**INFINITE SERIES**

An infinite series is a series that has an infinite number of terms. When working with infinite series, we have to be careful about the steps we take.

E.g.

We can see that the above is a GP in which a = 1 and r = ½.

Recall that the sum of n terms in a GP is given as:

The sum of the first n terms is therefore given by:

If n is a large number, will be a very large number and will be a very small number.

In fact, as n approaches infinity.

The sum of all the terms in this infinite series is therefore given by

i.e.

This result means that we can make the sum of the series as near to the value 2 as we want by taking a sufficiently large number of terms

However this is not always the case for infinite series. For example, taking a look at an AP, things are very different

Consider the infinite series 1+3+5+7+…

This series is an AP in which a =1 and d = 2.

In this case, if we chose a very large number (or if) we will have: which is not a definite numerical value and of little or no use to us.

This always happens with an AP i.e. if we try to find the sum to infinity of an Arithmetic Progression; we invariably obtain as the result, depending on the actual series

Note the following:

We cannot evaluate the sum of an infinite number of terms of an AP because the result is always infinite

We can sometimes evaluate the sum of an infinite number of terms of a GP. Since for such series

And provided that

Then as ,

In that case,

CONVERGENT AND DIVERGENT SERIES

A convergent series is one in which the sum approaches a definite value as n approaches infinity.

The opposite i.e. a series that its sum does not tend to a definite value (sum approaches infinity) as n approaches infinity is called a DIVERGENT SERIES

Consider the GP

We know that for a GP,

In this case, a = 1 and we have:

As ,

It is convergent

If tends to a definite value as, the series is convergent

If does not tend to a definite value as, the series is divergent.

Here is another series. Let us investigate this one:

This is also a GP with a = 1 and r= 3

You will see as you keep adding the numbers

You will see you’ll eventually get a very large number

The series is divergent

ABSOLUTE CONVERGENCE AND CONDITIONAL CONVERGENCE

A series is absolutely convergent if the absolute value of the series and the series itself are both convergent. For a series to be conditionally convergent, that means the series is convergent and the absolute value diverges

For divergence, both divergence.

To solve, we need to first check if the absolute value of the series diverges

If , converges, then converges as well. We can say that the series is absolutely convergent

If diverges, and converges, We say that the original series is conditionally convergent.

If both are divergent, we say the whole series is divergent.

Solving some examples,

First write out the terms

Next find the absolute value

We’re getting the 1 as the numerator is because, the series of is always going to yield and since it is in the absolute value, it will always be plus one on top.

From the absolute convergence theorem, if the absolute value of a series converges, then the series must also converge.

TEST FOR CONVERGENCE

1. Cauchy Test or Divergence test

2. Comparison test

3. D’Alembert’s ratio test

**CAUCHY TEST OR DIVERGENCE TEST**

A series cannot be convergent unless its terms ultimately tend to zero.

What that means is that as , the value of each term also approaches 0

Note: is the term while is the sum of n terms in the sequence.

If , then the series is divergent

However if the limit tends to 0, that doesn’t completely mean it is convergent. It means it is either convergent or divergent but if it’s not equal to zero, it has been confirmed that it is divergent

Therefore, the series is convergent because as tends to infinity, it is 0.

If we look at the series , this series is divergent because as n approaches infinity, it is not 0 instead it is another value (infinity)

If we look at

You will see that the above series is divergent even though the limit approaches 0.

Since that limit is not equal to 0, we conclude that the series is divergent.

COMPARISON TEST

A series of positive terms is convergent if its terms are less than the corresponding terms of a positive series which is known to be convergent and also if the series you are comparing with a divergent series, if the value is less, then it is divergent.

We can compare with series like

If the corresponding terms are less, it is convergent and vice versa

You can also compare with the “p” series

If, the series converges.

If , the series diverges.

So if you want to compare, you can compare with .

If you take a look at a harmonic series like, you’ll notice that it converges. In this case,

Examples

GEOMETRIC SERIES

INTEGRAL TEST

If you have a series, you can looking the corresponding integral where the function is just the replacement of n with x and if the integral converges then the series also converges

Alternating series test → AST

Take the limit as of . is everything apart from the negative side. That means

LIMIT COMPARISON TEST

ROOT TEST

THE ALEMBERT’S RATIO TEST FOR POSITIVE VALUES

This is also simply called the ratio test

Let be a series of positive terms.

First find an expression for

Next, find an expression for

Next, find the ratio of to i.e.

Next find the limit as n tends to infinity

If the limit is less than 1, the series converges

If is greater 1, it is divergent

If it is equal to 1, then it is either divergent or convergent and it is not confirmed

Examples

Test the series

First look for an equation of

Now an equation for,

Now find the ratio:

Now find the limit as n approaches infinity

Since it is less than 1, the series converges

Since , it is convergent

Check if the series with the Alembert’s ratio test

QUESTIONS

Determine if the given sequences converge or diverge, if it converges, then find their limits

Applying L’hopital rule,

Applying L’hopital’s rule,

Recall that

Therefore, the sequence is divergent and the limit doesn’t exist

Since the limit is equal to 1, it is either convergent or divergent and therefore needs another test to confirm

To determine whether it is divergent or convergent

We can use the ratio test

Since is always positive no matter the value of n, and we can bring out that expression from the absolute brackets

Then we find the limit

There’s no value of n in the expression so we get

Remember that for the series to be convergent, the absolute value has to be less than 1

That means the series is convergent in the interval,

Notice that in the above, we had two variables x and n. The series is in terms of n but is also a function of x. So we got . That means the whole series is convergent in the open interval

RADIUS AND INTERVAL OF CONVERGENCE

If the ratio test gives you 0, this means that the series is always converging and that it converges for all x values.

The radius of convergence R, is

The interval of convergence is

If the ratio test yields ,the series diverges for x values except a particular x. It will only converge when , Radius of convergence . Interval of convergence is just that value at which it converges . The value of c is also the where the power series is centered at.

If the ratio test yields a value like an expression in the form

R is the radius of convergence

Note that if you get , the series diverges

The interval of convergence

Possibilities:

Note that the only time you apply the idea radius of convergence is if the Alembert’s ratio test yields something that has a value x in it

Example

This means that the radius of convergence is infinity and the series is convergent for any value of x

Examples to try:

When you solve, you’ll notice that the answer is , Now we know that the radius of convergence for this will be 0. So what value of x is it convergent.

You’ll notice that the only value of x for which the series is convergent is 0.

3. This is the equation,

For this question, interval of convergence is and the series converges at , Radius of convergence is 0,

4.

5.

For this, the radius of convergence is 1

Interval of convergence,

6.

R=1



Find the radius of convergence and interval of convergence of the following:

R=0

Apply the Alembert’s ratio test to the series:

Applying Lhopital rule,

Since it is equal to 1, it is therefore divergent or convergent

Which of the following converge and which diverge?