss for split-phase is one-fourth that of single-phase

$$P_{\text{Loss}} = 2 |I_L|^2 H^2 R_w \text{ (single-phase configuration power loss)}$$

Three-Phase

- Higher-capacity systems may use three-phase distribution
 - three-phase inverter (or three single-phase inverters connected together)
 - three-phase gen set
- Industrial and agricultural users may require three-phase service
 - pumps, motors, large loads
- Two configurations: delta, wye

Three-Phase Wye

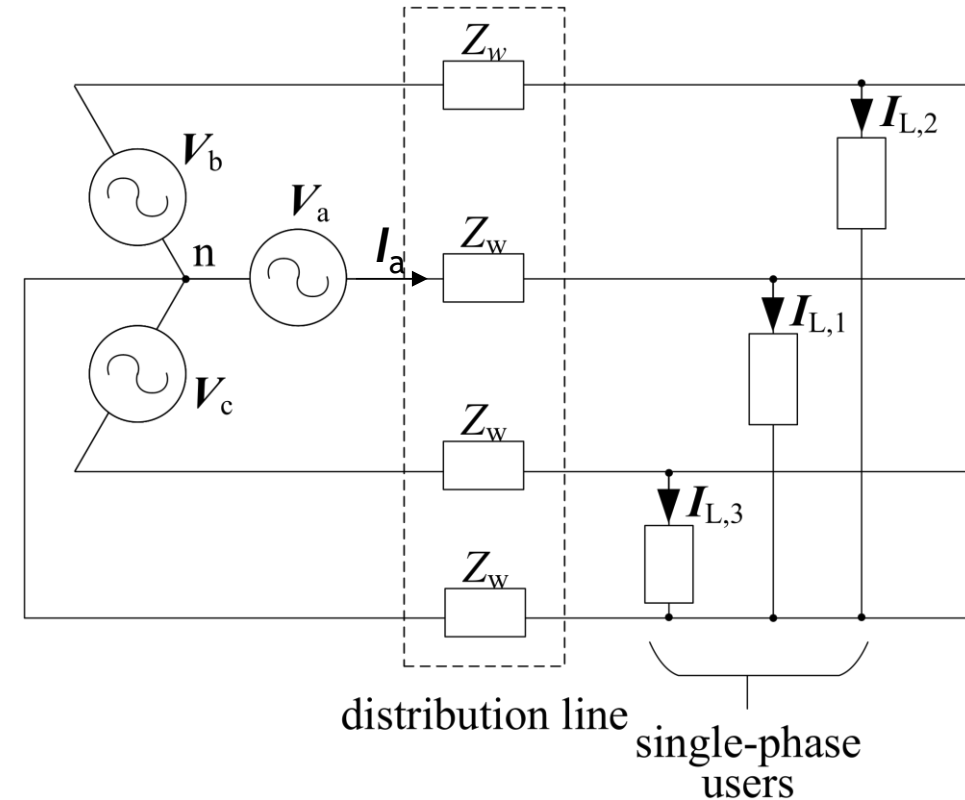
single-phase users can be thought of as one three-phase user/load

- Most efficient distribution configuration
- Assume:
 - load on each phase is identical
 - loads located at end of each line
- By three-phase theorem, the neutral current is zero (and so no voltage drop)

