**SCALARS AND VECTORS**

Apart from classifying physical quantities into fundamental and derived quantities, we can also classify them into scalar and vector quantities.

Scalar Quantities are those quantities which have only magnitude (but no direction). They can be described completely by their magnitude (without direction) Examples of scalar quantities include mass, length, time, temperature, electric current, luminous intensity, amount of substance (mole), distance, altitude, work done, energy, power, density, potential (any form of potential), capacitance, resistance, inductance and pressure etc.

Vector quantities are those quantities which have both magnitude and a specified direction. Examples of vectors include displacement, velocity acceleration, momentum, temperature gradient, impulse, moment (or torque), weight, all forces (tension, up thrust, friction etc.), all fields and field intensities (electric field, magnetic field and gravitational fields etc.) and flux.

**IDEAS FOR REMEMBERING SCALARS AND VECTORS**

3 p’s of scalars which are

Power, Pressure and Potential

3 M’s of vectors

Momentum, Moment and Magnetic field Intensity

Close Substitute

**Scalars – Vectors**

Distance – Displacement

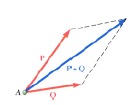
Temperature – Temperature Gradient

Speed – Velocity

Mass – Weight

**VECTORS**

Vectors are parameters possessing magnitude and direction which add according to the parallelogram law.



**VECTOR CLASSIFICATION**

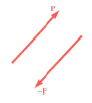
Fixed (or bound) vectors: These have well defined points of application that cannot be changed without affecting an analysis calculation of the vector

Free vectors: These vectors may be freely moved in space without changing their effect on an analysis

Sliding vectors: These vectors may be applied anywhere along their line of action without affecting an analysis

Equal vectors: Two vectors are said to be equal if they have the same magnitude and direction.

Negative vectors: The negative of a vector (A), usually written as (-A) is a vector that has the same magnitude as A but acts in an opposite direction



Force definition:

It is the action of one body on another characterized by its point of application, magnitude, line of action and sense (direction)

Experiment shows that the effect of two forces may be represented by a single resultant force

The resultant is equivalent to the diagonal of a parallelogram which contains the two forces in adjacent legs

Concurrent forces: These are forces that pass through the same point

**VECTOR REPRESENTATION**

Vectors are represented in boldface letters like **A** or with an arrow over the letter or with an underscore . The magnitude (modulus or length) of the vector can be written as . A straight line with an arrow can be used to represent a vectorIn this topic, we’ll be treating vectors as forces. Force can be defined as the action of a body on another; characterized by its point of application, magnitude, line of action and

**UNIT VECTOR**

A unit vector is a (dimensionless) vector with a magnitude of 1. It describes a direction in space and has no other physical significance. In the X-Y coordinate system, the vector I is defined as a vector whose direction is towards the X-axis and J is a unit vector in the direction of the Y-axis.

Generally, in the xyz plane, a vector A can be expressed as

Unit vectors can be represented by i (in the direction (or parallel) to the x axis), j (in the direction of the y axis) and k (in the direction of the z axis).

are defined as perpendicular unit vectors

are defined as the scalar magnitudes or scalar components of the (unit) vector component

This can easily be expressed assince the unit vectors have units of one each but the unit vectors are added to show direction since vectors have both magnitude and direction.

**MAGNITUDE OF A VECTOR**

Thus is also called the length or modulus of a vector

The magnitude of a vectoris expressed asis defined mathematically as

E.g. if

Find the magnitude of A

The unit vector of a vector A is expressed as

That is the vector divided by its magnitude will produce a unit vector of the vector

**BRACKET NOTATION OF VECTORS**

A vector can also be represented using bracket notation

Example1: Given a vector , how is it represented in bracket notation?

The representation becomes . and are known as **terminal points** while the initial points are assumed to be at the origin

**ADDITION AND SUBTRACTION OF VECTORS**

Vectors can be added and subtracted from one another based on their unit vectors (I, j and k).

Special methods are employed in addition and subtraction of vectors e.g. graphical method (polygon and parallelogram method) and algebraic method

Note: When two vectors are added together, the resulting vector is called resultant vector as it has the same effect as two or more vectors taken together

Graphical method: This involves scale drawing e.g. polygon method. Let’s take vectors u and v to be drawn to scale as

**ADDITION OF VECTORS**

Trapezoidal rule for vector addition

In mathematics (numerical analysis), the trapezoidal rule AKA the trapezoid rule or trapezium rule; is a technique for approximating the definite integral

In Euclidean geometry, a convex quadrilateral with at least one pair of parallel sides is referred to as a trapezium in pure English but it is referred to as a trapezoid in north American English in places like the US and Canada. The parallel sides are called the bases while the other sides are called the legs or the lateral sides if they are not parallel. If they are parallel, then there are two pairs of bases in the trapezoid.

A scalene trapezoid is one with none of the sides of equal length

is a vector from the initial point of u to terminal of v

If the terminal point of the resultant is equal to the initial point of , then the resultant will have a value of 0. Just like in displacement

Draw your Diagram OHHHH!!!!!

Where is said to be the resultant of both vectors

Subtraction of vectors

When a vector v is subtracted from a vector u as in u-v, it will be written as where is simply taken as the opposite vector of v

Example,

Given that v and u are vectors drawn to scale as

Diagram

Evaluate v-u using the polygon method

TAKE NOTE OF THE FOLLOWING

If we have two points A (Ax, Ay, Az) and B (Bx, By, Bz) and a vector (r) joins the two points, r can be written as:

Algebraic method

This is also known as component method. Unlike the geometric addition of vectors treated earlier, vectors in component form are added and subtracted algebraically

Example 1:

Given vector and vector Evaluate vector

Subtraction of vectors

Given two points P and Q in space that have Cartesian coordinates (-3, 1, 4) and (2, -2, 5) respectively; determine the vector PQ

Magnitude of a vector in component form

Recall that I said a vector can be described by its components e.g. . How about when given a vector in its component form; how do we evaluate its magnitude?

We simply root the square of all its unit components

Example: Give the vectors and , what is their magnitude?

**STEPS TO SOLVING VECTOR PROBLEMS**

Representing the vectors as a parallelogram and measuring or by transforming it to a triangle and then using the sine and cosine rules

For example, for a vector and another

Their sum or difference is always a vector and is expressed as

If and, find A+B

Find A-B

Find 3A-2B

What is the final answer of AB –CB+DA+2CD Answer: CD

What is the final answer of Answer: 6OZ

If and , find and

Find the values of for which the points X and P are confident given that

If A is point and has a position a, and B is a point with position b, and P divides the line between A and B in the ratio m:n

OP is the postion vector of the point P

If a point P with a position vector r divides two vectors a and b in the ratio (m: n). The position vector ® can be gotten as follows

If the ratio (m:n) is positive, then the point P lies between the two vector points (A and B) (i.e. it divides the vectors internally) and if it is negative, it lies outside A and B

Also, if the value of m and n are equal, the position vector is given as

**Direction Cosines**

This refers to the angle that a component of a vector makes with its corresponding axis i.e. the angle Ax makes with the x-axis

Direction angles are the angles that a vector makes with the x, y and z components respectively. Direction cosines (of a vector A) are the direction angles of which are

Example, a vector shown below, calculate its direction cosines

**LAWS OF VECTOR ALGEBRA**

Let a, b and c be vectors and n and m be scalars

Note: If we have a vector a and another vector b is represented as

Where t is a scalar, it will be seen that b is parallel to a

**DOT PRODUCT**

This is also known as scalar product. This is because the result of this product gives a scalar value. A value with magnitude but no direction

If

And

If the anglebetween the two vectors A and B is given or the angle is to be found, the dot product can also be gotten as follows:

Given thatand

Find and the angle between the two vectors

From

is the projection of A in the direction of the vector B. Therefore dot product or scalar product is the product of the projection of the length of A in the direction B with the length of B or projection of B in the direction of A with the length of A

From the above, we will see that the product of two perpendicular vectors is zero

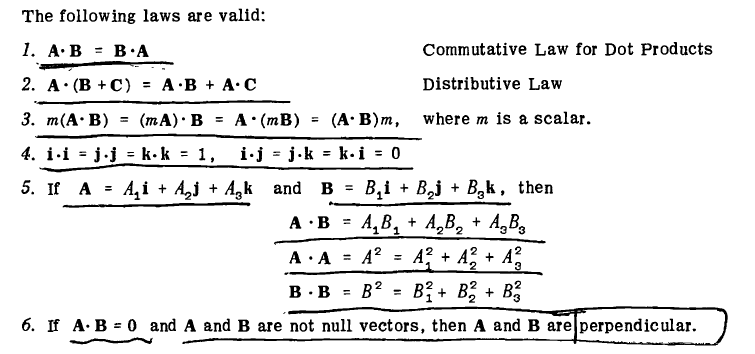
Note: If the cross product of two vectors is 0, the two vector are orthogonal (or perpendicular)

