

Polytope

URL, Mathematician,

February 2021

1 Introduction

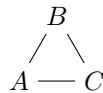
Welcome to the [Polytope Discord](#)!

2 What is a polytope?

A **polytope** is a general name for **polygons** (2D), **polyhedra** (3D), **polychora** (4D), and so on for any dimension. As with many terms used on the Polytope Discord, the word “polytope” can have a few definitions which are not completely equivalent. All of the commonly-used ones, however, agree on the following:

- A polytope in n dimensions (known hereafter as an n -polytope) is made of **facets** which are $(n - 1)$ -polytopes.
 - Polychora (4-polytopes) have facets which are polyhedra (3-polytopes),
 - whose facets are polygons (2-polytopes),
 - whose facets are line segments (1-polytopes),
 - whose facets are points (0-polytopes)!
- More stuff here, although the next section might break up the list, so it might be good not to format the whole def in terms of a list so that we can fit in more explanations.

For example, consider the following triangle ABC.



It is a polygon which contains three line segments, or **edges**, as they are known when mentioned as part of a larger polytope. It also contains three points, or **vertices**, or even **verts** for short. (In the polytope world, abbreviations are everywhere!) More specifically, its facets are the edges \overline{AB} , \overline{AC} , and \overline{BC} , and their facets are the vertices A, B, and C.

We keep track of what has what as a facet using a **Hasse diagram**.

3 Regular polytopes

There are multiple definitions for when a polytope is **regular**, but they all require every element (vertices, edges, faces, etc.) to “look the same.”

4 Uniform polytopes

Intuitively, a polytope is **uniform** when all of its facets are regular and all of its vertices “look the same.” To see what we mean, let’s look at a few examples.

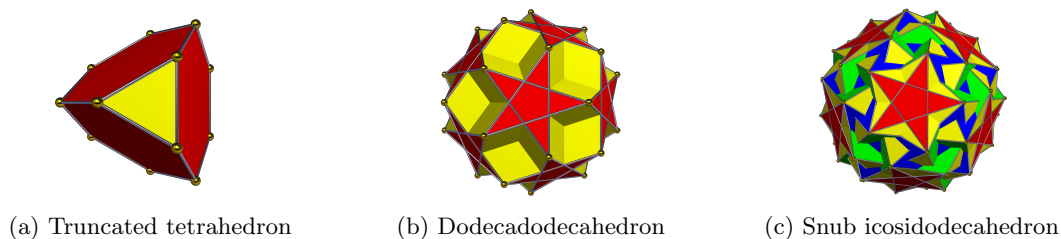


Figure 1: Three examples of uniform polytopes.

5 CRF polytopes

A polytope is called **convex regular-faced**, or **CRF** for short, when it is convex (without dents, holes or self-intersections) and all of its faces are regular. Let’s look at a few examples.

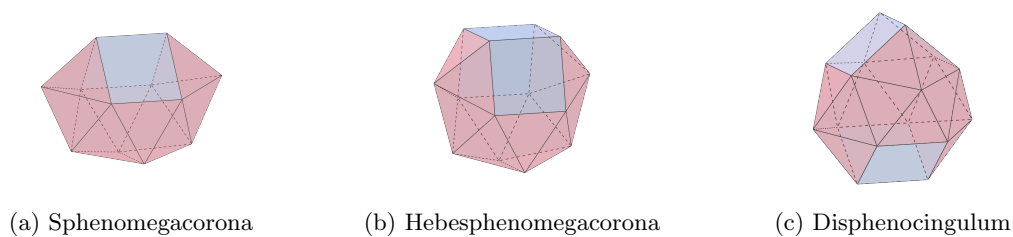


Figure 2: Test images!