- Voltage drop

$$V_{drop} = |I_a Z_w| = |I_{L,1} Z_w|$$

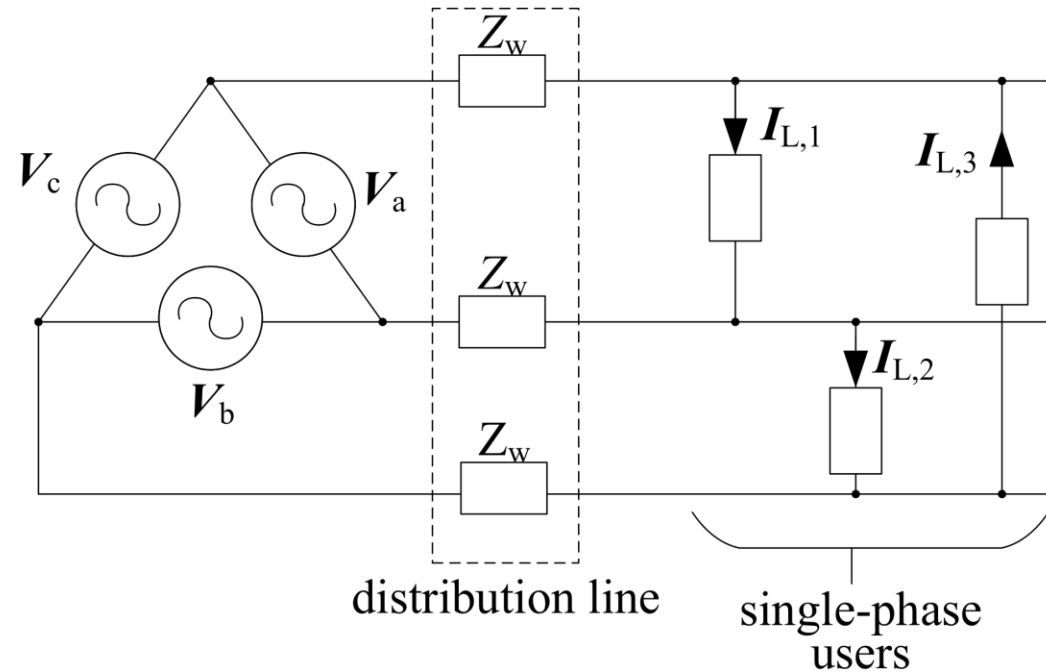
$$V_{drop} = \frac{H}{3} |I_L Z_w| \quad \text{for } H \text{ homes (a three-phase load counts as three homes)}$$



Three-Phase Delta

- Assume:
 - load on each phase is identical
 - loads located at end of each line
- No neutral current
- Voltage drop caused by current in two of the conductors

single-phase users can be thought of as one three-phase user/load



Three-Phase Delta

Voltage drop of load 1 is found by

$$V_a = I_a Z_W + V_{L,1} - I_b Z_W \text{ (KVL)}$$

$$V_a - V_{L,1} = I_a Z_W - I_b Z_W$$

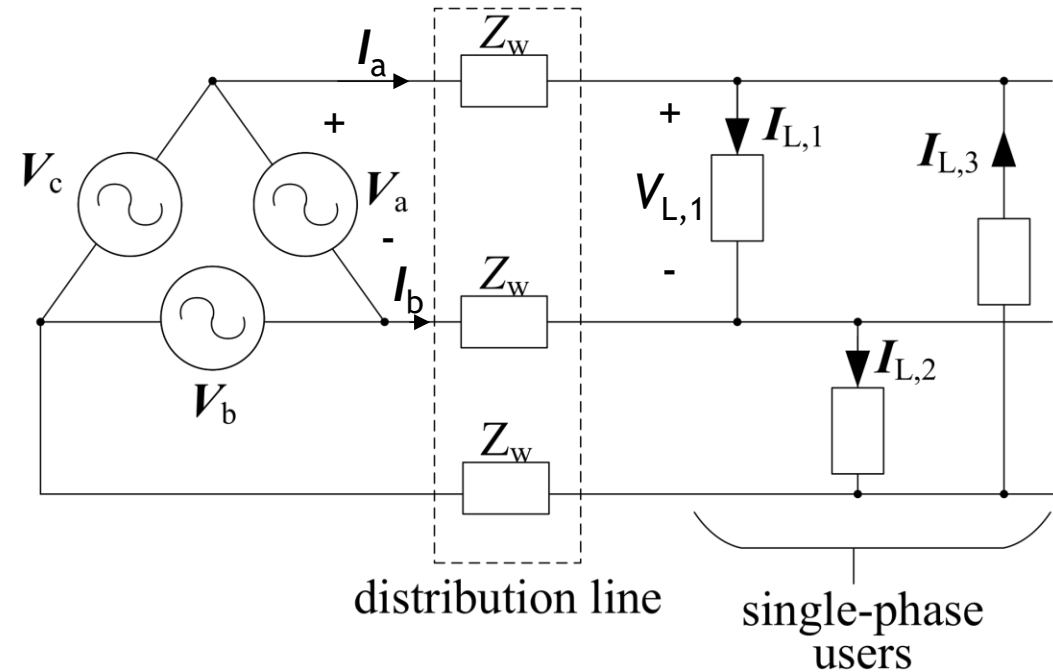
$$V_{\text{drop}} = |I_a Z_W - I_b Z_W|$$

where

$$I_b = I_a \times 1 \angle -120^\circ$$

so that

$$V_{\text{drop}} = |I_a Z_W - I_a (1 \angle -120^\circ) Z_W|$$



Three-Phase Delta

Voltage drop of load 1 is found by

$$V_{\text{drop}} = |I_a Z_w - I_a (1 \angle -120^\circ) Z_w| = |Z_w (I_a - I_a (1 \angle -120^\circ))|$$