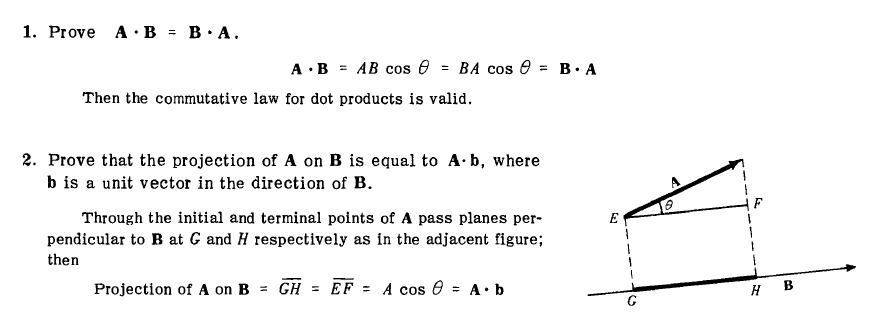
CHARACTERISTICS OF SCALAR PRODUCT

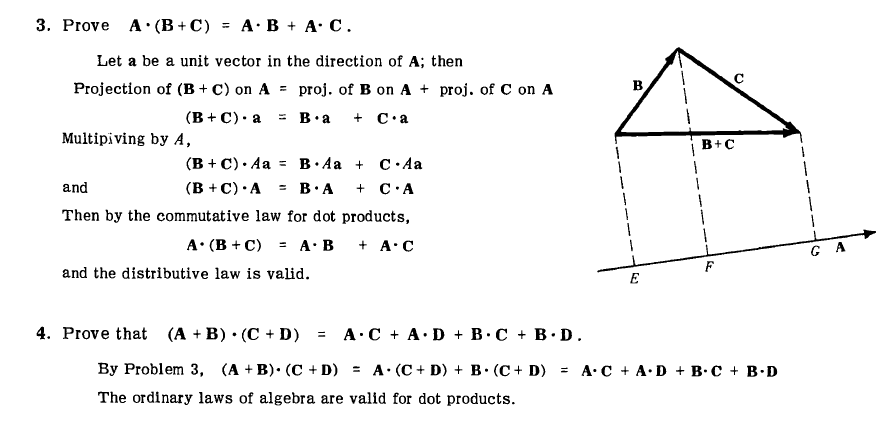
They are commutative PQ = QP

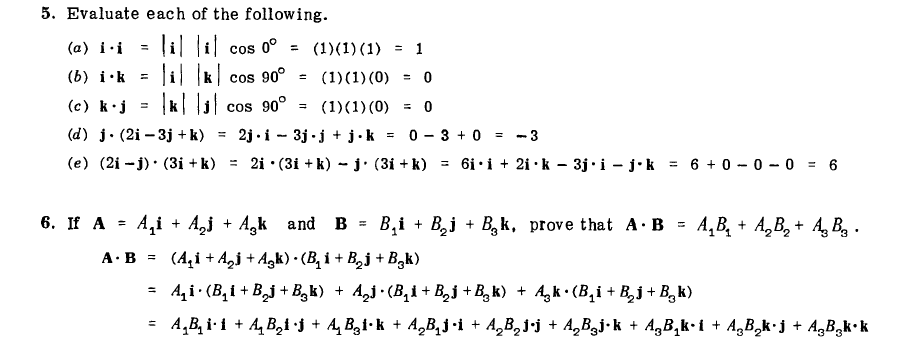
They are distributive

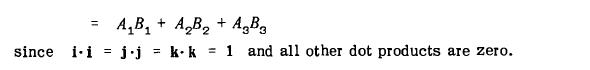
They are not associative (PQ)S = undefined











**CROSS PRODUCT**

This is also known as Vector product because its result is a vector. The concept of moment of a force about a point is more easily understood through applications of the vector product or cross product

If

And

Iffindand the angle between them.

The vector product of two vectors P and Q is defined as the vector V which satisfies the following conditions:

Line of action of P is perpendicular to plane containing P and Q

Magnitude of V is P X Q

Direction of V is obtained from the right hand rule

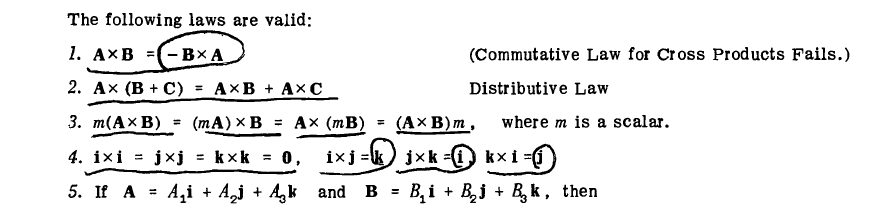
Note: If C is the result of the cross product of two vectors A and B, C will be perpendicular to A and also perpendicular to B

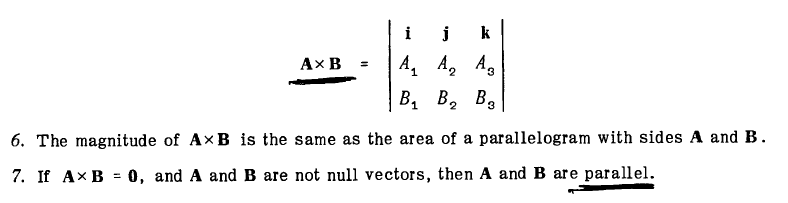
**CHARACTERISTICS OF VECTOR PRODUCTS**

Are not commutative

Are distributive

Are not associative





Practice questions

Find

Answer: Undefined

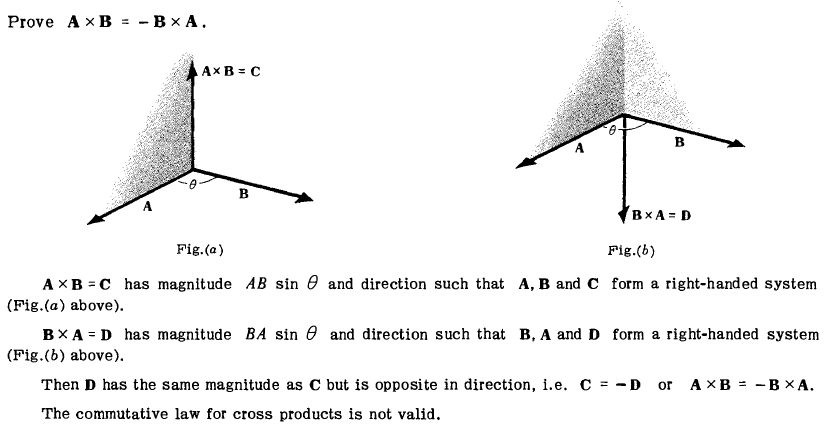
Find the angle between the vectors

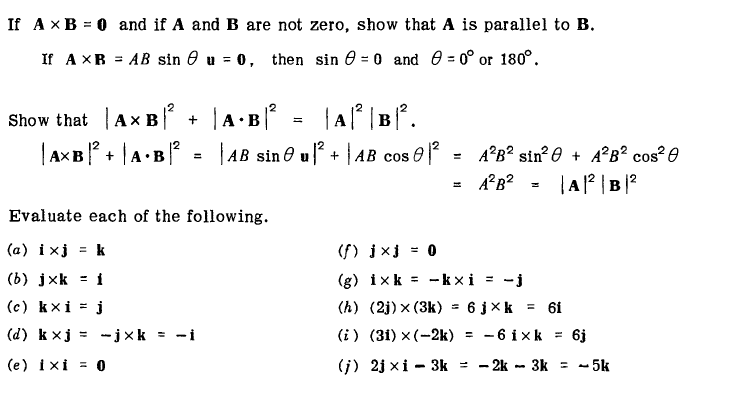
Answer: 66.6

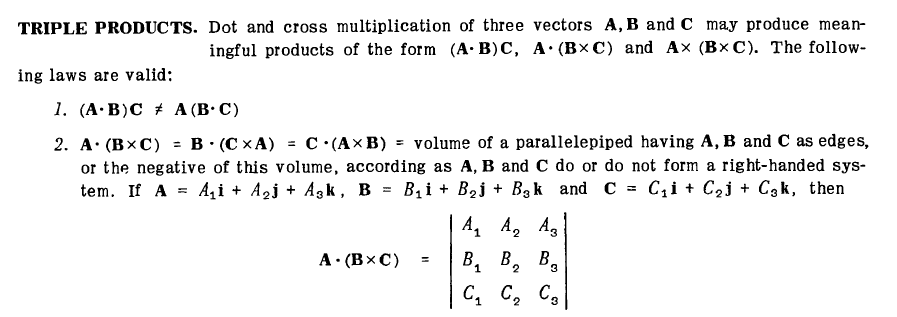
The resultant of 2 vectors A and B is a vector C and and. Find B, its magnitude and the angle it makes with the positive x-axis.

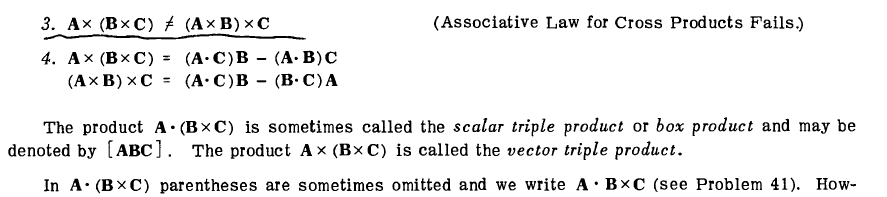
The resultant of 2 vectors is the sum of the two vectors

The anglethat a vector makes with the positive x-axis can be gotten from the formula

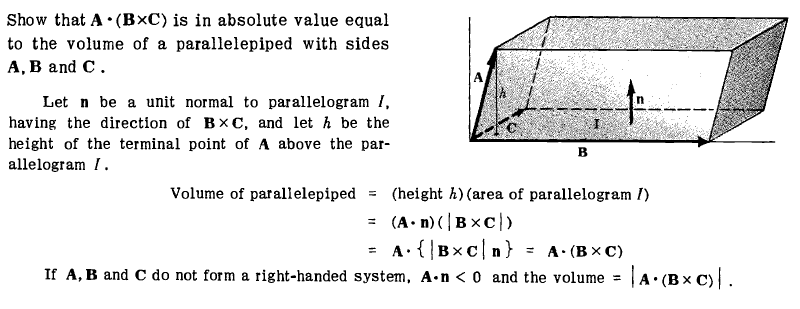


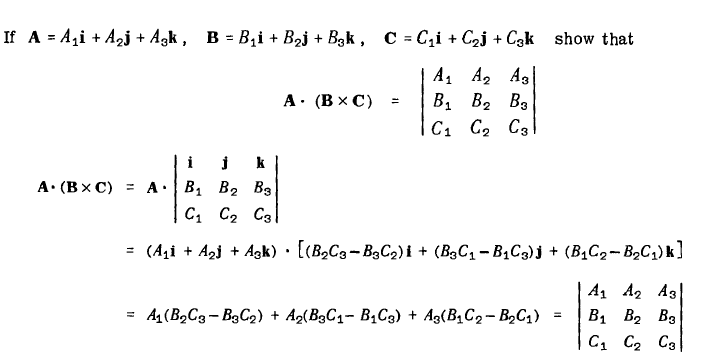












**RESULTANT FORCE/ VECTORS**

Experimental evidence shows that the combined effect of two (or more) forces may be represented by a single resultant force.

