2D **Singly** Scattered MC Simulation

Under the guidance of **Prof Pawel Moskal**

Detector Geometry

$26.5^\circ = c = \tan^{-1}\left(\frac{w}{2r}\right)$ → acceptance angle

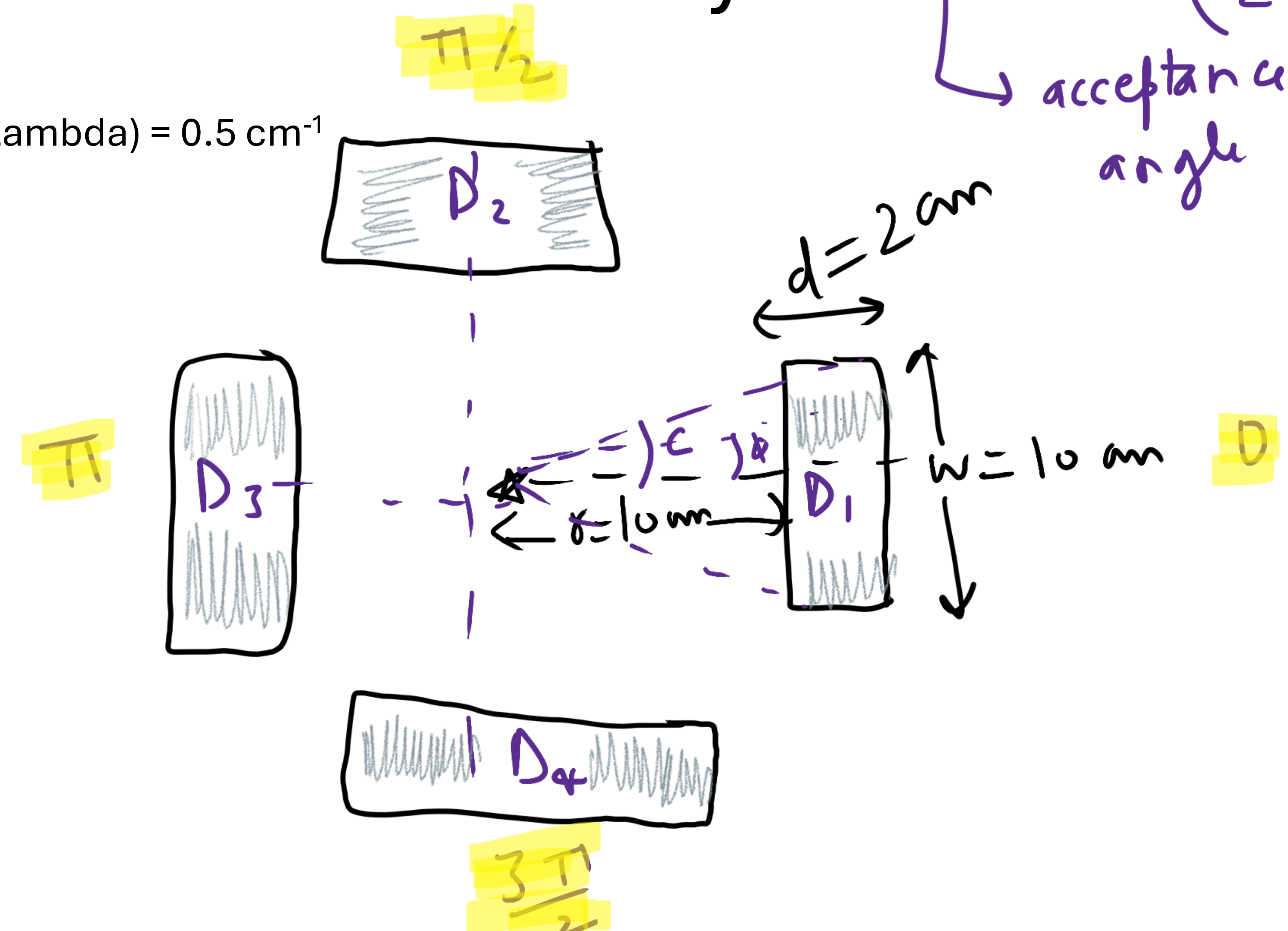
Attenuation Coefficient (Lambda) = 0.5 cm^{-1}

