AnalyticalCaseStudy

June 30, 2022

1 General procedure:

1. Exact mean values with different rates are found using: $\langle T\left(r\right)\rangle = \frac{1-\tilde{T}(r)}{r\tilde{T}(r)}$
2. The function $\langle T\left(r\right)\rangle$ is evaluated using different fits to the selected points.
3. The evaluated value at a selected reference point is compared to the exact value at this point.
4. The same process is repeated, using sampled mean values instead of exact values as selected points for the fit.

1.1 Frechet distribution

The selected distribution: $Pr(t) = t^{-2}exp\left(-t^{-1}\right)$.

The mean and standard deviation diverge; we will compare to $\langle T\left(0.001\right)\rangle = 6.8$.

The Laplace transform may be found using the modified Bessel function: $\tilde{T}(r) = 2\sqrt{r}K_1\left(2\sqrt{r}\right)$ (https://aip.scitation.org/doi/10.1063/1.4893338).

1.2 Gamma distribution

The selected distribution: $Pr(t) = \frac{1}{\Gamma(k)\theta^k}t^{k-1}exp\left(-\frac{t}{\theta}\right)$.

$\mu = k\theta$, $\sigma = \sqrt{k}\theta$, $\tilde{T}(r) = (1+\theta r)^{-k}$

We chose $k = 0.25$, $\theta = 25$, which leads to $CV = \frac{\sigma}{\mu} = \frac{1}{\sqrt{k}} = 2$

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Text(0.75, 9, 'Gamma')
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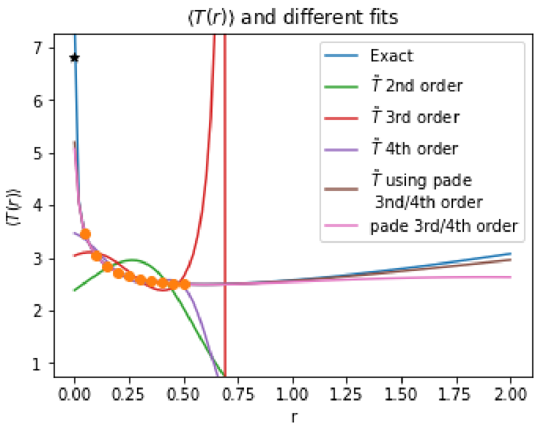
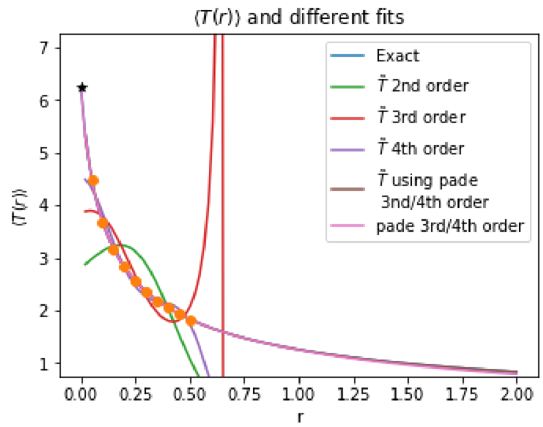
Frechet