Resultant vector can be defined as a single vector that can be used for replacing two or more vectors

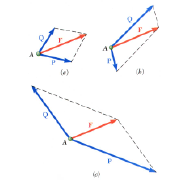
The resultant of two vectors is simply the addition two vectors.

That is if we have two vectors:

And

The resultant of these two vectors is just their sum.

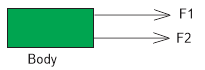
The forces that make up the resultant are called the vector force components therefore, vector force components can be defined as two or more force vectors which together have the same effect as a single vector.



Finding the resultant of vectors using their magnitude

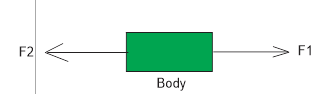
**Vectors acting at zero degrees (0)**: These are vectors acting in the same direction. Their resultant can be obtained by simply adding their magnitude

Resultant Force



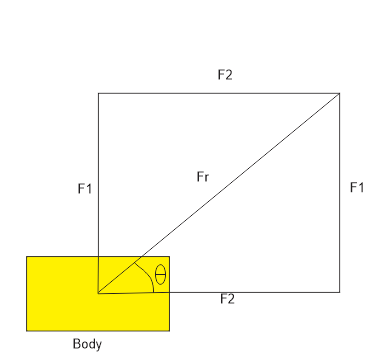
**Vectors acting at 180**: These are vectors that act in opposite direction. Their resultant can be obtained by finding the difference between the magnitude of the vectors

If F1 > F2,



Towards (i.e. towards the east)

**Vectors acting at right angle (90) or Rectangular vector components**: The resultant of such vectors can be obtained using the Pythagoras theorem



From Pythagoras’ theorem,

The following Pythagorean triplets can be used for solving questions on vectors

(3, 4, 5), (5, 12, 13), (6, 8, 10), (7, 24, 25), (8, 15, 17), (9, 12, 15), (10, 24, 26)

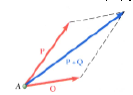
For equal forces,

Therefore, for Equal forces

And

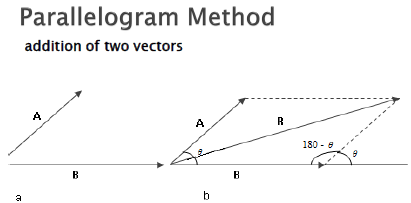
The equation implies that for two equal forces, the angle between one of the forces and the resultant force is 45

**Parallelogram law of forces (vectors)**: This is best used for angles other than 0, 90, and 180 degrees. This law states that when two forces are acting on a body, the lines of forces are used to represent the adjacent sides of a parallelogram.



It can also be stated as if two vectors acting on a particle at the same time be represented in magnitude and direction by two adjacent sides of a parallelogram drawn from a point their resultant vector is represented in magnitude and direction by the diagonal of the parallelogram drawn from the same point

The diagonal of the parallelogram gives the resultant force (vector)



Applying cosine rule,

But

Also,

Also, note that the difference of two vectors A and B (A – B) is defined as the vector sum of A and –B

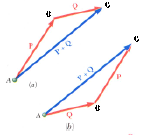
B and –B have the same magnitude but they act in opposite direction (at 180 degrees to each other)

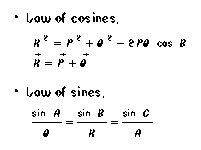
**NB**: The maximum (resultant) vector is obtained when the angle between the two vectors is zero degree

The minimum resultant is obtained when the angle between the vectors is one hundred and eighty degrees (180)

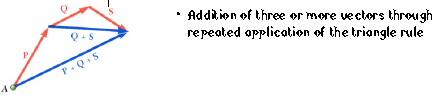
The resultant force decreases as the value of theta increases and vice versa

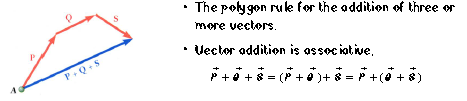
**Triangle rule of vector addition:**



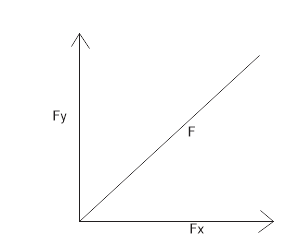


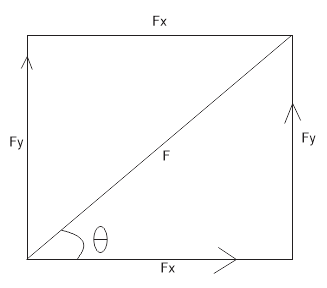
**ADDITION OF MULTIPLE VECTORS**





**RESOLUTION OF VECTORS**





A vector can be resolved horizontally or vertically as shown below. These horizontal and vertical resolutions are called the components of the vector.

**HORIZONTAL RESOLUTION**

Is called the horizontal component of the vector (force)

**VERTICAL RESOLUTION**

Is called the vertical component of the vector

**RESOLVING MULTIPLE FORCES**

The following steps are used to obtain the resultant of multiple forces

Resolve all the forces horizontally and find their sum

Resolve all the forces vertically and find their sum

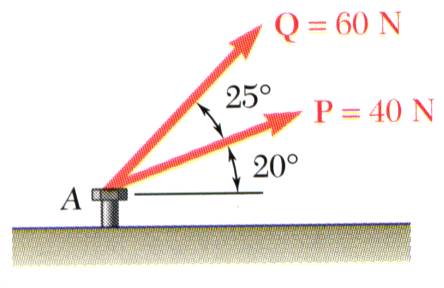
Apply the formula,

To obtain the direction (angle), the formula below is used.

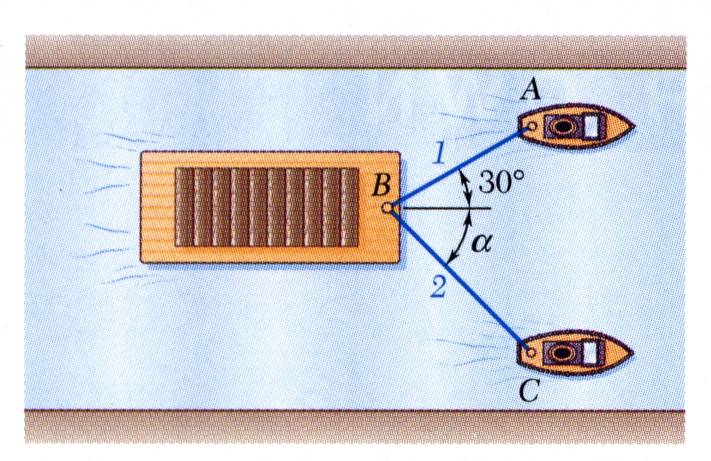
When resolving vectors, the angleused is always the angle the vector makes with the horizontal (the x-axis).

In the resolution of vectors depending on the position it can either be negative or positive. For example, if resolving horizontally, if the force is towards the negative x-axis, then instead of adding the vector, it is written as and if it is on the positive x-axis we will have. Similarly, when resolving vertically, if it is on the positive y-axis we have and if it is on the negative y-axis

EG



The two forces act on a bolt at *A*. Determine their resultant (and direction of their resultant)



A barge is pulled by two tugboats. If the resultant of the forces exerted by the tugboats is 5000 lbf directed along the axis of the barge, determine

1. the tension in each of the ropes for a = 45o,
2. the value of a for which the tension in rope 2 is a minimum.

SOLUTION:

* Find a graphical solution by applying the Parallelogram Rule for vector addition. The parallelogram has sides in the directions of the two ropes and a diagonal in the direction of the barge axis and length proportional to 5000 lbf.
* Find a trigonometric solution by applying the Triangle Rule for vector addition. With the magnitude and direction of the resultant known and the directions of the other two sides parallel to the ropes given, apply the Law of Sines to find the rope tensions.
* The angle for minimum tension in rope 2 is determined by applying the Triangle Rule and observing the effect of variations in a.

MORE QUESTIONS

* An airplane travels on a bearing of 100° at an airspeed of 190 km/h while a wind is blowing 48 km/h from 220°. Find the ground speed of the airplane and the direction of its track, or course, over the ground.

**VECTOR EQUATION OF STRAIGHT LINES**

Equation of a line passing through a point () and parallel to a vector ()

Diagram

The diagram above depicts a straight line passing through point () with position vector () and is parallel to the vector

This is because parallel vectors are scalar multiples of each other hence

This equation is known as a straight line vector

By equating I, j and k components, on both sides the vector equation of the straight line through P1 and parallel to a leads to equations called **parametric equations**.

Example: Determine the vector equation of the straight line passing through the point with position vector and parallel to the vector

Answer:

Example: Determine the vector equation of a straight line passing through the point (5, 6, -3) and parallel to the vector (2, 8, 4)

Obtain the equation of the line passing through the points A () and parallel to the vector ()

Verify if the point (-23, 5, 8) lies on the line

To verify that, we input the values into the equation

Recall that

Since we got the same value of t for all equations, therefore the point lies on the line.

