organized

June 29, 2022

1 General procedure:

1. Exact mean values with different rates are found using: $\langle T(r)\rangle = \frac{1-\tilde{T}(r)}{r\tilde{T}(r)}$
2. The function $\langle T(r)\rangle$ is evaluated using different fits to the selected points.
3. The evaluated value at a selected reference point is compared to the exact value at this point.
4. The same process is repeated, using sampled mean values instead of exact values as selected points for the fit.

1.1 Frechet distribution

The selected distribution: $Pr(t) = t^{-2}exp\left(-t^{-1}\right)$.

The mean and standard deviation diverge; we will compare to $\langle T(0.001)\rangle = 6.8$.

The Laplace transform may be found using the modified Bessel function: $\tilde{T}(r) = 2\sqrt{r}K_1\left(2\sqrt{r}\right)$ (https://aip.scitation.org/doi/10.1063/1.4893338).

1.2 Gamma distribution

The selected distribution: $Pr(t) = \frac{1}{\Gamma(k)\theta^k}t^{k-1}exp\left(-\frac{t}{\theta}\right)$.

$\mu = k\theta$, $\sigma = \sqrt{k}\theta$, $\tilde{T}(r) = (1+\theta r)^{-k}$

We chose $k = 0.25$, $\theta = 25$, which leads to $CV = \frac{\sigma}{\mu} = \frac{1}{\sqrt{k}} = 2$

