Vessel dynamic model:

$$\dot{x} = v_x \cos\theta - v_y \sin\theta$$

$$\dot{y} = v_x \sin\theta + v_y \cos\theta$$

$$\dot{z} = v_z$$

$$\dot{\theta} = \omega$$

$$\dot{v}_x = a\_v\_x\, v_x + a\_v\_x\_2\, v_x|v_x| + a\_v\_yw\, v_y\omega + b\_v\_x\, u_{v_x} + d_{v_x}$$

$$\dot{v}_y = a\_v\_y\, v_y + a\_v\_y\_2\, v_y|v_y| + a\_v\_wx\, v_x\omega + b\_v\_y\, u_{v_y} + d_{v_y}$$

$$\dot{v}_z = a\_v\_z\, v_z + a\_v\_z\_2\, v_z|v_z| + b\_v\_z\, u_{v_z} + d_{v_z}$$

$$\dot{\omega} = a\_v\_w\, \omega + a\_v\_w\_2\, \omega|\omega| + a\_v\_xy\, v_x v_y + b\_v\_w\, u_\omega + d_\omega$$

where

$$a\_v\_x = -0.5265,\ a\_v\_y = -0.5357,\ a\_v\_z = -1.0653,\ a\_v\_w = -4.2579$$

$$a\_v\_x\_2 = -1.1984,\ a\_v\_y\_2 = -5.1626,\ a\_v\_z\_2 = -1.7579e-05,\ a\_v\_w\_2 = -2.4791e-08$$

$$a\_v\_yw = -0.5350,\ a\_v\_wx = -1.2633,\ a\_v\_xy = -0.9808$$

$$b\_v\_x = 1.2810,\ b\_v\_y = 0.9512,\ b\_v\_z = 0.7820,\ b\_v\_w = 2.6822$$

(x, y, z) and θ are the robot's position and orientation in the (fixed) global coordinate system. Whereas (v_x, v_y, v_z) and ω are the robot's linear velocity and rotation velocity in the body coordinate system. $(d_{v_x}, d_{v_y}, d_{v_z}, d_\omega)$ are disturbance/model uncertainty.

We model $(d_{v_x}, d_{v_y}, d_{v_z}, d_\omega)$ as Gaussian random variables. Using experiment data, the mean and covariances of $(d_{v_x}, d_{v_y}, d_{v_z}, d_\omega)$ are given as

$$\text{mean: } \begin{pmatrix} -0.01461447 & -0.02102184 & -0.00115958 & 0.05391866 \end{pmatrix}$$

$$\text{cov: } \begin{pmatrix} 2.89596342e-02 & 5.90296868e-03 & -4.22672521e-05 & -6.38837738e-03 \\ 5.90296868e-03 & 2.05937494e-02 & 8.59805304e-05 & 2.92258483e-03 \\ -4.22672521e-05 & 8.59805304e-05 & 2.44296056e-03 & 1.64117342e-03 \\ -6.38837738e-03 & 2.92258483e-03 & 1.64117342e-03 & 3.71338116e-01 \end{pmatrix}$$