Find the coordinates of the point on the line with x-coordinate of 3

Therefore, the coordinates of the point are

Find the co-ordinates of the point on the line each distant 5 units from A

Let the points be B and C

Find the equation of a line passing through the point A(-1,0,3) and parallel to the vector u(5,3,-4). Find the point where this line intersects the xy-plane

At the point where it intersects the x-y-plane,

Therefore the point is

EQUATION OF A STRAIGHT LINE PASSING THROUGH TWO GIVEN POINTS

In this case, nothing really changes; we are simply going to evaluate the parallel vector to the straight line .

Equation of a straight line passing through two points () and () is expressed as

Example: Determine the vector equation of the straight line passing through the points and

Answer:

Example: Determine the vector equation of a straight line passing through the points and

PERPENDICULAR DISTANCE OF A POINT FROM A STRAIGHT LINE

The diagram depicts a straight line passing through points A and B and is parallel to the vector b. The line … can be seen to be at a perpendicular distance from point c

The distance d from a point c to a line that passes through r1 and r2 is given as

Where

SHORTEST DISTANCE BETWEEN TWO SKEW LINES

Skew lines are non-parallel lines that do not intercept. However, to both lines, there is a common perpendicular. The shortest distance is given as that perpendicular

The distance (d) between two lines and is given as

Example, determine the perpendicular distance between two skew lines where

Answer:

Example: Determine the distance between the lines and

Answer: 8.39841254841

Find the shortest distance between the two lines A and B where:

EQUATION OF A PLANE

A plane in space is completely specified if we know one point in it, together with a vector which is perpendicular to the plane called the normal vector

Finding the equation of a plane with a given point and a (normal) vector

In a diagram, we have a point on the plane. The vector n, (the normal vector) is perpendicular to . Also, we have a position vector (vector that starts from the origin) to . There is another point P on the plane and a position vector r that goes to P. When the diagram is drawn it will be seen that there is another vector from to P which we will call (r - )

The normal vector (n) is perpendicular to . Since they are perpendicular, their dot product is 0.

At the end of the calculation, we will see that we have an equation like

Determine the vector equation and hence the Cartesian equation of the plane passing through the point with position vector and perpendicular to the vector

Solution:

Ans:

Find the equation of a plane passing through the point with a normal

Find the equation of the plane through the point (1,-1,2) and normal to 2i-3j+4k

FINDING THE EQUATION OF A PLANE WITH THREE GIVEN POINTS:

In order to write the equation of a plane or define a plane, we need a point on a plane and a vector perpendicular to the plane. This perpendicular vector is called the normal vector.

If you are given 3 points, we can define the plane using these points but we don’t have the normal vector.

Find an equation of the plane that passes through the points P (2, 1, 4), Q (4, -2, 7) and R (5, 3, -2)

Let

Let

The normal vector will be the cross products of the vectors a and b.

From there, we can find the equation of the plane and any of the given points P, Q or R.

Also note that if we have three points P (ap, bp, cp), Q(aq, bq, cq) and R(ar, br, cr) the equation of the plane can be expressed as

Determine the Cartesian equation of the plane passing through the three points (0, 2, -1), (3, 0 , 1) and (-3, -2, 0)

POINT OF INTERSECTION BETWEEN A LINE AND A PLANE

We have a plane and a line intersects it at P (x, y, z) coordinate. If we are given a parametric equation e.g. (x = 3 + 4t, y = 5 – 2t, z = 4 + 7t) and the equation of the plane e.g. (2x + 4y – z = 1).

What we have to first do is to substitute the parameters of the parametric equation into the equation of the plane

We get, t = 3.

We substitute the value of t into the parametric equation

So,

So, y = 5 – 2t = 5 – 2(3) = -1

So, z = 4 + 7t = 4 + 7(3) = 25

Therefore, the point of intersection P is P (15, -1, 25) between a line and the plane.

At what point does the line passing through P(2, 1, 3) and Q(5, 2, 1) intersect the plane x – 3y – 5z = 4

First, to define a line, we have to define a vector. The vector is given as (a, b, c):

Next, we define parametric equations. The general parametric equations are

are any of the given points P or Q. After solving, we get t = 2 and we can continue solving to get our answer

All the equations above can be expressed in a general form

If we are given the equation of a straight line as , and a plane as

Then,

The value of the point t(x, y, z) is the point of intersection

Determine the point of intersection of the plane, whose vector equation is and the straight line passing through the point , which is parallel to the vector .

First we find the equation of the line

NB: When given the equation of a plane, the values of the normal vector can be gotten from the coefficients of the equation. Given the equation ax + by +cz = d as the equation of a plane, the normal vector of the plane is n = <a, b, c> and these values are called the direction numbers.

**ANGLE BETWEEN TWO (NON-PARALLEL) PLANES**

The angle between two planes is also the angle between the normal vectors of the two planes.

Given two planes 2x – 3y + 4z = 5 and 3x + 5y – 2z = 7, Find the angle between the two planes;

First, you have to find their normal:

So, n1 = <2, -3, 4> and n2 = <3, 5, -2>

PERPENDICULAR DISTANCE BETWEEN A POINT AND A PLANE

A plane has an equation of . There is a point on the plane. The position vector of this point is . A line passes through this point and this line is perpendicular to the plane. The equation of this line is

Reason being that since the line is perpendicular to the plane, it will be a multiple of the normal because it will be parallel to the normal and the line passes through a point P\_1.

There is another point where the line meets the plane. The point has a position vector

Its magnitude , will be the perpendicular distance (p) between the point P1 from the plane

The perpendicular distance between a point () and the plane is expressed as

Also note that from the above, we have

Determine the perpendicular distance, p, of the point () from the plane whose Cartesian equation is .

Find the distance from point () to the plane that passes through the point () and has a normal

Equation of the plane

LINE OF INTERSECTION OF TWO PLANES

Suppose we are given two non-parallel planes whose vector equations are:

Their line of intersection will be perpendicular to and also perpendicular to , since these are the normal to the planes

And recall that a line that is perpendicular to two different vectors is parallel to the cross product of the two vectors. Thus, this line of intersection of two planes is parallel to , and then we get the vector equation

EXTERNAL AND INTERNAL FORCES

Forces acting on a body can be divided into external and internal forces

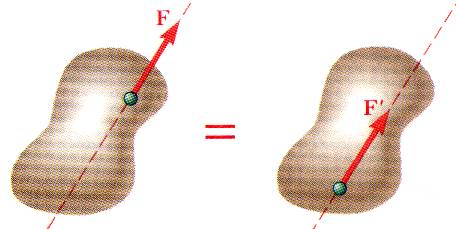
External forces like a push or pull or friction all act on a body

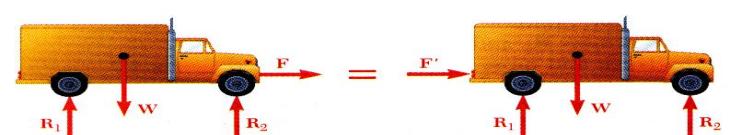
The weight of a body is an example of an internal force.

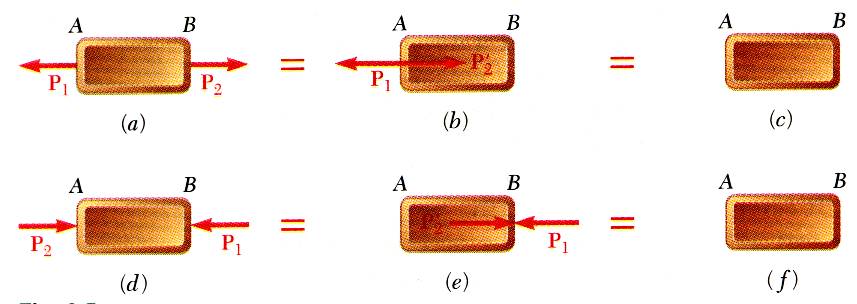
If the external force is unopposed by the internal forces or by other external forces or if the force is greater than the opposing forces then the force can impart a motion of translation or rotation or both

PRINCIPLE OF TRANSMISSIBILITY:

The equilibrium or motion state of a body are not affected by transmitting (sliding or moving) a force along its line of action. For example if a pull force makes a car to move. If the force is applied to the back of the car (i.e. a push force), then the car will still move at the same velocity. So you can see that by transmitting the force along its line of action to the back of the car, it doesn’t affect the motion or equilibrium of the car and it also doesn’t affect other forces acting on the body

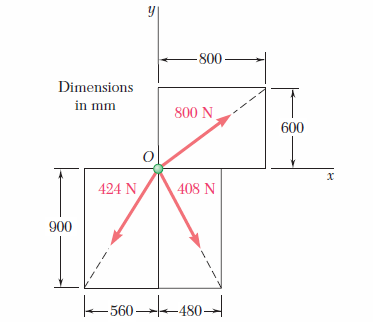


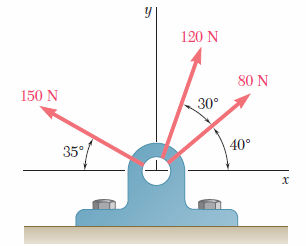


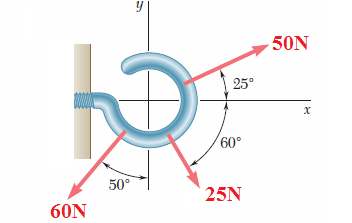


MORE QUESTIONS

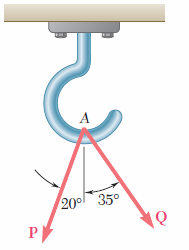
1. Determine the *x and y components of each of the* forces shown. Also find the resultants of the forces.



