Polytope

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1 Introduction

Welcome to the Polytope Discord!

1.1 What is a polytope?

A **polytope** is a general name for **polygons** (2D), **polyhedra** (3D), **polychora** (4D), and so on for any dimension. As with many of the terms that the Polytope Discord uses, the word "polytope" can have a few definitions which are not completely equivalent. All of the commonly-used ones, however, agree on the following:

- A polytope in n dimensions (known hereafter as an n-polytope) is made of **facets** which are $(n-1)$-polytopes.
  Polychora (4-polytopes) are made of polyhedra (3-polytopes),
  which are made of polygons (2-polytopes),
  which are made of line segments (1-polytopes),
  which are made of points (0-polytopes)!

2 Regular polytopes

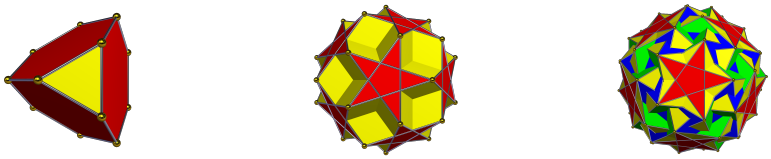
There are multiple definitions for when a polytope is **regular**, but they all require every element (vertices, edges, faces, etc.) to "look the same."

3 Uniform polytopes

Intuitively, a polytope is **uniform** when all of its facets are regular and all of its vertices "look the same." To see what we mean, let's look at a few examples.

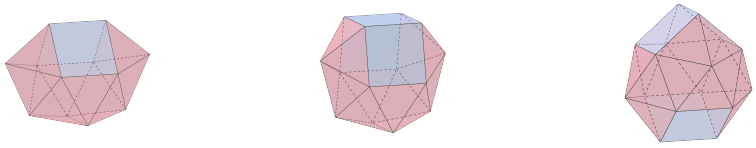
4 CRF polytopes

A polytope is called **convex regular-faced**, or **CRF** for short, when it is convex (without dents, holes or self-intersections) and all of its faces are regular. Let's look at a few examples.



(a) Truncated tetrahedron (b) Dodecadodecahedron (c) Snub icosidodecahedron

Figure 1: Three examples of uniform polytopes.



(a) Sphenomegacorona (b) Hebesphenomegacorona (c) Disphenocingulum

Figure 2: Test images!