$$= |Z_w (I_a \times \sqrt{3} \angle 30^\circ)|$$

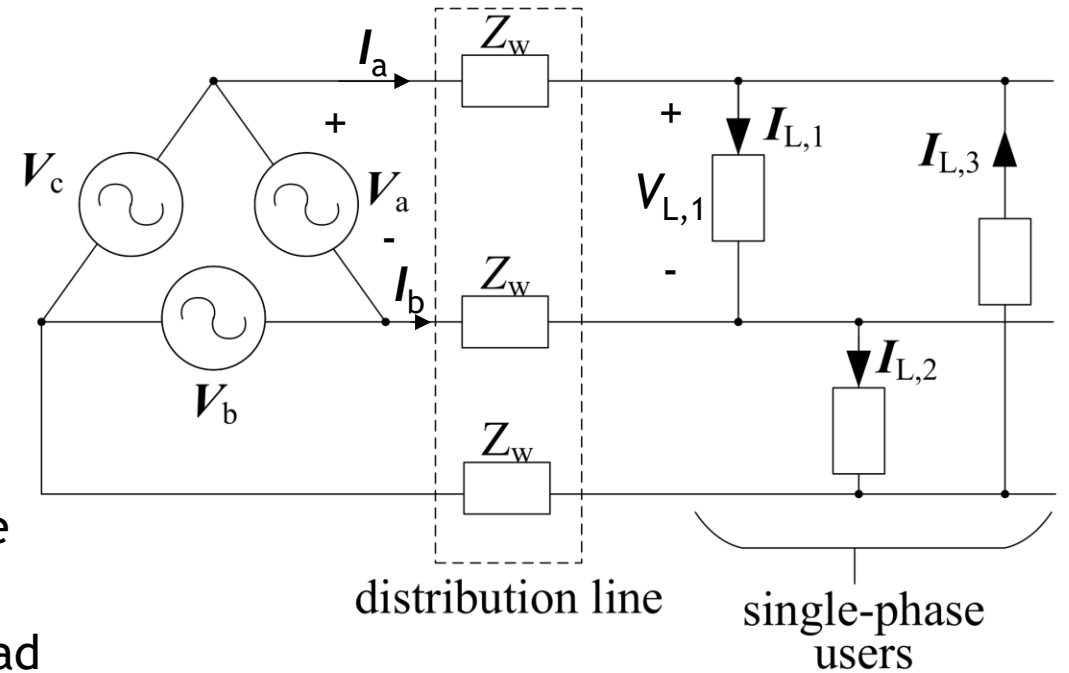
where

$$I_a = I_{L,1} \times \sqrt{3} \angle -30^\circ$$

relationship between line and phase current in delta-systems

$$V_{\text{drop}} = |3Z_w I_{L,1}| = H |Z_w I_L|$$

for H users (a three-phase load counts as three users)



Example 17.3

A three-phase wye distribution line serves a large three-phase motor for a mill. The impedance of each line is $Z_W = 0.7 \, \Omega$. The motor draws 21 kVA at a power factor of 0.80 lagging. The line-to-line voltage at the mill is 415 V. Compute the voltage drop and power loss.

Example 17.3

A three-phase wye distribution line serves a large three-phase motor for a mill. The impedance of each line is $Z_w = 0.7 \, \Omega$. The motor draws 21 kVA at a power factor of 0.80 lagging. The line-to-line voltage at the mill is 415 V. Compute the voltage drop and power loss.

A three-phase motor is conceptually the same as three single phase loads, so that $H = 3$. Current in each phase:

$$S_\phi = \frac{21,000}{3} = 7000 \text{ VA}$$

$$|I_L| = \frac{|S_\phi|}{|V_{\phi,L}|} = \frac{7,000}{\left| \frac{415}{\sqrt{3}} \right|} = 29.2 \text{ A}$$

Example 17.3

A three-phase wye distribution line serves a large three-phase motor for a mill. The impedance of each line is $Z_w = 0.7 \, \Omega$. The motor draws 21 kVA at a power factor of 0.80 lagging. The line-to-line voltage at the mill is 415 V. Compute the voltage drop and power loss.

