oPs Decay

Mathematics

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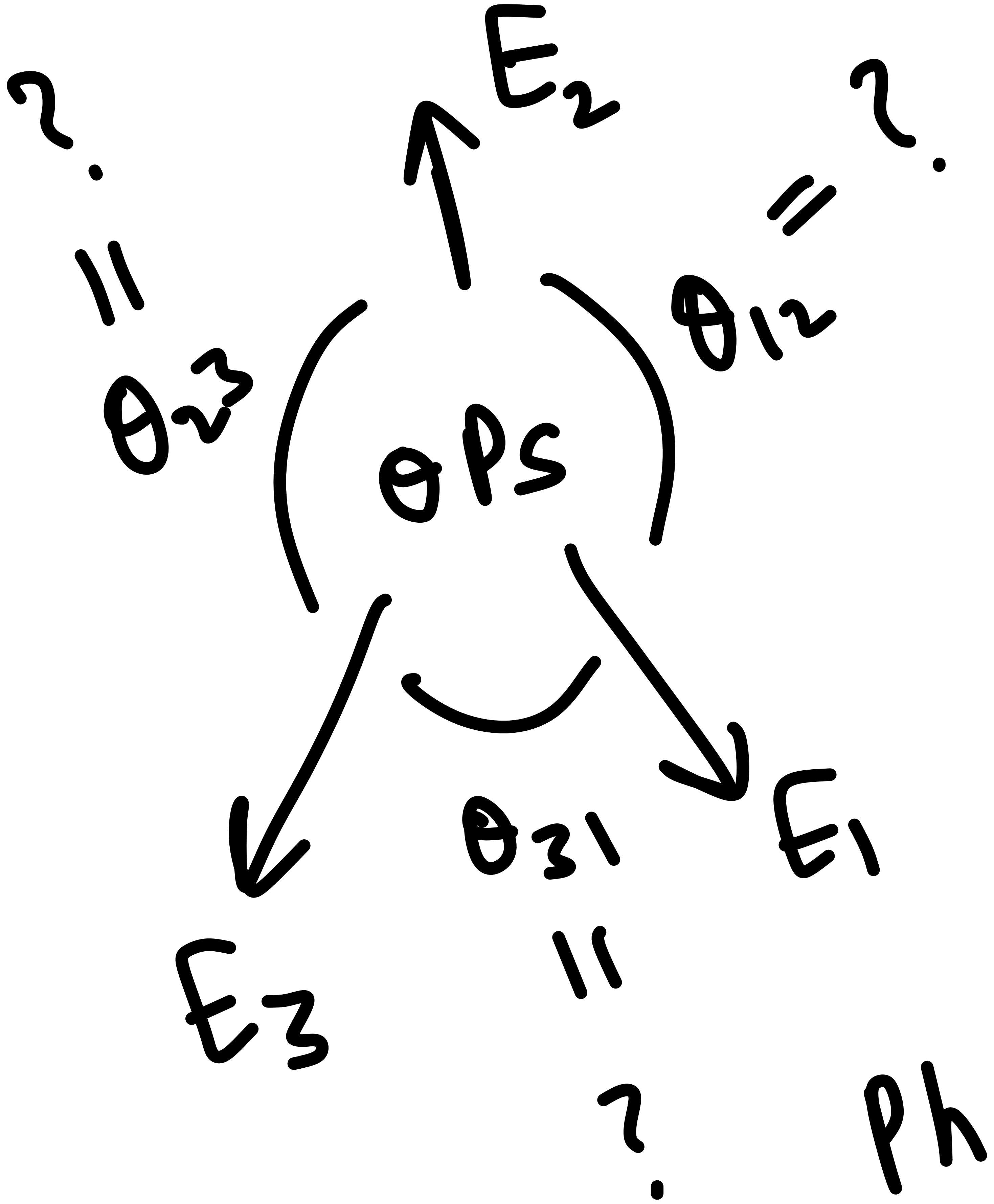
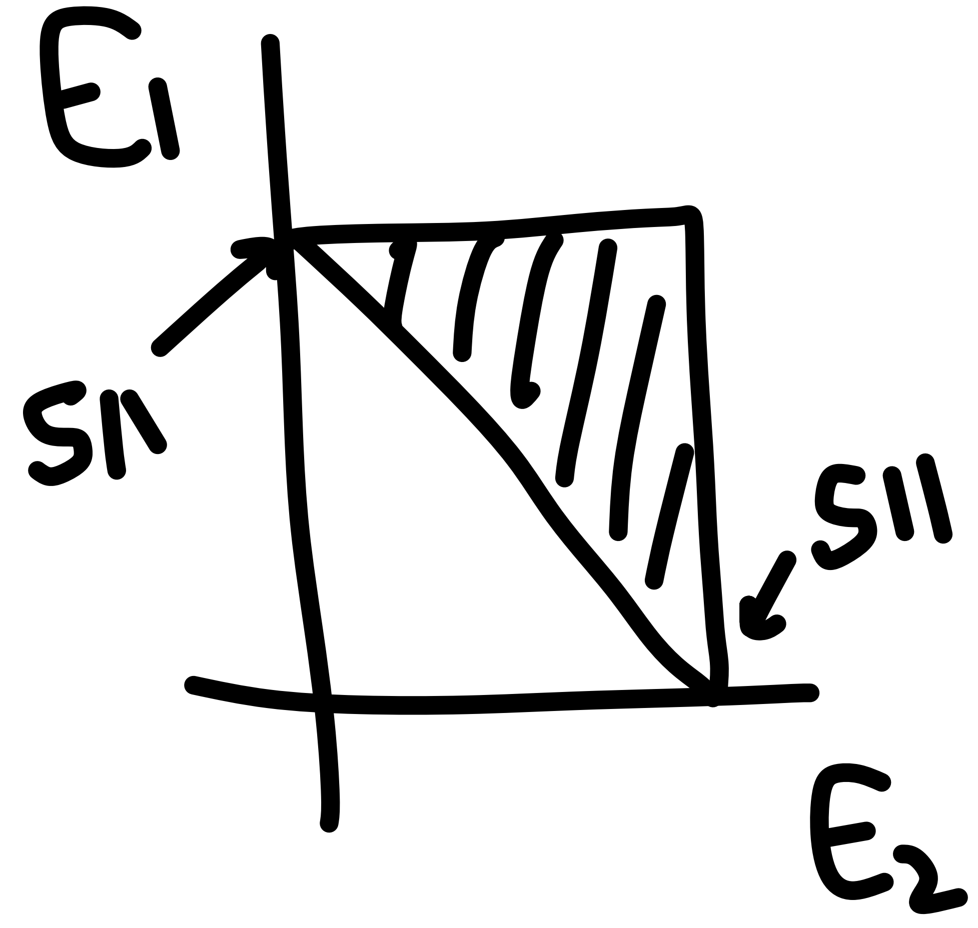