Thickness (d) = 2.0 cm

Width (w) = 10.0 cm

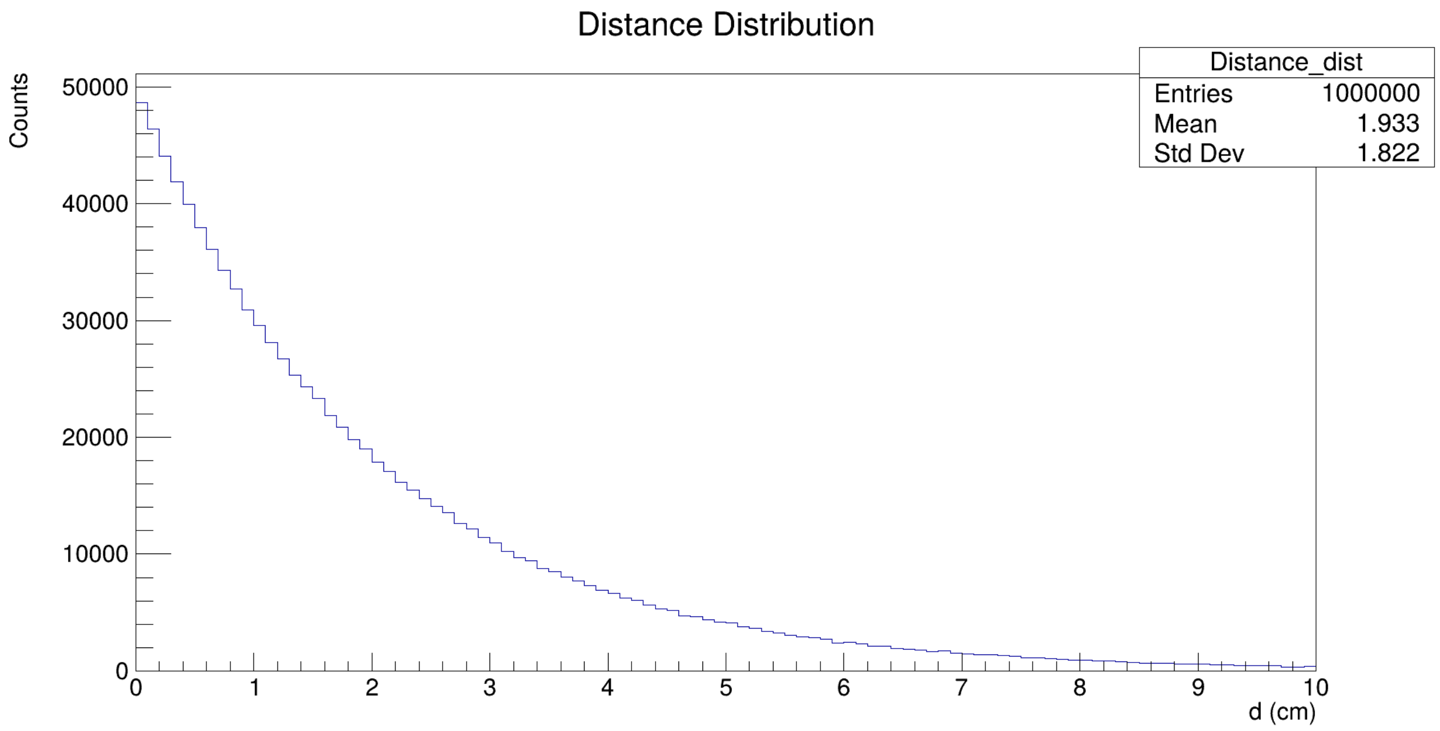
Radius (r) = 10.0 cm

$\phi \sim U(0, 2\pi)$



Simulate Distance

(Inverse Transform Sampling)



Aim: Sampling distance from the Exponential Distribution.