For a three-phase wye line:

$$V_{\text{drop}} = \frac{1}{3} H |I_L| |Z_w| = \frac{1}{3} 3 |29.2| |0.7| = 20.45 \, \text{V}$$

$$P_{\text{loss}} = \frac{1}{3} (H |I_L|)^2 R_w = \frac{1}{3} (3 \times 29.2)^2 0.7 = 1.79 \, \text{kW}$$

Configuration Comparison

Configuration	Voltage Drop	Power Loss
Single-Phase, Two Wire	$2H \mathbf{I}_L Z_w $	$2(H \mathbf{I}_L)^2 R_w$
Split-Phase (Single-Phase, Three Wire)	$\frac{1}{2}H \mathbf{I}_L Z_w $	$\frac{1}{2}(H \mathbf{I}_L)^2 R_w$
Three-Phase, Wye (Four Wire)	$\frac{1}{3}H \mathbf{I}_L Z_w $	$\frac{1}{3}(H \mathbf{I}_L)^2 R_w$
Three-Phase, Delta (Three Wire)	$H \mathbf{I}_L Z_w $	$(H \mathbf{I}_L)^2 R_w$

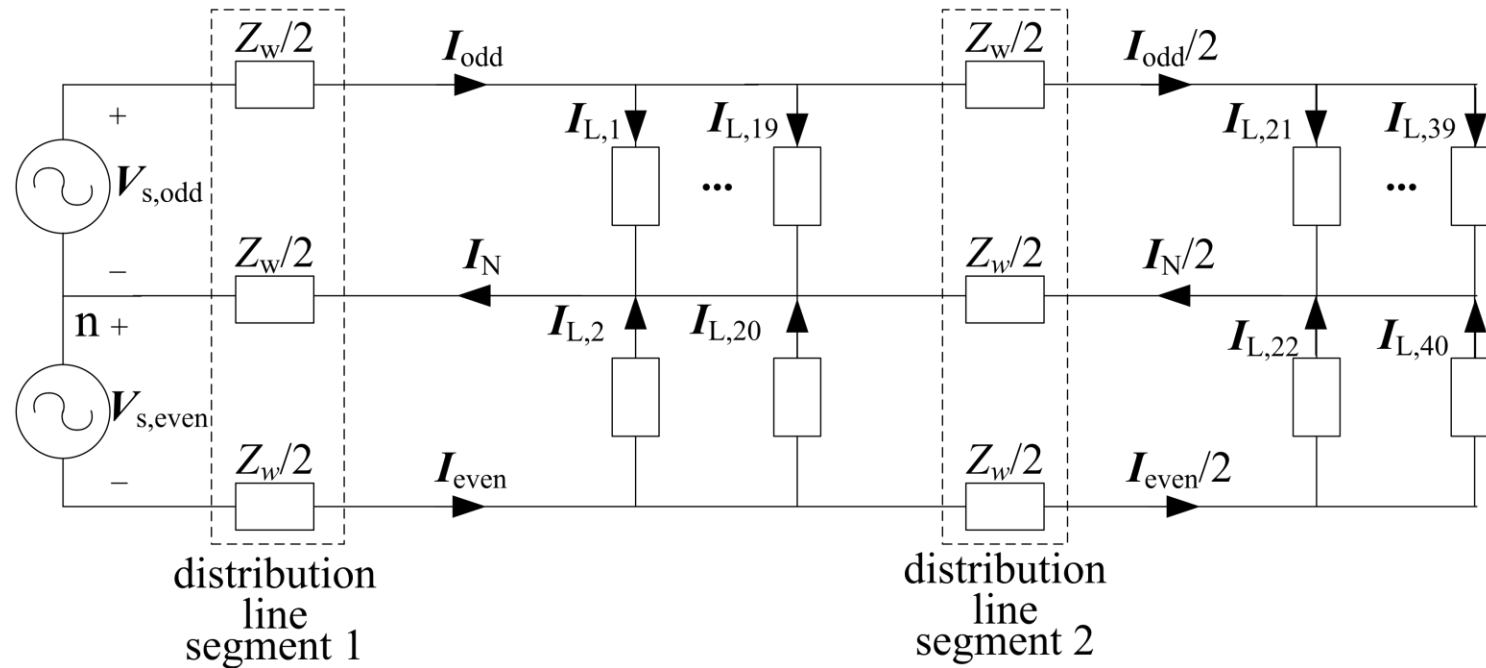
lowest voltage drop and power loss achieved with three-phase wye connection

keep in mind that different configurations require different number of conductors

Real World Conditions

- Load typically not lumped at end of distribution line
 - lower losses and voltage drop if load is spread across line and closer to the sending end
- Load imbalance
 - aside from three-phase loads, loads will be imbalanced (homes will not consume the same load at each moment)
 - imbalance causes more current in neutral conductor and for uneven voltage at homes
 - losses and voltage drop can increase

Load Location



circuit model of a split-phase distribution line
with half of the load in the middle of the line