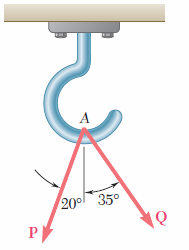




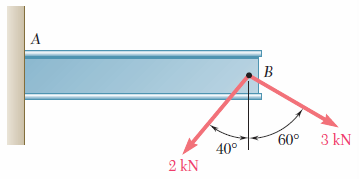
1. Two forces P and Q are applied as shown at point A of a hook support. Knowing that P = 75N and Q=125N, determine graphically the magnitude and direction of their resultant using
2. The parallelogram law
3. The triangle law



1. Two forces P and Q are applied as shown at point A of a hook support. Knowing that P = 75N and Q=125N, determine graphically the magnitude and direction of their resultant using
2. The parallelogram law
3. The triangle law



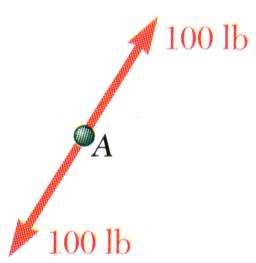
1. Two forces are applied at point B of beam AB. Determine graphically the magnitude and direction of their resultant using
2. The parallelogram law
3. The triangle rule



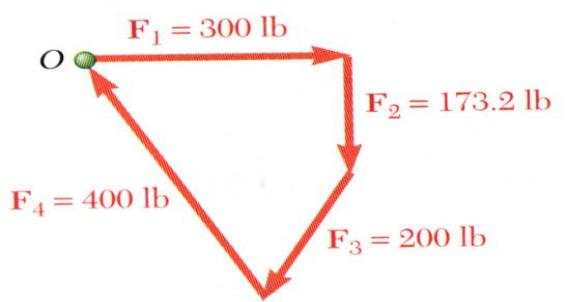
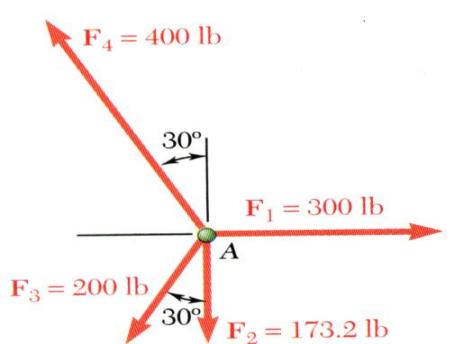
EQUILIBRIUM

When the resultant of all forces acting on a particle is zero, the particle is in equilibrium

According to Newton’s first law, if the resultant force on a particle is zero, the particle will remain at a rest or will continue at constant speed in a straight line



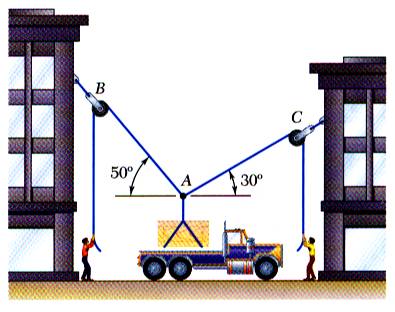
* Particle acted upon by two forces:
  + equal magnitude
  + same line of action
  + opposite sense



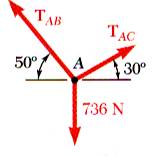
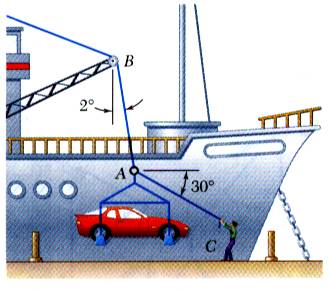
* Particle acted upon by three or more forces:
  + graphical solution yields a closed polygon
  + algebraic solution



*Space Diagram*: A sketch showing the physical conditions of the problem



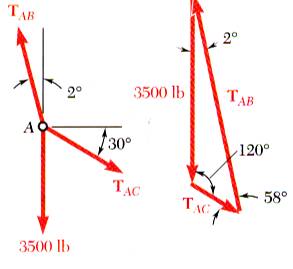
*Free-Body Diagram*: A sketch showing only the forces on the selected particle.



In a ship-unloading operation, a 3500-lb automobile is supported by a cable. A rope is tied to the cable and pulled to center the automobile over its intended position. What is the tension in the rope?

SOLUTION:

* Construct a free-body diagram for the particle at the junction of the rope and cable.
* Apply the conditions for equilibrium by creating a closed polygon from the forces applied to the particle.
* Apply trigonometric relations to determine the unknown force magnitudes.



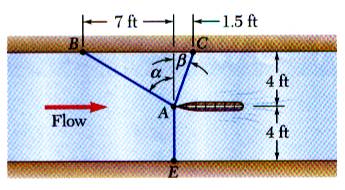
SOLUTION:

* Construct a free-body diagram for the particle at *A*.
* Apply the conditions for equilibrium.
* Solve for the unknown force magnitudes.







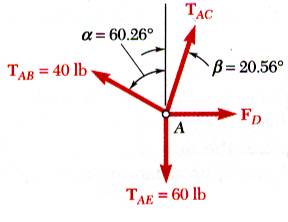


It is desired to determine the drag force at a given speed on a prototype sailboat hull. A model is placed in a test channel and three cables are used to align its bow on the channel centerline. For a given speed, the tension is 40 lb in cable *AB* and 60 lb in cable *AE*.

Determine the drag force exerted on the hull and the tension in cable *AC*.

SOLUTION:

* Choosing the hull as the free body, draw a free-body diagram.
* Express the condition for equilibrium for the hull by writing that the sum of all forces must be zero.
* Resolve the vector equilibrium equation into two component equations. Solve for the two unknown cable tensions.



SOLUTION:

* Choosing the hull as the free body, draw a free-body diagram.





* Express the condition for equilibrium for the hull by writing that the sum of all forces must be zero.



.(a) Find the shortest distance between the two lines A and B. where

A = (2, 0, 7) + t(1, 1, 1), B = (1, 2, 4) + s(0, 1, 2).

Also find the angle between A and B.

(b) Find the equation of a plane through the point (4, 3, 1) and with normal (i -2j +3k).

2. (i) An engineering student of the University of Lagos is jugging at a speed of 50m/s from the senate building to the gate i.e. from a point A to B which is 300m directly opposite the senate building. A wind blows against the student at 20m/s. If the student always jugs in the direction parallel to AB, find how he gets to point B. In what direction must the student jug in order to cross directly from A to B.

Find the distance from the point (0, 1, 0) to the plane that passes through the point (4, 3, 1) and has a normal (i -2j + 3k).

Obtain the equation of the line through the point A(1,-3,2) and parallel to the vector (-12,4,3)

i.) verify that the point (-23,5,8) lies on this line.

ii.) find the co-ordinates of the point on the line with x-co-ordinates 3.

iii.) find the co-ordinates of two points on the line each distant 5 units from A.

4. Show that

5. Solve for the vector **u**, if

6. Obtain the vector equation of the line through the point A(-1,0,3) and parallel to the vector u=(5,3,-4). Find the point where this line intersects the xy-plane.

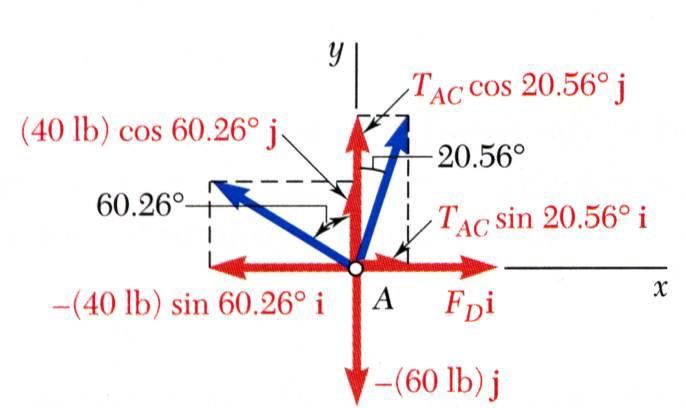
7. Obtain the equation of the plane through the point (1,-1,2) and normal to 2i-3j+4k.

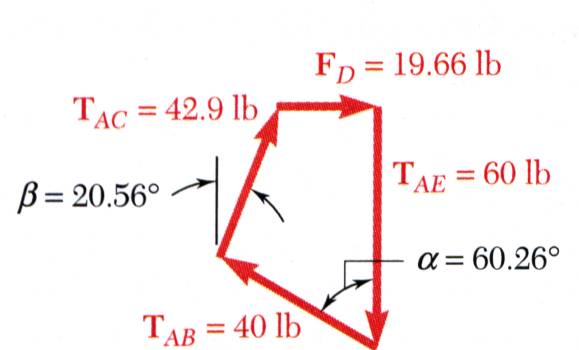
8. Write vector V as a linear combination of thee unit vectors I and j given that the vector V has a magnitude of 16 and an angle of 30 with the positive x-axis

Solution: v has a magnitude of 16 and is inclined at 30 degrees to the horizontal

x-component

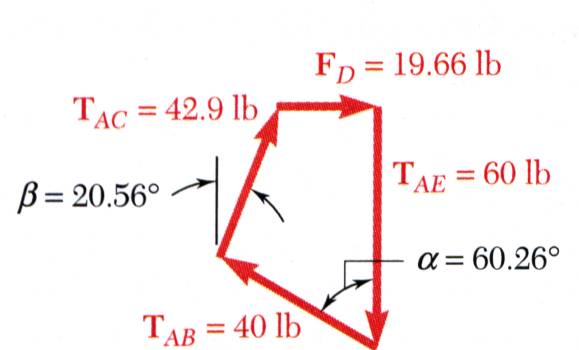
Find the magnitude of the vector shown below and determine the angle that it makes with the positive x axis





* Resolve the vector equilibrium equation into two component equations. Solve for the two unknown cable tensions.





