**INFINITE SERIES**

An infinite series is a series that has an infinite number of terms. When working with infinite series, we have to be careful about the steps we take.

E.g.

We can see that the above is a GP in which a = 1 and r = ½.

Recall that the sum of n terms in a GP is given as:

The sum of the first n terms is therefore given by:

If n is a large number, will be a very large number and will be a very small number.

In fact, as n approaches infinity.

The sum of all the terms in this infinite series is therefore given by

i.e.

This result means that we can make the sum of the series as near to the value 2 as we want by taking a sufficiently large number of terms

However this is not always the case for infinite series. For example, taking a look at an AP, things are very different

Consider the infinite series 1+3+5+7+…

This series is an AP in which a =1 and d = 2.

In this case, if we chose a very large number (or if) we will have: which is not a definite numerical value and of little or no use to us.

This always happens with an AP i.e. if we try to find the sum to infinity of an Arithmetic Progression; we invariably obtain as the result, depending on the actual series

Note the following:

We cannot evaluate the sum of an infinite number of terms of an AP because the result is always infinite

We can sometimes evaluate the sum of an infinite number of terms of a GP. Since for such series

And provided that

Then as ,

In that case,

CONVERGENT AND DIVERGENT SERIES

A convergent series is one in which the sum approaches a definite value as n approaches infinity i.e. as

The opposite i.e. a series that its sum does not tend to a definite value (sum approaches infinity) as n approaches infinity is called a DIVERGENT SERIES

Consider the GP

We know that for a GP,

In this case, a = 1 and we have:

As ,

It is convergent

If tends to a definite value as, the series is convergent

If does not tend to a definite value as, the series is divergent.

Here is another series. Let us investigate this one:

This is also a GP with a = 1 and r= 3

You will see as you keep adding the numbers

You will see you’ll eventually get a very large number ()

The series is divergent

TEST FOR CONVERGENCE

Cauchy test or divergence test:

A series cannot be convergent unless its terms ultimately tend to zero i.e.

What that means is that as , the value of each term also approaches 0

Note: is the term while is the sum of n terms in the sequence.

If

Then the series is divergent

However if the limit tends to 0, that doesn’t completely mean it is convergent. It means it is either convergent or divergent but if it’s not equal to zero, it has been confirmed that it is divergent

Therefore, the series

Is convergent because as tends to infinity, it is 0

If we look at the series

This series is divergent because as n approaches infinity, it is not 0 instead it is another value (infinity)

If we look at

You will see that the above series is divergent even though the limit approaches 0.

The comparison test:

A series of positive terms is convergent if its terms are less that the corresponding terms of a positive series which is known to be convergent and vice-versa

We can compare with series like

If the corresponding terms are less, it is convergent and vice versa

You can also compare with

If, the series converges

If , the series diverges

So if you want to compare, you can compare with p = 2

The Alembert’s ratio test for positive terms

Let be a series of positive terms.

First find an expression for

Next, find an expression for

Next find the ratio of to i.e.

Next find the limit as n tends to infinity

If the limit is less than 1, the series converges

If is greater 1, it is divergent

If it is equal to 1, then it is either divergent or convergent and it is not confirmed

Examples

Test the series

First look for an equation of

Now an equation for,

Now find the ratio:

Now find the limit as n approaches infinity

Since it is less than 1, the series converges

Since , it is convergent

Check if the series with the Alembert’s ratio test

QUESTIONS

Determine if the given sequences converge or diverge, if it converges, then find their limits

Applying Lhopital rule,

Applying L’hopital’s rule,

Recall that

Therefore, the sequence is divergent and the limit doesn’t exist

Since the limit is equal to 1, it is either convergent or divergent and therefore needs another test to confirm

To determine whether it is divergent or convergent

We can use the ratio test

Since is always positive no matter the value of n, and we can bring out that expression from the absolute brackets

Then we find the limit

There’s no value of n in the expression so we get

Remember that for the series to be convergent, the absolute value has to be less than 1

That means the series is convergent in the interval,

Notice that in the above, we had two variables x and n. The series is in terms of n but is also a function of x. So we got . That means the whole series is convergent in the open interval

Possibilities:

The series converges when x = a, Here, the radius of convergence is zero as only one value is allowed for convergence

The series converges for all values of x, here the radius of convergence is infinity as the series converges for any value of x

For some possible value R, when, the series is convergent and when, it is divergent.

That number R is called the radius of convergence

Note that the only time you apply the idea radius of convergence is if the Alembert’s ratio test yields something that has a value x in it

Example

This means that the radius of convergence is infinity and the series is convergent for any value of x

KNOWING WHICH TEST TO USE

Geometric Series:

Integral test:

If you have a series, you can looking the corresponding integral where the function is just the replacement of n with x and if the integral converges then the series also converges

Alternating series test

Divergence test

Comparing test

Limit comparison test

Root test

Ratio test:

Find the radius of convergence and interval of convergence of the following:

R=0

Apply the Alembert’s ratio test to the series:

Applying Lhopital rule,

Since it is equal to 1, it is therefore divergent or convergent



Which of the following converge and which diverge?