全国硕士研究生招生考试

管综数学极简模式

构造数列

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出现 $a_{n+1} = qa_n + d$

凑配成 $a_{n+1} + c = q(a_n + c)$，其中 $c = \dfrac{d}{q-1}$

1.设数列$\{a_n\}$满足$a_1 = 2$，$a_{n+1} = 3a_n + 2$，则$a_{99} = $ 【 】

A.$3^{100} - 1$

B.3^{99}

C.3^{99}-1

D.2×3^{99}

E.$3^{98} - 1$

数列·构造数列

1.设数列$\{a_n\}$满足$a_1 = 2$，$a_{n+1} = 3a_n + 2$，则$a_{99} =$ 【 C 】

A.$3^{100} - 1$

B.3^{99}

C.3^{99}-1

D.2×3^{99}

E.$3^{98} - 1$

【解析】因为$a_{n+1} = 3a_n + 2, a_{n+1} + c = q(a_n + c)$，其中$c = \dfrac{d}{q-1} = \dfrac{2}{3-1} = 1$，得等比数列$\{a_n + 1\}$公比为3,首项为$a_1$+1=3. 则$a_{99} + 1 = 3 \times 3^{99-1}$, a_{99}=$3^{99} - 1$,故选C.

2.（2019）设数列$\{a_n\}$满足$a_1 = 0$，$a_{n+1} - 2a_n = 1$，

则数列$a_{100} = $　【　】

A.$2^{99} - 1$

B.2^{99}

C.$2^{99} + 1$

D.$2^{100} - 1$

E.$2^{100} + 1$

数列·构造数列

2.（2019）设数列$\{a_n\}$满足$a_1 = 0$，$a_{n+1} - 2a_n = 1$，

则数列$a_{100} = $ 【 A 】

A.$2^{99} - 1$

B.2^{99}

C.$2^{99} + 1$

D.$2^{100} - 1$

E.$2^{100} + 1$

$a_{n+1} = 2a_n + 1$

$\Downarrow$

$a_{n+1} + c = 2(a_n + c)$

$c = \dfrac{d}{q-1} = \dfrac{1}{2-1} = 1$

$\therefore \{a_n + 1\}$ 为公比$=2$，首项为$a_1 + 1 = 0 + 1 = 1$ 的等比数列

则 $a_n + 1 = 1 \times 2^{n-1} \Rightarrow a_n = 2^{n-1} - 1$

故 $a_{100} = 2^{100-1} - 1 = 2^{99} - 1$