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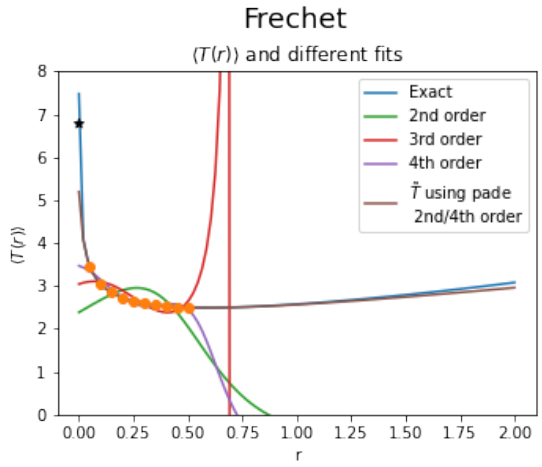
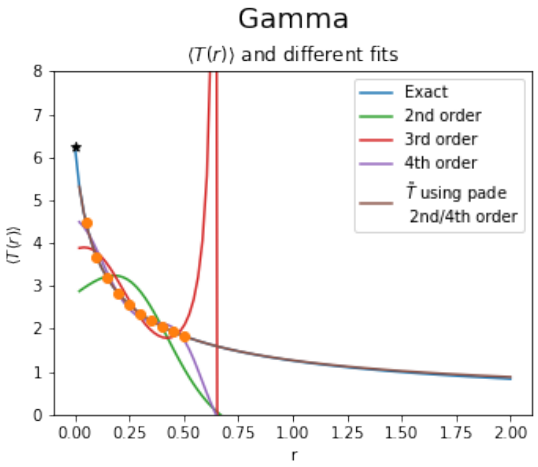
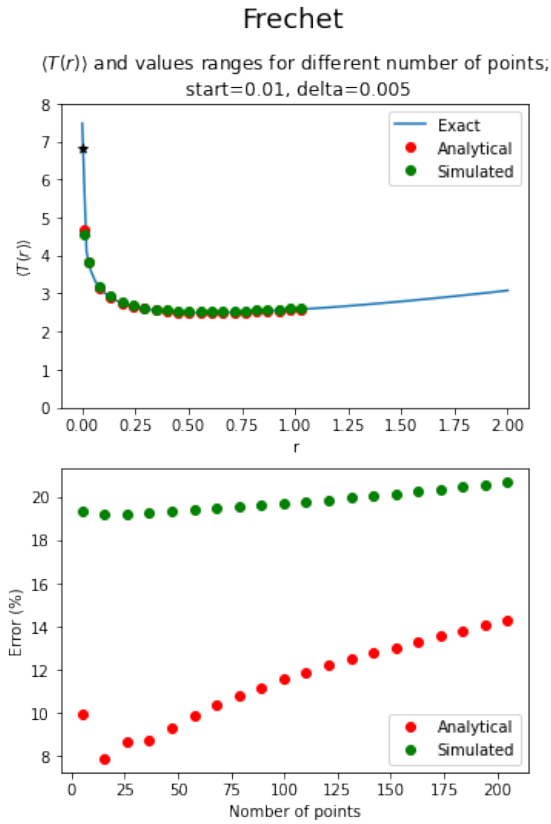
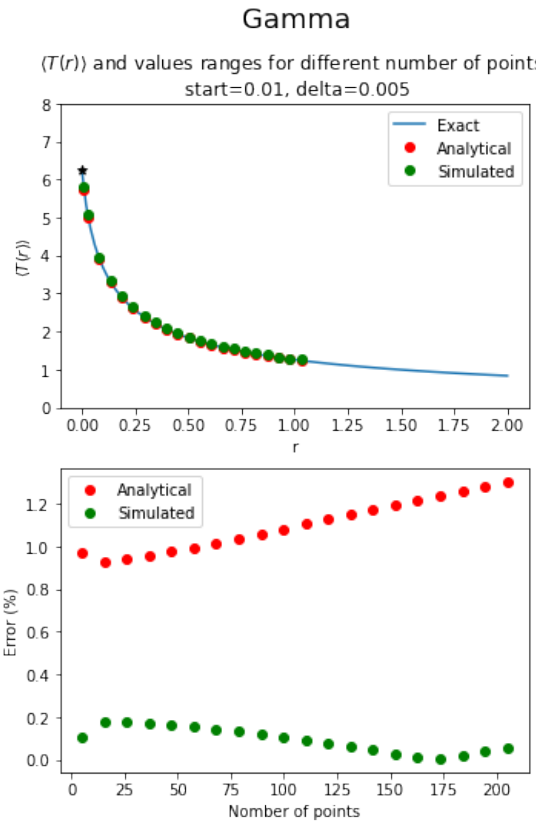
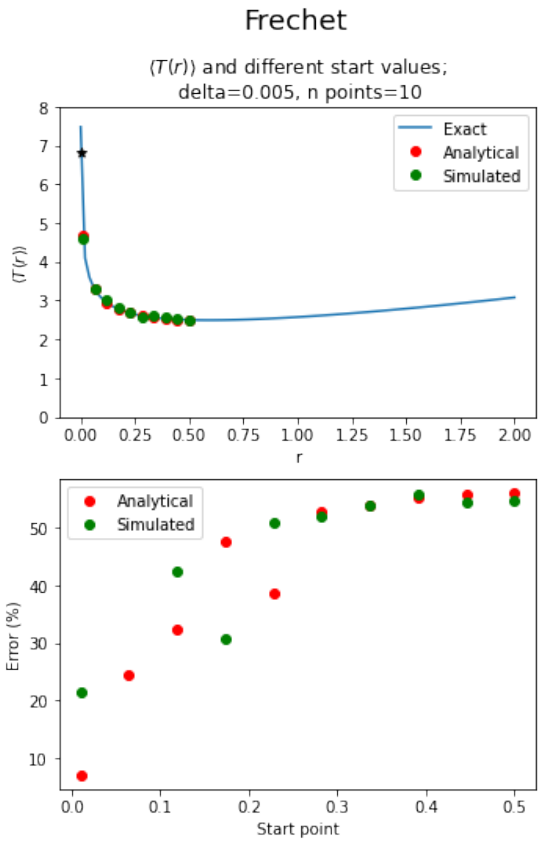
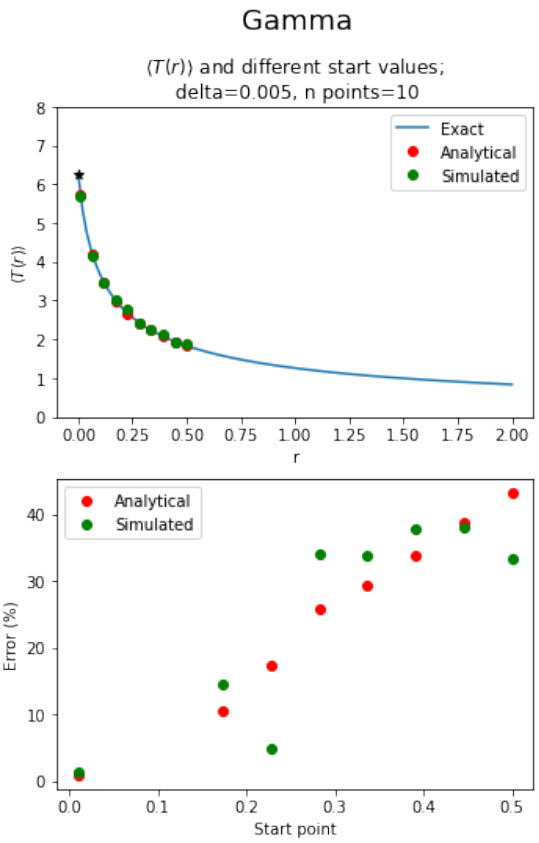

