Solutions

Question 1 (Formulation):

Let x_1 indicates the number of units of P1 to be produced,

And x_2 indicates the number of units of P2 to be produced

- We want to maximize the stock by the end of the week.
- The stock contains both P1 and P2.

Objective function:

Maximize $F(x_1, x_2) = (x_1 + 30 - 70) + (x_2 + 65 - 100)$

(Stock at the beginning of the week) (To be dispatched at the end of the week)

Maximize $F(x_1, x_2) = x_1 + x_2 - 40 - 35$

$= x_1 + x_2 - 75$

The constraints imposed are related to the capacity of machine A and B as well as the minimum of units of P1 and P2 to be produced so that we satisfy at least the order which is 70 units of P1 and 100 units of P2.

Therefore:

For machine A: $20x_1 + 30x_2 <= 45$ hours

⇨ $20x_1 + 30x_2 <= 2700$ (minutes)

For machine B: $40x_1 + 50x_2 <= 65$ hours

⇨ $40x_1 + 50x_2 <= 3900$

The minimum units of P1 is 70 - 30 = 40 units

(Order) (Already in stock)

⇨ $x_1 >= 40$

The minimum units of P2 is 100 – 65 = 35 units

⇨ $x_2 >= 35$

The problem is then can be formulated as

Maximize $F(x_1, x_2) = x_1 + x_2 - 75$

Such that,

- $20x_1 + 30x_2 <= 2700$ .... (C1)
- $40x_1 + 50x_2 <= 3900$ .... (C2)
- $x_1 >= 40$ ....(C3)
- $x_2 >= 35$ ....(C4)

Question 2:

Let's find the solution to the given mathematical program.

Let's use a graphical method to show how the space of solutions will reduce as we process each of the constraints.

The steps are as follows:

1. Transform the constraints into equations and draw the corresponding lines by using instantiation of the variables x_1 and x_2.
2. Find the intersection point of all lines.
3. Evaluate the objective function on each of the points found.
4. Select the point that evaluate the objective function to the maximum.

Step 1:

Constraint 1 (C1):

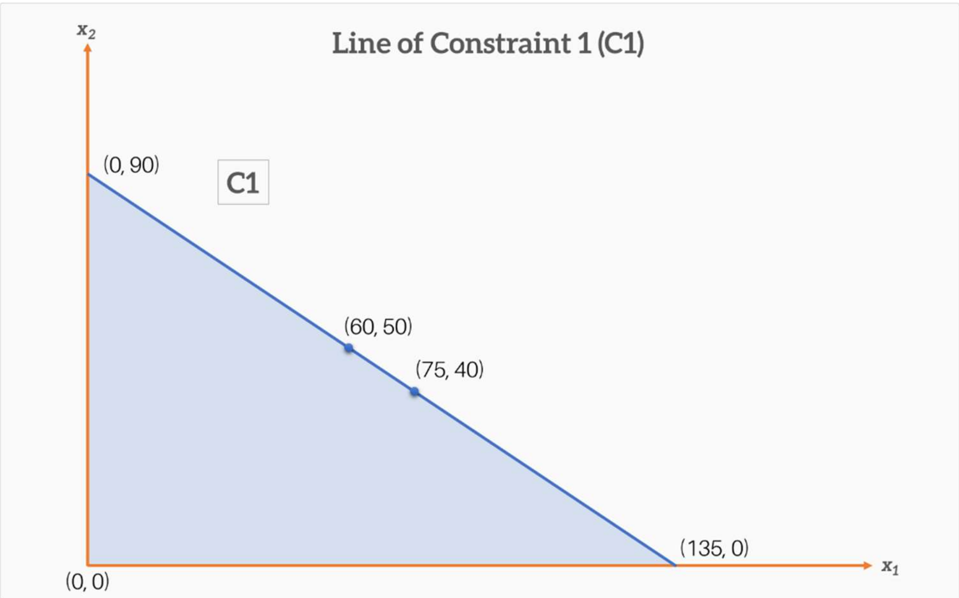
$20x_1 + 30x_2 = 2700$
If $x_1 = 60$, $x_2 = ?$
$20 \times 60 + 30x_2 = 2700$
⇨ $30x_2 = 2700 - 1200$
⇨ $30x_2 = 1500$
⇨ $x_2 = 50$

We need a second point so that we can draw a line connection the line of the first constraint C1.

