Problem Set 1 Solutions Template

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####Useful Formatting Notes.

It is best to enclose any in-line equations, including math operators, within two $ symbols, e.g. $0.40 + 0.02 = 0.42$. The following operators may be useful: $\times$, $\cdot$, $\cap$, $\cup$, $\neq$, $\geq$, and $\leq$. To create a superscript, A^C. To create a subscript, P_X. To use the square root symbol, $\sqrt{x}$.

To typeset fractions, use the command $\frac{numerator}{denominator}$.

For your convenience, the following syntax is given:

$$Var(X) = E(X-\mu)^2 = \sum_i^k P(X = x_i)(x_i - \mu)^2$$

$$P(X = k) = \frac{e^{-\lambda} \cdot \lambda^k}{k!}$$

###Problem 1.

a) 0, 1, 2, 3.

b) Possible values with their probabilities and values of W:

Value	Probability	Value of W
DDD	$(1-0.27)^3 * (0.27)^0 = 0.3890$	0
DFD	$(1-0.27)^2 * (0.27)^1 = 0.1438$	1
DFF	$(1-0.27)^1 * (0.27)^2 = 0.0532$	2
DDF	$(1-0.27)^2 * (0.27)^1 = 0.1438$	1
FDD	$(1-0.27)^2 * (0.27)^1 = 0.1438$	1
FFD	$(1-0.27)^1 * (0.27)^2 = 0.0532$	2
FFF	$(1-0.27)^0 * (0.27)^3 = 0.0196$	3
FDF	$(1-0.27)^1 * (0.27)^2 = 0.0532$	2

c)

Value of W	0	1	2	3
Probability	.3890	.4314	.1596	.0196

###Problem 2.

$$\begin{aligned} E(X) =& sumofallxp \\ =& 0 * .05 + 1 * .04 + 2 * .20 + 3 * .40 + 4 * .31 \\ =& 2.88 \end{aligned}$$

$$\begin{aligned} Var(X) =& sqrt(sumofallx^2 * p - mean^2) \\ =& sqrt((.05) * 0^2 + (.04) * 1^2 + (.20) * 2^2 + (.40) * 3^2 + (.31) * 4^2) \\ =& sqrt(0 + .04 + .80 + 3.60 + 4.96) \\ =& sqrt(9.40) \\ =& 3.06594194335 \end{aligned}$$

###Problem 3.

No because every single day is not independent of each other. One day's pollution may affect the next day's pollution. Also the probability of pollution passing a certain level may be different depending on the day.

###Problem 4.

a) i.

```
dbinom(20, 50, 0.43)
```

```
## [1] 0.1044492
```

ii.

```
dbinom(30, 50, 0.57)
```

```
## [1] 0.1044492
```

b) i.

```
pbinom(10, 20, 0.43)
```

```
## [1] 0.8051091
```

ii.

```
1 - pbinom(10, 20, 0.43)
```

```
## [1] 0.1948909
```

c) Each adult is independent from each other and that the probability stays constant at 0.43.

###Problem 5.

a)

$$P(X = 2) = (e^(- 2) * 2^2)/2! = 0.2706$$

$$P(X <= 2) = (e^(- 2) * 2^2)/2! + (e^(- 2) * 2^1)/1! + (e^(- 2) * 2^0)/0! = 0.6766$$

$$P(X >= 3) = 1 - ((e^(- 2) * 2^2)/2! + (e^(- 2) * 2^1)/1! + (e^(- 2) * 2^0)/0!) = 0.3233$$

b)

```
dpois(2, 2)
```

```
## [1] 0.2706706
```

```
ppois(2, 2)
```

```
## [1] 0.6766764
```

```
ppois(3, 2, lower.tail = FALSE) + dpois(3,2)
```

```
## [1] 0.3233236
```

###Problem 6.

a)

```
exp = 8 * 1400000/1000000
exp
```

```
## [1] 11.2
```

b)

```
ppois(14, 8, lower.tail = FALSE)
```

```
## [1] 0.01725699
```

c)

```
lambda3 = 8 * 450000/1000000

ppois(9, lambda3, lower.tail = FALSE)
```

```
## [1] 0.004024267
```

The probability is 0.4024%.

d) The official is incorrect because even if the 0.4024% result happened in Brooklyn, it also includes the probability that the rest of NYC had 0 cases of osteosarcoma, which is not reflected in the answer to c.

###Problem 7.

a)

```
1 - pnorm(2.6)
```

```
## [1] 0.004661188
```

b)

```
pnorm(1.35)
```

```
## [1] 0.911492
```

c)

```
pnorm(3.10) - pnorm(-1.70)
```

```
## [1] 0.9544669
```

d)

```
qnorm(0.85)
```

```
## [1] 1.036433
```

e)

```
qnorm(0.20)
```

```
## [1] -0.8416212
```

###Problem 8.

```
1 - pnorm(2.5)
```

```
## [1] 0.006209665
```

###Problem 9. 0.2 to 6.2.

###Problem 10.

a)

```
sd = (240-185)/39
1-pnorm(sd)
```

```
## [1] 0.07923199
```

b)

```
sd2 = (200-185)/39
pnorm(sd) - pnorm(sd2)
```

```
## [1] 0.2710292
```

###Problem 11.

a)

```
lambda1 = 1 * 4000000/5000
ppois(380,lambda1)
```

```
## [1] 1.10302e-61
```

b)

```
ppois(449, lambda1, lower.tail = FALSE)
```

```
## [1] 1
```

c)

```
lambda2 = 1 * 1500000/5000
lambda2*2
```

```
## [1] 600
```