$E_1 \sim U(0, 511)$

$E_2 \sim U(511 - E_1, 511)$

$E_3 \sim 2m_e - E_1 - E_2$



$E_1 + E_2 > 511$

(keV)

Photons $\Longrightarrow$ Rest mass = 0

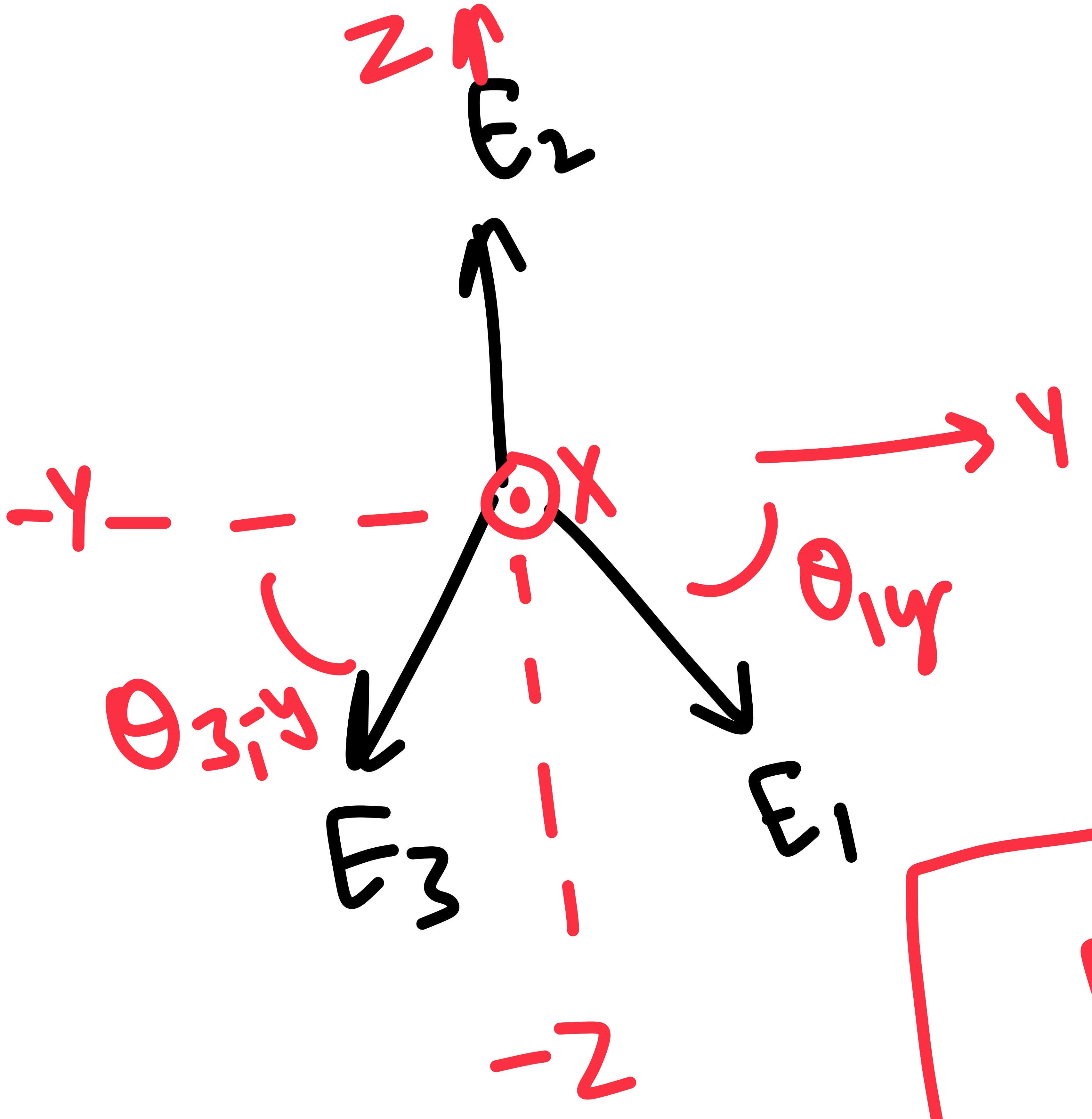
$\Downarrow$

$$|\vec{p}| = \frac{E}{c}$$

$$|\vec{p}| = E$$

$\because$ Natural units $(c = 1)$

(Planar Decay)



$$\vec{p}_{e^-} + \vec{p}_{e^+} = 0$$
$$\parallel$$
$$\vec{p}_1 + \vec{p}_2 + \vec{p}_3$$

$$\Uparrow$$
$$\vec{p}_1 + \vec{p}_2 = -\vec{p}_3$$

$$\boxed{\begin{aligned} E_2 &= E_1 \sin\theta_{1y} + E_3 \sin\theta_{3,-y} \\ E_1\cos\theta_{1y} &= E_3\cos\theta_{3,-y} \end{aligned}}$$

$$\theta_{12} = \theta_{1y} + \frac{\pi}{2} \; ; \; \theta_{23} = \theta_{3,-y} + \frac{\pi}{2} \; ; \; \theta_{13} = 2\pi - \theta_{12} - \theta_{23}$$
$$= \pi - \theta_{1y} - \theta_{3,-y}$$

$$\begin{bmatrix} E_2 = E_1 \sin x + E_3 \sin y \\ E_1 \cos x = E_3 \cos y \end{bmatrix} \quad (x, y) = ?$$