1. Find the CDF of exponential distribution F(x).
2. Generate pseudorandom number u ~ random(0,1).
3. Calculate $x = F^{-1}(u)$.

$[0, 2\pi) \rightarrow \phi$ (# Domain Shifting)

$\phi = \phi \,\%\, 2\pi$

$(-ve)$

Add 2π

Angle Check (4 Sides)

(ϕ)

Det	Min^m	Max^m
* D_1	$\phi \le c$	$\phi \ge 2\pi - c$
D_2	$\frac{\pi}{2} - c$	$\frac{\pi}{2} + c$
D_3	$\pi - c$	$\pi + c$
D_4	$\frac{3\pi}{2} - c$	$\frac{3\pi}{2} + c$

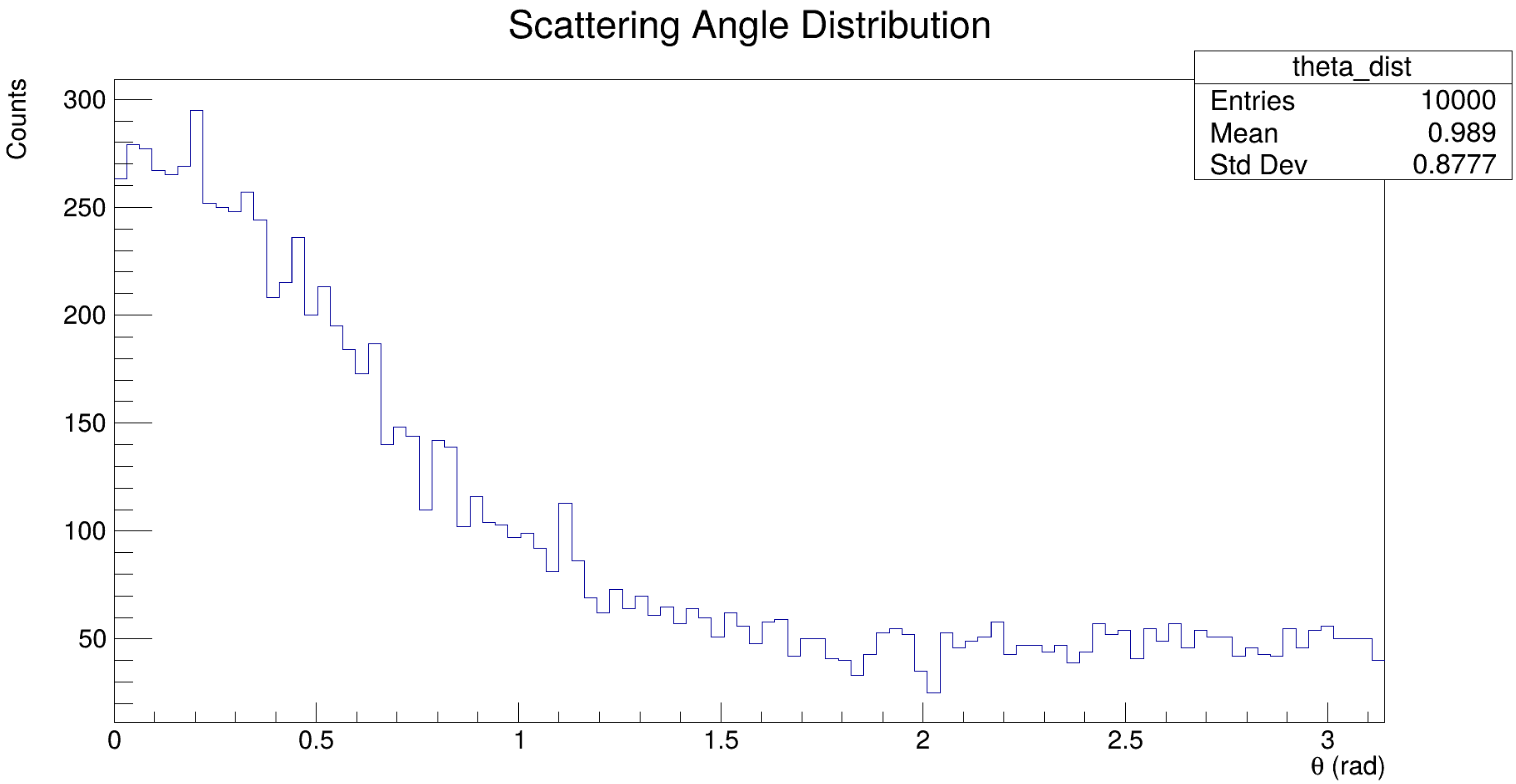
$c = \tan^{-1}\left(\frac{w}{2\gamma}\right)$ [special case]