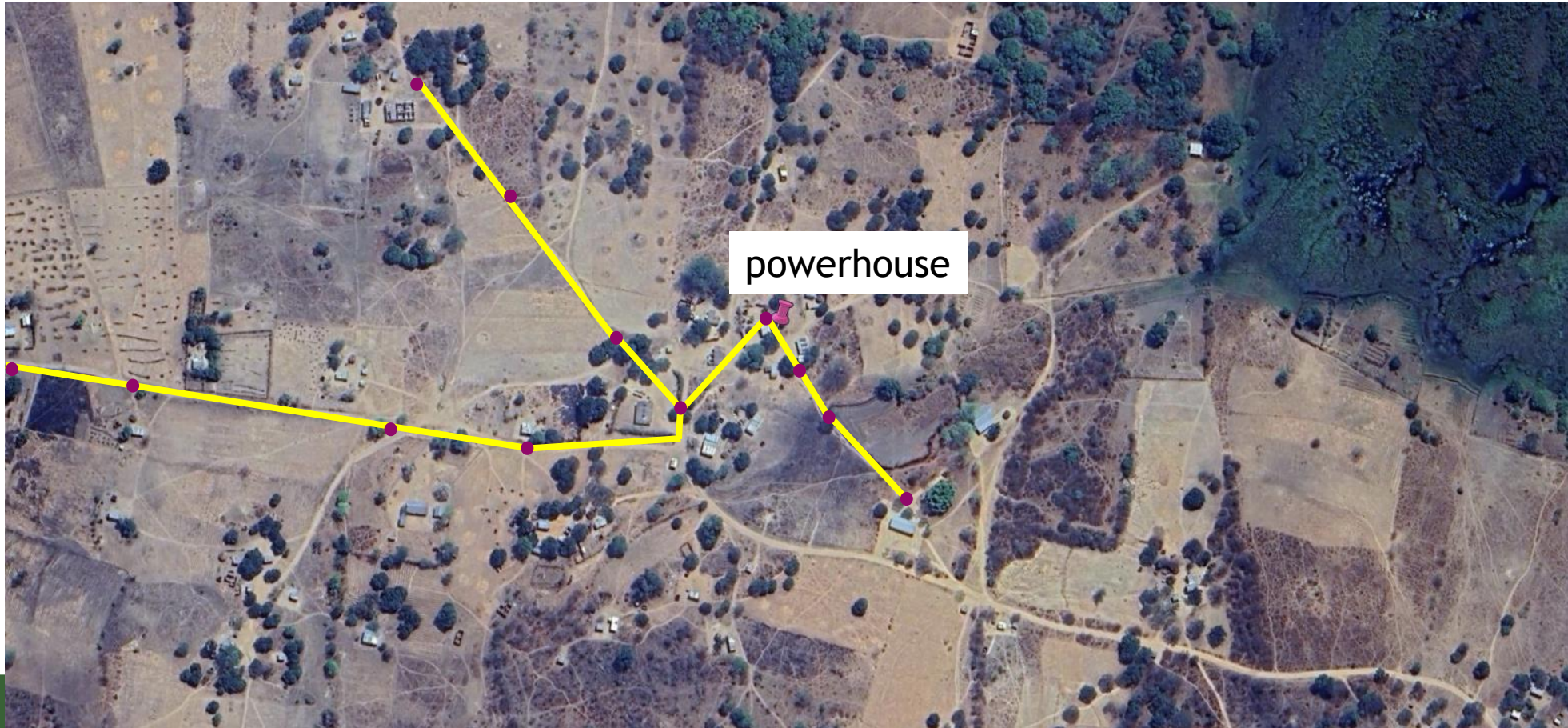
Distribution Design

- Design of distribution system includes selecting
 - voltage level and type (AC/DC)
 - method (overhead or underground)
 - topology (hub-and-spoke, trunk-and-branch, etc.)
 - configuration (single, split, three-phase)
- Conductor selection
- Distribution line routing

Distribution Line Routing

- Determine where lines are to be constructed
 - identify location for poles and transformers (if needed)
 - minimize total material cost (length of line)
- General strategy: locate lines along roads to minimize obstruction and to facilitate maintenance
- Calculate voltage drop and power loss of proposed design, re-design if necessary
 - software such as Xendee is commonly used