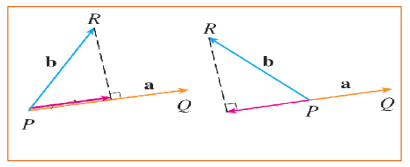


This equation is satisfied only if each component of the resultant is equal to zero

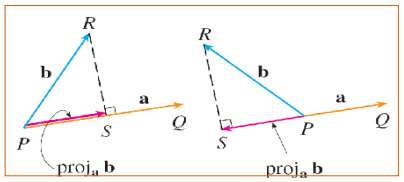


PROJECTIONS

The figure shows representations PQ and PR of two vectors a and b with the same initial point P



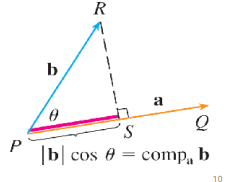
The vector representation PS os called the vector projection of b onto a and is denoted by . You can think of it as a shadow of b on a



SCALAR PROJECTION

The scalar projection of b onto a (also called the component of b along a) is defined to be the signed magnitude of the vector projection. This is the number . Here, theta is the angle between a and b

This is denoted as and the value will be negative if



From the equation

We can see that the dot product of a and b can be interpreted as the length (or magnitude) of a times the scalar projection of b onto a

The component of b along a can be computed by taking the dot product of b with unit vector in the direction of a

Scalar projection of b onto a:

Vector projection of b onto a:

Or

Take note:

The shortest distance between two lines A and B of parametric equations

Is given as

The equation of a plane through a point r1and a normal n is given as

R = r1 + tn

E.g. the equation of the plane through a point (4, 3, 1) and with nomal (i-2j+3k)

R = (4, 3, 1) + t(1 , -2, 3)

DIFFERENTIATION OF VECTOR EQUATIONS

We want to consider vectors that are functions of parameter (t) assuming that “t” is time. Differentiating the components with respect to time

Recall the definition of a derivative,

A vector valued function

Vectors can be differentiated

A the vector function correspondends to a curve

The first derivative

If we have a vector,

If we take the derivative,

One thing we know is that the derivative corresponds to a tangent in the direction the of the tangent

The unit vector of the tangent vector

Dot product

I . I = 1

J . J = 1

K . K = 1

i.j = 0

i.k= 0

k.j = 0

Vector operator

Gradient

FROM THE SLIDES

We can differentiate vectors from the basic rule below

If the point P has a position vector r(t), r’(t) = tangent at that point P

If we also have another point Q with a position vector , the vector connecting PQ will be and this vector passes through the points P and Q therefore called a secant vector

Recall that a secant line is one that passes through two given points on a graph

If, (tangent line) will have the same direction as PQ (secant line); PQ is the vector between p and Q while is the vector after Q that is in the same direction as PQ

As h approaches 0, it appears that is the tangent vector

The unit tangent vector

If you look at this carefully you will see the mean value theorem in it

The mean value theorem says, there’s a point c where

Let a = the point P

Let b = the point Q

Recall that the derivative of a function is the slope of the function. Therefore,

From mean value theorem,

Slope of tangent line = slope of secant line

Find the derivative of

Find the unit tangent vector at the point where t = 0.

Solution

At t=0,

The unit tangent

For the curve , find and sketch the position vector and the tangent vector

Position

CENTROIDS, CENTER OF MASS AND CENTER OF GRAVITY

Applications of centroids, centre of gravity and centre of mass

In the designing and the structure for supporting an overhead water tank

In determining the stability of sport utility vehicles (SUV)

In designing the ground support structure for the goal post

The balancing of missiles in the air after they are fired.

The luggage compartment of tour buses located at the bottom of the bus in order to lower the centroid/center of mass of the bus in order to maintain stability

Racing cars are built to be low and broad for stability especially when navigating sharp turns at high speed without the risk of toppling

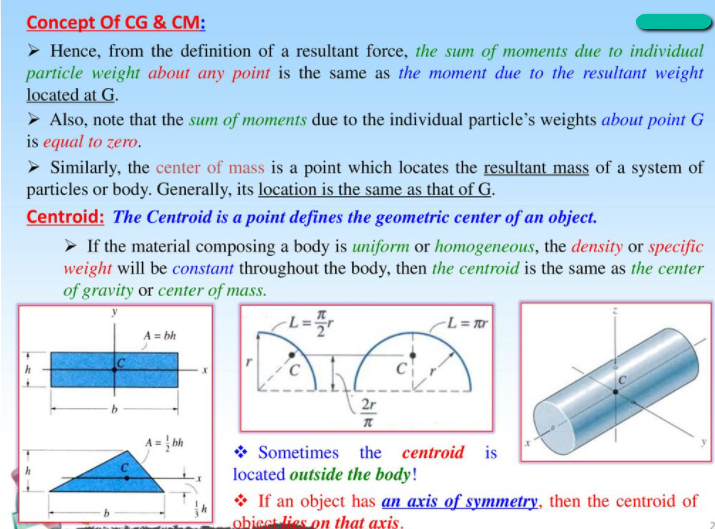
CONCEPT OF CENTER OF GRAVITY (CG) AND CENTRE OF MASS (CM)

Hence, from the definition of a resultant force, the sum of moments due to individual particle weight about any point is the same as the moment due to the resultant weight located at G.

Also note that the sum of moments due to the individual particle’s weights about point G is zero

Similarly, the center of mass is a point which locates the resultant mass of a system of particles or body. Generally, its location is the same as that of G.

Centroid: The centroid is a point that defines the geometric center of an object. If the material composing a body is uniform or homogeneous, the density or specific weight will be constant throughout the body, then the centroid is the same as the center of gravity or center of mass



WAYS OF SOLVING CENTROIDS.

For a body on a 1d value, if you look at a metre rule on a fulcrum with some load on it, you will see is that the fulcrum will be placed at where the resultant of these weights act and that is the center of gravity.

The distance between the center of gravity and the beginning of the rule is expressed as

Where represents the weight of each element and represents the distance between the beginning of the rule and the center of gravity of that particle

CENTROIDS BY INTEGRATION

Bodies are made up of an infinite number of particles of weight i.e. the weight of each particle. These weights are parallel to each other and the total weight of all the bodies will be the total sum of all the weights of the individual particles concentrated on a single point called the center of gravity .

The location of the center of gravity can be located from each of the orthogonal (perpendicular) directions (or axes) by knowing that the moment caused by weight from a random point of reference (or datum) which will be the distance from the center of gravity times the weight

The moment about the x-axis for each particle is equal to

is the distance between the centroid of that particle and the y-axis

So the total moment of the object about the x-axis will be

The y-coordinate of the centroid will be given as

The same applies for all the coordinates

Remember that

are called the coordinates of the center of gravity

For a body in motion, we do not really care about the weight instead, we care about the centre of mass of the body

We can cancel the g out taking “g” as constant

Centroid of a volume:

Centroid of An Area:

The 3d can be simplified to 2d if z is taken as constant

**CENTROIDS OF AREAS BY INTEGRATION FROM SLIDES**

The following steps should be taken when solving centroid questions:

**USING A VERTICAL STRIP**:

First, take out a vertical strip of the shape.

The width of this strip will be

The height of the strip will be assumed to be y. My own reasoning concept being that each strip will have a height based on the equation of y.

Next mark out the assumed centre of the strip. Since that will be the centroid of the strip since it is a rectangle

Next, find the equation/equation of the graph as y as a function of x

If there are two equations, the upper one is the second equation and the lower one is the first equation.

We’ll call the y component of the centroid and it will be equal to 2 . If there is only one equation of the line,

Next, find the area of the strip

For single equations:

For two equations:

Next, we find the area of the shape using integration

Next find the area of the whole shape.

Next substitute the equation for the value of y.

The area of the whole shape is assumed as the sum of areas of all suspected strips taken out of the shape

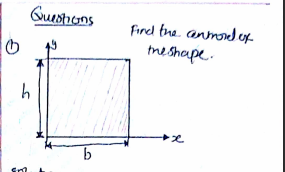
Next we calculate the moment about x. In the moment about x, since its x position is going to be constant and its y position will vary, the moment about x is expressed as sum of all moments about the x axis

Similarly, the moment about y will be expressed as

Most times,

Most times,

To find the coordinate of the centroid G



Solution:

First we take out a vertical strip,

The height of the strip will be y,

The width of the strip will be dx

The area of that strip will be dA, given as

Next, we find the equation of y since it wasn’t given to us

The y intercept “c” = h

The total area of the shape will be a sum of areas of all individual strips

From the diagram,

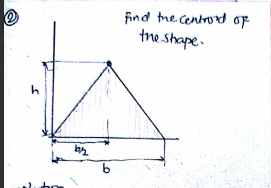
Next,

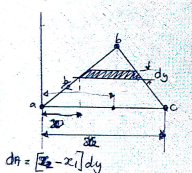
Next, we find the momentum about the x-axis

Next, we find the momentum about the y axis

The position of the centroid is given as

Answer:





For this case, we are going to use the horizontal slip since that will be easier to solve with

If we take out a horizontal slip:

The height of the slip will be dy

The width of the slip will be x

And the area of the slip will be dA, given as

But taking a good look at that image, we would see that there are two possible equations i.e. one equation from point A to B and another from point B to A

For points to point

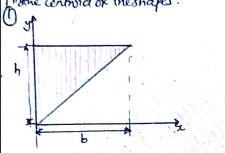
For points to point

And now, instead of area of the slip to be just

It is now going to be;

Next, we find the momenta

Answer:



First, we take out a vertical strip:

But there are also two possible equations of y

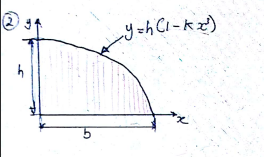
Therefore,

Let the upper equation be

Let the lower equation be

m

Answer:



To get dA in this question,

We fisrt say, when y = 0 and x = b

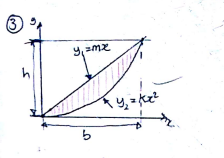
That is used to get the value of k in the equation.