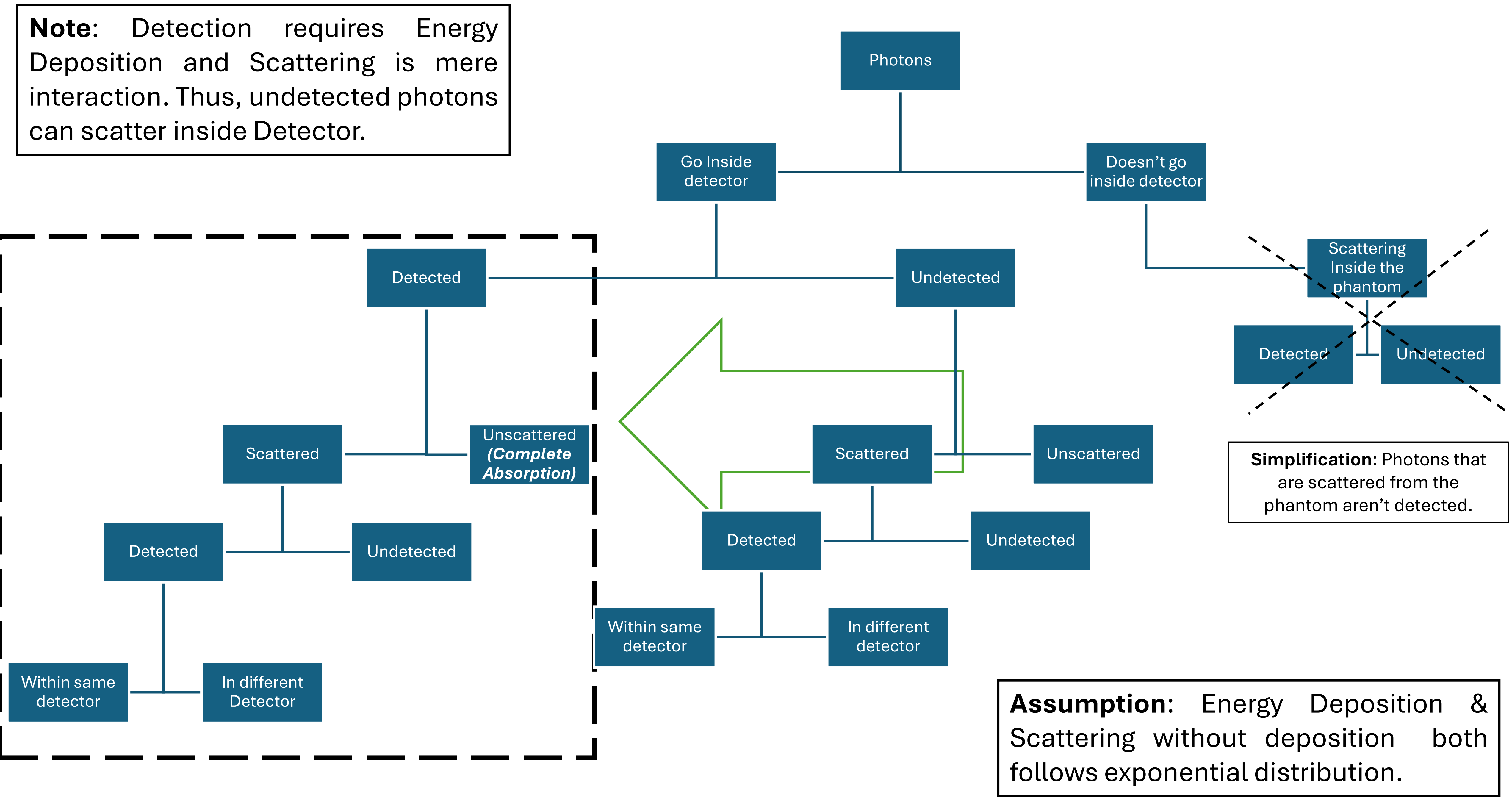
$\phi \sim U(0, 2\pi);$

Simulate Theta (Rejection Sampling Method)