Distribution Line Routing



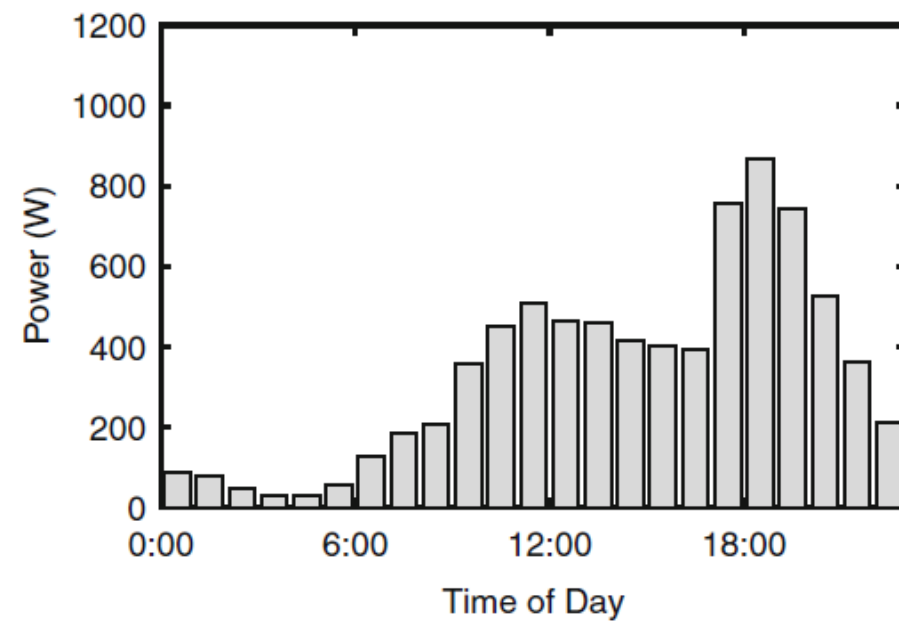
Satellite image Source: Google Earth, data attribution: Airbus



Distribution Design Example: Mwase

Load Estimation & Characterization (see Chap. 16)

- Survey method used to produce load profile
 - Individual loads are aggregated into single load profile
 - Assumed weekday is the same as weekend
- Load growth estimate: 5% per year



Parameter	Initial	After five years
Avg. daily load (kWh/day)	7.875	10.05
Peak individual load (kW)	4.11	5.24
Coincidence factor	0.37	0.37
Peak aggregate load (kW)	1.52	1.94
Power factor	0.85	0.85

Distribution Design Example

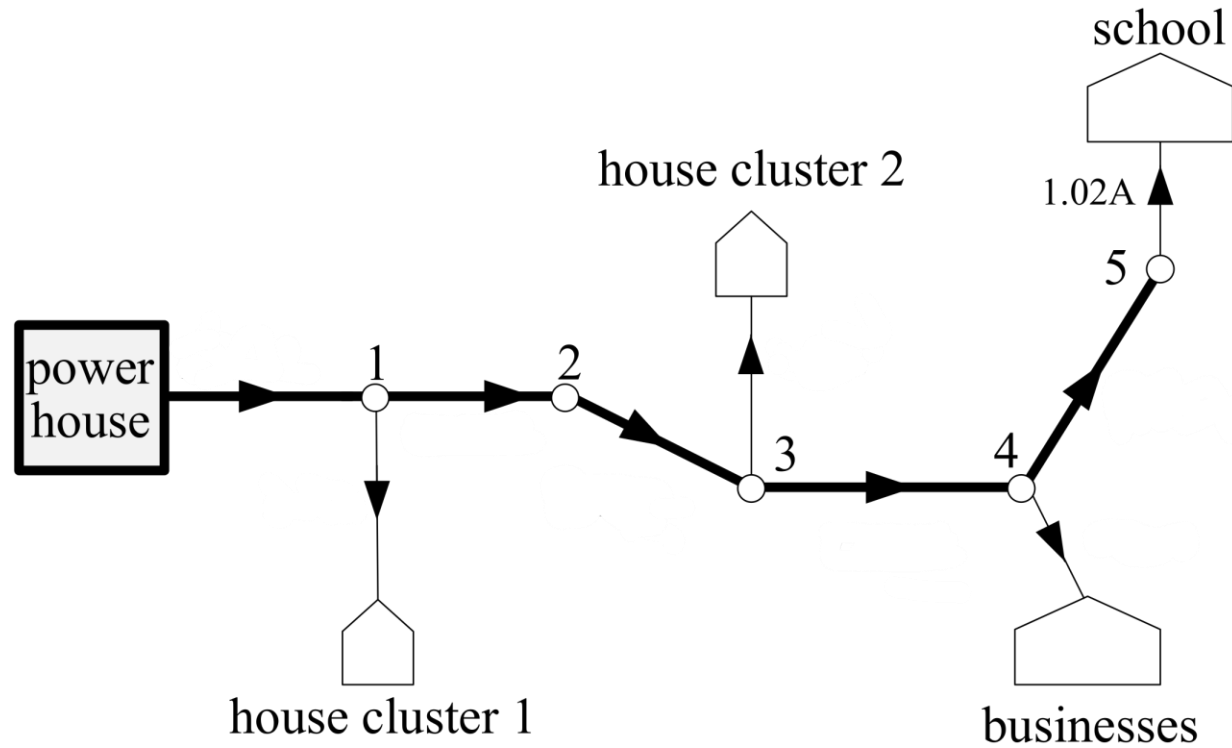
- Voltage level and type: 230 VAC
- Method: overhead
- Topology: radial (trunk-and-branch)
- Configuration: single-phase two wire

