MOMENT OF INERTIA

We have treated centroids of plane shapes

MASS MOMENT OF INERTIA

This is a measure of an object’s resistance to angular acceleration.

The moment of inertia of shape is the measure of the shapes resistance to bending. It is also defined as the second moment of a shape along a particular axis. The moments about the axes are denoted by (or even when it comes to solid (3d) shapes)

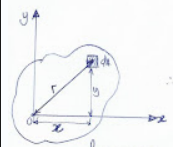
Recall that for moments,

Similarly for moment of inertia,

There are two methods of solving moment of Inertia and they are the integration method and the geometry method

POLAR MOMENT OF INERTIA

This is the quantity used to describe resistance to torsional deformation (deplection), in cylindrical objects (or segments of cylindrical objects) with an invariant cross section and no significant warping or out of plane deformation



RADIUS OF GYRATION

The radius of gyration is defined as the distance from the axis of rotation to a point where the total area of the body is supposed to be concentrated so that the moment of inertia about the axis may remain the same

, this generally known as the radius of gyration

MOMENT OF INERTIA BY INTEGRATION

In this method, the same way we took a strip (vertical or horizontal), find the center of the strip and then integrate the center of the strip through the whole shape. We also take a strip (vertical or horizontal), find the moment of inertia of the strip through the whole shape. The method is as simple as the method of centroid by Integration.

The following steps are taken when solving moment of inertia questions.

First, we take out our strip (vertical or horizontal) in this case vertical.

This vertical strip will have an area of

However, we can also decide to take a horizontal strip from the vertical one. This horizontal strip will have a height of and will still have a width of . Therefore, the area of this new horizontal strip will be

Area of the original slip, will be

We will see that width of the strip will be constantly dx

Therefore, the area will be given as

The moment of inertia of just the vertical strip will be given as

As already explained, the width will be constantly dx so the moment of inertia of that vertical strip will be:

Then, the moment of inertia of the whole shape will then be the addition of all these strips

Now we integrate with respect to x and in this case since we are finding the moment of inertia of the whole body, dx is not constant

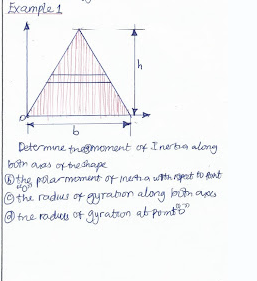
This is the moment of inertia of the shape about the x-axis

For the moment of inertia about the y axis,

The above formulae are used to find the moment of inertia of a shape with vertical strips

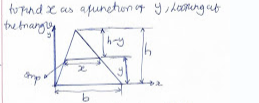
For horizontal strips,

QUESTIONS



So taking a horizontal slip, since that will make it easier for us to solve

Recall that for a horizontal slip, we are usually integrating with respect to the y-axis; therefore, we find a x as a function of y



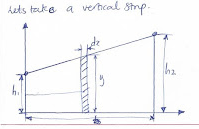
From similar triangles,

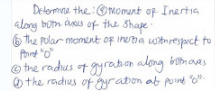
Recall that b and h are both constants

Polar moment of inertia:

Radii of gyration

Question2:





Since we have a vertical strip, we deal with respect to x

If you look at the above image, there are two possible equations of y

For the upper line,

Area of the strip

For vertical strip:

More questions:

Determine the:

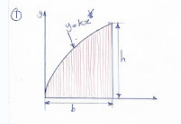
Moment of inertia along both axes

The polar moment of inertia at point 0

The radius of gyration along both axes

The radius of gyration at point 0

Of the following shapes:

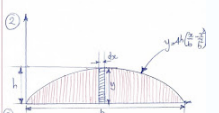


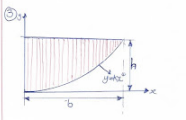
First, find the value of k

So we can say at point (b, h)

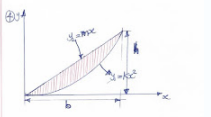
At the end, we are supposed to get the following values:

Recall that for vertical strip:

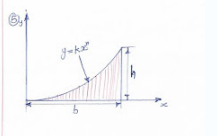


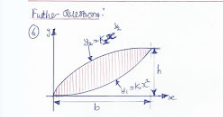


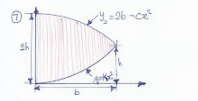
Using horizontal strips,



Area of the strip,

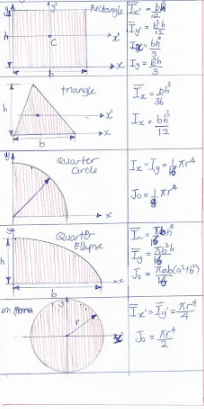






MOMENT OF INERTIA BY GEOMETRY

Moment of inertia of some standard shapes



For a rectangle, the moment of inertia depends on the point of reference of the x-axis and the y-axis

If the y and x axes pass through the centre of the rectangle then the moment of inertia is given as

However if the y axis is on the left of the rectangle and the x-axis is the base of the rectangle, then the moment of inertia will be given as

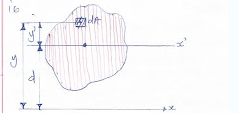
Triangle:

Quarter Ellipse:

Circle:

To solve moment of inertia by geometry, we need to know the parallel axis theorem

The parallel axis theorem states that the moment of inertia of a shape along any axis parallel to the centroidal axis of that shape is equal to the moment of inertia along the centroidal axis plus the area of the product of the area of the shape and the distance between the centroidal axis and the new axis

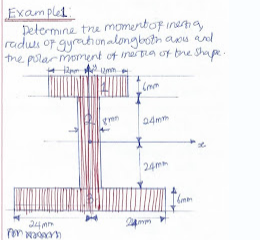


Here, is the distance between the x-axis and the centroidal x-axis

From the parallel axis theorem, the value of y will be equal to the centroidal axis plus the distance between y and the centroidal axis

Recall that moment about the y axis is given as

In this case, the moment is about the centroidal axis and as we know the moment about the centroidal axis is 0. Also take note that the centroid of something that has a line of symmetry lies on the line of symmetry and the moment about that centroidal axis is equal to 0



First, we can divide the whole shapes into smaller shapes (just like when solving centroids)



So we then find the centroid around the x-axis of the shape

Recall that if the y and x axes pass through the centre of the rectangle then the moment of inertia is given as

Distance between the centroid of that individual shape and the centroid of the whole shape

For shape two, the vertical rectangle,

Shape three, the lower horizontal rectangle

So all these moments we have gotten are the centroidal moments of each piece of the whole shape. Now, to find the centroid in relation to the axis of the shape, we do:

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For the y-axis,

MOMENT OF INERTIA OF A FEW SHAPES

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| SHAPE |  |  |  |  |  |
| Rectangle |  |  |  |  |  |
| Triangle |  |  |  |  |  |
| Circle |  |  |  |  |  |
| Semi-circle |  |  |  |  |  |
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