$$\frac{d\sigma}{d\Omega} = \frac{1}{2} r_e^2 \left(\frac{\lambda}{\lambda'}\right)^2 \left[\frac{\lambda}{\lambda'} + \frac{\lambda'}{\lambda} - \sin^2(\theta)\right]$$

```
f(theta) = Klein-Nishina Formula as a function of theta;
f_max = f(0);
f = random(0,f_max);
theta ~ random(0,pi);
if (f<f_max)
     return theta;
```


Scattering Angle Distribution
theta_dist
Entries 10000
Mean 0.989
Std Dev 0.8777
Counts
300
250
200
150
100
50
0
0.5
1
1.5
2
2.5
3
θ (rad)



Note: Detection requires Energy Deposition and Scattering is mere interaction. Thus, undetected photons can scatter inside Detector.
Photons
Go Inside detector
Doesn't go inside detector
Detected
Undetected
Scattering Inside the phantom
Detected
Undetected
Scattered
Unscattered *(Complete Absorption)*
Scattered
Unscattered
Detected
Undetected
Detected
Undetected
Within same detector
In different Detector
Within same detector
In different detector
Simplification: Photons that are scattered from the phantom aren't detected.
Assumption: Energy Deposition & Scattering without deposition both follows exponential distribution.

Detected

$$N_{tot} = N_{det} + N_{undet}$$

$$N_{det} = N_{det\_scat} + \boxed{N_{det\_unscat}}\,^{*}$$

$$N_{det\_scat} = N_{det\_scat\_same} + N_{det\_scat\_other}$$

Verification

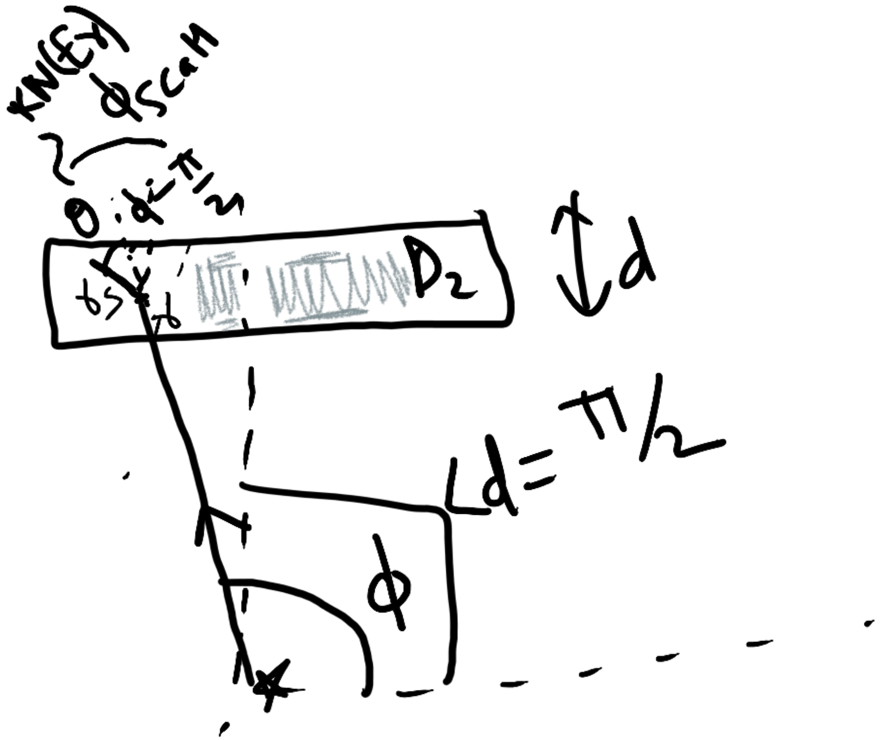
$$\left[N_{det\_unscat} = N_{det} - N_{det\_scat\_same} - N_{det\_scat\_other}\right]$$

