**SEQUENCES, SERIES AND POWER SERIES**

A sequence is an array or series of numbers that is governed by certain rules. These numbers are usually represented as with a definite number order of arrangement. The set of numbers corresponds with natural numbers and are formed according to a definite rule. Each value in the sequence is called a term

**MONOTONIC SEQUENCES**

A monotonic sequence is a sequence that is always moving in one direction i.e. always increasing or always decreasing.

For always increasing

The slope of this graph is

For always decreasing

The slope of this graph is

If the graph is increasing and decreasing then it is not monotonic

e.g.

Is the equation a monotonic increasing or decreasing sequence

Because 0 is less than 6 we can say that is a monotonic sequence

The equation . Is it a monotonic sequence? And if it is, is it increasing or decreasing

**BOUNDED SEQUENCES**

Let’s say the sequence , we say that the sequence is bounded above i.e. it has an upper bound (or ceiling).

For example if we draw the graph of the series, if we see that the graph never goes above a certain value (M), we say that the series is bounded above and that it has an upper bound at M

However, if , we say the sequence has a floor or lower bound i.e. it is bounded below

For this, , the sequence is Bounded i.e. it is bounded above and bounded below

If the sequence is bounded (i.e. above and below) and it is monotonic (i.e. always increasing or always decreasing) then the sequence converges

Taking a look at this equation

Since the sign of the first derivative is negative, the sequence is decreasing

If you try to solve using , and you get something like

if the condition is true, then it is monotonic decreasing

The equation is bounded above at 1 (M)

It is also bounded below at 0 (M)

Also,

Next question: Determine if the sequence is monotonic, bounded and whether it converges

Answer: It is bounded below at -1 and above at 1. It is not monotonic.

Since it is not monotonic even though it is bounded, because it is not monotonic, it is not convergent. It is divergent

Consider the sequence

You can plug in values like 10 , 20

Therefore it converges

To prove it,

Is this monotonic? Well yes it is. Even though it increases at first, the remaining part of the sequence reduces

If you solve using , you get, . This means that the sequence is decreasing for

The sequence is bounded above at 4.5 and bounded below at 0

And because it is bounded and monotonic, the sequence is convergent

QUESTIONS

Prove that the sequence with nth term is

Monotonic increasing, has a limit, is bounded above, is bounded below and is bounded

Prove that the sequence with nth term is monotonic increasing, bounded and thus has a limit

For monotonic increasing,

Use an appropriate test for monotonicity to determine if the sequences below increases or decreases

A sequence is said to have a limit if it is monotonic and it is bounded

For a monotonic increasing,

At

Therefore, it is monotonic decreasing

Test for limit,

For monotonic increasing,

Substituting

Therefore it is monotonic increasing

A dog chasing a rat. At start, the dog (at A) is 20m behind the rat (at B). They both run at steady speeds, but the dog can run twice as fast as the rat. The rat runs in a straight line.

When the dog reaches B, the rat at C, when the dog reaches C, the rat is at D and so on

Dog initially at A

Every time the dog and rat run, they use the same time

So when rat runs from B to C,

Dog runs from A to B

When dog runs from B to C, rat runs from C to D

When rat runs from D to E, dog runs from C to D

Show that the distances AB, BC, CD, … form a GP and find its common ration

Hence find the total distance the dog must run before it catches the rat

Total distance = 20m + sum of the whole distance

Total distance covered =

The first and second terms of a gp are p and 2p respectively, where p is a positive constant. The sum of the first n terms is greater than 1000p. Show that

In another case, p and 2p are the first and second terms of an AP respectively. The nth term is 336 and the sum of the first n terms is 7224. Write down two equations in n and p and hence find n and p

But,

The first three terms of an AP are 4, x and y respectively. The first three terms of the geometric sequence are x, y and 18 respectively. It is given that x and y are positive.

Find the value of x and y

Find the fourth term of each progression

In an arithmetic progression, the first term is a and the common difference is 3. The nth term is 94 and the sum of the first n terms is 1420; find n and a

A person wants to buy a pension which will provide her with an income of 500,000 at the end of the next n years. Show that with a steady interest of 5% per annum, the pension should cost her

Find a simple formula for calculating their sum and find its value when n = 10, 20, 30, 40, 50

As part of a fund-raising campaign, I have been given some books of raffle tickets to sell. Each book has the same number of tickets and all the tickets I have been given are numbered in sequence. The number of the ticket on the front of the 5th book in 205 and that on the front of the 19th book is 373

By writing the number of the ticket on the front of the first book as a and the number of the tickets in each book as d, write down two equations involving a and d

From these two equations find how many tickets are in each book and the number on the front of the first book I have been given

The last ticket I have been given is numbered 492. How many books have I been given?

A tank is filled with 20litres of water. Half the water is removed and replaced with anti-freeze and thoroughly mixed. Half this mixture is then removed and replaced with anti-freeze. This process continues

Find the first five terms in the sequence of amounts of water in the tank at each stage

Find the first five terms in the sequence of amounts of anti-freeze in the tank at each stage

Is either of these sequences geometric? Explain

A company producing salt from sea water changed to a new process. The amount of salt obtained each week increased by 2% of the amount obtained in the preceding week. It is given that in the first week after the change the company obtained 8000kg of salt

Find the amount of salt obtained in the 12th week after the change

Find the total amount of salt obtained in the first 12 weeks after the change

The common ratio of a GP is 0.99. Express the sum of the first 100 terms as a percentage of the sum to infinity, giving your answer correct to 2sf

Each year, the value of a certain rare stamp increases by 5% of its value at the beginning of the year. A collector bought the stamp for $10000 at the beginning of 2005. Find its value at the beginning of 2015 correct to the nearest $100

The sum of the first n terms of an arithmetic progression is. Find the 1st term and the common difference of the progression

An organ has 8 tubes. The lengths of these tubes are a geometrical progression and the length of the shortest tube is exactly half that of the longest which is 66cm long. What is the total length of all the tubes in the organ to the nearest cm?

Determine how it will take to pey an interest free loan $6625 by montly payments initially of $10 and increasing by $5 each month. How much is th final payment?

If the loan is $10000, find the smallest number of months that would be needed to repay it.

The third term of an AP is 34 and the 17th term is -8. Find the sum of the first 20 terms

For the series , find the 6th term and the sum of the first 10 terms