**POWER SERIES**

What are power series?

In mathematics, a power series is the form

or .

Here, represents the coefficient of the nth term and c is a constant where the function is centered at

The a’s are called coefficients

The sum of the series can be represented in a function

This results in something looking like a polynomial with infinite number of terms

The domain of the function

Domain = set of x for which f(x) converges

Examples

All the coefficients are equal to 1

The series f(x) will be convergent when -1<x<1

**EXAMPLES OF POWER SERIES**

Taylor Series

Maclaurin Series

Laurent Series

**SOME REASONS WHY SOME SERIES ARE NOT POWER SERIES**

Fractional powers cannot be referred to as power series

If its complex differentiability is not an open set

If the function doesn’t change

**TAYLOR SERIES**

Derive from first principle , the power series of

Derivation and final answer

We are going to use the idea the power series to create a method for representing other functions i.e.

So if we could represent f(x) with a power series centered at , it would look like

We are going to come up with a formula for the coefficients for

Our goal is to define the coefficient in terms of the function we want to represent

So it’s like coming up with a formula for the coefficients of the series based on the original function

First, we substitute the value of ‘c’ into the equation

Next we find the derivatives

Here,

This, is called the Taylor series of the function f(x) centred at

When the series is centered at 0 i.e. , it has a special name called the **Maclaurin series**

We can also look at a portion of the series called the nth – degree Taylor Polymonial

For me, mathematics is a collection of examples; a theorem is a statement about a collection of examples and the purpose of proving theorems is to classify and explain the examples. John B Conway

Taylor series is taking non polynomial functions and finding polynomials that approximate functions near a value of x.

QUESTIONS

1. Prove that And that the series is valid for all values of x

Solution:

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Recall that for a taylor series, we express in the form

For this case, the value of c is equal to 0.

First term, , , ,

Similarly for n = 1

1. Deduce the power series of
2. Show that for if x is small, . Approximately
3. Find the taylor series for the function , centered at . Answer:
4. Find the taylor series of , centered at . Answer:
5. Find the maclaurin series for the function . Answer:
6. Find the maclaurin series for using the maclaurin series for . Hint: Differentiate the series of the i.e. either you differentiate the whole equation or you differentiate the sum. Answer:
7. Find the maclaurin series for
8. Find the maclaurin series for
9. Find the maclaurin series for
10. Find the maclaurin series for

EXPRESSING FUNCTIONS IN POWER SERIES

Express as a power series

a = 1, r = -x

Express as a power series

Answer:

3.

Answer:

4. for when centered at 0 and when centered at 1

5. centered at c=3