Angle of the detector $(0, \frac{\pi}{2}, \pi, \frac{3\pi}{2})$

Main Loop $(i : 0 \to N_{tot})$

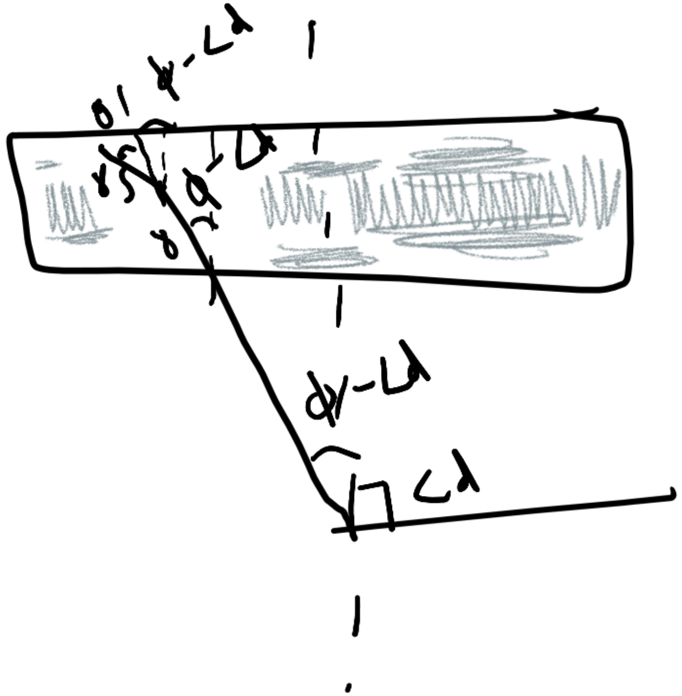
```
{ r ~ exp(λ);
  φ ~ U(0,2π);

bool ini_det = (|r cos(φ - ∠d)| < d)
              && (check_Angle(φ))

if(ini_det) {
    N_det ++ ;
    θ ~ KN(Eγ) ;
    φ_scat = (θ + φ) % 2π ;
    rs = exp(λ);

  if((rs|cos(φ_scat - ∠d)| + r|cos(φ - ∠d)|) < d)
    { N_det_scat_same ++ ; }

  else{  if(check_angle(φ_scat)) -> ∠d2
    { if( rs cos(φ_scat - ∠d2) < d)
        { N_det_scat_other ++ ; }
  } } } else{ N_undet ++ ; }
                outside
```

Within



(without zoom)

(with zoom)



$$d \begin{cases} \gamma_s \cos(\phi + \theta - \angle d) \\ \gamma \cos\phi \end{cases}$$

Simulation Results (Efficiencies/Fractions)

$\frac{N\_det}{N\_tot} = e =$ Efficiency: 0.38179 ± 0.00153632 $\rightarrow$ 38%

Fraction scattered in same detector: 64.1085 $= \frac{N_{det\_scat\_same}}{N_{det}}$

$\frac{N\_detscat\_other}{N_{det}} =$ Fraction scattered to other detector: 8.0725

Fraction undetected: 61.821 $= \frac{N_{det}}{N_{tot}} \rightarrow 62.7\%$

$$e = 4\frac{(2c)}{4\pi}(1 - e^{-\lambda d}) = 37.31\% \quad \checkmark \quad \underbrace{\left(+ \underbrace{e_{scat}}_{\text{within}} + \underbrace{e_{scat}}_{\text{other}}\right)}_{?}$$

$$P_{undet} = 1 - \frac{4(2c)}{4\pi} \times (1 - e^{-\lambda d}) = 62.7\%$$

Future Aspect

Using the singly scattered data for finding attenuation coefficients.

(Part of my PhD thesis)

Thank You