Frechet
⟨T(r)⟩ and different fits
Exact
2nd order
3rd order
4th order
T̃ using pade
2nd/4th order
⟨T(r)⟩
r

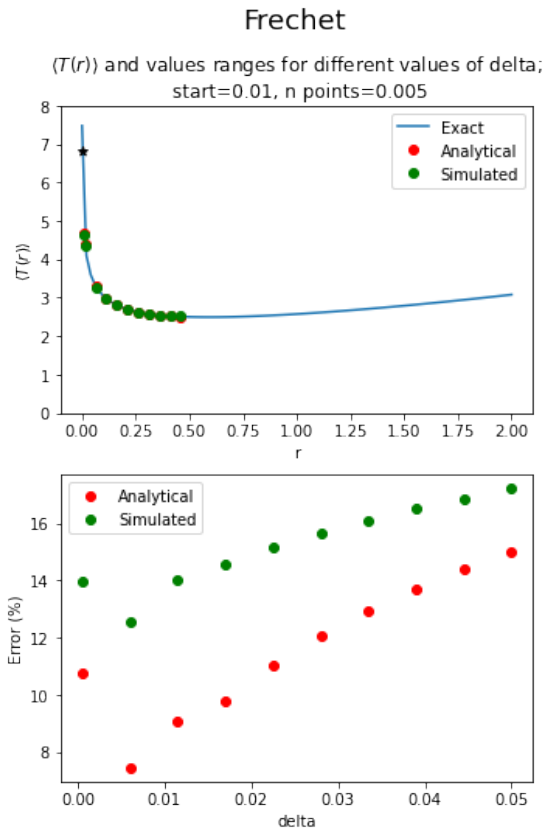

Gamma
⟨T(r)⟩ and different fits
Exact
2nd order
3rd order
4th order
T̃ using pade
2nd/4th order
⟨T(r)⟩
r