Distribution Design Conductor

Conductor: uninsulated ACSR (aluminum conductor, steel reinforced), 0.3 m spacing

Cross-sectional (mm ²)	Resistance (Ohm/km)	Reactance (Ohm/km)	Ampacity
13	2.25	0.31	105

Distribution Design: Routing



Voltage Drop and Loss Analysis

- Software often used
- Important to understand basic approach and calculations using as manual (and simplified) approach
- Compute voltage drop and losses based on peak load at each node
- Assumptions
 - nominal voltage (230 V) at each node
 - power factor is the same at each node (0.85)

1. Estimate Load Current at each node

$$|I_{L,n}| = \frac{P_{L,n}}{PF_{L,n} \times |V_s|} \quad |I_{L,1}| = \frac{200}{0.85 \times |230|} = 1.02 \text{ A}$$

Node	Length (km)	Estimated Coincident Peak Load (W)	Estimated Load Current $ I_{L,n} $	Segment $ Z_{w,s} $ (Ω)	Segment $ I_s $ (A)	Segment Voltage Drop (V)	Segment Power Loss (W)
5	0.15	200	1.02				
4	0.10	1300	6.65				
3	0.08	100	0.51				
2	0.05	0	0				
1	0.05	300	1.53				

Totals

2. Calculate the line segment impedance magnitude

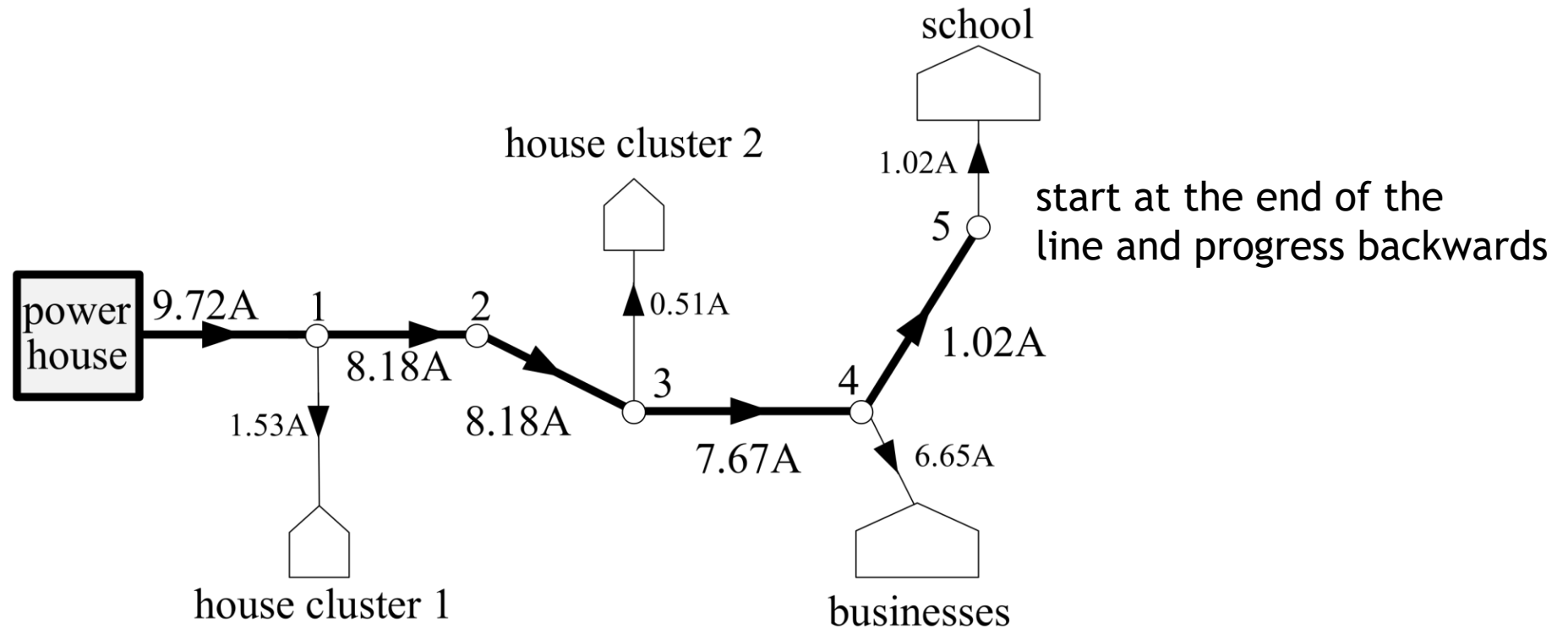
line impedance: $2.25 + j0.31$ Ohms/km

$$\sqrt{(2.25 \times 0.15)^2 + (0.31 \times 0.15)^2} = 0.341 \text{ m}$$

Node	Length (km)	Estimated Coincident Peak Load (W)	Estimated Load Current $ I_{L,n} $	Segment $ Z_{w,s} $ (Ω)	Segment $ I_s $ (A)	Segment Voltage Drop (V)	Segment Power Loss (W)
5	0.15	200	1.02	0.341			
4	0.10	1300	6.65	0.227			
3	0.08	100	0.51	0.182			
2	0.05	0	0	0.114			
1	0.05	300	1.53	0.114			

Totals

3. Calculate line segment current via KCL at each node



3. Calculate line segment current via KCL at each node

Node	Length (km)	Estimated Coincident Peak Load (W)	Estimated Load Current $ I_{L,n} $	Segment $ Z_{w,s} $ (Ω)	Segment $ I_s $ (A)	Segment Voltage Drop (V)	Segment Power Loss (W)
5	0.15	200	1.02	0.341	1.02		
4	0.10	1300	6.65	0.227	7.67		
3	0.08	100	0.51	0.182	8.18		
2	0.05	0	0	0.114	8.18		
1	0.05	300	1.53	0.114	9.72		

Totals

4. Calculate the voltage drop at each segment