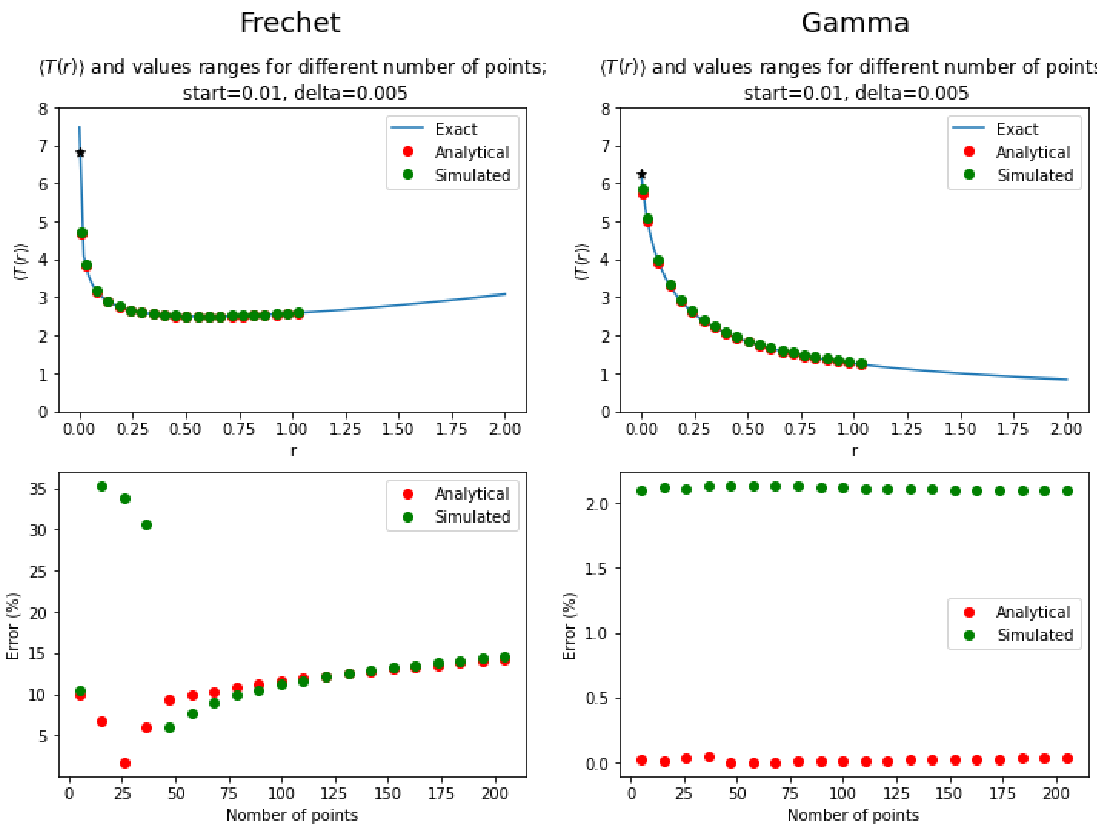
Gamma



Tried different fitting functions using the orange exact points on the function $\langle T\left(r\right)\rangle$:

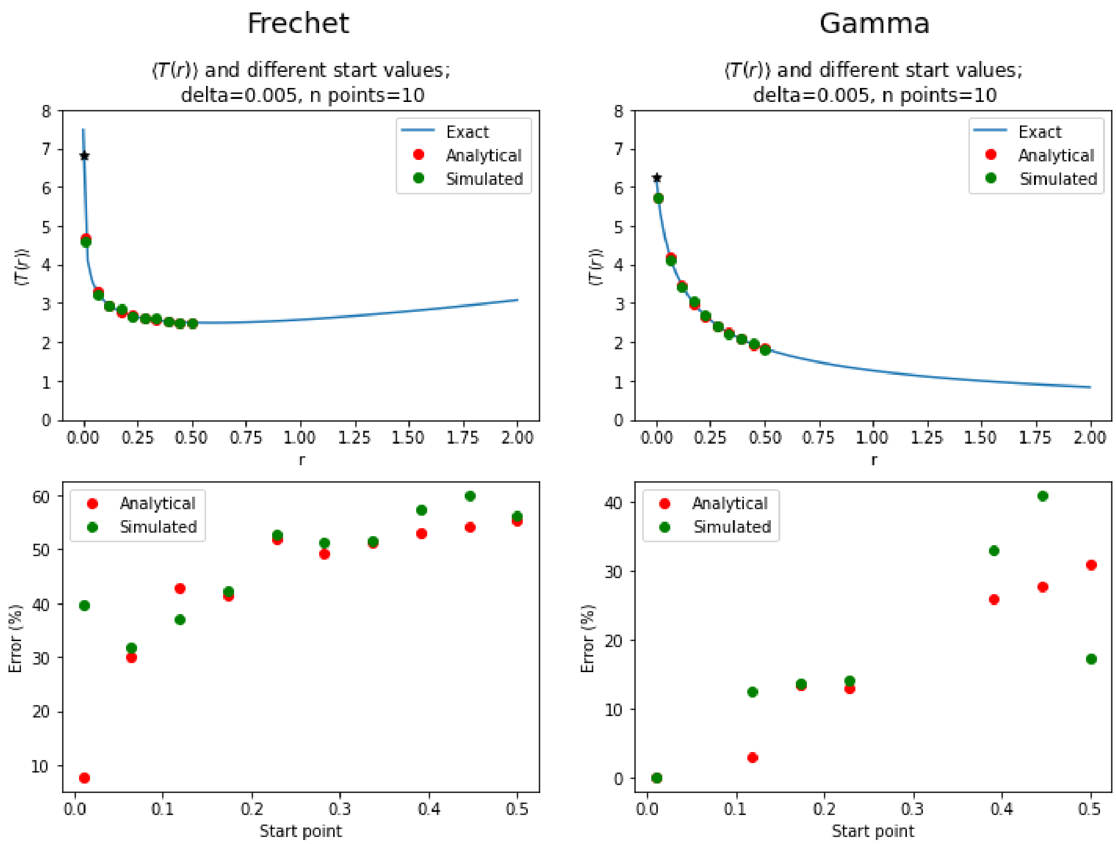
1. Evaluating $\tilde{T}$ in the expression $\langle T\left(r\right)\rangle = \frac{1-\tilde{T}(r)}{r\tilde{T}(r)}$ using a Taylor expansion fits very poorly.
2. Evaluating $\tilde{T}$ in this expression using a Pade function, and evaluating $\langle T\left(r\right)\rangle$ directly using a Pade of the same order, fits well.