Let $x_2 = 40$, $x_1 = ?$

$20x_1 + 30 \times 40 = 2700$

⇨ $20x_1 = 2700 - 1200$
⇨ $20x_1 = 1500$
⇨ $x_1 = 75$



To know where the space of feasible solution, evaluate the constraint by picking any point and check if the constraint is satisfied.

Let's choose (0, 0) : 20 * 0 + 30 * 30 <= 2700

0 <= 2700

Therefore, the space of feasible solutions should be below the line.

We do the same thing with constraint 2. We should any value for x_1 and calculate x_2 or vice-versa. We need 2 points to be able to draw the line for constraint 2.

Constraint 2 (C2):

$40x_1 + 50x_2 = 3900$

Let $x_1 = 60$:

$40 * 60 + 50x_2 = 3900$

⇨ $x_2 = 30$

Thus, we get the first point (60,30)

Let $x_2 = 50$:

$40\, x_1 + 50 * 50 = 3900$

⇨ $x_1 = 35$

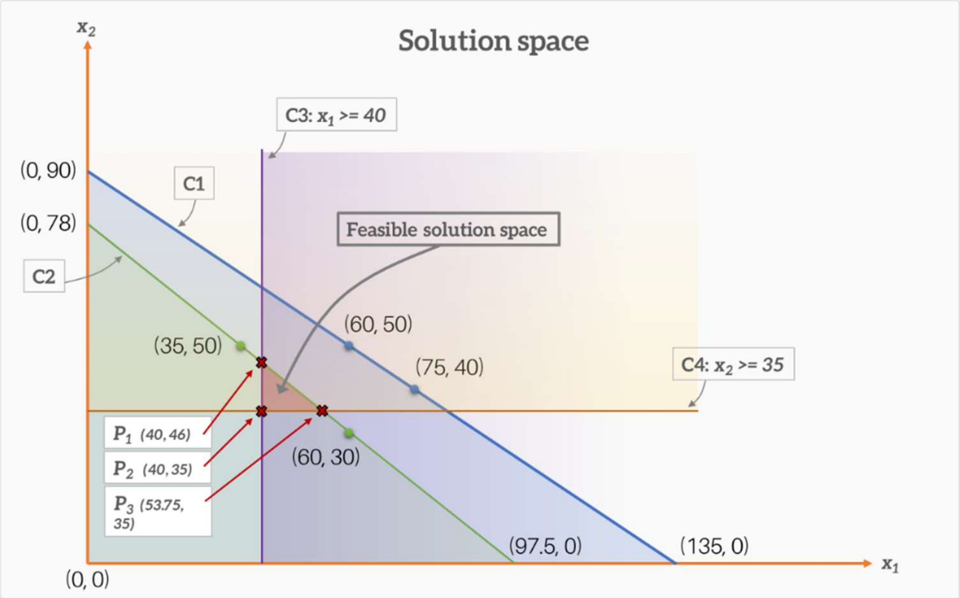
Thus, we get the second point (35, 50) [See the figure below]

Constraint 3 (C3):

$x_1 >= 40$

Constraint 4 (C4):

$x_2 >= 35$



Solution must be one of the 3 points that form the intersection between the lines representing the constraints.

p_1 is the intersection point between C2 and C3

p_2 is the intersection point between C3 and C4

p_3 is the intersection point between C2 and C4

Calculating p_1:

C2: $40x_1 + 50x_2 = 3900$

C3: $x_1 = 40$

- ⇨ $40 \times 40 + 50x_2 = 3900$
- ⇨ $x_2 = 46$

$p_1 = (40, 46)$

Calculating p_2:

C3: $x_1 = 40$

C4: $x_2 = 35$

$p_2 = (40, 35)$

Calculating p_3:

C2: $40x_1 + 50x_2 = 3900$

C4: $x_2 = 35$

- ⇨ $40x_1 + 50 \times 35 = 3900$
- ⇨ $x_1 = 53.75$

$p_3 = (53.75, 35)$

Now let's find the point that yields the maximum of the objective function

$F(x_1, x_2) = x_1 + x_2 - 75$

For $p_1 = (40, 46)$, we get:

$F(40, 46) = 40 + 46 - 75 = 11$

For $p_2 = (40, 35)$, we get:

$F(40, 35) = 40 + 35 - 75 = 0$

For $p_3 = (53.75, 35)$, we get:

$F(53.75, 35) = 53.75 + 35 - 75 = 13.75$

Thus, the solution is (53.75, 35)

Answer: The company needs to produce 53.75 units of P1 and 35 units of P2 to obtain the maximum of units in the stock at the end of the week.