

Chapter 23: Catalan Number Algorithm

Section 23.1: Catalan Number Algorithm Basic Information

Catalan numbers algorithm is Dynamic Programming algorithm.

In combinatorial mathematics, the [Catalan numbers](#) form a sequence of natural numbers that occur in various counting problems, often involving recursively-defined objects. The Catalan numbers on nonnegative integers n are a set of numbers that arise in tree enumeration problems of the type, 'In how many ways can a regular n -gon be divided into $n-2$ triangles if different orientations are counted separately?'

Application of Catalan Number Algorithm:

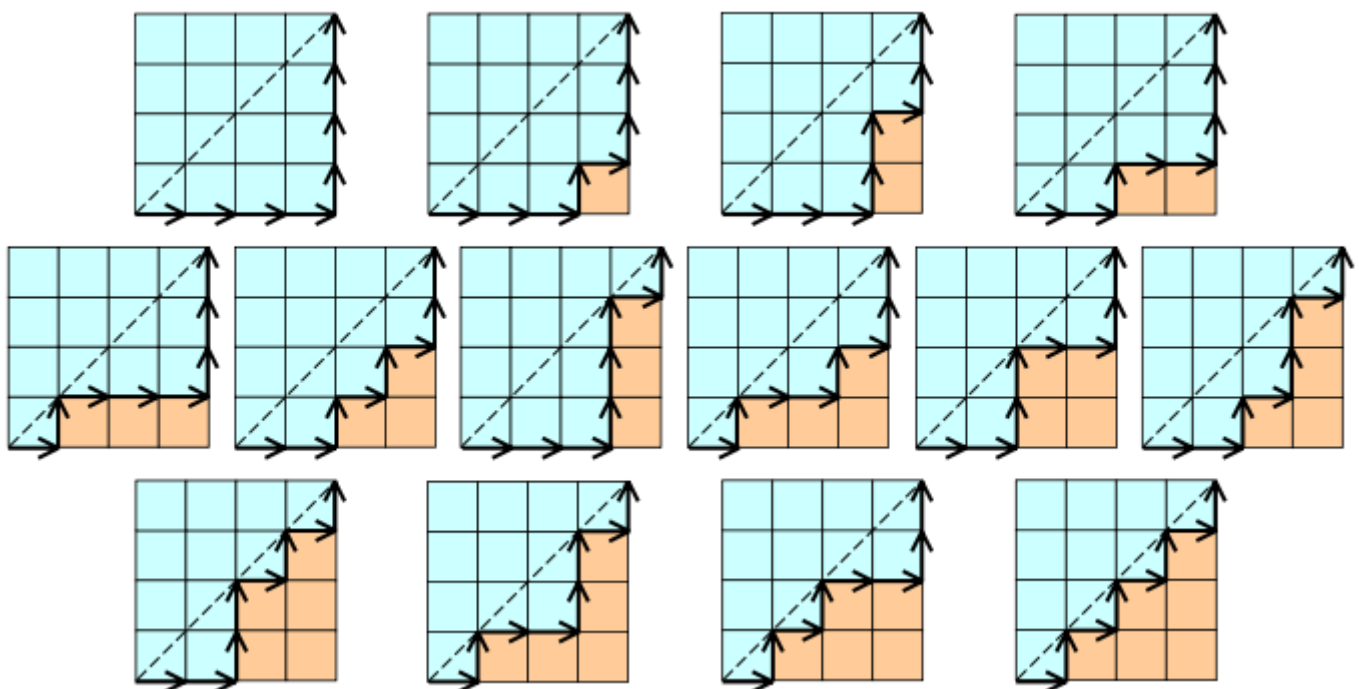
1. The number of ways to stack coins on a bottom row that consists of n consecutive coins in a plane, such that no coins are allowed to be put on the two sides of the bottom coins and every additional coin must be above two other coins, is the n th Catalan number.
2. The number of ways to group a string of n pairs of parentheses, such that each open parenthesis has a matching closed parenthesis, is the n th Catalan number.
3. The number of ways to cut an $n+2$ -sided convex polygon in a plane into triangles by connecting vertices with straight, non-intersecting lines is the n th Catalan number. This is the application in which Euler was interested.

Using zero-based numbering, the n th Catalan number is given directly in terms of binomial coefficients by the following equation.

$$C_n = \frac{1}{n+1} \binom{2n}{n} = \frac{(2n)!}{(n+1)!n!} = \prod_{k=2}^n \frac{n+k}{k} \quad \text{for } n \geq 0.$$

Example of Catalan Number:

Here value of $n = 4$. (Best Example - From Wikipedia)



Auxiliary Space: $O(n)$

Time Complexity: $O(n^2)$