At point (b, 0)

But k =

A = 3bh/4

Answer:



Therefore,

At point (b, h)

Therefore,

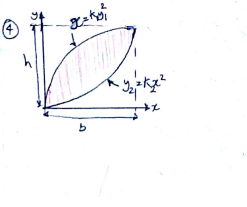
Here, dA = (y1 – y2)dx

Y1 = mx

M = h/b

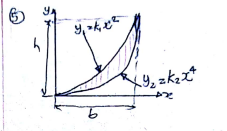
Y1= hx/b

Answer: b/2 , 2h/5

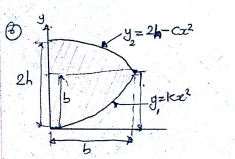


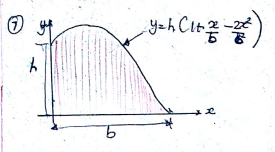
at point (h, b)

Answer: 9b/20, 9h/20

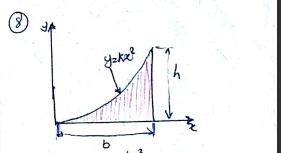


Answer: 5b/8, b/3

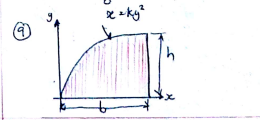




Answer: 2/5b, 12h/25



Answer: 3b/4 3h/10



Answer: 3b/5, 3h/8

CENTROID BY GEOMETRY AND CENTROIDS OF SIMPLE SHAPES

If Gravity is assumed to be constant, the center of mass is the center of gravity and for bodies of uniform material and thickness, the center of gravity is located at the center of area which we call the centroid.

For bodies that are symmetrical about two axes like squares, circles and rectangles, the centroid is at the center of the shape of or at the point of intersection of the two lines of symmetry.

For bodies that have one line of symmetry, the centroid will lie of that line of symmetry. The centroid of a semicircle from its base (diameter) is given as where r is the radius

Right angled triangles (or triangles in general) have their centroid 1/3 of the height (from the base line) and 1/3 of the base from the (height line)

This method of solving centroids is quite easy but can be tricky a times.

Note the following:

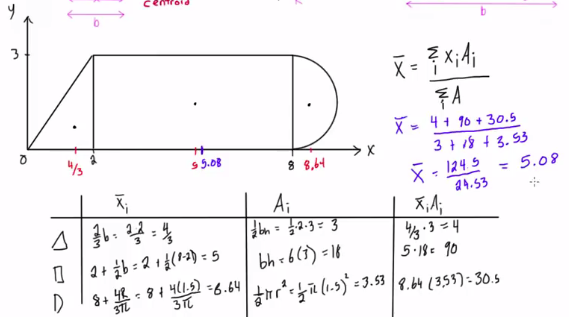
|  |  |  |  |
| --- | --- | --- | --- |
| Shape |  |  |  |
| Triangle |  |  |  |
| Quarter Circle or semi-circle |  |  |  |
| Quarter ellipse or semi-circle |  |  |  |
| Parabola |  |  |  |
| Parabolic Spandrel |  |  |  |
| Rectangle |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |

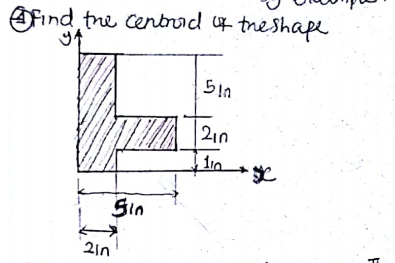
For a parabola, the above formula is if the base of the parabola is the x-axis. However if the base of the parabola is the y-axis, then the formulae will be

Take note that the following formulae apply when the point of reference for the x-axis is the base and that of the y-axis

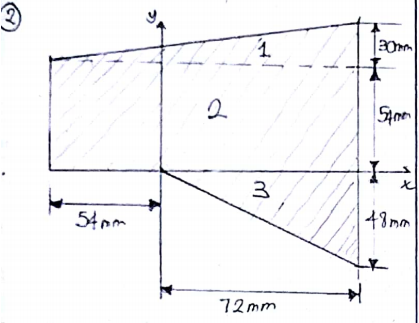
CENTROIDS OF COMPOUND OR COMPOSITE SHAPES

For 2d shapes, we can break the complex shapes into simpler shapes

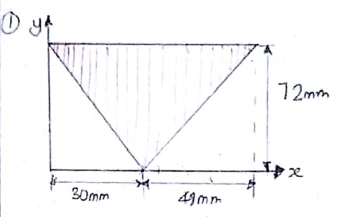




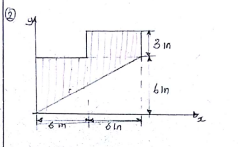
Answer: (1,68, 3.45)



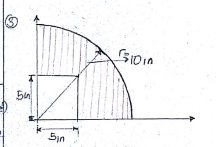
Answer: (19.27, 26.58)

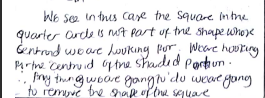


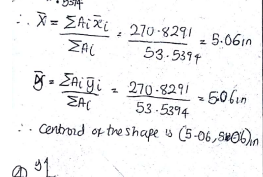


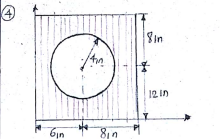


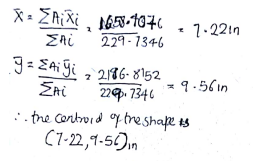


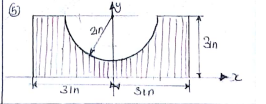


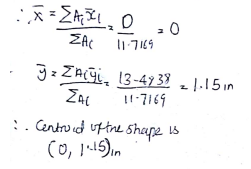


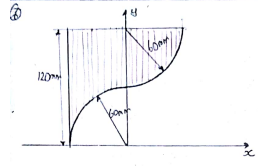




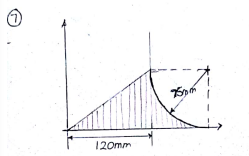


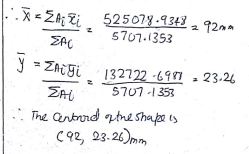


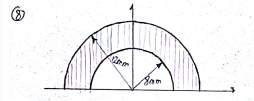




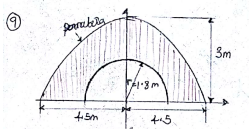


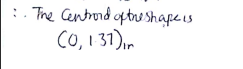


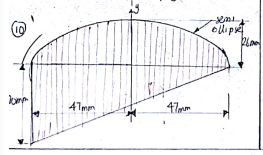




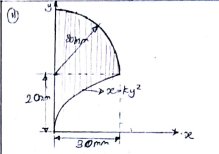




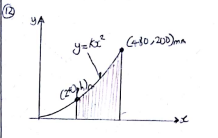


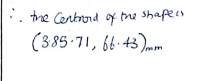


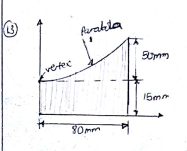


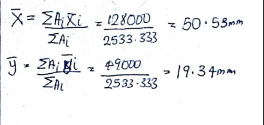


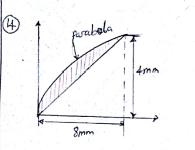


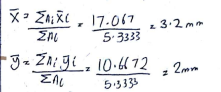


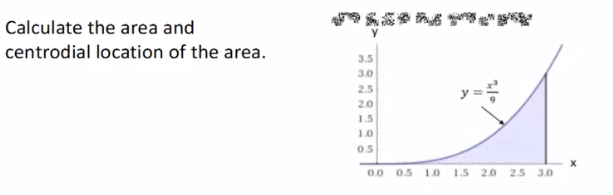




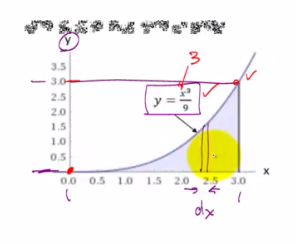








Next we take out a small section of the figure (in the form of a rectangular shape). The base of the shape will have a width dx and its height will be a certain value (y)



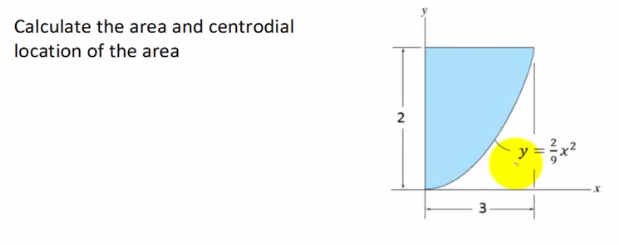
Now, the area of that chunk will be

Recall that the equation of the graph is

From the graph, we integrate from the lowest value of x in the function (0) to the highest value (3).