We will use the Pade fit for $\tilde{T}$ in the next plots.



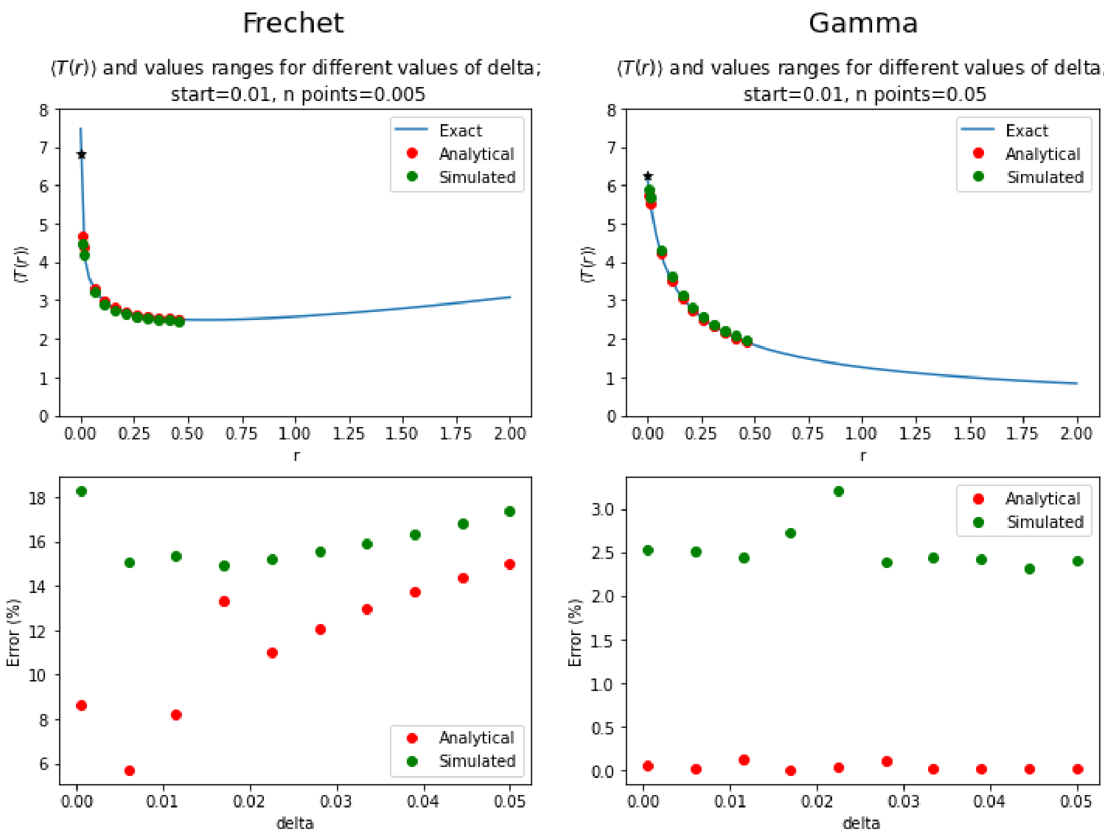
Repeated the prediction process for the same reset rate $r = 0.01$ and delta “jumps” forwards, with different number of “jumps”. This number doesn’t seem to have a significant influence on the error in the chosen region.

