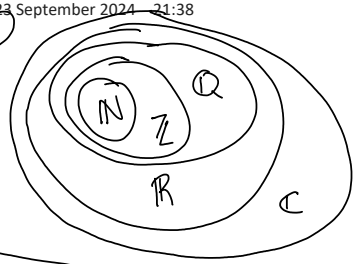
1 Complex numbers

23 September 2024 21:38

DONE

(1)



(2)

1) $x^2+1=0$

$(x-i)(x+i)=0$

$\therefore x=\pm i$ //

2) $x^2+2x+10=0$

$(x+1)^2-1+10=0$

$(x+1)^2=-9$

$x=-1\pm\sqrt{-9}$

$x=-1\pm 3i$ //

(3)

$2x^2+8x+9=0$

$2(x^2+4x)=-9$

$(x+2)^2-4=-\frac{9}{2}$

$(x+2)^2=-\frac{1}{2}$

$x=-2\pm\sqrt{\frac{1}{2}}\sqrt{-1}$

$x=-2\pm\frac{\sqrt{2}}{2}i$ //

(3)

1) $f(x)=x^3-6x^2+37x-58$

$f(2)=0 \quad \therefore$ Solve $f(x)=0$

$\therefore (x-2)$ is a factor

$x-2 \;\big|\; x^3-6x^2+37x-58 \;\big|\; x^2-4x+29$

$-x^3-2x^2$

$-4x^2+37x$

$-4x^2+8x$

$29x-58$

$-29x-58$

$\therefore f(x)=(x-2)(x^2-4x+29)$

$(x-2)^2-4+29=0$

$x=2\pm\sqrt{-25}$

$x=2\pm 5i$

$\therefore x=2,\ x=2\pm 5i$ //

(4)

$f(x)=x^4-6x^3+25x^2-28x-60=0$

$f(-1)=f(3)=0 \quad \therefore$ Solve $f(x)=0$

$\therefore$ Factors are $(x-3)$, $(x+1)$

$(x+1)(x-3)=x^2-2x-3$

$x^2-2x-3 \;\big|\; x^4-6x^3+25x^2-28x-60 \;\big|\; x^2-4x+20$

$-x^4-2x^3-3x^2$

$-4x^3+28x^2-28x$

$-4x^3+8x^2+12x$

$20x^2-40x-60$

$-20x^2-40x-60$

$x^2-4x+20=0$

$(x-2)^2-4+20=0$

$(x-2)^2=-16$

$x=2\pm 4i$

$\therefore x=-1,\ 3,\ 2\pm 4i$ //

(5)

$f(x)=x^6+10x^4+29x^2+20$

$f(i)=f(2i)=0 \quad$ Solve $f(x)=0$

$x=i \implies x=-i$

$x=2i \implies x=-2i$

$x=\pm i \qquad x=\pm 2i$

$x^2=-1 \qquad x^2=-4$

$x^2+1=0 \qquad x^2+4=0$

$(x^2+1)(x^2+4)$

x^4+5x^2+4

$x^4+0x^3+5x^2+0x+4 \;\big|\; x^6+0x^5+10x^4+0x^3+29x^2+0x+20 \;\big|\; x^2+5$

$-x^6+0x^5+5x^4+0x^3+4x^2$

$5x^4+0x^3+25x^2+0x+20$

$-5x^4+0x^3+25x^2+0x+20$

$\therefore f(x)=(x^2+1)(x^2+4)(x^2+5)$

$\therefore x=\pm i,\ \pm 2i,\ \pm\sqrt{5}i$ //