3 Complex conjugates

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DONE

① ≻ $z^2 - 4z + 29 = 0$

$(z-2)^2 + 25 = 0$

$z = 2 \pm 5i$

$z = 2+5i \qquad z^* = 2-5i$

Complex conjugate

≻ $z = a+bi \quad z^* = a-bi$

$zz^* = (a+bi)(a-bi)$

$= a^2+b^2$

② ≻ $z^2 - (z^*)^2$

$(5i+1)^2 - (5i-1)^2$

$(25i^2+10i+1) - (25i^2-10i+1)$

$25i^2 - 25i^2 + 10i + 10i + 1 - 1$

$20i$ //

≻ $\frac{z}{w^*} = \frac{5i+1}{2+i}$

$\frac{(5i+1)(2-i)}{(2+i)(2-i)} = \frac{10i - 5i^2 + 2 - i}{4 - i^2}$

$\frac{9i+7}{4+1} = \frac{7}{5} + \frac{9}{5}i$

③ $\boxed{\text{If } z \text{ is a complex root, then } z^* \text{ is also a root (in polynomials)}}$

$\alpha = 2+4i$

$\beta = 2-4i$

$\gamma = 3$

④

1) $\alpha = 2+3i$

$\therefore \beta = 2-3i$

$\therefore (z-(2+3i))(z-(2-3i)) = 0$

$z^2 - z(2-3i) - z(2+3i) + (2+3i)(2-3i) = 0$

$z^2 - 2z + 3zi - 2z - 3zi + 4 + 9 = 0$

$z^2 - 4z + 13 = 0$ //

2) $\alpha = 3+i$

$\beta = 3-i$

$\gamma = 4-3i$

$\delta = 4+3i$

$(z-(3+i))(z-(3-i))(z-(4-3i))(z-(4+3i)) = 0$

$[z^2 - z(3-i) - z(3+i) + (3+i)(3-i)][z^2 - z(4+3i) - z(4-3i) + (4-3i)(4+3i)] = 0$

$[z^2 - 3z + zi - 3z - zi + 9 + 1][z^2 - 4z - 3iz - 4z + 3iz + 16 + 9] = 0$

$(z^2 - 6z + 10)(z^2 - 8z + 25) = 0$

$(z^2 - 6z + 10)(z^2 - 8z + 25) = 0$

$z^4 - 8z^3 + 25z^2$

$-6z^3 + 48z^2 - 150z$

$+10z^2 - 80z + 250 = 0$

$z^4 - 14z^3 + 83z^2 - 230z + 250 = 0$ //