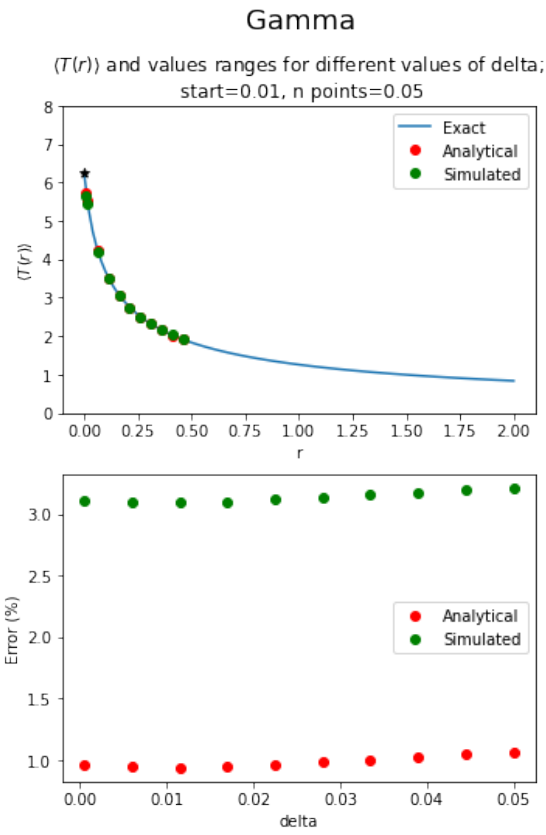

Frechet
⟨T(r)⟩ and values ranges for different number of points;
start=0.01, delta=0.005
Exact
Analytical
Simulated
⟨T(r)⟩
r
Error (%)
Nomber of points

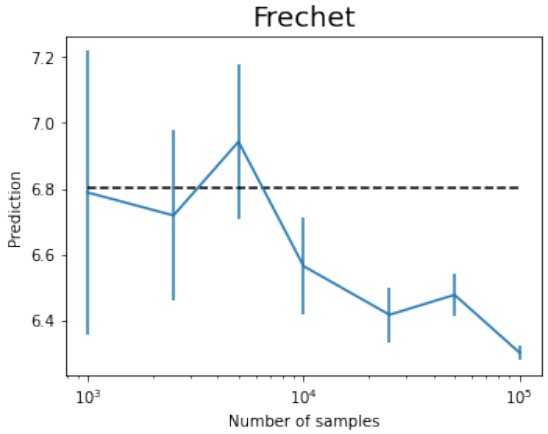

Gamma
⟨T(r)⟩ and values ranges for different number of points;
start=0.01, delta=0.005
Exact
Analytical
Simulated
⟨T(r)⟩
r
Error (%)
Nomber of points

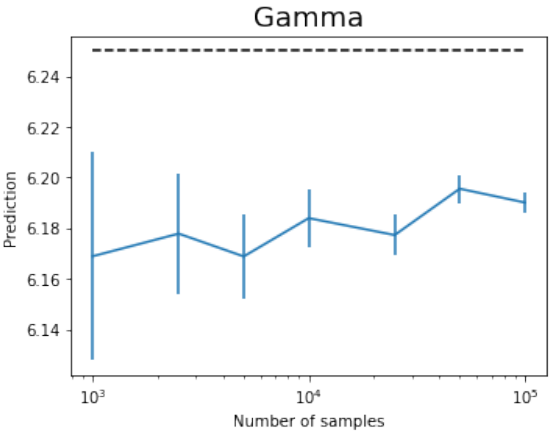

Frechet
⟨T(r)⟩ and different start values;
delta=0.005, n points=10
Exact
Analytical
Simulated
⟨T(r)⟩
r
Analytical
Simulated
Error (%)
Start point


Gamma
⟨T(r)⟩ and different start values;
delta=0.005, n points=10
Exact
Analytical
Simulated
⟨T(r)⟩
r
Analytical
Simulated
Error (%)
Start point


Frechet
⟨T(r)⟩ and values ranges for different values of delta;
start=0.01, n points=0.005
Exact
Analytical
Simulated
⟨T(r)⟩
r
Analytical
Simulated
Error (%)
delta


Gamma
⟨T(r)⟩ and values ranges for different values of delta;
start=0.01, n points=0.05
Exact
Analytical
Simulated
⟨T(r)⟩
r
Analytical
Simulated
Error (%)
delta


Frechet
Prediction
Number of samples


Gamma
Prediction
Number of samples