use the voltage drop equation that corresponds to the distribution configuration

$$V_{\text{drop}} = 2 |I_L| H |Z_w| = 2 |1.02| 1 |0.341| = 0.70 \text{ V}$$

loads have been lumped together at each node, so let $H = 1$

Node	Length (km)	Estimated Coincident Peak Load (W)	Estimated Load Current $ I_{L,n} $	Segment $ Z_{w,s} $ (Ω)	Segment $ I_s $ (A)	Segment Voltage Drop (V)	Segment Power Loss (W)
5	0.15	200	1.02	0.341	1.02	0.70	
4	0.10	1300	6.65	0.227	7.67	3.49	
3	0.08	100	0.51	0.182	8.18	2.97	
2	0.05	0	0	0.114	8.18	1.86	
1	0.05	300	1.53	0.114	9.72	2.21	

Totals 11.22

voltage drop at the end of the line

5. Calculate the power loss for each segment

$$P_{\text{Loss}} = 2(H |I_L|)^2 R_w = 2(1 \times 1.02)^2 (2.25 \times 0.15) = 0.71\text{W}$$

use the power loss equation that corresponds to the distribution configuration

loads have been lumped together at each node, so let $H = 1$

Node	Length (km)	Estimated Coincident Peak Load (W)	Estimated Load Current $ I_{L,n} $	Segment $ Z_{w,s} $ (Ω)	Segment $ I_s $ (A)	Segment Voltage Drop (V)	Segment Power Loss (W)
5	0.15	200	1.02	0.341	1.02	0.70	0.71
4	0.10	1300	6.65	0.227	7.67	3.49	26.49
3	0.08	100	0.51	0.182	8.18	2.97	24.11
2	0.05	0	0	0.114	8.18	1.86	15.07
1	0.05	300	1.53	0.114	9.72	2.21	21.25

Totals 11.22 87.63

total power loss

Distribution Line Example

If voltage drop or power loss is too large or if segment exceeds conductor ampacity, then re-design

- select larger conductor
- change configuration type
- increase voltage (careful not to exceed voltage maximum)
- change routing

Summary

- Distribution systems are important considerations for mini-grids, metro-grids, and mesh/swarm grids
- Design decisions: AC/DC, overhead/underground, topology, configuration (single, split, three-phase)
- Different configurations result in different voltage drop and losses (three-phase wye best use of conductors)
- Routing design includes specifying route for each line, poles, and transformers (if necessary)
- Simplified voltage drop and power loss calculation method presented