(Iterative sol^n) Numerical sol^n : Newton-Raphson Method

$$f(x,y) = E_1 \sin x + E_3 \sin y - E_2 = 0$$

$$g(x,y) = E_1 \cos x - E_3 \cos y = 0$$

$$J = \begin{bmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} \\ \frac{\partial g}{\partial x} & \frac{\partial g}{\partial y} \end{bmatrix} = \begin{bmatrix} E_1 \cos x & E_3 \cos y \\ -E_1 \sin x & E_3 \sin y \end{bmatrix}$$

$(x_0, y_0) \Rightarrow$ Initial guess

$$\begin{bmatrix} x_{n+1} \\ y_{n+1} \end{bmatrix} = \begin{bmatrix} x_n \\ y_n \end{bmatrix} - J^{-1}(x_n, y_n) \begin{bmatrix} f(x_n, y_n) \\ g(x_n, y_n) \end{bmatrix}$$

$$|J| = E_1 E_3 \sin(x+y)$$

$$J^{-1} = \begin{bmatrix} E_3 \sin y & -E_3 \cos y \\ E_1 \sin x & E_1 \cos x \end{bmatrix} \frac{1}{E_1 E_3 \sin(x+y)}$$

Update Rule :→

$$x_{n+1} = x_n - \frac{\sin y_n \cdot f(x_n, y_n) - \cos y_n \, g(x_n, y_n)}{E_1 \sin(x_n + y_n)}$$

$$y_{n+1} = y_n - \frac{\sin x_n \cdot f(x_n, y_n) + \cos y_n \, g(x_n, y_n)}{E_3 \sin(x_n + y_n)}$$

Convergence: $|f|$ & $|g| <$ tolerance (10^{-6})

* **Note** :→

ⓐ If $\sin(x_n + y_n) \approx 0$

then method fails. (singularity)

ⓑ If $E_1 = E_3$ (symmetry)

then $x = y = \sin^{-1}\left(\frac{E_2}{2E_1}\right)$

T.P :→ $E_{\gamma_i} + E_{\gamma_j} > m_e \rightarrow 511\ \text{KeV}$ in ops → 3γ

$(c=1)$

Proof :→

$$E_{\gamma_1} + E_{\gamma_2} + E_{\gamma_3} = 2m_e$$

$$\vec{p}_{\gamma_1} + \vec{p}_{\gamma_2} + \vec{p}_{\gamma_3} = 0$$

* **Invariant Mass cond^n** :→

$$m_{12}^2 = (E_{\gamma_1} + E_{\gamma_2})^2 - |\vec{p}_{\gamma_1} + \vec{p}_{\gamma_2}|^2 \geq 0$$

$$\left(\because \vec{p}_{\gamma_3} = -(\vec{p}_{\gamma_1} + \vec{p}_{\gamma_2}) \right)$$

$|\vec{p}_{\gamma_3}|^2 \rightarrow E_{\gamma_3}^2$

$$m_{12}^2 = (E_{\gamma_1} + E_{\gamma_2})^2 - E_{\gamma_3}^2 \geq 0$$

$E_{\gamma_3} \rightarrow (2m_e - E_{\gamma_1} - E_{\gamma_2})^2$

$$\Rightarrow \boxed{E_{\gamma_1} + E_{\gamma_2} \geq m_e}$$

Hence proved

(Appendix)

Invariant Mass cond^n :→

$(c=1)$

Mass of sys. of particles $= m_{inv}^2 = \left(\sum_i E_i\right)^2 - \left|\sum_i \vec{p_i}\right|^2 \geq 0$

↓

same in all inertial frames

ex:→ 2 particles ; $p_1 = (E_1, \vec{p_1})$

$p_2 = (E_2, \vec{p_2})$

$$m_{inv}^2 = (E_1 + E_2)^2 - |\vec{p_1} + \vec{p_2}|^2$$