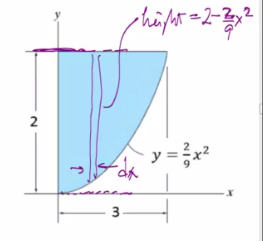
Remember that

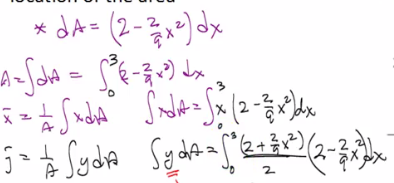
When dealing with centroids, the value of will be half of the assumed rectangle that was created

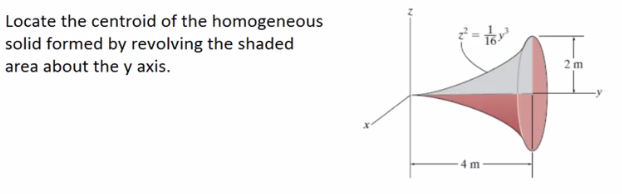


For this side, the height of the strip drawn will be

The reason we have that two is because if you look at the curve you will see that the curve is reducing in height and so as the value of x increases, the value of y reduces







For example, cutting out a little part of the shape, we will have a small disk (cylinder) . This disk will have a radius of R and it will have a thickness of dy

Note that (when finding the center of area), if the object is symmetrical about the y axis and that y axis is your reference point, then the x coordinate will be zero. Likewise for the y and if the object is symmetrical about the x-axis the y coordinate will be zero if that line of symmetry is your reference point

The centroid of a triangle with a base b and a height h, the diagonal line (hypotenuse will be described by a straight line equation

The above tells us that the width of any rectangle given as (x) and has a distance y from the base,

The centroid from the bottom our point of reference is h/3

The centroid of a triangle will be 1/3 the height measured from the base of the triangle and also 1/3 of the width

Centroid is the geometric centre of a body (line, area or volume)

Integration for continuous system/body

Summation for discrete system

Center of gravity

The area of a semi elliptical shape is given as

And the centroid is

Where “a” is the radius and b is the height.

If we are to work on a plane, we are working with 2d objects and then

If the gravity of the place is constant (or assumed to be), we can substitute

Then that means the center of gravity of that object will also be the centerof mass of the object.

If the body has uniform density i.e. the density is constant, we can then say that the

Then we substitute that into the equation to get the center of volume of the object

Also, if we have an object of uniform thickness; we know that this object is composed of mini-particles that make up the object.

The center of gravity is gonna be based on the idea of a relationship between the thickness of the plate and the areas of each particles

So we divide the above formula of the volumes by the thickness

The center of area is called the centroid

Examples: Find the center of gravity of a rod that is 14inches in length. It has 10inches aluminium and 4 inches steel and has a thickness of 1inch

Al: , Steel:

QUESTIONS

The point through which the whole weight of the body acts is called \_\_\_\_\_\_\_\_\_\_\_\_ Answer: Centre of gravity

The point at which the total area of a plane figure is assumend to be concentrated is called \_\_\_\_\_\_\_\_\_ Answer: Centroid

Where will the centre of gravity of a uniform rod lie? Answer: At its middle point

Where does the center of gravity of a circle lie? Answer: At its centre

Where will the centre of gravity of an I section will be if the dimension of upper web is 2X10cm, lower web 2X20cm and that of the flange is 2X15cm. If the y axis passes through the centre of the section?

A body is constituted of how many particles? Answer: Infinite

A body’s all small particles have their own weights which is being applied by them to the body, which adds up to the total weight of the body; true or false? Answer true

The small weights that are being applied by all the infinite particles of the body act \_\_\_\_\_\_\_\_\_\_\_ to each other; Answer: parallel

The toal of all the weights of the small particles add up to give the total body weight. This weight is the force vector which is being passed by \_\_\_\_\_\_\_\_\_

The centre of gravity is the ratio of \_\_\_\_\_\_\_\_\_ to \_\_\_\_\_\_\_\_\_\_?Answer: Product of centroid and weight to the total weight

The x-axis coordinate and the y-axis coordinate of the centre of gravity are having different types of calculations to calculate them; true or false? Answer: False?

The centre of mass is the ratio of what to what?

The centre of volume is the ratio of what to what?

Volume is best given by the ratio of what to what? Mass to density

MOMENT OF INERTIA

We have treated centroids of plane shapes

MASS MOMENT OF INERTIA

This is a measure of an object’s resistance to angular acceleration.

The moment of inertia of shape is the measure of the shapes resistance to bending. It is also defined as the second moment of a shape along a particular axis. The moments about the axes are denoted by (or even when it comes to solid (3d) shapes)

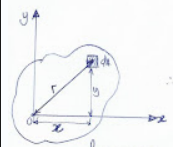
Recall that for moments,

Similarly for moment of inertia,

There are two methods of solving moment of Inertia and they are the integration method and the geometry method

POLAR MOMENT OF INERTIA

This is the quantity used to describe resistance to torsional deformation (deplection), in cylindrical objects (or segments of cylindrical objects) with an invariant cross section and no significant warping or out of plane deformation



RADIUS OF GYRATION

The radius of gyration is defined as the distance from the axis of rotation to a point where the total area of the body is supposed to be concentrated so that the moment of inertia about the axis may remain the same

, this generally known as the radius of gyration

MOMENT OF INERTIA BY INTEGRATION

In this method, the same way we took a strip (vertical or horizontal), find the centre of the strip and then integrate the centre of the strip through the whole shape. We also take a strip (vertical or horizontal), find the moment of inertia of the strip through the whole shape. The method is as simple as the method of centroid by Integration.

The following steps are taken when solving moment of inertia questions.

First, we take out our strip (vertical or horizontal) in this case vertical.

This vertical strip will have an area of

However, we can also decide to take a horizontal strip from the vertical one. This horizontal strip will have a height of dy and will still have a width of dx. Therefore, the area of this new horizontal strip will be

Area of the original slip, will be

We will see that width of the strip will be constantly dx

Therefore, the area will be given as

The moment of inertia of just the vertical strip will be given as

As already explained, the width will be constantly dx so the moment of inertia of that vertical strip will be:

Then, the moment of inertia of the whole shape will then be the addition of all these strips

Now we integrate with respect to x and in this case since we are finding the moment of inertia of the whole body, dx is not constant

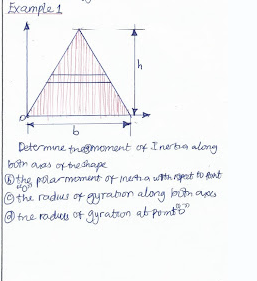
This is the moment of inertia of the shape about the x-axis

For the moment of inertia about the y axis,

The above formulae are used to find the moment of inertia of a shape with vertical strips

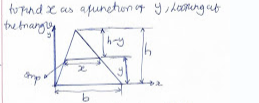
For horizontal strips,

QUESTIONS



So taking a horizontal slip, since that will make it easier for us to solve

Recall that for a horizontal slip, we are usually integrating with respect to the y-axis; therefore, we find a x as a function of y



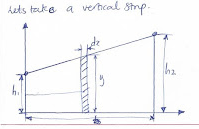
From similar triangles,

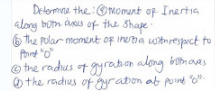
Recall that b and h are both constants

Polar moment of inertia:

Radii of gyration

Question2:





Since we have a vertical strip, we deal with respect to x

If you look at the above image, there are two possible equations of y

For the upper line,

Area of the strip

For vertical strip:

More questions:

Determine the:

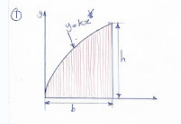
Moment of inertia along both axes

The polar moment of inertia at point 0

The radius of gyration along both axes

The radius of gyration at point 0

Of the following shapes:

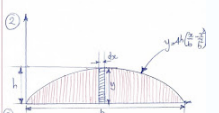


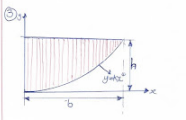
First, find the value of k

So we can say at point (b, h)

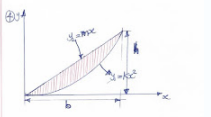
At the end, we are supposed to get the following values:

Recall that for vertical strip:

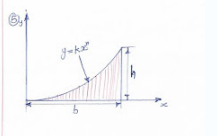


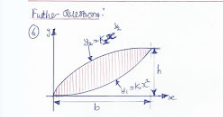


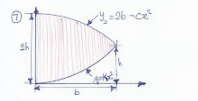
Using horizontal strips,



Area of the strip,

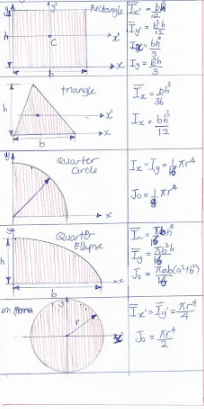






MOMENT OF INERTIA BY GEOMETRY

Moment of inertia of some standard shapes



For a rectangle, the moment of inertia depends on the point of reference of the x-axis and the y-axis

If the y and x axes pass through the centre of the rectangle then the moment of inertia is given as

However if the y axis is on the left of the rectangle and the x-axis is the base of the rectangle, then the moment of inertia will be given as

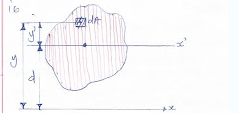
Triangle:

Quarter Ellipse:

Circle:

To solve moment of inertia by geometry, we need to know the parallel axis theorem

The parallel axis theorem states that the moment of inertia of a shape along any axis parallel to the centroidal axis of that shape is equal to the moment of inertia along the centroidal axis plus the area of the product of the area of the shape and the distance between the centroidal axis and the new axis

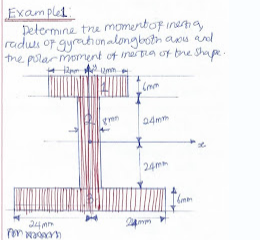


Here, is the distance between the x-axis and the centroidal x-axis

From the parallel axis theorem, the value of y will be equal to the centroidal axis plus the distance between y and the centroidal axis

Recall that moment about the y axis is given as

In this case, the moment is about the centroidal axis and as we know the moment about the centroidal axis is 0. Also take note that the centroid of something that has a line of symmetry lies on the line of symmetry and the moment about that centroidal axis is equal to 0



First, we can divide the whole shapes into smaller shapes (just like when solving centroids)



So we then find the centroid around the x-axis of the shape

Recall that if the y and x axes pass through the centre of the rectangle then the moment of inertia is given as

Distance between the centroid of that individual shape and the centroid of the whole shape

For shape two, the vertical rectangle,

Shape three, the lower horizontal rectangle

So all these moments we have gotten are the centroidal moments of each piece of the whole shape. Now, to find the centroid in relation to the axis of the shape, we do:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |

For the y-axis,