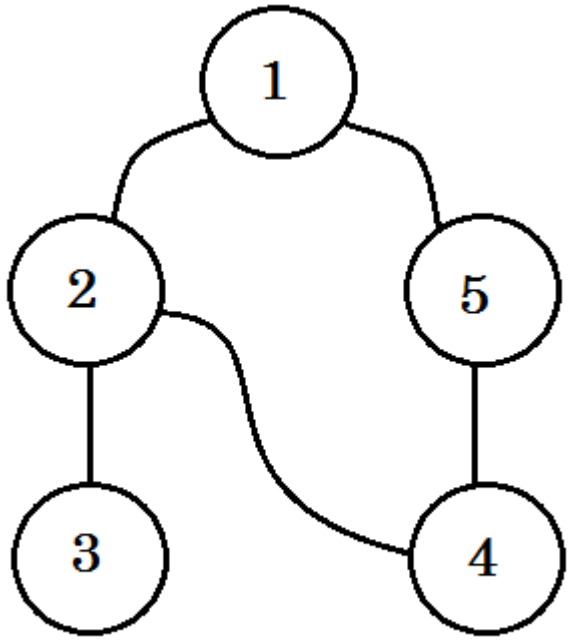
Chapter 42: Depth First Search

Section 42.1: Introduction To Depth-First Search

Depth-first search is an algorithm for traversing or searching tree or graph data structures. One starts at the root and explores as far as possible along each branch before backtracking. A version of depth-first search was investigated in the 19th century French mathematician Charles Pierre Trémaux as a strategy for solving mazes.

Depth-first search is a systematic way to find all the vertices reachable from a source vertex. Like breadth-first search, DFS traverse a connected component of a given graph and defines a spanning tree. The basic idea of depth-first search is methodically exploring every edge. We start over from a different vertices as necessary. As soon as we discover a vertex, DFS starts exploring from it (unlike BFS, which puts a vertex on a queue so that it explores from it later).

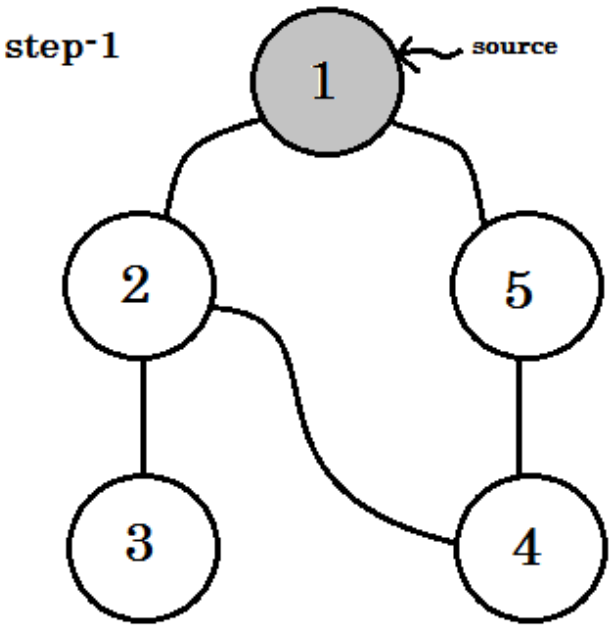
Let's look at an example. We'll traverse this graph:

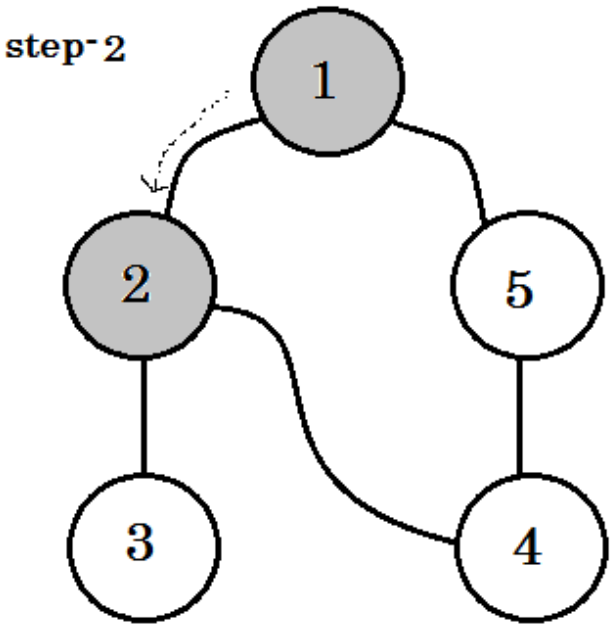


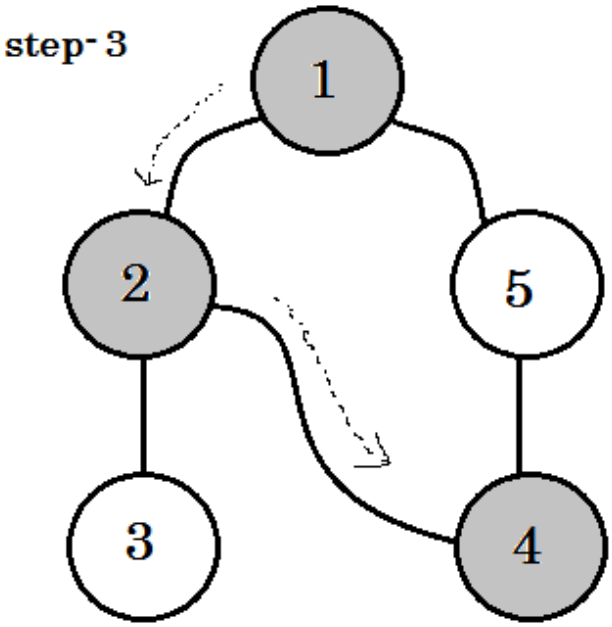
We'll traverse the graph following these rules:

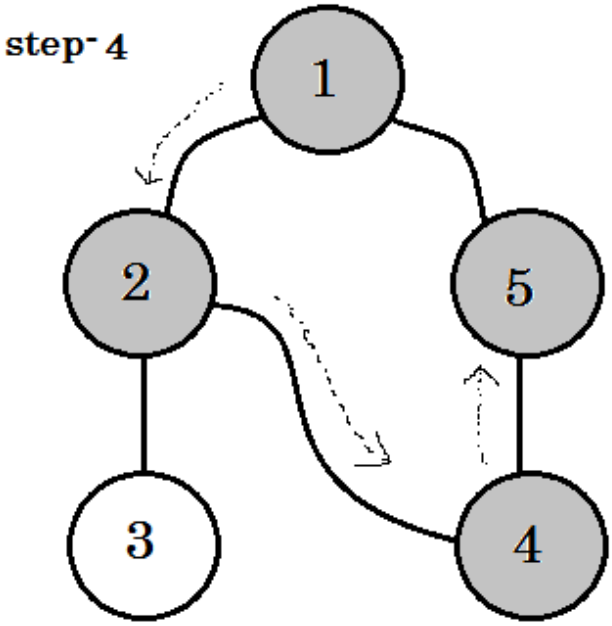
- We'll start from the source.
- No node will be visited twice.
- The nodes we didn't visit yet, will be colored white.
- The node we visited, but didn't visit all of its child nodes, will be colored grey.
- Completely traversed nodes will be colored black.

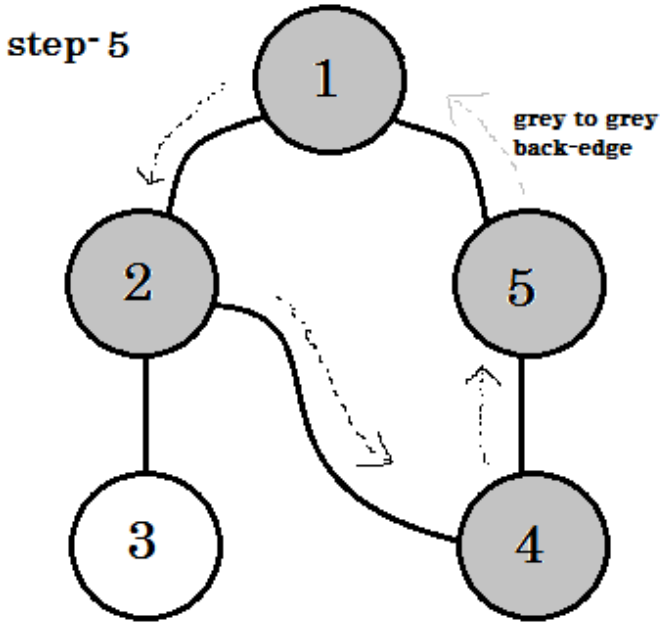
Let's look at it step by step:

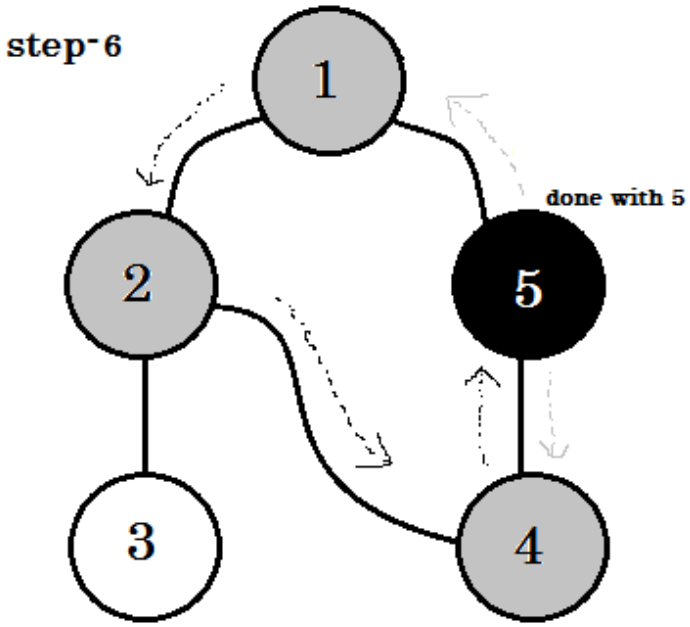

step-1
source
1
2
5
3
4

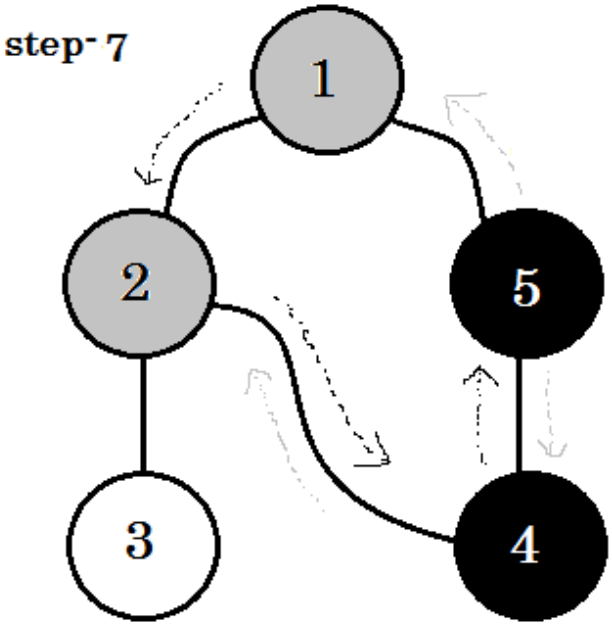

step-2
1
2
5
3
4

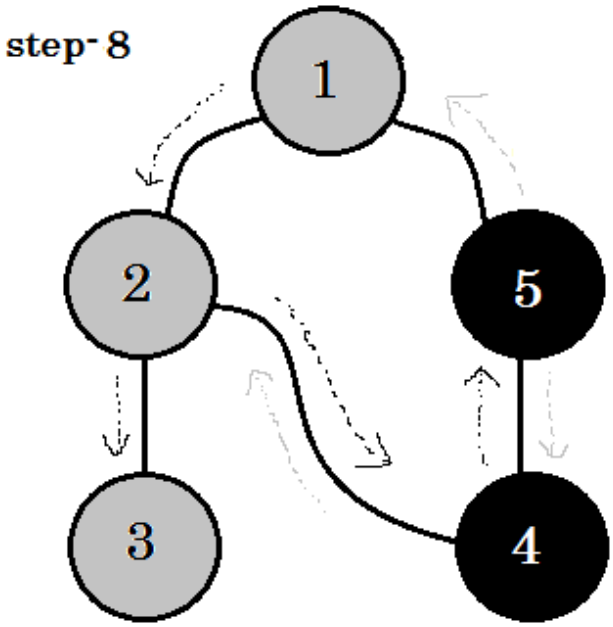

step-3
1
2
5
3
4

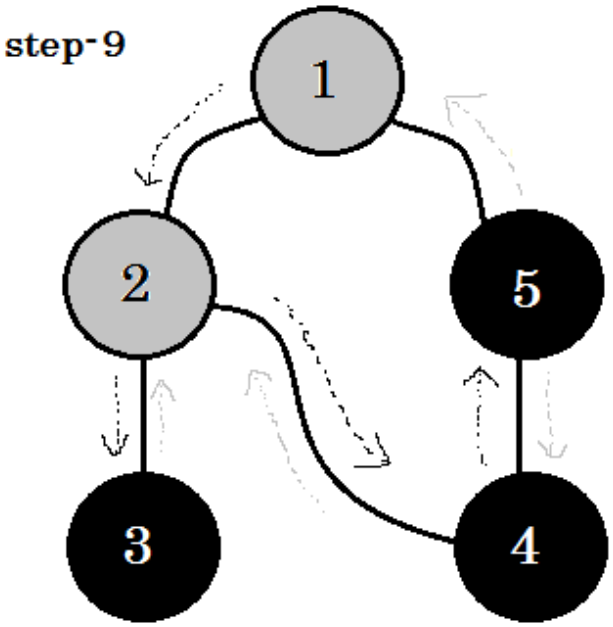

step-4
1
2
5
3
4

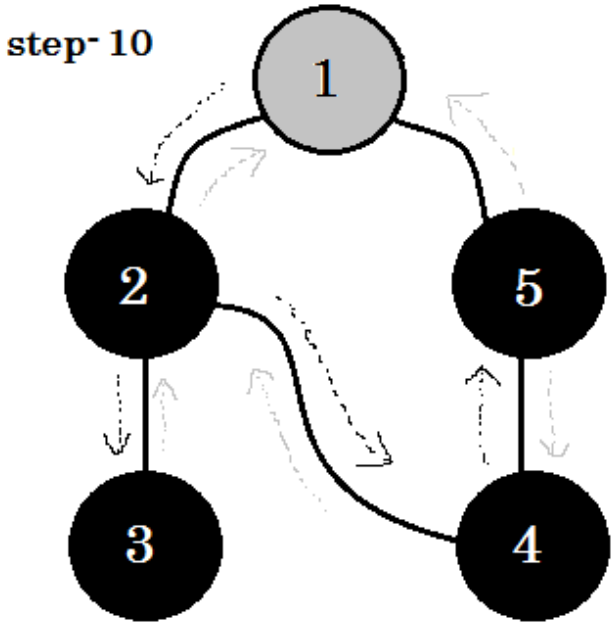

step- 5
1
grey to grey
back-edge
2
5
3
4

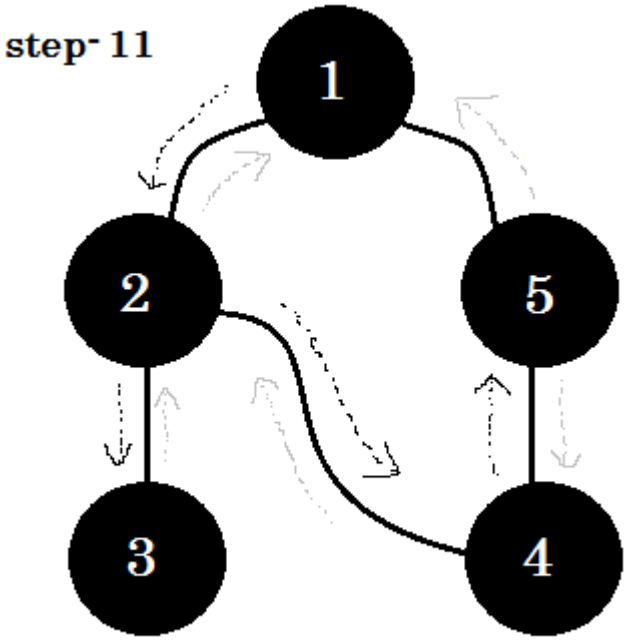

step- 6
1
done with 5
2
5
3
4


step- 7
1
2
5
3
4


step- 8
1
2
5
3
4







We can see one important keyword. That is **backedge**. You can see. **5-1** is called backedge. This is because, we're not yet done with **node-1**, so going from another node to **node-1** means there's a cycle in the graph. In DFS, if we can go from one gray node to another, we can be certain that the graph has a cycle. This is one of the ways of detecting cycle in a graph. Depending on **source** node and the order of the nodes we visit, we can find out any edge in a cycle as **backedge**. For example: if we went to **5** from **1** first, we'd have found out **2-1** as backedge.

The edge that we take to go from gray node to white node are called **tree edge**. If we only keep the **tree edge**'s and remove others, we'll get **DFS tree**.

In undirected graph, if we can visit a already visited node, that must be a **backedge**. But for directed graphs, we must check the colors. *If and only if we can go from one gray node to another gray node, that is called a backedge*.

In DFS, we can also keep timestamps for each node, which can be used in many ways (e.g.: Topological Sort).

1. When a node **v** is changed from white to gray the time is recorded in **d[v]**.

2. When a node **v** is changed from gray to black the time is recorded in **f[v]**.

Here **d[]** means *discovery time* and **f[]** means *finishing time*. Our pesudo-code will look like:

```
Procedure DFS(G):
for each node u in V[G]
    color[u] := white
    parent[u] := NULL
end for
time := 0
for each node u in V[G]
    if color[u] == white
        DFS-Visit(u)
    end if
end for

Procedure DFS-Visit(u):
color[u] := gray
time := time + 1
d[u] := time
for each node v adjacent to u
    if color[v] == white
        parent[v] := u
        DFS-Visit(v)
    end if
end for
color[u] := black
time := time + 1
f[u] := time
```

Complexity:

Each nodes and edges are visited once. So the complexity of DFS is **O(V+E)**, where **V** denotes the number of nodes and **E** denotes the number of edges.

Applications of Depth First Search:

- Finding all pair shortest path in an undirected graph.
- Detecting cycle in a graph.
- Path finding.
- Topological Sort.
- Testing if a graph is bipartite.
- Finding Strongly Connected Component.
- Solving puzzles with one solution.