Simulated and analytical points for the fit yeald similar results.



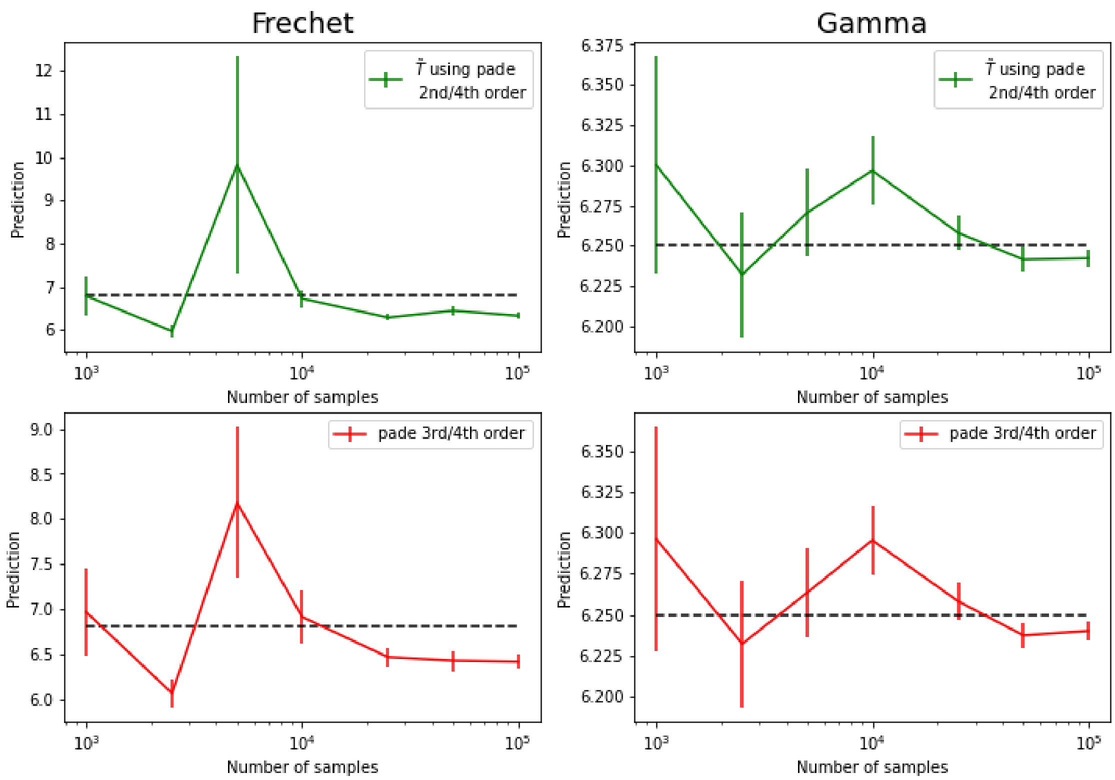
Repeated the prediction process for the same delta and number for “jumps” forwards, with different starting rates. As predicted, the error increases significantly as the starting rate increases.

Simulated and analytical points for the fit yeald similar results.



Repeated the prediction process for the same reset rate $r = 0.01$ and number "jumps" forwards, with different delta values for the "jumps". These values don't seem to have a significant influence on the error in the chosen region.

Simulated and analytical points for the fit yeald similar results.



Made predictions using different number of samples. Repeated each number of samples 50 times to get the error of the prediction. The calculated error decreases with the number of samples, but the prediction doesn't converge to the analytical value.

Evaluating $\tilde{T}$ using a Pade function, and evaluating $\langle T(r) \rangle$ using a Pade gives similar results.